The Market for Used Capital

Endogenous Irreversibility and Reallocation over the Business Cycle

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Abstract

Capital reallocation is strongly procyclical in the data, but in standard business-cycle models with heterogeneous firms it is countercyclical. In this paper I argue that endogenizing the price of used capital solves this puzzle. First, I show empirically that for several sectors the price of used investment goods relative to new is procyclical. Second, I build an equilibrium model of endogenous partial irreversibility, with heterogeneous firms facing aggregate and idiosyncratic productivity shocks. Used investment goods are imperfect substitutes for new investment because of firm-level capital specificity and this creates a downward-sloping demand for used capital that shifts in response to aggregate shocks. The model generates a procyclical resale price and procyclical reallocation. In a recession, when the price of used capital is low, both static and dynamic real-options effects induce unproductive firms to sell fewer assets to more productive firms. This generates an amplification mechanism for measured aggregate TFP. Finally, the model shows that endogenous irreversibility smooths aggregate investment and generates a countercyclical cross-sectional dispersion of returns from capital, consistent with the empirical evidence.

1 Introduction

1.1 Motivation

Firms buy and sell large amounts of used investment goods both directly on secondary markets for equipment and plants and indirectly through acquisitions. Over the business cycle, this volume of capital reallocation is volatile and positively correlated with aggregate output.[1] Why is this the case? Can the cyclicality of reallocation be efficient? How do equilibrium dynamics in the market for used capital affect macroeconomic variables such as aggregate TFP and investment? This paper addresses these questions by first showing new evidence on prices of used real assets and then building a dynamic general equilibrium model with heterogeneous firms facing aggregate and idiosyncratic uncertainty.

Importantly, the market for used capital reallocates assets from less productive to more productive firms, as Maksimovic and Phillips (2011) document. Hence, more reallocation in booms means that more capital flows to highly productive firms when the economy is expanding, while downturns are associated with a smaller flow of assets towards their most productive use. This suggests that understanding the cyclicality of capital reallocation may be a step towards a theory of the cyclical movements in aggregate TFP. Particularly in the aftermath of the Great Recession, it seems important to understand the drivers of this reallocative process, which policy-makers in the UK see as an important condition for the onset of a strong recovery in productivity.[2]

Despite its relevance, the procyclicality of capital reallocation has so far been a puzzle for the macroeconomic literature, for at least two reasons. First, existing DSGE models of investment with heterogeneous firms (e.g. Khan and Thomas, 2008, 2013) imply a negative correlation between output and reallocation. In these models, unproductive firms want to disinvest by a larger amount in recessions, because their profitability falls following a negative aggregate shock. As these are one-sector models of the economy, demand for their used capital comes from both consumers and other firms. This demand is perfectly elastic, as the standard assumption is either full reversibility of investment, or a constant level of partial irreversibility, i.e. the relative price of used capital is assumed to be constant and less than 1. Hence, an outward shift in supply of used capital from disinvesting firms necessarily leads

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[1]Eisfeldt and Rampini (2006) show that reallocation of physical capital among US firms, which amounts to approximately 30% of total investment, is strongly procyclical. The cyclical component of their reallocation series, composed of Sales of Plants, Property and Equipment plus Acquisitions from Compustat, is very volatile (about 7 times the volatility of output) and positively correlated with US GDP (with a correlation coefficient of .56). Other measures of capital reallocation point to the same stylized fact: sales of used corporate assets for the UK from the ONS Capital Expenditure Survey are also procyclical and more disaggregated evidence on the market for used commercial ships shows the same pattern of cyclicality. Section 2 and Appendix A present the empirical evidence.

[2]For instance, Ben Broadbent, Deputy Governor of the Bank of England, attributes low labor productivity in the slow recovery post-2008 in the UK to a lack of capital reallocation (Broadbent, 2012, Barnett et al., 2014).

to more reallocation. This gives more reallocation in recessions and less in booms. Second, as Eisfeldt and Rampini (2006) point out, several measures of dispersion of returns on capital are higher in recessions than in booms, suggesting that benefits from reallocation are countercyclical. Hence we should expect to see more reallocation during downturns, when higher dispersion makes reallocation of capital towards its most productive use more beneficial.

In order to explain the puzzle, this paper starts by presenting a new stylized fact: the relative price of used capital goods, far from being constant, is actually volatile and strongly procyclical, suggesting that partial investment irreversibility is to a great extent a market equilibrium outcome. Recessions are bad times to disinvest, as more firms would like to sell their assets to downsize but the demand side coming from investing firms is weak.

Starting from this observation, I build an equilibrium model of partially irreversible investment, where the resale price of capital is endogenous. I assume a degree of capital specificity at the firm level: after installation, capital becomes a different good with respect to the output (and consumption) good, partially specific to the firm who owns it. Not only is it useless for consumers, but also an imperfect substitute with respect to new investment for other firms. This assumption allows me to rationalize the procyclicality of the resale price and capital reallocation in an otherwise standard business-cycle model. In a recession, used capital is relatively cheap, because more firms would like to disinvest and downsize, while expanding firms cannot fully benefit from the abundance of used capital on the market, because this capital is to an important extent specific to the firms that operated it previously.

The model emphasizes both a static and a dynamic real-options mechanism that induce procyclical reallocation. Let us examine the static mechanism first. A lower resale price associated with a recession increases the target level of capital of a disinvesting firm, hence reducing its desired level of disinvestment. Intuitively, after a negative aggregate productivity shocks there are two opposing forces on the disinvestment decision: both the internal value of capital for the firm and its market value fall. In equilibrium, when new and used capital as sufficiently poor substitutes, the latter effect dominates and sales of used capital fall. Next, let us introduce the dynamic real-options effects. Consider again a firm that is hit by a negative idiosyncratic shock in a recession and evaluates the opportunity to disinvest. In a dynamic environment, this firm needs to compare the price at which it can sell its assets in the current period with the price it would get by waiting one more period. In a recession, the current resale price falls, and so does the future expected resale price. However, if there is a positive probability of exiting the recession in the near future, the future expected resale price falls by less than the current one, and it may be better to wait, hold on to the assets and disinvest later by selling them at a higher price. This dynamic effect holds in general when the underlying stochastic process is mean-reverting and it generates an option value from waiting to disinvest that further decreases and delays the reallocation of capital in bad times.

The procyclicality of reallocation is matched by a countercyclical dispersion of returns from capital, consistently with a growing body of empirical evidence (e.g. Bloom et al., 2012). In the model, this happens because large unproductive firms downsize by less in recessions and hence their marginal product remains low relative to that of more productive firms. Several papers have interpreted the increase in the dispersion of returns associated with downturns as a symptom of the worsening of financial frictions, leading to the policy implication that credit expansions and non-conventional monetary policy can facilitate reallocation and stimulate the recovery. In contrast, the present paper shows that lack of reallocation and high dispersion of returns in recessions can be efficient outcomes in an economy where capital is partially specific at the firm level and hence used assets are imperfect substitutes for new ones.

While theories based on time-varying financial frictions may explain capital misallocation in recessions, they do not have implications for the cyclicality of the relative price of used capital. In contrast, my theory of capital specificity is able to explain both dynamics in prices and quantities traded on secondary markets for capital. The important and challenging question of how much dispersion in marginal products is due to financial or to real frictions is beyond the scope of this paper. However, because procyclical reallocation can be explained as an equilibrium outcome of a model without financial frictions, a policy implication of the paper is that credit expansions may in fact be less relevant for reallocation than previously thought. It should be noted that reallocation has fallen in every recession since we have data for it (1970's), that is also in recessions for which the financial component was arguably less important than in the Great Recession. A contribution of this paper is to present a real model where only one aggregate shock can generate both standard business-cycle facts and procyclical reallocation.

