Endogenous Uncertainty and Credit Crunches*

Ludwig Straub
MIT

Robert Ulbricht
Toulouse School of Economics

August 26, 2015
[click here for latest draft]

Preliminary and Incomplete

Abstract

We develop a theory of endogenous uncertainty where the ability of investors to learn about firm-level fundamentals declines during financial crises. At the same time, higher uncertainty reinforces financial distress of firms, giving rise to "belief traps"—a persistent cycle of uncertainty, pessimistic expectations, and financial constraints, through which a temporary shortage of funds can develop into a long-lasting funding problem for firms. At the macro-level, belief traps provide a rationale for the long-lasting recessions that typically entail financial crises. In our model, financial crises are characterized by high levels of credit misallocation, an increased cross-sectional dispersion of growth rates, endogenously increased pessimism, uncertainty and disagreement among investors, highly volatile asset prices, and high risk premia. A calibration of our model to U.S. micro data on investor beliefs matches the slow recovery after the 08/09 crisis remarkably well.

Keywords: Credit crises, endogenous uncertainty, financial frictions, learning, misallocation, output and belief dispersion, persistence of pessimism.

JEL Classification: D83, E32, E44, G01.

*This paper supersedes an earlier working paper circulated in 2012 under the title "Credit Crunches, Information Failures, and the Persistence of Pessimism". *Email Addresses:* straub@mit.edu, robert.ulbricht@tse-fr.eu.

1 Introduction

The recent financial crisis has been characterized by a sharp rise in investors' uncertainty about firm-level fundamentals. Figure 1 documents this by plotting a closely related proxy—the average disagreement among analysts over firm-specific earnings forecasts.[1] Before 2007, disagreement among analysts does not show a clear cyclical pattern. Between 2007 and 2013, however, the financial crisis and the subsequent slow recovery were accompanied by a sharp and *countercyclical* rise in analysts' uncertainty.[2]

In this paper, we develop a theory of endogenous uncertainty that causes investors' uncertainty to increase precisely during financial crises. At the same time, higher uncertainty reinforces the impact of exogenous financial shocks on firms, unleashing a feedback loop through which a temporary shortage of funds can develop into a long-lasting funding problem for firms. At the macro level, financial shocks—even if short-lived—lead to resource misallocation which may persist even after financial stress has subsided. The theory thus provides a rationale for the long-lasting recessions that typically entail financial crises (Reinhart and Rogoff, 2009; Hall, 2014; Ball, 2014). A calibration of the model to U.S. micro data on investor beliefs is able to capture important characteristics of the slow recovery that followed the 2008/09 financial crisis.

The theory is based on two main ingredients. First, investors have only limited information about each firm's fundamentals. Their evaluation of the fundamentals is based on noisy business indicators that include signals related to the production and employment choices of firms. Second, investors' beliefs about a firm's fundamentals shape the credit supply to that firm. When investors are optimistic and their uncertainty is low, funding will be more generous than when investors are pessimistic and their uncertainty is high. This follows naturally, for example, when investors assess the repayment probabilities of loans or the value of the assets used to secure these loans.

In this environment, the ability of investors to learn about a firm's fundamentals depends on the degree to which a firm's actions (investments, employment, production, etc) reflect these fundamentals. When firms are short of funds and their actions become increasingly dictated by financial constraints, the actions carry less information and investors' learn less about these firms' fundamentals. Accordingly, a shortage of funds increases investors' uncertainty and lets them rely more on their prior expectations. With investors' beliefs feeding back into credit supply, this opens the door to "belief traps"—a persistent (and perfectly rational) surge of uncertainty, coupled with pessimistic expectations and tight credit constraints that are virtually decoupled from a firm's fundamentals.

We explore the macro implications of this two-way interaction between beliefs and financial

[1]Forecasts are about end-of-quarter earnings per share, obtained from the IBES database. Disagreement is defined as cross-analyst dispersion based on an average of 7 analysts per firm, and is aggregated over an average of 2483 U.S. firms per quarter (see Appendix C for further details). Similar proxies are widespread in the literature (e.g., Bachmann et al., 2013); in particular, our proxy is closely related to Senga (2015), who also looks at the cross-analyst dispersion *within* firms. In Section 5 we provide a formal argument in favor of this practice.

[2]From 1985 to 2006, the contemporaneous correlation of analysts' uncertainty with GDP (bandpass-filtered at frequencies corresponding to 6–32 quarters) was +0.18 (not significant at the 5 percent level), whereas from 2007 to 2015 this correlation is -0.57 (significant at the 1 percent level). Using the HP-filter yields similar results.

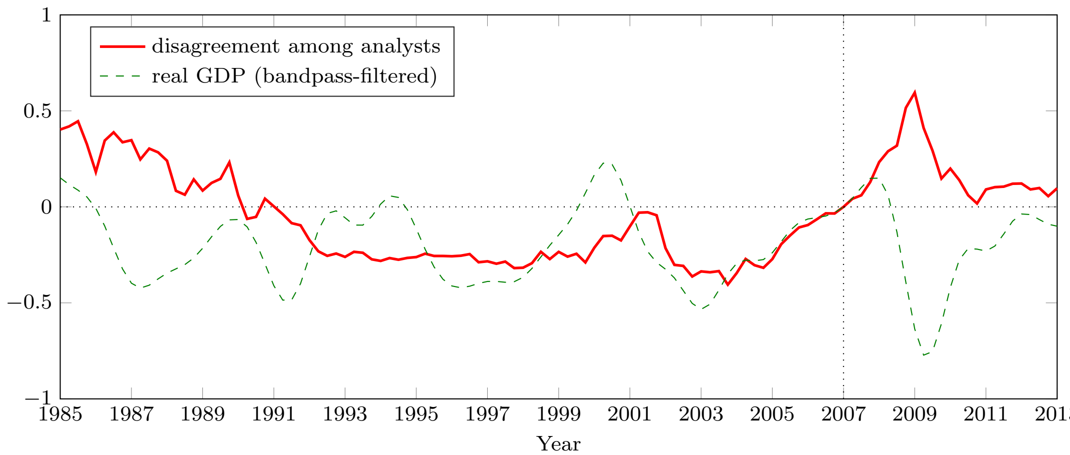

Figure 1: Average disagreement among analysts and real GDP in the U.S. The disagreement (or cross-analyst dispersion) is computed using IBES data (see Appendix C for details). GDP is bandpass-filtered at frequencies corresponding to 6–32 quarters. Both series are normalized relative to 2007Q1 and scaled to have a unit range. Without the normalization, disagreement at the peak of the 2008/09 crisis increased by 88 percent compared to its pre-crisis average.

constraints in a neoclassical economy with a financial sector. The financial sector provides firms with credit based on its beliefs about their productivities, and further subjects firms to both idiosyncratic and aggregate credit supply shocks. In the model, an aggregate credit supply shock makes it more likely for firms to become financially constrained, increasing the fraction of constrained firms. This increases credit misallocation, which manifests itself in higher efficiency and labor wedges.[3] These wedges and the corresponding drops in output and labor are characterized by a high degree of internal persistence that is driven by the stagnation of firms that have fallen into belief traps.

Closely related to the endogenous nature of uncertainty, financial crises in our model are marked by a number of further characteristics that are typical for such episodes, namely: high levels of (i) pessimism and (ii) disagreement among investors; (iii) highly volatile asset prices; (iv) high risk-premia; and (v) an increased cross-sectional dispersion of growth rates (e.g., Bloom et al., 2014, Fig. 2). In our model, the real effects of a financial crisis are captured by an aggregate and *commonly known* credit supply shock.

To give intuitions for the five effects listed above, consider first the impact of such a shock on pessimism. Even though productivities are unaffected and signals are on average unbiased, such a shock increases the average pessimism in our model because it is precisely those firms about which investors are pessimistic that are most likely to be pushed into belief traps. Since expectations adjust significantly more slowly for firms in belief traps compared to unconstrained firms, average pessimism increases. Next, disagreement rises as firms place more weight on private information when learning from firms' actions becomes less informative; asset price volatility spikes as firms place more weight on asset price signals; and risk premia increase as a direct consequence of high uncertainty. Finally, the self-reinforcing nature of belief traps causes a sharp divergence of the dynamics of a firm that

[3]For simplicity, the model abstracts from physical capital and instead works with a working-capital requirement. In a richer version of our model, credit constraints would manifest themselves as investment wedges as well.

is marginally constrained compared to one that is marginally unconstrained, helping increase the cross-sectional dispersion in firm growth rates. In sum, the endogenous uncertainty in our model can naturally explain the comovement of various related measures of uncertainty (see also the discussion in an earlier working paper version of this paper, Straub and Ulbricht, 2012, and Kozeniauskas et al., 2014).

We explore the quantitative potential of belief traps in a calibration of the model to the U.S. economy. Calibrating models with information frictions is notoriously hard due to the scarcity of reliable information about beliefs. Here we address this problem by explicitly exploiting micro data on analyst forecasts made at the firm-level to discipline investors' beliefs in our model. Equipped with these data we use a similar approach as in David et al. (2015) and construct a number of target moments that pin down the information parameters in our model.

We conduct two experiments in the calibrated model. First, we explore how a temporary credit shock (with a half-life of four quarters) propagates through the economy, and then compare it with counterfactual responses, where we keep investor uncertainty constant. While such a four quarter shock to the counterfactual economy produces a short-lived recession with only a half-life of 2 quarters, the same shock produces a long-lasting recession with a half-life of 10 quarters in the endogenous uncertainty economy. We interpret the difference between these two responses as the contribution of belief traps to the internal persistence of financial shocks.

Second, we compare the quantitative predictions of our calibrated model with U.S. data from the 2008/09 financial crisis. To do this, we feed our model data from the St. Louis Fed Financial Stress Index to capture the relatively short-lived distress within the financial sector. We scale the magnitude of the shock to match a fraction of 20 percent of firms that has reported to be constrained by financial factors in the third quarter of 2008 (Campello et al., 2010). Comparing the resulting model responses to U.S. data, our model matches the observed series remarkably well. In particular, the model-implied efficiency wedge is able to account for 78 percent of the observed drop in TFP, and, in combination with a countercyclical rise in the labor wedge, explains 74 percent of the observed drop in output. Qualitatively, the model also matches the dynamics of analysts' disagreement and expectations, and asset price volatility.

At a methodological level, uncertainty is state-dependent in this paper because learning from financially constrained firms gives rise to a *nonlinear* signal structure, where, all else equal, signals about more constrained firms' fundamentals are less informative. In a related contribution (Straub and Ulbricht, 2014), we show that signal nonlinearities generally imply state-dependent posterior uncertainty. One technical challenge in analyzing the dynamic properties of our model is that nonlinear Gaussian signal structures do not pair with any (reasonable) conjugate prior distribution. In this paper, we address this issue by developing a simple approximative approach, which captures the key features of the nonlinear learning problem while preserving tractability.

In our comparisons to the data, we focus on firm-level forecast dispersion as a measure of uncertainty—since it precisely measures the micro-level uncertainty of investors that we focus on in our model—and show what the effects of higher uncertainty of this kind are on the real economy.

Many other accounts of both higher uncertainty and the key role it played during the recent crisis have been given. Perhaps most prominently, Olivier Blanchard (2009) speculated at the height of the recent financial crisis that "the crisis would largely go away" if it were not for uncertainty. Similarly, Bloom et al. (2014) reported how uncertainty was repeatedly recognized by the Federal Open Market Committee as a key driver of both, the recession that followed the dot-com bubble in 2001, and the recent Great Recession; and in an empirical study, Stein and Stone (2013) find that uncertainty, proxied by options-implied volatilities, approximately doubled in the 2007–2009 crisis, accounting for one third of the decline in U.S. capital investments and hirings during that period.[4] Our account of sharply increased firm-level forecast dispersion complements these studies.

Related literature This paper is part of an emerging literature exploring the role of endogenous uncertainty for business cycles. Most notably, this literature includes van Nieuwerburgh and Veldkamp (2006), Ordoñez (2013), and Fajgelbaum et al. (2015).[5] In these papers, aggregate economic activity determines the quality of information regarding the current aggregate state of the economy. Van Nieuwerburgh and Veldkamp (2006) show that this can make firm investments strategic complements—higher economic activity generates more information which encourages more investment)—which they use to explain the asymmetric nature of business cycles. Ordoñez (2013) relates the business cycle asymmetry in a very similar setup to the degree of financial development, arguing that greater financial frictions exacerbate the asymmetry. The paper by Fajgelbaum et al. (2015) combines the insight in van Nieuwerburgh and Veldkamp (2006) with fixed costs of investment. This increases the degree of strategic complementarity in firms' investments and is able to generate multiple steady states.

Our paper stands out in this literature in that it links *financial crises* and uncertainty through a novel mechanism, providing a rationale for Figure 1. In our model, it is not the level of economic activity that determines how much information about firms' fundamentals is revealed; rather it is the degree to which firms' actions (investments, employment, production, etc) actually reflect these fundamentals. This insight naturally implies that actions from financially constrained firms carry less information than actions from unconstrained firms.

The endogenous uncertainty literature faces two difficult challenges. First, often an unrealistically high drop in economic activity is needed to generate a significant drop in information quality (for a similar observation, see also Kozlowski et al., 2015); and, second, it is hard to discipline these models due to the widespread lack of data on beliefs. We attempt to make progress on both of these issues. Regarding the first, two remedies help us: Agents in our model learn about *firm-specific* fundamentals, not economy aggregates[6]; and learning breaks down when a firm is constrained, not when economic activity comes to a stand-still. This implies that small variations

[4] See also, Stock and Watson (2012), Caldara et al. (2013), and Gilchrist et al. (2014) for further evidence regarding the importance of uncertainty during financial crisis.

[5] Studies of endogenous uncertainty in financial market settings include Veldkamp (2005), Yuan (2005), Albagli (2011), and Sockin (2015).

[6] See Bachmann and Moscarini (2011) and Senga (2015) for alternative models with learning about firm-specific fundamentals.

in average uncertainty (in our calibration 8 percent during the recent financial crisis), can have severe consequences. On the second issue, we follow a calibration strategy to target moments in data on explicit beliefs. In this effort to use explicit belief data, we are related to Coibion and Gorodnichenko (2012, 2014) who discipline models of informational rigidities using data from the Survey of Professional Forecasters.

Our paper also relates to a recent literature around Christiano et al. (2014), Arellano et al. (2012), and Gilchrist et al. (2014), which stresses the effects of exogenous uncertainty (or risk) shocks in the presence of financial market imperfections. This literature complements our approach by emphasizing the importance of uncertainty in the financial sector, but treats uncertainty as exogenous.[7] In support of a financial transmission channel, Caldara et al. (2013) and Gilchrist et al. (2014) present evidence that uncertainty strongly affects investments *via* increasing credit spreads, but has virtually no impact on investments when controlling for credit spreads.

The key predictions of our model are broadly consistent with a recent empirical literature that measures the effects of financial constraints. Giroud and Mueller (2015) show that establishments of firms that are more likely to be financially constrained were heavily affected by falling collateral values (house prices). In fact, they show that the entire correlation of employment loss and house prices is explained by these arguably financially constrained firms. Similar in spirit, Chodorow-Reich (2013) documents that firms borrowing from less healthy lenders experience relatively steeper declines in employment during the financial crisis, consistent with the interpretation that these firms faced tighter financial constraints (see also Chaney et al., 2012). Our model clarifies how an intense but relatively short-lived financial crisis can still translate into *persistent* financial constraints for firms, making it much harder for them to weather such periods and retain their employment and capital.

Outline The plan for the rest of the paper is as follows. The next section introduces the model economy. Section 3 characterizes the equilibrium and explores the core mechanism of how to learn from financially constrained firms. Section 4 studies the workings of belief traps for a single island in isolation. Section 5 analyzes the model's response to aggregate shocks and compares it to data on the 2008/09 financial crisis. Section 6 concludes.

2 Model

The economy consists of a representative household, a continuum of "islands", a continuum of competitively monopolistic firms located on each of these islands, and a financial sector. There are two frictions. First, firms produce subject to a working capital constraint, where the tightness of the constraint depends on how "local" (island-specific) fundamentals are perceived by the

[7] Two other related strands of the literature study the propagation of exogenous uncertainty through real options as in Bloom (2009), Bloom et al. (2014), and Bachmann and Bayer (2009, 2013), and through risk premia as in the time-varying (disaster) risk literature (e.g., Gabaix, 2012; Gourio, 2012). Related to the latter, Kozlowski et al. (2015) explore a model where agents learn about tail-risks and where belief revisions after short-lived financial shocks can have long-lasting effects. Similar, Nimark (2014) presents a mechanism that increases uncertainty after rare events, if news selectively focus on outliers.

financial market. Second, in order to form these expectations, financial markets have access to only limited information about each island's business fundamentals. Time is discrete and indexed by $t \in \{0, 1, 2, \dots\}$. Islands are indexed by $i \in I$. Firms are indexed by $(i, j) \in I \times J = [0, 1]^2$.

Households The preferences of the representative household are given by

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t U(C_t, N_t),$$

with separable isoelastic preferences over consumption C_t and labor supply N_t,

$$U(C_t, N_t) = \log C_t - \frac{1}{1+\zeta} N_t^{1+\zeta},$$

where $\zeta \geq 0$ is the inverse of the Frisch elasticity of labor supply and $\beta \in (0, 1)$ is the discount factor. C_t is a composite consumption good given by

$$C_t = \left[\int_{I \times J} C_{ij,t}^{\frac{\xi-1}{\xi}} \, \mathrm{d}j \right]^{\frac{\xi}{\xi-1}},$$

where $C_{ij,t}$ is the consumption of good (i, j) at time t, and $\xi > 1$.

The representative household provides the financial sector with funding L_t in an economy-wide lending market. For simplicity, lending is assumed to be made within periods, implying an infinitely elastic credit supply and a risk-free rate of $R_t = 1$. The budget constraint of the household is

$$\int_{I \times J} P_{ij,t} C_{ij,t} \, \mathrm{d}(i, j) \leq W_t N_t + (R_t - 1) L_t + \int_{I \times J} \Pi_{ij,t} \, \mathrm{d}(i, j), \tag{1}$$

where $P_{ij,t}$ is the price of good (i, j), W_t is the nominal wage rate, and $\Pi_{ij,t}$ are profits of firm (i, j).

In equilibrium households choose consumption, lending, and hours worked to maximize expected utility subject to their budget constraint. From the household's optimization problem it follows that demand for good (i, j) is

$$C_{ij,t} = \left(\frac{P_{ij,t}}{P_t} \right)^{-\xi} C_t,$$

where

$$P_t = \left[\int_{I \times J} P_{ij,t}^{1-\xi} \, \mathrm{d}(i, j) \right]^{1/(1-\xi)}$$

is the economy-wide price index. Throughout, we normalize $P_t = 1$, defining the composite consumption good to be the numeraire.

Firms Each good $(i, j) \in I \times J$ is produced by a monopolistically competitive firm which has access to a linear production technology

$$Y_{ij,t} = A_{ij,t} N_{ij,t},$$

where $A_{ij,t}$ is the firm's productivity, and $N_{ij,t}$ is the firm's employment. Wages must be paid before production takes place and are financed by within-period working capital loans of size $L_{ij,t} = W_t N_{ij,t}$ that are intermediated by an economy-wide financial sector.[8] The financial sector provides these loans at the risk-free rate R_t, but firms face an island-specific credit limit $\bar{L}_{i,t}$ on the maximum loan capacity.

In equilibrium, firms plan their production to maximize profits,

$$\Pi_{ij,t} = P_{ij,t} Y_{ij,t} - W_t N_{ij,t},$$

subject to the credit limit $W_t N_{ij,t} = L_{ij,t} \leq \bar{L}_{i,t}$. From the firms' optimization problem it follows that firms access working capital loans

$$L_{ij,t} = \min\{A_{ij,t}, \bar{A}_{i,t}\}^{\xi-1}\, \Omega_t, \tag{2}$$

where

$$\Omega_t = \left(\frac{\xi - 1}{\xi}\right)^{\xi} \frac{C_t}{W_t^{\xi-1}}$$

summarizes the state of the aggregate economy, and where

$$\bar{A}_{i,t} = \left(\frac{\bar{L}_{i,t}}{\Omega_t}\right)^{1/(\xi-1)} \tag{3}$$

formulates the local credit limit $\bar{L}_{i,t}$ in terms of "productivity-units".

In our model, we abstract from physical capital as a factor of production. Together with assuming that lending takes place *within*-periods, this improves tractability and further sharpens our predictions by ruling out any source of persistence beyond the information channel explored in this paper.

Productivities Productivities have an island-specific component $A_{i,t}$ and a firm-specific component $\epsilon_{ij,t}$. There is no uncertainty about the economy-wide aggregate productivity distribution.

