Specific and General Types of Human Capital

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Abstract

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We define specific (general) human capital as the set of occupations whose use is spread in a limited (wide) set of industries. Using EU Labor Force Survey we identify these human capital types and analyze their employment and education. This exercise yields persistent assignment of occupations into specific and general human capital types. The share of specific human capital has considerable variation across countries and has declined over time almost everywhere. In a stylized model, we assess the effect of uncertainty on human capital portfolio composition and the effect of the composition on propagation of sectoral shocks.

Keywords: Specific and General Human Capital Types; Sectoral Shocks; Output Volatility

JEL classification: E24; E30; I20; J23; J24; O41

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1 Introduction

In this paper, we utilize a novel way to horizontally differentiate across skill types in order to analyze the impact of human capital composition on aggregate economic performance. Similarly to Vourvachaki et al. (2014), we define two distinct types of human capital: "general" and "specific." As general human capital, we define a set of skills that enable individuals to perform generic tasks that are required for production in a wide range of industries (e.g., services skills of managers, manual skills of cleaners). In contrast, specific human capital is defined as a set of skills that enable one to perform highly specialized tasks in a few industries (e.g., the cognitive skills of doctors, manual skills of craft workers).

We use the EU Labor Force Survey (EU LFS) database to identify specific and general human capital types and to analyze their employment and education fields and levels. The empirical exercise yields remarkably stable assignment of occupations into specific and general human capital types. For example, according to the assignment, occupations such as Market-oriented Skilled Agricultural and Fishery Workers and Extraction and Building Trades Workers correspond to specific human capital in almost all countries and years in our sample. In turn, occupations such as General Managers and Sales and Services Elementary Occupations correspond to general human capital in almost all countries and years.

The share of the individuals employed in specific human capital occupations varies between and within countries. Moreover, it has declined over time almost everywhere. Both between and within industry shifts have contributed to this negative trend. Approximately 64 percent of variation in the share of specific human capital is because the share of almost all industries, which use specific human capital more intensively, has declined over time. The remaining variation stems from more intensive use of general human capital in almost all industries. Industries which use specific human capital very intensively are, for example, Agriculture, Hunting, and Fishing; Education; and Construction. In turn, industries which do not use it intensively are Financial Intermediation; Transport, Storage, and Communication; and Hotels and Restaurants. The ranking of industries according to their intensity of use of specific human capital is remarkably stable across countries and years.

Regarding the education of these human capital types, the graduates of education fields such as Teacher Training and Education Science; Health and Welfare; and Agriculture and Veterinary are usually employed in specific human capital occupations. The graduates of education fields such as Social Sciences, Business, and Law; Services; and Science, Mathematics, and Computing usually have general human capital occupations. The level of education (skills) is very similar across the specific and general human capital types, which agrees with our horizontal differentiation of skills.

We build a stylized multi-sector model to illustrate the distortions which arise if there are unanticipated shocks and how this horizontal differentiation of human capital can matter for aggregate economic performance. We assume that general human capital is required for production in two sectors and is mobile across these sectors to capture the inherent flexibility of general human capital in the model. We call these sectors h and l and their outputs Y_h and Y_l. Specific human capital is required for production in sector h only. The outputs of these sectors are aggregated into consumption goods with a CES function. Sectors h and l are subject to i.i.d. shocks which happen in between hiring and compensating inputs. Our analysis focuses on the effects of the variances of these shocks (uncertainty) on the demand for and supply of human capital types and on the effect of human capital portfolio composition on prorogation of these shocks to final output fluctuations.

The elasticity of substitution between Y_h and Y_l turns to be important for our analysis. For brevity, we summarize our results for the case when Y_h and Y_l are gross-complements. The opposites of these results hold when they are gross-substitutes.

We show that the share of general human capital allocation to sector h increases (declines) with the volatility of shocks to sector h (l). The relative inverse demand for (i.e., relative expected wage rate of) general human capital declines (increases) with the volatility of shocks to sector h (l).

We consider two countries which have different endowments of specific and general human capital types, but have the same expected output. In the country which has higher amount of specific human capital the volatility of shocks to sector h (l) has lower (higher) contribution to the volatility of final output. Therefore, the volatility of final output in this country is higher if, for example, there are no shocks to sector h.

Further, we turn off one of the sectoral shocks and consider a planner which has an option to marginally increase either the amount of general human capital or specific human capital at no cost/at the same cost. If the planner increases the amount of general human capital then the volatility of final output does not necessarily increase less than if it increases the amount of specific human capital. To make the trade-offs more comparable, we impose a condition that the marginal increase in either of human capital types should also deliver the same marginal increase in expected final output. In such a case, if the planner increases the amount of general human capital then the volatility of final output increases more (less) than increasing the amount of specific human capital if there are no shocks to sector l (h). Clearly, if the planner has a concave objective function then it would prefer investing in specific human capital if there are no shocks to sector l. In this respect, it would under-invest in specific human capital if it does not anticipate shocks to sector h.

There is a large body of empirical evidence documenting that output volatility in many countries has declined in recent years (see, for example, Stock and Watson, 2005, and the references therein). According to our theoretical inference, the secular decline in the share of specific human capital can be one of the factors contributing to the documented trends in output volatility.

Finally, we close the model in a trivial manner and assume that the economy is populated by one-period lived identical households. The representative household has concave utility from consumption of final goods. At the beginning of the period the household needs to decide how much specific and general human capital to acquire given the expected wage rates. The costs of acquiring both types skills are in terms of final goods.

According to the inference for inverse demands, the household will decrease (increase) its relative supply of general human capital and increase (decrease) the allocation of general human capital to sector h if the volatility of shocks to sector h (l) increases. Therefore, if the shocks to sector h cannot be anticipated, then the household will over-invest in general human capital and allocate less than optimal amount of general human capital to sector h.

Our paper is related to studies which horizontally differentiate among types of skills and examine the role of such differences for economic outcomes (e.g., Hanushek et al., 2011; Acemoglu and Autor, 2011; Autor and Dorn, 2013). It also broadly relates to studies that examine the intra- and inter-temporal trade-offs between different types of human capital in environments with uncertainty, introduction of new technologies, or trade. Such mechanisms are analyzed in Autor and Dorn (2013), Krueger and Kumar (2004a,b), Gould et al. (2001), and Hummels et al. (2014), among others. There is also a large number of studies which assess whether human capital is firm, industry, and/or occupation specific (e.g., Helwege, 1992; Neal, 1995; Kambourov and Manovskii, 2009a,b; Sullivan, 2010; Ritter, 2014). From the perspective of these latter studies, specific human capital can be called industry-specific human capital. In contrast to these latter studies, however, we focus on relative transferability of skills across industries using concentration measures rather than changes in earnings. In the model, we assume that specific human capital is not transferable across industries. We contribute to all these groups of studies by introducing a novel way for horizontally differentiating among types of skills and analyzing the effect of human capital portfolio composition in terms of these skills on economic performance. We also contribute by identifying skills for which industry and occupational specificity can be rather hard to distinguish.

Gervais et al. (2008) is one of the closest studies to ours in terms of the theoretical analysis. Gervais et al. (2008) considers an economy where firms hire firm-specific and general human capital as in Becker (1962). Firms are subject to idiosyncratic productivity shocks and receive signals about next period values shocks. Firm-specific human capital is more productive but cannot be hired/fired after the shocks. General human capital can be hired and fired. Gervais et al. (2008) show that output is higher in economies with higher precision of signal because the amount of specific human capital is higher. However, output declines more in these economies after an unexpected decline in the precision of the signal. The latter effect arises because unexpected decline of the precision in their framework does not alter human capital portfolio composition and implies higher misallocation of resources in these economies. Contrary to Gervais et al. (2008), we do not assume that specific human capital is necessarily more productive than general human capital. Moreover, we focus on distortions which arise if shocks are unanticipated and on assessing propagation of sectoral shocks in a stylized economy where factor inputs are not mobile after shocks. In additional results section, we also consider an economy where general human capital can be reallocated after the shocks and analyze the dependence of the elasticity of final output with respect to sectoral shocks on human capital portfolio composition. The results we derive are similar to our earlier results for the variance of final output. However, they are for certain limiting values of the elasticity of substitution between sectoral outputs.

A large number of studies theoretically examines the determinants of aggregate volatility (e.g., see Acemoglu and Zilibotti, 1997; Horvath, 2000; Koren and Tenreyro, 2013). These studies have highlighted the importance of, for example, financial, sectoral, and product variety diversification. Our paper illustrates the possible importance of human capital portfolio composition, mobility of factor inputs, and substitutability of sectoral/industrial output.[1]

The paper is organized as follows. Section 2 discusses the composition of specific and general human capital. Section 3 presents the model and its results. Section 4 concludes.

2 Specific and General Human Capital

We treat each occupation as a set of skills which enable the performance of tasks that are necessary as a part of the production process. In this respect, occupations define the labor services input in the production in each industry. To the extent that industries differ in their technological needs in terms of the types of labor services, their demand for occupations would also be different. We classify an occupation as "specific human capital" if it is used in a limited set of industries, i.e., its employment share exhibits a high degree of concentration across industries. Accordingly, we classify an occupation as "general human capital" if it is used in the production of a wide variety of products, i.e., its employment share has a high degree of dispersion.

Ideally, we would need data on industries' technological demand for occupations to identify the degree of an occupation's "industry specificity." In the data, however, we commonly observe the demand and supply together. Given that our classification is

[1] As it will become apparent in the model section, our results are not specific to human capital portfolio composition. They apply to the portfolio of mobile assets/factors in general.

based on relative concentrations of occupations across industries, actually it is sufficient to have that industries' demands for different occupations and the supplies of these skills to any particular industry are not disproportionately affected by frictions and distortions in the economy, if any. Hereafter, we assume that this is the case.

We employ data from the EU LFS database (yearly files of 2012 issue) to identify specific and general human capital types and summarize how they are used and produced in EU countries. We retrieve from this database information on the number of people in labor force in each country and year, their occupation in the main job (2-digit ISCO-88), the industry in which they are employed (1-digit NACE Rev. 1), and their education level and field of education (1-digit ISCED-97). Table 6 offers our sample of countries and years.

Using these data we compute the number of individuals employed in each occupation-industry cell for each country and year. From this matrix, we derive the distributions of *within-occupation employment share across industries, within-industry employment share across occupations*, and *total employment shares* by occupation. Table 1 reports the results from an ANOVA exercise for within-occupation employment share. The variation of within-occupation employment share is mostly driven by industry and occupation differences, while time and country differences are less important. Table 2 reports within-occupation employment share for each occupation and industry, where we take the averages over countries and years. Figure 1 illustrates the trends in the share of employment in each occupation, where we take country-level and year- and country-level averages (see also Table 25 in Data Appendix).

For each country and year, we use the distribution of within-occupation employment shares to calculate five concentration statistics: coefficient of variation (CV), and Herfindahl (HI), Gini, Theil, and generalized entropy (with parameter 2; GE) indices. According to simple ANOVA exercises, the concentration measures vary significantly across occupations but are remarkably stable across countries and years. Moreover, the rank correlations among the concentration measures are almost perfect (see Tables 19-24 in Data Appendix).

We average each of these concentration measures over countries and years. For each of the averages, we create a dummy variable which equals 1 for higher than median values of the averaged concentration measures. An occupation is classified as specific human capital if the average of these five dummy variables is greater than 0.5 and as general otherwise. This ranking of the different occupations is offered in Table 3 together with the values of averaged concentration measures.

For example, our classification suggests that occupations such as Market-oriented Skilled Agricultural and Fishery Workers and Extraction and Building Trades Workers correspond to specific human capital. In turn, occupations such as General Managers and Sales and Services Elementary Occupations correspond to general human capital. This ranking is stable across countries and years, with very few exceptions, in line with ANOVA exercises and rank correlations. To illustrate this, we perform a similar assignment within countries using time averaged concentration measures and within years using country averages. Table 4 reports the number of times that an occupation is assigned into specific human capital type in the sample of countries and in the sample of years.[2]

We use this assignment to calculate the share of individuals employed in specific human capital occupations out of total employment in each country and year in our sample. We call this simply "the share of specific human capital" and present the results in Figure 2. The share of specific human capital displays considerable time and country variation (see also Table 5 for an ANOVA exercise). Country-level basic statistics for the share of specific human capital are offered in Table 6 (see Table 26 for correlations).

The share of specific human capital displays negative trend in almost all countries.[3] We average it over the countries and offer the results in Figure 3. On average, the share of specific human capital has declined in the countries in our sample by approximately 6 percentage points during the period of 1992–2010.[4]

We compute the share of specific human capital in industries to assess its use. Table 7 offers the share of specific human capital in each industry, where we take country and time averages. For example, specific human capital is very intensively used in industries Agriculture, Hunting, and Fishing (1-digit NACE A-B) and Construction. It is not used intensively in industries Transport, Storage, and Communication, and Financial Intermediation. Figure 4 plots the share of specific human capital in each industry, where we take country and country-year averages. Tables 8 and 9 present time-averaged share of specific human capital in each country and industry and country-averaged share in each year and industry. The ranking of industries according to their use of specific human capital appears to be quite stable over time and across countries, which is further confirmed with country-level rank correlations as reported in Table 27 in Data Appendix.

The country-level trends in the share of specific human capital can be decomposed into changes in the employment shares of industries (between-industry) and industry-level changes in the shares of specific human capital (within-industry). Let $\omega^s_{c,t}$ and $\omega^s_{c,i,t}$ be the shares of specific human capital in country c at time t and in industry i in country c at time t. In turn, let $\omega_{c,i,t}$ be the share of employment in industry i in country c at

[2]We perform also similar assignment for each country and year. In line with the results in Table 4, this assignment has very little variation across countries and years. It is also highly correlated with our original assignment ($\rho = 0.911$).

[3]The EU LFS database is based on stratified sampling. We use the available sample weights in all our calculations. However, these weights might not be very precise for the level of disaggregation we are interested in. This can be responsible for some of the variation (e.g., breaks) in the share of specific human capital in Figure 2.

[4]Kambourov and Manovskii (2008) find that industry mobility of workers has increased over time in the US. If the share of specific human capital displays similar negative trend in the US, then that negative trend could be a potential explanation for the higher industry mobility.

time t. We have that

$$\Delta\omega^s_{c,t} = \sum_i \bar{\omega}^s_{c,i}\Delta\omega_{c,i,t} + \sum_i \bar{\omega}_{c,i}\Delta\omega^s_{c,i,t}, \quad (1)$$

where we use Δ to denote first difference operator and bar to denote the average over two consecutive periods.

We perform this decomposition for each country in the sample and average the changes in the share of specific human capital and its between- and within-industry components across countries and years. The average yearly change in the share of specific human capital is equal to -0.0042. In turn, the between- and within-industry components are equal to -0.0027 and -0.0015, correspondingly. This means that on average the share of industries which use specific human capital more intensively has declined over time (64 percent of variation) and industries have started using general human capital more intensively (36 percent of variation).[5]

These between- and within-industry components have non-trivial variation in sample countries. Table 10 offers the basics statistics of results from this decomposition for each country in the sample (see for decomposition for each sample year Table 28 Data Appendix). For example, the mean of within-industry component is not uniformly negative across countries. It is positive and relatively large for France, which indicates that industries in France have increased their specific human capital intensity over time.

Further, we retrieve from the EU LFS database information on the workers' fields of studies for the highest degree (1-digit ISCED-97) and their levels of education. The levels of education are classified into three groups: pre-primary to lower-secondary (low-level; ISCED-97 0-2), secondary to post-secondary and non-tertiary (medium-level; ISCED-97 3-4), and tertiary (high-level; ISCED-97 5-6). These data are used to identify how the workers' background in terms of educational field maps onto occupations in the labor market and the education levels of specific and general human capital types.

We calculate the number of workers in each occupation-education field cell and the within-education field share of workers across different occupations. This share varies a lot with occupations and shows very little variation over time and countries according to Table 11. We sum this share across specific human capital occupations and across general human capital occupations and average these sums across countries and years. Table 12 offers for each education field the share of workers who have specific human capital occupation and their highest education degree in that field out of total number of employed individuals who have their highest degrees in that field. More than 70 percent of the graduates of education field Teacher Training and Education Science are employed in specific human capital occupations. Meanwhile, less than 20 percent of the graduates of

[5]Around half of between-industry component is attributable to the decline of Agriculture, Hunting, and Fishing industry. Within-industry component is quite similar across industries.

education field Social Sciences, Business, and Law are employed in specific human capital occupations.[6] These disparities in shares suggest that the differences of skills between general and specific human capital types are not solely associated with differences in occupations. They can be also associated with formal education and, especially, with the fields of education.

We list the levels of education/skills for all occupations in Table 13. This table offers the share of employed individuals in each "level of education"-occupation cell out of total number of individuals in each occupation, which we have averaged over countries and years.[7] Table 14 offers basic statistics for the levels of skills across specific and general human capital occupations. The distribution of skill-levels across the two types of human capital is such that no human capital type is singled out as exclusively low- or highly skilled. As an illustration, 70 percent of workers who have specific human capital have completed the pre-primary to lower-secondary education, while 27 percent are graduates of tertiary education, as opposed to 75% and 25% respectively for the workers with general human capital. These differences are very minor, but are statistically significant.[8]

The EU LFS database also provides us with information about employed individuals' professional status at the job in terms of being self-employed, employee, and family worker, if they have second jobs, and their age and gender. For each of category of these variables we compute the share of employed individuals in specific human capital occupations out of total number of employed individuals in specific human capital occupations. We do the same for general human capital occupations. Table 15 provides basic statistics for these shares and tests the differences in means. The differences tend to be very small but statistically significant. For example, among individuals who have specific human capital the share of self-employed is slightly higher and the share of employees is slightly lower than among individuals who have general human capital. Moreover, the share of individuals who have more than 1 jobs and the share of females are slightly lower among individuals who have specific human capital than among individuals who have general human capital.

For each country and year, we compute employment shares in industries where the share of specific human capital is higher than the 50th, 70th, and 90th percentiles of its

[6]Tables 29, 30, and 31 in Data Appendix offer the share of workers in each education field for sample countries and sample years and the share of workers in each education-occupation cell out of total number in education fields, averaged across countries and time.

[7]Tables 32 and 33 in Data Appendix offer for each country and for each year the shares of employed individuals in each level of highest attained education out of total number of employed individuals which we have averaged across years and countries correspondingly.

[8]For the majority of sample countries we can use 3-digit disaggregation of occupations. We prefer 2-digit disaggregation since it allows us to focus on sets of skills which are neither very broad nor narrow so that they can be used in many industries and have relatively high scope of difference. We perform similar assignment for 1-digit and 3-digit occupations and report the results in Tables 34 and 36 in Data Appendix. Similarly to 2-digit occupations, we observe negative trends and uniform levels of education (see Figures 5 and 6 and Tables 35 and 37 in Data Appendix).

industry-level distribution in the country and year. Table 16 offers correlations of the employment shares in these industries with the share of specific human capital. Table 17 offers the results from regressions where the dependent variable is the share of specific human capital and explanatory variables are the employment shares in these industries. The employment shares in these industries appear to be important explanatory variables and account for approximately 70 percent of the variation in the share of specific human capital.[9] Finally, we run simple OLS regressions where the dependent variables is the logarithm of real GDP per-capita and the main explanatory variable is the share of specific human capital. The data for GDP per capita we obtain from the WDI database. The results are presented in Table 18. The share of specific human capital and GDP per capita appear to be strongly negatively correlated.[10]

The Model

General human capital is used in production in a wide range of industries, whereas the use of specific human capital is concentrated in a few industries. In this sense, general human capital is mobile and can serve as an *exante* insurance against future industry-and sector-level shocks.[11] According to Figure 2, at any point in time there is considerable variation across countries in terms of the human capital portfolio composition. Countries then differ in terms of relative abundance of the immobile factor input. In turn, according to Figures 2 and 3, the share of immobile factor input has changed over time and has declined on average.

In this section, we offer a simplistic model where we write the production side so that to capture the inherent flexibility of general human capital and inflexibility of specific human capital in terms of employment in different sectors. We analyze the effect of uncertainty on allocations of general human capital and relative demand for general human capital. We identify the distortions which arise in case shocks are not anticipated.

We also analyze the effect of human capital portfolio composition on propagation of sectoral shocks to final output fluctuations. Finally, we close the model in a trivial manner and discuss potential determinants behind the negative trend in the share of specific human capital.

There are two intermediate goods sectors: h and l. These sectors produce homogenous

[9] Similar inference holds if we smooth the series of the share of specific human capital using third degree polynomial approximation.

[10] By definition and identification our classification of human capital types is different than the abstract, manual, and routine skills classification used by Autor and Dorn (2013). Nevertheless, we check if the trends observed for the share of specific human capital repeat for the shares of employment in occupations requiring abstract, manual, and routine skills in Appendix - Abstract, Manual, and Routine Skills.

[11] To back this up, ideally we would need to have either no switches across occupations or that switches are costly. Kambourov and Manovskii (2008, 2009b) and Longhi and Brynin (2010) provide evidence of large occupational tenure and limited occupational mobility.

h-goods (Y_h) and l-goods (Y_l). The producers of the h-goods have a nested-CES production function. Their inputs are specific and general types of human capital (H_s, H_g) and K, which we call physical capital. In turn, the producers of l-goods have a single input of general human capital.[12] The final goods producers have a CES production function. Their inputs are h-goods (Y_h) and l-goods (Y_l) and they produce homogenous goods (Y). Perfect competition prevails in all markets, and all producers maximize their instantaneous profits. For the time being, we keep all factor inputs in a fixed supply.

The production function of the representative final goods producer is

$$Y = \lambda \left[\gamma_1 Y_h^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1 - \gamma_1) Y_l^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{2}$$

where $\varepsilon_1 > 0$ is the elasticity of substitution between h- and l-goods, $\gamma_1 \in (0, 1)$, and λ is a random variable with log-normal distribution $\ln \mathcal{N}(\mu_z, \sigma_z^2)$. We set the price of final goods as the numeraire and denote the prices of the h- and l-goods by p_{Y_h} and p_{Y_l}. From the usual profit maximization problem it follows then that the (inverse) demand functions for h- and l-goods are given by

$$p_{Y_h} = \omega_{Y_h}^{Y} \frac{Y}{Y_h}, \tag{3}$$

$$p_{Y_l} = \left(1 - \omega_{Y_h}^{Y}\right) \frac{Y}{Y_l}, \tag{4}$$

where $\omega_{Y_h}^{Y}$ is the share of Y_h compensation:

$$\omega_{Y_h}^{Y} = \frac{\gamma_1 Y_h^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\gamma_1 Y_h^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1 - \gamma_1) Y_l^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}. \tag{5}$$

The production function of the representative h-goods producer is

$$Y_h = \lambda_h \left[\gamma_2 \left(u_h^g H_g\right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + (1 - \gamma_2) Y_m^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}}, \tag{6}$$

where

$$Y_m = \left[\gamma_3 K^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + (1 - \gamma_3) H_s^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}}, \tag{7}$$

u_h^g is the share of general human capital in h-sector, and γ_2, $\gamma_3 \in (0, 1)$. In this nested-CES production function, $\varepsilon_2 > 0$ is the elasticity of substitution between Y_m and general human capital. It describes the elasticity of substitution between physical capital and general human capital and the elasticity of substitution between specific and general

[12] According to Table 7, sector h can be thought to be comprised of 1-digit NACE Rev. 1 industries A-B, C, D, E, F, G, L, M, N, and Q and sector l of the reminder.

human capital. For brevity, we will say that ε_2 *is the elasticity of substitution between the pairs* K *and* H_g *and* H_s *and* H_g. In turn, $\varepsilon_3 > 0$ is the elasticity of substitution between physical capital and specific human capital. The shocks to h-goods production are given by λ_h, which has a log-normal distribution $\ln \mathcal{N}\left(\mu_{z_h}, \sigma^2_{z_h}\right)$.

We denote the wage rates of specific and general types of human capital by w_s and w_g and the return on K by r. The h-goods producer's (inverse) demand functions for physical capital and specific and general human capital are given by

$$r = \omega^{Y_h}_{Y_m} \omega^{Y_m}_{K} \frac{p_{Y_h} Y_h}{K}, \tag{8}$$

$$w_s = \omega^{Y_h}_{Y_m} \left(1 - \omega^{Y_m}_{K}\right) \frac{p_{Y_h} Y_h}{H_s}, \tag{9}$$

$$w_g = \left(1 - \omega^{Y_h}_{Y_m}\right) \frac{p_{Y_h} Y_h}{u^g_h H_g}, \tag{10}$$

where $\omega^{Y_h}_{Y_m}$ and $\omega^{Y_m}_{K}$ are shares:

$$\omega^{Y_h}_{Y_m} = \frac{(1-\gamma_2)\, Y_m^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}{\gamma_2 \left(u^g_h H_g\right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + (1-\gamma_2)\, Y_m^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}, \tag{11}$$

$$\omega^{Y_m}_{K} = \frac{\gamma_3 K^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}{\gamma_3 K^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + (1-\gamma_3)\, H_s^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}. \tag{12}$$

The representative l-goods producer has the following production technology

$$Y_l = \lambda_l \left(u^g_l H_g\right)^{\gamma_4}, \tag{13}$$

where u^g_l is the share of general human capital in l-sector and $\gamma_4 \in (0,1)$. The shocks to l-goods production are given by λ_l, which has a log-normal distribution $\ln \mathcal{N}\left(\mu_{z_l}, \sigma^2_{z_l}\right)$.

The usual profit maximization problem implies that l-goods producer's (inverse) demand function for general human capital is given by

$$w_g = \gamma_4 \frac{p_{Y_l} Y_l}{u^g_l H_g}. \tag{14}$$

Any profits are distributed to the households, which are discussed at the end of the section. Firms hire inputs before observing the values of the shocks. They compensate inputs after the realization of the shocks.

In equilibrium, the shares of general human capital across the sectors sum to one

$$1 = u^g_h + u^g_l. \tag{15}$$

Moreover, the expected wage rates of general human capital should be equal across hand l-sectors. Therefore, from (10), (14), (3), and (4) it follows that

$$\mathbb{E}\left[\frac{p_{Y_l}Y_l}{u_h^g H_g}\left[\gamma_4\frac{u_h^g}{u_l^g}-\left(1-\omega_{Y_m}^{Y_h}\right)\frac{\gamma_1}{1-\gamma_1}\left(\frac{Y_h}{Y_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\right]=0.$$

This equation can be rewritten in the following way

$$\gamma_4\frac{u_h^g}{u_l^g}=\left(1-\omega_{Y_m}^{Y_h}\right)\frac{\mathbb{E}\left[\frac{\gamma_1}{1-\gamma_1}\left(\frac{Y_h}{Y_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\lambda_l\left[\frac{\gamma_1}{1-\gamma_1}\left(\frac{Y_h}{Y_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}+1\right]^{\frac{1}{\varepsilon_1-1}}\right]}{\mathbb{E}\left[\lambda_l\left[\frac{\gamma_1}{1-\gamma_1}\left(\frac{Y_h}{Y_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}+1\right]^{\frac{1}{\varepsilon_1-1}}\right]}. \tag{16}$$

To keep things simple, we assume that the last term in this equation is equal to

$$\mathbb{E}\left[\frac{\gamma_1}{1-\gamma_1}\left(\frac{Y_h}{Y_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right].$$ [13]

Further, we use (6) and (13) to rewrite this equation as

$$\left(u_h^g\right)^{1-\frac{\varepsilon_2-1}{\varepsilon_2}}\left(1-u_h^g\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4-1}=\mathbb{E}\left[\left(\frac{\lambda_h}{\lambda_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\frac{\gamma_2}{\gamma_4}\frac{\gamma_1}{1-\gamma_1}\left(H_g\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}-\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4} \tag{17}$$
$$\times\left[\gamma_2\left(u_h^g H_g\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}+\left(1-\gamma_2\right)Y_m^{\frac{\varepsilon_2-1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}\frac{\varepsilon_1-1}{\varepsilon_1}-1}.$$

The expression in (17) characterizes the equilibrium share of general human capital in sector h (u_h^g). The following proposition describes the behavior of u_h^g in response to changes in K, H_s, and H_g.

Proposition 1. *1. The share of general human capital in sector h (u_h^g) declines with K and H_s when $\varepsilon_2 > \varepsilon_1$ and increases with them when $\varepsilon_1 > \varepsilon_2$.*

2. It increases with H_g when $\varepsilon_2 > \varepsilon_1 > 1$ or $\varepsilon_2 > \varepsilon_1$ and $\gamma_4 = 1$ and declines with it when $1 > \varepsilon_1 > \varepsilon_2$ or $\varepsilon_1 > \varepsilon_2$ and $\gamma_4 = 1$.

Proof. See Proofs Appendix. □

For example, when $\varepsilon_2 > \varepsilon_1$, h- and l-goods are less substitutable than the pairs K and H_g and H_s and H_g in the production of h-goods. The share of general human capital in sector h (u_h^g) declines with K and H_s because of this.

In (17) shocks λ_h and λ_l enter into the expected value operator with an exponent of $(\varepsilon_1 - 1)/\varepsilon_1$. This implies the following proposition.

[13] Adimitedly, this is not a trivial assumption. We relax this assumption and use numerical methods to check our results.

Proposition 2. *1. The share of general human capital in h-sector declines with μ_{z_h} and increases with μ_{z_l} when h- and l-goods are gross complements $(1 > \varepsilon_1)$. Moreover, it increases with $\sigma^2_{z_l}$.*

2. The share of general human capital in h-sector increases with μ_{z_h} and declines with μ_{z_l} when h- and l-goods are gross substitutes $(\varepsilon_1 > 1)$. Moreover, it increases with $\sigma^2_{z_h}$.

Proof. See Proofs Appendix. □

The substitutability between Y_h and Y_l is decisive for these results because shocks λ_h and λ_l are Hicks-neutral in Y_h and Y_l. For example, u_h^g declines with μ_{z_h} when Y_h and Y_l are gross-complements because, *ceteris paribus*, higher μ_{z_h} implies higher μ_{Y_h}.

The *ex ante* relative inverse demand $(\mathbb{E}[w_g]/\mathbb{E}[w_s])$ for general human capital can be derived using (9), (10), and (11). We denote it by $\tilde{w}_g$, and it is given by

$$\tilde{w}_g = \frac{\gamma_2}{1-\gamma_2}\frac{1}{1-\omega_K^{Y_m}}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} H_s. \tag{18}$$

This expression implies that the previous proposition also applies to the relative (inverse) demand for general human capital.

Corollary 1. *1. The relative (inverse) demand for general human capital increases with μ_{z_h} and declines with μ_{z_l} when $1 > \varepsilon_1$. Moreover, it declines with $\sigma^2_{z_l}$.*

2. The relative (inverse) demand for general human capital declines with μ_{z_h} and increases with μ_{z_l} when $\varepsilon_1 > 1$. Moreover, it declines with $\sigma^2_{z_h}$.

The relative (inverse) demand for general human capital also depends on the amounts of factor inputs in the following way:

Proposition 3. *The relative (inverse) demand for general human capital increases with K when $\varepsilon_3 > 1 \geq \varepsilon_2 > \varepsilon_1$ and declines with K when $\varepsilon_1 > \varepsilon_2 \geq 1 > \varepsilon_3$. It declines with H_g and increases with H_s.*

Proof. See Proofs Appendix. □

The elasticity of substitution between H_g and K in the production of h-goods is governed by ε_2. The elasticity of substitution between H_s and K is ε_3. This proposition states that when h- and l-goods and general human capital and physical capital are gross complements $(1 \geq \varepsilon_2 > \varepsilon_1)$ and physical capital and specific human capital are gross substitutes $(\varepsilon_3 > 1)$ then the relative demand for general human capital increases with K. In turn, the relative demand for general human capital declines with K when h- and l-goods and general human capital and physical capital are gross substitutes ($\varepsilon_1 > \varepsilon_2 \geq 1$) and physical capital and specific human are gross complements ($1 > \varepsilon_3$).

In this framework, the composition of human capital portfolio matters for the contribution of volatility of sectoral shocks to the volatility of final output.

Proposition 4. *Consider two countries where H_s and H_g are different but the level of expected output is the same.*

1. *In the country where H_s is higher the variance of λ_h has a lower contribution to the variance of final output when $1 > \varepsilon_1$. It has a higher contribution in that country when $\varepsilon_1 > 1$.*

2. *In the country where H_s is higher the variance of λ_l has a higher contribution to the variance of final output when $1 > \varepsilon_1$. It has lower a contribution when $\varepsilon_1 > 1$.*

Proof. See Proofs Appendix. □

All the proofs for variances are performed using first order Taylor series approximations of Y around deterministic values of shocks $\{\lambda\}$.

This result holds because H_s is immobile across h- and l-sectors and increasing H_s increases Y_h/λ_h. Therefore, $(Y_h/\lambda_h)^{\frac{\varepsilon-1}{\varepsilon}}$ declines with H_s when $1 > \varepsilon_1$ and increases with it when $\varepsilon_1 > 1$. To keep the expected level of output constant then $(Y_l/\lambda_l)^{\frac{\varepsilon-1}{\varepsilon}}$ has to increase when $1 > \varepsilon_1$ and it has to decline when $\varepsilon_1 > 1$. These two quantities multiply the volatilities of λ_h and λ_l, respectively, because they stand in front of these shocks in Y.

For example, this result implies that in the country where H_s is higher the volatility of final output is higher if either $1 > \varepsilon_1$ and $\sigma^2_{z_l} > \sigma^2_{z_h} = 0$ or $\varepsilon_1 > 1$ and $\sigma^2_{z_h} > \sigma^2_{z_l} = 0$. It is lower if either $1 > \varepsilon_1$ and $\sigma^2_{z_h} > \sigma^2_{z_l} = 0$ or $\varepsilon_1 > 1$ and $\sigma^2_{z_l} > \sigma^2_{z_h} = 0$. This result also implies the following corollary.

Corollary 2. *Consider two countries where H_s and H_g are different but the level of expected output is the same. Suppose that $\varepsilon_1 > 1$ and $\sigma^2_{z_h} > \sigma^2_{z_l}$ (i.e., the coefficient of variation of λ_h is higher than the coefficient of variation of λ_l). Further, suppose that the share of Y_h is higher than or equal to the share of Y_l:*

$$\omega^Y_{Y_h} \geq 0.5.$$

In such a case, the volatility of final output is higher in the country where H_s is higher. The volatility of final output is also higher in that country in case when $1 > \varepsilon_1$, $\sigma^2_{z_l} > \sigma^2_{z_h}$, and $0.5 \geq \omega^Y_{Y_h}$.