Furthermore, the model highlights important equilibrium real-options effects on investment. Consider again a recession. Used capital becomes cheaper, so that overall investment can be made at a lower cost. However, investment is also expected to be harder to reverse in the future (if the recession is expected to persist). These contrasting effects can either amplify or dampen the response of investing firms to aggregate shocks depending on the properties of the idiosyncratic and aggregate shock processes. In the quantitative section of the paper, I show that one of the aggregate implications of endogenous irreversibility is a significant smoothing of the aggregate investment series, bringing its volatility and autocorrelation closer to the empirical counterparts. Hence, the mechanism presented in the paper can be seen as a plausible microfoundation for an aggregate capital adjustment cost.

The next subsection discusses the related literature. The rest of the paper is organized as follows. Section 2 presents data on reallocation and the price of used investment goods over the business cycle. Section 3 introduces the key model assumptions in a simple static model. Section 4 extends the model to discuss dynamics and equilibrium real-options effects. Section 5 presents a fully-fledged DSGE model and Section 6 discusses the quantitative results. Section 7 concludes.

1.2 Related literature

Using Compustat data, Eisfeldt and Rampini (2006) show that Sales of Plants, Property and Equipment, as well as Acquisitions, are highly procyclical and argue that this is a puzzle given that the benefits from reallocation, as measured for instance by dispersion in TFP or dispersion in utilization rates, appear to be countercyclical. Their conclusion is that there must be a countercyclical degree of reallocation frictions. In this sense, one can see the present paper as microfounding this conclusion by explicitly modelling a market for used capital and showing that the equilibrium resale price falls in bad times.

The empirical evidence I present on the price of used capital fills an important gap in the empirical literature on capital adjustment costs. Inference on investment irreversibility is typically indirect, based on firms' investment and disinvestment rates rather than directly on prices (e.g. Cooper and Haltiwanger, 2006). Furthermore, irreversibility is generally assumed to be a constant technological friction. By looking at sectors that allow a direct comparison of the price of new and used assets, I establish that partial irreversibility is to a large extent a market equilibrium outcome and that it varies significantly with the business cycle.

The most closely related papers are Khan and Thomas (2013) and Cui (2014). Both papers build DSGE models with heterogeneous firms that feature constant partial irreversibility (defined by a constant resale price of capital below one) and collateral constraints. When feeding the model with aggregate TFP shocks, they cannot generate a procyclical response of reallocation, because disinvesting firms face a constant resale price and disinvest by more in recessions and less in booms. However, Cui (2014) shows that the procyclicality of reallocation can be obtained by introducing credit shocks, i.e. an exogenous tightening of the borrowing constraint. After such shocks, unproductive firms hold on to their capital and use its return to pay back their debt and deleverage. Regarding the question on the source of business cycles, Cui (2014) interprets the procyclicality of reallocation as evidence in favor of credit shocks.

The results in the present paper suggest that this conclusion may depend on the assumption of a constant resale price of capital. In fact, I show that the procyclicality of this price can reconcile the Eisfeldt and Rampini (2006) findings without resorting to exogenous credit shocks. However, I would stress that this does not necessarily mean that credit shocks are not important in driving business-cycle fluctuations. It only implies that procyclical reallocation is less of a puzzle, as it can be rationalized in a more standard business-cycle model, where only one aggregate shock drives both standard business-cycle facts and reallocation.[3] Furthermore, the present model provides a useful framework that can be extended to include financial frictions in the form of collateral constraints. Following aggregate shocks, the availability of credit

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[3]Section 6 shows that the main mechanism is robust to both aggregate and investment-specific productivity shocks.

would change endogenously with movements in the price of used capital.

Caunedo (2014) also considers an economy with heterogeneous firms and investment irreversibility and shows that the dispersion in marginal products that arises in equilibrium does not need to be inefficient. In this paper, I show that also the cyclical movements of such dispersion of returns (high dispersion in recessions) are not necessarily a symptom of time-varying financial frictions. Cooper and Schott (2013) consider a similar framework with heterogeneous firms and introduce an exogenous time-varying probability of being able to reallocate capital. They show that exogenous shocks to this probability may induce procyclical reallocation. Gilchrist et al. (2014) treat the resale price of capital as an exogenous shock process and show that a fall in this price, combined with collateral constraints, can replicate a recession associated with a liquidity crisis. My contribution with respect to these papers is to explicitly model the market for used capital and to endogenize the resale price and show that both this price and reallocation respond positively to aggregate TFP shocks in equilibrium.

A related strand of literature is that on real-options theory, starting with the seminal work of Dixit and Pindyck (1994) and Abel et al. (1996). This literature typically assumes exogenous stochastic paths for the prices at which a firm can buy and sell capital. As the resale price is assumed to be strictly less than the buying price, part of the investment is sunk and uncertainty regarding future productivity (or equivalently the future output price) leads to the presence of option values connected with the opportunity to wait and invest in the future. In this paper, I compute the equilibrium effects of aggregate shocks on these option values and show that with an endogenous resale price the value of the option to resell (put option) can be procyclical, contrary to what arises in partial equilibrium.

The key assumption in this paper is imperfect substitutability between new and used capital. In their seminal work on capital reallocation, Ramey and Shapiro (2001) provide an extreme example of this friction. They report that during the liquidation of an aerospace plant, a wind tunnel that could generate a 270 miles/hour wind was sold to a company that rented it to bicycle helmet designers, who only needed low speeds and did not value it as much as the aerospace firm who sold it. Edgerton (2011) uses evidence from tax depreciation reforms in the US to estimate the elasticity of substitution between new and used capital in the production function and finds values in the range between 1 and 10 for sectors such as farming, construction and aircraft. Jovanovic and Yatsenko (2012) build a vintage capital model to study technology adoption decisions and assume that different vintages of capital enter the production function in a Constant Elasticity of Substitution (CES) form. Eisfeldt and Rampini (2007) show that that in the data firms of all sizes invest both in new and in used capital and build a model assuming that new and used investment goods differ because used capital is cheaper, but requires more maintenance in the future, inducing financially constrained firms to buy a higher ratio of used to new items. In this paper, I abstract from financial frictions and focus on the role of capital specificity.

An alternative attempt to endogenize the resale price of capital relies on asymmetric information, especially lemons problems in secondary markets (Eisfeldt, 2004, Kurlat, 2013, Li and Whited, 2014). In my empirical evidence I focus on the aircraft sector, for which asymmetric information is unlikely to be relevant, as the maintenance history of each aircraft is public information. Furthermore, in models of asymmetric information the fraction of lemons does not necessarily increase in recessions, as would be required to explain a procyclical resale price and procyclical reallocation. As Eisfeldt (2004) argues, one can imagine a case where the fraction of lemons decreases in recessions, because more sellers owning good quality assets are forced to downsize, leading to a higher resale price and more reallocation. Perri and Quadrini (2014) follow a different approach to endogenize the resale price of capital: they assume that the value of used capital depends on whether it is sold to other firms or to consumers. In the latter case, the price is lower. In this paper, I assume that used capital is useless for consumer and focus instead on its imperfect substitutability with new investment.

Furthermore, this paper contributes to the literature on the link between micro-level irreversibility and smooth aggregate investment. Using a partial equilibrium model, Bertola and Caballero (1994) argued that irreversibility at the micro level is a plausible explanation for what in the aggregate looks like a convex adjustment cost. However, this result did not seem to pass the test of general equilibrium. Veracierto (2002) considers a model with constant partial irreversibility and concludes that consumption smoothing forces undo all the effects of irreversibility and the property of the aggregate investment series are almost identical, independently of the level of the resale price of capital. In this paper I show that endogenizing irreversibility reaffirms the result of Bertola and Caballero (1994). What matters is not the average level of the resale price, but its correlation with aggregate shocks: investment becomes more irreversible exactly at the time when disinvesting firms would like to disinvest by more and this induces more caution in investment decisions. Importantly, because the price of used capital is procyclical, the total cost of investment (new and used) is also procyclical and this further smooths investment decisions by making capital cheaper in recessions and more expensive in booms.

Finally, it is worth emphasizing that the literature on DSGE models with heterogeneous firms obtains results that imply a small or insignificant role for heterogeneity and changes in the cross-sectional distribution of firms, especially following aggregate TFP shocks. Most notably, Veracierto (2002) and Khan and Thomas (2008) show that the aggregate behavior of these model is remarkably close to that of representative-firm RBC models. The present paper is an example of a model where heterogeneity is important in order to microfound and understand an aggregate observation: without firms changing their idiosyncratic productivity levels over time, the market for used capital would not open and we could not rationalize the data on reallocation and the smoothing of the aggregate investment series.