Conditional on $A_{i,t}$, within-island productivities are i.i.d. and log-normally distributed:

$$\log A_{ij,t} = \log A_{i,t} + \epsilon_{ij,t},$$

where $\epsilon_{ij,t} \sim \mathcal{N}(0, \sigma_\epsilon^2)$. As becomes clear below, this within-island dispersion is introduced for technical reasons only and should be thought of as being small. With this in mind, our preferred interpretation is that one island corresponds to one firm in the data. The remainder of the setup is tailored towards this interpretation, placing the island-specific component of firms' productivities

[8] While we refer to the funding of firms as credit throughout this paper, the story is also consistent with other means of finance such as corporate bonds or equity finance (see also our discussion below). Since our model does not allow for firm entry and exit, we do abstract, however, from the possibility of firms saving their way out of constraints. See Cooley and Quadrini (2001), Khan and Thomas (2013), Moll (2014), and Siemer (2014) for studies exploring firm dynamics in the presence of financial constraints.

at the center of our analysis.[9] We assume that $\{A_{i,t}\}$ follows a Markov process, with transition probabilities

$$\text{Prob}(\log A_{i,t} \leq a | A_{i,t-1}) = F_a(a|A_{i,t-1}).$$

We leave this process undetermined for now in order to make a few general statements in Section 3. Later we will assume $\log A_{i,t}$ to be AR(1).

Financial sector To streamline the exposition of this paper and to focus on the mechanism at its core we adopt a reduced form representation of the financial sector throughout the main body of the paper. Appendix B contains one possible microfoundation, but the principles underlying the following specification are more general.

Our formulation is based on the idea that credit is more readily available when financial markets believe firm fundamentals to be more favorable. This feature is, implicitly or explicitly, present in much of the previous macro-finance as a borrower's funding position naturally depends on the lenders' beliefs about the repayment amount (which for both debt—either credit or bonds—and equity increases in the borrower's fundamental).[10] For instance, suppose that firms pledge part of their future income as collateral. Then the expectation and uncertainty over the value of the collateral will naturally affect the available credit. In our microfoundation, we formalize this intuition using an approximate CRRA-normal asset market that prices securities issued by firms to raise funds. Confining the technical details to Appendix B, we here adopt the resulting credit rule that directly specifies a firm's credit limit as a function of "market beliefs" and credit shocks. We would like to stress, however, that the underlying ideas extend beyond our specific microfoundation.

Specifically, suppose there is a continuum of investors whose average belief about an island's productivity determines the availability of working capital loans on that island. Letting $\bar{\mathbb{E}}_t\{\cdot\}$ denote the investors' average expectation given their information sets (defined below), and letting $\hat{\sigma}^2_{i,t}$ denote their uncertainty, the adopted credit rule (expressed in productivity units) is linear in beliefs $(\bar{\mathbb{E}}_t\{\log A_{i,t}\}, \hat{\sigma}^2_{i,t})$ and given by

$$\log \bar{A}_{i,t} = \underbrace{\bar{\mathbb{E}}_t\{\log A_{i,t}\}}_{\text{market belief}} - \underbrace{\pi_\sigma \hat{\sigma}^2_{i,t}}_{\text{uncertainty}} + \underbrace{\upsilon_t + \pi_\sigma \hat{\sigma}^2_{i,t}\eta_{i,t}}_{\text{financial shocks}} + \pi_0, \tag{4}$$

where υ_t and $\eta_{i,t}$ are aggregate and island-specific credit shocks, and π_σ and π_0 are positive constants. Since these two credit supply shocks are meant to be a stand-in for some of the ways in which financial sector shocks translate affect the real economy, we also refer to υ_t and $\eta_{i,t}$ as "financial

[9] Notice, that variations in local productivities $A_{i,t}$ can be equivalently interpreted as shifts in relative demand across islands and are more generally meant as a stand-in for a variety of shocks that shape local business conditions.

[10] In the case of unsecured debt, fundamentals affect the repayment amount via the expected repayment probability, for secured debt that repayment amount further depends on fundamentals via the expected collateral value. Seminal frameworks where beliefs implicitly shape the supply of funds are, e.g., the perfect information models by Kiyotaki and Moore (1997) and Bernanke et al. (1999), where borrowers refinance themselves by writing debt contracts. The debt contracts are then valued according to lenders' beliefs, which in these perfect information models coincide with borrowers' beliefs. In models with imperfect information or heterogeneous priors, this dependence on lenders' beliefs is more explicit, for example in the models by Simsek (2013*a*,*b*) or Fostel and Geanakoplos (2008, 2015).

shocks".

The first two terms in (4) reflect the already discussed idea that credit should naturally be more available when investors are optimistic about the productivity of firms and when uncertainty is low compared to when they are pessimistic and uncertainty is high. The third and fourth term further allow an island's credit supply to depend on aggregate and local financial shocks that are orthogonal to the productive potential given by $A_{i,t}$.[11] We assume that local shocks $\eta_{i,t}$ follow an autoregressive process, such that

$$\eta_{i,t} \sim \mathcal{N}(\rho_\eta \eta_{i,t-1}, \sigma_\eta^2),$$

with persistence parameter $0 \leq \rho_\eta < 1$. Because all aggregate shocks are common knowledge across all agents and there is no intertemporal savings technology in our model, there is no need at this point to specify any particular process for υ_t.

The credit limit in (4) adjusts *one-to-one* with the market belief. This ensures that *known* variations in the productivity of a firm change the availability of credit in the same way they change the firm's demand for credit and, therefore, do not affect the degree to which a firm is constrained.[12] This property also makes sure that under perfect information, there is no internal propagation in our model, as firms can only be constrained in periods with negative aggregate financial shocks υ_t but immediately cease to be so as shocks fade out. On another note, one might expect the credit rule (4) to also depend on future productivities. In our setup with within-period lending, however, it is natural that the only income a firm can pledge are its current end-of-period revenues, which is why (4) depends on the market belief over current log productivity (see also Appendix B).[13]

Information To complete the description of the model, we need to specify the information available to the representative household, to firms, and to the investors in the financial sector. As stated before, we assume full information about all aggregate variables of the economy, allowing us to solve the household problem in a completely standard way.[14] To focus on the propagation of financial shocks, we further assume that $A_{ij,t}$ is known by firm (i, j) at date t, so that—like the household—firms follow the same decision rules as they would under full information.

We are left to specify the investors' information sets on which their average expectations $\bar{\mathbb{E}}_t\{\cdot\}$ and uncertainty $\hat{\sigma}_{i,t}^2$ are based on. The critical assumption is that investors do not directly observe the local (average) productivities $\{A_{i,t}\}$. While there is no aggregate uncertainty, investors are

[11]Here we scale the local shock $\eta_{i,t}$ relative to the uncertainty $\hat{\sigma}_{i,t}^2$ in order to keep the learning problem tractable. The scaling also naturally emerges from our microfoundation where $\eta_{i,t}$ corresponds to a local supply shock in the collateral market, which, as in other CARA-normal models, manifests itself via a shift in the risk-premia. One implication of this scaling is that for $\eta_{i,t} > 1$ the credit limit increases in $\hat{\sigma}_{i,t}^2$. This peculiarity is, however, of little consequence, since in either case the credit limit is unlikely to bind for large positive realizations of $\eta_{i,t}$.

[12]Similarly, variations in the aggregate state Ω_t do not affect the available credit limit *measured in productivity units*, and hence do not affect the degree to which firms are constrained.

[13]Intuitively, we expect forward looking behavior to amplify the feedback mechanism that leads to belief traps in our model, since, similar to Kiyotaki and Moore (1997), falling in a belief trap affects many future periods as well; and this can feed back into lower credit limits today if the credit rule is forward looking.

[14]While this assumption highlights the independence of our results from any aggregate uncertainty, it is not crucial for our results. See Straub and Ulbricht (2012) for an earlier version of this paper where learning was with respect to aggregate business conditions.

thus unable to directly identify the productivity of any given island. Index the set of investors by $k \in K = [0, 1]$. For any island i, each investor has access to three distinct signals regarding that island's productivity. First, he observes the public history of working capital $\{L_{i,s}\}_{s \leq t-1}$ invested in the island up to date $t-1$, perturbed by some noise. This is meant as a stand-in for observing news about investments and other production-related signals, which are often thought to be valuable indicators about productivities and other fundamentals. We use $s^l_{i,t}$ to denote this signal and F_l to denote the conditional distribution of $s^l_{i,t}$, such that

$$\mathrm{Prob}(s^l_{i,t} \leq s | L_{i,t-1}) = F_l(s | L_{i,t-1}).$$

Again, we make no assumptions on the signal distribution F_l for now. After our general results in Section 3, F_l will be log-normal.

Second, investor k has access to a private signal $s^p_{ik,t}$ that directly communicates $A_{i,t}$ perturbed by some noise, and has the conditional distribution

$$\mathrm{Prob}(s^p_{ik,t} \leq s | A_{i,t}) = F_p(s | A_{i,t}).$$

This signal is meant as a stand-in for all direct information about $A_{i,t}$ as well as endogenous sources (possibly relating to the productive sector) that have a precision that is unaffected by the credit channel that we highlight in this paper. To simplify the analysis, we assume that investors are short-lived so that their private information does not persist from one period to the next. This allows us to avoid dealing with Townsend's (1983) infinite regress problem.

Finally, investors observe the history of credit limits $\bar{A}_{i,t}$ on island i up to date t, which endogenously aggregates some of the information dispersed across investors. In our microfoundation in Appendix B, credit limits come from security prices, so that one can think of $\bar{A}_{i,t}$ as information contained in these market prices.

In sum, the complete information set available to investor k is given by

$$\mathcal{I}_{k,t} = \{s^p_{ik,t}\}_i \cup \{s^l_{i,s}, \bar{A}_{i,s}, \upsilon_s\}_{i, s \leq t},$$

where the history of aggregate credit supply shocks $\{\upsilon_s\}^t_{s=0}$ completely determines the aggregate variables of the economy. Given their information sets, investors' average expectation and average uncertainty are defined by

$$\bar{\mathbb{E}}_t\{\log A_{i,t}\} = \int_K \mathbb{E}\{\log A_{i,t} | \mathcal{I}_{k,t}\} \, \mathrm{d}k$$

and

$$\hat{\sigma}^2_{i,t} = \int_K \mathrm{Var}\{\log A_{i,t} | \mathcal{I}_{k.t}\} \, \mathrm{d}k.$$

3 Equilibrium

The equilibrium in our economy is a competitive equilibrium of a standard neoclassical economy with two modifications. First, firms operate subject to a working capital requirement. Second, the maximum available working capital is governed by a credit supply rule, which depends on how local productivities—or, more generally, business conditions, including e.g. relative demand—are perceived by the financial sector (i.e. investors). In the next subsection, we characterize the dynamics of output and employment in the economy conditional on a given distribution of credit limits $\{\bar{A}_{i,t}\}$. In Subsection 3.2, we then study the learning problem of investors so as to determine the equilibrium credit limits and fully characterize the equilibrium of our economy.

3.1 Output and employment

Recalling that firms and households effectively operate under full information, we can use their optimality conditions to obtain the following characterization of the economy's aggregate dynamics.

Proposition 1. *In equilibrium, economy-wide hours and output are given by*

$$N_t = (1 - \tau_t^N)^{1/(1+\zeta)}$$
$$Y_t = (1 - \tau_t^A) A^{\mathit{eff}} N_t,$$

where

$$A^{\text{eff}} = \left[\int_{I \times J} A_{ij,t}^{\xi - 1} \, \mathrm{d}(i,j) \right]^{1/(\xi - 1)}$$

is the constant efficient aggregate productivity level that would obtain if the marginal products of labor were equalized across all firms. The labor wedge $1 - \tau_t^N$ is given by

$$1 - \tau_t^N = \frac{\xi - 1}{\xi} \frac{\int_{I \times J} A_{ij,t}^{\xi - 1} R_{ij,t}^{-\xi} \, \mathrm{d}(i,j)}{\int_{I \times J} A_{ij,t}^{\xi - 1} R_{ij,t}^{1-\xi} \, \mathrm{d}(i,j)} < 1$$

and the efficiency wedge $1 - \tau_t^A$ by

$$1 - \tau_t^A = \frac{1}{A^{\text{eff}}} \frac{\left(\int_{I \times J} A_{ij,t}^{\xi - 1} R_{ij,t}^{1-\xi} \, \mathrm{d}(i,j) \right)^{\xi/(\xi - 1)}}{\int_{I \times J} A_{ij,t}^{\xi - 1} R_{ij,t}^{-\xi} \, \mathrm{d}(i,j)} < 1,$$

where $R_{ij,t}$ is the shadow interest rate at which constrained firms value additional funds:[15]

$$R_{ij,t} = \max \left\{ 1, \left(\frac{A_{ij,t}}{\bar{A}_{i,t}} \right)^{(\xi - 1)/\xi} \right\}.$$

[15]That is, $R_{ij,t}$ equals one plus firm (i, j)'s multiplier on the credit limit $\bar{L}_{ij,t}$. Equivalently, $R_{ij,t}$ is precisely the rate at which a firm would borrow if there were a competitive, firm-specific credit market with a limited supply of $\bar{L}_{ij,t}$.

Proposition 1 provides a simple characterization of the aggregate dynamics in terms of a labor wedge and an efficiency wedge.[16] A positive labor wedge reflects an inefficiently low labor demand by firms whose working capital constraint is binding. The efficiency wedge in turn reflects that in the presence of credit constraints marginal productivities are not equalized across firms, decreasing the effective (Solow) productivity in the economy through credit misallocation. If all firms were unconstrained (or, equivalently, if the shadow rate $R_{ij,t} = R_t = 1$ for all firms), then the economy would only face the usual labor wedge due to monopolistic competition ($\tau_t^N = 1/\xi$) and no efficiency wedge ($\tau_t^A = 0$). However, with non-trivial within-island heterogeneity, $\sigma_\epsilon > 0$, there are always some firms that are constrained in the cross-section, so we generally have that $\tau_t^A > 0$ and $\tau_t^N > 1/\xi$.

The second part of the proposition states that the wedges depend only on the joint cross-sectional distribution of productivities $A_{ij,t}$ and credit limits $\bar{A}_{i,t}$. While the distribution across $A_{ij,t}$ is exogenous, the distribution over $\bar{A}_{i,t}$ is endogenous. In particular, it depends on the beliefs of the financial market. The next subsection characterizes the beliefs, completing the characterization of the equilibrium.

3.2 Equilibrium beliefs

The information sets $\{\mathcal{I}_{k,t}\}$ underlying the investors' belief formation contain three distinct signals: $\{s_{i,t}^l\}$, $\{s_{ik,t}^p\}$ and $\{\bar{A}_{i,t}\}$. Before turning to the full information extraction problem, we first explore the information contained in the working capital signal $s_{i,t}^l$. Crucially, Section 3.2.1 establishes that the working capital signal is inherently *nonlinear* in the presence of financial constraints and shows how this implies that tighter constraints inhibit learning. To stress the generality of this effect, we do not built on parametric assumptions on the information structure until this point. In order to bridge the gap to solving the full learning problem, Section 3.2.2 then refines the information structure and presents a simple linearization method that allows us to capture all the main effects induced by the nonlinearity of the working capital signal while preserving tractability. Finally, Section 3.2.3 characterizes the solution to the full information extraction problem.

As this section is somewhat technical, readers who are mainly interested in the economic mechanism should feel free to skip to Section 4 after reading Section 3.2.1.

3.2.1 Learning from financially constrained firms

The information loss caused by financial constraints is due to the fact that, for each island, working capital responds nonlinearly to changes in credit limits. The basic intuition for this is simple and generalizes to many other types of constraints: Whenever firms are financially constrained, their choices will be guided less by their information about fundamentals, and are instead dictated by the financial constraint. Therefore, markets aggregate less information.[17]

[16] As usual, the labor wedge, $1 - \tau_t^N$, amounts to the household's marginal rate of substitution divided by the economy's marginal product of labor; and the efficiency wedge, $1 - \tau_t^A$, amounts to the economy's marginal product of labor divided by the efficient productivity A^{eff}. See Chari et al. (2007) for details.

[17] In the present setup, financial frictions impact firms' choices via credit limits, but results would be the same if the available supply of funds were limited and firms were affected via rising credit rate spreads. More generally,

Fix an island i. From (2), aggregating over firms, total working capital on that island is given by,

$$L_{i,t} = \Omega_t \int_J \min\{A_{ij,t}, \bar{A}_{i,t}\}^{\xi-1} \,\mathrm{d}j. \tag{5}$$

The following proposition characterizes island i's log working capital $l_{i,t} \equiv \log L_{i,t}$ as a function of log productivity $a_{i,t} \equiv \log A_{i,t}$ and the log credit limit $\bar{a}_{i,t} \equiv \log \bar{A}_{i,t}$.

Proposition 2. *Working capital $l_{i,t}$ on island i takes the form*

$$l_{i,t} = \log \Omega_t + (\xi - 1)\bar{a}_{i,t} + \mathcal{L}(a_{i,t} - \bar{a}_{i,t}), \tag{6}$$

where $\mathcal{L} : \mathbb{R} \to \mathbb{R}_-$ is a smooth, strictly concave, and increasing function, with $\lim_{x\to-\infty} \mathcal{L}(x) = -\infty$, $\lim_{x\to\infty} \mathcal{L}(x) = 0$, $\lim_{x\to-\infty} \mathcal{L}'(x) = (\xi - 1)$, $\lim_{x\to\infty} \mathcal{L}'(x) = 0$. In particular,

(a) in the absence of credit constraints, $\bar{a}_{i,t} \to \infty$, the equilibrium sensitivity of working capital to fundamentals, $\partial l_{i,t}/\partial a_{i,t}$, is constant in the credit limit $\bar{a}_{i,t}$ (in the sense that $\lim_{\bar{a}_{i,t}\to\infty} \partial l_{i,t}/\partial a_{i,t}$ exists and is nonzero), and island fundamentals $a_{i,t}$.

(b) in the presence of credit constraints, $\bar{a}_{i,t} < \infty$, the equilibrium sensitivity of working capital to fundamentals, $\partial l_{i,t}/\partial a_{i,t}$, is increasing in the credit limit $\bar{a}_{i,t}$ and decreasing in island fundamentals $a_{i,t}$.

Equation (6) is derived in closed form in the appendix. The decomposition in (6) demonstrates that working capital depends on three terms: First, it depends positively on economy-wide business conditions Ω_t. Second, it depends on the credit limit $\bar{a}_{i,t}$ imposed on firms, since loose credit limits naturally translate into higher business activity. Finally, and crucially, working capital depends on the "credit tightness" $(a_{i,t} - \bar{a}_{i,t})$ on island i: The island-specific fundamental $a_{i,t}$ drives island i's demand for credit, while $\bar{a}_{i,t}$ measures the credit limit (both in productivity units). If credit is not tight, there are sufficient funds for most firms on the island to operate without being financially constrained. In this case, the equilibrium is governed mainly by the demand for credit $a_{i,t}$ and working capital is sensitive to fluctuations in demand. If, however, credit is tight, a significant fraction of firms on the island is financially constrained. Then, island i's working capital is mostly determined by the credit limit $\bar{a}_{i,t}$ and hence almost insensitive to fluctuations in fundamentals $a_{i,t}$.

The sensitivity of working capital $\partial l_{i,t}/\partial a_{i,t}$ is key in our model as it determines the information content of the working capital signal $s^l_{i,t}$. This is because, when the sensitivity is small, the magnitude of the noise induced by the conditional distribution F_l of the working capital signal $s^l_{i,t}$ given a certain amount of working capital $l_{i,t-1}$ will become large in relative terms, and vice-versa if the sensitivity is large. The goal of the following paragraphs is to formalize this intuition.

To this end, we impose a small set of assumptions on the economy's information structure. First, we assume that the (conditional) distribution F_l of $s^l_{i,t}$ gives rise to a "stochastically monotone"

tightened credit conditions reduce the responsiveness of firms to fundamentals regardless of their origin and regardless of whether they manifest themselves through quantity constraints or increased credit spreads.

relationship of $s^l_{i,t}$ and $l_{i,t-1}$, in the sense that conditional on a signal realization $s^l_{i,t} = s$, the posterior over $l_{i,t-1}$ is increasing in s with respect to the monotone likelihood ratio property (MLRP). This ensures that the signal structure F_l associates large signals with large values for working capital. Our second assumption is that the signal structure F_l does not become more accurate for larger signal realizations, so that the decrease in the sensitivity of working capital does not become mechanically overturned by an exogenous reduction in noise. Formally, we require that the variance of the posterior of $l_{i,t-1}$ conditional on signal $s^l_{i,t} = s$ be nondecreasing in s. In particular, the variance can be constant.

Based on these two assumptions, we use the results of Straub and Ulbricht (2014) to prove the following result.

Proposition 3. *Suppose the posterior distribution of log working capital $l_{i,t-1}$ given signal $s^l_{i,t} = s$ is (i) increasing in s in the sense of the MLRP and (ii) its variance is nondecreasing in s. Then,*

- *(a) conditional on a level of the constraint $\bar{a}_{i,t-1}$, posterior uncertainty $\mathrm{Var}\{a_{i,t-1}|s^l_{i,t} = s\}$ is increasing in the signal realization s, and*
- *(b) conditional on a realization of the working capital signal, $s^l_{i,t} = s$, posterior uncertainty $\mathrm{Var}\{a_{i,t-1}|s^l_{i,t} = s\}$ is decreasing in the credit limit $\bar{a}_{i,t-1}$.*

Proposition 3 has two parts, which relate to the two determinants of credit tightness $a_{i,t} - \bar{a}_{i,t}$: The first part formalizes the above intuition. By assumption (i) in Proposition 3, a large realization of the working capital signal $s^l_{i,t}$ corresponds (stochastically) to a large working capital $l_{i,t-1}$, which—keeping $\bar{a}_{i,t-1}$ constant—corresponds to a large underlying value of $a_{i,t-1}$. In Proposition 2(b) we showed that the sensitivity of working capital was decreasing in $a_{i,t-1}$, which causes a reduction in the informational content contained in $s^l_{i,t}$, as measured by an increase of the posterior variance $\mathrm{Var}\{a_{i,t-1}|s^l_{i,t} = s\}$. In sum, higher credit demand, as signaled through high $s^l_{i,t}$ relative to a fixed $\bar{a}_{i,t}$, decreases the informativeness of the working capital signal.[18]

The second part of Proposition 3 focuses in turn on changes in the credit limit of island i, given by changes in $\bar{a}_{i,t-1}$. Again, we can let Proposition 2 guide us through the intuition: Suppose that $\bar{a}_{i,t-1}$ increases. By Proposition 2(b) this heightens the sensitivity of working capital $l_{i,t-1}$ to changes in fundamentals $a_{i,t-1}$, causing an increase in the informational content contained in $s^l_{i,t}$, as measured by a decrease in the posterior variance $\mathrm{Var}\{a_{i,t-1}|s^l_{i,t} = s\}$. Therefore, a higher credit limit $\bar{a}_{i,t-1}$, ceteris paribus, increases the informativeness of the working capital signal.