According to Table 7, mostly services industries are very intensive in general human capital (e.g., 1-digit NACE industries H, I, J, and K) and in this sense these industries could represent Y_l. Their output (and employment) share is usually lower than 0.5 and they tend to be less volatile than the other industries (e.g., see Koren and Tenreyro, 2007). This proposition then implies that, if $\varepsilon_1 > 1$, the secular decline in the share of specific human capital, as observed in our sample countries, can explain the negative trend in output volatility documented by, for example, Stock and Watson (2005).[14]

Further, we consider a planner (e.g., household, policy-maker) who has an option to marginally increase either H_g or H_s at no cost (or at the same cost). We establish the following results for the cases when $\sigma_z^2 = 0$ and either $\sigma_{z_h}^2 > \sigma_{z_l}^2 = 0$ or $\sigma_{z_l}^2 > \sigma_{z_h}^2 = 0$.

Proposition 5. *Suppose the planner has an option to marginally increase either H_g or H_s by the same amount. In such a case, increasing H_g does not necessarily increase the volatility of final output less than so does increasing H_s, and*

$$\frac{\partial \sigma_Y^2}{\partial H_g} / \frac{\partial \sigma_Y^2}{\partial H_s} < 1$$

depends on the values of model parameters.

Proof. See Proofs Appendix. □

Proposition 6. *Suppose the planner has an option to increase either H_g or H_s by amounts that deliver the same marginal increase in expected final output,*

$$\frac{\partial \mu_Y}{\partial H_g} = \frac{\partial \mu_Y}{\partial H_s}. \tag{19}$$

1. *When $\sigma_{z_h}^2 > \sigma_{z_l}^2 = 0$ and $1 > \varepsilon_1$ ($\varepsilon_1 > 1$) the volatility of the final output increases more (less) with H_g than with H_s.*
2. *When $\sigma_{z_l}^2 > \sigma_{z_h}^2 = 0$ and $1 > \varepsilon_1$ ($\varepsilon_1 > 1$) the volatility of the final output increases less (more) with H_g than with H_s.*

Proof. See Proofs Appendix. □

Suppose that the objective function of the planner (e.g., the utility function of the households) is increasing and concave in final output. In such a case, the planner would like to insure against the sectoral shocks. The results in this proposition imply that under condition (19) when $\sigma_{z_l}^2 > \sigma_{z_h}^2 = 0$ and $1 > \varepsilon_1$ the planner would prefer investing in H_g. It would over-invest in H_s if it does not anticipate the shocks and treats λ_l as a deterministic variable at its mean value. Clearly, it would overinvest in H_s also in case when $\sigma_{z_h}^2 > \sigma_{z_l}^2 = 0$ and $\varepsilon_1 > 1$ if it does not anticipate the shocks and treats λ_l as a deterministic variable at its mean value.

[14] We present an attempt to estimate ε_1 in Appendix - Elasticity of Substitution. We obtain both lower and higher than 1 values for ε_1.

We close the model and endogenize the supply of human capital types in a trivial manner. The economy is populated by a mass one of one period lived and identical households. The households own all assets and have strictly increasing, concave, and twice continuously differentiable utility from consumption (C) of final goods. At the beginning of the period they are endowed with K amount of physical capital and no human capital. They need to decide how much specific and general human capital to acquire. The costs of acquiring both types skills (S) are in terms of final goods. The production happens after the households supply physical capital and both types of human capital.

The representative household then solves the following problem

$$\begin{aligned}
&\max_{C,S_s,S_g} \mathbb{E}\left[u\left(C\right)\right] \\
&s.t. \\
&C + S_s + S_g = rK + w_s H_s + w_g H_g, \\
&H_s = \lambda_H S_s, \\
&H_g = \lambda_H S_g,
\end{aligned}$$

where S_s and S_g are the expenses for acquiring the corresponding skills and $\lambda_H > 0$ is an exogenous productivity level.

Clearly, it is optimal to supply all K, and the household chooses the amounts of H_s and H_g so that

$$\begin{aligned}
&\mathbb{E}\left[w_s\right] = \mathbb{E}\left[w_g\right] = \lambda_H, \qquad (20)\\
&\tilde{w}_g = 1.
\end{aligned}$$

The allocations of general human capital and the levels of output and human capital types can be solved from (17), (15), (2), (6), (13), (7), (5), (11), (12), (9), (10), and (20).

The supply of human capital fixes the ratio of expected wages. Therefore, for example, H_g/H_s declines and u_h^g increases with $\sigma^2_{\lambda_l}$ when $1 > \varepsilon_1$ according to (18), (20), Proposition 2 and Corollary 1. The household then will over-invest in general human capital, H_g, if it does not anticipate the shocks to λ_l and treats λ_l as constant at its mean it. Moreover, it will allocate more than optimal amount of general human capital to l-goods production.

We observe that the share of specific human capital has declined over time in almost all countries in the sample. The model offered above can match this observation in a straightforward manner. Suppose that physical capital (K) grows over time. Further, h- and l-goods and general human capital and physical capital are gross complements ($1 \geq \varepsilon_2 > \varepsilon_1$) while physical capital and specific human capital are gross substitutes ($\varepsilon_3 > 1$). In such a case, the relative demand for general human capital grows over time. Therefore, the households will acquire increasingly more general skills relative to specific skills, and the share of specific human capital will decline. Clearly, changes in the means of λ_h and λ_l can also drive this pattern. For example, over time the households will acquire increasingly more general skills relative to specific skills if $1 > \varepsilon_1$ and μ_{λ_h} grows relative to μ_{λ_l}.[15] In this reduced form model, the changes in K and $\mu_{\lambda_h}/\mu_{\lambda_l}$ can be thought to represent, for example, changes in the production of h-goods, because of factor accumulation or biased technical change and obsolescence, and changes of industrial composition in terms of sectors h and l (Appendix A.1 shows that increasing K increases Y_h/Y_l).[16]

Supply side factors can also be responsible for the negative trend in the share of specific human capital. In this model, H_g/H_s will increase if the productivity of schooling general human capital grows over time relative to the productivity of schooling specific human capital. According to Table 12 more than 70% of the graduates from the education field of Science, Mathematics, and Computing attain general human capital. It could be then reasonable to expect that the relative productivity in schooling of general human capital has increased if technical change implied by the introduction of ICT increases the efficiency in the education process in this education field, relative to other fields.[17]

According to our data, the share of specific human capital has declined over time within and between industries. Our model can match this observation too. In the model, the share of general human capital can be decomposed as

$$\begin{aligned}\frac{H_g}{H_g+H_s} &= \frac{u_l^g H_g}{u_l^g H_g}\frac{u_l^g H_g}{H_g+H_s} + \frac{u_h^g H_g}{u_h^g H_g + H_s}\frac{u_h^g H_g + H_s}{H_g+H_s} \\ &\equiv \omega_l^g \omega_l + \omega_h^g \omega_h.\end{aligned}$$

In this expression ω_l^g and ω_h^g are the shares of general human capital in l- and h-sector and ω_l and ω_h are the shares of industries in terms of employment out of total employment. The data suggests that ω_h^g and ω_l have grown. By construction this implies that ω_h has declined. Moreover, by construction ω_l^g is equal to 1 and does not vary.

Proposition 7. *For brevity, we define the following function*

$$\mathbb{I}(x,y) = \begin{cases} x & if \quad x > 0, \\ y & otherwise. \end{cases}$$

[15] We call K physical capital and assume that the production of Y_l requires no K. Given the way we model K, the mean value of λ_l can be thought to represent the amount of physical capital in the production of Y_l.

[16] As discussed above, changes in variances of sectoral shocks can also affect the relative demand for general human capital and drive this pattern.

[17] We take no stance on which of these channels is more likely to be behind the trend in Figure 3. We also do not take a firm stance on parameter values, although we present estimation results for ε_1 and γ_1 in Appendix - Elasticity of Substitution.

In order for ω_h^g and ω_l to grow with K it is sufficient to have ε_1 higher than

$$\mathbb{I}\left(\frac{(1-\gamma_2)\left[1+\omega_{Y_h}^Y(1-\gamma_2)\right]-1}{(1-\gamma_2)\left[1+\omega_{Y_h}^Y(1-\gamma_2)\right]},0\right)$$

and lower than

$$\mathbb{I}\left(\frac{\frac{H_s}{H_g+H_s}\frac{1}{1-u_h^g}+(1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g}-\frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^Y\right]}{(1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g}-\frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^Y\right]},+\infty\right).$$

Proof. See Proofs Appendix. □

It is easy to check that the interval for ε_1 where ω_h^g and ω_l grow with K includes both lower than 1 and greater than 1 values of ε_1.

Additional Results

We further consider a version of the model where firms hire inputs after observing the values of shocks and $\varepsilon_2=\gamma_4=1$ keeping the reminder of the model intact. The following propositions are true for such an economy.

Proposition 8. *1. The elasticity of final output Y with respect to λ_h (λ_l) increases less (more) with a marginal (percentage) increase in H_s than with a marginal (percentage) increase in H_g if $1>\varepsilon_1$.*

2. The elasticity of final output Y with respect to λ_h (λ_l) increases more (less) with a marginal (percentage) increase in H_s than with a marginal (percentage) increase in H_g if $\varepsilon_1=1+$ or $\varepsilon_1=+\infty$.

Proof. See Proofs Appendix. □

Proposition 9. *Consider two countries where H_s and H_g are different but the level of expected output is the same.*

1. *In the country where H_s is higher the elasticity of final output with respect to λ_h (λ_l) is lower (higher) if $1>\varepsilon_1$.*
2. *In the country where H_s is higher the elasticity of final output with respect to λ_h (λ_l) is higher (lower) if $\varepsilon_1=1+$ or $\varepsilon_1=+\infty$.*

Proof. See Proofs Appendix. □

3 Conclusions

In this paper, we consider industry-specificity as a distinct source of human capital heterogeneity that is defined irrespective of the skill-level accumulated through education. Accordingly, we define specific and general human capital types treating occupations as types of skills. Specific human capital is the set of skills/occupations whose use is spread in a limited set of industries. In turn, general human capital is the set of skills which are used in a wide range of industries.

We use EU Labor Force Survey to identify these human capital types and analyze their employment and education. Our empirical exercise yields remarkably persistent assignment of occupations into specific and general human capital types. We find that the share of employment in specific human capital occupations varies significantly across countries and has declined over time almost everywhere.

This negative trend is attributable to both within and between industry shifts. Industries have started using general human capital more intensively and the share of industries which use specific human capital more intensively has declined. Importantly, the ranking of industries according to their intensity of use of specific human capital is also remarkably stable.

??Education fields??We also find that education levels of specific and general types of human capital are very uniform. This agrees with our horizontal differentiation.

Finally, in a multi-sector model we assess the effect of human capital portfolio composition on the propagation of sectoral shocks and the effect of uncertainty on the composition. In the model, we split industries into two sectors, h and l, according to the intensity of use of specific human capital. General human capital is required for production in both sectors. Specific human capital is required for production in sector h only. The outputs of these sectors, Y_h and Y_l, are aggregated into consumption goods with a CES function.

Suppose Y_h and Y_l are gross-complements. Among countries where expected output is the same, in the country where the amount of specific human capital is higher the volatility of shocks to sector h (l) has lower (higher) contribution to the volatility of final output. Therefore, the volatility of final output in this country is lower if, for example, there are no shocks to l.

Further, we turn off shocks either to sector h or l and consider a planner who has an option to marginally increase either the amount of general human capital or specific human capital at no cost (or the same cost) and at the same benefit in terms of expected final output. We show that increasing the amount of general human capital increases the volatility of final output more (less) than increasing the amount of specific human capital if there are no shocks to sector l (h). Clearly, if the planner has a concave objective function then it would prefer investing in specific human capital if there are no shocks to sector l. It would under-invest in general human capital if it does not anticipate shocks to sector h.

The share of specific human capital increases (decreases) with the volatility of shocks to sector h (sector l). It decreases (increases) with the mean of shocks to sector h (sector l). The opposites of these results hold when Y_h and Y_l are gross-substitutes.

Our theoretical framework can be also used to gain an insight into what can drive the decline in the share of specific human capital. Such a negative trend can stem from biased technical change in the production of Y_h and Y_l goods and/or in schooling of human capital types. It can also stem from factor accumulation.??Industrial composition??

Tables

Table 1: *ANOVA for Within-Occupation Share Across Industries*

Source	Partial SS	df	MS	F	P-stat
Model	463.487	84	5.518	201.380	0.000
Occupation	46.383	25	1.855	67.720	0.000
Industry	380.486	15	25.366	925.790	0.000
Country	11.995	26	0.461	16.840	0.000
Year	1.226	18	0.068	2.490	0.001
Residual	3382.424	123450	0.027		
Total	3845.911	123534	0.031		

Note: This table reports the results from an ANOVA exercise for the share of workers in each occupation-industry cell out of total employment in each occupation in all industries. The variation in the data is at occupation-industry-country-year level, and we perform the ANOVA exercise along each of these dimensions. Number of Obs. = 123535; Adj. R-squared = 0.120.

Table 2: *The Employment Shares of Occupations in Industries*

	Industry (NACE)															
Occupation (ISCO-88)	A-B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
11	0.020	0.007	0.026	0.010	0.016	0.026	0.007	0.025	0.017	0.037	0.755	0.033	0.053	0.112	0.005	0.045
12	0.018	0.006	0.222	0.016	0.067	0.190	0.036	0.066	0.091	0.111	0.057	0.049	0.043	0.037	0.001	0.013
13	0.063	0.002	0.104	0.005	0.080	0.388	0.134	0.047	0.019	0.089	0.010	0.017	0.022	0.041	0.005	0.002
21	0.006	0.011	0.227	0.030	0.080	0.048	0.003	0.050	0.040	0.382	0.075	0.023	0.014	0.015	0.001	0.014
22	0.025	0.002	0.023	0.003	0.002	0.074	0.003	0.003	0.004	0.044	0.038	0.024	0.764	0.011	0.003	0.004
23	0.003	0.001	0.005	0.001	0.002	0.005	0.003	0.003	0.003	0.011	0.016	0.930	0.022	0.012	0.005	0.003
24	0.006	0.003	0.094	0.007	0.012	0.057	0.006	0.030	0.067	0.249	0.188	0.039	0.073	0.160	0.001	0.021
31	0.015	0.011	0.292	0.038	0.080	0.062	0.004	0.122	0.018	0.185	0.074	0.025	0.039	0.042	0.002	0.008
32	0.028	0.002	0.035	0.005	0.003	0.069	0.005	0.003	0.003	0.025	0.029	0.024	0.774	0.023	0.003	0.005
33	0.006	0.010	0.015	0.004	0.005	0.038	0.008	0.014	0.008	0.016	0.042	0.666	0.247	0.046	0.012	0.005
34	0.007	0.002	0.121	0.008	0.021	0.192	0.012	0.052	0.135	0.143	0.155	0.023	0.054	0.071	0.001	0.009
41	0.010	0.004	0.156	0.015	0.036	0.157	0.009	0.150	0.076	0.109	0.137	0.038	0.056	0.044	0.001	0.006
42	0.004	0.003	0.031	0.013	0.008	0.230	0.075	0.166	0.226	0.063	0.044	0.013	0.064	0.077	0.002	0.003
51	0.005	0.001	0.014	0.002	0.002	0.023	0.276	0.031	0.003	0.043	0.117	0.058	0.286	0.121	0.021	0.003
52	0.004	0.001	0.051	0.001	0.006	0.881	0.019	0.008	0.005	0.018	0.002	0.002	0.003	0.010	0.003	0.001
61	0.866	0.002	0.018	0.002	0.010	0.019	0.005	0.003	0.001	0.019	0.027	0.007	0.010	0.036	0.016	0.002
71	0.006	0.017	0.108	0.017	0.741	0.029	0.004	0.010	0.001	0.026	0.018	0.008	0.010	0.011	0.003	0.002
72	0.012	0.011	0.481	0.038	0.095	0.223	0.004	0.068	0.002	0.025	0.020	0.005	0.007	0.013	0.002	0.005
73	0.010	0.007	0.762	0.010	0.029	0.087	0.007	0.013	0.006	0.056	0.020	0.012	0.030	0.032	0.011	0.023
74	0.012	0.002	0.733	0.003	0.033	0.158	0.025	0.005	0.002	0.011	0.006	0.006	0.013	0.019	0.006	0.004
81	0.018	0.049	0.746	0.078	0.033	0.028	0.005	0.015	0.007	0.027	0.014	0.018	0.014	0.023	0.007	0.012
82	0.012	0.004	0.859	0.006	0.019	0.042	0.005	0.011	0.002	0.017	0.005	0.003	0.014	0.026	0.003	0.005
83	0.046	0.014	0.128	0.011	0.116	0.096	0.007	0.496	0.003	0.018	0.027	0.004	0.014	0.024	0.002	0.004
91	0.011	0.003	0.068	0.011	0.013	0.071	0.102	0.043	0.012	0.174	0.073	0.121	0.122	0.091	0.092	0.003
92	0.828	0.010	0.040	0.016	0.021	0.039	0.010	0.009	0.009	0.033	0.056	0.015	0.036	0.057	0.026	0.012
93	0.016	0.010	0.368	0.011	0.231	0.149	0.008	0.093	0.003	0.033	0.038	0.006	0.029	0.022	0.003	0.009

Note: We compute the share of workers in each occupation (2-digit ISCO-88) and industry (1-digit NACE Rev. 1) out of total employment in the occupation in all industries for each country and year. This table reports country and year average of these shares. See Table 3 and Table 7 for the definitions of occupations and industries.

Table 3: *Concentrations of Occupations in Industries and the Assignment of Occupations into Specific and General Human Capital Types*

2-digit ISCO-88: Occupation Name	HI	CV	Gini	Theil	GE	Specific (= 1; General = 0)
11: Legislators and Senior Officials	0.646	3.064	0.866	1.986	4.610	1
12: Corporate Managers	0.153	1.184	0.556	0.582	0.709	0
13: General Managers	0.245	1.707	0.693	0.969	1.440	0
21: Physical, Mathematical, and Engineering Science Professionals	0.247	1.736	0.695	0.960	1.454	0
22: Life Science and Health Professionals	0.606	3.012	0.860	1.848	4.300	1
23: Teaching Professionals	0.871	3.687	0.915	2.421	6.393	1
24: Other Professionals	0.176	1.363	0.639	0.758	0.890	0
31: Physical and Engineering Science Associate Professionals	0.182	1.399	0.624	0.731	0.941	0
32: Life Science and Health Associate Professionals	0.631	3.064	0.859	1.882	4.491	1
33: Teaching Associate Professionals	0.604	2.973	0.874	1.958	4.268	1
34: Other Associate Professionals	0.145	1.171	0.585	0.624	0.651	0
41: Office Clerks	0.133	1.075	0.548	0.541	0.550	0
42: Customer Services Clerks	0.208	1.532	0.670	0.874	1.144	0
51: Personal and Protective Services Workers	0.251	1.729	0.710	1.009	1.483	0
52: Models, Salespersons, and Demonstrators	0.788	3.486	0.899	2.232	5.734	1
61: Market-oriented Skilled Agricultural and Fishery Workers	0.776	3.433	0.892	2.212	5.633	1
71: Extraction and Building Trades Workers	0.574	2.921	0.840	1.739	4.047	1
72: Metal, Machinery, and Related Trades Workers	0.316	2.049	0.765	1.213	2.001	0
73: Precision, Handicraft, Printing, and Related Trades Workers	0.616	3.022	0.865	1.896	4.376	1
74: Other Craft and Related Trades Workers	0.612	2.998	0.870	1.931	4.333	1
81: Stationary-plant and Related Operators	0.601	2.974	0.852	1.840	4.261	1
82: Machine Operators and Assemblers	0.755	3.400	0.885	2.142	5.474	1
83: Drivers and Mobile-plant Operators	0.303	1.996	0.740	1.120	1.896	0
91: Sales and Services Elementary Occupations	0.140	1.123	0.554	0.563	0.611	0
92: Agricultural, Fishery, and Related Laborers	0.732	3.320	0.890	2.169	5.290	1
93: Labourers in Mining, Construction, Manufacturing, and Transport	0.269	1.846	0.742	1.116	1.630	0

Note: This table offers the values of country- and year-averaged concentration measures for each occupation (2-digit ISCO-88) and the assignment of occupations into specific and general human capital types. An occupation corresponds to specific human capital if Specific dummy variable, which is offered in the last column of the table, is equal to 1. This dummy variable is constructed in the following manner. For each of the country- and year-averaged concentration measures (columns 2–6) we define a dummy variable which equals 1 for the values of the concentration measure which are higher than its median. We average these dummy variables over the concentration measures and set Specific dummy variable to 1 if the average is greater than 0.5, and to 0 otherwise. We obtain the same assignment if we weight observations using the number of hours worked in the reference week.??For each country and year we drop observations for occupations which are coded only at 1-digit level.??

Table 4: *Specific Human Capital Identified Separately for Each Country and Year*

2-digit ISCO-88: Occupation Name	Count in Countries	Count in Years
11: Legislators and Senior Officials	24	19
22: Life Science and Health Professionals	27	19
23: Teaching Professionals	27	19
32: Life Science and Health Associate Professionals	26	19
33: Teaching Associate Professionals	27	19
51: Personal and Protective Services Workers	2	0
52: Models, Salespersons, and Demonstrators	27	19
61: Market-oriented Skilled Agricultural and Fishery Workers	27	19
71: Extraction and Building Trades Workers	26	16
72: Metal, Machinery, and Related Trades Workers	2	0
73: Precision, Handicraft, Printing and Related Trades Workers	27	19
74: Other Craft and Related Trades Workers	25	19
81: Stationary-plant and Related Operators	26	19
82: Machine Operators and Assemblers	27	19
83: Drivers and Mobile-plant Operators	1	0
92: Agricultural, Fishery, and Related Laborers	26	19

Note: In this table, we offer the results from an exercise where we use our methodology to identify specific and general human capital types separately for each country and for each year. The second column offers the number of times that an occupation is assigned into specific human capital type in sample 27 countries. The third column offers the number of times that an occupation is assigned into specific human capital type in sample 19 years. For the second column, we average the concentration measures over years in each country and define a dummy variable for each of the averaged concentration measure which equals 1 for the values of the concentration measure which are higher than its median. Finally, we average these dummy variables over the concentration measures and call an occupation specific human capital if this average is greater than 0.5, and general otherwise. ISCO 11 occupation is classified as general human capital in Switzerland, Italy, and the UK. ISCO 32 is classified as general human capital in Spain. ISCO 51 is classified as specific human capital in Denmark and Sweden. ISCO 71 is classified as general human capital in Germany. ISCO 72 is classified as specific human capital in Germany and Spain. ISCO 74 is classified as general human capital in Luxembourg and Sweden. ISCO 81 is classified as general human capital in Denmark. ISCO 83 is classified as specific human capital in the UK. Our data do not contain ISCO 92 for France. For the third column, we repeat the exercise taking averages over years instead of countries. ISCO 71 is classified as general human capital in 1997, 2001, and 2002. See Table 3 for the original assignment of occupations into specific and general human capital types.

Table 5: *ANOVA for the Share of Specific Human Capital*

Source	Partial SS	df	MS	F	P-stat
Model	1.154	44	0.026	45.450	0.000
Country	1.055	26	0.041	70.350	0.000
Year	0.170	18	0.009	16.330	0.000
Residual	0.212	368	0.001		
Total	1.366	412	0.003		

Note: This table reports the results from an ANOVA exercise for the share of specific human capital. The variation in the data are at country-year level, and we perform the ANOVA exercise along each of these dimensions. Number of Obs. = 413; Adj. R-squared = 0.826. See Table 3 for the assignment of occupations into specific and general human capital types.

Table 6: *Sample Countries and Years, and Basic Statistics for the Share of Specific Human Capital*

Country	Sample Period	Obs.	Mean	SD	Min	Max
Austria	1995–2010	16	0.337	0.034	0.291	0.369
Belgium	1993–2010	18	0.327	0.016	0.304	0.357
Cyprus	1999–2010	12	0.326	0.016	0.292	0.347
Czech Republic	1997–2010	14	0.359	0.014	0.337	0.379
Denmark	1992–2010	19	0.317	0.009	0.301	0.332
Estonia	1998–2010	13	0.346	0.010	0.325	0.363
Finland	1997–2010	14	0.324	0.028	0.295	0.367
France	1992–2010	19	0.315	0.020	0.279	0.363
Germany	2002–2010	9	0.293	0.008	0.283	0.306
Greece	1992–2010	19	0.456	0.035	0.409	0.517
Hungary	1998–2010	13	0.383	0.013	0.362	0.401
Iceland	1995–2010	16	0.398	0.018	0.372	0.433
Ireland	1992–2010	19	0.351	0.065	0.298	0.463
Italy	1992–2010	19	0.393	0.044	0.327	0.449
Latvia	1998–2010	13	0.349	0.038	0.276	0.386
Lithuania	1998–2010	13	0.413	0.030	0.388	0.475
Luxembourg	1992–2010	19	0.283	0.077	0.222	0.581
Netherlands	1992–2010	19	0.272	0.016	0.246	0.299
Norway	1996–2010	15	0.340	0.009	0.320	0.350
Poland	2002–2010	9	0.457	0.012	0.439	0.478
Portugal	1998–2010	13	0.373	0.050	0.335	0.469
Slovakia	1998–2010	13	0.379	0.011	0.368	0.403
Slovenia	1996–2010	15	0.423	0.026	0.378	0.466
Spain	1993–2010	18	0.353	0.024	0.308	0.388
Sweden	1997–2010	14	0.320	0.010	0.307	0.339
Switzerland	1996–2010	15	0.344	0.011	0.321	0.356
UK	1992–2010	19	0.273	0.012	0.257	0.292

Note: This table offers our sample of countries and years, and basic statistics for the share of workers in specific human capital occupations out of total employment (the share of specific human capital) in each country in the sample. See Table 3 for the assignment of occupations into specific and general human capital types.

Table 7: *The Share of Specific Human Capital in Industries*

1-digit NACE: Industry Name	Share
A-B: Agriculture, Hunting, and Fishing	0.797
C: Mining and Quarrying	0.325
D: Manufacturing	0.417
E: Electricity, Gas and Water Supply	0.190
F: Construction	0.571
G: Wholesale and Retail Trade; Repair of Goods	0.412
H: Hotels and Restaurants	0.057
I: Transport, Storage, and Communication	0.027
J: Financial Intermediation	0.014
K: Real Estate, Renting, and Business Activities	0.073
L: Public Administration; Social Security	0.128
M: Education	0.694
N: Health and Social Work	0.473
O: Other Community and Personal Service Activities	0.117
P: Households with Employed Persons	0.117
Q: Extra-territorial Organizations and Bodies	0.169

Note: This table offers the share of workers in specific human capital occupations out of total employment in industries (1-digit NACE). The data are averaged across countries and years. See Table 3 for the assignment of occupations into specific and general human capital types.

Table 8: *The Employment Share of Specific Human Capital in Countries and Industries*

Countries	Industries A-B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
Austria	0.939	0.307	0.343	0.165	0.551	0.400	0.030	0.022	0.009	0.049	0.117	0.747	0.474	0.109	0.025	0.078
Belgium	0.891	0.320	0.353	0.148	0.577	0.362	0.027	0.031	0.008	0.047	0.162	0.769	0.501	0.071	0.076	0.102
Cyprus	0.927	0.230	0.408	0.109	0.588	0.429	0.063	0.006	0.005	0.038	0.086	0.795	0.556	0.080	0.006	0.100
Czech Republic	0.575	0.378	0.426	0.245	0.556	0.427	0.034	0.022	0.007	0.061	0.138	0.658	0.619	0.106	0.127	0.166
Denmark	0.819	0.169	0.326	0.222	0.597	0.364	0.083	0.018	0.007	0.073	0.147	0.680	0.389	0.107	0.072	0.399
Estonia	0.637	0.235	0.447	0.218	0.551	0.397	0.084	0.023	0.029	0.078	0.105	0.597	0.509	0.089	0.263	0.369
Finland	0.904	0.259	0.336	0.204	0.500	0.395	0.022	0.013	0.010	0.105	0.088	0.626	0.415	0.154	0.267	0.163
France	0.896	0.268	0.416	0.143	0.624	0.308	0.061	0.031	0.014	0.077	0.130	0.685	0.387	0.177	0.075	0.162
Germany	0.846	0.407	0.299	0.219	0.605	0.336	0.081	0.019	0.006	0.086	0.131	0.710	0.440	0.133	0.052	0.138
Greece	0.989	0.421	0.532	0.111	0.756	0.359	0.025	0.007	0.006	0.029	0.155	0.865	0.591	0.077	0.037	0.122
Hungary	0.641	0.397	0.468	0.329	0.577	0.513	0.074	0.041	0.006	0.064	0.147	0.644	0.551	0.099	0.349	0.281
Iceland	0.799	0.571	0.500	0.178	0.657	0.510	0.169	0.040	0.016	0.120	0.122	0.692	0.437	0.157		0.173
Ireland	0.483	0.271	0.396	0.123	0.525	0.477	0.035	0.040	0.036	0.109	0.142	0.701	0.482	0.129	0.063	0.216
Italy	0.881	0.321	0.474	0.207	0.605	0.460	0.036	0.022	0.010	0.071	0.144	0.754	0.597	0.085	0.023	0.160
Latvia	0.682	0.170	0.420	0.203	0.447	0.391	0.064	0.050	0.033	0.101	0.145	0.573	0.555	0.139	0.235	0.244
Lithuania	0.840	0.274	0.486	0.222	0.491	0.404	0.071	0.034	0.017	0.106	0.126	0.601	0.589	0.098	0.206	0.110
Luxembourg	0.874	0.376	0.402	0.186	0.560	0.358	0.048	0.046	0.057	0.050	0.146	0.789	0.427	0.106	0.012	0.086
Netherlands	0.400	0.158	0.320	0.154	0.574	0.360	0.050	0.026	0.010	0.072	0.157	0.701	0.424	0.142	0.140	0.150
Norway	0.926	0.238	0.419	0.143	0.524	0.500	0.048	0.013	0.007	0.077	0.090	0.700	0.325	0.101	0.056	1.000
Poland	0.962	0.450	0.445	0.235	0.539	0.522	0.067	0.016	0.006	0.065	0.058	0.649	0.561	0.122	0.057	0.327
Portugal	0.918	0.525	0.554	0.141	0.609	0.391	0.022	0.025	0.010	0.052	0.156	0.592	0.312	0.100	0.008	0.177
Slovakia	0.516	0.429	0.481	0.285	0.536	0.500	0.044	0.027	0.007	0.072	0.099	0.636	0.575	0.106	0.065	0.358
Slovenia	0.945	0.430	0.503	0.202	0.466	0.451	0.057	0.028	0.005	0.054	0.095	0.694	0.572	0.088	0.161	0.196
Spain	0.888	0.450	0.462	0.259	0.601	0.364	0.026	0.017	0.011	0.046	0.115	0.777	0.427	0.082	0.023	0.187
Sweden	0.791	0.347	0.425	0.229	0.599	0.394	0.115	0.020	0.014	0.130	0.126	0.635	0.293	0.152	0.172	
Switzerland	0.930	0.310	0.354	0.180	0.591	0.455	0.095	0.055	0.032	0.109	0.146	0.755	0.536	0.231	0.196	0.158
UK	0.700	0.255	0.320	0.169	0.525	0.390	0.052	0.025	0.014	0.063	0.112	0.550	0.348	0.114	0.306	0.234

Note: For each industry (1-digit NACE), this table offers the yearly average share of employed individuals in specific human capital occupations out of total employment in the industry in each country. We have no observations for 1-digit NACE industry P in Iceland and industry Q in Sweden. See Table 3 for the assignment of occupations into specific and general human capital types and Table 7 for the definitions of industries.

Table 9: *The Employment Share of Specific Human Capital in Years and Industries*

Year	A-B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1992	0.705	0.310	0.447	0.247	0.595	0.335	0.112	0.094	0.099	0.210	0.251	0.628	0.550	0.183	0.061	0.363
1993	0.786	0.306	0.434	0.209	0.589	0.402	0.068	0.033	0.014	0.070	0.180	0.737	0.481	0.102	0.056	0.224
1994	0.837	0.345	0.431	0.186	0.593	0.402	0.071	0.031	0.017	0.071	0.172	0.739	0.476	0.103	0.109	0.185
1995	0.838	0.379	0.436	0.163	0.596	0.403	0.057	0.029	0.014	0.069	0.162	0.733	0.474	0.120	0.084	0.197
1996	0.865	0.311	0.425	0.188	0.583	0.404	0.048	0.025	0.011	0.061	0.151	0.727	0.477	0.112	0.075	0.159
1997	0.846	0.365	0.422	0.188	0.580	0.409	0.042	0.024	0.009	0.067	0.145	0.707	0.475	0.127	0.104	0.134
1998	0.787	0.345	0.430	0.195	0.570	0.409	0.056	0.026	0.015	0.078	0.133	0.689	0.490	0.121	0.124	0.201
1999	0.784	0.325	0.433	0.201	0.567	0.413	0.060	0.028	0.014	0.072	0.126	0.701	0.496	0.120	0.103	0.131
2000	0.794	0.307	0.430	0.186	0.573	0.421	0.052	0.028	0.013	0.064	0.126	0.697	0.494	0.114	0.112	0.117
2001	0.789	0.326	0.435	0.205	0.574	0.425	0.054	0.028	0.012	0.065	0.122	0.692	0.481	0.111	0.111	0.161
2002	0.800	0.324	0.426	0.205	0.575	0.427	0.052	0.026	0.011	0.068	0.122	0.691	0.480	0.110	0.111	0.138
2003	0.798	0.349	0.422	0.201	0.571	0.428	0.053	0.022	0.015	0.061	0.120	0.685	0.470	0.113	0.114	0.244
2004	0.791	0.373	0.413	0.191	0.567	0.406	0.057	0.024	0.013	0.070	0.118	0.689	0.471	0.116	0.156	0.177
2005	0.794	0.316	0.409	0.204	0.569	0.409	0.057	0.027	0.012	0.068	0.114	0.694	0.471	0.114	0.132	0.154
2006	0.792	0.288	0.408	0.186	0.573	0.409	0.056	0.025	0.012	0.065	0.114	0.688	0.463	0.112	0.133	0.168
2007	0.778	0.306	0.402	0.182	0.568	0.410	0.057	0.024	0.011	0.065	0.114	0.686	0.460	0.109	0.130	0.157
2008	0.776	0.311	0.396	0.175	0.565	0.409	0.056	0.025	0.012	0.068	0.114	0.687	0.453	0.110	0.126	0.162
2009	0.804	0.306	0.393	0.165	0.552	0.411	0.059	0.021	0.011	0.092	0.115	0.678	0.452	0.121	0.132	0.123
2010	0.802	0.310	0.396	0.163	0.554	0.416	0.063	0.023	0.011	0.091	0.112	0.681	0.450	0.128	0.124	0.134

Note: For each industry (1-digit NACE), this table offers country-level average share of employed individuals in specific human capital occupations out of total employment in the industry in each year. See Table 3 for the assignment of occupations into specific and general human capital types and Table 7 for the definitions of industries.