2 Empirical evidence

2.1 Capital reallocation

Figure 1 shows the cyclical components of the Eisfeldt and Rampini (2006) capital reallocation series for the period 1971-2011, composed of annual Compustat data on Sales of Plants, Property and Equipment (SPPE) plus Acquisitions (all deflated using the US GDP deflator) and filtered using a Hodrick-Prescott (HP) filter with smoothing parameter equal to 6.25 (as suggested by Ravn and Uhlig, 2002, for annual data). Capital reallocation is very volatile (approximately 7 times as volatile as US GDP) and positively correlated with output, with a correlation coefficient of .56.[4]

I confirm the evidence on the procyclicality of capital reallocation by looking at two other data sources (both of which exclude acquisitions): UK sales of second-hand investment goods and global sales of second-hand commercial ships. In the UK, data on second-hand investment from the Survey of Capital Expenditures of the ONS show positive correlation between sales volumes and GDP. In particular during the recent recession, sales of second-hand investment goods were historically low, as shown in Figure 13 in Appendix A.

The market for second-hand commercial ships also provides a useful source of data on second-hand sales. Differently from the above-mentioned data sources, data on trading of used ships are divided into prices and quantities traded (number of sales) and do not depend on aggregation across types of investment goods.[5] By looking at these industry-level data, the following picture emerges: high trading volumes are associated with the period of economic expansion leading to 2007, and an abrupt fall in the number of sales coincides with the start of the Great Recession. This is illustrated in Figure 14 in Appendix A.

2.2 The price of used capital

A new stylized fact emerges from the analysis of sectoral evidence on the resale price of capital: the price of used investment goods is more volatile and more procyclical than the price of new investment goods. I construct or gather price indices from sectors that allow direct comparison of the value of new and used items in the same asset class. These sectors are

- commercial aircraft

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[4]Acquisitions represent the larger component of the capital reallocation series (approximately two thirds of the total). However, each of the two components (SPPE and Acquisitions) is significantly procyclical. In this paper, I will not distinguish between bundled and unbundled sales of used capital and I will refer to the sum of the two as capital reallocation, following Eisfeldt and Rampini (2006). Using a different data source, i.e. the Longitudinal Research Database compiled by the US Census Bureau, Maksimovic and Phillips (2001) find that the fraction of manufacturing plants involved in M&A activity goes from 3.89% in an average year to 6.19% in expansion years.

[5]Price indices and sales numbers are compiled by Clarkson and VesselsValue.
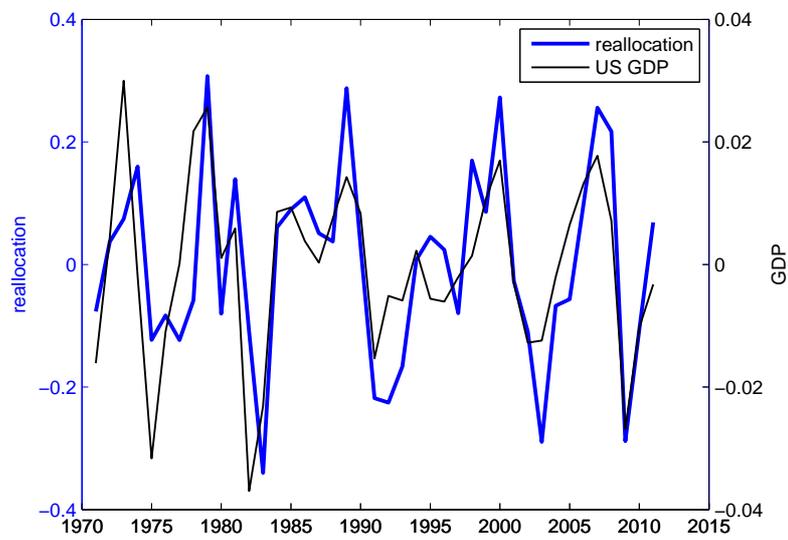
Figure 1: Capital reallocation and US GDP (cyclical components)

Log-deviations from HP trend (smoothing parameter = 6.25) of (i) the Eisfeldt and Rampini (2006) capital reallocation series, composed of Sales of Property, Plants and Equipment and Acquisitions from Compustat, deflated using the US GDP deflator, (ii) US real GDP. Yearly frequency.

- commercial ships

- vehicles and trucks

- construction equipment.

While this is only suggestive evidence related to these specific sectors, the pattern of cyclicality in these four sectors is remarkably similar, showing a much stronger reaction of the resale price of capital to business-cycle shocks relative to the price of new investment goods. I will now describe the evidence related to the aircraft sector. Appendix A reports the evidence for ships, vehicles and construction equipment.

Starting with a dataset on the value of all Western-built commercial aircraft from 1967 to 2009, I construct a price index of used and new aircraft. This dataset is compiled by a specialized consulting company that evaluates aircraft based on actual transactions prices for which the seller was not bankrupt. It includes prices of all the different vintages of 38 types of aircraft, starting from their first production year onwards. The observation unit is an aircraft of type j, vintage v in year t, with price p_{jvt}. To construct the index, I divide the data into prices of new aircraft ($v = t$) and prices of used aircraft ($v < t$). I deflate all prices using the US GDP deflator. Then I create dummy variables for year, age and type (and interaction terms) and run a regression of $log(p_{jvt})$ on these dummies. In each subsample (new and used), the coefficients on the time dummies are the quality-age-adjusted price index of aircraft. Finally, I detrend the series using an HP filter, with a smoothing coefficient of 6.25.[6] Figure 2 plots the price index of new aircraft, that of used aircraft and US GDP as a measure of the business cycle. It is evident that the cyclical component of the price of used aircraft is more volatile than that of new aircraft. It is also more strongly correlated with GDP. Table 1 reports standard deviations and coefficients of correlation of these series.

I interpret variations in the relative price of used assets as evidence in favor of capital specificity and against the standard assumption of perfect substitutability between new and used capital. Consistently with this interpretation, Gavazza (2011) suggests a reason why capital specificity may be playing an important role in determining the volatility of the price of used aircraft. Carriers typically operate a very small number of models in order to exploit economies of scale in maintenance and staff training costs and they are unwilling to substitute into other models when there is an increase in the supply of used aircraft due to aggregate shocks, leading to a fall in the value of used aircraft. By looking at cross-sectional evidence on the prices of different models, he finds support for this theory: the volatility of resale prices of more specific models of aircraft (e.g. Boeing 747, which can operate on a limited range of routes) is significantly higher than the volatility of more flexible models that can be used on a larger range of routes (e.g. Boeing 737).

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[6]Robustness exercises with different smoothing parameters lead to very similar results.

Evidence on secondary markets for commercial ships, vehicles and trucks and construction equipment is consistent with the main finding: the price of used capital relative to new is volatile and procyclical. Appendix A presents price series for these sectors.
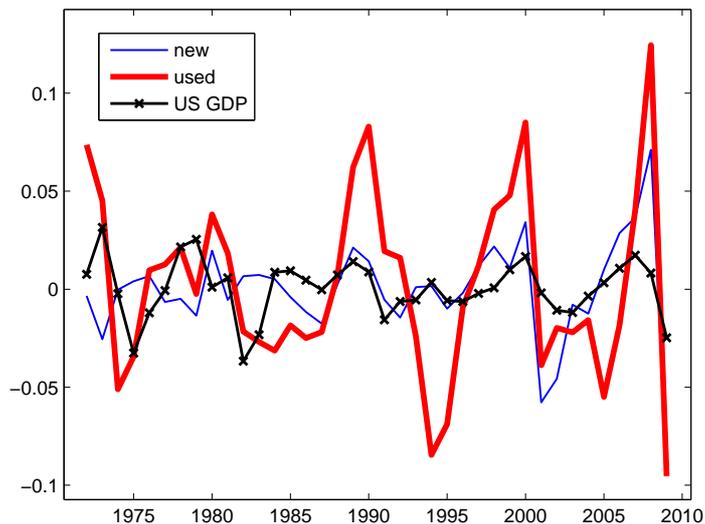

Figure 2: Aircraft prices and US GDP (cyclical components)

Log-deviations from HP trend (smoothing parameter = 6.25) of (i) price index of new aircraft, (ii) price index of used aircraft, (iii) US real GDP. Aircraft prices are deflated using the GDP deflator. Yearly frequency.