3.2.2 Approximate Gaussian updating

We now approach the full information extraction problem. One technical challenge in analyzing this learning problem is that the nonlinear working capital signal generally does not conjugate with reasonable prior distributions. To address this problem, we apply a specific linear approximation to

[18] It is important to keep in mind that variations in $a_{i,t}$ will also affect the equilibrium credit limit $\bar{a}_{i,t}$. The determinants of credit tightness, taking into account the endogeneity of $\bar{a}_{i,t}$, will be explored in Section 4.1.

the investors' learning problem that is able to preserve the key properties of the *non*linear updating problem derived above.

Specifically, we focus on a standard Gaussian information structure, with a Gaussian AR(1) process for $a_{i,t}$,

$$F_a(a|a_{i,t-1}) = \Phi\left(\frac{a - \rho_a a_{i,t-1}}{\sigma_a}\right), \tag{7}$$

where $\rho_a \in (0,1)$, and normally distributed signals $s^l_{i,t}$ and $s^p_{ik,t}$ with

$$F_l(s|l_{i,t-1}) = \Phi\left(\frac{s - l_{i,t-1}}{\sigma_\psi}\right), \tag{8}$$

$$F_p(s|a_{i,t}) = \Phi\left(\frac{s - a_{i,t}}{\sigma_p}\right), \tag{9}$$

and where $\sigma_\psi > 0$ and $\sigma_p > 0$. Given the Gaussian structure, the working capital signal can be written as

$$s^l_{i,t} = \mathcal{L}(a_{i,t-1} - \bar{a}_{i,t-1}) + \psi_{i,t}, \tag{10}$$

where $\psi_{i,t} \sim \mathcal{N}(0, \sigma^2_\psi)$. Here we dropped the term $(\Omega_{t-1} + (\xi - 1)\bar{a}_{i,t-1})$, which given information set $\mathcal{I}_{k,t}$ is constant and hence irrelevant for signal inference. The first term in (10) reflects fundamental variations in the signal that are driven by credit tightness; $\psi_{i,t}$ reflects the noise in the signal structure.

Based on this decomposition, we approximate the learning problem as follows. After observing a realization of the working capital signal $s^l_{i,t}$, agents linearize the function $\mathcal{L}$ to do standard Gaussian updating. We let the linearization point, however, depend on the realization of $s^l_{i,t}$: Large signal realizations should correspond to larger values of the fundamental $a_{i,t-1}$, and hence to *flatter* regions of the concave function $\mathcal{L}$. To reflect this in the linearization, the linearization point is taken to be the "face value" of $s^l_{i,t}$—the implied credit tightness if there was no noise in the signal structure, $s^{\text{face}}_{i,t} \equiv \mathcal{L}^{-1}(s^l_{i,t})$.[19] According to this definition, the face value of $s^l_{i,t}$ exactly becomes the agent's belief about the true tightness of credit,[20] $a_{i,t-1} - \bar{a}_{i,t-1}$, in the limit of the signal $s^l_{i,t}$ becoming perfectly informative. When $s^l_{i,t}$ is not perfectly informative, the fact that higher signal realizations have face values in regions where $\mathcal{L}$ is flatter lets agents attribute a higher uncertainty to the information contained in the face value $s^{\text{face}}_{i,t}$. This is formalized in the following lemma.

Lemma 1. *Suppose investors linearize $\mathcal{L}$ around the signal's "face-value" $s^{\text{face}}_{i,t}$ to assess the likelihood of observing $s^l_{i,t}|a_{i,t-1}$. Then, agents update as if $s^{\text{face}}_{i,t}$ was a "fictitious" Gaussian signal, distributed according to $\mathcal{N}(\mu_l, \sigma^2_l)$ with*

$$\mu_l = a_{i,t-1} - \bar{a}_{i,t-1}$$
$$\sigma_l = \frac{\sigma_\psi}{\mathcal{L}'(s^{\text{face}}_{i,t})},$$

[19] In the case where $s^l_{i,t} > \mathcal{L}(\infty)$, we let $s^{\text{face}}_{i,t} = \infty$.

[20] Recall that while $\bar{a}_{i,t-1}$ is perfectly known, $a_{i,t-1}$ is not.

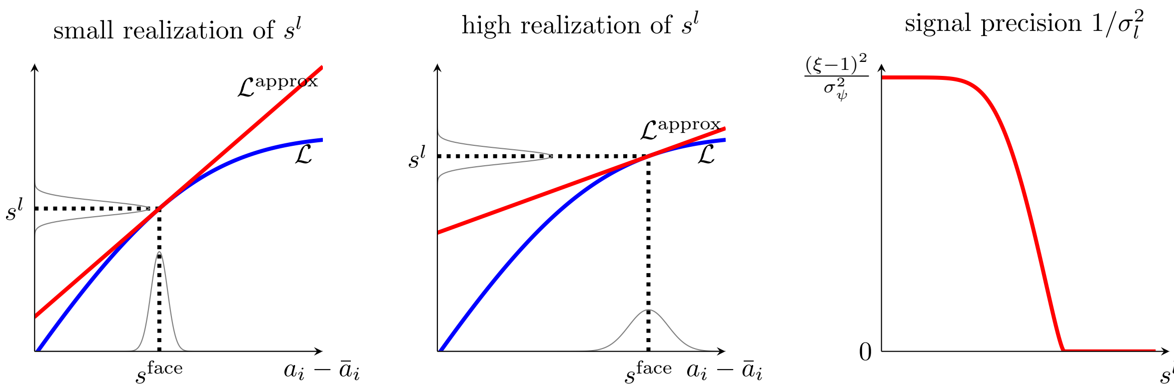

Figure 2: Approximate Gaussian updating. The nonlinear signal structure gives rise to signal-dependent uncertainty that is preserved using the linearization (red line). The right panel displays the resulting signal precision as a function of the signal realization s^l.

and where agents update as if σ_l were exogenous.

The updating behavior described in Lemma 1 is graphically depicted in Figure 2. It is evident how larger signal realizations—driven by increases in either credit tightness $a_{i,t-1} - \bar{a}_{i,t-1}$ or noise $\psi_{i,t}$—lead the agent to suspect the actual fundamental state in regions where $\mathcal{L}$ is flatter, rendering the agent more uncertain about the state's position. Notice that this approximate Gaussian updating requires the function $\mathcal{L}$ to be differentiable, which is the reason for having a small but positive within-island dispersion of productivities in the model.

The following proposition summarizes the ability to learn from the approximate working capital signal.

Proposition 4. *The standard deviation of the approximate Gaussian working capital signal $\sigma_l(s^l_{i,t-1})$ is increasing, with $\lim_{s\to-\infty}\sigma_l(s) = \sigma_\psi/(\xi-1)$ and $\lim_{s\to+\infty}\sigma_l(s) = \infty$. Consequently, given a realization of the noise term $\psi_{i,t}$, the precision of the working capital signal declines in credit tightness $a_{i,t} - \bar{a}_{i,t}$.*

The right panel of Figure 2 illustrates the resulting relation between the signal precision $1/\sigma_l^2$ and the realization $s^l = \mathcal{L}(a_{i,t-1} - \bar{a}_{i,t-1}) + \psi_{i,t}$. In the limit where firms on island i are essentially unconstrained ($\bar{a}_{i,t-1} \gg a_{i,t-1}$), the information content in the signal is equivalent to the exogenous noise, $\sigma_l = \sigma_\psi/(\xi-1)$. In the opposite case where most firms on island i are constrained ($\bar{a}_{i,t-1} \ll a_{i,t-1}$), observing the working capital contains no information, $\sigma_l = \infty$. Henceforth, we will use $\sigma_{l,i,t}$ to abbreviate $\sigma_l(s^l_{i,t})$.

3.2.3 Full learning problem

The two remaining ingredients for the full learning problem are prior beliefs and the information contained in the credit limit $\bar{A}_{i,t}$. Regarding the first ingredient, since investors were assumed to be one-period lived, their private information dies after one period as well, so each generation of investors has prior beliefs based on the *public* history of signals $\tilde{\mathcal{I}}_t \equiv \mathcal{I}_{k,t} \backslash \{s^p_{ik,t}\}_i = \{s^l_{i,s}, \bar{A}_{i,s}, \upsilon_s\}_{i,s\leq t}$.

Concordantly, we denote with a "tilde" the expectation and variance of public beliefs $\tilde{\mathbb{E}}_t\{\,\cdot\,\} \equiv \mathbb{E}\{\,\cdot\,|\tilde{\mathcal{I}}_t\}$ and $\tilde{\sigma}^2_{i,t} \equiv \mathrm{Var}\{a_{i,t}|\tilde{\mathcal{I}}_t\}$. In conjunction with $s^l_{i,t}$ (which is a signal about $a_{i,t-1}$), these public beliefs define the prior at date t. Projecting $\mathbb{E}\{a_{i,t-1}|\tilde{\mathcal{I}}_{t-1}, s^l_{i,t}\}$ forward (in time) to obtain a prior estimate of $a_{i,t}$, it can be shown that the relevant precisions of the prior and the working capital signal are given by $\delta_{i,t}\tilde{\sigma}^{-2}_{i,t-1}$ and $\delta_{i,t}\sigma^{-2}_{l,i,t}$, where

$$\delta_{i,t} = \left(\rho_a^2 + (\tilde{\sigma}^{-2}_{i,t-1} + \sigma^{-2}_{l,i,t})\sigma_a^2\right)^{-1}$$

measures the decay in past information due to the stochastic progression in $a_{i,t}$ (see the proof to Proposition 5 for details).

Regarding the information contained in $\bar{A}_{i,t}$, applying techniques similar to those used when solving a standard CARA-Normal asset pricing equilibrium (e.g., Hellwig, 1980) one finds that observing the credit limit $\bar{A}_{i,t}$ is informationally equivalent to receiving the signal

$$s^{\bar{a}}_{i,t} = a_{i,t} + \sigma_p^2\pi_\sigma\left(\eta_{i,t} - \rho_\eta\tilde{\mathbb{E}}_{t-1}\{\eta_{i,t-1}\}\right),$$

which has time-varying noisiness $\sigma^2_{\bar{a},i,t} = \rho_\eta^2\tilde{\sigma}^2_{i,t-1} + \sigma_\eta^2(\sigma_p^2\pi_\sigma)^2$.

After these preparations, we can now use the Kalman filter to recursively filter through all public information up to period $t-1$, and then use the filter one last time taking into account the private information available in period t. The following proposition summarizes the result.

Proposition 5. *In equilibrium, average financial market beliefs are given by*

$$\bar{\mathbb{E}}_t\{a_{i,t}\} = \frac{\hat{\sigma}^2_{i,t}}{\sigma_p^2}a_{i,t} + \frac{\hat{\sigma}^2_{i,t}}{\tilde{\sigma}^2_{i,t}}\tilde{\mathbb{E}}_t\{a_{i,t}\}$$
$$\hat{\sigma}^2_{i,t} = \left(\sigma_p^{-2} + \tilde{\sigma}^{-2}_{i,t}\right)^{-1},$$

with public beliefs given by

$$\tilde{\mathbb{E}}_t\{a_{i,t}\} = \tilde{\sigma}^2_{i,t}\begin{bmatrix}\delta_{i,t}\sigma^{-2}_{l,i,t} & \sigma^{-2}_{\bar{a},i,t} & \delta_{i,t}\tilde{\sigma}^{-2}_{i,t-1}\end{bmatrix} \times \begin{bmatrix}\rho_a(s^{\text{face}}_{i,t} + \bar{a}_{i,t-1}) \\ s^{\bar{a}}_{i,t} \\ \rho_a\tilde{\mathbb{E}}_{t-1}\{a_{i,t-1}\}\end{bmatrix}$$
$$\tilde{\sigma}^2_{i,t} = \left(\delta_{i,t}\sigma^{-2}_{l,i,t} + \sigma^{-2}_{\bar{a},i,t} + \delta_{i,t}\tilde{\sigma}^{-2}_{i,t-1}\right)^{-1}$$
$$\tilde{\mathbb{E}}_t\{\eta_{i,t}\} = \frac{1}{\sigma_p^2\pi_\sigma}\left(s^{\bar{a}}_{i,t} - \tilde{\mathbb{E}}_t\{a_{i,t}\}\right) + \rho_\eta\tilde{\mathbb{E}}_{t-1}\{\eta_{i,t-1}\}.$$

The intuition for the equations in Proposition 5 is as follows. Since the signal structure is Gaussian in our framework, expectations are convex combinations of signals. In particular, public expectations are a convex combination of the working capital signal, the credit limit signal, and the prior expectation. Investors' private expectations are very similar, except that they also include a term $(\hat{\sigma}^2_{i,t}/\sigma^2_p)a_{i,t}$ coming from (average) private signals.

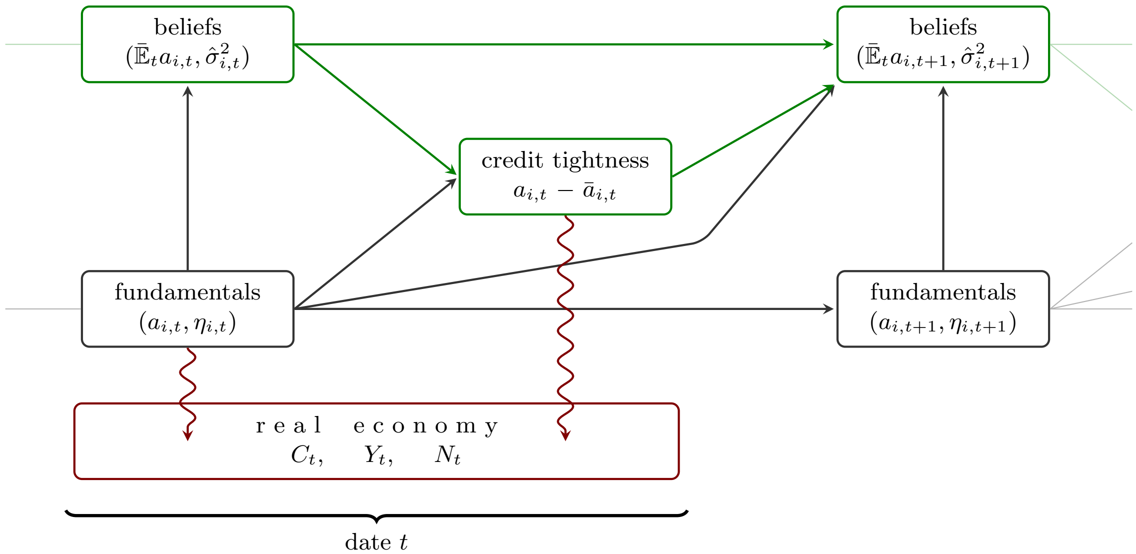

Figure 3: Schematic illustration of equilibrium dynamics. Green arrows: Internal persistence through feedback between beliefs and credit tightness. Red arrows: Effect of fundamentals and credit tightness on real aggregates.

The key ingredient in this otherwise standard Kalman filtering problem is that the noise in the working capital signal, $\sigma^2_{l,i,t}$, is *endogenous*. When this noise increases, investors optimally shift weight away from the working capital signal towards the three other signals. This affects their posterior expectations, as well as their posterior uncertainty: Expectations $\bar{\mathbb{E}}_t\{a_{i,t}\}$ become "sticky" in that now more weight is on the prior expectation. And posterior uncertainty $\hat{\sigma}^2_{i,t}$ naturally increases since one of the signals, the working capital signal, loses some of its precision. Both effects are crucial to understand the propagation of shocks in the model.

3.3 Computation and illustration of equilibrium

The previous two subsections provide a complete characterization of the equilibrium in the model economy. As established in Proposition 1, the economy's aggregate quantities only depend on the joint cross-sectional distribution of productivities $A_{ij,t}$ and credit limits $\bar{A}_{ij,t}$. By the credit rule (4), the latter is pinned down by exogenous shocks and investors' average beliefs, $\bar{\mathbb{E}}_t\{a_{i,t}\}$ and $\hat{\sigma}^2_{i,t}$, which by Proposition 5 are in turn recursively determined as functions of exogenous shocks and productivities. Combing Propositions 1 and 5, it follows that the equilibrium in our economy is unique and entirely backward looking.

Figure 3 illustrates the resulting equilibrium dynamics for a given island i, and how it connects with the aggregate real economy. The state of an island i at date t is characterized by the island's (exogenous) fundamentals $(a_{i,t}, \eta_{i,t})$, and (endogenous) beliefs, summarized by $\bar{\mathbb{E}}_t(a_{i,t}, \eta_{i,t})$ and $\hat{\sigma}^2_{i,t}$.[21] Fundamentals propagate according to the exogenous laws of motion of $a_{i,t}$ and $\eta_{i,t}$ (horizontal black arrow). Beliefs endogenously propagate through Bayesian updating but the degree of *internal*

[21]Here we drop the dependence on the aggregate state υ_t which could be defined as part of the fundamentals. Moreover, we note that the link from $(a_{i,t}, \eta_{i,t})$ to $(a_{i,t+1}, \eta_{i,t+1})$ is stochastic, in the sense that shocks enter along this link.

persistence in beliefs is crucially determined by the island's credit tightness $a_{i,t} - \bar{a}_{i,t}$ (green arrows). The red arrows illustrate what matters for the aggregate real economy: When many islands suffer from tight credit limits, this translates into labor and efficiency wedges, affecting aggregate variables.

In the next section, we study how the dynamics of beliefs in our economy can significantly slow down the island-level economic adjustment after negative shocks, bearing in mind how this matters for the aggregate economy (through the red arrows in Figure 3).

4 Belief traps

We now turn to the feedback mechanism between credit constraints and beliefs that lies at the core of our contribution. Much of the insights can be gained at the island-level, which we explore in this section. The dynamics of the aggregate economy will be explored in Section 5.

4.1 Pessimism, uncertainty, and credit constraints

Proposition 4 documents that, *ceteris paribus*, the ability to learn about an island's productivity deteriorates in the credit tightness $a_{i,t} - \bar{a}_{i,t}$. Using the credit rule (4), credit tightness can be written in terms of beliefs and fundamentals,

$$\underbrace{a_{i,t} - \bar{a}_{i,t}}_{\text{credit tightness}} \;=\; \underbrace{a_{i,t} - \bar{\mathbb{E}}_{i,t}\{a_{i,t}\}}_{\text{optimism/pessimism}} + \underbrace{\pi_\sigma \hat{\sigma}^2_{i,t}}_{\text{uncertainty}} - \underbrace{\upsilon_t - \pi_\sigma \hat{\sigma}^2_{i,t}\eta_{i,t}}_{\text{credit supply shocks}} - \pi_0. \tag{11}$$

Apart from being influenced by exogenous credit supply shocks, credit tightness depends on current beliefs, $\bar{\mathbb{E}}_{i,t}\{a_{i,t}\}$ and $\hat{\sigma}^2_{i,t}$. In particular, pessimistic beliefs ($\bar{\mathbb{E}}_{i,t}\{a_{i,t}\} < a_{i,t}$) and high uncertainty $\hat{\sigma}^2_{i,t}$ each reduce the credit limit relative to $a_{i,t}$ and hence increase credit tightness. By virtue of Proposition 4, tighter credit in turn feeds back into higher levels of uncertainty in the working capital signal,

$$\underbrace{\sigma_{l,i,t+1}}_{\text{signal uncertainty}} \;=\; \sigma_l\big(\,\mathcal{L}\;\underbrace{(a_{i,t} - \bar{a}_{i,t})}_{\text{credit tightness}} + \psi_{i,t+1}\,\big), \tag{12}$$

and through Bayesian updating (see Proposition 5), further translates into future pessimistic expectations $\bar{\mathbb{E}}_{i,t+1}\{a_{i,t+1}\}$ and future uncertainty $\hat{\sigma}^2_{i,t+1}$.[22] In this sense, pessimism and uncertainty mutually reinforce each other when credit is tight (see also the green arrows in Figure 3).