Table 10: *Industry-Level Decomposition of the Trends in the Share of Specific Human Capital for Each Country*

		Between Industries				Within Industries				Total			
Country	Obs.	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
Austria	15	-0.002	0.005	-0.015	0.009	-0.003	0.015	-0.054	0.007	-0.005	0.018	-0.069	0.016
Belgium	17	-0.001	0.004	-0.008	0.007	-0.001	0.011	-0.025	0.030	-0.002	0.012	-0.032	0.030
Cyprus	11	0.000	0.004	-0.007	0.006	-0.005	0.008	-0.015	0.008	-0.005	0.011	-0.023	0.013
Czech Republic	13	-0.002	0.002	-0.007	0.001	0.001	0.007	-0.015	0.014	-0.001	0.008	-0.018	0.015
Denmark	18	-0.001	0.003	-0.009	0.005	0.001	0.009	-0.015	0.024	0.000	0.010	-0.018	0.026
Estonia	12	-0.003	0.006	-0.013	0.006	0.000	0.011	-0.014	0.020	-0.002	0.009	-0.021	0.013
Finland	13	-0.002	0.002	-0.007	0.001	-0.003	0.013	-0.043	0.006	-0.006	0.013	-0.046	0.004
France	18	-0.003	0.004	-0.016	0.002	0.004	0.025	-0.025	0.100	0.001	0.022	-0.030	0.084
Germany	8	-0.001	0.003	-0.007	0.002	0.000	0.003	-0.005	0.005	-0.002	0.004	-0.007	0.008
Greece	18	-0.005	0.005	-0.017	0.000	-0.001	0.004	-0.012	0.005	-0.006	0.007	-0.029	0.003
Hungary	12	-0.002	0.002	-0.006	0.001	-0.001	0.004	-0.006	0.006	-0.002	0.005	-0.010	0.006
Iceland	15	-0.003	0.010	-0.021	0.022	0.000	0.009	-0.016	0.019	-0.003	0.013	-0.018	0.025
Ireland	18	-0.003	0.008	-0.023	0.003	-0.004	0.026	-0.092	0.038	-0.008	0.027	-0.107	0.015
Italy	18	-0.003	0.005	-0.015	0.009	-0.004	0.011	-0.046	0.004	-0.007	0.013	-0.055	0.001
Latvia	12	-0.003	0.007	-0.011	0.013	-0.004	0.017	-0.045	0.020	-0.006	0.020	-0.051	0.027
Lithuania	12	-0.006	0.024	-0.080	0.011	0.001	0.009	-0.014	0.017	-0.004	0.023	-0.068	0.020
Luxembourg	18	-0.004	0.008	-0.023	0.007	-0.015	0.059	-0.250	0.019	-0.020	0.064	-0.273	0.012
Netherlands	18	-0.001	0.007	-0.012	0.017	-0.002	0.005	-0.007	0.014	-0.003	0.008	-0.016	0.014
Norway	14	-0.002	0.004	-0.014	0.002	0.002	0.009	-0.014	0.017	-0.001	0.009	-0.019	0.014
Poland	8	-0.006	0.011	-0.022	0.019	0.001	0.004	-0.004	0.006	-0.005	0.013	-0.024	0.022
Portugal	12	-0.009	0.018	-0.066	0.000	-0.002	0.006	-0.011	0.008	-0.011	0.020	-0.072	0.006
Slovakia	12	-0.002	0.004	-0.007	0.007	0.002	0.010	-0.010	0.028	0.000	0.013	-0.013	0.034
Slovenia	13	-0.002	0.009	-0.019	0.011	0.000	0.010	-0.009	0.022	-0.003	0.016	-0.028	0.030
Spain	17	-0.004	0.003	-0.013	0.001	-0.001	0.003	-0.008	0.006	-0.005	0.003	-0.010	0.000
Sweden	13	-0.001	0.004	-0.008	0.011	-0.001	0.007	-0.016	0.014	-0.002	0.007	-0.020	0.010
Switzerland	14	-0.003	0.009	-0.026	0.008	0.001	0.008	-0.020	0.015	-0.002	0.010	-0.029	0.015
UK	18	-0.001	0.002	-0.004	0.003	-0.001	0.003	-0.010	0.005	-0.002	0.004	-0.013	0.005

Note: This table offers the basic statistics for between- and within-industry decomposition of changes in the share of specific human capital (1) for each country. Columns 1-4 and 5-8 offer the basic statistics of between- and within-industry components. Columns 9-12 offer the basic statistics of total change in the share of specific human capital. Figures 4 and 2 show that there are spikes in the share of specific human capital. These spikes can stem from imperfections in sampling weights at this level of disaggregation. For example, in our data there are very large persistent changes in the number of employees in occupations 82 and 93 in between 2000 and 2001 in the Czech Republic. Such imperfections can bias these decompositions in ambiguous directions. The effects such biases are likely to be alleviated when we take country and/or year averages.

Table 11: *ANOVA for the Within-Education Field Share Across Occupations*

Source	Partial SS	df	MS	F	P-stat
Model	174.557	266	0.656	400.910	0.000
Occupation	34.120	25	1.365	833.780	0.000
Occupation x Education Field	139.625	200	0.698	426.500	0.000
Education Field	0.000	8	0.000	0.010	1.000
Country	0.003	26	0.000	0.060	1.000
Year	0.000	7	0.000	0.000	1.000
Residual	75.033	45840	0.002		
Total	249.591	46106	0.005		

Note: This table reports the results from an ANOVA exercise for the share of workers in each highest-degree education field-occupation cell out of total number of workers who have their highest degree in that education field. The variation in the data are at occupation-education field-country-year level, and we perform the ANOVA exercise along each of these dimensions and the interaction of education fields and occupations. The data for education fields are available for the period of 2003–2010. Number of obs = 46107; Adj. R-squared = 0.698. See Table 3 for the list of occupations and Table 12 for education fields.

Table 12: *The Share of Specific Human Capital in Education Fields*

1-digit ISCED-97: Education Field Name	Share of Specific Human Capital
1: Teacher Training and Education Science	0.722
2: Humanities, Languages, and Arts	0.369
3: Social Sciences, Business, and Law	0.170
4: Science, Mathematics, and Computing	0.277
5: Engineering, Manufacturing, and Construction	0.314
6: Agriculture and Veterinary	0.543
7: Health and Welfare	0.650
8: Services	0.216
9: Unknown	0.296

Note: This table offers for each education field (1-digit ISCED-97) the country-year averaged share of workers who have specific human capital occupation and their highest degree in that education field out of total number of workers who have their highest degrees in that field. The data for education fields are available for the period of 2003–2010. See Table 3 for the assignment of occupations into specific and general human capital types.

Table 13: *Skill-Levels Across Occupations*

	Skill-level (ISCED-97 0-2; 3-4; 5-6)		
Occupation (ISCO-88)	Low-skilled	Medium-skilled	Highly-skilled
11	0.101	0.332	0.574
12	0.093	0.361	0.544
13	0.241	0.490	0.264
21	0.018	0.157	0.832
22	0.007	0.083	0.916
23	0.013	0.116	0.868
24	0.033	0.212	0.751
31	0.093	0.564	0.340
32	0.071	0.477	0.449
33	0.096	0.494	0.418
34	0.111	0.550	0.334
41	0.185	0.641	0.170
42	0.213	0.627	0.157
51	0.306	0.598	0.091
52	0.318	0.590	0.086
61	0.495	0.433	0.066
71	0.377	0.577	0.043
72	0.288	0.645	0.063
73	0.286	0.621	0.097
74	0.391	0.555	0.049
81	0.402	0.535	0.063
82	0.446	0.500	0.047
83	0.444	0.516	0.032
91	0.553	0.395	0.043
92	0.616	0.349	0.052
93	0.540	0.416	0.037

Note: This table offers for each occupation the share of workers in each level of highest attained education out of total number of workers in that occupation, which we have averaged over countries and years. There are three levels of highest attained education: pre-primary to lower-secondary (low-skilled; ISCED-97 0-2), secondary to post-secondary non-tertiary (medium-skilled; ISCED-97 3-4), and tertiary (highly-skilled; ISCED-97 5-6). See Table 3 for the list of occupations.

Table 14: *Skill-Levels of Specific and General Human Capital Types*

		Specific Human Capital				General Human Capital					
Level of Education	Obs.	Mean	SD	Min	Max	Mean	SD	Min	Max	Diff. in Means	SE
Low (ISCED-97 0-2)	402	0.295	0.176	0.031	0.849	0.247	0.149	0.032	0.767	0.048***	(0.012)
Medium (ISCED-97 3-4)	402	0.436	0.176	0.050	0.814	0.503	0.146	0.155	0.810	-0.067***	(0.011)
High (ISCED-97 5-6)	402	0.265	0.089	0.056	0.431	0.245	0.079	0.069	0.508	0.020***	(0.006)

Note: This table offers basic statistics for the share of workers with low-, medium-, and high-level skills/education who have specific human capital occupation out of total employment in specific human capital occupations and the share of workers with low-, medium-, and high-level skills/education who have general human capital occupation out of total employment in general human capital occupations. The data are for all countries and years. There are three levels of highest attained education: pre-primary to lower-secondary (low-skilled; ISCED-97 0-2), secondary to post-secondary non-tertiary (medium-skilled; ISCED-97 3-4), and tertiary (highly-skilled; ISCED-97 5-6). The last two columns of the table use two-sided t-test to test the significance of differences in means. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. See Table 3 for the assignment of occupations into specific and general human capital types.

Table 15: *Professional Status, Number of Jobs, Gender, and Age of Specific and General Human Capital Types*

			Specific Human Capital				General Human Capital					
Variables	Categories	Obs.	Mean	SD	Min	Max	Mean	SD	Min	Max	Diff. in Means	SE
Professional Job Status	Self-employed	415	0.168	0.072	0.046	0.379	0.126	0.058	0.041	0.327	0.041***	(0.005)
	Employee	415	0.799	0.101	0.411	0.936	0.865	0.065	0.640	0.955	-0.067***	(0.006)
	Family Worker	411	0.034	0.040	0.000	0.214	0.009	0.008	0.000	0.038	0.025***	(0.002)
Number of Jobs	1 Job	415	0.949	0.033	0.806	0.996	0.954	0.032	0.725	0.992	-0.005***	(0.002)
	More than 1 Jobs	415	0.050	0.033	0.004	0.194	0.045	0.029	0.007	0.176	0.005***	(0.002)
Gender	Male	415	0.543	0.058	0.394	0.705	0.565	0.042	0.490	0.699	-0.022***	(0.004)
	Female	415	0.457	0.058	0.295	0.606	0.435	0.042	0.301	0.510	0.022***	(0.004)
Age	17	415	0.037	0.026	0.003	0.101	0.025	0.018	0.003	0.069	0.012***	(0.002)
	22	415	0.092	0.019	0.047	0.154	0.083	0.017	0.045	0.157	0.009***	(0.001)
	27	415	0.124	0.018	0.082	0.176	0.126	0.019	0.082	0.166	-0.002***	(0.001)
	32	415	0.132	0.017	0.092	0.180	0.138	0.018	0.095	0.179	-0.006***	(0.001)
	37	415	0.134	0.014	0.098	0.172	0.141	0.015	0.100	0.177	-0.006***	(0.001)
	42	415	0.133	0.013	0.093	0.172	0.138	0.014	0.110	0.177	-0.005***	(0.001)
	47	415	0.127	0.014	0.096	0.169	0.130	0.016	0.093	0.176	-0.003***	(0.001)
	52	415	0.108	0.016	0.066	0.143	0.110	0.017	0.068	0.154	-0.003***	(0.001)
	57	415	0.075	0.019	0.036	0.117	0.075	0.019	0.027	0.122	0.000	(0.001)
	62	415	0.037	0.018	0.004	0.091	0.033	0.017	0.005	0.088	0.004***	(0.001)

Note: This table offers basic statistics for the share of workers in each category of variables Professional Job Status, Number of Jobs, Gender, Age who have specific human capital occupation out of total employment in specific human capital occupations and the share of workers who have general human capital occupation out of total employment in general human capital occupations. The data are for all countries and years. The last two columns of the table use two-sided t-test to test the significance of differences in means. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. See Table 3 for the assignment of occupations into specific and general human capital types.

Table 16: *Correlations Among the Share of Specific Human Capital and Employment Shares in Industries with high Share of Specific Human Capital Employment*

	1	2	3
1. The Share of Specific Human Capital			
2. Employment Share in P50 Industries	0.687		
3. Employment Share in P70 Industries	0.332	0.284	
4. Employment Share in P90 Industries	0.775	0.602	0.409

Note: This table offers pairwise correlations among the share of specific human capital and employment shares in industries where the employment share of specific human capital is higher than its 50th, 70th, and 90th percentile in industries (within years and countries). Employment Share in P[] Industries is the share of employment in industries where the employment share of specific human capital is higher than its []th percentile. All correlations are significant at 10% level.

Table 17: *Regression Results for the Share of Specific Human Capital and Employment Shares in Industries with high Share of Specific Human Capital Employment*

	(1)	(2)	(3)	(4)
Employment Share in P50 Industries	0.362***	0.710***	0.339***	0.760***
	(0.070)	(0.160)	(0.067)	(0.205)
Employment Share in P70 Industries	-0.001	-0.141	0.002	-0.13
	(0.072)	(0.147)	(0.074)	(0.175)
Employment Share in P90 Industries	0.828***	0.527***	0.835***	0.497**
	(0.112)	(0.175)	(0.115)	(0.221)
Country Fixed Effects	N	Y	N	Y
Year Fixed Effects	N	N	Y	Y
Number of Obs.	401	401	401	401
R2	0.680	0.890	0.684	0.893

Note: In regressions reported in this table, the dependent variable is the share of specific human capital. Employment Share in P[] Industries is the share of employment in industries where the employment share of specific human capital is higher than its []th percentile. The variation in the data is at country-year level. Regressions are estimated using the OLS method. Robust standard errors are in parentheses. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level.

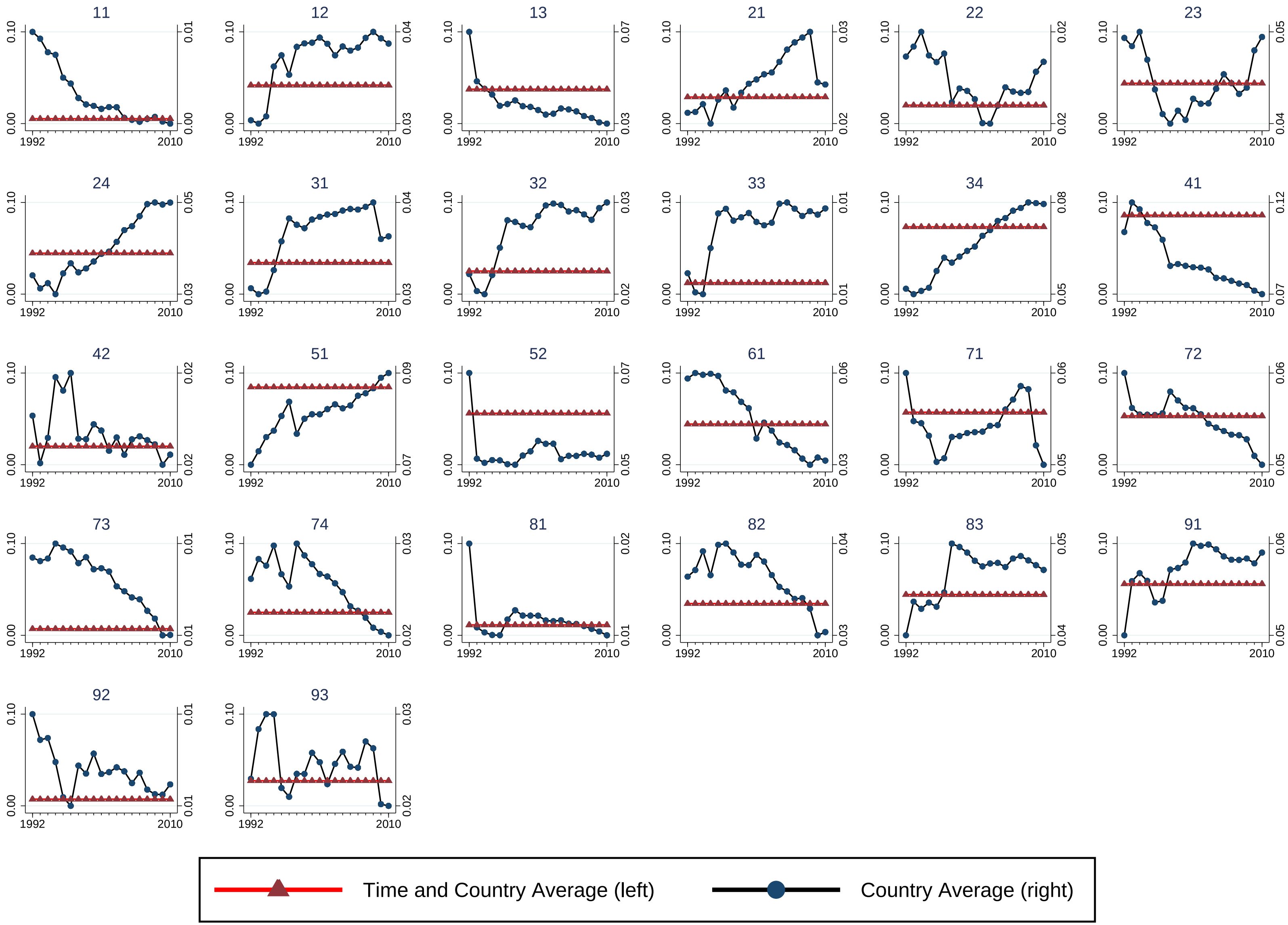
Table 18: *Regression Results for GDP per capita and the Share of Specific Human Capital*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Share of Specific Human Capital	-3.810***	-3.924***	-2.835***	-1.385**	-3.808***	-3.878***	-0.725***	-0.706**
	(0.367)	(0.401)	(0.504)	(0.550)	(0.349)	(0.363)	(0.258)	(0.277)
Share of Medium- and Highly Skilled Employees	N	Y	N	Y	N	Y	N	Y
Country Fixed Effects	N	N	Y	Y	N	N	Y	Y
Year Fixed Effects	N	N	N	N	Y	Y	Y	Y
Number of Obs.	415	402	415	402	415	402	415	402
R2	0.304	0.383	0.925	0.942	0.323	0.404	0.968	0.969

Note: In regressions reported in this table, the dependent variable is the logarithm of real (PPP-adjusted) GDP per capita, which we obtain from the WDI database. The main explanatory variable is the share of specific human capital. In columns (2), (4), (6), and (8) we include as additional explanatory variables the shares of employed individuals who report to have secondary to post-secondary non-tertiary education (medium-skilled; ISCED-97 3-4) and tertiary education (highly-skilled; ISCED-97 5-6) out of total number of employed individuals who report their education level. The variation in the data is at country-year level. Regressions are estimated using the OLS method. Robust standard errors are in parentheses. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. We obtain similar results if we use a third degree polynomial approximation for the share of specific human capital for each country.

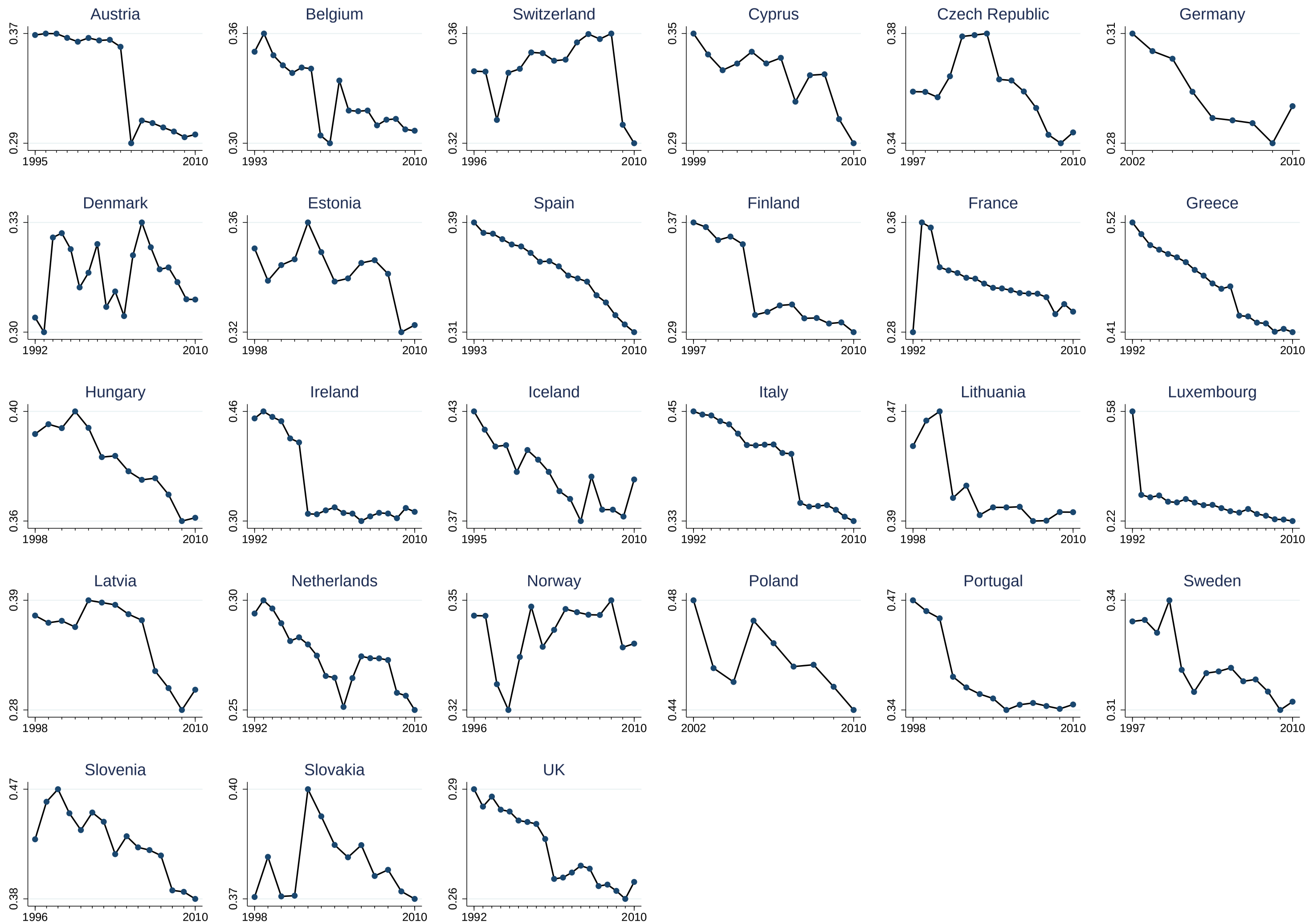
Figures

Figure 1: *The Employment Shares of Occupations*



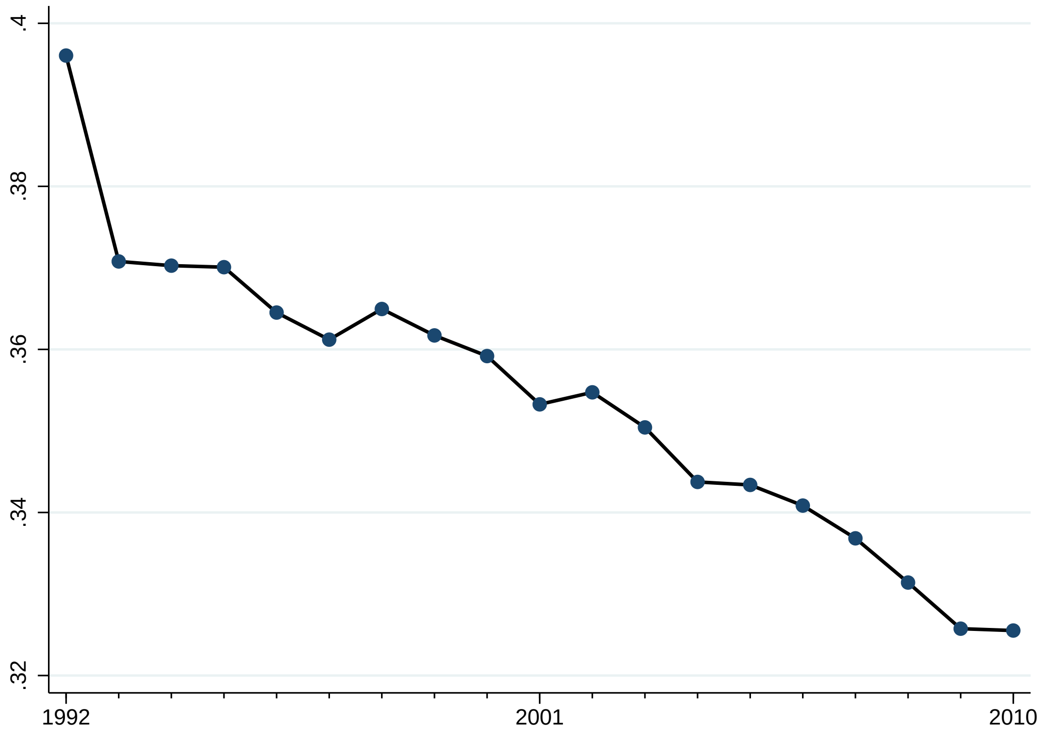
Note: This figure illustrates country- and year-level and country-level averages of the number of workers in 2-digit ISCO-88 occupations out of total employment. See Table 3 for the definitions of occupations.

Figure 2: *The Share of Specific Human Capital in Sample Countries*



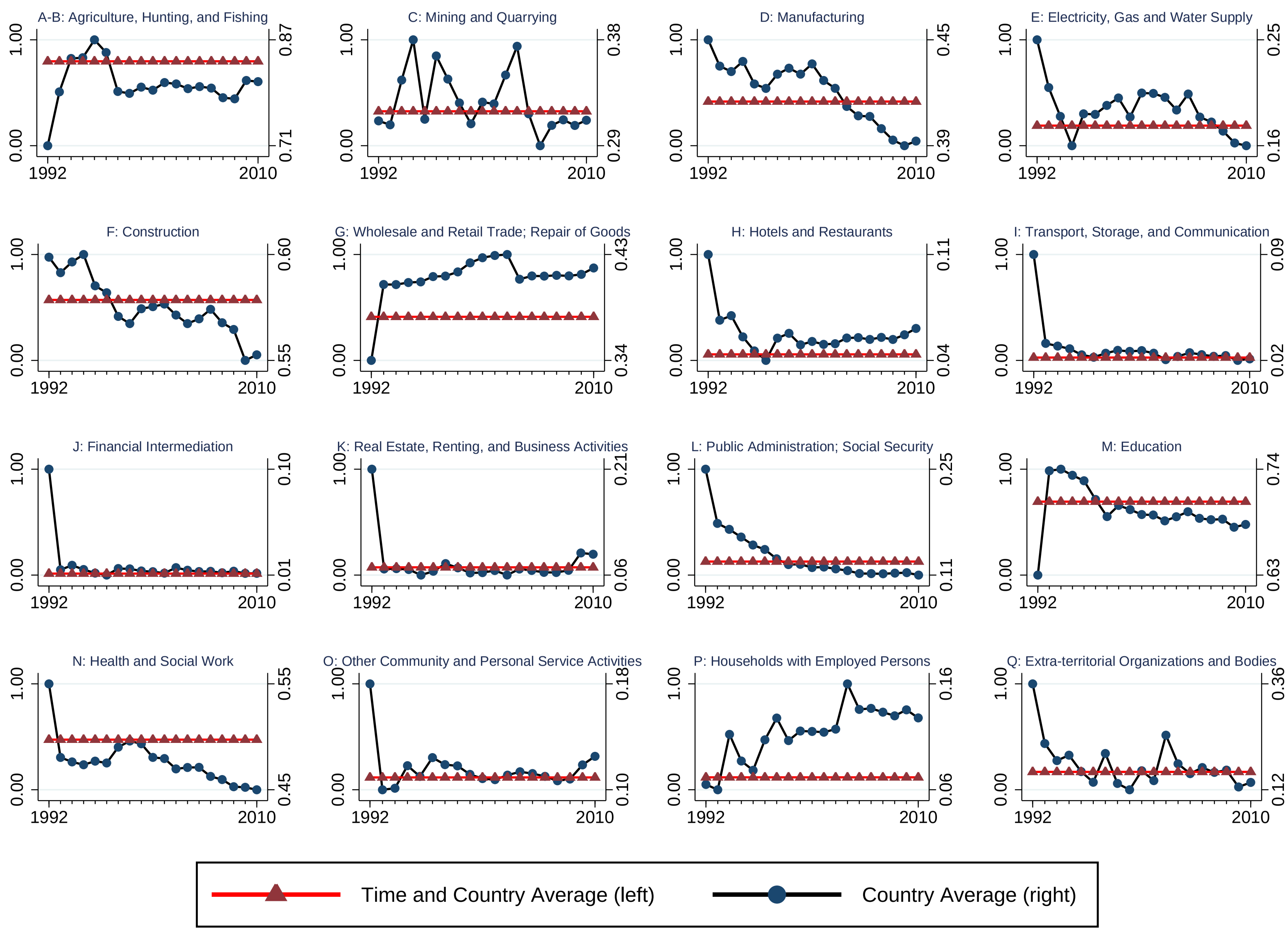
Note: This figure offers the share of workers in specific human capital occupations out of total employment in each country in our sample. See Table 3 for the assignment of occupations into specific and general human capital types. We obtain virtually the same figure when we weight observations using the number of hours worked in the reference week. Vourvachaki et al. (2014) use data from Czech Labor Force Survey (2007Q2) and from Jeong et al. (2008) and find that the share of specific human capital has steadily declined in the Czech Republic in the period of 1994–2007, which somewhat contrasts with this figure. Such a difference can stem from imperfections in sampling weights at this level of disaggregation. For example, in our data there are very large persistent changes in the number of employees in occupations 82 and 93 in between 2000 and 2001. Our results for the Czech Republic are very similar to Vourvachaki et al. (2014) when we predict employment in these occupations before 2001 using their values from 2001 onwards and polynomials of time. The persistence of the assignment indicates that these irregularities are not important. In the Czech Republic, the assignment is time invariant.

Figure 3: *The Average Employment Share of Specific Human Capital in Sample Countries*



Note: This figure offers country-averaged value of the share of specific human capital. Employment weighted average value displays similar negative trend and percentage change over time. We pull countries c and years t and run a regression of the following form: $\textit{Share of Specific Human Capital}_{c,t} = \alpha + \beta t + \eta_{c,t}$. The coefficient in front of time trend t is highly significant and negative. See Table 3 for the assignment of occupations into specific and general human capital types.

Figure 4: *The Average Employment Share of Specific Human Capital in Industries*



Note: For each industry, this figure offers the country- and year-average and country-average share of employed individuals in specific human capital occupations out of total employment in the industry. See Table 3 for the assignment of occupations into specific and general human capital types.

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Appendix Further Results

Appendix - Abstract, Manual, and Routine Skills

By definition and identification our classification of human capital types is different than the abstract, manual, and routine skills classification used by Autor and Dorn (2013). Nevertheless, we check if the trends observed for the share of specific human capital repeat for the shares of employment in occupations requiring abstract, manual, and routine skills. We match our 2-digit ISCO-88 occupations with 5 groups of occupations in Table 2 of Autor and Dorn (2013) and assign occupations into abstract, manual, and routine types. Table 38 in Data Appendix presents the assignment.[18] According to it, 11 occupations require abstract skills, 7 require manual skills, and 8 require routine skills. Out of the first set 5 are specific human capital occupations. Out of each of the second and third sets 4 are specific human capital occupations. This uniformity should not be very surprising. In our classification, skill levels are quite uniform according to Table 14. In contrast, according to Figure 2 of Autor and Dorn (2013), skill levels correlate negatively with the order: abstract, manual, and routine skills.[19]

We compute employment shares in occupations requiring abstract, manual, and routine skills for each country and year. Table 39 in Data Appendix offers the basic statistics for these shares. Table 40 in Data Appendix offers correlations among these shares and the share of specific human capital. Further, we take the averages of these shares across sample countries and illustrate their behavior over time, together with the share of specific human capital, in Figure 7 of Data Appendix. The share of specific human capital is firmly positively correlated with the share of employment in occupations requiring routine skills and negatively correlated with the share of employment in occupations requiring abstract skills. It is also positively correlated with the share of employment in occupations requiring manual skills. We obtain very similar correlations if we exclude occupations requiring either routine or manual skills from occupations corresponding to specific human capital.

Appendix - Elasticity of Substitution

The values of the elasticity of substitution between h- and l-goods ε_1 is decisive for the most of our results. In this section, we present an attempt to estimate ε_1.

Following current practice (e.g., see Herrendorf et al., 2013), we use equations (2), (3), and (4) to estimate ε_1:

$$Y = \lambda \left[\gamma_1 Y_h^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1) Y_l^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}},$$

$$p_{Y_h} = \omega_{Y_h}^{Y} \frac{Y}{Y_h},$$

$$p_{Y_l} = \left(1 - \omega_{Y_h}^{Y}\right) \frac{Y}{Y_l}.$$

In these equations, Y_l is the real output in sectors which are (very) intensive in general human capital input and Y_h is the real output in the remaining sectors. Variable λ is

[18] This assignment can be noisy because of differences in occupation coding and matching between ISCO-88 and 5 groups of occupations in Table 2 of Autor and Dorn (2013).