Series	Standard Deviation	Corr. with *new*	Corr. with *used*	Corr. with GDP
new	0.0342	1	-	-
used	0.0799	0.4781	1	-
GDP	0.0239	0.4090	0.5647	1

Table 1: New and used aircraft prices: second order moments

Standard deviations and correlation coefficients of the cyclical components of the price index of new aircraft, the price index of used aircraft and US real GDP. Yearly frequency, HP smoothing parameter = 6.25.

2.3   Discussion

Looking again at the Eisfeldt and Rampini (2006) reallocation series (Figure 1), in light of this evidence on the cyclicality of resale prices one may ask whether deflating

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their series with an index of used capital prices (instead of the GDP deflator) would explain the procyclicality of the volume of reallocation. In other words: is the cyclicality in the volume of reallocation only due to the cyclicality of the price of used capital? A back-of-the-envelope calculation suggests a negative answer. Under the assumption that the sectors discussed above (and in Appendix A) are representative of the whole economy, it is possible to compare the cyclical movements in these price series (cyclical deviations from trend in a ballpark of 10%) with that of the Eisfeldt and Rampini series (approximately 20% above and below trend in booms and recessions respectively). This suggests that part of the volatility in the Eisfeldt and Rampini series is certainly due to prices, but approximately half of this volatility may be due to movements in the quantity traded, consistently with the observation on the quantities traded on the market for used commercial ships.

Overall, the empirical evidence suggests that recessions are bad times to disinvest, because the resale price of capital is low. By treating the resale price as a constant parameter, the previous theoretical literature on investment irreversibility has not drawn any distinction between the case of a firm that needs to downsize during an expansion or a firm that needs to downsize in a downturn. However, the price of these two types of transactions can be quite different.

The assumption of perfect substitutability between new and used assets, which is implicit in the literature, is inconsistent with the evidence presented on the relative price of used capital: even an infinitesimal decrease in this price would lead investing firms to jump to a corner solution and demand only used capital, which is counterfactual. For instance, US Census ACES data on capital expenditures show a stable ratio between used and new investment expenditures, with a standard deviation of approximately 1%. The assumption of perfect substitutability between new and used assets, which is implicit in the literature, is inconsistent with the evidence presented on the relative price of used capital: even an infinitesimal decrease in this price would lead investing firms to jump to a corner solution and demand only used capital, which is counterfactual. For instance, US Census ACES data on capital expenditures show a stable ratio between used and new investment expenditures, with a standard deviation of approximately 1%.

3   A simple model of capital reallocation

Building on the empirical evidence presented above, this section introduces a simple static model that features imperfect substitutability between new and used investment goods and allows the derivation of analytical results on the response of capital reallocation to exogenous changes in aggregate productivity.

Section 4 extends this simple setup to include dynamic real-options effects and section 5 embeds the mechanism in a dynamic general equilibrium model with aggregate and idiosyncratic productivity shocks.

3.1 Technological assumptions

There is a continuum of firms $j \in [0, 1]$, all of which are endowed with an initial capital level k_0. They produce a homogeneous output good with production function

$$y_j = z s_j k_j^{\alpha}, \tag{1}$$

where z is an aggregate productivity parameter, s_j is an idiosyncratic shock with cdf $F(s_j)$ and $\alpha \in (0, 1)$ is the returns-to-scale parameter.

Each firm uses its specific type of capital in order to produce the output good. Before production, firms can adjust their capital level k_j according to their productivity. Firms that decide to decrease their capital stock can sell some of their capital on the market for used capital. On the other hand, firms that decide to increase their capital level can invest using newly produced capital (supplied inelastically by a representative consumer) or used capital sold by disinvesting firms. New capital can be freely specialized. In contrast, used capital is partially specific to its previous owner. As a consequence, expanding firms cannot make the whole investment buying used capital and they need to bundle it instead with some newly produced output good in order to make it specific to their firms. Hence the substitutability between new and used investment goods is imperfect. This can be rationalized in a world where investment goods needed to build a plant are of different types. Some of them are fairly generic and can be easily purchased as used and put in production in a different plant. Some others have to be specifically designed for the production of a particular business line. In this environment, substitutability is imperfect and firms will only be willing to substitute towards more used capital if this becomes cheaper.[7] However, investing firms always have the choice to buy only new goods.

Formally, the investment technology is given by a perfect substitutes aggregator of new capital and a CES aggregator of used and new capital.[8]

$$k_j - k_0 = \widetilde{i}_{j,new} + g(i_{j,new}, i_{j,used}) \tag{2}$$

$$g(i_{j,new}, i_{j,used}) = [\eta^{\frac{1}{\epsilon}} (i_{j,new})^{\frac{\epsilon-1}{\epsilon}} + (1-\eta)^{\frac{1}{\epsilon}} (i_{j,used})^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

where $\widetilde{i}_{j,new}$ and $i_{j,new}$ are new investment goods and $i_{j,used}$ is used capital sold by disinvesting firms. $\eta \in (0, 1)$ is a parameter that determines the average ratio between

[7]The sale of a GM pick-up trucks production plant in Shreveport, Louisiana, to three-wheeled electric car manifacturer Elio Motors provides a recent example of capital reallocation with imperfect susbstitutability. Elio chose to acquire the plant because it was ready for reuse as well as more convenient than building a new plant from scratch. However, they clearly need to substitute part of the GM machinery with specific equipment for the production of their product. (source: cnn.com, April 2014)

[8]Differently from Jovanovic and Yatsenko (2012) and Edgerton (2011), and in order to make my model computationally tractable, I assume that the imperfect substitutability is in the investment technology rather than in the production function, which allows to endogenize the resale price while at the same time keeping track of only one type of capital as a state variable in the full dynamic model.

new and used investment, while $\epsilon > 0$ is the elasticity of substitution between new and used investment goods.

The price of a unit of new capital in terms of the output good is 1, while the cost of a unit of used capital is equal to the sum of the price of used capital q_t and a per-unit reallocation cost γ. Hence, the CES price index associated with a composite g of new and used investment goods is

$$Q = [\eta + (1 - \eta)(q + \gamma)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}. \tag{4}$$

Clearly, firms will choose the cheapest option between a fully new investment $\widetilde{i}_{j,new}$ and a bundle g. As long as $q < 1 - \gamma \Rightarrow Q < 1$, the bundle is the cheapest option and firms choose to make a fraction of their investment using used capital and set $\widetilde{i}_{j,new} = 0$. However, if the price of used capital became hypothetically higher then the price of new capital, firms would optimally buy only new capital. Throughout the paper, in equilibrium it will always be the case that $q < 1 - \gamma$, hence I will focus on this case in the following analysis.

I interpret the elasticity ϵ as an inverse measure of capital specificity. When $\epsilon \to \infty$, new and used capital are perfect substitutes and the model nests a standard model of partial irreversibility with constant resale price $q = 1 - \gamma$. On the other hand, when $\epsilon = 0$, the technology does not allow any substitutability between new and used capital.

With imperfect substitutability, for each price of used capital q, there is a well defined optimal ratio of used to new investment, in contrast to models that assume perfect substitutability. Increasing the ratio of used to new investment goods above this optimal level would be suboptimal, because the investing firm would be buying a larger amount of capital that was specific to another firm, relative to the new capital than can be freely specialized.

Finally, I assume that used capital is useless for consumers and the market for used capital clears between investing and disinvesting firms only. While it is true that some types of capital like cars and computers could be useful for consumers after having been used by firms, for most other kinds of equipment and for plants this is impossible. This assumption is equivalent to the assumption of total irreversibility in the aggregate, as for instance in Sargent (1980). Investment is partially reversible from an individual firm's point of view, because it can be sold to another firm, but in the aggregate, used capital cannot be retransformed into consumption.