Before we proceed, it is worth noting that the dependence of the signal uncertainty $\sigma_{l,i,t}$ on credit tightness $(a_{i,t} - \bar{a}_{i,t})$ is highly nonlinear. This is because for $a_{i,t} \ll \bar{a}_{i,t}$, the majority of firms in island i are far away from the credit limit so that small changes in the credit supply bear little effect. As credit tightens, however, the *marginal* fraction of firms that is affected further increases until the median firm is effected. In line with our interpretation of islands as firms, we think about

[22] To see why tighter credit translates into more pessimistic future beliefs, first note that from (11), credit is tight precisely in states where expectations tend to be pessimistic. The link to future pessimistic expectations then arises, since a higher signal uncertainty implies slower revisions in pessimism due to a greater "stickiness" in beliefs (see Section 3.2.3).

the within-island productivity dispersion to be small, so that $\bar{a}_{i,t}$ essentially defines a threshold where for slightly smaller realizations in $a_{i,t}$ most firms are unconstrained and for slightly larger realizations most firms are constrained.

Belief dynamics To deepen our understanding of the described feedback loop, it is useful to first explore how beliefs for a specific island evolve absent any shocks, i.e. with $a_{i,s} = \eta_{i,s} = \psi_{i,s} = 0$ and $\upsilon_s = \bar{\upsilon}$ for all s, and some constant $\bar{\upsilon}$. In this case, the laws of motion for investors' expectations and uncertainty can be expressed as[23]

$$\bar{\mathbb{E}}_{t+1}\{a_{i,t+1}\} - \bar{\mathbb{E}}_t\{a_{i,t}\} = g_{\mathbb{E}}(\bar{\mathbb{E}}\{a_{i,t}\}, \hat{\sigma}_{i,t}) \tag{13}$$

and

$$\hat{\sigma}^2_{i,t+1} - \hat{\sigma}^2_{i,t} = g_{\hat{\sigma}}(\bar{\mathbb{E}}\{a_{i,t}\}, \hat{\sigma}_{i,t}). \tag{14}$$

We illustrate the state space in the left panel of Figure 4. The red line corresponds to the constant expectations locus ($g_{\mathbb{E}} = 0$), which is equivalent to $\bar{\mathbb{E}}\{a_{i,t}\} = 0$ as $a_{i,t}$ is mean-reverting. The blue line corresponds to the constant uncertainty locus ($g_{\hat{\sigma}} = 0$). The latter is "Z"-shaped, because higher levels of time t uncertainty $\hat{\sigma}^2_{i,t}$ not only directly increase time $t+1$ uncertainty $\hat{\sigma}^2_{i,t+1}$ but also indirectly through tighter credit limits and less learning from the working capital signal. This implies that for moderate levels of $\bar{\mathbb{E}}\{a_{i,t}\}$, there are multiple stationary values of uncertainty (for which $g_{\hat{\sigma}} = 0$): for high levels of uncertainty, credit is tight and information is scarce, reinforcing a high level of uncertainty, and vice versa. Intersecting the two loci, we see that this no-shock phase diagram can have a unique or multiple steady states, depending on the location of the uncertainty locus.

The three example paths in the left panel of Figure 4 illustrate the no-shock evolution of the beliefs from different starting points when there is a unique steady state. The key aspect in which the three paths differ is the degree of persistence and the amount of uncertainty induced along the path. The path starting with the (relatively) most optimistic expectations rapidly converges back to the unique steady state. The two other paths, however, behave distinctly differently: By starting to the left of the kink of the blue locus, investors are sufficiently pessimistic to induce a critical level of credit tightness ($\bar{a}_{i,t} < 0$) so that learning breaks down. This implies that along the two paths investors accumulate higher and higher levels of uncertainty and, accordingly, pessimism starts fading out slower and slower—two tendencies that jointly reinforce each other through tighter and tighter credit limits. Caused by the decreasing velocity of expectation dynamics in the neighborhood of such a "belief trap", its effects can be very persistent. Only after a significant amount of time do the paths cross to the right of the blue locus ($\bar{a}_{i,t} > 0$) and uncertainty drops back to its steady

[23] Of course, in equilibrium the beliefs will be fully determined by the history $\{a_{i,s}, \eta_{i,s}, \psi_{i,s}, \upsilon_s\}_{s \leq t}$. The equations given here are more general in that they describe the precise path how beliefs converge to their steady state values for any hypothetical starting point. To see why $g_{\mathbb{E}}$ depends only on $\bar{\mathbb{E}}\{a_{i,t}\}$ and $\hat{\sigma}_{i,t}$, note that given the steady state property we can write $g_{\bar{\mathbb{E}}}(\bar{\mathbb{E}}\{a_{i,t}\}, \hat{\sigma}_{i,t}) = (\rho_a \delta_{i,t+1} \hat{\sigma}^2_{i,t+1} / \hat{\sigma}^2_{i,t} - 1)\bar{\mathbb{E}}\{a_{i,t}\}$. Substituting for $\hat{\sigma}_{i,t+1}$, $\delta_{i,t+1}$ and $\sigma_{l,i,t+1}$ using Propositions 4 and 5 then yields the result.

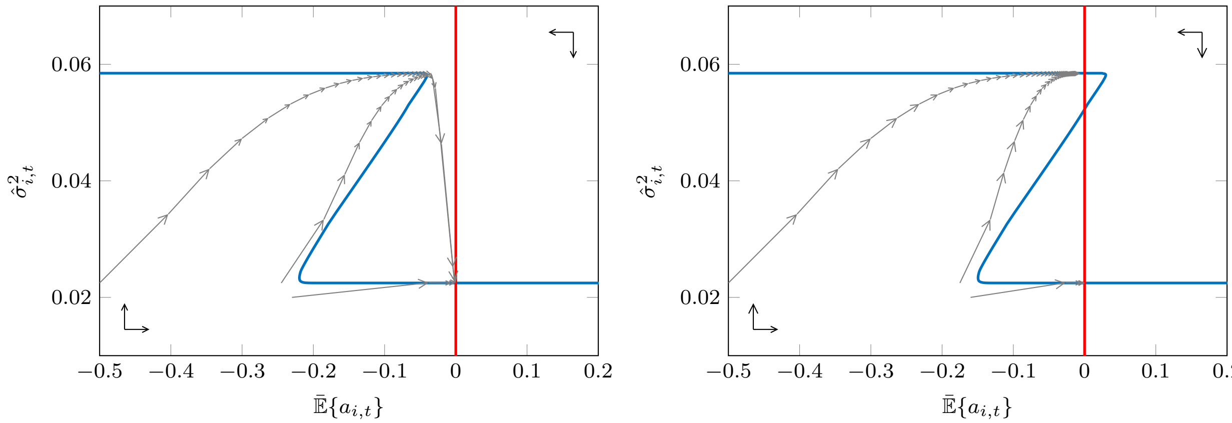

Figure 4: Phase diagrams for island level beliefs, in the absence of shocks. Z-shaped blue lines are $(\Delta\hat{\sigma}_{i,t} = 0)$-loci; vertical red lines are $(\Delta\bar{\mathbb{E}}\{a_{i,t}\} = 0)$-loci. *Left:* Case with a unique steady state. *Right:* Case with multiple steady states.

state value.

When the value of $\pi_0 + \upsilon_t$ is smaller than in the left panel, it can also happen that the blue locus intersects the red locus multiple times, leading to a second stable steady state (see the right panel of Figure 4 for an example). If this is the case, belief traps are infinitely persistent in the absence of shocks. (In either case, the logic from Section 3.3 applies and the equilibrium is unique.)

Reintroducing shocks We now discuss how the logic developed in the absence of shocks applies to the general case where the state includes a stochastic process of fundamentals. In the no-shock case it was clear that belief traps emerge when the state was to the left of the uncertainty locus or, equivalently, when $a_{i,t} - \bar{a}_{i,t} < 0$ (the combination of beliefs, for which $a_{i,t} = \bar{a}_{i,t}$, defines the upward-sloping arm of the uncertainty locus in Figure 4 above). Shocks enter this picture in two ways. First, for any sequence $\{a_{i,s}, \eta_{i,s}, \psi_{i,s}, \upsilon_s\}_{s\leq t}$, Proposition 5 pins down a pair $(\bar{\mathbb{E}}_t\{a_{i,t}\}, \hat{\sigma}_{i,t})$ and thus effectively selects a particular starting point in Figure 4. Second, shocks may also have a direct impact on $(a_{i,t} - \bar{a}_{i,t})$ in addition to their impact on $(\bar{\mathbb{E}}_t\{a_{i,t}\}, \hat{\sigma}_{i,t})$, which in Figure 4 corresponds to a (temporary) horizontal shift of the uncertainty locus.

In our model, shocks to $\eta_{i,t}$ are the only shocks for which these two effects are reinforcing each other. Consider for instance an adverse shock to $\eta_{i,t}$. On the one hand, such a shock directly reduces $\bar{a}_{i,t}$ and hence increases credit tightness $(a_{i,t} - \bar{a}_{i,t})$. This shifts the uncertainty locus to the right since it needs less pessimistic beliefs to sustain a particular level of uncertainty when credit is exogenously tighter. On the other hand, since investors learn from $\bar{a}_{i,t}$, an adverse $\eta_{i,t}$ shock moves expectations $\bar{\mathbb{E}}_t\{a_{i,t}\}$ to the left, reinforcing the exogenous tightening of credit. If the combination of these two effects is strong enough so that $a_{i,t} < \bar{a}_{i,t}$, then learning breaks down, triggering belief trap dynamics very similar to the ones discussed above.

In contrast, shocks to $a_{i,t}$ and $\psi_{i,t}$ both induce variations in $\bar{\mathbb{E}}_t\{a_{i,t}\}$ that are partially offsetting the direct impact on credit tightness $(a_{i,t} - \bar{a}_{i,t})$. For instance a positive innovation in $a_{i,t}$ naturally increases firms' demand for credit and thus increases $(a_{i,t} - \bar{a}_{i,t})$. At the same time, however, investors also receive signals about the productivity innovation that increase $\bar{\mathbb{E}}_t\{a_{i,t}\}$ and thereby partially offset the tightening of credit. An interesting observation is that depending on the ability of investors to learn about unconstrained firms, they may still fail to fully compensate for the increased demand of credit. This is akin to an "arrival of an unknown technology shock": Only firms themselves know about their inherent productivity innovations, while investors are initially uncertain and first need to learn about them. If this leaves a firm to lack the funds to run its newest technology, nobody will learn about its productivity, triggering a belief trap.[24]

Finally, shocks to v_t are common knowledge and do not change beliefs *per se*. They do, however, affect the island's credit supply and therefore affect the credit tightness $(a_{i,t} - \bar{a}_{i,t})$ similar to island-specific shocks to $\eta_{i,t}$. For small shocks to v_t and/or sufficiently optimistic investors, this bears little effects on any outcome. If, however, investors happen to be sufficiently pessimistic, an adverse shock to v_t unleashes its power and plunges the island into a belief trap, similar to island-specific credit shocks. In the cross-section of all islands, precisely the pessimistic ones are therefore affected. This is exactly the channel through which an aggregate *common knowledge* shock can cause a prolonged period of high uncertainty.

4.2 Simulation of an island-specific credit shock

So far, we explained that the two key ingredients to a belief trap are pessimistic expectations and uncertainty, and that credit shocks are the most direct way to enter into such a trap.[25] Here, we augment our previous exploration and numerically illustrate the power of belief traps in our economy by simulating an island's response to an adverse credit shock to $\eta_{i,t}$. The simulation (and all other in the main body of this paper) uses the parametrization presented in Section 5.1.

Belief traps versus counterfactual In particular, consider a single island that is hit by a $-2\sigma_\eta$ shock to $\eta_{i,t}$, while the aggregate financial state v_t is set to a constant and the economy is in its stochastic steady state. It is instructive to compare the (island-specific) dynamics of the model to such a shock with the counterfactual dynamics that would emerge given the same shock but where the signal precision $\sigma_{l,i,t}^{-2}$ is fixed in all periods at the pre-shock level (at which the island was essentially unconstrained). Figure 5 displays the model dynamics (solid black lines) alongside the counterfactuals (dashed red lines).

It can be seen that upon impact the financial shock affects the model economy in the same way as it affects the counterfactual response. This is because investors observe *lagged* working capital, so that an increase in uncertainty affects the dynamics with a delay of at least 1 period. On impact,

[24]Shocks in $\psi_{i,t}$ are similar to technology shocks, except that the rightward shift in the uncertainty locus is not caused by an increased credit demand of firms, but by investors perceiving credit demand to be closer to the credit limit. Since this reduces the precision of the working capital signal, expectations have to be less pessimistic to induce a certain level of uncertainty—similar to a rightward shift in the uncertainty locus.

[25]Of course, belief traps will in general emerge from a combination of shocks. Here we focus on credit shocks because they conveniently combine an impulse in pessimism and credit tightening. We note, however, that both ingredients by themselves would be enough to enter a belief trap (but would require larger realizations). Alternatively, one could yield similar results with a belief-neutral credit shock combined with a pure pessimism shock.

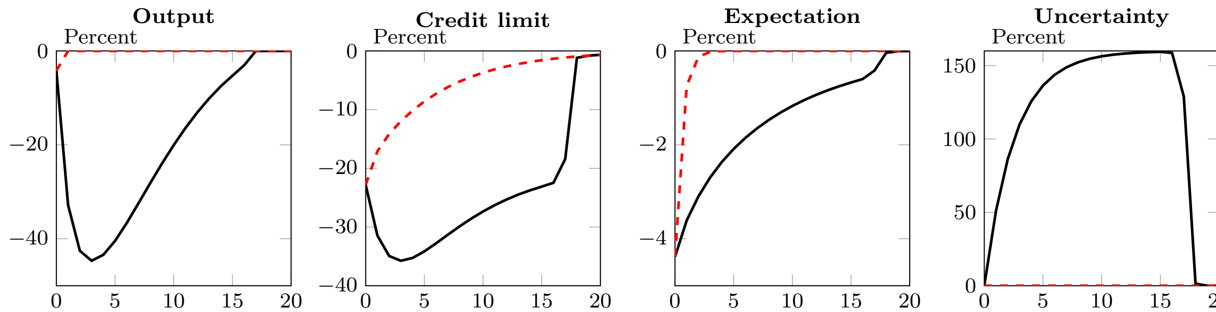

Figure 5: Impulse responses to island-specific financial shock. Solid black lines are impulse responses to a $-2\sigma_\eta$ shock to $\eta_{i,t}$. Dashed red lines are counterfactual responses to the same shock where the information precision is exogenously fixed at its unconstrained level. All responses are in percentage deviations.

the financial shock decreases the credit limit $\bar{a}_{i,t}$ and leads to more pessimistic expectations $\mathbb{E}_t\{a_{i,t}\}$ since traders learn from $\bar{a}_{i,t}$.

The difference between model and counterfactual emerges in the second period after impact, when the initial decline in the credit limit $\bar{a}_{i,t}$ slows down learning in the model economy. This triggers the aforementioned self-reinforcing cycle of uncertainty and pessimism—a belief trap. As can be seen in the first panel, this decline causes a persistent decline in output, which stands in stark contrast to the small impact on output in the counterfactual case.[26] As in the belief trap paths in Figure 4, uncertainty *accumulates* over time when reliable information ceases to arrive, explaining the hump-shaped response of uncertainty, credit limits, and output.

Since there is a unique steady state in the considered simulation, beliefs and credit limits will eventually recover. Once expectations cross to the right of the uncertainty locus (see Figure 4) and the island becomes sufficiently unconstrained so that a sufficient amount of new information is aggregated, there is a sharp drop in uncertainty. At this point, the island has essentially left the "belief trap", and further recovery proceeds quickly.

Marginally constrained versus marginally unconstrained islands We conclude our analysis with an illustration of the dynamics of a "marginally constrained" island compared to a "marginally unconstrained" island. Based on our previous discussion, we can define a threshold $\bar{\eta}_{i,t}$ such that for all $\eta_{i,t} \leq \bar{\eta}_{i,t}$ credit is tight ($\bar{a}_{i,t} < a_{i,t}$), whereas for all $\eta_{i,t} > \bar{\eta}_{i,t}$ the majority of firms in island i are unconstrained. In particular, letting $\mathcal{S}_{i,t} \equiv (a_{i,t}, \bar{\mathbb{E}}_t\{a_{i,t}\}, \hat{\sigma}^2_{i,t})$, we define $\bar{\eta}_{i,t} = \bar{\eta}(\mathcal{S}_{i,t}, \upsilon_t)$ such that for all $\eta_{i,t} \leq \bar{\eta}_{i,t}$ at least fraction $u = 1/2$ of firms in island i are constrained.[27] The number of constrained firms on island i is captured by $u = \Phi((a_{i,t} - \bar{a}_{i,t})/\sigma_\epsilon)$, yielding for the

[26] Underlying the discrepancy in the output between model and counterfactual is an inherent nonlinearity in the propagation of credit limits to output. Intuitively, for small variations in the credit limit, only firms in the right tail of the productivity distribution are limited in their production, which under standard assumptions on the productivity distribution are few in numbers. For larger variations in the credit limit, however, the marginally constrained firm within an island moves closer to the median where the probability density is larger. Accordingly, the marginal impact of a decline in an island's credit limit on its output is necessarily increasing until output is significantly affected.

[27] Here the precise value for u is not crucial as long as it is not too close to 0 since in our calibration within-island firm heterogeneity is very limited. Setting $u = 1/2$ is convenient as it simplifies the expression for $\bar{\eta}$.

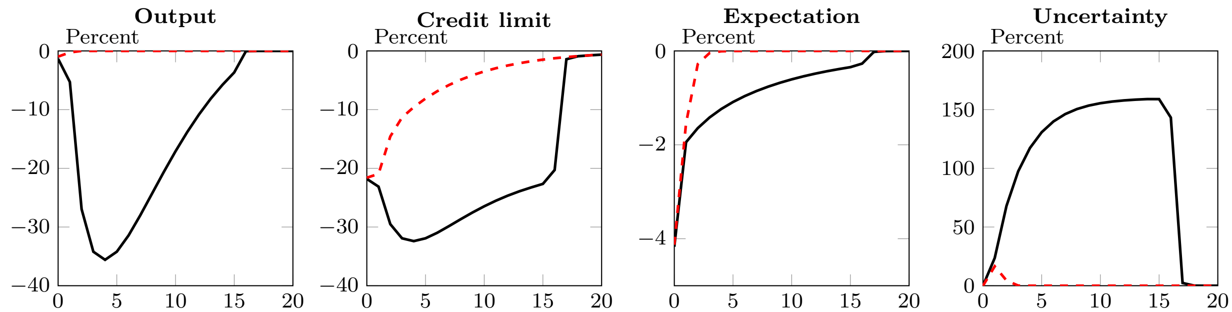

Figure 6: Impulse responses to island-specific financial shock so that island is "marginally constrained" . Solid black lines and dashed red lines are impulse responses to $-1.90\sigma_\eta$ and $-1.88\sigma_\eta$ shocks to $\eta_{i,t}$, respectively. All responses are in percentage deviations.

threshold

$$\bar{\eta}(\mathcal{S}_{i,t}, \upsilon_t) = \frac{1}{\pi_\sigma \hat{\sigma}_{i,t}^2} \left(a_{i,t} - \bar{\mathbb{E}}_{i,t}\{a_{i,t}\} - \upsilon_t - \pi_0 \right) + 1. \tag{15}$$

Figure 6 displays impulse response functions for two ex-ante identical islands, but where one is hit by a shock to $\eta_{i,t}$ slightly above $\bar{\eta}_{i,t}$, whereas the other is hit by a shock to $\eta_{i,t}$ slightly below $\bar{\eta}_{i,t}$. By design, the responses for the marginally constrained island (solid black lines) closely resemble the belief trap dynamics seen in Figure 5. In contrast, the responses for the marginally unconstrained island (dashed red lines) show little sign of an information breakdown or belief trap, similar to the counterfactual case discussed above.

In sum, there is *de facto* a discontinuity in the dynamics of the marginally constrained island and the marginally unconstrained island. While output of the marginally unconstrained island is virtually unaffected by the reduction in the credit limit, a slightly more constrained island remains constrained for a considerable amount of time, with severe consequences for resource misallocation and output.

5 Aggregate financial shocks

We now turn to the economy's response to an aggregate financial shock to υ_t. In the model, υ_t matters through its effect on the threshold $\bar{\eta}_p(\mathcal{S}_{i,t}, \upsilon_t)$ that determines the likelihood of an island to fall into a belief trap. From (15), $\bar{\eta}$ is strictly decreasing in υ_t. During a financial crisis when υ_t is small, firms are therefore more prone to idiosyncratic financial shocks in the sense that it takes smaller realizations of $\eta_{i,t}$ for a firm to become constrained. In this section, we explore the consequences of an aggregate drop in υ_t using numerical simulations.

5.1 Parametrization

We interpret one period as a quarter. The inverse Frisch elasticity of labor supply ζ is set to 0.5 and the elasticity of substitution between consumption goods ξ is set to 4. The productivity parameters are set to $\rho_a = 0.9$ and $\sigma_a = 0.15$, so that islands can be interpreted as firms. These parameters are

Table 1: Parameter values used in simulation

Parameter	Value	Description
ζ	0.50	Inverse Frisch elasticity of labor supply; set to standard value used in business cycle literature.
ξ	4.00	Elasticity of substitution among goods; set to standard value used in business cycle literature.
ρ_η	0.84	Persistence of credit supply shocks; set to $(1/2)^{1/4}$, reflecting a half-life of 4 quarters.
ρ_a	0.90	Persistence of island-average productivities; calibrated to firm-level productivity data.
σ_a	0.15	Dispersion of productivity innovations across islands; calibrated to firm-level productivity data.
σ_ϵ	0.01	Dispersion of productivities within islands; set to a small number to be consistent with interpreting islands as firms.
σ_ψ	0.23	Standard deviation of firm signal; calibrated to match firm-level forecast data (see text).
σ_p	0.47	Standard deviation of private signal; calibrated to match firm-level forecast data (see text).
σ_η	0.63	Standard deviation of financial noise; calibrated to match firm-level forecast data (see text).
π_σ	7.50	Credit rule coefficient; chosen based on our microfoundation in Appendix B for a relative risk aversion of 1.5.

consistent with the existing literature on firm-level dynamics.[28] To interpret islands as firms, we further keep the within-island dispersion low, setting σ_ϵ to 0.01.