[19] Autor and Dorn (2013) use wage data to measure skill levels.

productivity level, and p_h and p_l are the relative prices of Y_h and Y_l. We assume that $\ln \lambda(t)$ is a smooth function of time so that it can be represented by a polynomial of the following form

$$g(t) = \delta_1 t + \delta_2 t^2 + \delta_3 t^3,$$

where $\{\delta_i\}$ are real numbers.

We normalize output Y and price levels dividing them to their geometric averages. Further, we take the logarithms of these equations and take the first differences of price equations so that we get

$$\ln\left(\frac{Y}{\bar{Y}}\right) = \frac{\varepsilon_1}{\varepsilon_1 - 1}\ln\left[\overline{\omega_{Y_h}^Y}\left(\frac{Y_h}{\bar{Y}_h}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \overline{\left(1-\omega_{Y_h}^Y\right)}\left(\frac{Y_l}{\bar{Y}_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right] + \tilde{g}(t), \tag{21}$$

$$\Delta\ln\left(\frac{Y/\bar{Y}}{Y_h/\bar{Y}_h}\right) = \varepsilon_1\Delta\ln\left(\frac{p_h}{\bar{p}_h}\right) + (1-\varepsilon_1)\Delta\tilde{g}(t), \tag{22}$$

$$\Delta\ln\left(\frac{Y/\bar{Y}}{Y_l/\bar{Y}_l}\right) = \varepsilon_1\Delta\ln\left(\frac{p_l}{\bar{p}_l}\right) + (1-\varepsilon_1)\Delta\tilde{g}(t), \tag{23}$$

where we use bars to denote geometric averages and

$$\tilde{g}(t) = g(t) - \frac{1}{T}\sum_{t=1}^{T}\tilde{g}(t),$$

$$\overline{\omega_{Y_h}^Y} = \gamma_1\left(\frac{\exp(\tilde{g}(t))\bar{Y}_h}{\bar{Y}}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}},$$

$$\overline{\left(1-\omega_{Y_h}^Y\right)} = (1-\gamma_1)\left(\frac{\exp(\tilde{g}(t))\bar{Y}_l}{\bar{Y}}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}.$$

We use data from the EU KLEMS database and estimate this system of equations jointly for each sample country. The identification of parameters is based on within country variation. In this database, there are no data for Iceland, Norway, and Switzerland. There are data for Australia, Korea, Japan, and the US, however. We carry our estimations also for these countries.

We use the total industrial real output for Y and real output in h- and l-sectors for Y_h and Y_l. In line with Table 7, h-sectors are NACE industries A-B, C, D, E, F, G, L, M, and N, and l-sectors are industries H, I, J-K, O, and P.[20] The data for prices and nominal output in the EU KLEMS database are at 1-digit level of aggregation. We use (current) nominal value-added weights to aggregate prices and obtain prices in h- and l-sectors:

$$p_h = \sum_{i\in h}\frac{p_i Y_i}{pY}p_i,$$

$$p_l = \sum_{i\in l}\frac{p_i Y_i}{pY}p_i.$$

[20] As an auxiliary exercise, we obtain from the EU KLEMS database average hourly wage rates in h- and l-sectors for each country and year. We find that the wage rates tend to be very similar across these industries in sample countries. This serves a further confirmation that our classification of human capital types is rather horizontal.

The base year for prices is 1995 in the EU KLEMS database. For each country, we transform price series so that the base year is the first year in the sample. [21]

For each country, the values of $\overline{\omega_{Y_h}^{Y}}$ and $\overline{\left(1-\omega_{Y_h}^{Y}\right)}$ are obtained using the geometric averages of compensation shares of h- and l-sectors:

$$\overline{\omega_{Y_h}^{Y}} = \overline{\left(\frac{p_h Y_h}{Y}\right)},$$
$$\overline{\left(1-\omega_{Y_h}^{Y}\right)} = \overline{\left(\frac{p_l Y_l}{Y}\right)}.$$

After obtaining the estimate of ε_1 and $g\left(t\right)$ we use equation

$$\gamma_1 = \overline{\left(\frac{p_h Y_h}{Y}\right)} \left(\frac{\exp\left(\frac{1}{T}\sum_{t=1}^{T} g\left(t\right)\right)\bar{Y}_h}{\bar{Y}}\right)^{-\frac{\varepsilon_1 - 1}{\varepsilon_1}} \tag{24}$$

to obtain the estimate of γ_1.

The estimations of equations (21)-(23) are carried using non-linear seemingly unrelated regressions routine in STATA. Tables 41 and 42 summarize our estimation results. The results depend on the initial value of the elasticity that we specify. Usually, the non-linear estimator converges to a point less than one for ε_1 when we specify initial value that is less than 1. It converges to a point greater than 1 otherwise. The less than 1 estimated values of ε_1 seem to be preferable in terms of the Root Square Mean Error.

[21] Clearly, $Y = Y_h + Y_l$ in the first/base year. Therefore, the first sample year is dropped in estimations.

Data Appendix

Data Appendix - Tables

Table 19: *ANOVA for the Coefficient of Variation (CV)*

Source	Partial SS	df	MS	F	P-stat
Model	8317.808	69	120.548	940.610	0.000
Occupation	8176.574	25	327.063	2552.020	0.000
Country	131.939	26	5.075	39.600	0.000
Year	8.774	18	0.487	3.800	0.000
Residual	1366.941	10666	0.128		
Total	9684.748	10735	0.902		

Note: This table reports the results from an ANOVA exercise for the Coefficient of Variation (CV). The variation in the data are at occupation-country-year level, and we perform the ANOVA exercise along each of these dimensions. Number of Obs. = 10736; Adj. R-squared = 0.858.

Table 20: *ANOVA for the Herfindahl Index (HI)*

Source	Partial SS	df	MS	F	P-stat
Model	658.395	69	9.542	724.600	0.000
Occupation	643.871	25	25.755	1955.770	0.000
Country	13.333	26	0.513	38.940	0.000
Year	1.239	18	0.069	5.230	0.000
Residual	140.457	10666	0.013		
Total	798.852	10735	0.074		

Note: This table reports the results from an ANOVA exercise for the Herfindahl Index (HI). The variation in the data are at occupation-country-year level, and we perform the ANOVA exercise along each of these dimensions. Number of Obs. = 10736; Adj. R-squared = 0.823.

Table 21: *ANOVA for the Gini Index*

Source	Partial SS	df	MS	F	P-stat
Model	163.457	69	2.369	1067.090	0.000
Occupation	160.169	25	6.407	2885.940	0.000
Country	3.105	26	0.119	53.800	0.000
Year	0.136	18	0.008	3.390	0.000
Residual	23.678	10666	0.002		
Total	187.135	10735	0.017		

Note: This table reports the results from an ANOVA exercise for the Gini Index. The variation in the data are at occupation-country-year level, and we perform the ANOVA exercise along each of these dimensions. Number of Obs. = 10736; Adj. R-squared = 0.873.

Table 22: *ANOVA for the Theil Index*

Source	Partial SS	df	MS	F	P-stat
Model	4219.624	69	61.154	903.920	0.000
Occupation	4128.522	25	165.141	2440.950	0.000
Country	84.644	26	3.256	48.120	0.000
Year	6.222	18	0.346	5.110	0.000
Residual	721.602	10666	0.068		
Total	4941.226	10735	0.460		

Note: This table reports results from an ANOVA exercise for the Theil Index. The variation in the data are at occupation-country-year level, and we perform the ANOVA exercise along each of these dimensions. Number of Obs. = 10736; Adj. R-squared = 0.853.

Table 23: *ANOVA for Generalized Entropy Index (GE)*

Source	Partial SS	df	MS	F	P-stat
Model	41179.771	69	596.808	727.670	0.000
Occupation	40277.858	25	1611.114	1964.380	0.000
Country	843.607	26	32.446	39.560	0.000
Year	58.375	18	3.243	3.950	0.000
Residual	8747.871	10666	0.820		
Total	49927.642	10735	4.651		

Note: This table reports the results from an ANOVA exercise for the Generalized Entropy Index (with parameter 2; GE). The variation in the data are at occupation-country-year level, and we perform the ANOVA exercise along each of these dimensions. Number of Obs. = 10736; Adj. R-squared = 0.824.

Table 24: *Rank Correlations Among Concentration Measures*

Variable	1	2	3	4
1 Herfindahl Index (HI)				
2 Coefficient of Variation (CV)	0.999			
3 Gini Index	0.987	0.987		
4 Theil Index	0.994	0.994	0.998	
5 Generalized Entropy Index (GE)	0.999	1.000	0.987	0.994

Note: This table offers pairwise rank correlations among the concentration measures computed for the within-occupation employment shares distribution. The measures are Herfindahl index (HI), coefficient of variation (CV), and Gini, Theil, and Generalized Entropy (with parameter 2; GE) indices. The variation in the data are at occupation-country-year level. Number of Obs. = 10736; All correlations are significant at 1% level.

Table 25: *The Employment Shares of Occupations in Years*

Occupation (ISCO-88)	Year 1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
11	0.011	0.011	0.009	0.009	0.007	0.007	0.006	0.005	0.005	0.005	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004
12	0.032	0.032	0.033	0.040	0.041	0.039	0.043	0.043	0.043	0.044	0.043	0.041	0.043	0.042	0.042	0.044	0.045	0.044	0.043
13	0.072	0.050	0.046	0.044	0.039	0.039	0.041	0.038	0.038	0.037	0.035	0.035	0.037	0.037	0.036	0.034	0.033	0.031	0.031
21	0.025	0.025	0.026	0.023	0.026	0.027	0.025	0.027	0.028	0.028	0.029	0.029	0.030	0.032	0.033	0.033	0.034	0.028	0.028
22	0.022	0.023	0.023	0.022	0.022	0.022	0.019	0.020	0.020	0.019	0.018	0.018	0.019	0.020	0.020	0.020	0.020	0.021	0.022
23	0.047	0.046	0.047	0.046	0.044	0.042	0.042	0.042	0.042	0.043	0.043	0.043	0.044	0.045	0.044	0.044	0.044	0.046	0.047
24	0.037	0.034	0.035	0.033	0.037	0.040	0.038	0.039	0.040	0.042	0.042	0.044	0.047	0.048	0.050	0.053	0.053	0.053	0.053
31	0.027	0.026	0.026	0.029	0.032	0.035	0.034	0.034	0.035	0.035	0.036	0.036	0.036	0.036	0.036	0.037	0.037	0.033	0.033
32	0.021	0.019	0.019	0.021	0.023	0.025	0.025	0.025	0.025	0.026	0.027	0.027	0.027	0.026	0.026	0.026	0.025	0.026	0.027
33	0.009	0.008	0.008	0.011	0.013	0.013	0.012	0.013	0.013	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.013	0.013
34	0.056	0.054	0.056	0.057	0.062	0.066	0.065	0.067	0.068	0.070	0.073	0.075	0.078	0.079	0.081	0.082	0.084	0.084	0.083
41	0.105	0.121	0.117	0.110	0.108	0.101	0.087	0.088	0.087	0.086	0.086	0.085	0.080	0.080	0.079	0.077	0.077	0.074	0.072
42	0.021	0.019	0.020	0.023	0.022	0.023	0.020	0.020	0.021	0.020	0.020	0.020	0.019	0.020	0.020	0.020	0.020	0.019	0.019
51	0.073	0.076	0.079	0.080	0.083	0.086	0.079	0.082	0.083	0.083	0.084	0.085	0.084	0.085	0.087	0.087	0.088	0.091	0.092
52	0.073	0.055	0.054	0.055	0.054	0.054	0.054	0.056	0.056	0.059	0.058	0.058	0.055	0.055	0.055	0.056	0.056	0.055	0.056
61	0.058	0.059	0.059	0.059	0.058	0.054	0.054	0.051	0.050	0.041	0.046	0.043	0.040	0.039	0.038	0.036	0.034	0.036	0.035
71	0.064	0.058	0.058	0.056	0.053	0.053	0.056	0.056	0.056	0.057	0.057	0.057	0.057	0.059	0.060	0.062	0.062	0.055	0.053
72	0.063	0.056	0.055	0.055	0.055	0.055	0.059	0.057	0.056	0.056	0.055	0.053	0.052	0.052	0.051	0.051	0.050	0.047	0.046
73	0.008	0.008	0.008	0.009	0.009	0.009	0.008	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.006	0.006	0.005	0.005
74	0.027	0.029	0.028	0.031	0.027	0.026	0.031	0.030	0.028	0.027	0.027	0.026	0.025	0.023	0.022	0.021	0.020	0.020	0.019
81	0.021	0.011	0.010	0.010	0.010	0.012	0.013	0.012	0.012	0.012	0.012	0.011	0.012	0.011	0.011	0.011	0.010	0.010	0.010
82	0.035	0.036	0.038	0.035	0.039	0.039	0.038	0.036	0.036	0.038	0.037	0.035	0.034	0.034	0.033	0.033	0.032	0.029	0.029
83	0.037	0.041	0.040	0.041	0.040	0.042	0.047	0.047	0.046	0.045	0.045	0.045	0.045	0.045	0.046	0.046	0.045	0.045	0.044
91	0.047	0.054	0.055	0.054	0.051	0.051	0.055	0.055	0.056	0.058	0.058	0.058	0.058	0.057	0.056	0.056	0.057	0.056	0.057
92	0.011	0.009	0.009	0.008	0.006	0.006	0.008	0.007	0.009	0.007	0.008	0.008	0.008	0.007	0.007	0.007	0.006	0.006	0.007
93	0.027	0.030	0.031	0.031	0.026	0.025	0.027	0.027	0.028	0.028	0.026	0.028	0.028	0.027	0.027	0.029	0.029	0.025	0.025

Note: We compute the share of workers in each occupation (2-digit ISCO-88) out of total employment for each country and year. This table reports country-level average of these shares. See Table 3 for the names of occupations.

Table 26: *Correlations for the Share of Specific Human Capital*

Country	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1 Austria																										
2 Belgium	0.58																									
3 Cyprus	0.61	0.53																								
4 Czech Republic	0.63	0.12	0.59																							
5 Denmark	-0.06	0.24	0.43	0.06																						
6 Estonia	0.62	0.40	0.65	0.75	0.00																					
7 Finland	0.77	0.43	0.59	0.27	-0.03	0.30																				
8 France	0.82	0.80	0.73	0.54	0.15	0.57	0.84																			
9 Germany	0.74	0.83	0.61	0.90	0.19	0.64	0.55	0.68																		
10 Greece	0.94	0.80	0.73	0.53	-0.11	0.56	0.89	0.53	0.87																	
11 Hungary	0.85	0.40	0.79	0.77	0.07	0.72	0.78	0.87	0.87	0.88																
12 Iceland	0.79	0.59	0.17	0.36	0.05	0.42	0.79	0.81	0.47	0.86	0.68															
13 Ireland	0.49	0.76	-0.33	-0.01	-0.08	-0.14	0.48	0.53	-0.36	0.81	0.07	0.73														
14 Italy	0.97	0.74	0.69	0.63	-0.09	0.64	0.83	0.56	0.82	0.98	0.91	0.85	0.75													
15 Latvia	0.59	0.43	0.70	0.84	0.36	0.72	0.43	0.78	0.83	0.65	0.78	0.45	-0.25	0.64												
16 Lithuania	0.69	0.43	0.50	0.15	-0.10	0.22	0.84	0.74	0.63	0.79	0.64	0.71	0.17	0.71	0.38											
17 Luxembourg	0.85	0.79	0.79	0.55	-0.26	0.59	0.90	-0.14	0.77	0.69	0.90	0.81	0.58	0.59	0.74	0.76										
18 Netherlands	0.37	0.75	0.53	0.10	0.19	0.27	0.59	0.59	-0.03	0.78	0.38	0.53	0.81	0.68	0.48	0.31	0.57									
19 Norway	-0.39	-0.41	-0.20	0.07	0.10	0.03	-0.43	-0.30	-0.15	-0.38	-0.32	-0.28	0.21	-0.31	-0.12	-0.79	-0.36	0.01								
20 Poland	0.56	0.77	0.59	0.61	-0.09	0.76	0.52	0.62	0.43	0.54	0.71	0.14	-0.11	0.54	0.62	0.66	0.80	0.15	0.08							
21 Portugal	0.74	0.53	0.56	0.16	-0.02	0.28	0.91	0.77	0.86	0.86	0.67	0.77	0.15	0.77	0.37	0.94	0.82	0.42	-0.80	0.35						
22 Slovakia	0.15	0.40	0.42	0.55	0.21	0.54	-0.37	0.15	0.85	0.06	0.21	-0.17	-0.38	0.14	0.55	-0.19	0.04	-0.09	0.11	0.72	-0.27					
23 Slovenia	0.75	0.57	0.75	0.59	0.13	0.68	0.83	0.86	0.74	0.81	0.92	0.64	0.26	0.78	0.78	0.64	0.92	0.62	-0.32	0.72	0.72	0.13				
24 Spain	0.87	0.78	0.85	0.65	0.15	0.69	0.84	0.85	0.83	0.96	0.95	0.78	0.70	0.94	0.84	0.69	0.98	0.79	-0.25	0.71	0.75	0.21	0.91			
25 Sweden	0.68	0.43	0.67	0.23	0.09	0.36	0.90	0.81	0.46	0.82	0.69	0.70	0.37	0.74	0.52	0.88	0.83	0.66	-0.43	0.37	0.89	-0.24	0.75	0.80		
26 Switzerland	-0.01	-0.09	0.63	0.41	0.28	0.59	-0.10	0.06	0.22	-0.06	0.31	-0.20	-0.07	0.04	0.39	-0.19	0.06	0.22	0.52	0.65	-0.25	0.49	0.15	0.18	0.00	
27 UK	0.73	0.86	0.67	0.17	0.07	0.28	0.87	0.53	0.78	0.93	0.65	0.81	0.82	0.87	0.53	0.88	0.66	0.85	-0.42	0.27	0.93	-0.13	0.70	0.89	0.92	-0.16

Note: This table offers pairwise correlations among the shares of specific human capital in sample countries. The variation in the data is at country-year level. See Table 3 for the assignment of occupations into specific and general human capital types.

Table 27: *Rank Correlations for the Share of Specific Human Capital in Industries*

Country	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1 Austria																										
2 Belgium	0.97																									
3 Cyprus	0.94	0.94																								
4 Czech Republic	0.95	0.95	0.92																							
5 Denmark	0.97	0.97	0.97	0.94																						
6 Estonia	0.94	0.93	0.93	0.94	0.94																					
7 Finland	0.94	0.93	0.90	0.91	0.94	0.92																				
8 France	0.94	0.94	0.91	0.93	0.93	0.93	0.95																			
9 Germany	0.97	0.96	0.94	0.95	0.97	0.94	0.95	0.94																		
10 Greece	0.96	0.97	0.95	0.96	0.97	0.95	0.94	0.96	0.97																	
11 Hungary	0.97	0.97	0.96	0.96	0.98	0.96	0.95	0.94	0.97	0.98																
12 Iceland	0.89	0.88	0.89	0.84	0.91	0.87	0.90	0.88	0.91	0.88	0.89															
13 Ireland	0.86	0.86	0.83	0.89	0.86	0.84	0.89	0.87	0.87	0.86	0.87	0.81														
14 Italy	0.95	0.94	0.92	0.94	0.94	0.93	0.95	0.95	0.96	0.97	0.95	0.89	0.87													
15 Latvia	0.93	0.92	0.91	0.94	0.93	0.92	0.90	0.92	0.91	0.93	0.94	0.80	0.82	0.91												
16 Lithuania	0.94	0.95	0.92	0.96	0.93	0.95	0.91	0.92	0.94	0.96	0.95	0.84	0.83	0.94	0.94											
17 Luxembourg	0.95	0.96	0.92	0.94	0.94	0.91	0.92	0.95	0.96	0.96	0.94	0.89	0.86	0.93	0.88	0.92										
18 Netherlands	0.92	0.92	0.91	0.93	0.93	0.89	0.90	0.90	0.90	0.91	0.93	0.82	0.92	0.89	0.90	0.88	0.89									
19 Norway	0.92	0.92	0.90	0.89	0.94	0.90	0.97	0.93	0.94	0.93	0.94	0.90	0.87	0.94	0.89	0.90	0.91	0.89								
20 Poland	0.93	0.91	0.91	0.92	0.93	0.92	0.95	0.92	0.96	0.93	0.95	0.89	0.87	0.96	0.90	0.92	0.90	0.87	0.94							
21 Portugal	0.93	0.94	0.88	0.90	0.92	0.92	0.94	0.95	0.94	0.95	0.93	0.88	0.85	0.92	0.87	0.91	0.94	0.87	0.93	0.88						
22 Slovakia	0.93	0.92	0.90	0.97	0.92	0.93	0.91	0.92	0.93	0.93	0.94	0.83	0.92	0.92	0.92	0.92	0.91	0.93	0.89	0.91	0.88					
23 Slovenia	0.94	0.94	0.93	0.95	0.94	0.92	0.92	0.93	0.95	0.96	0.95	0.88	0.84	0.95	0.93	0.95	0.93	0.87	0.92	0.94	0.89	0.91				
24 Spain	0.95	0.96	0.92	0.95	0.95	0.95	0.95	0.96	0.96	0.98	0.96	0.89	0.86	0.95	0.91	0.95	0.96	0.89	0.93	0.91	0.97	0.92	0.94			
25 Sweden	0.94	0.93	0.91	0.90	0.94	0.93	0.97	0.96	0.95	0.95	0.94	0.92	0.87	0.95	0.88	0.90	0.93	0.88	0.96	0.93	0.96	0.89	0.92	0.96		
26 Switzerland	0.95	0.93	0.94	0.92	0.96	0.92	0.94	0.95	0.93	0.94	0.95	0.87	0.86	0.93	0.93	0.91	0.91	0.92	0.92	0.93	0.89	0.91	0.92	0.92	0.93	
27 UK	0.96	0.95	0.92	0.91	0.96	0.94	0.97	0.95	0.97	0.96	0.96	0.91	0.88	0.95	0.90	0.92	0.94	0.91	0.96	0.94	0.96	0.91	0.92	0.96	0.97	0.94

Note: This table offers the rank correlations among the shares of specific human capital in industries (1-digit NACE) in sample countries and years. In a country, the variation in the data is at industry-year level. All correlations are significant at 1% level. See Table 3 for the assignment of occupations into specific and general human capital types.

Table 28: *Industry-Level Decomposition of the Trends in the Share of Specific Human Capital for Each Year*

		Between Industries				Within Industries				Total			
Year	Obs.	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max
1993	8	-0.008	0.011	-0.023	0.011	-0.016	0.101	-0.250	0.100	-0.025	0.105	-0.273	0.084
1994	10	-0.002	0.004	-0.007	0.004	0.001	0.009	-0.006	0.024	-0.001	0.011	-0.011	0.026
1995	10	-0.003	0.004	-0.008	0.007	-0.003	0.008	-0.025	0.005	-0.006	0.010	-0.030	0.006
1996	12	-0.003	0.003	-0.012	0.001	-0.004	0.007	-0.022	0.006	-0.007	0.008	-0.026	0.001
1997	15	-0.002	0.004	-0.013	0.008	0.001	0.008	-0.007	0.022	-0.002	0.009	-0.011	0.030
1998	18	-0.004	0.006	-0.016	0.004	-0.004	0.024	-0.092	0.019	-0.008	0.026	-0.107	0.011
1999	24	-0.002	0.005	-0.017	0.008	0.000	0.006	-0.014	0.017	-0.003	0.009	-0.019	0.020
2000	25	-0.002	0.005	-0.013	0.009	0.000	0.010	-0.025	0.019	-0.003	0.010	-0.032	0.014
2001	25	-0.006	0.021	-0.080	0.011	0.000	0.008	-0.016	0.017	-0.006	0.021	-0.072	0.015
2002	25	-0.001	0.006	-0.012	0.013	-0.001	0.014	-0.043	0.030	-0.002	0.016	-0.046	0.034
2003	27	-0.002	0.007	-0.022	0.017	-0.002	0.007	-0.015	0.017	-0.004	0.010	-0.026	0.017
2004	26	-0.002	0.006	-0.017	0.007	-0.004	0.015	-0.054	0.014	-0.007	0.018	-0.069	0.014
2005	27	-0.001	0.007	-0.021	0.019	0.000	0.005	-0.011	0.009	0.000	0.008	-0.014	0.022
2006	27	0.000	0.006	-0.011	0.022	-0.002	0.006	-0.018	0.008	-0.003	0.008	-0.022	0.025
2007	27	-0.001	0.003	-0.006	0.006	-0.003	0.010	-0.045	0.008	-0.004	0.011	-0.051	0.013
2008	27	-0.003	0.004	-0.019	0.002	-0.002	0.005	-0.016	0.006	-0.005	0.007	-0.028	0.004
2009	27	-0.008	0.006	-0.026	0.000	0.002	0.010	-0.015	0.038	-0.006	0.010	-0.029	0.015
2010	27	-0.002	0.003	-0.013	0.006	0.001	0.007	-0.012	0.020	0.000	0.008	-0.012	0.021

Note: This table offers the basic statistics for between- and within-industry decomposition of changes in the share of specific human capital (1) for each year. Columns 1-4 and 5-8 offer the basic statistics of between- and within-industry components. Columns 9-12 offer the basic statistics of total change in the share of specific human capital. Figures 4 and 2 show that there are spikes in the share of specific human capital. These spikes can stem from imperfections in sampling weights at this level of disaggregation. For example, in our data there are very large persistent changes in the number of employees in occupations 82 and 93 in between 2000 and 2001 in the Czech Republic. Such imperfections can bias these decompositions in ambiguous directions. The effects such biases are likely to be alleviated when we take country and/or year averages.

Table 29: *The Share of Workers in Education Fields and Countries*

	Education Field (ISCED-97)								
	1	2	3	4	5	6	7	8	9
Country									
Austria	0.043	0.039	0.283	0.012	0.368	0.049	0.039	0.120	0.003
Belgium	0.082	0.037	0.238	0.062	0.251	0.020	0.104	0.050	0.014
Cyprus	0.049	0.099	0.293	0.059	0.105	0.003	0.040	0.061	0.005
Czech Republic	0.027	0.018	0.189	0.014	0.522	0.054	0.037	0.091	0.000
Denmark	0.024	0.036	0.269	0.041	0.264	0.035	0.147	0.047	0.000
Estonia	0.055	0.021	0.124	0.015	0.349	0.048	0.060	0.099	0.001
Finland	0.021	0.020	0.211	0.023	0.298	0.019	0.135	0.095	0.000
France	0.006	0.085	0.329	0.073	0.300	0.042	0.097	0.052	0.008
Germany	0.062	0.031	0.297	0.018	0.310	0.028	0.095	0.061	0.000
Greece	0.041	0.093	0.214	0.052	0.248	0.023	0.088	0.020	0.002
Hungary	0.070	0.016	0.210	0.018	0.464	0.041	0.047	0.078	0.000
Iceland	0.018	0.040	0.167	0.037	0.270	0.015	0.084	0.080	0.004
Ireland	0.036	0.063	0.252	0.100	0.158	0.019	0.083	0.083	0.006
Italy	0.056	0.087	0.317	0.074	0.245	0.024	0.057	0.050	0.091
Latvia	0.050	0.052	0.179	0.027	0.309	0.039	0.043	0.078	0.000
Lithuania	0.064	0.022	0.180	0.022	0.401	0.058	0.062	0.089	0.001
Luxembourg	0.053	0.058	0.158	0.067	0.134	0.015	0.064	0.057	0.023
Netherlands	0.093	0.026	0.305	0.028	0.195	0.033	0.179	0.093	0.012
Norway	0.124	0.082	0.218	0.053	0.094	0.025	0.085	0.022	0.069
Poland	0.036	0.023	0.200	0.043	0.405	0.073	0.025	0.080	0.002
Portugal	0.052	0.143	0.233	0.171	0.109	0.011	0.012	0.027	0.129
Slovakia	0.030	0.012	0.173	0.016	0.533	0.064	0.039	0.058	0.001
Slovenia	0.047	0.015	0.249	0.008	0.411	0.027	0.030	0.098	0.001
Spain	0.072	0.053	0.312	0.070	0.238	0.021	0.105	0.041	0.015
Sweden	0.054	0.052	0.222	0.017	0.297	0.027	0.173	0.078	0.015
Switzerland	0.042	0.030	0.302	0.029	0.271	0.017	0.054	0.088	
UK	0.022	0.062	0.143	0.059	0.116	0.008	0.072	0.033	0.451

Note: This table offers for each education field (1-digit ISCED-97) the yearly average share of employed individuals who have their highest degree in that education field out of total number of employed individuals (who have reported their highest level of education). The data for education fields are available for the period of 2003–2010. See Table 12 for the names of education fields.

Table 30: *The Share of Workers in Education Fields and Years*

	Education Field (ISCED-97)								
	1	2	3	4	5	6	7	8	9
Year									
2003	0.044	0.049	0.222	0.039	0.282	0.029	0.078	0.066	0.109
2004	0.038	0.053	0.204	0.036	0.272	0.026	0.075	0.055	0.047
2005	0.047	0.048	0.248	0.054	0.295	0.033	0.082	0.075	0.038
2006	0.045	0.055	0.224	0.041	0.288	0.028	0.075	0.069	0.034
2007	0.056	0.047	0.218	0.051	0.290	0.034	0.073	0.062	0.032
2008	0.043	0.045	0.258	0.045	0.292	0.031	0.069	0.071	0.031
2009	0.064	0.048	0.237	0.043	0.297	0.031	0.077	0.067	0.026
2010	0.059	0.045	0.249	0.044	0.251	0.035	0.082	0.073	0.028

Note: This table offers for each education field (1-digit ISCED-97) country-averaged share of employed individuals who have their highest degree in that education field out of total number of employed individuals (who have reported their highest level of education). See Table 12 for the names of education fields.

Table 31: *Within-Education Field Share Across Occupations*

	Education Field (ISCED-97)								
	1	2	3	4	5	6	7	8	9
Occupations (ISCO-88)									
11	0.004	0.006	0.008	0.006	0.004	0.005	0.002	0.009	0.004
12	0.041	0.047	0.087	0.073	0.052	0.042	0.023	0.034	0.048
13	0.017	0.030	0.038	0.027	0.036	0.058	0.013	0.038	0.047
21	0.010	0.015	0.015	0.223	0.104	0.015	0.004	0.008	0.027
22	0.004	0.002	0.002	0.043	0.002	0.085	0.280	0.002	0.009
23	0.558	0.202	0.022	0.125	0.014	0.020	0.019	0.022	0.044
24	0.043	0.181	0.172	0.059	0.017	0.023	0.038	0.020	0.058
31	0.008	0.036	0.016	0.092	0.091	0.025	0.015	0.035	0.042
32	0.009	0.006	0.007	0.021	0.005	0.036	0.296	0.011	0.019
33	0.119	0.018	0.005	0.007	0.004	0.004	0.021	0.009	0.017
34	0.048	0.121	0.199	0.080	0.047	0.055	0.048	0.083	0.092
41	0.031	0.075	0.166	0.067	0.039	0.042	0.028	0.073	0.088
42	0.008	0.019	0.036	0.015	0.007	0.008	0.008	0.033	0.019
51	0.040	0.051	0.051	0.039	0.037	0.044	0.138	0.302	0.096
52	0.016	0.043	0.083	0.031	0.029	0.037	0.018	0.060	0.045
61	0.005	0.012	0.010	0.007	0.021	0.270	0.005	0.025	0.018
71	0.004	0.013	0.007	0.009	0.131	0.028	0.002	0.023	0.045
72	0.003	0.013	0.006	0.012	0.140	0.021	0.002	0.023	0.043
73	0.001	0.033	0.002	0.003	0.007	0.002	0.005	0.003	0.014
74	0.003	0.009	0.006	0.005	0.032	0.012	0.002	0.018	0.018
81	0.001	0.004	0.003	0.006	0.019	0.011	0.001	0.007	0.012
82	0.003	0.020	0.013	0.013	0.043	0.024	0.005	0.023	0.033
83	0.004	0.011	0.009	0.010	0.060	0.058	0.003	0.061	0.042
91	0.016	0.024	0.028	0.020	0.032	0.037	0.019	0.058	0.087
92	0.001	0.001	0.002	0.001	0.004	0.018	0.001	0.004	0.004
93	0.003	0.008	0.009	0.009	0.024	0.023	0.003	0.019	0.028

Note: This table offers for each education field (1-digit ISCED-97) the country-year average share of individuals in each occupation (2-digit ISCO-88) who have their highest degree in that field, out of total number of workers who have their highest degree in that field. See Table 3 for the list of occupations and Table 12 for education fields.

Table 32: *Skill-Levels Across Countries*

Country	Skill-level (ISCED-97 0-2; 3-4; 5-6) Low-skilled	Medium-skilled	Highly-skilled
Austria	0.196	0.648	0.156
Belgium	0.278	0.378	0.344
Cyprus	0.290	0.385	0.325
Czech Republic	0.070	0.789	0.141
Denmark	0.224	0.499	0.277
Estonia	0.102	0.558	0.340
Finland	0.201	0.462	0.337
France	0.290	0.448	0.262
Germany	0.149	0.587	0.264
Greece	0.435	0.357	0.208
Hungary	0.147	0.653	0.199
Iceland	0.433	0.321	0.246
Ireland	0.325	0.374	0.301
Italy	0.465	0.407	0.128
Latvia	0.129	0.634	0.237
Lithuania	0.084	0.563	0.353
Luxembourg	0.362	0.388	0.250
Netherlands	0.275	0.447	0.278
Norway	0.158	0.520	0.323
Poland	0.101	0.683	0.215
Portugal	0.725	0.145	0.130
Slovakia	0.057	0.794	0.149
Slovenia	0.185	0.622	0.193
Spain	0.516	0.200	0.285
Sweden	0.185	0.521	0.293
Switzerland	0.190	0.554	0.255
UK	0.316	0.397	0.286

Note: This table offers for each country the share of workers in each level of highest attained education out of total number of employed individuals, which we have averaged over years. There are three levels of highest attained education: pre-primary to lower-secondary (low-skilled; ISCED-97 0-2), secondary to post-secondary non-tertiary (medium-skilled; ISCED-97 3-4), and tertiary (highly-skilled; ISCED-97 5-6).