3.2   Optimal investment and reallocation decisions

The solution to the firms' optimal investment problem can be easily characterized:

- If they are sufficiently productive, they will invest, buying a bundle of new and used capital at price Q. This will happen if $s_j \geq s^I = \frac{Q}{\alpha z k_0^{\alpha-1}}$. In this case, the

optimal capital level is given by

$$k_j = \left(\frac{\alpha z s_j}{Q}\right)^{\frac{1}{1-\alpha}}.$$

- If they are sufficiently unproductive, they will disinvest, selling part of their capital at price q. This will happen if $s_j < s^D = \frac{q}{\alpha z k_0^{\alpha-1}}$. In this case, the optimal capital level is given by

$$k_j = \left(\frac{\alpha z s_j}{q}\right)^{\frac{1}{1-\alpha}}.$$

- Firms with intermediate productivity $s^D \leq s_j < s^I$ will choose to keep their capital level at k_0, as their marginal product of capital lies between the purchasing price Q and the selling price q.

3.3 Equilibrium in the market for used capital

Given a chosen amount of total investment, investing firms minimize their expenditure by buying a composite of new and used capital. By solving a standard CES expenditure minimization problem and integrating over the measure of investing firms we get total demand for used capital:

$$D_{used} = (1 - \eta) \left(\frac{q + \gamma}{Q}\right)^{-\epsilon} \int_{s^I} \left[\left(\frac{\alpha z s}{Q}\right)^{\frac{1}{1-\alpha}} - k_0\right] dF(s). \tag{5}$$

On the other hand, total supply of used capital, coming from disinvesting firms is

$$S_{used} = \int^{s^D} \left[k_0 - \left(\frac{\alpha z s}{q}\right)^{\frac{1}{1-\alpha}}\right] dF(s). \tag{6}$$

The market-clearing condition $D_{used} = S_{used}$ defines implicitly the equilibrium price as a function of the aggregate productivity parameter z, $q = q(z; \epsilon)$, where I emphasize that this equilibrium price mapping depends on the elasticity of substitution ϵ. The following proposition relates this elasticity to the effect of aggregates shocks on irreversibility and reallocation.

Proposition 1. *(i) $q(z; \epsilon)$ is increasing in z. (ii) There exists an $\bar{\epsilon} > 0$ such that for $\epsilon < \bar{\epsilon}$ the elasticity of q with respect to z is greater than 1 and reallocation is increasing in z.*

The proof, in Appendix B, is based on an application of the Implicit Function Theorem to the market-clearing condition. To derive intuition on the mechanism, consider without loss of generality a marginal decrease in z. At a given resale price, disinvesting firms would like to sell more capital, because their optimal target level is now lower. This implies that the supply schedule of used capital (6) shifts out in a standard price-quantity space. Similarly, investing firms want to invest less and demand (5) shifts in. The price of used capital q must fall to clear the market, and the price index of investment Q will also decrease, although by less, because it is a CES average of 1 (the price of new capital) and $(q + \gamma)$. This reflects the fact that investing firms cannot fully benefit from the cheaper used capital on offer, because its specificity makes it less valuable for them.

For sufficiently low elasticity of substitution, q will fall by more than the initial fall in z, the threshold $s^D = q/\alpha z k_0^{\alpha-1}$ will decrease and the choice of new capital level conditional on s_j for disinvesting firms will increase, inducing less reallocation. Importantly, this is in contrast to a model where the resale price is constant, which implies that the disinvesting threshold would increase following a fall in z, and total reallocation would necessarily be higher.

We will see in the following sections that the reaction of the resale price potentially leads to amplification of exogenous aggregate productivity shocks. The mechanism works as follows: measured aggregate productivity (the Solow residual obtained assuming an aggregate production function) falls by more that the initial negative shock because the decrease in trading in the market for used capital leads unproductive firms to remain large relative to what they would be in a world with fixed resale price, and as a consequence a larger fraction of aggregate capital is operated by firms with lower productivity.

Furthermore, under a reasonable assumption on the distribution F, the model generates a countercyclical dispersion of marginal products. Assume for simplicity that F is uniform on $[s_{min}, s_{max}]$. Consider again a marginal decrease in z. Out of the inaction region, investing firms set their marginal product equal to Q and disinvesting firms set it equal to q. In the inaction region, the marginal product of each firm stays equal to its initial value. When aggregate productivity falls, the distance between q and Q increases, because q decreases by more than Q (Q is a CES average of $q + \gamma$ and 1). This has two effects. First, the difference between the marginal product of investing firms and that of disinvesting firms increases. Second, the mass of firms in the inaction region increases. Both effects necessarily lead to higher dispersion of marginal products.

The previous literature has often interpreted the countercyclical dispersion of returns from capital as a consequence of a worsening of financial frictions in recessions, with productive firms not getting enough external finance to implement high return projects (see for instance Cui, 2014, Chen and Song, 2013). This model highlights a different channel: part of the increase in the dispersion of returns in recessions is due to imperfect substitutability between new and old capital and hence can be fully

efficient and should not be addressed with expansionary credit policies.

Figure 3 provides a graphical representation of the solution to the model and its comparative statics. Consider the upper horizontal line. Each point on the line is the marginal product of capital of a firm, evaluated at k_0. Firms with initial marginal product larger than Q invest up to the point where their marginal product equals Q. Likewise, firms with initial marginal product below the resale price q disinvest up to the point where their marginal product equals q. These investment and disinvestment decisions are represented by the curved arrows pointing towards the two prices. Firms with intermediate initial marginal product are in the inaction region and remain at k_0. The lower horizontal line corresponds to a decrease in z. Note that both Q and q decrease, but the former decreases by less than the latter. Hence, the inaction region becomes larger.

In conclusion, this simple static model shows that in an economy with a sufficient degree of capital specificity a fall in aggregate productivity leads to an even larger decrease in the resale price of capital, a decrease in reallocation and an increase in the dispersion of marginal products.
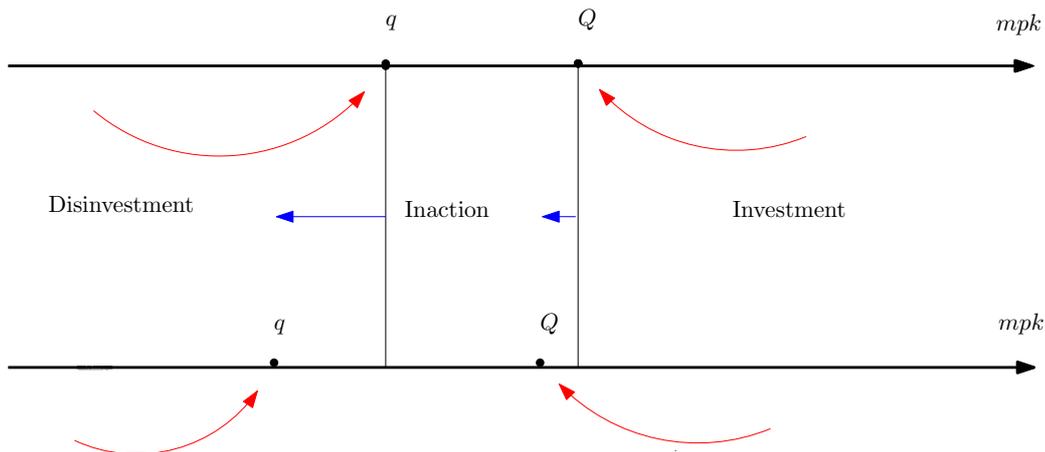

Figure 3: Graphical representation of the static mechanism

Firms' investment/disinvestment decisions and inaction region. Upper horizontal line: benchmark solution. Lower horizontal line: marginal decrease in aggregate productivity z.