It remains to specify the parameters of the credit rule and the parameters entering the learning problem. Notice that π_0 only matters by shifting $\{\upsilon_t\}$, so that we can normalize $\pi_0 = 0$. Our choice for π_σ is motivated using our micro-foundation in Appendix B, which maps a relative risk aversion of 1.5 for investors into $\pi_\sigma = 7.5$. Next, we set $\rho_\eta = 0.84$, reflecting a four quarter half-life of the island-specific credit supply shocks. Finally, for the learning parameters, we use forecasts about earnings per share (EPS) by financial analysts from the IBES database to construct calibration targets based on the financial market's ability to learn at the firm-level.[29] We exploit data on the

[28] See, for example, Gilchrist et al. (2014, Appendix 4).

[29] Our approach is similar to David et al. (2015), but differs in that we calibrate learning by investors about firms, whereas David et al. calibrate the learning of firms from financial markets. The existence of financial analysts' forecasts about firms allows us to calibrate our model more directly based on belief data.

following 4 panel variables[30]:

$$\bar{\mu}_{i,t} \equiv \text{average cross-analyst belief about firm } i\text{'s end-of-quarter EPS}$$
$$\sigma_{i,t}^{\text{cross}} \equiv \text{cross-analyst belief dispersion about firm } i\text{'s end-of-quarter EPS}$$
$$\text{EPS}_{i,t} \equiv \text{end-of-quarter realization of EPS}$$
$$\Delta p_{i,t} \equiv \text{log returns of firm } i\text{'s stock (adjusted for splits and dividends)}$$

To isolate the firm-specific components in these series, we extract a time-fixed effects from each of them, with the exception of $\sigma_{i,t}^{\text{cross}}$ (for which we target the sample mean). We choose σ_ψ, σ_p and σ_η to jointly match (i) the average belief dispersion[31] $\mathbb{E}\{(\sigma_{i,t}^{\text{cross}})^2\}/\mathbb{V}\{\text{EPS}_{i,t}\}$, where $\mathbb{E}$ and $\mathbb{V}$ denote the sample mean and variance; (ii) the signal-to-noise ratio of stock prices $\mathbb{V}\{\Delta p_{i,t}\}/\mathbb{V}\{\text{noise}_{i,t}\}$, where $\text{noise}_{i,t}$ are the residuals from regressing $\Delta p_{i,t-1}$ on $\text{EPS}_{i,t}$ and firm-level fixed effects[32]; and (iii) the correlation between average cross-analysts beliefs and actual realizations $\text{Corr}\{\bar{\mu}_{i,t}, \text{EPS}_{i,t}\}$.

Intuitively, the first of these moments determines the contribution of investors' private signals $s_{k,t}^p$ relative to all other signals, the second moment pins down the predictive power of market prices, and the third moment parametrizes the overall information available to investors. Tables 1 and 2 summarize the target moments and the calibrated variance parameters. In line with the asset pricing literature, the signal-to-noise ratio of prices is close to unity, reflecting a low correlation between prices and fundamentals.[33] The dispersion of beliefs and the correlation of beliefs and actuals, however, suggests that learning from the other sources is significantly more efficient. Taken together the learning parameters imply a moderate posterior uncertainty that averages to about one fifth of the unconditional uncertainty at the steady state. The robustness of our results to variations in the parametrization are explored in Appendix D.

5.2 Simulation of an aggregate credit shock

We conduct two numerical experiments. In this subsection, we illustrate the model's implication by simulating its response to a temporary shock to υ_t that decays at a geometric rate. This allows us to explore its implications in a controlled environment. The next subsection then chooses a different

[30]In the calibration, we compare pre-crisis model moments to monthly data prior to the financial crisis (1984Q2–2006Q4). To reduce the sensitivity of our calibration to outliers, we trim for each month the 2% tails of all variables. The resulting panel contains on average 2053 firms per month. Price data is adjusted for dividends and splits and is obtained from the CRSP database. See Appendix C for further details. The model moments are computed at the stochastic steady state with a constant $\upsilon_t = \bar{\upsilon}$ set so that 2.5 percent of firms are constrained.

[31]We normalize the average dispersion relative to $\mathbb{V}\{\text{EPS}_{i,t}\}$ to make it unitless, allowing us to directly compare it to the dispersion of investors' beliefs in our model without relying on further structural assumptions.

[32]We use lagged prices, since we are interested in calibrating the predictive power of stock prices for not yet realized earnings. In the model, we interpret the credit supply signal $s_{i,t}^{\bar{a}}$ as a natural counterpart, since it similarly aggregates information through its dependence on average expectations. Appendix B establishes this equivalence more formally. Paralleling the treatment of the data, we do not assume any prior knowledge when computing the signal-to-noise ratio of $s_{i,t}^{\bar{a}}$; i.e., we match the (average) unconditional ratio, $\text{Var}\{s_{i,t}^{\bar{a}}\}/\text{Var}\{s_{i,t}^{\bar{a}}|a_{i,t}\} = \text{Var}\{a_{i,t}\}/\sigma_{\bar{a},i,t}^2 + 1$, to the data.

[33]The calibration implies a relative contribution of the credit supply signal to investors' learning between 2 and 17 percent (with an average of 3 percent at the stochastic steady state). This is broadly consistent with David et al. (2015) who find that the information contained in stock market prices (the equivalent to the credit supply signal in our microfounded model in Appendix B) contributes between 2 and 8 percent to learning about firm fundamentals.

Table 2: Calibration targets for learning parameters (based on IBES data, see text)

Moment		Value
Average of firm-specific cross-analyst dispersion	$\mathbb{E}\{(\sigma_{i,t}^{\text{cross}})^2\}/\mathbb{V}\{\text{EPS}_{i,t}\}$	0.0206
Signal-to-noise ratio of stock returns	$\mathbb{V}\{\Delta p_{i,t}\}/\mathbb{V}\{\text{noise}_{i,t}\}$	1.1059
Correlation of firm-specific average belief and realized EPS	$\text{Corr}\{\bar{\mu}_{i,t}, \text{EPS}_{i,t}\}$	0.8697

Note: All target moments are exactly matched by the calibrated learning parameters. The model moments are computed at the stochastic steady state for a constant value of the aggregate credit shock $\upsilon_0 = \upsilon_1 = \cdots = \bar{\upsilon}$, chosen so that 2.5% of firms are constrained.

process for υ_t that is aimed to replicate the distress within the financial sector during the 2008/09 crisis.[34]

We let the initial aggregate financial state $\upsilon_0 = \cdots = \upsilon_{t-1} = \bar{\upsilon}$ be such that 2.5 percent of firms are constrained at the stochastic steady state. At date t the economy is hit by an aggregate shock Δ that reduces υ_t and decays with a half-life of 4 quarters:

$$\upsilon_{t+s} = \bar{\upsilon} - (0.5)^{s/4}\,\Delta.$$

The size of the initial impact Δ is chosen, so that 20 percent of firms are constrained at the peak of the crisis, consistent with the number of firms that reported to be "very affected" by difficulties in accessing the credit market during the recent financial crisis (Campello et al., 2010).[35]

Figure 7 depicts the responses of aggregate output, employment, the efficiency wedge $(1-\tau_t^A)$, the labor wedge $(1-\tau_t^N)$, average expectations $\int_I \bar{\mathbb{E}}_t\{a_{i,t}\}\,\mathrm{d}i$ and average uncertainty $\int_I \hat{\sigma}_{i,t}^2\,\mathrm{d}i$, the economy-wide fraction of constrained firms, and the economy-wide average "credit spread" $\int_{I\times J}(R_{ij,t}-1)\,\mathrm{d}(i,j)$. All responses are reported in percentage deviations from the steady state, except for the fraction of constrained firms (which is reported in percentage points) and the credit spread (which is reported in percentage points relative to the steady state).

The solid black lines show the responses in the endogenous uncertainty economy. To illustrate the effects of the belief trap mechanism, we also plot counterfactual responses where we exogenously fix $\sigma_{l,i,t}$ at their unconstrained level, shutting down any amplification and persistence stemming from belief traps. The counterfactual responses reflect the direct impact of tighter credit limits on the economy, whereas the difference between the counterfactual and the endogenous uncertainty economy is due to the belief trap mechanism.

By construction, the simulated shock increases the fraction of constrained firms upon impact, which is further reflected in an increase in the average credit spread. Tighter constraints then lead to credit and resource misallocation, illustrated by increases in the efficiency and labor wedges, and

[34] The simulations are implemented by fixing a large cross-section of islands and then running the recursion outlined in Section 3.3 for each island and the exogenous process for υ_t.

[35] Campello et al. (2010) conduct a survey among CFOs, finding that that 35% of firms report that they experience less access to credit in the 3rd quarter of 2008, 27% report that they experience higher costs of funds, and 18% state that they have difficulties in accessing a credit line. Asked about how much credit constraints affected their operations, 56% of firms report to be "somewhat affected" by difficulties in accessing the credit market in the third quarter of 2008, whereas 20% of firms report to be "very affected".

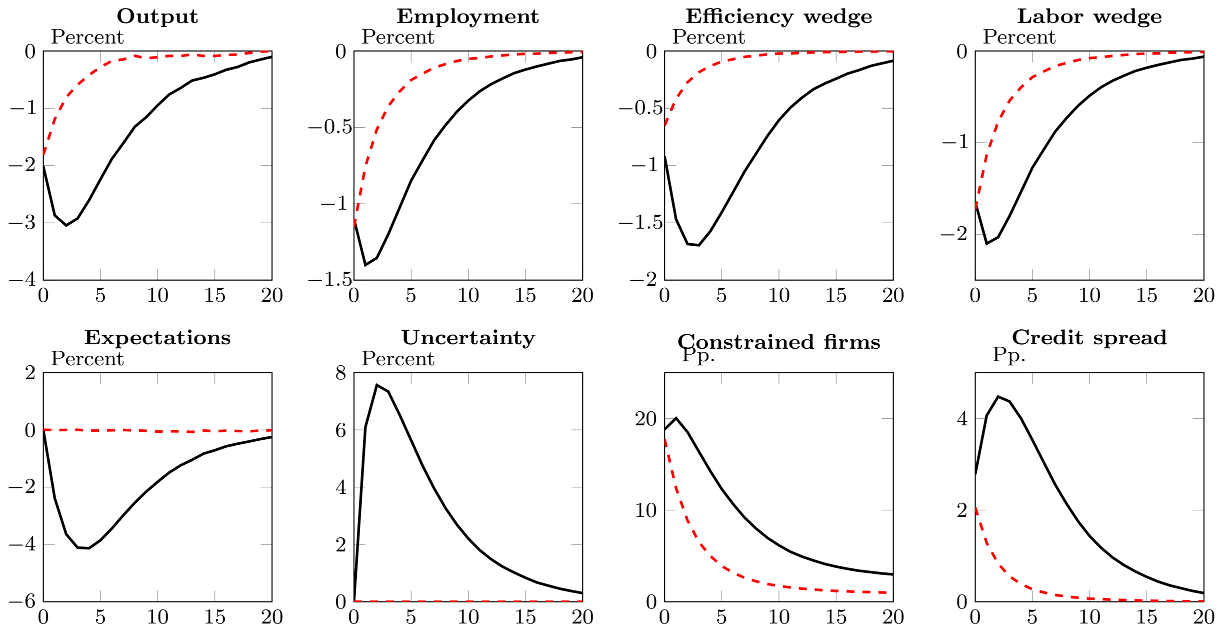

Figure 7: Impulse responses to aggregate financial shock. Solid black lines are impulse responses of the aggregated (or averaged) endogenous uncertainty economy; dashed red lines are counterfactual responses where the signal precision is exogenously fixed at its unconstrained level. All responses, except the last two plots, are in percentage deviations.

further causing aggregate output and employment to fall (see first row of Figure 7).

Notice that upon impact, there is no conceptional difference between the counterfactual responses and the endogenous uncertainty economy's—all visible differences are due to variations in the steady state distributions between the two economies. Starting with the first period after the initial impact, however, the responses between the model and the counterfactual persistently diverge as learning in the endogenous uncertainty economy is inhibited for islands facing tighter credit constraints (see our explanations in Section 4 above).

Internal persistence On an aggregate level, the disproportionately long-lasting contraction of firms in belief traps results in a discrepancy between the underlying aggregate financial shock, which was set to a half-life of 4 quarters, and the persistence in the endogenous responses of the economy. This is illustrated in Table 3, which lists the half-lives of the simulated responses in output and employment. It is evident that the endogenous uncertainty model has a high degree of internal persistence, implying that the half-life of output (10 quarters) and hours (8 quarters) significantly outlasts the financial shock that caused the crisis. The small internal persistence of the same shocks in the counterfactual economy (2 quarters) illustrates how in the absence of belief traps the fundamental impact of financial constraints quickly dissipates. For a comparison, the table also reports the half-lives for a simulation where only 10 percent of firms are constrained at the peak of the crisis, which is similar in magnitudes, reflecting that the economy scales approximately proportionally in the fraction of firms being constrained (see below).

Table 3: Half-life of output and employment to an aggregate financial shock with a half-life of 4 quarters. The size of the shock is calibrated, so that 10 or 20 percent of firms are constrained at the peak of the crisis.

	10% constrained		20% constrained	
	model	counterfactual	model	counterfactual
financial shock	4	4	4	4
output	11	2	10	2
hours	10	3	8	2

Endogenous pessimism An interesting implication of our belief trap mechanism is that the average pessimism in the economy endogenously increases as υ_t falls, even though υ_t is common knowledge and does not directly affect learning. The reason for this is statistical selection: When investors are pessimistic about an island, that island is more likely to become constrained. At the same time, expectations about constrained islands are endogenously persistent due to the belief trap mechanism. When investors are, by contrast, optimistic about an island, this relaxes credit constraints, so that the signals become more informative and investors are more likely to learn about their too optimistic views. This asymmetry between optimism and pessimism causes the economy-wide average expectation to fall when credit limits tighten (see the first plot in the second row in Figure 7).

Cross-sectional dispersion A fact often stressed about the recent financial crisis is that the cross-sectional dispersion of growth rates has drastically increased compared to the pre-crisis level (e.g., Bloom et al. 2014). This stands in contrast to what a simple model of the financial crisis based on financial constraints would predict: Financially constrained firms can only respond less to productivity innovations, which increases comovement. The belief trap mechanism in this paper opposes this effect by increasing the discrepancy between constrained and unconstrained firms. Depending on the parametrization of our model, either of the two effects can dominate (see Appendix D). The first panel of Figure 8 shows the response for our baseline calibration. Unlike some of the alternative parametrizations explored in the appendix, the "constraints"-effect still dominates here at the beginning of the crisis, causing the dispersion to initially fall, before it is overturned by increasing dispersion in the aftermath of the crisis. In contrast, dispersion is unambiguously reduced in the counterfactual due to the absence of belief traps.

Disagreement, volatility, and risk premia Another set of corollaries endogenously related to the increase of uncertainty in our model are an increasing disagreement of investors and an increasing volatility and risk premia of stock prices. This is in line with the available data and further provides a foundation for the common praxis to use volatility and disagreement as proxies for uncertainty (e.g., Bloom, 2009; Bachmann et al., 2013).[36]

[36] Increases in disagreement and volatility during the recent financial crisis are eminent, e.g., based on the IBES survey data described above and the VIX volatility index (see also the next subsection). Also see Carlin et al., 2013 for empirical evidence (based on MBS trading and forecast data) that there is a strong positive relationship between disagreement, return volatility, and risk premia.

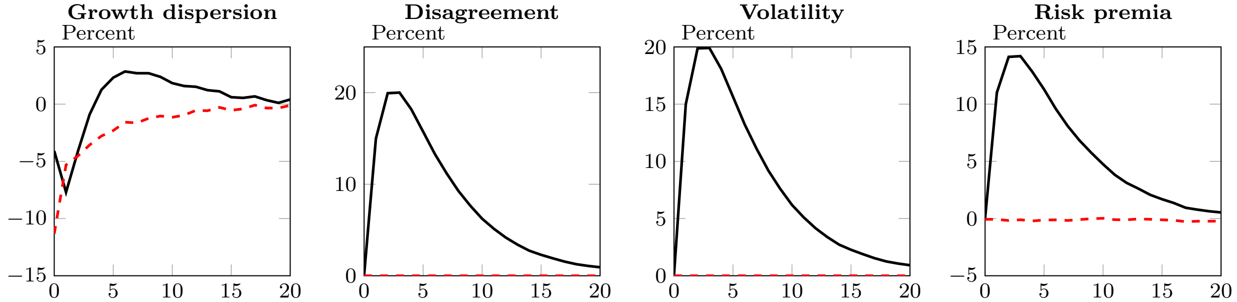

Figure 8: Further impulse responses to an aggregate financial shock. Solid black lines are impulse responses in the endogenous uncertainty economy; dashed red lines are counterfactual responses where the signal precision is exogenously fixed at its unconstrained level. All responses are in percentage deviations.

The increase in the dispersion of investors, defined by $\mathrm{Var}\{\mathbb{E}\{a_{i,t}|\mathcal{I}_{k,t}\}\}$ (variance with respect to k), directly follows from Proposition 5. When less can be learned from the firm signal, $s^l_{i,t}$, Bayesian agents respond by increasing the weight on their private information $s^p_{k,t}$, causing an increase in the cross-investor dispersion of beliefs.[37] For volatility and risk premia, we build on our microfoundation where $\bar{a}_{i,t}$ reflects variations in stock prices. Accordingly, we define volatility as the variance of $\bar{a}_{i,t}$ conditional on the complete history up to date $t-1$ and conditional on υ_t. Similar to disagreement, volatility increases when less can be learned from firms, since this causes investors to increase the weight on the market signal $s^{\bar{a}}_{i,t}$, making them more exposed to financial noise shocks $\eta_{i,t}$. Finally, risk premia in our model correspond to the term $\pi_\sigma(1-\eta_{i,t})\hat{\sigma}^2_{i,t}$ in our credit rule, which by definition increases when investors' uncertainty increases.[38]

The second to last panel in Figure 8 illustrates these effects for the simulated credit supply shock. Notably, none of the three series increases in the counterfactual where uncertainty is fixed at the pre-crisis level.

Asymmetric impact of financial shocks While the model's response to a credit supply shock scales almost linearly with the *fraction* of firms that are constrained during the crisis[39], the model's response is highly nonlinear and asymmetric in the *magnitude* of the exogenous shock to υ_t. Figure 9 illustrates this, relating the output loss at the peak of a crisis to the magnitude of the initial shock (measured in percentage deviations) and the corresponding increase in the fraction of firms that becomes constrained on impact (relative to the steady state). Two things can be noted. First, even very large positive shocks have only a muted impact on the economy. This is because there is no "over-borrowing" in our model, so that positive shocks to υ_t merely ensure that almost all islands are unconstrained, but do not lead to "credit booms". Second, negative but small credit

[37] More formally, the disagreement across investors can be shown to satisfy $\mathrm{Var}\{\mathbb{E}\{a_{i,t}|\mathcal{I}_{ik,t}\}|i,t\} = \hat{\sigma}^4_{i,t}/\sigma^2_p$. In Figure 8 we report the economy-wide average disagreement, given by $\int_I \hat{\sigma}^4_{i,t}/\sigma^2_p \,\mathrm{d}i$.

[38] Notice the positive correlation between $\eta_{i,t}$ and $\hat{\sigma}^2_{i,t}$. This explains why the average risk-premia increases more (in percentage terms) than the average uncertainty does, even though $\eta_{i,t}$ averages to zero in the cross-section.

[39] To see what drives the linearity, observe that absent general equilibrium effects, the output loss is approximately given by the fraction of island in a belief trap times the average output loss among those islands. The linearity then follows because the endogenous tightening of credit limits caused by pessimism and uncertainty for islands in a belief trap is large compared to the direct effect of the credit supply shock.