Table 33: *Skill-Levels Across Years*

	Skill-level (ISCED-97 0-2; 3-4; 5-6)		
	Low-skilled	Medium-skilled	Highly-skilled
Year			
1992	0.493	0.342	0.164
1993	0.452	0.340	0.208
1994	0.440	0.343	0.218
1995	0.414	0.382	0.204
1996	0.350	0.434	0.217
1997	0.328	0.448	0.224
1998	0.275	0.505	0.220
1999	0.283	0.484	0.233
2000	0.278	0.484	0.238
2001	0.273	0.492	0.234
2002	0.255	0.508	0.237
2003	0.250	0.507	0.243
2004	0.238	0.504	0.258
2005	0.229	0.505	0.267
2006	0.230	0.500	0.271
2007	0.228	0.497	0.275
2008	0.223	0.494	0.283
2009	0.214	0.492	0.294
2010	0.206	0.491	0.302

Note: This table offers for each year the share of workers in each level of highest attained education out of total number of employed individuals, which we have averaged across countries. There are three levels of highest attained education: pre-primary to lower-secondary (low-skilled; ISCED-97 0-2), secondary to post-secondary non-tertiary (medium-skilled; ISCED-97 3-4), and tertiary (highly-skilled; ISCED-97 5-6).

Table 34: *Concentrations of 1-digit ISCO-88 Occupations*

1-digit ISCO-88: Occupation Name Occupations	HI	CV	Gini	Theil	GE	Highly Concentrated
1: Legislators, Senior Officials and Managers	0.151	1.185	0.559	0.577	0.694	0
2: Professionals	0.189	1.449	0.649	0.785	1.000	1
3: Technicians and Associate Professionals	0.132	1.069	0.542	0.531	0.542	0
4: Clerks	0.126	1.027	0.530	0.505	0.499	0
5: Service Workers and Shop and Market Sales Workers	0.241	1.719	0.711	0.987	1.410	1
6: Skilled Agricultural and Fishery Workers	0.781	3.443	0.893	2.222	5.667	1
7: Craft and related Trades Workers	0.306	2.020	0.770	1.236	1.922	1
8: Plant and Machine Operators and Assemblers	0.321	2.062	0.761	1.211	2.037	1
9: Elementary Occupations	0.120	0.953	0.488	0.437	0.444	0

Note: This table offers the assignment of 1-digit ISCO-88 occupations into "highly concentrated" and "not highly concentrated" groups. For each country, year, and occupation, we compute 5 concentration measures (HI, CV, Gini, Theil, GE) for the distribution of within-occupation share across industries. We average the values of concentration measures across countries and years. The country- and year-averaged values of the concentration measures are offered in columns 2-6 of this table. Clearly, the values of concentration measures on average are lower than the values of concentration measures for 2-digit ISCO-88 offered in Table 3. We define dummy variables for each of the concentration measures which are equal to 1 for the values of the concentration measures that are higher than or equal to their medians. We take the average of these dummy variables and define Highly Concentrated dummy variable which is equal to 1 if the average is greater than 0.5, and to 0 otherwise. Column 7 offers the value of this dummy variable for each occupation. We call an occupation highly concentrated if Highly Concentrated dummy variable is equal to 1, not highly concentrated otherwise. Similarly to 2-digit occupations, concentration measures for 1-digit occupations vary a lot across occupations and much less across countries and years and have very high rank correlations. We perform a similar assignment into highly concentrated group for each country and year. This assignment has almost no year and country variation and is highly correlated with the assignment offered in this table ($\rho = 0.764$).

Table 35: *Skill-Levels of Highly Concentrated and Less Concentrated 1-Digit ISCO-88 Occupations*

		Highly Concentrated				Not Highly Concentrated					
Level of Education	Obs.	Mean	SD	Min	Max	Mean	SD	Min	Max	Diff. in Means	SE
Low (ISCED-97 0-2)	402	0.288	0.176	0.03	0.857	0.236	0.139	0.042	0.719	0.052***	(0.011)
Medium (ISCED-97 3-4)	402	0.455	0.178	0.052	0.827	0.508	0.142	0.196	0.797	-0.053***	(0.011)
High (ISCED-97 5-6)	402	0.252	0.08	0.061	0.549	0.251	0.101	0.056	0.577	0.001	(0.006)

Note: This table offers basic statistics for the share of employed individuals with low-, medium-, and high-level skills/education who have highly concentrated 1-digit occupations out of total employment in highly concentrated occupations. It also offers these basic statistics for the share of employed individuals who not have highly concentrated occupations. The data are for all countries and years. There are three levels of highest attained education: pre-primary to lower-secondary (low-skilled; ISCED-97 0-2), secondary to post-secondary non-tertiary (medium-skilled; ISCED-97 3-4), and tertiary (highly-skilled; ISCED-97 5-6). The last two columns of the table test uses two-sided t-test to test the significance of differences in means. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. These results are sensitive to the inclusion of the median concentrated occupations, Professionals, in the group of highly concentrated occupations. See Table 34 for the assignment of occupations into highly concentrated and not highly concentrated groups.

Table 36: *Concentrations of 3-digit ISCO-88 Occupations*

3-digit ISCO-88 Occupations	HI	CV	Gini	Theil	GE	Highly Concentrated	2-digit ISCO-88 Occupations	Specific
111	0.658	3.053	0.849	1.950	4.666	1	11	1
114	0.701	3.216	0.898	2.184	4.932	1	11	1
119	0.673	3.229	0.881	1.997	4.887	1	11	1
121	0.169	1.314	0.612	0.706	0.839	0	12	0
122	0.157	1.233	0.580	0.633	0.745	0	12	0
123	0.183	1.376	0.625	0.752	0.949	0	12	0
131	0.250	1.729	0.696	0.982	1.479	0	13	0
211	0.340	2.142	0.790	1.351	2.205	0	21	0
212	0.236	1.653	0.711	0.988	1.284	0	21	0
213	0.301	1.963	0.736	1.136	1.880	0	21	0
214	0.264	1.820	0.730	1.079	1.593	0	21	0
221	0.268	1.818	0.745	1.163	1.615	0	22	1
222	0.677	3.211	0.890	2.057	4.859	1	22	1
223	0.908	3.769	0.926	2.539	6.669	1	22	1
231	0.844	3.607	0.917	2.420	6.165	1	23	1
232	0.943	3.854	0.927	2.607	6.974	1	23	1
233	0.930	3.820	0.929	2.605	6.864	1	23	1
234	0.640	3.105	0.900	2.129	4.559	1	23	1
235	0.568	2.842	0.842	1.788	3.998	1	23	1
241	0.228	1.632	0.669	0.886	1.304	0	24	0
242	0.502	2.681	0.849	1.741	3.493	0	24	0
243	0.555	2.754	0.871	1.933	3.709	1	24	0
244	0.263	1.744	0.699	1.032	1.583	0	24	0
245	0.407	2.389	0.829	1.559	2.723	0	24	0
246	0.469	2.554	0.865	1.772	3.053	0	24	0
247	0.606	2.961	0.847	1.843	4.299	1	24	0
311	0.213	1.575	0.676	0.882	1.185	0	31	0
312	0.225	1.623	0.683	0.931	1.281	0	31	0
313	0.288	1.905	0.770	1.276	1.772	0	31	0
314	0.662	3.120	0.889	2.086	4.686	1	31	0
315	0.327	2.042	0.744	1.209	2.077	0	31	0
321	0.302	1.957	0.756	1.220	1.871	0	32	1
322	0.526	2.769	0.854	1.757	3.668	1	32	1
323	0.916	3.792	0.925	2.544	6.752	1	32	1
331	0.837	3.603	0.919	2.433	6.156	1	33	1
332	0.743	3.338	0.907	2.279	5.338	1	33	1
333	0.677	3.176	0.897	2.141	4.837	1	33	1
334	0.496	2.641	0.832	1.674	3.433	0	33	1
341	0.268	1.832	0.744	1.129	1.621	0	34	0
342	0.265	1.826	0.727	1.067	1.601	0	34	0
343	0.168	1.276	0.587	0.654	0.832	0	34	0
344	0.549	2.829	0.851	1.762	3.847	1	34	0
345	0.919	3.765	0.926	2.555	6.648	1	34	0
346	0.533	2.756	0.844	1.750	3.761	1	34	0
347	0.313	2.023	0.781	1.295	1.969	0	34	0
411	0.143	1.143	0.563	0.585	0.634	0	41	0
412	0.243	1.586	0.640	0.870	1.429	0	41	0
413	0.265	1.839	0.745	1.135	1.603	0	41	0
414	0.538	2.750	0.835	1.716	3.724	1	41	0

Table 36 – (Continued)

3-digit ISCO-88 Occupations	HI	CV	Gini	Theil	GE	Highly Concentrated	2-digit ISCO-88 Occupations	Specific
419	0.145	1.170	0.563	0.581	0.655	0	41	0
421	0.369	2.219	0.796	1.401	2.423	0	42	0
422	0.190	1.417	0.640	0.809	1.002	0	42	0
512	0.548	2.820	0.831	1.694	3.853	1	51	0
513	0.618	3.028	0.878	1.963	4.395	1	51	0
514	0.764	3.428	0.898	2.210	5.564	1	51	0
516	0.484	2.607	0.819	1.601	3.345	0	51	0
521	0.890	3.728	0.918	2.445	6.507	1	52	1
522	0.785	3.480	0.899	2.228	5.715	1	52	1
523	0.907	3.722	0.926	2.514	6.483	1	52	1
611	0.631	3.025	0.860	1.920	4.505	1	61	1
612	0.864	3.654	0.917	2.449	6.330	1	61	1
613	0.955	3.876	0.929	2.636	7.051	1	61	1
614	0.839	3.597	0.915	2.388	6.088	1	61	1
615	0.909	3.731	0.927	2.540	6.534	1	61	1
711	0.408	2.343	0.826	1.511	2.563	0	71	1
712	0.674	3.200	0.878	2.008	4.843	1	71	1
713	0.558	2.853	0.836	1.723	3.923	1	71	1
714	0.522	2.748	0.850	1.741	3.635	0	71	1
721	0.491	2.666	0.850	1.710	3.387	0	72	0
722	0.699	3.245	0.875	2.045	5.046	1	72	0
723	0.338	2.132	0.784	1.327	2.177	0	72	0
724	0.230	1.658	0.710	0.988	1.318	0	72	0
731	0.534	2.777	0.857	1.811	3.747	1	73	1
732	0.801	3.521	0.914	2.357	5.869	1	73	1
733	0.607	2.982	0.877	1.983	4.289	1	73	1
734	0.731	3.341	0.900	2.191	5.296	1	73	1
741	0.598	2.974	0.886	2.035	4.230	1	74	1
742	0.693	3.219	0.887	2.100	4.990	1	74	1
743	0.681	3.168	0.885	2.085	4.886	1	74	1
744	0.860	3.680	0.925	2.485	6.378	1	74	1
811	0.698	3.225	0.898	2.177	4.975	1	81	1
812	0.938	3.847	0.930	2.612	6.948	1	81	1
813	0.970	3.931	0.935	2.697	7.247	1	81	1
814	0.815	3.534	0.913	2.373	5.929	1	81	1
815	0.794	3.491	0.904	2.297	5.796	1	81	1
816	0.321	2.008	0.759	1.261	2.028	0	81	1
817	0.931	3.841	0.929	2.592	6.924	1	81	1
821	0.844	3.625	0.912	2.388	6.202	1	82	1
822	0.888	3.712	0.924	2.519	6.506	1	82	1
823	0.871	3.677	0.924	2.511	6.399	1	82	1
824	0.696	3.192	0.897	2.168	4.884	1	82	1
825	0.782	3.471	0.906	2.262	5.682	1	82	1
826	0.683	3.161	0.886	2.115	4.902	1	82	1
827	0.790	3.471	0.907	2.314	5.754	1	82	1
828	0.756	3.389	0.896	2.208	5.495	1	82	1
829	0.573	2.910	0.850	1.793	4.051	1	82	1
831	0.730	3.321	0.887	2.154	5.313	1	83	0
832	0.391	2.318	0.769	1.308	2.599	0	83	0
833	0.280	1.885	0.755	1.169	1.718	0	83	0
911	0.584	2.838	0.857	1.923	4.163	1	91	0
912	0.205	1.489	0.640	0.754	1.035	0	91	0

Table 36 – (Continued)

3-digit ISCO-88 Occupations	HI	CV	Gini	Theil	GE	Highly Concentrated	2-digit ISCO-88 Occupations	Specific
913	0.176	1.346	0.619	0.724	0.893	0	91	0
914	0.236	1.657	0.677	0.929	1.373	0	91	0
915	0.193	1.415	0.620	0.761	1.030	0	91	0
916	0.260	1.702	0.676	0.978	1.537	0	91	0
921	0.731	3.321	0.887	2.138	5.283	1	92	1
931	0.598	2.951	0.851	1.843	4.241	1	93	0
932	0.542	2.784	0.839	1.734	3.786	1	93	0
933	0.286	1.920	0.766	1.230	1.759	0	93	0

Note: This table offers the assignment of 3-digit ISCO-88 occupations into "highly concentrated" and "not highly concentrated" groups. The assignment is performed in the following manner. For each country and year, we drop occupations which comprise 1 percent of observations in the sample of employed individuals and occupations which have less than 10 percent employment in the corresponding 2-digit code. We also drop occupation which are coded only at 1- and 2-digit levels. Further, for each country, year, and occupation, we compute 5 concentration measures (HI, CV, Gini, Theil, GE) for the distribution of within-occupation share across industries. We average the values of concentration measures across countries and years. The country- and year-averaged values of the concentration measures are offered in columns 2-6 of this table. Clearly, the values of concentration measures on average are higher than the values of concentration measures for 2-digit ISCO-88 offered in Table 3. We define dummy variables for each of the concentration measures which are equal to 1 for the values of the concentration measures that are higher than their medians. We take the average of these dummy variables and define Highly Concentrated dummy variable which is equal to 1 if the average is greater than 0.5, and to 0 otherwise. Column 7 offers the value of this dummy variable for each occupation. We call an occupation highly concentrated if Highly Concentrated dummy variable is equal to 1, not highly concentrated otherwise. In columns 8 and 9 we offer the corresponding 2-digit ISCO-88 codes and their assignment into specific and general human capital types from Table 3. Similarly to 2-digit occupations, concentration measures for 3-digit occupations vary a lot across occupations and much less across countries and years and have very high rank correlations. We perform a similar assignment into highly concentrated group for each country and year. This assignment has almost no year and country variation and is highly correlated with the assignment offered in this table ($\rho = 0.746$). Poland and Slovenia are excluded from the sample of countries because we do not have 3-digit ISCO-88 for them.

Table 37: *Skill-Levels of Highly Concentrated and Less Concentrated 3-Digit ISCO-88 Occupations*

		Highly Concentrated				Not Highly Concentrated					
Level of Education	Obs.	Mean	SD	Min	Max	Mean	SD	Min	Max	Diff. in Means	SE
Low (ISCED-97 0-2)	378	0.316	0.184	0.039	0.864	0.239	0.141	0.032	0.738	0.077***	(0.012)
Medium (ISCED-97 3-4)	378	0.454	0.179	0.051	0.843	0.485	0.141	0.171	0.796	-0.031***	(0.012)
High (ISCED-97 5-6)	378	0.225	0.075	0.065	0.401	0.272	0.086	0.071	0.536	-0.047***	(0.006)

Note: This table offers basic statistics for the share of employed individuals with low-, medium-, and high-level skills/education who have highly concentrated 3-digit occupations out of total employment in highly concentrated occupations. It also offers these basic statistics for the share of employed individuals who not have highly concentrated occupations. The data are for all countries and years. There are three levels of highest attained education: pre-primary to lower-secondary (low-skilled; ISCED-97 0-2), secondary to post-secondary non-tertiary (medium-skilled; ISCED-97 3-4), and tertiary (highly-skilled; ISCED-97 5-6). The last two columns of the table uses two-sided t-test to test the significance of differences in means. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. See Table 36 for the assignment of occupations into highly concentrated and not highly concentrated groups. Poland and Slovenia are excluded from the sample of countries because we do not have 3-digit ISCO-88 for them.

Table 38: *Assignment of Occupations (ISCO-88) into Groups Requiring Abstract, Manual, and Routine Skills*

Occupations (ISCO-88)	Abstract (A); Manual (M); Routine (R)	Specific (1); General (0)
11	A	1
12	A	0
13	A	0
21	A	0
22	A	1
23	A	1
24	A	0
31	A	0
32	A	1
33	A	1
34	A	0
41	R	0
42	R	0
51	M	0
52	M	1
61	M	1
71	M	1
72	M	0
73	R	1
74	R	1
81	R	1
82	R	1
83	R	0
91	R	0
92	M	1
93	M	0

Note: This table offers the assignment of 2-digit ISCO-88 occupations into abstract (A), manual (M), and routine (R) types. This assignment is performed using the 5 groups of occupations in Table 2 of Autor and Dorn (2013).

Table 39: *Basic Statistics for the Share of Employment in Abstract-, Manual-, and Routine-Skills Occupations*

	Obs.	Mean	SD	Min	Max
Share of Abstract-Skills	415	0.366	0.057	0.214	0.519
Share of Manual-Skills	415	0.285	0.042	0.162	0.386
Share of Routine-Skills	415	0.330	0.051	0.186	0.538
Share of Specific Human Capital	415	0.349	0.057	0.222	0.581

Note: This table offers the basic statistics for the shares of employment in occupations requiring abstract, manual, and routine skills. It also offers basic statistics for the share of specific human capital. The variation in the data is at country-year level. See Table 38 for the assignment of occupations into abstract-, manual-, and routine-skills occupations and into specific and general human capital types.

Table 40: *Correlations Among the Shares of Employment in Abstract-, Manual-, and Routine-Skills Occupations and the Share of Specific Human Capital*

	Within and Between Years			Between Years		
	1	2	3	1	2	3
1. Share of Abstract-Skills						
2. Share of Manual-Skills	-0.550*			-0.909*		
3. Share of Routine-Skills	-0.709*	-0.037		-0.832*	0.733*	
4. Share of Specific Human Capital	-0.646*	0.116*	0.765*	-0.910*	0.817*	0.967*

Note: This table offers pairwise correlations among the shares of employment in occupations requiring abstract, manual, and routine skills and the share of specific human capital. In the first panel, variation in the data is at country-year level, and the number of observations is 413. In the second panel, we take country averages. The variation in the data is at year level, and the number of observations is 19. * indicates significance at the 1% level. See Table 38 for the assignment of occupations into abstract-, manual-, and routine-skills occupations and into specific and general human capital types.

Table 41: *Estimated values of* ε_1 *and* γ_1 *- Initial* $\varepsilon_1 \in (0,1)$

Country	Sample Period	ε_1	δ_1	δ_2	δ_3	γ_1	RMSE EQ. (21)	RMSE EQ. (22)	RMSE EQ. (23)
Austria	1971–2007	0.317***	-0.013***	0.000***	0.000	0.390	0.004	0.006	0.017
Belgium	1971–2007	0.430***	-0.005***	0.000*	0.000	0.461	0.004	0.009	0.020
Cyprus	1996–2007	0.578***	-0.005***	0.001***	0.000	0.410	0.001	0.014	0.022
Czech Republic	1996–2007	0.642***	-0.002	0.000**	0.000	0.631	0.002	0.008	0.020
Denmark	1971–2007	0.458***	0.003***	-0.000***	0.000***	0.482	0.005	0.008	0.018
Estonia	1996–2007	0.751***	0.022***	-0.003***	0.000***	0.611	0.011	0.015	0.032
Finland	1971–2007	0.316***	-0.001	-0.000***	0.000***	0.412	0.003	0.006	0.017
France	1971–2007	0.333***	-0.007***	0.000***	0	0.328	0.004	0.006	0.014
Germany	1971–2007	0.475***	-0.007***	0.000***	-0.000**	0.497	0.004	0.008	0.021
Greece	1971–2007	0.554***	-0.003***	-0.000**	0.000***	0.528	0.004	0.009	0.023
Hungary	1992–2007	0.264	-0.016***	0.002***	-0.000***	0.354	0.004	0.014	0.040
Ireland	1971–2007	0.538***	0.004***	-0.001***	0.000***	0.607	0.008	0.014	0.035
Italy	1971–2007	0.279***	-0.001	-0.000***	0.000***	0.364	0.005	0.005	0.018
Latvia	1996–2007	0.263*	0	0.001	0	0.226	0.004	0.014	0.027
Lithuania	1996–2007	0.395***	0.006***	-0.001	0	0.539	0.005	0.015	0.041
Luxembourg	1971–2007	0.456***	-0.056***	0.002***	-0.000***	0.184	0.042	0.048	0.098
Netherlands	1971–2007	0.541***	-0.002**	-0.000**	0.000***	0.532	0.007	0.007	0.018
Poland	1996–2007	0.112***	0	0	0	0.105	0.006	0.010	0.019
Portugal	1971–2007	0.664***	0	-0.000***	0.000***	0.649	0.007	0.013	0.038
Slovakia	1996–2007	0.747***	0	0	0.000**	0.707	0.002	0.013	0.039
Slovenia	1996–2007	0.446***	0	0	0	0.507	0.000	0.008	0.022
Spain	1971–2007	0.336***	-0.009***	0.000***	0	0.430	0.003	0.007	0.018
Sweden	1971–2007	0.401***	-0.007***	0.000***	0	0.352	0.004	0.008	0.018
UK	1971–2007	0.439***	0	-0.000***	0.000***	0.441	0.009	0.011	0.030
Australia	1971–2007	0.543***	-0.001	-0.000***	0.000***	0.504	0.010	0.012	0.027
Japan	1974–2007	0.293***	-0.003***	0	0.000**	0.389	0.004	0.006	0.021
Korea	1971–2007	0.544***	0.005***	-0.000***	0.000***	0.685	0.010	0.008	0.029
US	1978–2007	0.413***	-0.007***	0	0.000***	0.362	0.004	0.008	0.017

Note: This table reports the results from the estimation of equations (21)-(23). Columns (8)-(10) report the Root Mean Square Error of the fit for each equation. The estimations are carried using non-linear seemingly unrelated regressions routine in STATA and initial values of ε_1 are selected from $(0,1)$. Adjusted R^2 of the first equations is greater than 0.99 in all estimations. Standard errors are robust to arbitrary heteroscedasticity and autocorrelation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. We compute the value of γ_1 from (24) using the estimated value of ε_1. This is the reason why no significance level is attached to γ_1.

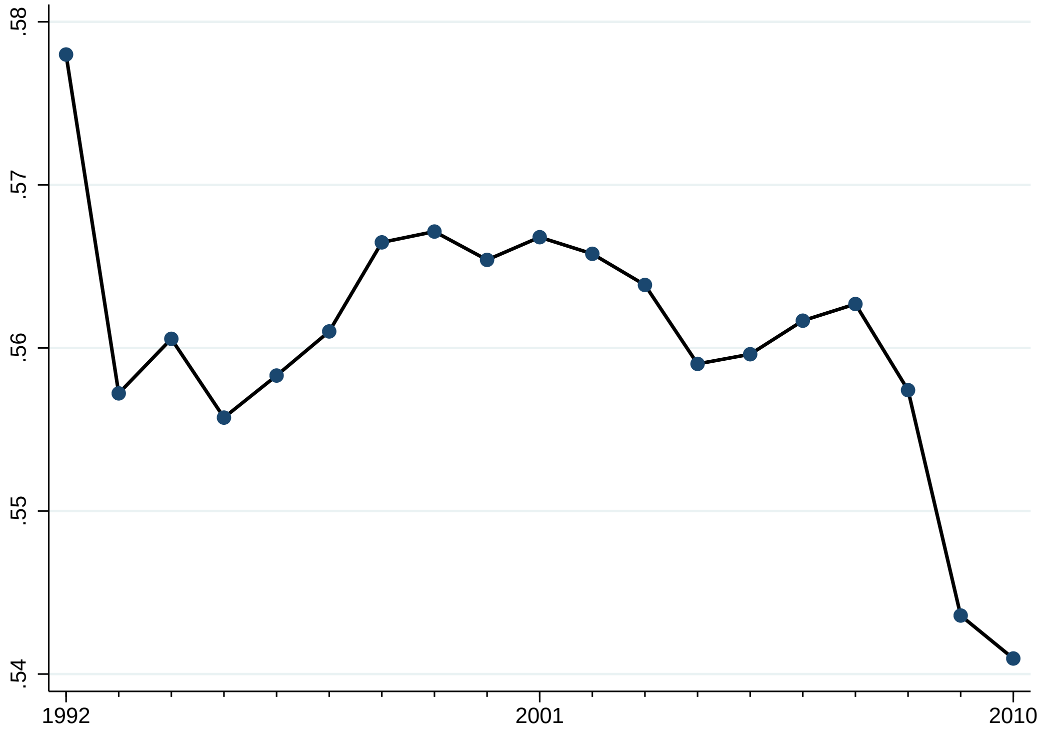
Table 42: *Estimated values of* ε_1 *and* γ_1 *- Initial* $\varepsilon_1 > 1$

Country	Sample Period	ε_1	δ_1	δ_2	δ_3	γ_1	RMSE EQ. (21)	RMSE EQ. (22)	RMSE EQ. (23)
Austria	1971–2007	1.559***	-0.005***	0	0	0.792	0.021	0.011	0.030
Belgium	1971–2007	1.490***	0	0	0	0.788	0.018	0.012	0.032
Cyprus	1996–2007	1.153***	0.004***	0	0	0.648	0.004	0.016	0.023
Czech Republic	1996–2007	1.058***	0	0	0	0.749	0.003	0.008	0.022
Denmark	1971–2007	1.548***	0.002	-0.000**	0.000***	0.783	0.016	0.011	0.023
Estonia	1996–2007	1.098***	0.057***	-0.007***	0.000***	0.682	0.020	0.020	0.042
Finland	1971–2007	1.404***	0.002	-0.000***	0.000***	0.809	0.014	0.010	0.031
France	1971–2007	1.525***	-0.004***	0	0	0.770	0.014	0.010	0.021
Germany	1971–2007	1.739***	0	0	0	0.810	0.016	0.011	0.027
Greece	1971–2007	1.344***	0	0	0	0.791	0.015	0.012	0.035
Hungary	1992–2007	1.158***	0.008	-0.001	0	0.765	0.007	0.018	0.058
Ireland	1971–2007	1.279***	0.003	-0.001***	0.000***	0.773	0.016	0.016	0.043
Italy	1971–2007	1.436***	0.003	-0.000***	0.000***	0.795	0.020	0.012	0.037
Latvia	1996–2007	1.096***	0.010***	-0.001	0.000**	0.701	0.006	0.019	0.041
Lithuania	1996–2007	1.230***	0.024**	-0.003**	0.000**	0.790	0.011	0.017	0.054
Luxembourg	1971–2007	2.931***	0.011	-0.001**	0.000**	0.737	0.082	0.095	0.111
Netherlands	1971–2007	1.421***	0	0	0.000**	0.779	0.019	0.012	0.026
Poland	1996–2007	1.140***	0.041***	-0.005***	0.000***	0.768	0.012	0.014	0.042
Portugal	1971–2007	1.336***	-0.001	-0.000**	0.000***	0.784	0.012	0.013	0.039
Slovakia	1996–2007	1.151***	0.004	0	0	0.785	0.005	0.013	0.042
Slovenia	1996–2007	1.165***	0.002	0	0	0.763	0.002	0.009	0.026
Spain	1971–2007	1.568***	-0.005***	0	0	0.804	0.019	0.010	0.029
Sweden	1971–2007	1.246***	-0.004**	0	0	0.736	0.013	0.015	0.033
UK	1971–2007	1.562***	0.006	-0.001**	0.000***	0.782	0.032	0.020	0.047
Australia	1971–2007	1.466***	-0.004	0	0	0.767	0.021	0.014	0.026
Japan	1974–2007	1.494***	0.004*	-0.000**	0.000***	0.788	0.016	0.013	0.035
Korea	1971–2007	1.260***	0.005*	-0.000***	0.000***	0.829	0.019	0.009	0.036
US	1978–2007	1.462***	-0.003*	0	0.000*	0.744	0.017	0.013	0.025

Note: This table reports the results from the estimation of equations (21)-(23). Columns (8)-(10) report the Root Mean Square Error of the fit for each equation. The estimations are carried using non-linear seemingly unrelated regressions routine in STATA and initial values of ε_1 are selected above 1. Adjusted R^2 of the first equations is greater than 0.99 in all estimations. Standard errors are robust to arbitrary heteroscedasticity and autocorrelation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. We compute the value of γ_1 from (24) using the estimated value of ε_1. This is the reason why no significance level is attached to γ_1.

Data Appendix - Figures

Figure 5: *The Average Employment Share of Highly Concentrated 1-digit ISCO-88 Occupations in Sample Countries*

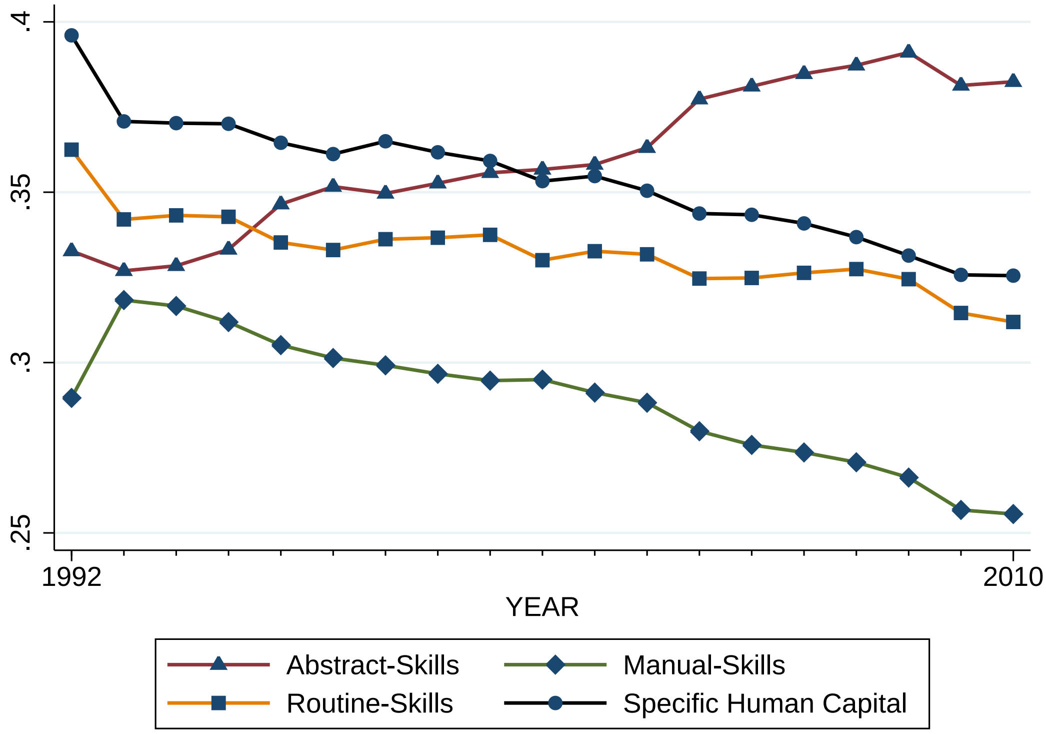


Note: This figure offers country-averaged value of the share of highly concentrated 1-digit ISCO-88 occupations. See Table 36 for the assignment of occupations into highly concentrated and not highly concentrated groups. The coefficient in front of time trend t is highly significant and negative in regressions of a form $\textit{Share of Highly Concentrated 1-digit Occupations}_{c,t} = \alpha + \beta t + \eta_{c,t}$. This figure is sensitive to the inclusion of the median concentrated occupations, Professionals, in the group of highly concentrated occupations.

Figure 6: *The Average Employment Share of Highly Concentrated 3-digit ISCO-88 Occupations in Sample Countries*

Note: This figure offers country-averaged value of the share of highly concentrated 3-digit ISCO-88 occupations. See Table 36 for the assignment of occupations into highly concentrated and not highly concentrated groups. The coefficient in front of time trend t is highly significant and negative in regressions of a form $\textit{Share of Highly Concentrated 3-digit Occupations}_{c,t} = \alpha + \beta t + \eta_{c,t}$. Poland and Slovenia are excluded from the sample of countries because we do not have 3-digit ISCO-88 for them.

Figure 7: *The Average Employment Shares in Abstract-, Manual-, and Routine-Skills Occupations in Sample Countries*



Note: This figure offers the shares of employment in occupations requiring abstract, manual, and routine skills averaged over the sample countries. It also offers the share of specific human capital averaged over the sample countries. See Table 38 for the assignment of occupations into abstract-, manual-, and routine-skills occupations and into specific and general human capital types.

Proofs Appendix

Proof of Proposition 1: We use $F_1\left(\right)$ to denote

$$F_1\left(u_h^g, H_g, Y_m\right) = \left(u_h^g\right)^{1-\frac{\varepsilon_2-1}{\varepsilon_2}} \left(1-u_h^g\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4-1} - \mathbb{E}\left[\left(\frac{\lambda_h}{\lambda_l}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right] \frac{\gamma_2}{\gamma_4}\frac{\gamma_1}{1-\gamma_1}$$
$$\times \left(H_g\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}-\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4} \left[\gamma_2\left(u_h^g H_g\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \left(1-\gamma_2\right) Y_m^{\frac{\varepsilon_2-1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}\frac{\varepsilon_1-1}{\varepsilon_1}-1}.$$

According to (17) we have that $F_1\left(u_h^g, H_g, Y_m\right) \equiv 0$.