4 Aggregate shocks and equilibrium real option values

This section presents a two-period model with uncertainty and forward-looking firms, where investment and reallocation decisions depend not only on current prices, but also, importantly, on future expected prices at which firms can buy and sell investment

goods. The model extends the seminal work of Abel et al. (1996) by imposing equilibrium in the market for used capital. Abel et al. (1996) assume exogenous streams of purchasing and resale prices of capital and focus on solving the individual firm's problem of partially irreversible investment under idiosyncratic uncertainty. They show that with partial irreversibility, as part of the investment is sunk, the firm has an option value from waiting until future productivity is known (or equivalently the future output price).

Here, I consider instead a continuum of firms hit by idiosyncratic and aggregate shocks and I impose that the market for used capital clears in all states. This allows me to obtain results on the equilibrium effects of aggregate shocks on firms' real option values in an environment where all firms make their investment decisions taking these options into account. As the resale price is positively correlated with the aggregate productivity shock, the option value to sell capital is procyclical, contrary to what happens in partial equilibrium, where this option is more valuable in recessions. After a negative persistent aggregate TFP shock, capital is not only less productive, but is also perceived as more irreversible, as its expected future resale price falls. This section ends with a discussion of how such equilibrium effects of aggregate shocks on real option values affect both investment and reallocation.

4.1 Two-period model

There is a continuum of firms with idiosyncratic productivity s_{jt} producing with technology

$$y_{jt} = z_t s_{jt} k_{jt}^\alpha, \tag{7}$$

for $t = 1, 2$. Both z_t and s_{jt} follow a positively autocorrelated process. In particular, aggregate productivity takes either of two values $\{z^L, z^H\}$ and a Markov transition matrix gives conditional probabilities for time $t = 2$. I assume $Pr\{z_2 = z_1\} > 1/2$. The idiosyncratic shock at $t = 1$ is drawn from a log-normal distribution with mean $-\sigma_1^2/2$ and variance σ_1^2. At time 2, the shocks satisfies $log(s_2) = \rho\, log(s_1) + v_2$, with $v_2 \sim N(-\sigma_2^2/2, \sigma_2^2)$.

Firms start the initial period with a common level of capital k_0, observe the realizations of the two productivity shocks, (s_{j1}, z_1), and are allowed to choose their capital level k_{j1} before starting production. If they invest, they can purchase a combination of new capital and used capital, which is being sold by disinvesting firms who decide to decrease their capital level. As in the static model of the previous section, the investment technology is given by (2) and the price index of investment goods is given by (4) in both periods, except that q and Q will now have a t subscript. The reallocation cost γ is constant.

After the investment/disinvestment activity is concluded, production takes place. Abstracting from physical depreciation for simplicity of exposition, firms start the second period with an initial level of capital k_{j1}, observe the realizations of s_{j2} and

z_2 and are again allowed to adjust their capital level before production. Then production takes place again and at the end of the period firms receive a terminal value proportional to their capital level, χk_{j2}, with $\chi \geq 0$.

4.2 Value of a firm

Because $q_t < Q_t$, in both periods some firms will invest, some will disinvest and some will keep their capital stock unchanged because the marginal product of their capital lies between the two prices. Let's start by characterizing the value of a firm after its choice of capital at $t = 1$. The next subsection will then move backwards and solve for the optimal choice of k_{j1}. By anticipating optimal behavior at $t = 2$, the value of a firm can be decomposed into a component that assumes no further adjustment in the second period, a component that depends on the opportunity to buy more capital in the second period (call option) and a component that depends on the possibility to sell some capital in the second period (put option). The call option will be exercised only for sufficiently high s_{j2} and the put option for sufficiently low s_{j2}. In the following derivation I will drop the subscript j for notational convenience and consider a generic firm. At $t = 1$, after observing the pair (s_1, z_1) and choosing k_1, the value of the firm is

$$V(k_1, s_1, z_1) = z_1 s_1 k_1^\alpha + \beta \left(\mathrm{E}_1 z_2 s_2 k_1^\alpha + \chi k_1 \right) + \beta C(k_1, s_1, z_1) + \beta P(k_1, s_1, z_1) \quad (8)$$

where E_1 is a conditional expectation operator that sums over the future realizations of z and integrates over the distribution of s_2, conditional on (s_1, z_1). The value of the firm consists of the value of producing in both periods with k_1, i.e. without doing any further adjustment at $t = 2$, plus the call option value of increasing the capital stock, $C(k_1, s_1, z_1)$ and the put option value of selling part of the capital stock in the second period, $P(k_1, s_1, z_1)$.

The call option value is given by

$$C(k_1, s_1, z_1) = \mathrm{E}_{z_1} \int_{s_2^I(k_1, z_2)} \left\{ [z_2 s_2 (k_2 (s_2, z_2))^\alpha + \chi k_2 (s_2, z_2)] \right.$$
$$\left. - [z_2 s_2 k_1^\alpha + \chi k_1] - Q_2 [k_2 (s_2, z_2) - k_1] \right\} dF (s_2 | s_1) \quad (9)$$

where E_{z_1} sums over realizations of z_2, with probabilities conditional on z_1. This option will be exercised at $t = 2$ if idiosyncratic productivity turns out to be above the threshold $s_2^I(k_1, z_2) = \frac{Q_2 - \chi}{\alpha z_2 k_1^{\alpha-1}}$, in which case the firm will invest and go to a capital level given by

$$k_2 = \left(\frac{\alpha z s_{j2}}{Q_2 - \chi} \right)^{\frac{1}{1-\alpha}} > k_1.$$

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Similarly, the put option is

$$P(k_1, s_1, z_1) = \mathrm{E}_{z_1} \int^{s_2^D(k_1, z_2)} \{[z_2 s_2 (k_2 (s_2, z_2))^\alpha + \chi k_2 (s_2, z_2)]$$
$$- [z_2 s_2 k_1^\alpha + \chi k_1] + q_2 [k_1 - k_2 (s_2, z_2)]\} \, dF(s_2 | s_1) \quad (10)$$

and will be exercised at $t = 2$ if idiosyncratic productivity turns out to be below the threshold $s_2^D(k_1, z_2) = \frac{q_2 - \chi}{\alpha z_2 k_1^{\alpha-1}}$ by selling capital up to the level

$$k_2 = \left(\frac{\alpha z s_{j2}}{q_2 - \chi} \right)^{\frac{1}{1-\alpha}} < k_1.$$

It is easy to see that the value of the call option is decreasing in the realizations of Q_2, while the put option is increasing in the realizations of q_2. While a higher Q_2 makes it harder to expand tomorrow, a higher q_2 makes it easier to downsize, should it be desirable.

4.3   Optimal investment and reallocation decisions

We can now characterize the optimal choice of capital level in the first period. At $t = 1$, firms compare the marginal value of their initial capital level k_0 with the purchasing price Q_1 and the selling price q_1. Call V_k the partial derivative of V with respect to its first argument.

- Firms who receive an idiosyncratic shock such that $V_k(k_0, s_1, z_1) \geq Q_1$ will choose to invest and their optimal capital level $k_1(s_1, z_1)$ satisfies

$$\frac{V_k(k_1, s_1, z_1)}{Q_1} = \frac{W_k(k_1, s_1, z_1) + C_k(k_1, s_1, z_1) + P_k(k_1, s_1, z_1)}{Q_1} = 1$$

  where I have emphasized that the marginal value of capital is composed by the present discounted value of their marginal product assuming no further adjustment, which I call W_k, the marginal call C_k and the marginal put P_k. Note that W_k and P_k are positive, while C_k is negative as increasing the capital level implies exercising part of the call option value.

- For firms with sufficiently low idiosyncratic productivity, $V_k(k_0, s_1, z_1) < q_1$. They will disinvest and choose $k_1(s_1, z_1)$ by solving

$$\frac{V_k(k_1, s_1, z_1)}{q_1} = \frac{W_k(k_1, s_1, z_1) + C_k(k_1, s_1, z_1) + P_k(k_1, s_1, z_1)}{q_1} = 1$$

- Firms with intermediate productivity, such that $q_1 \leq V_k(k_0, s_1, z_1) < Q_1$, are in the inaction region and optimally keep $k_1(s_1, z_1) = k_0$.