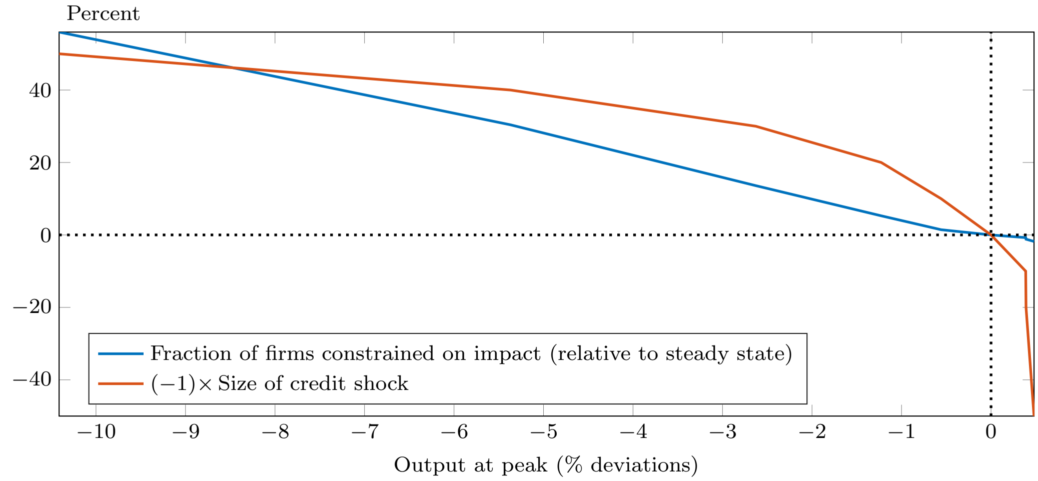

Figure 9: Output loss at the peak of the crisis in relation to the magnitude of the underlying credit supply shock to υ_t (in percent) and the corresponding fraction of firms that was constrained on impact.

supply shocks are less severe than the large shocks simulated in this section, since they map into disproportionately less firms that become constrained. Similar to, e.g., Brunnermeier and Sannikov (2012), this nonlinearity generates a discrepancy between high-frequency day-to-day fluctuations in financial markets, which have little impact on the real economy, and rare tail events, which cause pronounced recessions.

5.3 Application to the 2008/09 financial crisis

We now compare our model to data from the 2008/09 financial crisis. To this end, we conduct a second simulation similar to the one in the previous subsection, but where we replace the geometrically decaying shock to υ_t with one that is aimed to resemble the disturbances in the financial system seen during the recent crisis. Specifically, starting from the stochastic steady state (see above), we now simulate a sequence $\{\upsilon_{t+s}\}$,

$$\upsilon_{t+s} = \bar{\upsilon} - \Delta_s,$$

where Δ_s is set proportional to the St. Louis Fed's financial stress index (STLFSI), which is designed to capture the "financial stress" component underlying the performance of the financial sector.[40] In our simulation, we treat the STLFSI as a disturbance in credit supply that is intrinsic to the financial sector, but we do not take a stand on the original cause or the propagation of that cause within the financial sector. The bottom left panel in Figure 10 plots the evolution of the STLFSI between 2007 and 2013. Since the series does not have a natural unit, we scale $\{\Delta_s\}$ so that at the peak of the crisis 20 percent of firms in the economy are constrained. This is broadly consistent

[40] The STLFSI is based on a principal components analysis, where "financial stress" is taken as the most important factor explaining the comovement of several financial indicators, including 6 interest rate series, 5 yield spreads and 2 volatility indices. See https://www.stlouisfed.org/On-The-Economy/2014/June/What-Is-the-St-Louis-Fed-Financial-Stress-Index for details.

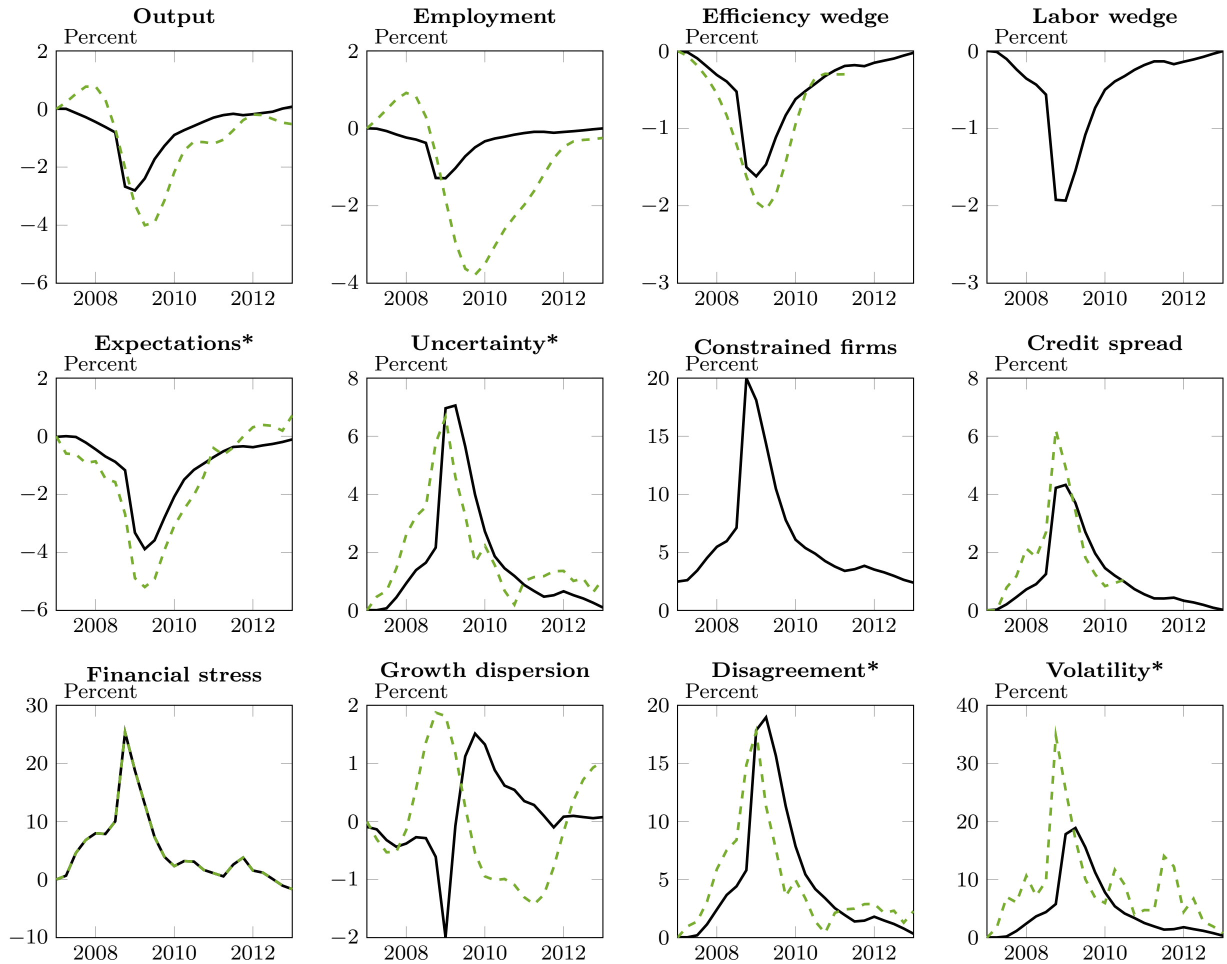

Figure 10: Simulation of 2008/09 financial crisis. Solid black lines are simulated responses of the aggregated (or averaged) endogenous uncertainty economy; dashed green lines are corresponding data series. The number of constrained firms and the credit spread are in percentage points, all other responses are in percentage deviations. Data series marked with * proxy their respective model counterparts without sharing the same units. In order to retain the same scale, the data series were scaled down by a factor 10.

with the empirical evidence on the fraction of firms affected by the recent financial crisis (c.f,. Footnote 35).[41] The remainder of the economy follows the parametrization discussed in Section 5.1.

Figure 10 shows the resulting model responses (black solid lines) along with the corresponding data moments in the U.S. (green dashed lines).[42] The bottom left panel displays the St. Louis financial stress index to which we calibrated the exogenous shock v_t. The other panels display

[41] Since our economy scales approximately linear in the fraction of firms constrained at the peak, variations in the scaling of $\{\Delta_s\}$ affect mainly the magnitude but not the persistence of the crisis.

[42] See Appendix C for a detailed description of the data. The data on output, employment, and the efficiency wedge (measured using TFP data) are detrended using a (6,32) band-pass filter. Data on credit spreads is obtained from Gilchrist and Zakrajšek (2012), defining the average spread between corporate bonds and a hypothetical Treasure security that mirrors the cash flow of the corporate bond. Expectations, uncertainty and disagreement are computed using IBES data. The measure for volatility is the VIX. Output growth dispersion is based on COMPUSTAT data. Volatility, expectations, uncertainty, and disagreement are all an order of magnitude larger in the data than in the model, likely due to the fact that the EPS data are an imperfect proxy for fundamentals. To make the graphs readable we scaled those data series down by a factor 10.

the endogenous responses. First, pointing to the core of this paper's mechanism, it can be seen that firm-level uncertainty in the data (proxied by the average cross-investor standard deviation of firm-specific forecasts in the IBES database) shows a remarkably similar shape to the predicted uncertainty. Similarly, average expectations in the data (measured by the average firm-specific forecast in the EPS database) shows a similar shape to the one predicted by the model.

In regard to the transmission to the real economy, the model predicts an increase in credit spreads by 433 basis points, compared to an increase in the credit spread by 621 measured in the data. The increased spreads then translate into an increase in the efficiency and labor wedges. Comparing the predicted decline in the model's efficiency wedge to the measured drop in Solow residuals in the data, the model accounts for 78 percent of the observed fall in aggregate productivity at the peak of the crisis. In comparison, the counterfactual without belief traps (not plotted here) only accounts for 29 percent. Similar to standard RBC models, the model underpredicts the fall in employment, due to the decline in the wage rate that leads unconstrained firms to increase hours throughout the crisis. Nevertheless, the models explains 74 percent in the drop in output at the peak (43 percent in the counterfactual).[43] At the same time, the model is able to explain the persistent decline in these variables after financial stress has faded out in 2009Q3.

Finally, the bottom row plots the model and data series on the cross-sectional growth dispersion, disagreement (the square of our uncertainty measure[44]), and volatility (measured by the VIX index in the data). It can be seen that both disagreement and volatility have similar shapes in the data. The dispersion in output growth rates in the model first drops due to the homogeneously negative change induced by a large tightening in financial constraints. After the initial impact, however, the emergence of belief traps and the corresponding firms' decline in sales increase the dispersion. The data also show an increase in the dispersion, although it peaks before the model-implied dispersion does.

6 Concluding remarks

to be written

[43]Of course, the absence of capital in our model suggests caution when interpreting the output loss. For this reason, we view the efficiency wedge as a better benchmark.

[44]In our model, disagreement across traders is proportional to the square of $\hat{\sigma}_t^2$.

A Mathematical appendix

A.1 Proof of Proposition 1

Firms' optimality implies that

$$\begin{aligned} P_{ij,t} &= \frac{\xi}{\xi-1}\frac{W_t R_{ij,t}}{A_{ij,t}} \\ Y_{ij,t} &= P_{ij,t}^{-\xi} C = \left(\frac{\xi-1}{\xi}\right)^{\xi}\left(\frac{A_{ij,t}}{W_t R_{ij,t}}\right)^{\xi} C_t \\ L_{ij,t} &= W_t N_{ij,t} = \left(\frac{\xi-1}{\xi}\right)^{\xi}\left(\frac{A_{ij,t}}{W_t}\right)^{\xi-1} R_{ij,t}^{-\xi} C_t, \end{aligned} \tag{16}$$

where $R_{ij,t} \equiv \max\left\{1, (A_{ij,t}/\bar{A}_{ij,t})^{1-1/\xi}\right\}$.

Aggregating over labor demands $N_{ij,t} = \frac{Y_{ij,t}}{A_{ij,t}}$ and per-island output $Y_{ij,t}$, yields

$$N_t = \left[(1-\tau_t^A)A^{\text{eff}}\right]^{\xi-1}\left[(1-\tau_t^N)W_t^{-1}\right]^{\xi} C_t \tag{17}$$

$$Y_t = \left[(1-\tau_t^A)A^{\text{eff}}\right] N_t, \tag{18}$$

where

$$A^{\text{eff}} = \left[\int_{I\times J} A_{ij,t}^{\xi-1}\,\mathrm{d}(i,j)\right]^{1/(\xi-1)}$$

is the economy-wide efficient productivity, and where

$$1-\tau_t^A = \frac{\text{MPN}_t}{\text{MPN}_t^{\text{opt}}} = \frac{Y_t/N_t}{A^{\text{eff}}} = \frac{1}{A^{\text{eff}}}\frac{\left(\int_{I\times J} A_{ij,t}^{\xi-1}R_{ij,t}^{1-\xi}\,\mathrm{d}(i,j)\right)^{\xi/(\xi-1)}}{\int_{I\times J} A_{ij,t}^{\xi-1}R_{ij,t}^{-\xi}\,\mathrm{d}(i,j)}$$

and

$$1-\tau_t^N = \frac{\text{MRS}_t}{\text{MPN}_t} = \frac{W_t}{(1-\tau_t^A)A^{\text{eff}}} = \frac{\xi-1}{\xi}\frac{\int_{I\times J} A_{ij,t}^{\xi-1}R_{ij,t}^{-\xi}\,\mathrm{d}(i,j)}{\int_{I\times J} A_{ij,t}^{\xi-1}R_{ij,t}^{1-\xi}\,\mathrm{d}(i,j)},$$

define the aggregate efficiency and labor wedge. Because $A_{ij,t}$ is normally distributed around $A_{i,t}$ there always exists a positive measure of firms (in each island) with $R_{ij,t} > 1$, and thus $1-\tau_t^N < (\xi-1)/\xi$. To see that $(1-\tau_t^A) < 1$, rearrange to obtain

$$\int_{I\times J} A_{ij,t}^{\xi-1}R_{ij,t}^{1-\xi}\,\mathrm{d}(i,j) < \left[\int_{I\times J} A_{ij,t}^{\xi-1}R_{ij,t}^{-\xi}\,\mathrm{d}(i,j)\right]^{(\xi-1)/\xi}\left[\int_{I\times J} A_{ij,t}^{\xi-1}\,\mathrm{d}(i,j)\right]^{1/\xi}$$

Defining $x_{ij} \equiv A_{ij,t}^{(\xi-1)^2/\xi}R_{ij,t}^{-(\xi-1)}$, $y_{ij} \equiv A_{ij,t}^{(\xi-1)/\xi}$, $p = \xi/(\xi-1)$, and $q = \xi$ this can be rewritten as

$$\int x_{ij}y_{ij}\,\mathrm{d}(i,j) < \left[\int x_{ij}^p\,\mathrm{d}(i,j)\right]^{1/p}\left[\int y_{ij}^q\,\mathrm{d}(i,j)\right]^{1/q}.$$

Since $1/p + 1/q = 1$, this is an immediate consequence of Hölder's inequality. The strictness follows since $R_{ij,t} > 1$ for a positive measure of indices (i, j).

To compute the aggregates, note that household optimization yields

$$C_t N_t^{\zeta} = W_t. \tag{19}$$

Collecting equations and setting $C_t = Y_t$, aggregate employment, output, and the wage rate are pinned down by the solution to (17), (18) and (19), yielding

$$\begin{aligned} W_t &= (1 - \tau_t^N)(1 - \tau_t^A) A^{\text{eff}} \\ N_t &= (1 - \tau_t^N)^{1/(1+\zeta)} \\ Y_t &= (1 - \tau_t^A) A^{\text{eff}} N_t. \end{aligned}$$

A.2 Proof of Proposition 2

Note that the integral in (5) implicitly uses the cdf $\Phi\left(\frac{a_{ij,t} - a_{i,t}}{\sigma_\epsilon}\right)$ as a measure. So, another way to express $L_{i,t}$ is as

$$L_{i,t} = \Omega_t \int_{-\infty}^{\infty} e^{(\xi-1)\min\{u, \bar{a}_{i,t}\}} \, \mathrm{d}\Phi\left(\frac{u - a_{i,t}}{\sigma_\epsilon}\right). \tag{20}$$

This integral can be explicitly solved to give

$$l_{i,t} = \log \Omega_t + (\xi - 1)\bar{a}_{i,t} + \mathcal{L}(a_{i,t} - \bar{a}_{i,t})$$

where

$$\mathcal{L}(x) \equiv \log\left[e^{(\xi-1)x + \frac{1}{2}(\xi-1)^2\sigma_\epsilon^2} \Phi\left(-\frac{x}{\sigma_\epsilon} - (\xi - 1)\sigma_\epsilon\right) + \Phi\left(\frac{x}{\sigma_\epsilon}\right)\right].$$

It is easy to see that $\mathcal{L}$ is smooth, and, using L'Hospital's rule, that $\lim_{x\to-\infty} \mathcal{L}(x) = -\infty$, $\lim_{x\to+\infty} \mathcal{L}(x) = 0$. Next we prove that any function of the form

$$h(x) = \log\left[e^{x+\frac{1}{2}\sigma^2} \Phi\left(-\frac{x}{\sigma} - \sigma\right) + \Phi\left(\frac{x}{\sigma}\right)\right]$$

for $\sigma > 0$ is strictly increasing and strictly concave. Notice that any result about h can easily be translated into one of $\mathcal{L}$ since $\mathcal{L}(x) = h((\xi - 1)x)$ with $\sigma = (\xi - 1)\sigma_\epsilon$.

The first derivative of $h(x)$ is given by

$$h'(x) = \left(1 + \frac{\Phi\left(\frac{x}{\sigma}\right)}{e^{x+\frac{1}{2}\sigma^2} \Phi\left(-\frac{x}{\sigma} - \sigma\right)}\right)^{-1}$$

which is clearly positive, and strictly between 0 and 1. Using L'Hospital's rule it is straightforward to see that $\lim_{x\to+\infty} h'(x) = 0$ and $\lim_{x\to-\infty} h'(x) = 1$, hence $\lim_{x\to+\infty} \mathcal{L}'(x) = 0$ and $\lim_{x\to-\infty} \mathcal{L}'(x) = \xi - 1$.

The second derivative of $h(x)$ is given by

$$h''(x) = -h'(x)\left(1 - h'(x)\right)\left[\frac{1}{\sigma}\frac{\phi(\frac{x}{\sigma})}{\Phi(\frac{x}{\sigma})} + \frac{1}{\sigma}\frac{\phi(-\frac{x}{\sigma}-\sigma)}{\Phi(-\frac{x}{\sigma}-\sigma)} - 1\right]$$

which, using the fact[45] that $\frac{\phi(z)}{\Phi(z)} > -z$ for any z proves that $h''(x) < 0$, so $\mathcal{L}$ is concave. Parts (a) and (b) of Proposition 2 follow immediately from the properties of $\mathcal{L}$.

A.3 Proof of Proposition 3

Both proofs rely on Straub and Ulbricht (2014). Before we prove the two parts, notice that equation (6) can be solved for $a_{i,t}$ as function of $l_{i,t}$ and $\bar{a}_{i,t}$,

$$a_{i,t} = g(l_{i,t}, \bar{a}_{i,t}),$$

where, due to the properties of $\mathcal{L}$, g is increasing and strictly convex in $l_{i,t}$, and decreasing in $\bar{a}_{i,t}$. Moreover, g as a function of $l_{i,t}$ becomes "less convex" as $\bar{a}_{i,t}$ increases, in the following sense: The function $G(a) = g(g(\cdot, \bar{a}_1)^{-1}(a), \bar{a}_2)$ is concave and increasing for $\bar{a}_1 \leq \bar{a}_2$. Also note that G is differentiable with derivative between 0 and 1 since $\lim_{a\to-\infty} G'(a) = 1$.

Now consider part (a). Define two random variables X and Y as having distributions equal to $l_{i,t}|(s^l_{i,t} = s_1)$ and $l_{i,t}|(s^l_{i,t} = s_2)$ with $s_1 < s_2$.[46] As we know that $\text{Var}\{l_{i,t}|s^l_{i,t} = s\}$ is nondecreasing in s but $l_{i,t}|(s^l_{i,t} = s)$ is MLRP-increasing in a, Assumption 1 in Straub and Ulbricht (2014) is satisfied. Therefore, we can apply the strict version of Theorem 1 using g as strictly convex function of $l_{i,t}$ to yield $\text{Var}\{a_{i,t}|s^l_{i,t} = s_1\} < \text{Var}\{a_{i,t}|s^l_{i,t} = s_2\}$, which proves the result.

For part (b), let $\bar{a}_1 < \bar{a}_2$ and define G as above. Define random variables Z_j with distributions $a_{i,t}|(s^l_{i,t} = s, \bar{a}_{i,t} = \bar{a}_j)$ for $j = 1, 2$. Without loss assume that Z_1 and Z_2 are perfectly rank-correlated, with $G(Z_1) = Z_2$. This, together with the fact that $0 \leq G'(z) \leq 1$ lets us apply Lemma 3 in Appendix B in Straub and Ulbricht (2014) using $h = G^{-1}$, concluding that

$$\text{Var}\{Z_2\} < Cov(Z_2, G^{-1}(Z_2)) < Cov(G^{-1}(Z_2), G^{-1}(Z_2)) = \text{Var}\{Z_1\},$$

or in other words, that $\text{Var}\{a_{i,t}|(s^l_{i,t} = s, \bar{a}_{i,t})$ is decreasing in $\bar{a}_{i,t}$ for any given fixed s.

A.4 Proof of Lemma 1

Suppose the working capital signal $s^l_{i,t}$ realizes at some s. If agents linearize the function $\mathcal{L}$ around the face value $s^{\text{face}} = \mathcal{L}^{-1}(s)$, this means that they replace $\mathcal{L}$ by the following linearized function in

[45] One way to prove this fact is using a continued fraction expansion of $\Phi(x)$, or equivalently, the complimentary error function. See https://en.wikipedia.org/wiki/Error_function#Approximation_with_elementary_functions and Cuyt et al. (2008).