The partial derivatives of $F_1\left(u_h^g, H_g, Y_m\right)$ are given by

$$\frac{\partial F_1\left(u_h^g, H_g, Y_m\right)}{\partial u_h^g} = \frac{1}{u_h^g}\left(u_h^g\right)^{1-\frac{\varepsilon_2-1}{\varepsilon_2}} \left(1-u_h^g\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4-1}$$
$$\times \left[1-\frac{\varepsilon_1-1}{\varepsilon_1} + \left(\frac{\varepsilon_1-1}{\varepsilon_1}-\frac{\varepsilon_2-1}{\varepsilon_2}\right)\omega_{Y_m}^{Y_h} + \left(1-\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4\right)\frac{u_h^g}{1-u_h^g}\right],$$
$$\frac{\partial F_1\left(u_h^g, H_g, Y_m\right)}{\partial H_g} = -\frac{1}{H_g}\left(u_h^g\right)^{1-\frac{\varepsilon_2-1}{\varepsilon_2}} \left(1-u_h^g\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4-1}$$
$$\times \left[-\left(\frac{\varepsilon_1-1}{\varepsilon_1}-\frac{\varepsilon_2-1}{\varepsilon_2}\right)\omega_{Y_m}^{Y_h} + \frac{\varepsilon_1-1}{\varepsilon_1}\left(1-\gamma_4\right)\right],$$
$$\frac{\partial F_1\left(u_h^g, H_g, Y_m\right)}{\partial Y_m} = -\frac{1}{Y_m}\left(u_h^g\right)^{1-\frac{\varepsilon_2-1}{\varepsilon_2}} \left(1-u_h^g\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4-1}\left(\frac{\varepsilon_1-1}{\varepsilon_1}-\frac{\varepsilon_2-1}{\varepsilon_2}\right)\omega_{Y_m}^{Y_h}.$$

Therefore,

$$\frac{\partial u_h^g}{\partial H_g} = \frac{u_h^g}{H_g}\frac{-\left(\frac{\varepsilon_1-1}{\varepsilon_1}-\frac{\varepsilon_2-1}{\varepsilon_2}\right)\omega_{Y_m}^{Y_h} + \frac{\varepsilon_1-1}{\varepsilon_1}\left(1-\gamma_4\right)}{1-\frac{\varepsilon_1-1}{\varepsilon_1} + \left(\frac{\varepsilon_1-1}{\varepsilon_1}-\frac{\varepsilon_2-1}{\varepsilon_2}\right)\omega_{Y_m}^{Y_h} + \left(1-\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4\right)\frac{u_h^g}{1-u_h^g}},$$
$$\frac{\partial u_h^g}{\partial Y_m} = \frac{u_h^g}{Y_m}\frac{\left(\frac{\varepsilon_1-1}{\varepsilon_1}-\frac{\varepsilon_2-1}{\varepsilon_2}\right)\omega_{Y_m}^{Y_h}}{1-\frac{\varepsilon_1-1}{\varepsilon_1} + \left(\frac{\varepsilon_1-1}{\varepsilon_1}-\frac{\varepsilon_2-1}{\varepsilon_2}\right)\omega_{Y_m}^{Y_h} + \left(1-\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4\right)\frac{u_h^g}{1-u_h^g}}.$$

Multiplying the denominators and numerators of these expressions by $\varepsilon_1\varepsilon_2$ gives

$$\frac{\partial u_h^g}{\partial H_g} = \frac{u_h^g}{H_g}\frac{-\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h} + \varepsilon_2\left(\varepsilon_1-1\right)\left(1-\gamma_4\right)}{\varepsilon_2 + \left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}, \tag{25}$$

$$\frac{\partial u_h^g}{\partial Y_m} = \frac{u_h^g}{Y_m}\frac{\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}}{\varepsilon_2 + \left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}, \tag{26}$$

$$\frac{\partial u_h^g}{\partial H_s} = \left(1-\omega_K^{Y_m}\right)\frac{Y_m}{H_s}\frac{\partial u_h^g}{\partial Y_m}. \tag{27}$$

The denominator in (25) and (26) is positive. This can be easily checked noticing that the denominator increases with ε_1 and is positive for the limiting value $\varepsilon_1 = 0$. Therefore, u_h^g increases (declines) with Y_m if and only if $\varepsilon_1 > \varepsilon_2$ ($\varepsilon_2 > \varepsilon_1$). This implies that u_h^g increases (declines) with K and H_s if and only if $\varepsilon_1 > \varepsilon_2$ ($\varepsilon_2 > \varepsilon_1$) since Y_m increases with these inputs. In turn, u_h^g increases (declines) with H_g if the numerator in (25) is positive (negative). It is sufficient to have $\varepsilon_2 > \varepsilon_1 > 1$ in order the numerator to be positive and $1 > \varepsilon_1 > \varepsilon_2$ in order it to be negative. Moreover, if $\gamma_4 = 1$ then the numerator is positive if and only if $\varepsilon_2 > \varepsilon_1$. It is negative if and only if $\varepsilon_1 > \varepsilon_2$.

The ratio Y_h/Y_l increases with K. To show this we note that Y_m increases with K and evaluate the sign of the following partial derivative

$$\frac{\partial}{\partial Y_m}\frac{Y_h}{Y_l} = \frac{Y_l\frac{\partial Y_h}{\partial Y_m} - Y_h\frac{\partial Y_l}{\partial Y_m}}{(Y_l)^2}.$$

The sign of the numerator in this expression is the same as the sign of

$$\left(1 - \omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial Y_m} + \omega_{Y_m}^{Y_h}\frac{1}{Y_m} + \gamma_4\frac{u_h^g}{1-u_h^g}\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial Y_m}.$$

We denote

$$d = \varepsilon_1\left(1 - u_h^g\right) + \varepsilon_2\left[\varepsilon_1 - (\varepsilon_1 - 1)\gamma_4\right]u_h^g + (\varepsilon_1 - \varepsilon_2)\gamma_4 u_h^g.$$

Using the expression for $\partial u_h^g/\partial Y_m$ it can be shown that d has the same sign as the partial derivative of Y_h/Y_l with respect to Y_m. It can be easily shown that d increases with ε_1 and is equal to zero when $\varepsilon_1 = 0$. Therefore, Y_h/Y_l increases with K.

??These results are confirmed in numerical exercises where we use (16) instead of (17).??

Proof of Proposition 2: For brevity, we use Λ and $F_2\,()$ to denote

$$\begin{aligned}\Lambda &= \mathbb{E}\left[(\lambda_h/\lambda_l)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right],\\ &= \exp\left(\frac{\varepsilon_1-1}{\varepsilon_1}\left(\mu_{z_h} - \mu_{z_l}\right) + \frac{1}{2}\left(\frac{\varepsilon_1-1}{\varepsilon_1}\right)^2\left(\sigma_{z_h}^2 + \sigma_{z_l}^2\right)\right),\end{aligned}$$

and

$$\begin{aligned}F_2\left(u_h^g,\Lambda\right) &= \left(u_h^g\right)^{1-\frac{\varepsilon_2-1}{\varepsilon_2}}\left(1-u_h^g\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4-1} - \frac{\gamma_2}{\gamma_4}\frac{\gamma_1}{1-\gamma_1}\left(H_g\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}-\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4}\\ &\times\left[\gamma_2\left(u_h^g H_g\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + (1-\gamma_2)\,Y_m^{\frac{\varepsilon_2-1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}\frac{\varepsilon_1-1}{\varepsilon_1}-1}\Lambda.\end{aligned}$$

The partial derivatives of $F_2\left(u_h^g,\Lambda\right)$ are

$$\begin{aligned}\frac{\partial F_2\left(u_h^g,\Lambda\right)}{\partial u_h^g} &= \frac{\partial}{\partial u_h^g}F_1\left(u_h^g, H_g, Y_m\right),\\ \frac{\partial F_2\left(u_h^g,\Lambda\right)}{\partial \Lambda} &= -\left(u_h^g\right)^{1-\frac{\varepsilon_2-1}{\varepsilon_2}}\left(1-u_h^g\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}\gamma_4-1}\frac{1}{\Lambda}.\end{aligned}$$

Therefore,

$$\frac{\partial u_h^g}{\partial \Lambda} = \frac{u_h^g}{\Lambda}\frac{\varepsilon_1\varepsilon_2}{\varepsilon_2 + (\varepsilon_1-\varepsilon_2)\,\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1 - (\varepsilon_1-1)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}. \tag{28}$$

This implies that u_h^g increases with Λ. Therefore, u_h^g increases with μ_{z_h} and declines with μ_{z_l} when h- and l-goods are gross substitutes ($\varepsilon_1 > 1$). It declines with μ_{z_h} and increases with μ_{z_l} when h- and l-goods are gross complements ($\varepsilon_1 < 1$). Clearly, it also increases with σ_{z_h} when $\varepsilon_1 > 1$ and with σ_{z_l} when $\varepsilon_1 < 1$. ??These results are confirmed in numerical exercises where we use (16) instead of (17). However, according to our numerical results u_h^g does not increase with σ_{z_l} when $\varepsilon_1 > 1$ and with σ_{z_h} when $\varepsilon_1 < 1$ if we use (16) instead of (17).??

Proof of Proposition 3: Consider the derivative of the relative (inverse) demand for general human capital (18) with respect to K. It is given by

$$\begin{aligned}\frac{\partial \tilde{w}_g}{\partial K} &= \frac{\gamma_2}{1-\gamma_2}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} H_s \frac{\partial}{\partial K}\frac{1}{1-\omega_K^{Y_m}} \\ &+ \frac{\gamma_2}{1-\gamma_2}\frac{1}{1-\omega_K^{Y_m}}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} H_s \frac{\partial}{\partial K}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}} \\ &+ \frac{\gamma_2}{1-\gamma_2}\frac{1}{1-\omega_K^{Y_m}}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}} H_s \frac{\partial}{\partial K}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}},\end{aligned}$$

where

$$\begin{aligned}\frac{\partial}{\partial K}\frac{1}{1-\omega_K^{Y_m}} &= \frac{\varepsilon_3-1}{\varepsilon_3}\frac{\omega_K^{Y_m}}{1-\omega_K^{Y_m}}\frac{1}{K}, \\ \frac{\partial}{\partial K}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}} &= -\frac{1}{\varepsilon_2}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}}\frac{1}{u_h^g}\omega_K^{Y_m}\frac{Y_m}{K}\frac{\partial u_h^g}{\partial Y_m}, \\ \frac{\partial}{\partial K}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} &= -\frac{\varepsilon_2-1}{\varepsilon_2}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}\frac{1}{Y_m}\omega_K^{Y_m}\frac{Y_m}{K}.\end{aligned}$$

Therefore,

$$\frac{\partial \tilde{w}_g}{\partial K} = \tilde{w}_g \omega_K^{Y_m}\frac{1}{K}\left[\frac{\varepsilon_3-1}{\varepsilon_3} - \frac{1}{\varepsilon_2}\frac{1}{u_h^g}Y_m\frac{\partial u_h^g}{\partial Y_m} - \frac{\varepsilon_2-1}{\varepsilon_2}\right] \tag{29}$$

$$= \tilde{w}_g \omega_K^{Y_m}\frac{1}{K}\left\{\frac{\varepsilon_3-1}{\varepsilon_3} - \frac{\varepsilon_2-1}{\varepsilon_2} - \frac{1}{\varepsilon_2}\frac{(\varepsilon_1-\varepsilon_2)\,\omega_{Y_m}^{Y_h}}{\varepsilon_2 + (\varepsilon_1-\varepsilon_2)\,\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1 - (\varepsilon_1-1)\,\gamma_4\right]\frac{u_h^g}{1-u_h^g}}\right\} \tag{30}$$

A sufficient condition to have $\frac{\partial \tilde{w}_g}{\partial K} > 0$ then is

$$\varepsilon_3 > 1 \geq \varepsilon_2 > \varepsilon_1.$$

A sufficient condition to have $\frac{\partial \tilde{w}_g}{\partial K} < 0$ is

$$\varepsilon_3 < 1 \leq \varepsilon_2 < \varepsilon_1.$$

The derivative of the relative demand function with respect to H_g is given by

$$\frac{\partial \tilde{w}_g}{\partial H_g} = \frac{\gamma_2}{1-\gamma_2}\frac{1}{1-\omega_K^{Y_m}}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} H_s \frac{\partial}{\partial H_g}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}}, \tag{31}$$

where

$$\begin{aligned}\frac{\partial}{\partial H_g}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}} &= -\frac{1}{\varepsilon_2}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}}\frac{1}{H_g} \\ &\times \left\{\frac{\varepsilon_2 + \varepsilon_2\left[\varepsilon_1 - (\varepsilon_1-1)\,\gamma_4\right]\frac{u_h^g}{1-u_h^g} + \varepsilon_2\,(\varepsilon_1-1)\,(1-\gamma_4)}{\varepsilon_2 + (\varepsilon_1-\varepsilon_2)\,\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1 - (\varepsilon_1-1)\,\gamma_4\right]\frac{u_h^g}{1-u_h^g}}\right\}.\end{aligned}$$

Clearly,

$$\frac{\partial}{\partial H_g}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}} < 0,$$

which implies that

$$\frac{\partial \tilde{w}_g}{\partial H_g} < 0.$$

The derivative of the demand function with respect to H_s is given by

$$\begin{aligned}
\frac{\partial \tilde{w}_g}{\partial H_s} &= \tilde{w}_g \frac{1}{H_s} + \frac{\gamma_2}{1-\gamma_2}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} H_s \frac{\partial}{\partial H_s}\frac{1}{1-\omega_K^{Y_m}} \\
&+ \frac{\gamma_2}{1-\gamma_2}\frac{1}{1-\omega_K^{Y_m}}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} H_s \frac{\partial}{\partial H_s}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}} \\
&+ \frac{\gamma_2}{1-\gamma_2}\frac{1}{1-\omega_K^{Y_m}}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}} H_s \frac{\partial}{\partial H_s}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}},
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial}{\partial H_s}\frac{1}{1-\omega_K^{Y_m}} &= -\frac{\varepsilon_3-1}{\varepsilon_3}\frac{\omega_K^{Y_m}}{1-\omega_K^{Y_m}}\frac{1}{H_s}, \\
\frac{\partial}{\partial H_s}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}} &= -\frac{1}{\varepsilon_2}\left(\frac{1}{u_h^g H_g}\right)^{\frac{1}{\varepsilon_2}}\frac{1}{u_h^g}\left(1-\omega_K^{Y_m}\right)\frac{Y_m}{H_s}\frac{\partial u_h^g}{\partial Y_m}, \\
\frac{\partial}{\partial H_s}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} &= -\frac{\varepsilon_2-1}{\varepsilon_2}\left(\frac{1}{Y_m}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}\frac{1}{Y_m}\left(1-\omega_K^{Y_m}\right)\frac{Y_m}{H_s}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{\partial \tilde{w}_g}{\partial H_s} &= \tilde{w}_g \frac{1}{H_s}\left[1-\frac{\varepsilon_3-1}{\varepsilon_3}\omega_K^{Y_m} - \frac{1}{\varepsilon_2}\left(1-\omega_K^{Y_m}\right)\frac{Y_m}{u_h^g}\frac{\partial u_h^g}{\partial Y_m} - \frac{\varepsilon_2-1}{\varepsilon_2}\left(1-\omega_K^{Y_m}\right)\right] \\
&= \tilde{w}_g \frac{1}{H_s}\left\{1-\frac{\varepsilon_3-1}{\varepsilon_3}\omega_K^{Y_m} - \frac{\varepsilon_2-1}{\varepsilon_2}\left(1-\omega_K^{Y_m}\right) - \frac{1}{\varepsilon_2}\left(1-\omega_K^{Y_m}\right)\right. \\
&\times \left.\frac{\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}\left(1-u_h^g\right)}{\varepsilon_2\left(1-u_h^g\right)+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}\left(1-u_h^g\right)+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]u_h^g}\right\}
\end{aligned} \tag{32}$$

This expression declines as the term $(\varepsilon_3-1)/\varepsilon_3$ increases. Clearly, $(\varepsilon_3-1)/\varepsilon_3$ increases with ε_3 and tends to 1 as ε_3 tends to $+\infty$. Therefore,

$$\frac{\partial \tilde{w}_g}{\partial H_s} > \left.\frac{\partial \tilde{w}_g}{\partial H_s}\right|_{\varepsilon_3=+\infty}.$$

We take the limiting value of ε_3 to obtain

$$\begin{aligned}
\left.\frac{\partial \tilde{w}_g}{\partial H_s}\right|_{\varepsilon_3=+\infty} &= \tilde{w}_g \frac{1}{H_s}\left(1-\omega_K^{Y_m}\right) \\
&\times \frac{1}{\left(1-u_h^g\right)}\frac{1+\left(\varepsilon_1-1\right)\left(1-\gamma_4\right)u_h^g}{\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}.
\end{aligned}$$

This is positive, which means that

$$\frac{\partial \tilde{w}_g}{\partial H_s} > 0.$$

These results are confirmed in numerical exercises where we use (16) instead of (17). We describe the numerical exercise in ??Numerical Results?? section.

Proofs of Proposition 4: We use letters μ and σ^2 to denote the means and variances of λ, λ_h, and λ_l. Since random variables $\{\lambda\}$ are log-normal their means, variances, and coefficients of variation are given by

$$\mu_\lambda = \exp\left(\mu_z + \frac{1}{2}\sigma_z^2\right),$$
$$\sigma_\lambda^2 = \left(\exp\left(\sigma_z^2\right) - 1\right)\mu_\lambda^2$$
$$\left(\frac{\sigma_\lambda}{\mu_\lambda}\right) = \left(\exp\left(\sigma_z^2\right) - 1\right)^{\frac{1}{2}}.$$

To prove the proposition we use the Delta Method. We consider the first order Taylor approximation of final output around the means of Y_h, Y_l, and λ (any deterministic point would suit):

$$Y \approx \mu_\lambda \left[\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1)\mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}} \tag{33}$$

$$+ \mu_\lambda \left[\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1)\mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}} \frac{\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1)\mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} \frac{1}{\mu_{Y_h}} (Y_h - \mu_{Y_h})$$

$$+ \mu_\lambda \left[\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1)\mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}} \frac{(1-\gamma_1) \mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1)\mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} \frac{1}{\mu_{Y_l}} (Y_l - \mu_{Y_l})$$

$$+ \mu_\lambda \left[\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1)\mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}} \frac{1}{\mu_\lambda} (\lambda - \mu_\lambda). \tag{34}$$

We denote

$$\tilde{\omega}_{Y_h}^Y = \frac{\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1)\mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}.$$

The share of general human capital in h-goods, as well as l-goods, production is not stochastic. Therefore, the expression above can be rewritten as a sum of deterministic and stochastic terms in the following manner:

$$
\begin{aligned}
Y \approx & -\mu_\lambda \left[\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1)\, \mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}} \\
& + \mu_\lambda \left[\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1)\, \mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}} \tilde{\omega}_{Y_h}^{Y} \frac{1}{\mu_{\lambda_h}} \lambda_h \\
& + \mu_\lambda \left[\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1)\, \mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}} \left(1 - \tilde{\omega}_{Y_h}^{Y}\right) \frac{1}{\mu_{\lambda_l}} \lambda_l \\
& + \mu_\lambda \left[\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1)\, \mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}} \frac{1}{\mu_\lambda} \lambda.
\end{aligned}
$$

The variance of the final output then is given by

$$
\begin{aligned}
\sigma_Y^2 = & \left\{ \mu_\lambda \left[\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1)\, \mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}} \right\}^2 \\
& \times \left\{ \left(\frac{\sigma_\lambda}{\mu_\lambda}\right)^2 + \left(\tilde{\omega}_{Y_h}^{Y}\right)^2 \left(\frac{\sigma_{\lambda_h}}{\mu_{\lambda_h}}\right)^2 + \left(1 - \tilde{\omega}_{Y_h}^{Y}\right)^2 \left(\frac{\sigma_{\lambda_l}}{\mu_{\lambda_l}}\right)^2 \right\}.
\end{aligned}
$$

We set $\sigma_\lambda^2 = 0$ and rewrite the variance of final output as

$$
\begin{aligned}
\sigma_Y^2 = & \left\{ \mu_\lambda \left[\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1)\, \mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]^{\frac{1}{\varepsilon_1 - 1}} \right\}^2 \\
& \times \left\{ \left(\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right)^2 \left(\frac{\sigma_{\lambda_h}}{\mu_{\lambda_h}}\right)^2 + \left[(1-\gamma_1)\, \mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]^2 \left(\frac{\sigma_{\lambda_l}}{\mu_{\lambda_l}}\right)^2 \right\}.
\end{aligned}
$$

This involves no loss of generality for the purposes of the current analysis since the contribution of σ_λ^2 is not affected by the composition of human capital portfolio when μ_Y is constant.

In order to prove the proposition we consider how μ_{Y_h} and μ_{Y_l} change under the variation in H_g or H_s which keeps μ_Y constant. It is clear that μ_{Y_h} and μ_{Y_l} move in opposite directions under such a variation.

Using the partial derivatives of μ_{Y_h} and μ_{Y_l} (42)-(45) and the partial derivatives of μ_Y with respect to μ_{Y_h} and μ_{Y_l} [see (45) and (44)] we rewrite the condition

$$
\frac{d\mu_Y}{dH_g} dH_g + \frac{d\mu_Y}{dH_s} dH_s = 0 \tag{35}
$$

as

$$
\begin{aligned}
0 = & \left\{ \left[\hat{\omega}_{Y_h}^{Y} \left(1 - \omega_{Y_m}^{Y_h}\right) - \gamma_4 \left(1 - \hat{\omega}_{Y_h}^{Y}\right) \frac{u_h^g}{1 - u_h^g} \right] \frac{H_g}{u_h^g} \frac{du_h^g}{dH_g} \right. \\
& \left. + \left[\gamma_4 \left(1 - \hat{\omega}_{Y_h}^{Y}\right) + \hat{\omega}_{Y_h}^{Y} \left(1 - \omega_{Y_m}^{Y_h}\right) \right] \right\} \frac{1}{H_g} dH_g \\
& + \left\{ \left[\hat{\omega}_{Y_h}^{Y} \left(1 - \omega_{Y_m}^{Y_h}\right) - \left(1 - \hat{\omega}_{Y_h}^{Y}\right) \gamma_4 \frac{u_h^g}{1 - u_h^g} \right] \left(1 - \omega_K^{Y_m}\right) \frac{Y_m}{u_h^g} \frac{du_h^g}{dY_m} \right. \\
& \left. + \hat{\omega}_{Y_h}^{Y} \omega_{Y_m}^{Y_h} \left(1 - \omega_K^{Y_m}\right) \right\} \frac{1}{H_s} dH_s,
\end{aligned}
$$

where

$$\hat{\omega}_{Y_h}^{Y} = \mathbb{E}\left[\omega_{Y_h}^{Y} Y\right] / \mathbb{E}\left[Y\right]$$

Further, we plug for $\frac{du_h^g}{dH_g}$ and $\frac{du_h^g}{dY_m}$ from (25) and (26) to obtain

$$\begin{aligned} \frac{1}{H_g} dH_g &= -\frac{1}{H_s} dH_s \\ &\times \frac{\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\left\{\hat{\omega}_{Y_h}^{Y}\varepsilon_1 + \left[\hat{\omega}_{Y_h}^{Y}\varepsilon_1\left(1-\gamma_4\right)\left(\varepsilon_2-1\right)+\gamma_4\varepsilon_2-\gamma_4\varepsilon_1\right]u_h^g\right\}}{\hat{\omega}_{Y_h}^{Y}\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_2\varepsilon_1\left(1-\gamma_4\right)+\gamma_4\left(1-\hat{\omega}_{Y_h}^{Y}\right)\omega_{Y_m}^{Y_h}\varepsilon_1+\gamma_4\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_2}. \end{aligned} \tag{36}$$

Given that all production functions are increasing in factor inputs we need to have that

$$\hat{\omega}_{Y_h}^{Y}\varepsilon_1 + \left[\hat{\omega}_{Y_h}^{Y}\varepsilon_1\left(1-\gamma_4\right)\left(\varepsilon_2-1\right)+\gamma_4\varepsilon_2-\gamma_4\varepsilon_1\right]u_h^g > 0, \tag{37}$$

so that $\frac{1}{H_g}dH_g$ and $\frac{1}{H_s}dH_s$ have different signs.

The change in the mean of Y_h is given by

$$\frac{1}{\mu_{Y_h}} d\mu_{Y_h} = \left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}du_h^g + \left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{H_g}dH_g + \omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}dH_s \tag{38}$$

Apparently, according to (35) this is positive (negative) if the change of μ_{Y_l},

$$\frac{1}{\mu_{Y_l}} d\mu_{Y_l} = \gamma_4\left(\frac{1}{u_l^g}du_l^g + \frac{1}{H_g}dH_g\right), \tag{39}$$

is negative (positive).

Using (25)-(27), (36), (39) and the total variation of u_h^g,

$$du_h^g = \frac{du_h^g}{dH_g}dH_g + \frac{du_h^g}{dH_s}dH_s,$$

it can be shown that

$$\begin{aligned} &\frac{1}{\gamma_4}\frac{1}{\mu_{Y_l}}d\mu_{Y_l}\frac{1}{\frac{1}{H_s}dH_s} \\ &= -\frac{1}{1-u_h^g}\frac{\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}}{\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}} \\ &\times \frac{\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\left\{\hat{\omega}_{Y_h}^{Y}\varepsilon_1 + \left[\hat{\omega}_{Y_h}^{Y}\varepsilon_1\left(1-\gamma_4\right)\left(\varepsilon_2-1\right)+\gamma_4\varepsilon_2-\gamma_4\varepsilon_1\right]u_h^g\right\}}{\hat{\omega}_{Y_h}^{Y}\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_2\varepsilon_1\left(1-\gamma_4\right)+\gamma_4\left(1-\hat{\omega}_{Y_h}^{Y}\right)\omega_{Y_m}^{Y_h}\varepsilon_1+\gamma_4\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_2} \\ &-\frac{u_h^g}{1-u_h^g}\frac{\left(1-\omega_K^{Y_m}\right)\omega_{Y_m}^{Y_h}\left(\varepsilon_1-\varepsilon_2\right)}{\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}. \end{aligned}$$

The sign of this expression is equivalent to the sign of the following sum

$$-\left\{\varepsilon_1\omega_{Y_m}^{Y_h}\left(1-u_h^g\right)+\varepsilon_2\left[1+u_h^g\left(\varepsilon_1-1\right)\left(1-\gamma_4\right)-\omega_{Y_m}^{Y_h}\left(1-u_h^g\right)\right]\right\}.$$

It turns out that the interior of the curly brackets is exactly the denominator in (25). Therefore,

$$\frac{d\mu_{Y_l}}{dH_s} < 0.$$

Therefore, higher H_s reduces μ_{Y_l} and increases μ_{Y_h}. This implies that when $1 > \varepsilon_1$ ($\varepsilon_1 > 1$) higher H_s reduces (increases) the contribution of $\sigma^2_{z_h}$ (or $\sigma^2_{\lambda_h}$) to σ^2_Y and increases (reduces) the contribution of $\sigma^2_{z_l}$ (or $\sigma^2_{\lambda_l}$) to σ^2_Y.

Consider two countries which produce the same output but have different amounts of H_s and H_g. This result means that if $1 > \varepsilon_1$ ($\varepsilon_1 > 1$) in the country where H_s is higher the volatility of final output because of shocks to h-sector is lower (higher). In turn, the volatility of final output because of shocks to l-sector is higher (lower).

This result implies that in the country where H_s is higher the volatility of final output is higher, for example, if either $1 > \varepsilon_1$ and $\sigma^2_{\lambda_l} > \sigma^2_{\lambda_h} = 0$ or $\varepsilon_1 > 1$ and $\sigma^2_{\lambda_h} > \sigma^2_{\lambda_l} = 0$. It is lower, for example, if either $1 > \varepsilon_1$ and $\sigma^2_{\lambda_{Y_h}} > \sigma^2_{\lambda_l} = 0$ or $\varepsilon_1 > 1$ and $\sigma^2_{\lambda_l} > \sigma^2_{\lambda_h} = 0$.

We further rewrite the variance of final output in the following manner

$$\sigma^2_Y = \left\{ \mu_\lambda \left[\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1)\, \mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}} \right\}^2 \times \left[\left(\tilde{\omega}^Y_{Y_h}\right)^2 \left(\frac{\sigma_{\lambda_h}}{\mu_{\lambda_h}}\right)^2 + \left(1 - \tilde{\omega}^Y_{Y_h}\right)^2 \left(\frac{\sigma_{\lambda_l}}{\mu_{\lambda_l}}\right)^2 \right].$$

Suppose that $\varepsilon_1 > 1$ and

$$\sigma^2_{z_h} > \sigma^2_{z_l}, \tag{40}$$

i.e., $\sigma_{\lambda_h}/\mu_{\lambda_h} > \sigma_{\lambda_l}/\mu_{\lambda_l}$. Further, suppose that the share of expected Y_h is higher than or equal to the share of expected Y_l:

$$\tilde{\omega}^Y_{Y_h} \geq 0.5. \tag{41}$$

In such a case, the volatility of final output is higher in the country where H_s is higher. The volatility of final output is also higher in case when $1 > \varepsilon_1$ and the inverses of (40) and (40) hold. This statement is also true for any deterministic points of Y_h, Y_l, and $\omega^Y_{Y_h}$.

Proofs of Propositions 5 and 6: First, we consider the case when $\sigma^2_{z_h} > \sigma^2_{z_l} = \sigma^2 = 0$ so that $\sigma^2_{\lambda_h} > \sigma^2_{\lambda_l} = \sigma^2_\lambda = 0$. The standard deviation of final output in this case is given by

$$\sigma_Y = \mu_\lambda \left[\gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\gamma_1)\, \mu_{Y_l}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right]^{\frac{1}{\varepsilon_1 - 1}} \gamma_1 \mu_{Y_h}^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \frac{\sigma_{\lambda_h}}{\mu_{\lambda_h}}.$$

Therefore, its partial derivatives with respect to H_s and H_g are given by

$$\frac{\partial \sigma_Y}{\partial H_g} = \frac{\sigma_Y}{\mu_{Y_h}} \left[\left(1 - \tilde{\omega}^Y_{Y_h}\right) \frac{1}{\varepsilon_1} \frac{1}{\mu_{Y_l}} \left(\mu_{Y_h} \frac{\partial \mu_{Y_l}}{\partial H_g} - \mu_{Y_l} \frac{\partial \mu_{Y_h}}{\partial H_g} \right) + \frac{\partial \mu_{Y_h}}{\partial H_g} \right],$$

$$\frac{\partial \sigma_Y}{\partial H_s} = \frac{\sigma_Y}{\mu_{Y_h}} \left[\left(1 - \tilde{\omega}^Y_{Y_h}\right) \frac{1}{\varepsilon_1} \frac{1}{\mu_{Y_l}} \left(\mu_{Y_h} \frac{\partial \mu_{Y_l}}{\partial H_s} - \mu_{Y_l} \frac{\partial \mu_{Y_h}}{\partial H_s} \right) + \frac{\partial \mu_{Y_h}}{\partial H_s} \right].$$

The ratio of the partial derivatives is given by

$$\frac{\frac{\partial \sigma_Y}{\partial H_g}}{\frac{\partial \sigma_Y}{\partial H_s}} = \frac{\left(1-\tilde{\omega}_{Y_h}^{Y}\right)\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_l}}\left(\mu_{Y_h}\frac{\partial \mu_{Y_l}}{\partial H_g}-\mu_{Y_l}\frac{\partial \mu_{Y_h}}{\partial H_g}\right)+\frac{\partial \mu_{Y_h}}{\partial H_g}}{\left(1-\tilde{\omega}_{Y_h}^{Y}\right)\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_l}}\left(\mu_{Y_h}\frac{\partial \mu_{Y_l}}{\partial H_s}-\mu_{Y_l}\frac{\partial \mu_{Y_h}}{\partial H_s}\right)+\frac{\partial \mu_{Y_h}}{\partial H_s}},$$

where

$$\frac{\partial \mu_{Y_l}}{\partial H_g} = \gamma_4\mu_{Y_l}\left(\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_g}\right), \tag{42}$$

$$\frac{\partial \mu_{Y_l}}{\partial H_s} = -\gamma_4\mu_{Y_l}\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_s}, \tag{43}$$

$$\frac{\partial \mu_{Y_h}}{\partial H_g} = \left(1-\omega_{Y_m}^{Y_h}\right)\mu_{Y_h}\left(\frac{1}{H_g}+\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_g}\right), \tag{44}$$

$$\frac{\partial \mu_{Y_h}}{\partial H_s} = \mu_{Y_h}\left[\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}+\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_s}\right]. \tag{45}$$

This implies that the ratio of the partial derivatives of the standard deviation is given by

$$\begin{aligned}
\frac{\frac{\partial \sigma_Y}{\partial H_g}}{\frac{\partial \sigma_Y}{\partial H_s}} &= \Bigg\{\left(1-\tilde{\omega}_{Y_h}^{Y}\right)\left[\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_g}\right)-\left(1-\omega_{Y_m}^{Y_h}\right)\left(\frac{1}{H_g}+\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_g}\right)\right] \\
&\quad +\varepsilon_1\left(1-\omega_{Y_m}^{Y_h}\right)\left(\frac{1}{H_g}+\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_g}\right)\Bigg\} \\
&\quad \times\Bigg(-\left(1-\tilde{\omega}_{Y_h}^{Y}\right)\left\{\gamma_4\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_s}+\left[\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}+\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_s}\right]\right\} \\
&\quad +\varepsilon_1\left[\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}+\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_s}\right]\Bigg)^{-1}
\end{aligned}$$

where

$$\begin{aligned}
\frac{1}{H_g}+\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_g} &= \frac{1}{H_g}\frac{\varepsilon_2\left[\left(\varepsilon_1-1\right)\left(1-\gamma_4\right)+1\right]}{1-u_h^g} \\
&\quad\times\frac{1}{\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}},
\end{aligned} \tag{46}$$

$$\begin{aligned}
\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_g} &= \frac{1}{H_g}\frac{1}{1-u_h^g} \\
&\quad\times\frac{\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}}{\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}},
\end{aligned} \tag{47}$$

$$\begin{aligned}
&\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}+\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_s} \\
&= \frac{1}{H_s}\left(1-\omega_K^{Y_m}\right)\omega_{Y_m}^{Y_h}\frac{\varepsilon_1+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}{\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}},
\end{aligned} \tag{48}$$

$$\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_g}\right)-\left(1-\omega_{Y_m}^{Y_h}\right)\left(\frac{1}{H_g}+\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_g}\right) \tag{49}$$
$$=\frac{1}{H_g}\frac{1}{1-u_h^g}\frac{\left[\varepsilon_2+(\varepsilon_1-\varepsilon_2)\,\omega_{Y_m}^{Y_h}\right]\gamma_4-\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_2\left[(\varepsilon_1-1)(1-\gamma_4)+1\right]}{\varepsilon_2+(\varepsilon_1-\varepsilon_2)\,\omega_{Y_m}^{Y_h}+\varepsilon_2\left[\varepsilon_1-(\varepsilon_1-1)\,\gamma_4\right]\frac{u_h^g}{1-u_h^g}}.$$

Plugging these expressions back into the ratio of the partial derivatives of standard deviations gives the following expression.

$$\begin{aligned}\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}}=&\frac{H_s}{H_g}\frac{1}{\left(1-\omega_K^{Y_m}\right)\omega_{Y_m}^{Y_h}}\times\\&\left(\left(1-\tilde{\omega}_{Y_h}^{Y}\right)\left\{\gamma_4\left[\varepsilon_2+(\varepsilon_1-\varepsilon_2)\,\omega_{Y_m}^{Y_h}\right]-\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_2\left[(\varepsilon_1-1)(1-\gamma_4)+1\right]\right\}\right.\\&\left.+\varepsilon_1\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_2\left[(\varepsilon_1-1)(1-\gamma_4)+1\right]\right)\\&\times\left(-\left(1-\tilde{\omega}_{Y_h}^{Y}\right)\left\{\varepsilon_1\left(1-u_h^g\right)+\gamma_4u_h^g\left(\varepsilon_1-\varepsilon_2\right)+\varepsilon_2\left[\varepsilon_1-(\varepsilon_1-1)\,\gamma_4\right]u_h^g\right\}\right.\\&\left.+\varepsilon_1\left\{\varepsilon_1\left(1-u_h^g\right)+\varepsilon_2\left[\varepsilon_1-(\varepsilon_1-1)\,\gamma_4\right]u_h^g\right\}\right)^{-1}\end{aligned}$$

Let for example $\varepsilon_1=\varepsilon_2=\gamma_4=1$. In such a case, the ratio of the partial derivatives is given by

$$\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}}=\frac{H_s}{H_g}\frac{1}{1-\omega_K^{Y_m}}\frac{1}{1-\gamma_2}\frac{1-\gamma_1(1-\gamma_2)}{\gamma_1}.$$

This can be greater or lower than 1 depending on parameter values, which implies that, in general, $\frac{\partial\sigma_Y}{\partial H_g}/\frac{\partial\sigma_Y}{\partial H_s}$ can be greater or lower than 1.