As in the static model, firms compare the marginal value of capital with Q if they consider investing and with q if they consider disinvesting. Differently from the static model, however, this marginal value now takes into account the variations in the option values induced by such investment and disinvestment activity.

The market for used capital clears, meaning that total disinvestment from unproductive firms equals investment in used capital coming from investing firms. The market-clearing equation is analogous to that in the previous section.

4.4   Put option in booms and recessions

As the evidence presented in section 2 suggests, the resale price of capital is more volatile and procyclical than the price of new capital. Hence, I will focus on the put option value and its reaction to shocks, although similar arguments can be made about equilibrium effects of shocks on the call option.

To understand how equilibrium real options affect investment and disinvestment behavior following business-cycle shocks, consider the marginal value of the put option P_k. Differentiating (10) with respect to the choice of capital and writing the expectation more explicitly, we get

$$P_k(k_1, s_1, z_1) = \sum_{z_2 \in \{z^L, z^H\}} Pr\{z_2 | z_1\} \int^{s_2^D(k_1, z_2)} \left[q_2(z_1, z_2) - \alpha z_2 s_2 k_1^{\alpha - 1} - \chi \right] \, dF(s_2 | s_1) \tag{11}$$

where I emphasize that the equilibrium resale price depends on both realizations of the aggregate shock. Hence, conditional on z_1, there are two possible outcomes for q_2 depending on the realization of z_2.

Intuitively, P_k is increasing in the expected value of the future resale price q_2, as this price adds value to a marginal unit of capital bought in the first period, in the case this unit has to be resold in the second period. Consider the two elements of the summation over z_2. If one keeps the resale price q_2 constant, it appears that the marginal put value of capital is decreasing in z_2, i.e. a decrease in the value of aggregate productivity leads necessarily to an increase in P_k. This is because P_k depends negatively on the marginal product of installed capital. However, the expected resale price is also endogenous in this model. In numerical examples, for sufficiently low elasticity of substitution between new and used capital ϵ, this equilibrium effect dominates the effect of exogenous changes in z_2, implying that the marginal put option value becomes procyclical. The intuition for this is that the value of a marginal unit of capital purchased in the first period depends positively both on its productivity in the second period and on its resaleability.

Figure 4 illustrates the payoff of the put option value in equation (10), evaluated at k_0, as a function of the initial idiosyncratic shock s_1 and for each of two values of z_1. I label the low realization of the aggregate state "recession" and the high realization "boom". This figure shows that this option value has a similar payoff function to a

financial option. Here the strike price is the resale price q_2 and the underlying asset value is the marginal value of the firm's capital. For a firm with very low s_1, k_0 is a relatively high capital level, so that assuming no adjustment at $t = 1$, it is likely that some disinvestment will be optimal at $t = 2$, given the autocorrelation of s_t. This explains a high put option value. On the other hand, for a firm with very high s_1, the optimal size is higher than k_0, so that if the firm keeps its capital level at k_0 it is unlikely that there will be any need for disinvesting at $t = 2$, which explains a put option value close to 0.

Figure 5 shows what happens to the option value after firms choose k_1, both when $z_1 = z^L$ and when $z_1 = z^H$. Low productivity firms exercised some of their initial put value by selling some capital. High productivity firms, on the other hand, invest and purchase some put option value. Firms with intermediate productivity are in the inaction region and optimally choose to keep $k_1 = k_0$, so their put option value after trading equals the initial put option value. This figure illustrates that when aggregate productivity is low, the put option value falls, because of the equilibrium effect on the expected resale price illustrated above, thus making investment more irreversible.
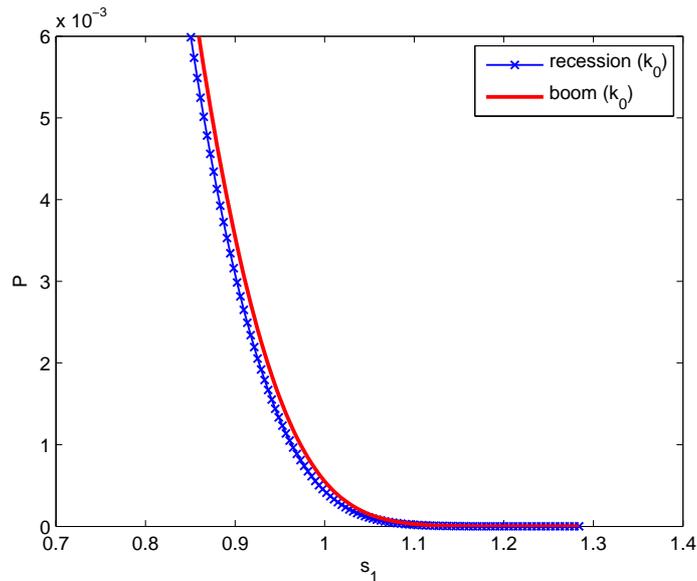

Figure 4: Put option value, before trading at $t = 1$

4.5   Equilibrium real options and investment

The model has rich implications in terms of the effects of aggregate shocks on investment. In a boom, capital is now attractive for two reasons. First of all, it is directly
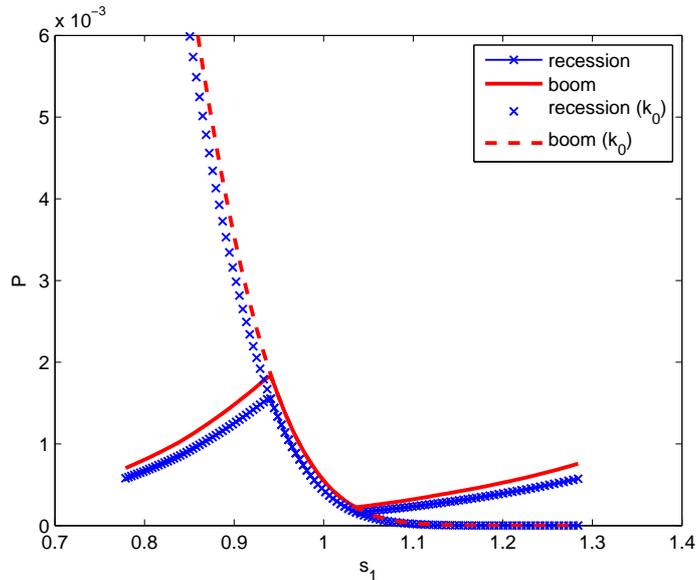
Figure 5: Put option value, after trading at $t = 1$

more productive. Second, it is easier to resell, in case a bad idiosyncratic shock hits the firm in the future. Third, it is more expensive today, because the current price of used assets increases and hence total investment comes at a higher price.

In the quantitative model presented in the next section, I investigate the aggregate effects of all these different incentives on investment and disinvestment decisions. It turns out that in general equilibrium endogenous irreversibility smooths aggregate investment, bringing its volatility and autocorrelation closer to the data, consistently with the original conjecture of Bertola and Caballero (1994) on the aggregate effects of micro-irreversibilities and in contrast to DSGE models where the resale price of capital is assumed to be constant.

4.6   Equilibrium real options and reallocation

For disinvesting firms, the first order condition for k_1 suggests a key comparison between the marginal value of capital and the current resale price. The ratio between the present discounted value of marginal returns and q_1 behaves similarly to the static model. Let us disregard variations in the marginal call value as they are small, given the relatively low volatility of Q_t and focus on the ratio $\frac{P_k}{q_1}$.

Unproductive firms compare the price they can get for their assets at $t = 1$, with the value from waiting to disinvest until the second period, which as we have seen is an increasing function of the expected value of q_2. This allows us to identify two forces that act in opposite directions. On the one hand, in a recession the marginal

put option falls, because the resale value of capital is going to fall at $t = 2$ with high probability (as the aggregate shock is persistent). This would imply that it is optimal to disinvest by more in the first period. On the other hand, also the current resale price in the first period is low. Importantly, with positive probability z_2 will be high and the resale price will increase, in which case it would be optimal to disinvest by less in the first period and wait until the second period.