[46] Since their distribution is all that matters for this result, the joint distribution of X and Y is allowed to be anything.

their information updating problem,

$$\mathcal{L}_{\text{linear}}(x) = \mathcal{L}(s^{\text{face}}) + \mathcal{L}'(s^{\text{face}})(x - s^{\text{face}}).$$

This then implies that agents perceive the nonlinear signal $s^l_{i,t}$ *as if* it came from the corresponding "fictitious" linearized signal,

$$\mathcal{L}_{\text{linear}}(a_{i,t-1} - \bar{a}_{i,t-1}) + \psi_{i,t},$$

or informationally equivalent to this, they update as if they saw the signal

$$\begin{aligned} s^{\text{face}}_{i,t} &= (\mathcal{L}_{\text{linear}})^{-1}(\mathcal{L}_{\text{linear}}(a_{i,t-1} - \bar{a}_{i,t-1}) + \psi_{i,t}) \\ &= a_{i,t-1} - \bar{a}_{i,t-1} + \frac{1}{\mathcal{L}'(s^{\text{face}})}\psi_{i,t}, \end{aligned}$$

realizing at $s^{\text{face}}_{i,t} = s^{\text{face}}$. This proves the lemma.

A.5 Proof of Proposition 4

We define

$$\sigma_l(x, \psi) = \left(\mathcal{L}'(\mathcal{L}^{-1}(\mathcal{L}(-x) + \psi))\right)^{-1} \sigma_\psi,$$

so that $\sigma_l = \sigma_l(\bar{a}_{i,t-1} - a_{i,t-1}, \psi_{i,t})$ with σ_l from Lemma 1. Obviously, because $\mathcal{L}$ is increasing and $\mathcal{L}'$ is decreasing, σ_l is increasing in its first and decreasing in its second argument.

Now, setting $\psi = 0$, we find that

$$\sigma_l(x, 0) = \left(\mathcal{L}'(-x)\right)^{-1} \sigma_\psi,$$

giving rise to $\lim_{x\to\infty} \sigma_l(x, 0) = \sigma_\psi/(\xi - 1)$ and $\lim_{x\to-\infty} \sigma_l(x, 0) = \infty$ using Proposition 2.

A.6 Proof of Proposition 5

Signals Consider the information set of trader k at time t, $\mathcal{I}_{k,t} = \{s^p_{ik,t}\}_i \cup \{s^l_{i,s}, \bar{A}_{i,s}, \upsilon_s\}_{i,s\le t}$. By definition, υ_t is orthogonal to $a_{i,t}$ and can thus be ignored for the purpose of learning about $a_{i,t}$. Given our approximation approach, the remaining elements of $\mathcal{I}_{k,t}$ are Gaussian signals so that we can characterize $\mathbb{E}\{a_{i,t}|\mathcal{I}_{k,t}\}$ using a standard Kalman filter. In particular, since $\tilde{\mathcal{I}}_t = \{s^l_{i,s}, \bar{A}_{i,s}, \upsilon_s\}_{i,s\le t}$ is common knowledge, we can characterize beliefs recursively by first filtering through the publicly observable history $\tilde{\mathcal{I}}_t$, and then applying the filter one last time to process the information contained in $\{s^p_{ik,t}, s^l_{i,t}, \bar{A}_{i,t}\}$.

From Lemma 1, $s^l_{i,t}$ is informational equivalent to observing $s^{\text{face}}_{i,t} \sim \mathcal{N}(a_{i,t-1} - \bar{a}_{i,t-1}, \sigma^2_{l,i,t})$ or $\tilde{s}^{\text{face}}_{i,t} \equiv s^{\text{face}}_{i,t} + \bar{a}_{i,t-1} \sim \mathcal{N}(a_{i,t-1}, \sigma^2_{l,i,t})$ since $\bar{a}_{i,t-1}$ is known. Credit limits $\{\bar{A}_{i,s}\}$ are thus the only endogenous signals that remain to be characterized. Stripping away informationally irrelevant quantities from (4), the information in $\bar{A}_{i,t}$ is equivalent to the one in

$$\bar{\mathbb{E}}_{i,t}\{a_{i,t}\} + \pi_\sigma \hat{\sigma}^2_{i,t}\eta_{i,t}. \tag{21}$$

The information content in (21) is endogenous and depends on the average expectation. To solve this fixed point, we postulate (and verify below) that (21) is informationally equivalent to a normal signal $s^{\bar{a}}_{i,t}$ with yet to be determined noise $\sigma_{\bar{a},i,t}$.

Law of motion of public beliefs Letting $\tilde{\mathbb{E}}_{t-1}\{a_{i,t-1}\}$ and $\tilde{\sigma}^2_{i,t-1}$ denote the (public) prior mean and variance given $\mathcal{I}^p_{t-1}$, we are now ready to update beliefs given $\tilde{s}^{\text{face}}_{i,t}$, $s^{\bar{a}}_{i,t}$, and $s^p_{k,t}$. Since $\tilde{s}^{\text{face}}_{i,t}$ is a signal about $a_{i,t-1}$, we split the updating into two steps, first forming expectations about $a_{i,t-1}$ using only $\tilde{s}^{\text{face}}_{i,t}$ and the prior. Standard Bayesian updating yields

$$\begin{aligned}
\mathbb{E}\{a_{i,t-1}|\mathcal{I}^p_{t-1}, \tilde{s}^{\text{face}}_{i,t}\} &= \frac{\tilde{\sigma}^{-2}_{i,t-1}\tilde{\mathbb{E}}_{t-1}\{a_{i,t-1}\} + \sigma^{-2}_{l,i,t}\tilde{s}^{\text{face}}_{i,t}}{\tilde{\sigma}^{-2}_{i,t-1} + \sigma^{-2}_{l,i,t}}, \\
\text{Var}\{a_{i,t-1}|\mathcal{I}^p_{t-1}, \tilde{s}^{\text{face}}_{i,t}\}^{-1} &= \tilde{\sigma}^{-2}_{i,t-1} + \sigma^{-2}_{l,i,t}
\end{aligned}$$

Projecting forward, we get

$$\begin{aligned}
\mathbb{E}\{a_{i,t}|\mathcal{I}^p_{t-1}, \tilde{s}^{\text{face}}_{i,t}\} &= \rho_a\frac{\tilde{\sigma}^{-2}_{i,t-1}\tilde{\mathbb{E}}_{t-1}\{a_{i,t-1}\} + \sigma^{-2}_{l,i,t}\tilde{s}^{\text{face}}_{i,t}}{\tilde{\sigma}^{-2}_{i,t-1} + \sigma^{-2}_{l,i,t}} \\
\text{Var}\{a_{i,t}|\mathcal{I}^p_{t-1}, \tilde{s}^{\text{face}}_{i,t}\}^{-1} &= \left(\tilde{\sigma}^{-2}_{i,t-1} + \sigma^{-2}_{l,i,t}\right)\delta_{i,t},
\end{aligned}$$

where

$$\delta_{i,t} = \left(\rho_a^2 + \left(\tilde{\sigma}^{-2}_{i,t-1} + \sigma^{-2}_{l,i,t}\right)\sigma_a^2\right)^{-1}.$$

Now treating $\mathbb{E}\{a_{i,t}|\mathcal{I}^p_{t-1}, \tilde{s}^{\text{face}}_{i,t}\}$ and $\text{Var}\{a_{i,t}|\mathcal{I}^p_{t-1}, \tilde{s}^{\text{face}}_{i,t}\}$ as prior, updating with respect to $s^{\bar{a}}_{i,t}$ yields

$$\tilde{\mathbb{E}}_t\{a_{i,t}\} = \tilde{\sigma}^2_{i,t}\begin{bmatrix}\delta_{i,t}\sigma^{-2}_{l,i,t} & \sigma^{-2}_{\bar{a},i,t} & \delta_{i,t}\tilde{\sigma}^{-2}_{i,t-1}\end{bmatrix} \times \begin{bmatrix}\rho_a\tilde{s}^{\text{face}}_{i,t} \\ s^{\bar{a}}_{i,t} \\ \rho_a\tilde{\mathbb{E}}_{t-1}\{a_{i,t-1}\}\end{bmatrix}$$

and

$$\tilde{\sigma}^2_{i,t} = \left(\delta_{i,t}\sigma^{-2}_{l,i,t} + \sigma^{-2}_{\bar{a},i,t} + \delta_{i,t}\tilde{\sigma}^{-2}_{i,t-1}\right)^{-1}.$$

Private and average beliefs If in addition, the private signal $s^p_{k,t}$ is observed, straightforward updating yields the following posterior, given by

$$\mathbb{E}\{a_{i,t}|\mathcal{I}^p_{t-1}, \tilde{s}^{\text{face}}_{i,t}, s^{\bar{a}}_{i,t}, s^p_{k,t}\} = \frac{\hat{\sigma}^2_{i,t}}{\sigma^2_p}s_{k,t} + \frac{\hat{\sigma}^2_{i,t}}{\tilde{\sigma}^2_{i,t}}\tilde{\mathbb{E}}_t\{a_{i,t}\} \tag{22}$$

and

$$\hat{\sigma}^2_{i,t} = \left(\sigma_p^{-2} + \tilde{\sigma}_{i,t}^{-2}\right)^{-1}. \tag{23}$$

Aggregating across agents, we have that

$$\bar{\mathbb{E}}_t\{a_{i,t}\} = \frac{\hat{\sigma}^2_{i,t}}{\sigma_p^2} a_{i,t} + \frac{\hat{\sigma}^2_{i,t}}{\tilde{\sigma}^2_{i,t}} \tilde{\mathbb{E}}_t\{a_{i,t}\}. \tag{24}$$

Characterizing $\sigma_{\bar{a},i,t}$ To complete the characterization, we still have to determine $\sigma_{\bar{a},i,t}$. For this, substitute (24) back into (21). Note, however, that the last term in (24) is common knowledge among investors, so that (21) is informationally equivalent to observing $\frac{\hat{\sigma}^2_{i,t}}{\sigma_p^2} a_{i,t} + \pi_\sigma \hat{\sigma}^2_{i,t} \eta_{i,t}$, or, equivalently, $a_{i,t} + \sigma_p^2 \pi_\sigma \eta_{i,t}$. Finally, subtracting the common knowledge term $\rho_\eta \tilde{\mathbb{E}}_{t-1}\{\eta_{i,t-1}\}$ from $\eta_{i,t}$, the $\bar{A}_{i,t}$ signal is equivalent to

$$s^{\bar{a}}_{i,t} \equiv a_{i,t} + \sigma_p^2 \pi_\sigma \left(\eta_{i,t} - \rho_\eta \tilde{\mathbb{E}}_{t-1}\{\eta_{i,t-1}\}\right). \tag{25}$$

Notice that the belief over $\eta_{i,t}$ evolves according to

$$\tilde{\mathbb{E}}_t\{\eta_{i,t}\} = \frac{1}{\sigma_p^2 \pi_\sigma}\left(s^{\bar{a}}_{i,t} - \tilde{\mathbb{E}}_t\{a_{i,t}\}\right) + \rho_\eta \tilde{\mathbb{E}}_{t-1}\{\eta_{i,t-1}\}.$$

We subtracted the prior belief over $\eta_{i,t}$ in (25) since

$$\left(\eta_{i,t} - \rho_\eta b^\eta_{i,t-1}\right)\Big| \mathcal{I}^p_{t-1} \sim \mathcal{N}(0, \sigma^2_{\eta,i,t})$$

where

$$\sigma^2_{\eta,i,t} = \sigma^2_\eta + \rho^2_\eta \mathrm{Var}\{\eta_{i,t-1} | \mathcal{I}^p_{t-1}\},$$

and

$$\mathrm{Var}\{\eta_{i,t-1}|\mathcal{I}^p_{t-1}\} = \mathrm{Var}\left\{\frac{s^q_{i,t-1} - a_{i,t-1}}{\sigma_p^2 \pi_\sigma}\middle| \mathcal{I}^p_{t-1}\right\} = \left(\sigma_p^2 \pi_\sigma\right)^{-2} \tilde{\sigma}^2_{i,t-1}.$$

Hence,

$$s^{\bar{a}}_{i,t} \sim \mathcal{N}(a_{i,t}, \sigma^2_{\bar{a},i,t}),$$

where

$$\sigma^2_{\bar{a},i,t} = \rho^2_\eta \tilde{\sigma}^2_{i,t-1} + \sigma^2_\eta (\sigma_p^2 \pi_\sigma)^2.$$

B A microfounded version of the credit limit (for online publication)

In this appendix we provide one possible way to derive the parametric specification of the "credit rule" (4) adopted in the main text, which links firms' credit conditions to investor beliefs. In the microfoundation firms sell securities based on their revenues to investors. Since the market price of these securities is determined by the beliefs of investors, this creates the proposed link. We note however that the link generalizes beyond this microfoundation (see Footnote **??** in the main text for details).

Section B.1 sets up the structure of the financial market in which firms can sell the securities, and links the credit limit $\bar{a}_{i,t}$ on island i through the security price to investor beliefs. Section B.2 then takes a linear approximation to the resulting relationship between credit limits and investor beliefs, yielding (4).

B.1 Investors and markets

In this microfoundation, we let the set of investors be 2-dimensional, $(i,k) \in [0,1]^2$.[47] As before, an investor (i,k) has information set $\mathcal{I}_{k,t}$, but is restricted to trade assets that come from island i. This partitioning of investors prevents full risk-sharing among investors, and gives a role for investor uncertainty about firm fundamentals to affect prices.[48]

Investors As explained in the main text, investors are one-period lived, but do observe past *public* information. Upon birth, each investor (i,k) is endowed with a basket of claims on a fraction of the revenues on island i. In addition to holding equity, investors can borrow from households within each period at the risk-free rate $R = 1$ and use this to fund firms on island i by purchasing securities. All equilibrium profits from these investments are assumed to be transferred back to households lump-sum. Each investor maximizes a CRRA utility function over his end-of-period wealth, $c^{1-\gamma}/(1-\gamma)$.[49]

Funding for firms We assume that firms can pledge a total fraction $\chi_t > 0$ of their revenues to investors.[50] Investors' intial endowments constitute a claim on one half of this, while the other half is being pledged to raise working capital, by issuing securities. Although we have fixed income securities in mind, e.g. corporate bonds, the technical steps below turn out to be slightly easier when firms sell equity-like claims on revenues. Henceforth we focus on equity, noting that completely analogous steps lead to essentially the same result when using debt-like securities.

In keeping with the static nature of the firms' decision problems, we restrict pledgability of revenues to the current period.[51] That is, we assume that firms mechanically (to avoid any adverse

[47] The only difference to the 1-dimensional set of investors in the main text is the notation. Anything we do above can be done with 2 or more dimensions of investors. We chose the simplest possible set in the main part of the paper.

[48] One might think about this kind of limited market participation as an "expert system" where for each island i there are "expert investors" that focus on investments on that island. In that sense, it is similar to the recent macro-finance literature around Brunnermeier et al. (2012).

[49] To be consistent with the main text where households receive the total profits of firms, we assume here that investors' end-of-period wealth is taxed away with a proportional consumption tax approaching 100%, and transferred to households. A proportional consumption tax ensures that investors' incentives are not distorted and hence investors behave as if there was no tax.

[50] The idea of limited pledgability is common among many macro and corporate finance models such as Tirole (2010). Notice that in practice, not all costs are funded by working capital. This fact is equivalent to a rise in χ_t, possibly above 1.

[51] Our model has a nonlinear learning mechanism which technically complicates any forward-looking behavior by

selection effects), sell half of all shares on the pledgable fraction of current period revenues[52]

$$\frac{1}{2}\chi_t X_{ij,t},$$

with $X_{ij,t}$ denoting the time t revenues of firm (i,j). We allow for aggregate shocks to χ_t to capture disturbances in the financial sector (which will be mapped into υ_t below). These disturbances could represent changes in the financial sector's ability to absorb risk, to lend, or to refinance. The payoff of all shares is realized at the end of the period.

Notice that for firms on constrained islands—islands where many firms need more than the proceeds from the security sales—the lack of sufficient net worth has similar consequences as in Kiyotaki and Moore (1997), albeit through a slightly different channel. Due to the financial constraints, these firms cannot produce as much as they would like, which decreases their revenues and hence also the payoffs from the securities, which tightens the constraints even further.

Pricing All firms within a certain island i have shares with identical payoff profiles conditional on an investor's information set. Therefore, investors perfectly hedge their investments *within the island* and invest an equal share of their wealth into each one of the firms. We can then write the utility maximization problem of an investor (i,k) living in period t as

$$\max_{\vartheta_k} \mathbb{E}_{k,t}\frac{c^{1-\gamma}}{1-\gamma}$$

for

$$c = \frac{1}{2}Q_{i,t} + \frac{1}{2}Q_{i,t}\vartheta_{ijk}\left(\frac{\chi_t \int X_{ij,t}\,\mathrm{d}j}{Q_{i,t}} - 1\right),$$

and where ϑ_k denotes the share of wealth $(Q_{i,t}/2)$ investor (i,k) invests into island i's firms' shares, and $Q_{i,t}$ denotes the price per share (that is common across firms). The first order condition of the investor's utility maximization problem is

$$\mathbb{E}_{k,t}c^{-\gamma}\left(\chi_t \int X_{ij,t}\,\mathrm{d}j - Q_{i,t}\right) = 0. \tag{26}$$

To avoid the asset price from revealing all information, there are noise traders with an inelastic, i.i.d. island-specific asset demand of $\eta_{i,t} \sim \mathcal{N}(\rho_\eta \eta_{i,t-1}, \sigma_\eta^2)$. The market clearing condition is then,

$$\frac{1}{2}\int \vartheta_k\,\mathrm{d}k + \eta_{i,t} = 1. \tag{27}$$

investors. Intuitively, we expect forward looking behavior to amplify the feedback mechanism that leads to belief traps in our model, since falling in a belief trap affects many future periods as well; and this can feed back into lower credit limits today if the credit rule is forward looking.

[52] Notice that no firm makes negative profits by being forced to sell assets. So selling half of all shares is strictly better than inactivity.

Credit limit From the proof of Proposition 1, revenues are given by

$$X_{ij,t} = \frac{\xi}{\xi - 1}\left(A_{ij,t}\min\{A_{ij,t}, \bar{A}_{i,t}\}^{\xi-1}\right)^{1-1/\xi}\Omega_t. \tag{28}$$

We think of the proceeds from selling the new securities as providing the firm with "credit". In this sense, the "credit limit" is related to the price raised by a firm on island i through (3):[53]

$$\frac{1}{2}Q_{i,t} = \bar{L}_{i,t} = \Omega_t \bar{A}_{i,t}^{\xi-1}. \tag{29}$$

Equations (26)–(29) describe an implicit mapping from island i's credit limit $\bar{A}_{i,t}$ to the set of investor beliefs about a firm's fundamentals on island i. Those beliefs are equal across all firms within an island i—so we can focus on any single firm (i, j)—and are described by the set of expectations $\{\mathbb{E}_{k,t}A_{ij,t}\}_k$ and the variance $\text{Var}_{k,t}A_{ij,t}$ (which is common across investors so we focus on some k here).

In this vein, we now introduce the following shortcut notations to improve the exposition. We denote beliefs (of various investors k) by $\mu_k \equiv \mathbb{E}_{k,t}A_{i,t}$ and $\sigma^2 \equiv \text{Var}_{k,t}A_{i,t}$. Further, we drop the subscripts from $\bar{A}_{i,t}$, $A_{ij,t}$, $Q_{i,t}$ and denote their logs with lower case $\bar{a}$, a_j, q. Also, define $x \equiv \log\int X_{ij,t}\,\mathrm{d}j$. Finally, we drop time subscripts from expectations $\mathbb{E}_{k,t}$, $\bar{\mathbb{E}}_t$ and the variance $\text{Var}_{k,t}$.

In the next section we derive a log-linearized solution to $\bar{a}$ as function of investor beliefs and show that it indeed yields our credit rule (4).

B.2 Deriving the credit rule approximation

We now formally linearize the (implicitly defined) function $\bar{a} = \bar{a}(\{\mu_k\}_k, \sigma^2, \log\chi_t)$ in the belief, around $\mu_k = \mu_0$ for some μ_0, around $\sigma^2 = 0$ and around $\log\chi_t = \log\chi_0$. We do this in a series of three steps. First we approximate investors' asset demands for small risks using the standard discrete time demand functions for CRRA utility (see, e.g., Campbell and Viceira, 2002, Section 2.1.3). Aggregating the individual asset demands and applying the market clearing condition, we can then write the price q as nonlinear function of beliefs. And finally, we linearize q around $\sigma^2 = 0$, $\mu_k = \mu_0$ and $\log\chi_t = \log\chi_0$, and relate q to $\bar{a}$ to derive the credit rule (4).

CRRA asset demand and asset price Approximating log investor utility as

$$\log\mathbb{E}_{k,t}\frac{c^{1-\gamma}}{1-\gamma} \approx (1-\gamma)\mathbb{E}_{k,t}\log c + \frac{1}{2}(1-\gamma)^2\text{Var}_{k,t}\log c$$

[53] An alternative, almost exactly equivalent, microfoundation is that firms have a certain loan-to-value cap until which they can take out loans, against an island-i collateral good.

we use the exact same steps as in Campbell and Viceira (2002, Section 2.1.3) to arrive at individual asset demands

$$\vartheta_k = \frac{\mathbb{E}_k x - q + \frac{1}{2}\mathrm{Var}_k x + \log\chi_t}{\gamma \mathrm{Var}_k x}.$$

Market clearing (27) lets us solve for an expression for the log price q,

$$q = \bar{\mathbb{E}} x - 2\gamma\left(1 - \frac{1}{4\gamma} - \eta_{i,t}\right)\mathrm{Var}\, x + \log\chi_t, \tag{30}$$

where we have adopted the information structure from Section 3.2.2 to ensure that $\mathrm{Var}_k x$ is constant across k.