In case when $\sigma_{z_l}^2>\sigma_{z_h}^2=\sigma_z^2=0$ so that $\sigma_{\lambda_l}^2>\sigma_{\lambda_h}^2=\sigma_\lambda^2=0$ the standard deviation of final output is given by

$$\sigma_Y=\mu_\lambda\left[\gamma_1\mu_{Y_h}^{\frac{\varepsilon_1-1}{\varepsilon_1}}+(1-\gamma_1)\,\mu_{Y_l}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^{\frac{1}{\varepsilon_1-1}}\left[(1-\gamma_1)\,\mu_{Y_l}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]\frac{\sigma_{\lambda_l}}{\mu_{\lambda_l}}.$$

Therefore, its partial derivatives of the standard deviation of final output are given by

$$\begin{aligned}\frac{\partial\sigma_Y}{\partial H_g}&=\frac{\sigma_Y}{\mu_{Y_l}}\left[\frac{1}{\varepsilon_1}\tilde{\omega}_{Y_h}^{Y}\frac{1}{\mu_{Y_h}}\left(\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_g}-\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_g}\right)+\frac{\partial\mu_{Y_l}}{\partial H_g}\right],\\\frac{\partial\sigma_Y}{\partial H_s}&=\frac{\sigma_Y}{\mu_{Y_l}}\left[\frac{1}{\varepsilon_1}\tilde{\omega}_{Y_h}^{Y}\frac{1}{\mu_{Y_h}}\left(\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_s}-\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_s}\right)+\frac{\partial\mu_{Y_l}}{\partial H_s}\right].\end{aligned}$$

The ratio of the partial derivatives is given by

$$\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}}=\frac{\frac{1}{\varepsilon_1}\tilde{\omega}_{Y_h}^{Y}\frac{1}{\mu_{Y_h}}\left(\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_g}-\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_g}\right)+\frac{\partial\mu_{Y_l}}{\partial H_g}}{\frac{1}{\varepsilon_1}\tilde{\omega}_{Y_h}^{Y}\frac{1}{\mu_{Y_h}}\left(\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_s}-\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_s}\right)+\frac{\partial\mu_{Y_l}}{\partial H_s}},$$

where the partial derivatives of μ_{Y_h} and μ_{Y_l} are given by (42)-(45). This implies that the ratio above is given by

$$\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}}=\frac{\tilde{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\left(1-\omega_{Y_m}^{Y_h}\right)\left(\frac{1}{H_g}+\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_g}\right)+\left(1-\tilde{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\right)\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_g}\right)}{\tilde{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\left[\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}+\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_s}\right]-\left(1-\tilde{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\right)\gamma_4\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_s}},$$

where the expressions in brackets are given by (46)-(48). Therefore, ratio of the partial derivatives can be rewritten as

$$\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}}=\frac{H_s}{H_g}\frac{1}{\omega^{Y_h}_{Y_m}}\left\{\tilde{\omega}^{Y}_{Y_h}\frac{1}{\varepsilon_1}\left(1-\omega^{Y_h}_{Y_m}\right)\varepsilon_2\left[\left(\varepsilon_1-1\right)\left(1-\gamma_4\right)+1\right]\right.$$
$$\left.+\left(1-\tilde{\omega}^{Y}_{Y_h}\frac{1}{\varepsilon_1}\right)\gamma_4\left[\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega^{Y_h}_{Y_m}\right]\right\}$$
$$\times\left(\tilde{\omega}^{Y}_{Y_h}\frac{1}{\varepsilon_1}\left(1-\omega^{Y_m}_{K}\right)\left\{\varepsilon_1\left(1-u^g_h\right)+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]u^g_h\right\}\right.$$
$$\left.-\left(1-\tilde{\omega}^{Y}_{Y_h}\frac{1}{\varepsilon_1}\right)\gamma_4\left(\varepsilon_1-\varepsilon_2\right)\left(1-\omega^{Y_m}_{K}\right)u^g_h\right)^{-1}.$$

Let for example $\varepsilon_1=\varepsilon_2=\gamma_4=1$. In such a case the above expression reduces to

$$\frac{\frac{\partial\sigma_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}}=\frac{H_s}{H_g}\frac{1}{1-\omega^{Y_m}_{K}}\frac{1}{1-\gamma_2}\frac{1-\gamma_1\left(1-\gamma_2\right)}{\gamma_1}.$$

This can be greater or lower than 1 depending on parameter values, which implies that similarly $\frac{\partial\sigma_Y}{\partial H_g}/\frac{\partial\sigma_Y}{\partial H_s}$ can be greater or lower than 1.

In case when $\sigma_z=\sigma_\lambda=0$ the variance of final output is given by

$$\sigma^2_Y=\left\{\mu_{\lambda_Y}\left[\gamma_1\mu_{Y_h}^{\frac{\varepsilon_1-1}{\varepsilon_1}}+\left(1-\gamma_1\right)\mu_{Y_l}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^{\frac{1}{\varepsilon_1-1}}\gamma_1\mu_{Y_h}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\frac{\sigma_{\lambda_h}}{\mu_{\lambda_h}}\right\}^2$$
$$+\left\{\mu_{\lambda_Y}\left[\gamma_1\mu_{Y_h}^{\frac{\varepsilon_1-1}{\varepsilon_1}}+\left(1-\gamma_1\right)\mu_{Y_l}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^{\frac{1}{\varepsilon_1-1}}\left(1-\gamma_1\right)\mu_{Y_l}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\frac{\sigma_{\lambda_l}}{\mu_{\lambda_l}}\right\}^2.$$

We denote

$$z_1=\mu_{\lambda_Y}\left[\gamma_1\mu_{Y_h}^{\frac{\varepsilon_1-1}{\varepsilon_1}}+\left(1-\gamma_1\right)\mu_{Y_l}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^{\frac{1}{\varepsilon_1-1}}\gamma_1\mu_{Y_h}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\frac{\sigma_{\lambda_h}}{\mu_{\lambda_h}},$$
$$z_2=\mu_{\lambda_Y}\left[\gamma_1\mu_{Y_h}^{\frac{\varepsilon_1-1}{\varepsilon_1}}+\left(1-\gamma_1\right)\mu_{Y_l}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^{\frac{1}{\varepsilon_1-1}}\left(1-\gamma_1\right)\mu_{Y_l}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\frac{\sigma_{\lambda_l}}{\mu_{\lambda_l}}.$$

According to the previous results, the partial derivatives of z_1, z_2, and σ^2_Y are given by

$$\frac{\partial z_1}{\partial H_g}=\frac{z_1}{\mu_{Y_h}}\left[\left(1-\tilde{\omega}^{Y}_{Y_h}\right)\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_l}}\left(\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_g}-\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_g}\right)+\frac{\partial\mu_{Y_h}}{\partial H_g}\right],$$
$$\frac{\partial z_1}{\partial H_s}=\frac{z_1}{\mu_{Y_h}}\left[\left(1-\tilde{\omega}^{Y}_{Y_h}\right)\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_l}}\left(\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_s}-\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_s}\right)+\frac{\partial\mu_{Y_h}}{\partial H_s}\right],$$
$$\frac{\partial z_2}{\partial H_g}=\frac{z_2}{\mu_{Y_l}}\left[\tilde{\omega}^{Y}_{Y_h}\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_h}}\left(\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_g}-\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_g}\right)+\frac{\partial\mu_{Y_l}}{\partial H_g}\right],$$
$$\frac{\partial z_2}{\partial H_s}=\frac{z_2}{\mu_{Y_l}}\left[\tilde{\omega}^{Y}_{Y_h}\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_h}}\left(\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_s}-\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_s}\right)+\frac{\partial\mu_{Y_l}}{\partial H_s}\right].$$

and

$$\frac{\partial \sigma_Y^2}{\partial H_g} = 2z_1^2 \frac{1}{\mu_{Y_h}} \left[\left(1 - \tilde{\omega}_{Y_h}^Y\right) \frac{1}{\varepsilon_1} \frac{1}{\mu_{Y_l}} \left(\mu_{Y_h} \frac{\partial \mu_{Y_l}}{\partial H_g} - \mu_{Y_l} \frac{\partial \mu_{Y_h}}{\partial H_g} \right) + \frac{\partial \mu_{Y_h}}{\partial H_g} \right]$$
$$+ 2z_2^2 \frac{1}{\mu_{Y_l}} \left[\tilde{\omega}_{Y_h}^Y \frac{1}{\varepsilon_1} \frac{1}{\mu_{Y_h}} \left(\mu_{Y_l} \frac{\partial \mu_{Y_h}}{\partial H_g} - \mu_{Y_h} \frac{\partial \mu_{Y_l}}{\partial H_g} \right) + \frac{\partial \mu_{Y_l}}{\partial H_g} \right],$$
$$\frac{\partial \sigma_Y^2}{\partial H_s} = 2z_1^2 \frac{1}{\mu_{Y_h}} \left[\left(1 - \tilde{\omega}_{Y_h}^Y\right) \frac{1}{\varepsilon_1} \frac{1}{\mu_{Y_l}} \left(\mu_{Y_h} \frac{\partial \mu_{Y_l}}{\partial H_s} - \mu_{Y_l} \frac{\partial \mu_{Y_h}}{\partial H_s} \right) + \frac{\partial \mu_{Y_h}}{\partial H_s} \right]$$
$$+ 2z_2^2 \frac{1}{\mu_{Y_l}} \left[\tilde{\omega}_{Y_h}^Y \frac{1}{\varepsilon_1} \frac{1}{\mu_{Y_h}} \left(\mu_{Y_l} \frac{\partial \mu_{Y_h}}{\partial H_s} - \mu_{Y_h} \frac{\partial \mu_{Y_l}}{\partial H_s} \right) + \frac{\partial \mu_{Y_l}}{\partial H_s} \right].$$

The ratio of the partial derivatives then is given by

$$\frac{\frac{\partial \sigma_Y^2}{\partial H_g}}{\frac{\partial \sigma_Y^2}{\partial H_s}} = \left\{ \left(1 - \tilde{\omega}_{Y_h}^Y\right) \frac{1}{\varepsilon_1} \frac{1}{\mu_{Y_l}} \left(\mu_{Y_h} \frac{\partial \mu_{Y_l}}{\partial H_g} - \mu_{Y_l} \frac{\partial \mu_{Y_h}}{\partial H_g} \right) + \frac{\partial \mu_{Y_h}}{\partial H_g} \right.$$
$$\left. + \left[\left(\frac{z_2}{z_1}\right)^2 \frac{\mu_{Y_h}}{\mu_{Y_l}} \right] \left[\tilde{\omega}_{Y_h}^Y \frac{1}{\varepsilon_1} \frac{1}{\mu_{Y_h}} \left(\mu_{Y_l} \frac{\partial \mu_{Y_h}}{\partial H_g} - \mu_{Y_h} \frac{\partial \mu_{Y_l}}{\partial H_g} \right) + \frac{\partial \mu_{Y_l}}{\partial H_g} \right] \right\}$$
$$\times \left\{ \left(1 - \tilde{\omega}_{Y_h}^Y\right) \frac{1}{\varepsilon_1} \frac{1}{\mu_{Y_l}} \left(\mu_{Y_h} \frac{\partial \mu_{Y_l}}{\partial H_s} - \mu_{Y_l} \frac{\partial \mu_{Y_h}}{\partial H_s} \right) + \frac{\partial \mu_{Y_h}}{\partial H_s} \right.$$
$$\left. + \left[\left(\frac{z_2}{z_1}\right)^2 \frac{\mu_{Y_h}}{\mu_{Y_l}} \right] \left[\tilde{\omega}_{Y_h}^Y \frac{1}{\varepsilon_1} \frac{1}{\mu_{Y_h}} \left(\mu_{Y_l} \frac{\partial \mu_{Y_h}}{\partial H_s} - \mu_{Y_h} \frac{\partial \mu_{Y_l}}{\partial H_s} \right) + \frac{\partial \mu_{Y_l}}{\partial H_s} \right] \right\}^{-1},$$

where

$$\left(\frac{z_2}{z_1}\right)^2 \frac{\mu_{Y_h}}{\mu_{Y_l}} = \left[\frac{\mu_{\lambda_h}}{\mu_{\lambda_l}} \frac{1 - \gamma_1}{\gamma_1} \frac{\sigma_{\lambda_l}}{\sigma_{\lambda_h}} \left(\frac{\mu_{Y_l}}{\mu_{Y_h}} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right]^2 \frac{\mu_{Y_h}}{\mu_{Y_l}}.$$

Clearly, the magnitude of $\frac{\partial \sigma_Y^2}{\partial H_g} / \frac{\partial \sigma_Y^2}{\partial H_s}$ depends on parameter values since so does the magnitude of this ratio when either of σ_{λ_h} and σ_{λ_l} is zero (i.e., either of σ_{z_h} and σ_{z_l} is zero).

In case, however, the marginal products of H_s and H_g are equal then the following condition also needs to hold

$$\frac{1}{\mu_Y} \frac{\partial \mu_Y}{\partial H_s} = \frac{1}{\mu_Y} \frac{\partial \mu_Y}{\partial H_g}, \tag{50}$$

where

$$\frac{1}{\mu_Y} \frac{d\mu_Y}{dH_s} = \hat{\omega}_{Y_h}^Y \left[\omega_{Y_m}^{Y_h} \left(1 - \omega_K^{Y_m}\right) \frac{1}{H_s} + \left(1 - \omega_{Y_m}^{Y_h}\right) \frac{1}{u_h^g} \frac{du_h^g}{dH_s} \right] - \left(1 - \hat{\omega}_{Y_h}^Y\right) \gamma_4 \frac{1}{1 - u_h^g} \frac{du_h^g}{dH_s},$$
$$\frac{1}{\mu_Y} \frac{d\mu_Y}{dH_g} = \hat{\omega}_{Y_h}^Y \left(1 - \omega_{Y_m}^{Y_h}\right) \left(\frac{1}{H_g} + \frac{1}{u_h^g} \frac{du_h^g}{dH_g} \right) + \left(1 - \hat{\omega}_{Y_h}^Y\right) \gamma_4 \left(\frac{1}{H_g} - \frac{1}{1 - u_h^g} \frac{du_h^g}{dH_g} \right).$$

These partial derivatives can be rewritten as

$$\frac{1}{\mu_Y}\frac{\partial\mu_Y}{\partial H_g} = \left(1-\hat{\omega}^Y_{Y_h}\right)\left[\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u^g_h}\frac{\partial u^g_h}{\partial H_g}\right)-\left(1-\omega^{Y_h}_{Y_m}\right)\left(\frac{1}{H_g}+\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_g}\right)\right] \quad (51)$$
$$+\left(1-\omega^{Y_h}_{Y_m}\right)\left(\frac{1}{H_g}+\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_g}\right),$$

$$\frac{1}{\mu_Y}\frac{\partial\mu_Y}{\partial H_s} = -\left(1-\hat{\omega}^Y_{Y_h}\right)\left\{\gamma_4\frac{1}{1-u^g_h}\frac{\partial u^g_h}{\partial H_s}\right. \quad (52)$$
$$\left.+\left[\omega^{Y_h}_{Y_m}\left(1-\omega^{Y_m}_K\right)\frac{1}{H_s}+\left(1-\omega^{Y_h}_{Y_m}\right)\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_s}\right]\right\}$$
$$+\left[\omega^{Y_h}_{Y_m}\left(1-\omega^{Y_m}_K\right)\frac{1}{H_s}+\left(1-\omega^{Y_h}_{Y_m}\right)\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_s}\right].$$

In case when $\sigma^2_{z_h}>\sigma^2_{z_l}=\sigma^2_z=0$ (i.e., $\sigma^2_{\lambda_h}>\sigma^2_{\lambda_l}=\sigma^2_\lambda=0$), this implies that the ratio of derivatives of volatilities is given by

$$\frac{\frac{\partial\sigma^2_Y}{\partial H_g}}{\frac{\partial\sigma_Y}{\partial H_s}} = \frac{\left(1-\tilde{\omega}^Y_{Y_h}\right)\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_l}}\left(\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_g}-\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_g}\right)+\frac{\partial\mu_{Y_h}}{\partial H_g}}{\left(1-\tilde{\omega}^Y_{Y_h}\right)\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_l}}\left(\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_s}-\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_s}\right)+\frac{\partial\mu_{Y_h}}{\partial H_s}}$$

where the partial derivatives of μ_{Y_h} and μ_{Y_l} are given by (42)-(45). It has been shown previously that

$$\frac{\frac{\partial\sigma^2_Y}{\partial H_g}}{\frac{\partial\sigma^2_Y}{\partial H_s}} = \left\{\left(1-\tilde{\omega}^Y_{Y_h}\right)\left[\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u^g_h}\frac{\partial u^g_h}{\partial H_g}\right)-\left(1-\omega^{Y_h}_{Y_m}\right)\left(\frac{1}{H_g}+\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_g}\right)\right]\right.$$
$$\left.+\varepsilon_1\left(1-\omega^{Y_h}_{Y_m}\right)\left(\frac{1}{H_g}+\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_g}\right)\right\}$$
$$\times\left(-\left(1-\tilde{\omega}^Y_{Y_h}\right)\left\{\gamma_4\frac{1}{1-u^g_h}\frac{\partial u^g_h}{\partial H_s}+\left[\omega^{Y_h}_{Y_m}\left(1-\omega^{Y_m}_K\right)\frac{1}{H_s}+\left(1-\omega^{Y_h}_{Y_m}\right)\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_s}\right]\right\}\right.$$
$$\left.+\varepsilon_1\left[\omega^{Y_h}_{Y_m}\left(1-\omega^{Y_m}_K\right)\frac{1}{H_s}+\left(1-\omega^{Y_h}_{Y_m}\right)\frac{1}{u^g_h}\frac{\partial u^g_h}{\partial H_s}\right]\right),$$

Suppose that the first order approximation of final output (33) is carried around deterministic points of λ_h and λ_l where $\tilde{\omega}^Y_{Y_h}=\hat{\omega}^Y_{Y_h}$. Therefore, according to (50) if $\varepsilon_1=1$ the partial derivatives of the volatilities are equal, $\frac{\partial\sigma^2_Y}{\partial H_g}=\frac{\partial\sigma^2_Y}{\partial H_s}$.

The ratio of the partial derivatives has the following form

$$z=\frac{x_1+\alpha x_2}{x_3+\alpha x_4},$$

where ε_1 is replaced with α. The derivative of this ratio with respect to α is

$$\frac{\partial z}{\partial\alpha}=\frac{x_2x_3-x_1x_4}{\left(x_3+\alpha x_4\right)^2}.$$

We denote

$$d=x_2x_3-x_1x_4.$$

For the ratio of the partial derivatives of σ_Y^2, d is given by

$$d = -\left(1-\omega_{Y_m}^{Y_h}\right)\left(\frac{1}{H_g}+\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_g}\right)\left(1-\hat{\omega}_{Y_h}^{Y}\right)$$
$$\times\left\{\gamma_4\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_s}+\left[\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}+\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_s}\right]\right\}$$
$$-\left(1-\hat{\omega}_{Y_h}^{Y}\right)\left[\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_g}\right)-\left(1-\omega_{Y_m}^{Y_h}\right)\left(\frac{1}{H_g}+\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_g}\right)\right]$$
$$\times\left[\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}+\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_s}\right].$$

If this difference is negative, $d<0$, then for $\varepsilon_1>1$ we have that $\frac{\partial\sigma_Y^2}{\partial H_g}/\frac{\partial\sigma_Y^2}{\partial H_s}<1$ and for $\varepsilon_1<1$ we have that $\frac{\partial\sigma_Y^2}{\partial H_g}/\frac{\partial\sigma_Y^2}{\partial H_s}>1$.

We denote

$$\tilde{d}=\left(\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_g}\right)\left[\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}+\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_s}\right]$$
$$+\frac{1}{1-u_h^g}\frac{\partial u_h^g}{\partial H_s}\left(1-\omega_{Y_m}^{Y_h}\right)\left(\frac{1}{H_g}+\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial H_g}\right).$$

It is clear that when $\tilde{d}>0$ then $d<0$.

To check the sign of $\tilde{d}$ we plug the partial derivatives of u_h^g into $\tilde{d}$ and multiply $\tilde{d}$ by H_sH_g and divide it to $\left(1-\omega_K^{Y_m}\right)$, $\omega_{Y_m}^{Y_h}$, and

$$\frac{1}{1-u_h^g}\frac{1}{\left\{\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}\right\}^2}$$

to obtain the following expression

$$\hat{d}=\left[\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}\right]\left\{\varepsilon_1+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}\right\}$$
$$+\frac{u_h^g}{1-u_h^g}\left(\varepsilon_1-\varepsilon_2\right)\left(1-\omega_{Y_m}^{Y_h}\right)\left[\varepsilon_2+\varepsilon_2\left(\varepsilon_1-1\right)\left(1-\gamma_4\right)\right].$$

The right-hand side of this expression apparently has the same sign as $\tilde{d}$. Moreover, clearly it is positive since it is equal to

$$\varepsilon_1\left[\varepsilon_1\omega_{Y_m}^{Y_h}+\varepsilon_2\left(1-\omega_{Y_m}^{Y_h}\right)\right]+\frac{u_h^g}{1-u_h^g}\left[\varepsilon_2+\varepsilon_2\left(\varepsilon_1-1\right)\left(1-\gamma_4\right)\right]\varepsilon_1>0.$$

Therefore, if $\varepsilon_1>1$ the ratio $\frac{\partial\sigma_Y}{\partial H_g}/\frac{\partial\sigma_Y}{\partial H_s}$ is less than one and if $\varepsilon_1<1$ the ratio $\frac{\partial\sigma_Y}{\partial H_g}/\frac{\partial\sigma_Y}{\partial H_s}$ is greater than one.

In case when $\sigma_{z_l}^2>\sigma_{z_h}^2=\sigma_z^2=0$ (i.e., $\sigma_{\lambda_l}^2>\sigma_{\lambda_h}^2=\sigma_\lambda^2=0$), it has been shown that the ratio of derivatives of volatilities is given by

$$\frac{\frac{\partial\sigma_Y^2}{\partial H_g}}{\frac{\partial\sigma_Y^2}{\partial H_s}}=\frac{\tilde{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_h}}\left(\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_g}-\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_g}\right)+\frac{\partial\mu_{Y_l}}{\partial H_g}}{\tilde{\omega}_{Y_h}^{Y}\frac{1}{\varepsilon_1}\frac{1}{\mu_{Y_h}}\left(\mu_{Y_l}\frac{\partial\mu_{Y_h}}{\partial H_s}-\mu_{Y_h}\frac{\partial\mu_{Y_l}}{\partial H_s}\right)+\frac{\partial\mu_{Y_l}}{\partial H_s}}$$

where the partial derivatives of μ_{Y_h} and μ_{Y_l} are given by (42)-(45). Therefore,

$$\begin{aligned}\frac{\frac{\partial\sigma_Y^2}{\partial H_g}}{\frac{\partial\sigma_Y^2}{\partial H_s}} &= \left[\tilde{\omega}_{Y_h}^Y\left(1-\omega_{Y_m}^{Y_h}\right)\left(\frac{1}{H_g}+\frac{1}{u_h^g}\frac{du_h^g}{dH_g}\right)+\left(1-\tilde{\omega}_{Y_h}^Y\right)\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{du_h^g}{dH_g}\right)\right.\\ &\left.+\left(\varepsilon_1-1\right)\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{du_h^g}{dH_g}\right)\right]\\ &\times\left\{\tilde{\omega}_{Y_h}^Y\left[\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{du_h^g}{dH_s}+\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}\right]\right.\\ &\left.-\left(1-\tilde{\omega}_{Y_h}^Y\right)\gamma_4\frac{1}{1-u_h^g}\frac{du_h^g}{dH_s}-\left(\varepsilon_1-1\right)\gamma_4\frac{1}{1-u_h^g}\frac{du_h^g}{dH_s}\right\}^{-1}.\end{aligned}$$

Suppose, again, that the first order approximation of final output (33) is carried around deterministic points of λ_h and λ_l where $\tilde{\omega}_{Y_h}^Y=\hat{\omega}_{Y_h}^Y$. This expression then can be rewritten as

$$\frac{\frac{\partial\sigma_Y^2}{\partial H_g}}{\frac{\partial\sigma_Y^2}{\partial H_s}}=\frac{\hat{\omega}_{Y_h}^Y\left(1-\omega_{Y_m}^{Y_h}\right)\left(\frac{1}{H_g}+\frac{1}{u_h^g}\frac{du_h^g}{dH_g}\right)-\hat{\omega}_{Y_h}^Y\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{du_h^g}{dH_g}\right)+\varepsilon_1\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{du_h^g}{dH_g}\right)}{\hat{\omega}_{Y_h}^Y\left[\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{du_h^g}{dH_s}+\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}\right]+\hat{\omega}_{Y_h}^Y\gamma_4\frac{1}{1-u_h^g}\frac{du_h^g}{dH_s}+\varepsilon_1\left(-\gamma_4\frac{1}{1-u_h^g}\frac{du_h^g}{dH_s}\right)}.$$

Clearly, according to (50) if $\varepsilon_1=1$ volatilities are equal, $\frac{\partial\sigma_Y^2}{\partial H_g}=\frac{\partial\sigma_Y^2}{\partial H_s}$.

Similar to the previous case, we compute d for the ratio of partial derivatives of volatilities:

$$\begin{aligned}d &= \gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{du_h^g}{dH_g}\right)\\ &\times\left\{\tilde{\omega}_{Y_h}^Y\left[\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}\frac{du_h^g}{dH_s}+\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}\right]+\tilde{\omega}_{Y_h}^Y\gamma_4\frac{1}{1-u_h^g}\frac{du_h^g}{dH_s}\right\}\\ &+\left[\tilde{\omega}_{Y_h}^Y\left(1-\omega_{Y_m}^{Y_h}\right)\left(\frac{1}{H_g}+\frac{1}{u_h^g}\frac{du_h^g}{dH_g}\right)-\tilde{\omega}_{Y_h}^Y\gamma_4\left(\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{du_h^g}{dH_g}\right)\right]\\ &\times\gamma_4\frac{1}{1-u_h^g}\frac{du_h^g}{dH_s}.\end{aligned}$$

It can be shown that

$$\frac{1}{\gamma_4\hat{\omega}_{Y_h}^Y}d=\frac{1}{H_g}\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{1-u_h^g}\frac{1}{u_h^g}\frac{du_h^g}{dH_s}+\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}\left(\frac{1}{H_g}-\frac{1}{1-u_h^g}\frac{du_h^g}{dH_g}\right).$$

We use the expressions for the derivatives of u_h^g to get

$$d=\frac{\gamma_4\varepsilon_1\frac{1}{H_gH_s}\frac{\hat{\omega}_{Y_h}^Y\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)}{1-u_h^g}}{\varepsilon_2+\left(\varepsilon_1-\varepsilon_2\right)\omega_{Y_m}^{Y_h}+\varepsilon_2\left[\varepsilon_1-\left(\varepsilon_1-1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}},$$

which implies that $d>0$. Therefore, when $\varepsilon_1>1$ we have that $\frac{\partial\sigma_Y^2}{\partial H_g}/\frac{\partial\sigma_Y^2}{\partial H_s}>1$, and if $\varepsilon_1<1$ we have that $\frac{\partial\sigma_Y^2}{\partial H_g}/\frac{\partial\sigma_Y^2}{\partial H_s}<1$.

Proof of Proposition 7: We ignore that λ, λ_h, and λ_l are stochastic and drop expectation operators everywhere. The total changes of ω_h^g and ω_l are given by

$$\begin{aligned}\frac{1}{\omega_h^g\left(1-\omega_h^g\right)}d\omega_h^g &= \frac{1}{H_g}dH_g - \frac{1}{H_s}dH_s + \frac{1}{u_h^g}du_h^g,\\ \frac{1}{\omega_l}d\omega_l &= \frac{H_s}{H_g+H_s}\left(\frac{1}{H_g}dH_g - \frac{1}{H_s}dH_s\right) - \frac{1}{1-u_h^g}du_h^g.\end{aligned}$$

Given that supply fixes the ratio of wages the total variation of wages should satisfy:

$$0 = \frac{\partial \frac{w_g}{w_s}}{\partial K}dK + \left(\frac{\partial u_h^g}{\partial \lambda_l}d\lambda_l + \frac{\partial u_h^g}{\partial \lambda_h}d\lambda_h\right)\frac{\partial \frac{w_g}{w_s}}{\partial u_h^g} + \frac{\partial \frac{w_g}{w_s}}{\partial H_g}dH_g + \frac{\partial \frac{w_g}{w_s}}{\partial H_s}dH_s$$

The partial derivatives of relative wages are given by (30), (31), and (32).

From demand and supply of specific human capital it also follows that

$$\lambda_H = \omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{\omega_{Y_h}^{Y}\lambda\left[\gamma_1 Y_h^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \left(1-\gamma_1\right)Y_l^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1-1}}}{H_s}.$$

Therefore,

$$\begin{aligned}0 &= \lambda_H\frac{1}{\omega_{Y_m}^{Y_h}}d\omega_{Y_m}^{Y_h} - \lambda_H\frac{1}{1-\omega_K^{Y_m}}d\omega_K^{Y_m} + \lambda_H\frac{1}{\omega_{Y_h}^{Y}}d\omega_{Y_h}^{Y} - \lambda_H\frac{1}{H_s}dH_s\\ &+ \lambda_H\omega_{Y_h}^{Y}\frac{1}{Y_h}dY_h + \lambda_H\left(1-\omega_{Y_h}^{Y}\right)\frac{1}{Y_l}dY_l.\end{aligned}$$

From (39) and (38) it follows that this expression can be rewritten in the following manner:

$$\begin{aligned}0 &= \lambda_H\frac{1}{\omega_{Y_m}^{Y_h}}d\omega_{Y_m}^{Y_h} - \lambda_H\frac{1}{\left(1-\omega_K^{Y_m}\right)}d\omega_K^{Y_m} + \lambda_H\frac{1}{\omega_{Y_h}^{Y}}d\omega_{Y_h}^{Y} - \lambda_H\frac{1}{H_s}dH_s\\ &+ \lambda_H\omega_{Y_h}^{Y}\left[\left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{u_h^g}du_h^g + \left(1-\omega_{Y_m}^{Y_h}\right)\frac{1}{H_g}dH_g + \omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{1}{H_s}dH_s\right.\\ &\left.+ \omega_{Y_m}^{Y_h}\omega_K^{Y_m}\frac{1}{K}dK\right] + \lambda_H\left(1-\omega_{Y_h}^{Y}\right)\gamma_4\left(-\frac{1}{1-u_h^g}du_h^g + \frac{1}{H_g}dH_g\right).\end{aligned} \tag{53}$$

Since only K changes, we use (26) and (5), (11), and (12), to evaluate the partial derivatives of the shares:

$$\begin{aligned}\frac{\partial\omega_{Y_h}^{Y}}{\partial K} &= \frac{\varepsilon_1-1}{\varepsilon_1}\omega_{Y_h}^{Y}\left(1-\omega_{Y_h}^{Y}\right)\left[\omega_{Y_m}^{Y_h}\omega_K^{Y_m}\frac{1}{K} + \left(1-\omega_{Y_m}^{Y_h} + \gamma_4\frac{u_h^g}{1-u_h^g}\right)\frac{1}{u_h^g}\frac{\partial u_h^g}{\partial K}\right],\\ \frac{\partial\omega_{Y_m}^{Y_h}}{\partial K} &= \frac{\varepsilon_2-1}{\varepsilon_2}\omega_{Y_m}^{Y_h}\left(1-\omega_{Y_m}^{Y_h}\right)\left(\omega_K^{Y_m}\frac{1}{K} - \frac{1}{u_h^g}\frac{\partial u_h^g}{\partial K}\right),\\ \frac{\partial\omega_K^{Y_m}}{\partial K} &= \frac{\varepsilon_3-1}{\varepsilon_3}\omega_K^{Y_m}\left(1-\omega_K^{Y_m}\right)\frac{1}{K}.\end{aligned}$$

Using these expressions, the expression for total change in the demand for specific human capital (53) can be rewritten as:

$$0 = \frac{K}{\omega_{Y_m}^{Y_h}} \frac{\partial \omega_{Y_m}^{Y_h}}{\partial K} - \frac{\omega_K^{Y_m}}{1-\omega_K^{Y_m}} \frac{K}{\omega_K^{Y_m}} \frac{\partial \omega_K^{Y_m}}{\partial K} + \frac{K}{\omega_{Y_h}^{Y}} \frac{\partial \omega_{Y_h}^{Y}}{\partial K} - \frac{K}{H_s}\frac{dH_s}{dK}$$
$$+ \omega_{Y_h}^{Y}\left[\left(1-\omega_{Y_m}^{Y_h}\right)\frac{K}{u_h^g}\frac{du_h^g}{dK} + \left(1-\omega_{Y_m}^{Y_h}\right)\frac{K}{H_g}\frac{dH_g}{dK} + \omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\frac{K}{H_s}\frac{dH_s}{dK}\right]$$
$$+ \omega_{Y_h}^{Y}\omega_{Y_m}^{Y_h}\omega_K^{Y_m} + \left(1-\omega_{Y_h}^{Y}\right)\gamma_4\left(-\frac{u_h^g}{1-u_h^g}\frac{K}{u_h^g}\frac{du_h^g}{dK} + \frac{K}{H_g}\frac{dH_g}{dK}\right).$$

The sum of the first three terms is given by

$$S_3 = \left[\frac{\varepsilon_2-1}{\varepsilon_2}\left(1-\omega_{Y_m}^{Y_h}\right) - \frac{\varepsilon_3-1}{\varepsilon_3} + \frac{\varepsilon_1-1}{\varepsilon_1}\left(1-\omega_{Y_h}^{Y}\right)\omega_{Y_m}^{Y_h}\right]\omega_K^{Y_m}$$
$$+ \left[\frac{\varepsilon_1-1}{\varepsilon_1}\left(1-\omega_{Y_h}^{Y}\right)\left(1-\omega_{Y_m}^{Y_h} + \gamma_4\frac{u_h^g}{1-u_h^g}\right) - \frac{\varepsilon_2-1}{\varepsilon_2}\left(1-\omega_{Y_m}^{Y_h}\right)\right]\frac{K}{u_h^g}\frac{\partial u_h^g}{\partial K}.$$

This implies that when $\varepsilon_2 = \gamma_4 = 1$ and $\varepsilon_3 = +\infty$, the following equation holds:

$$0 = \left[\frac{\varepsilon_1-1}{\varepsilon_1}\left(1-\omega_{Y_h}^{Y}\right)\omega_{Y_m}^{Y_h} + \omega_{Y_h}^{Y}\omega_{Y_m}^{Y_h} - 1\right]\omega_K^{Y_m} + \frac{K}{u_h^g}\frac{\partial u_h^g}{\partial K}$$
$$\times\left[\frac{\varepsilon_1-1}{\varepsilon_1}\left(1-\omega_{Y_h}^{Y}\right)\left(1-\omega_{Y_m}^{Y_h} + \frac{u_h^g}{1-u_h^g}\right) + \left(1-\omega_{Y_m}^{Y_h}\right)\omega_{Y_h}^{Y} - \left(1-\omega_{Y_h}^{Y}\right)\frac{u_h^g}{1-u_h^g}\right]$$
$$- \left[1-\omega_{Y_h}^{Y}\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)\right]\frac{K}{H_s}\frac{dH_s}{dK} + \left(1-\omega_{Y_m}^{Y_h}\omega_{Y_h}^{Y}\right)\frac{K}{H_g}\frac{dH_g}{dK}.$$

On the other hand, from (18) and (20) it follows that the total variation of wages is zero. This, together with (30), (31), and (32), implies that when $\varepsilon_2 = \gamma_4 = 1$ and $\varepsilon_3 = +\infty$ the following holds:

$$0 = \omega_K^{Y_m} - \frac{K}{H_g}\frac{dH_g}{dK} + \left(1-\omega_K^{Y_m}\right)\frac{K}{H_s}\frac{dH_s}{dK}$$

We solve for the percentage change of H_s using this expression and the one above:

$$\frac{K}{H_s}\frac{dH_s}{dK} = \left(\varepsilon_1-1\right)\omega_{Y_m}^{Y_h}\frac{1-\omega_{Y_h}^{Y}\omega_{Y_m}^{Y_h}}{\left(\varepsilon_1-1\right)\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g}}.$$

Therefore,

$$\frac{K}{H_g}\frac{dH_g}{dK} - \frac{K}{H_s}\frac{dH_s}{dK} = \omega_K^{Y_m}\frac{\frac{1}{1-u_h^g} + \left(\varepsilon_1-1\right)\omega_{Y_m}^{Y_h}\omega_{Y_h}^{Y}\omega_{Y_m}^{Y_h}}{\left(\varepsilon_1-1\right)\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g}}$$
$$\frac{K}{H_g}\frac{dH_g}{dK} - \frac{K}{H_s}\frac{dH_s}{dK} + \frac{K}{u_h^g}\frac{du_h^g}{dK} = \omega_K^{Y_m}\frac{\frac{1}{1-u_h^g} + \left(\varepsilon_1-1\right)\omega_{Y_m}^{Y_h}\omega_{Y_h}^{Y}\omega_{Y_m}^{Y_h} + \left(\varepsilon_1-1\right)\omega_{Y_m}^{Y_h}}{\left(\varepsilon_1-1\right)\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g}}$$

In terms of changes in shares then we have that

$$\frac{K}{\omega_h^g\left(1-\omega_h^g\right)}\frac{d\omega_h^g}{dK} = \frac{K}{H_g}\frac{dH_g}{dK} - \frac{K}{H_s}\frac{dH_s}{dK} + \frac{K}{u_h^g}\frac{du_h^g}{dK}$$

$$= \frac{\omega_K^{Y_m}\left\{\frac{1}{1-u_h^g} + (\varepsilon_1-1)(1-\gamma_2)\left[1+\omega_{Y_h}^Y(1-\gamma_2)\right]\right\}}{(\varepsilon_1-1)(1-\gamma_2)+\frac{1}{1-u_h^g}},$$

and

$$\frac{K}{\omega_l}\frac{d\omega_l}{dK} = \frac{\omega_K^{Y_m}\left\{\frac{H_s}{H_g+H_s}\left[\frac{1}{1-u_h^g} + (\varepsilon_1-1)(1-\gamma_2)(1-\gamma_2)\,\omega_{Y_h}^Y\right] - \frac{u_h^g}{1-u_h^g}(\varepsilon_1-1)(1-\gamma_2)\right\}}{(\varepsilon_1-1)(1-\gamma_2)+\frac{1}{1-u_h^g}},$$

where we have replaced $\omega_{Y_m}^{Y_h}$ by $1-\gamma_2$.

Clearly, $d\omega_h^g/dK$ is positive at least for

$$\varepsilon_1 > \frac{(1-\gamma_2)\left[1+\omega_{Y_h}^Y(1-\gamma_2)\right]-1}{(1-\gamma_2)\left[1+\omega_{Y_h}^Y(1-\gamma_2)\right]}.$$

In turn, if

$$\frac{u_h^g}{1-u_h^g} - \frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^Y < 0$$

then in order for both $d\omega_h^g/dK$ and $d\omega_l/dK$ to be positive it is sufficient to have

$$\varepsilon_1 > \frac{(1-\gamma_2)\left[1+\omega_{Y_h}^Y(1-\gamma_2)\right]-1}{(1-\gamma_2)\left[1+\omega_{Y_h}^Y(1-\gamma_2)\right]}.$$

However, if

$$\frac{u_h^g}{1-u_h^g} - \frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^Y > 0$$

then in order for $d\omega_l/dK$ to be positive it is sufficient to have

$$\frac{\frac{H_s}{H_g+H_s}\frac{1}{1-u_h^g} + (1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g} - \frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^Y\right]}{(1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g} - \frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^Y\right]} > \varepsilon_1.$$

When

$$\frac{u_h^g}{1-u_h^g} - \frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^Y > 0$$

these limits have the following relationship

$$\frac{\frac{H_s}{H_g+H_s}\frac{1}{1-u_h^g} + (1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g} - \frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^Y\right]}{(1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g} - \frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^Y\right]} > 1 > \frac{(1-\gamma_2)\left[1+\omega_{Y_h}^Y(1-\gamma_2)\right]-1}{(1-\gamma_2)\left[1+\omega_{Y_h}^Y(1-\gamma_2)\right]}.$$

We define a function $\mathbb{I}(.,.)$ as

$$\mathbb{I}(x,y) = \begin{cases} x & if \;\; x>0, \\ y & otherwise. \end{cases}$$

In general, in order for $d\omega_h^g/dK$ and $d\omega_l/dK$ to be positive it is sufficient to have ε_1 higher than

$$\mathbb{I}\left(\frac{(1-\gamma_2)\left[1+\omega_{Y_h}^Y(1-\gamma_2)\right]-1}{(1-\gamma_2)\left[1+\omega_{Y_h}^Y(1-\gamma_2)\right]},0\right)$$

and lower than

$$\mathbb{I}\left(\frac{\frac{H_s}{H_g+H_s}\frac{1}{1-u_h^g}+(1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g}-\frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^Y\right]}{(1-\gamma_2)\left[\frac{u_h^g}{1-u_h^g}-\frac{H_s}{H_g+H_s}(1-\gamma_2)\,\omega_{Y_h}^Y\right]},+\infty\right).$$

Clearly, this interval includes both lower than 1 and greater than 1 values of ε_1.

Proofs of Propositions 8 and 9: The derivatives of Y with respect to sectoral shocks are given by

$$\frac{\partial Y}{\partial\lambda_h}=\frac{\partial Y}{\partial Y_h}\left[\frac{\partial Y_h}{\partial\lambda_h}+\frac{\partial Y_h}{\partial u_h^g}\frac{\partial u_h^g}{\partial\lambda_h}\right]-\frac{\partial Y}{\partial Y_l}\frac{\partial Y_l}{\partial u_l^g}\frac{\partial u_h^g}{\partial\lambda_h},$$
$$\frac{\partial Y}{\partial\lambda_l}=\frac{\partial Y}{\partial Y_h}\frac{\partial Y_h}{\partial u_h^g}\frac{\partial u_h^g}{\partial\lambda_l}+\frac{\partial Y}{\partial Y_l}\left[\frac{\partial Y_l}{\partial\lambda_l}-\frac{\partial Y_l}{\partial u_l^g}\frac{\partial u_h^g}{\partial\lambda_l}\right],$$

We use (3), (4), (6), (13), (10), (14), and (28) to derive the elasticities of final output with respect to sectoral shocks. The elasticities add up to one and are given by:

$$\frac{\lambda_h}{Y}\frac{\partial Y}{\partial\lambda_h}=\frac{\varepsilon_1\omega_{Y_m}^{Y_h}\omega_{Y_h}^Y+\omega_{Y_h}^Y\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_1\varepsilon_2+\varepsilon_2\left[\varepsilon_1\omega_{Y_h}^Y-(\varepsilon_1-1)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}{\varepsilon_2+(\varepsilon_1-\varepsilon_2)\,\omega_{Y_m}^{Y_h}+\varepsilon_2\left[\varepsilon_1-(\varepsilon_1-1)\gamma_4\right]\frac{u_h^g}{1-u_h^g}},$$
$$\frac{\lambda_l}{Y}\frac{\partial Y}{\partial\lambda_l}=1-\frac{\lambda_h}{Y}\frac{\partial Y}{\partial\lambda_h}.$$

Both these elasticities should be greater than zero, which implies that they are bounded in between 0 and 1. Therefore, in equilibrium the following restrictions should hold

$$\varepsilon_1\omega_{Y_m}^{Y_h}\omega_{Y_h}^Y+\omega_{Y_h}^Y\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_1\varepsilon_2+\varepsilon_2\left[\varepsilon_1\omega_{Y_h}^Y-(\varepsilon_1-1)\gamma_4\right]\frac{u_h^g}{1-u_h^g}>0, \tag{54}$$

$$\varepsilon_1\omega_{Y_m}^{Y_h}\omega_{Y_h}^Y+\omega_{Y_h}^Y\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_1\varepsilon_2-\varepsilon_2\varepsilon_1\left(1-\omega_{Y_h}^Y\right)\frac{u_h^g}{1-u_h^g}<\varepsilon_2+(\varepsilon_1-\varepsilon_2)\,\omega_{Y_m}^{Y_h}. \tag{55}$$

In order to evaluate the partial derivatives of the elasticities with respect to stocks of human capital types we consider the partial derivatives of shares using (5) and (11). The partial derivatives of the shares are given by

$$\frac{\partial\omega_{Y_h}^Y}{\partial H_s}=\frac{\partial\omega_{Y_h}^Y}{\partial Y_h}\left[\frac{\partial Y_h}{\partial Y_m}\frac{\partial Y_m}{\partial H_s}+\frac{\partial Y_h}{\partial u_h^g}\frac{\partial u_h^g}{\partial H_s}\right]-\frac{\partial\omega_{Y_h}^Y}{\partial Y_l}\frac{\partial Y_l}{\partial u_l^g}\frac{\partial u_h^g}{\partial H_s},$$
$$\frac{\partial\omega_{Y_h}^Y}{\partial H_g}=\frac{\partial\omega_{Y_h}^Y}{\partial Y_h}\left[\frac{\partial Y_h}{\partial H_g}+\frac{\partial Y_h}{\partial u_h^g}\frac{\partial u_h^g}{\partial H_g}\right]+\frac{\partial\omega_{Y_h}^Y}{\partial Y_l}\left[\frac{\partial Y_l}{\partial H_g}-\frac{\partial Y_l}{\partial u_l^g}\frac{\partial u_h^g}{\partial H_g}\right],$$

and

$$\frac{\partial\omega_{Y_m}^{Y_h}}{\partial H_s}=\frac{\partial\omega_{Y_m}^{Y_h}}{\partial Y_m}\frac{\partial Y_m}{\partial H_s}+\frac{\partial\omega_{Y_m}^{Y_h}}{\partial u_h^g}\frac{\partial u_h^g}{\partial H_s},$$
$$\frac{\partial\omega_{Y_m}^{Y_h}}{\partial H_g}=\frac{\partial\omega_{Y_m}^{Y_h}}{\partial H_g}+\frac{\partial\omega_{Y_m}^{Y_h}}{\partial u_h^g}\frac{\partial u_h^g}{\partial H_g}.$$

Clearly, according to the formulas of the shares and production functions it is the case that

$$\frac{\partial \omega_{Y_h}^{Y}}{\partial Y_h} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \frac{1}{Y_h} \omega_{Y_h}^{Y} \left(1 - \omega_{Y_h}^{Y}\right),$$
$$\frac{\partial \omega_{Y_h}^{Y}}{\partial Y_l} = -\frac{\varepsilon_1 - 1}{\varepsilon_1} \frac{1}{Y_l} \omega_{Y_h}^{Y} \left(1 - \omega_{Y_h}^{Y}\right),$$

and

$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial Y_m} = \frac{\varepsilon_2 - 1}{\varepsilon_2} \frac{1}{Y_m} \omega_{Y_m}^{Y_h} \left(1 - \omega_{Y_m}^{Y_h}\right),$$
$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial u_h^g} = -\frac{\varepsilon_2 - 1}{\varepsilon_2} \frac{1}{u_h^g} \omega_{Y_m}^{Y_h} \left(1 - \omega_{Y_m}^{Y_h}\right),$$
$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial H_g} = -\frac{\varepsilon_2 - 1}{\varepsilon_2} \frac{1}{H_g} \omega_{Y_m}^{Y_h} \left(1 - \omega_{Y_m}^{Y_h}\right).$$

In turn, the partial derivatives of u_h^g are given by (25) and (27). Therefore, the partial derivatives of the shares are given by

$$\frac{\partial \omega_{Y_h}^{Y}}{\partial H_s} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \frac{\omega_{Y_h}^{Y}}{H_s} \left(1 - \omega_{Y_h}^{Y}\right) \left\{\omega_{Y_m}^{Y_h} \left(1 - \omega_K^{Y_m}\right) + \left[\left(1 - \omega_{Y_m}^{Y_h}\right) + \gamma_4 \frac{u_h^g}{1 - u_h^g}\right] \frac{H_s}{u_h^g} \frac{\partial u_h^g}{\partial H_s}\right\},$$
$$\frac{\partial \omega_{Y_h}^{Y}}{\partial H_g} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \frac{\omega_{Y_h}^{Y}}{H_g} \left(1 - \omega_{Y_h}^{Y}\right) \left\{\left(1 - \omega_{Y_m}^{Y_h} - \gamma_4\right) + \left[\left(1 - \omega_{Y_m}^{Y_h}\right) + \gamma_4 \frac{u_h^g}{1 - u_h^g}\right] \frac{H_g}{u_h^g} \frac{\partial u_h^g}{\partial H_g}\right\},$$

and

$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial H_s} = \frac{\varepsilon_2 - 1}{\varepsilon_2} \left(1 - \omega_{Y_m}^{Y_h}\right) \frac{\omega_{Y_m}^{Y_h}}{H_s} \left[\left(1 - \omega_K^{Y_m}\right) - \frac{H_s}{u_h^g} \frac{\partial u_h^g}{\partial H_s}\right],$$
$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial H_g} = -\frac{\varepsilon_2 - 1}{\varepsilon_2} \left(1 - \omega_{Y_m}^{Y_h}\right) \frac{\omega_{Y_m}^{Y_h}}{H_g} \left[1 + \frac{H_g}{u_h^g} \frac{\partial u_h^g}{\partial H_g}\right].$$

Plugging the values of $\frac{\partial u_h^g}{\partial H_g}$ and $\frac{\partial u_h^g}{\partial H_s}$ gives

$$\frac{\partial \omega_{Y_h}^{Y}}{\partial H_s} = \frac{\omega_{Y_h}^{Y}}{H_s} \left(1 - \omega_{Y_h}^{Y}\right) \left(1 - \omega_K^{Y_m}\right) \omega_{Y_m}^{Y_h} \times \frac{(\varepsilon_1 - 1) \left\{1 + \left[\varepsilon_2 (1 - \gamma_4) + \gamma_4\right] \frac{u_h^g}{1 - u_h^g}\right\}}{\varepsilon_2 + (\varepsilon_1 - \varepsilon_2) \omega_{Y_m}^{Y_h} + \varepsilon_2 \left[\varepsilon_1 - (\varepsilon_1 - 1) \gamma_4\right] \frac{u_h^g}{1 - u_h^g}}, \tag{56}$$

$$\frac{\partial \omega_{Y_h}^{Y}}{\partial H_g} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \left(1 - \omega_{Y_h}^{Y}\right) \frac{\omega_{Y_h}^{Y}}{H_g} \times \left[\frac{\left(1 - \omega_{Y_m}^{Y_h}\right) \left[\varepsilon_2 + \varepsilon_2 (\varepsilon_1 - 1)(1 - \gamma_4)\right] - \left[\varepsilon_2 + (\varepsilon_1 - \varepsilon_2) \omega_{Y_m}^{Y_h}\right] \gamma_4}{\varepsilon_2 + (\varepsilon_1 - \varepsilon_2) \omega_{Y_m}^{Y_h} + \varepsilon_2 \left[\varepsilon_1 - (\varepsilon_1 - 1) \gamma_4\right] \frac{u_h^g}{1 - u_h^g}} \frac{1}{1 - u_h^g}\right], \tag{57}$$

and

$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial H_s} = \frac{\varepsilon_2 - 1}{\varepsilon_2}\left(1-\omega_{Y_m}^{Y_h}\right)\frac{\omega_{Y_m}^{Y_h}}{H_s}\left(1-\omega_K^{Y_m}\right) \times \frac{\varepsilon_2 + \varepsilon_2\left(\varepsilon_1 - 1\right)\left(1-\gamma_4\right)u_h^g}{\varepsilon_2 + \left(\varepsilon_1 - \varepsilon_2\right)\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1 - \left(\varepsilon_1 - 1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}\frac{1}{1-u_h^g}, \tag{58}$$

$$\frac{\partial \omega_{Y_m}^{Y_h}}{\partial H_g} = -\frac{\varepsilon_2 - 1}{\varepsilon_2}\left(1-\omega_{Y_m}^{Y_h}\right)\frac{\omega_{Y_m}^{Y_h}}{H_g} \times \frac{\varepsilon_2 + \varepsilon_2\left(\varepsilon_1 - 1\right)\left(1-\gamma_4\right)}{\varepsilon_2 + \left(\varepsilon_1 - \varepsilon_2\right)\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1 - \left(\varepsilon_1 - 1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}\frac{1}{1-u_h^g}. \tag{59}$$

Let i be either s or g. This implies that the partial derivative of $\frac{\lambda_h}{Y}\frac{\partial Y}{\partial \lambda_h}$ with respect to H_i is given by

$$\frac{\partial}{\partial H_i}\left(\frac{\lambda_h}{Y}\frac{\partial Y}{\partial \lambda_h}\right) = \frac{\partial}{\partial H_i}\frac{\varepsilon_1\omega_{Y_m}^{Y_h}\omega_{Y_h}^{Y} + \omega_{Y_h}^{Y}\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_1\varepsilon_2 + \varepsilon_2\left[\varepsilon_1\omega_{Y_h}^{Y} - \left(\varepsilon_1 - 1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}}{\varepsilon_2 + \left(\varepsilon_1 - \varepsilon_2\right)\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1 - \left(\varepsilon_1 - 1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}},$$

The numerator of this expression is given by

$$\begin{aligned}
Z_{1,H_i} = &\left[\varepsilon_1\omega_{Y_m}^{Y_h} + \varepsilon_1\varepsilon_2\left(1-\omega_{Y_m}^{Y_h}\right) + \varepsilon_2\varepsilon_1\frac{u_h^g}{1-u_h^g}\right] \\
&\times\left\{\varepsilon_2 + \left(\varepsilon_1 - \varepsilon_2\right)\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1 - \left(\varepsilon_1 - 1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}\right\}\frac{\partial}{\partial H_i}\omega_{Y_h}^{Y} \\
&+ \varepsilon_1\omega_{Y_h}^{Y}\left(1-\varepsilon_2\right)\left\{\varepsilon_2 + \left(\varepsilon_1 - \varepsilon_2\right)\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1 - \left(\varepsilon_1 - 1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}\right\}\frac{\partial}{\partial H_i}\omega_{Y_m}^{Y_h} \\
&- \left(\varepsilon_1 - \varepsilon_2\right)\left\{\varepsilon_1\omega_{Y_m}^{Y_h}\omega_{Y_h}^{Y} + \omega_{Y_h}^{Y}\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_1\varepsilon_2 + \varepsilon_2\left[\varepsilon_1\omega_{Y_h}^{Y} - \left(\varepsilon_1 - 1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}\right\}\frac{\partial}{\partial H_i}\omega_{Y_m}^{Y_h} \\
&+ \left\{\left[\varepsilon_1\omega_{Y_h}^{Y} - \left(\varepsilon_1 - 1\right)\gamma_4\right]\left[\varepsilon_2 + \left(\varepsilon_1 - \varepsilon_2\right)\omega_{Y_m}^{Y_h}\right]\right. \\
&\left.- \varepsilon_1\omega_{Y_h}^{Y}\left[\varepsilon_1 - \left(\varepsilon_1 - 1\right)\gamma_4\right]\left[\omega_{Y_m}^{Y_h} + \left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_2\right]\right\}\varepsilon_2\frac{\partial}{\partial H_i}\frac{u_h^g}{1-u_h^g}
\end{aligned}$$

After some algebra, the numerator can be expressed as

$$\begin{aligned}
H_i Z_{1,H_i} = &\left[\varepsilon_1\omega_{Y_m}^{Y_h} + \varepsilon_1\varepsilon_2\left(1-\omega_{Y_m}^{Y_h}\right) + \varepsilon_2\varepsilon_1\frac{u_h^g}{1-u_h^g}\right] \\
&\times\left\{\varepsilon_2 + \left(\varepsilon_1 - \varepsilon_2\right)\omega_{Y_m}^{Y_h} + \varepsilon_2\left[\varepsilon_1 - \left(\varepsilon_1 - 1\right)\gamma_4\right]\frac{u_h^g}{1-u_h^g}\right\}H_i\frac{\partial}{\partial H_i}\omega_{Y_h}^{Y} \\
&- \varepsilon_2\left(\varepsilon_1 - 1\right)\left(\varepsilon_1\omega_{Y_h}^{Y}\left[1-u_h^g\left(1-\gamma_4\right)\right] + \left\{\varepsilon_2\left[\varepsilon_1\omega_{Y_h}^{Y}\left(1-\gamma_4\right) + \gamma_4\right] - \varepsilon_1\gamma_4\right\}u_h^g\right) \\
&\times\frac{1}{1-u_h^g}H_i\frac{\partial}{\partial H_i}\omega_{Y_m}^{Y_h} \\
&- \left(\varepsilon_1 - 1\right)\left\{\left[\varepsilon_1\omega_{Y_h}^{Y}\left(1-\gamma_4\right) + \gamma_4\right]\left(1-\omega_{Y_m}^{Y_h}\right)\varepsilon_2 + \varepsilon_1\omega_{Y_m}^{Y_h}\gamma_4\left(1-\omega_{Y_h}^{Y}\right)\right\}\varepsilon_2 H_i\frac{\partial}{\partial H_i}\frac{u_h^g}{1-u_h^g}
\end{aligned}$$

Clearly, when $\varepsilon_2 = \gamma_4 = 1$ the numerator can be rewritten as

$$H_i Z_{1,H_i} = \varepsilon_1 \frac{1}{1-u_h^g}\left[(\varepsilon_1 - 1)\,\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g}\right] H_i \frac{\partial \omega_{Y_h}^{Y}}{\partial H_i}$$
$$- (\varepsilon_1 - 1)\left[1 - \omega_{Y_m}^{Y_h} + \varepsilon_1 \omega_{Y_m}^{Y_h}\left(1 - \omega_{Y_h}^{Y}\right)\right] \frac{1}{1-u_h^g}\frac{1}{1-u_h^g} H_i \frac{\partial u_h^g}{\partial H_i}$$

which means that

$$H_s Z_{1,H_s} = (\varepsilon_1 - 1)\frac{\left(1-\omega_K^{Y_m}\right)\omega_{Y_m}^{Y_h}\frac{1}{\left(1-u_h^g\right)^2}}{(\varepsilon_1 - 1)\,\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g}}$$
$$\times \left\{\varepsilon_1\left[(\varepsilon_1 - 1)\,\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g}\right]\omega_{Y_h}^{Y}\left(1-\omega_{Y_h}^{Y}\right)\right.$$
$$\left. - (\varepsilon_1 - 1)\left[1 - \omega_{Y_m}^{Y_h} + \varepsilon_1 \omega_{Y_m}^{Y_h}\left(1-\omega_{Y_h}^{Y}\right)\right] u_h^g\right\}$$

$$H_g Z_{1,H_g} = -(\varepsilon_1 - 1)\frac{\omega_{Y_m}^{Y_h}\frac{1}{\left(1-u_h^g\right)^2}}{(\varepsilon_1 - 1)\,\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g}}$$
$$\times \left\{\varepsilon_1\left[(\varepsilon_1 - 1)\,\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g}\right]\omega_{Y_h}^{Y}\left(1-\omega_{Y_h}^{Y}\right)\right.$$
$$\left. - (\varepsilon_1 - 1)\left[1 - \omega_{Y_m}^{Y_h} + \varepsilon_1 \omega_{Y_m}^{Y_h}\left(1-\omega_{Y_h}^{Y}\right)\right] u_h^g\right\}$$

The difference between the elasticities is given by

$$\Delta_{\lambda_h}(\varepsilon_1 - 1)\frac{1}{\left(1-u_h^g\right)^2}\left[\frac{\left(2-\omega_K^{Y_m}\right)\omega_{Y_m}^{Y_h}}{1 + (\varepsilon_1 - 1)\,\omega_{Y_m}^{Y_h} + \frac{u_h^g}{1-u_h^g}}\right]$$
$$\times \left\{\varepsilon_1\left[(\varepsilon_1 - 1)\,\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g}\right]\omega_{Y_h}^{Y}\left(1-\omega_{Y_h}^{Y}\right)\right.$$
$$\left. - (\varepsilon_1 - 1)\left[\left(1 - \omega_{Y_m}^{Y_h}\right) + \varepsilon_1 \omega_{Y_m}^{Y_h}\left(1-\omega_{Y_h}^{Y}\right)\right] u_h^g\right\}.$$

Similar exercise for elasticities with respect to λ_l gives $\Delta_{\lambda_l} = -\Delta_{\lambda_h}$. If $\varepsilon_1 < 1$ then Δ_{λ_h} is negative and Δ_{λ_l} is positive. Therefore, the elasticity of final output with respect to shocks λ_h (λ_l) increases less (more) with a marginal (percentage) increase in H_s than with a marginal (percentage) increase in H_g if $\varepsilon_1 < 1$.

To analyze the case when $\varepsilon_1 > 1$, we denote by d the following expression:

$$d = \varepsilon_1\left[(\varepsilon_1 - 1)\,\omega_{Y_m}^{Y_h} + \frac{1}{1-u_h^g}\right]\omega_{Y_h}^{Y}\left(1-\omega_{Y_h}^{Y}\right) \tag{60}$$
$$- (\varepsilon_1 - 1)\left[\left(1 - \omega_{Y_m}^{Y_h}\right) + \varepsilon_1 \omega_{Y_m}^{Y_h}\left(1-\omega_{Y_h}^{Y}\right)\right] u_h^g.$$

Clearly, d is positive, for example, when ε_1 is very close to 1 from above. It also positive when $\varepsilon_1 = +\infty$ since $d > 0$ in that case it is equivalent to (54). Therefore, d is positive at least in the limit when $\varepsilon_1 = 1+$ and $\varepsilon_1 = +\infty$. This implies that Δ_{λ_h} is positive and Δ_{λ_l} is negative for $\varepsilon_1 = 1+$ and $\varepsilon_1 = +\infty$. The elasticity of final output with respect to shocks λ_h (λ_l) then increases more (less) with a marginal (percentage) increase in H_s than with a marginal (percentage) increase in H_g.

The total change of the elasticities is given by

$$d\left(\frac{\lambda_{Y_i}}{Y}\frac{\partial Y}{\partial \lambda_{Y_i}}\right)=\frac{\partial}{\partial H_s}\left(\frac{\lambda_{Y_i}}{Y}\frac{\partial Y}{\partial \lambda_{Y_i}}\right)dH_s+\frac{\partial}{\partial H_g}\left(\frac{\lambda_{Y_i}}{Y}\frac{\partial Y}{\partial \lambda_{Y_i}}\right)dH_g \tag{61}$$

We consider variation in H_s and H_g such that (expected) final output stays constant. This variation satisfies (36).

We again set $\varepsilon_2=\gamma_4=1$. The total variation of elasticise (61) for λ_h is a ratio where the numerator is given by

$$\begin{aligned}Z_{\lambda_h}=&\left\{\varepsilon_1\frac{1}{1-u_h^g}\left[1+(\varepsilon_1-1)\,\omega_{Y_m}^{Y_h}+\frac{u_h^g}{1-u_h^g}\right]\frac{\partial\omega_{Y_h}^{Y}}{\partial H_s}\right.\\&\left.-(\varepsilon_1-1)\left[\left(1-\omega_{Y_m}^{Y_h}\right)+\varepsilon_1\omega_{Y_m}^{Y_h}\left(1-\omega_{Y_h}^{Y}\right)\right]\frac{\partial}{\partial H_s}\frac{u_h^g}{1-u_h^g}\right\}dH_s\\&+\left\{\varepsilon_1\frac{1}{1-u_h^g}\left[1+(\varepsilon_1-1)\,\omega_{Y_m}^{Y_h}+\frac{u_h^g}{1-u_h^g}\right]\frac{\partial\omega_{Y_h}^{Y}}{\partial H_g}\right.\\&\left.-(\varepsilon_1-1)\left[\left(1-\omega_{Y_m}^{Y_h}\right)+\varepsilon_1\omega_{Y_m}^{Y_h}\left(1-\omega_{Y_h}^{Y}\right)\right]\frac{\partial}{\partial H_g}\frac{u_h^g}{1-u_h^g}\right\}dH_g,\end{aligned}$$

and the denominator is positive. For λ_l we have that $Z_{\lambda_l}=-Z_{\lambda_h}$. Using $\varepsilon_2=\gamma_4=1$ and (25), (27), (56), and (57) it can be rewritten as

$$\begin{aligned}Z_{\lambda_h}\frac{1}{dH_s}\frac{H_s}{H_g}=&(\varepsilon_1-1)\frac{\omega_{Y_m}^{Y_h}\left(1-\omega_K^{Y_m}\right)}{\left(1-u_h^g\right)^2\left[(\varepsilon_1-1)\,\omega_{Y_m}^{Y_h}+\frac{1}{1-u_h^g}\right]}\\&\times\left\{1+\frac{\left[\omega_{Y_h}^{Y}\varepsilon_1-(\varepsilon_1-1)\,u_h^g\right]\omega_{Y_m}^{Y_h}}{\left(1-\omega_{Y_m}^{Y_h}\right)+\varepsilon_1\omega_{Y_m}^{Y_h}\left(1-\omega_{Y_h}^{Y}\right)}\right\}\\&\times\left\{\left[(\varepsilon_1-1)\,\omega_{Y_m}^{Y_h}+\frac{1}{1-u_h^g}\right]\varepsilon_1\omega_{Y_h}^{Y}\left(1-\omega_{Y_h}^{Y}\right)\right.\\&\left.-u_h^g(\varepsilon_1-1)\left[\left(1-\omega_{Y_m}^{Y_h}\right)+\varepsilon_1\omega_{Y_m}^{Y_h}\left(1-\omega_{Y_h}^{Y}\right)\right]\right\}\end{aligned}$$

The expression in the first curly brackets is positive. Therefore, it is sufficient to look at the sign of the term in the second curly brackets. This term is identical to d in (60). It is positive for $\varepsilon_1<1$, $\varepsilon_1=1+$, and $\varepsilon_1=+\infty$. Therefore, the elasticity of final output with respect to shocks λ_h (λ_l) is lower (higher) in the country where H_s is higher if $\varepsilon_1<1$. If $\varepsilon_1=1+$ or $\varepsilon_1=+\infty$ then the elasticity of final output with respect to shocks λ_h (λ_l) is higher (lower) in the country where H_s is higher.