Linearization of asset price Substituting in (28) and (29) we rewrite (30) in terms of $\bar{a}$,

$$\bar{a} = \underbrace{\frac{1}{\xi-1}\bar{\mathbb{E}}\left\{\log\int e^{(\xi-1)a_j}\min\{1, e^{\bar{a}-a_j}\}^{(\xi-1)^2/\xi}\,\mathrm{d}j\right\}}_{=\int \mu_k dk + \int H^{(1)}(\bar{a}-\mu_k,\sigma^2,\sigma_\epsilon^2)dk} + \frac{1}{\xi-1}\log\left\{\frac{\xi}{\xi-1}\frac{1}{2}\chi_t\right\}$$
$$- 2\gamma\left(1-\frac{1}{4\gamma}-\eta_{i,t}\right)\frac{1}{\xi-1}\underbrace{\mathrm{Var}\left\{\log\int e^{(\xi-1)a_j}\min\{1, e^{\bar{a}-a_j}\}^{(\xi-1)^2/\xi}\,\mathrm{d}j\right\}}_{=\int H^{(2)}(\bar{a}-\mu_k,\sigma^2,\sigma_\epsilon^2)dk}$$

where we used the fact that investor k believes that $a - \mu_k \sim \mathcal{N}(0,\sigma^2)$. We now linearize $\bar{a}(\{\mu_k\}_k, \sigma^2, \log\chi_t)$ around $\mu_k = \mu_0$ for some μ_0, $\sigma^2 = 0$, and $\log\chi_t = \log\chi_0$. Notice that the two H functions also depend on the within-island dispersion of productivities σ_ϵ, which is very small in our calibration. For simplicity, we therefore also linearize with respect to σ_ϵ around $\sigma_\epsilon = 0$. Since the value of μ_0 turns out to be irrelevant, we set it to zero. Letting $\bar{a}^*$ denote the solution at the linearization point $\bar{a}^* \equiv \bar{a}(\{\mu_0\}, 0, \log\chi_0)$, we find the following slopes of the linearized function,

$$\begin{aligned}
\bar{a}_{\mu_k} &= 1 \\
\bar{a}_{\sigma^2} &= -\left(1 - H_1^{(1)}\right)^{-1} H_2^{(2)} 2\gamma\left(1 - \frac{1}{4\gamma} - \eta_{i,t}\right) \\
\bar{a}_{\log\chi} &= \left(1 - H_1^{(1)}\right)^{-1}\frac{1}{\xi-1}
\end{aligned}$$

where the only two non-zero partial derivatives of $H^{(1)}$ and $H^{(2)}$ can be computed as[54]

$$H_1^{(1)} = \frac{1}{\xi}(\xi-1)1_{\{\bar{a}^*<0\}}$$

$$H_2^{(2)} = (\xi-1)\left(1_{\{\bar{a}^*<0\}}\frac{1}{\xi^2} + 1_{\{\bar{a}^*>0\}}\right).$$

[54] This holds for $\bar{a}^* \neq 0$, which is the case in our calibration.

Both expressions depend on whether that island is mainly constrained ($\bar{a}^* < 0$) or mainly unconstrained ($\bar{a}^* > 0$) at the linearization point. In line with our calibration where 2.5% of firms are constrained at the steady state, the calibrated value of χ_0 ensures that $\bar{a}^* > 0$. The slope $\bar{a}_{\sigma_\epsilon^2}$ with respect to σ_ϵ^2 is not important for the calibrated model as σ_ϵ^2 is constant (it enters the intercept π_0 in (31) below). The solution at the linearization point is given as the unique solution to[55]

$$\bar{a}^* = \frac{1}{\xi}(\xi - 1)\min\{0, \bar{a}^*\} + \frac{1}{\xi - 1}\log\left\{\frac{\xi}{\xi - 1}\frac{1}{2}\chi_0\right\},$$

or equivalently,

$$\bar{a}^* = \left(1 + 1_{\frac{\xi}{\xi-1}\frac{1}{2}\chi_0 < 1}(\xi - 1)\right)\frac{1}{\xi - 1}\log\left\{\frac{\xi}{\xi - 1}\frac{1}{2}\chi_0\right\}.$$

Putting the pieces together, the linearized credit rule is

$$\bar{a} = \int \mu_k \,\mathrm{d}k - \pi_\sigma \sigma^2 + \upsilon_t + \pi_\sigma \sigma^2 \eta_{i,t} + \pi_0, \tag{31}$$

where we collected terms as $\upsilon_t = \left(1 - H_1^{(1)}\right)^{-1} \frac{1}{\xi-1}\log\chi_t$, $\pi_\sigma = \left(1 - H_1^{(1)}\right)^{-1} H_2^{(2)} 2\gamma\left(1 - \frac{1}{4\gamma}\right)$, $\pi_0 = \bar{a}^* + \bar{a}_{\sigma_\epsilon^2}\sigma_\epsilon^2 - \left(1 - H_1^{(1)}\right)^{-1}\frac{1}{\xi-1}\log\chi_0$, and rescaled the standard deviation of $\eta_{i,t}$ by a factor of $\left(1 - \frac{1}{4\gamma}\right)^{-1}$. This is precisely the credit rule in (4).

To calibrate the key parameter in the credit rule, π_σ, we note that due to $\bar{a}^* > 0$ in our calibration,

$$\pi_\sigma = (\xi - 1)\gamma\left(2 - \frac{1}{2\gamma}\right).$$

For a modest relative risk aversion of $\gamma = 1.5$ and $\xi = 4$, this gives $\pi_\sigma = 7.5$.

C Data

C.1 Data used for calibration

Our calibration of the learning parameters is based on the "Summary History" for US firms from the Institutional Brokers Estimate System (IBES). We use forecasts about current quarter earnings per share (EPS), which are available starting in the third quarter of 1984. From the original sample, we exclude all forecasts that are recorded prior to the beginning of the current fiscal period and that are recorded less than 1 week before the forecast period ends. To reduce the sensitivity to outliers, we trim the 2% tails for each month and variable. Returns are obtained from the CRSP database and are adjusted for splits and dividends. To assess the predictive power of prices towards future earnings, we merge the two data sets so that returns at month $t - 1$ are matched to EPS realizations at month t.[56] The resulting panel contains on average 2053 firms per month.

[55] Notice that the derivative of the right hand side with respect to $\bar{a}^*$ is positive but strictly bounded above by 1.

[56] Due to small timing issues in the two data sets, the implemented lag varies by ±3 days, resulting in a total lag between 28 to 33 days.

For our calibration, we compare the pre-crisis model moments (computed at the stochastic steady state with $\bar{\upsilon}$ chosen so that 2.5 percent of all firms are constrained) to the available data prior to the financial crisis (1984Q3–2006Q4). To isolate the firm-specific components in our data series, we extract a time-fixed effects from each of them, except for $\sigma_{i,t}^{\text{cross}}$ (for which we exploit the sample mean in the calibration).[57]

C.2 Data used for introduction and financial crisis plot

Here we describe the data sources for the time series shown in Figures 1 and 10. We measure output, employment, and efficiency using real GDP (GDPC96), total nonfarm employment (PAYEMS) and TFP (RTFPNAUSA632NRUG) data from the St. Louis Fed's FRED database. All three series are detrended using a (6,32) band-pass filter. Credit spreads data is obtained from Gilchrist and Zakrajšek (2012), defining the average spread between corporate bonds and a hypothetical treasure security that mirrors the cash flow of the corporate bond. The data sources for credit spreads and labor wedges only cover the periods until 2011 and 2012, respectively. The measure for volatility is the VIX. Output growth dispersion is measured using the dispersion in sales growth rates in the COMPUSTAT database.

Finally, for investors' expectations, uncertainty, and disagreement, we use the final data set prepared for our calibration (see above), looking at the cross-sectional sample average in a given quarter.[58] The resulting time series are adjusted using a stationary seasonal filter to get rid of a strong seasonal trend. Based on our model, disagreement $(\sigma_{i,t}^{\text{cross}})^2$ is proportional to $\hat{\sigma}_{i,t}^4$. Accordingly, we use $\sigma_{i,t}^{\text{cross}}$ to proxy for uncertainty, and use $(\sigma_{i,t}^{\text{cross}})^2$ for the disagreement time series.

D Robustness specifications (for online publication)

In this appendix we explore the role of the learning parameters and the credit rule coefficient for the response of the model economy to an aggregate credit shock. For this purpose, we repeat our simulation in Section 5.2 of a geometrically decaying aggregate shock for various specifications. For each specification, we set $\bar{\upsilon}$ and Δ so that 2.5 percent of the firms are constrained in the steady state and 20 percent of the firms are constrained at the peak of the crisis akin to our baseline simulation. Figure 11 displays the resulting model responses for output, employment, uncertainty, and the cross-sectional growth dispersion.

Specifically, the figure shows four sets of specifications. First, the blue lines correspond to a ± 20 percent change in σ_p, reflecting a relative increase (dotted lines) or decrease (dashed lines) of the private investor signals relative to the benchmark case (solid black lines). Similarly, the green lines correspond to a ± 20 percent change in σ_η, whereas the red lines correspond to a simultaneous ± 20 percent change in all three learning parameters (σ_ψ, σ_p and σ_η). Finally, the magenta lines

[57]Formally, we subtract from each data series the cross-sectional average for a given month (equivalent to regressing each variable on a time dummy and working with the residuals).

[58]Of course, here we cannot control for time fixed effects.

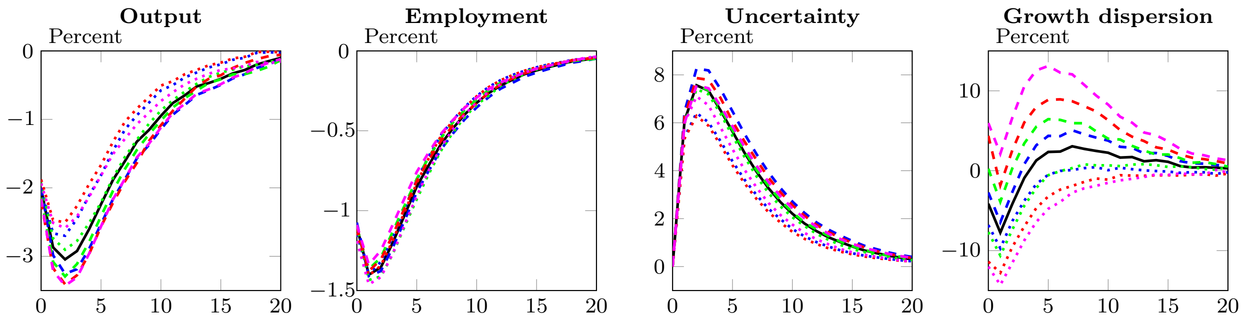

Figure 11: Impulse responses for output, employment, uncertainty, and the cross-sectional growth dispersion for alternative learning and credit rule parameters. The solid black lines correspond to the baseline parametrization. The dashed and dotted lines correspond to specifications with positive and negative changes in the parameters values, respectively. In particular, the blue lines correspond to a ± 20 percent change in σ_p; the green lines to a ± 20 percent change in σ_η; the red lines to a simultaneous ± 20 percent change in σ_ψ, σ_p and σ_η; and the magenta lines to a ± 40 percent change in π_σ. All impulse responses are in percentage deviations.

correspond to ± 40 percent changes in π_σ, reflecting values for the credit rule parameter based on our micro-foundation of the credit rule when investors have relative risk aversion of 1 and 2, respectively (see Appendix B).

It can be seen that variations in these parameters induce only small changes in the responses of output, employment, and uncertainty. The same holds true for all other model variables not depicted here. Specifically, looking at output, a variation in the relative signal precision (blue and green lines) results in responses that are within 0.5 and 0.25 percent point bands around the benchmark, respectively. Varying the informativeness of all signals at the same time, leads to slightly larger deviations from the benchmark response of maximal 0.65 percentage points. Changes in the credit rule coefficient lead to deviations from the benchmark response of maximal 0.5 percentage points.

In contrast to the small impact on the cross-sectional first moments, variations in the learning and credit rule parameters do have, however, important consequences for the cross-sectional dispersion of output growth. This can be seen in the forth panel of Figure 11. In particular, it can be seen that depending on the parametrization, the response in the growth dispersion may be both negative or positive throughout the whole impulse response path, or can be first decreased and the increased as is the case in our calibration.

References

Albagli, Elias. 2011. "Amplification of Uncertainty in Illiquid Markets." *mimeo.*

Arellano, Cristina, Yan Bai, and Patrick J. Kehoe. 2012. "Financial markets and fluctuations in uncertainty." *mimeo.*

Bachmann, Rüdiger, and Christian Bayer. 2009. "Firm-Specific Productivity Risk over the Business Cycle: Facts and Aggregate Implications." *mimeo.*

Bachmann, Rüdiger, and Christian Bayer. 2013. "'Wait-and-See' business cycles?" *Journal of Monetary Economics*, 60(6): 704–719.

Bachmann, Rüdiger, and Giuseppe Moscarini. 2011. "Business Cycles and Endogenous Uncertainty." *mimeo.*

Bachmann, Rüdiger, Steffen Elstner, and Eric R. Sims. 2013. "Uncertainty and Economic Activity: Evidence from Business Survey Data." *American Economic Journal: Macroeconomics*, 5(2): 217–49.

Ball, Laurence. 2014. "Long-Term Damage from the Great Recession in OECD Countries." *mimeo.*

Bernanke, Ben, Mark Gertler, and Simon Gilchrist. 1999. "The financial accelerator in a quantitative business cycle framework." In *Handbook of Macroeconomics.* Vol. 1, , ed. John B. Taylor and Michael Woodford, 1341–1393. Amsterdam, New York and Oxford:North-Holland.

Blanchard, Olivier. 2009. "(Nearly) nothing to fear but fear itself." *The Economist*, January 31: 84.

Bloom, Nicholas. 2009. "The Impact of Uncertainty Shocks." *Econometrica*, 77(3): 623–685.

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen Terry. 2014. "Really Uncertain Business Cycles." *mimeo.*

Brunnermeier, Markus K., and Yuliy Sannikov. 2012. "A macroeconomic model with a financial sector." *mimeo.*

Brunnermeier, Markus K., Thomas M. Eisenbach, and Yuliy Sannikov. 2012. "Macroeconomics with financial frictions: A survey." *mimeo.*

Caldara, Dario, Cristina Fuentes-Albero, Simon Gilchrist, and Egon Zakrajšek. 2013. "On the Identification of Financial and Uncertainty Shocks." *mimeo.*

Campbell, John Y, and Luis M Viceira. 2002. *Strategic asset allocation: portfolio choice for long-term investors.* Oxford University Press.

Campello, Murillo, John R. Graham, and Campbell R. Harvey. 2010. "The Real Effects of Financial Constraints : Evidence from a Financial Crisis." *Journal of Financial Economics*, 97(3): 470–487.

Carlin, Bruce I., Francis A. Longstaff, and Kyle Matoba. 2013. "Disagreement and asset prices." *mimeo.*

Chaney, Thomas, David Sraer, and David Thesmar. 2012. "The collateral channel: How real estate shocks affect corporate investment." *American Economic Review*, 102(6): 2381–2409.

Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan. 2007. "Business cycle accounting." *Econometrica*, 75(3): 781–836.

Chodorow-Reich, Gabriel. 2013. "The Employment Effects of Credit Market Disruptions: Firm-level Evidence from the 2008-9 Financial Crisis." *The Quarterly Journal of Economics*, 129(1): 1–59.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno. 2014. "Risk Shocks." *American Economic Review*, 104(1): 27–65.

Coibion, Olivier, and Yuriy Gorodnichenko. 2012. "What can survey forecasts tell us about informational rigidities?" *Journal of Political Economy*, 120(1): 116–159.

Coibion, Olivier, and Yuriy Gorodnichenko. 2014. "Information Rigidity and the Expectations Formation Process : A Simple Framework and New Facts." *American Economic Review*, forthcoming.

Cooley, Thomas F., and Vincenzo Quadrini. 2001. "Financial markets and firm dynamics." *American Economic Review*, 91(5): 1286–1310.

Cuyt, Annie A. M., Vigdis B. Petersen, Brigitte Verdonk, Haakon Waadeland, and William B. Jones. 2008. *Handbook of Continued Fractions for Special Functions.* Springer-Verlag.

David, Joel M., Hugo A. Hopenhayn, and Venky Venkateswaran. 2015. "Information, Misallocation and Aggregate Productivity." *working paper.*

Fajgelbaum, Pablo, Edouard Schaal, and Mathieu Taschereau-Dumouchel. 2015. "Uncertainty traps." *working paper.*

Fostel, Ana, and John Geanakoplos. 2008. "Leverage cycles and the anxious economy." *American Economic Review*, 98(4): 1211–1244.

Fostel, Ana, and John Geanakoplos. 2015. "Leverage and Default in Binomial Economies: A Complete Characterization." *Econometrica*, forthcoming.

Gabaix, Xavier. 2012. "Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance." *Quarterly Journal of Economics*, 127(2): 645–700.

Gilchrist, Simon, and Egon Zakrajšek. 2012. "Credit Spreads and Business Cycle Fluctuations." *American Economic Review*, 102(4): 1692–1720.

Gilchrist, Simon, Jae W. Sim, and Egon Zakrajšek. 2014. "Uncertainty, Financial Frictions, and Investment Dynamics." *mimeo.*

Giroud, Xavier, and Holger M. Mueller. 2015. "Firm Leverage and Unemployment during the Great Recession." *working paper.*

Gourio, François. 2012. "Disaster Risk and Business Cycles." *American Economic Review*, 102(6): 2734–2766.

Hall, Robert E. 2014. "Quantifying the Lasting Harm to the US Economy from the Financial Crisis." *NBER Macroeconomics Annual*, 29: 1–56.

Hellwig, Martin F. 1980. "On the aggregation of information in competitive markets." *Journal of Economic Theory*, 22(3): 477–498.

Khan, Aubhik, and Julia K. Thomas. 2013. "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity." *Journal of Political Economy*, 121(6): 1055–1107.

Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit cycles." *Journal of Political Economy*, 105(2): 211–248.

Kozeniauskas, Nicholas, Anna Orlik, and Laura L. Veldkamp. 2014. "Black Swans and the Many Shades of Uncertainty." *mimeo.*

Kozlowski, Julian, Laura L. Veldkamp, and Venky Venkateswaran. 2015. "The Tail that Wags the Economy: Belief-Driven Business Cycles and Persistent Stagnation." *working paper.*

Moll, Benjamin. 2014. "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?" *American Economic Review*, 104(10): 3186–3221.

Nimark, Kristoffer P. 2014. "Man-Bites-Dog Business Cycles." *American Economic Review*, 104(8): 2320–2367.

Ordoñez, Guillermo. 2013. "The Asymmetric Effects of Financial Frictions." *Journal of Political Economy*, 121(5): 844–895.

Reinhart, Carmen M., and Kenneth S. Rogoff. 2009. "The Aftermath of Financial Crises." *American Economic Review: Papers & Proceedings*, 99(2): 466–472.

Senga, Tatsuro. 2015. "A New Look at Uncertainty Shocks: Imperfect Information and Misallocation." *mimeo.*

Siemer, Michael. 2014. "Firm Entry and Employment Dynamics in the Great Recession." *working paper.*

Simsek, Alp. 2013*a*. "Belief Disagreements and Collateral Constraints." *Econometrica*, 81(1): 1–53.

Simsek, Alp. 2013*b*. "Speculation and risk sharing with new financial assets." *Quarterly Journal of Economics*, 128(3): 1365–1396.

Sockin, Michael. 2015. "Not so Great Expectations: A Model of Growth and Informational Frictions."

Stein, Luke C.D., and Elizabeth C. Stone. 2013. "The effect of uncertainty on investment, hiring, and R&D: Causal evidence from equity options." *mimeo.*

Stock, James H., and Mark W. Watson. 2012. "Disentangling the Channels of the 2007-2009 Recession." *Brookings Papers on Economic Activity*, 81–156.

Straub, Ludwig, and Robert Ulbricht. 2012. "Credit Crunches, Information Failures, and the Persistence of Pessimism." *mimeo.*

Straub, Ludwig, and Robert Ulbricht. 2014. "Variance Transformation Theorems and Applications." *mimeo.*

Tirole, Jean. 2010. *The theory of corporate finance.* Princeton University Press.

Townsend, Robert M. 1983. "Forecasting the forecasts of others." *The Journal of Political Economy*, 91(4): 546–588.

van Nieuwerburgh, Stijn, and Laura L. Veldkamp. 2006. "Learning asymmetries in real business cycles." *Journal of Monetary Economics*, 53(4): 753–772.

Veldkamp, Laura L. 2005. "Slow boom, sudden crash." *Journal of Economic Theory*, 124(2): 230–257.

Yuan, Kathy. 2005. "Asymmetric Price Movements and Borrowing Constraints: A Rational Expectations Equilibrium Model of Crises, Contagion, and Confusion." *The Journal of Finance*, 60(1): 379–412.