Because of mean reversion of the aggregate shock process, following a low realization of the aggregate TFP shock, the fall in the current price of used capital dominates the fall in its future expected value, generating a value of waiting to disinvest in the future, further delaying reallocation and amplifying the static effects analyzed in Section 3.

5   A DSGE model with endogenous irreversibility

This section presents an infinite-horizon general equilibrium model that combines all the static and dynamic effects illustrated in the previous sections and includes risk aversion and endogenous labor supply, allowing for a quantitative evaluation of the mechanism.

5.1   Households

There is a representative household who consumes the output good, supplies labor and owns shares in all firms in the economy. Her preferences are described by the utility function

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \left[log \left(c_t \right) - \psi n_t \right] \tag{12}$$

where c_t is consumption and n_t are hours worked.

The representative household's budget constraint is

$$c_t = w_t n_t + \pi_t \tag{13}$$

where π_t are aggregate profits.[9]

The labor supply schedule is defined by the first order condition that equates the marginal rate of substitution between hours and consumption to the wage w_t

$$\psi c_t = w_t. \tag{14}$$

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[9]Alternatively, one could write this budget constraint including the household's choice of buying and selling shares in all firms. In equilibrium, her portfolio would have to coincide with the distribution of firms in the economy and stock prices would be given by the firms' value functions below. The distinction between these two formulations is immaterial in terms of competitive equilibrium allocations and prices.

5.2 Firms

Consider now firms' optimization problem. In each period t, productivity of firm j is the product of an aggregate component $z_t \in \{z^L, z^H\}$ that follows a Markov chain with transition matrix T_z and an idiosyncratic component $s_{jt} \in \{s^L, s^H\}$ with Markov transition matrix T_s. The firm produces a homogenous output good with technology

$$y_{jt} = z_t s_{jt} k_{jt}^\alpha n_{jt}^\nu \tag{15}$$

with $\alpha + \nu < 1$, and chooses current labor demand and the future level of capital in order to maximize its value for the consumer taking prices q_t and w_t as given.[10] By assuming a flexible labor market with no adjustment costs, I can separate the labor demand choice from the investment decision in a very convenient way. I will first describe the intratemporal labor decision and then derive the implied return on capital in order to formulate the intertemporal investment problem.

Labor demand equates the marginal product of labor to the wage rate:

$$n_{jt} = \left(\frac{\nu z_t s_{jt} k_{jt}^\alpha}{w_t} \right)^{\frac{1}{1-\nu}} \tag{16}$$

Using (16), it is easy to derive an expression for output net of the wage bill as a function of the two productivity shocks, current capital level and wage:

$$y_{jt} - w_t n_{jt} = A(w_t) z_t^\theta s_{jt}^\theta k_{jt}^{\alpha\theta}, \tag{17}$$

where $A(w_t) = \left[\left(\frac{\nu}{w_t} \right)^{\frac{\nu}{1-\nu}} - w_t \left(\frac{\nu}{w_t} \right)^{\frac{1}{1-\nu}} \right]$, and $\theta = 1/(1-\nu)$. This transformation of the production function is used by firms in order to evaluate the return on investment in physical capital. In other words, the flexible labor demand decision is incorporated in their expectations as they know that in every period they will be free to reoptimize their required labor input.

Let m be the distribution of firms over individual capital level and idiosyncratic productivity. Both the price of used capital and the wage will depend on it, so that this distribution is now a state variable with its own law of motion.

$$m_{t+1} = \Gamma(m_t, z_t) \tag{18}$$

Let us focus on a recursive solution to the firm's problem, with state vector (k, s, z, m). After observing the state, each firm decides whether to invest or disinvest, and by how much. Switching to recursive notation, the value of an investing

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[10]Differently from the simple models presented above, here I assume that capital is chosen one period in advance. This assumption makes the model more easily comparable with standard business-cycle models.

firm is

$$V^i(k,s,z,m) = \max_{k' \geq (1-\delta)k} A(w) z^\theta s^\theta k^{\alpha\theta} - Q[k' - (1-\delta)k] + \beta E\left\{\frac{c}{c'} V(k',s',z',m')|s,z\right\}$$
(19)

and the value of a disinvesting firm is

$$V^d(k,s,z,m) = \max_{k' \leq (1-\delta)k} A(w) z^\theta s^\theta k^{\alpha\theta} - q[k' - (1-\delta)k] + \beta E\left\{\frac{c}{c'} V(k',s',z',m')|s,z\right\}$$
(20)

At the beginning of each period, the discrete choice between investment and disinvestment gives $V(k,s,z,m) = max\left\{V^i(k,s,z,m), V^d(k,s,z,m)\right\}$. Note that these Bellman equations implicitly define the value of the firm as the present discounted value of profits (i.e. output net of the wage bill and investment expenditure), evaluated using the representative household's stochastic discount factor.

5.3 General Equilibrium

Market clearing in the used capital market needs to be imposed in an analogous way to the simpler models in the previous sections. Investing firms demand new capital and used capital by solving a standard CES expenditure minimization problem and market clearing implies that total investment in used capital equals total disinvestment.

I can now define a recursive competitive equilibrium.

Definition 1. *A recursive competitive equilibrium is defined as a set of functions m, Γ, w, q, Q, π, C, N, V^i, V^d, V, n, k', i, i_{new}, i_{used}, d that solve the household's and firms' optimization problems and clear markets for the output good, labor and used capital:*

- *Consumption $C(z,m)$ and labor supply $N(z,m)$ solve the consumer's problem of maximizing (12) subject to (13)*

- *Firms labor demand $n(k,s,z,m)$ satisfies equation (16)*

- *The value functions V^i, V^d and V satisfy the functional equations (19), (20) and $V(k,s,z,m) = max\left\{V^i(k,s,z,m), V^d(k,s,z,m)\right\}$*

- *For investing firms, i.e. firms such that $V^i(k,s,z,m) \geq V^d(k,s,z,m)$ the policy function $k'(k,s,z,m)$ solves (19), investment is $i(k,s,z,m) = k'(k,s,z,m) - (1-\delta)k$ and is allocated to new and used investment goods according to the CES expenditure minimization first order condition:*

$$\frac{i_{used}(k,s,z,m)}{i_{new}(k,s,z,m)} = \frac{1-\eta}{\eta} \left(q(z,m) + \gamma\right)^{-\epsilon}$$

- For disinvesting firms, i.e. $V^i(k, s, z, m) < V^d(k, s, z, m)$, the policy function $k'(k, s, z, m)$ solves (20) and disinvestment is $d(k, s, z, m) = (1 - \delta)k - k'(k, s, z, m)$

- Aggregate profits are given by $\pi(z, m) = z \int sk^\alpha n^\nu \, dm(k, s) - w(z, m)N(z, m)$

$$-Q(z, m) \int i(k, s, z, m) \, dm(k, s) + q \int d(k, s, z, m) \, dm(k, s)$$

- The market for the output good clears:

$$C(z, m) = z \int sk^\alpha n^\nu \, dm(k, s) - Q(z, m) \int i(k, s, z, m) \, dm(k, s) +$$

$$q \int d(k, s, z, m) \, dm(k, s)$$

- The labor market clears: $N(z, m) = \int n(k, s, z, m) \, dm(k, s)$

- The market for used capital clears:

$$\int d(k, s, z, m) \, dm(k, s) = \int i_{used}(k, s, z, m) \, dm(k, s)$$

- The price functions $q(z, m)$ and $Q(z, m)$ satisfy equation (4)

- The transition function Γ defines the evolution of the distribution of firms m according to the policy function k' and the Markov transition matrices T_s and T_z

5.4   Calibration

Table 1 reports the choice of parameter values. When possible, these choices reflect the attempt to stay close to previous work on firm heterogeneity and investment for comparison purposes (in particular Khan and Thomas, 2013). A period coincides with a year: this choice is motivated by the fact that both data on capital reallocation and on micro-level investment are yearly.

Parameters β, ψ and δ correspond to a yearly interest rate of 4 percent, hours worked equal to .33 and an investment/capital ratio of 6.5%. The capital share α is then set to match a capital/output ratio around 2.5. The labor share ν is 60% as in US postwar data.

Both aggregate and idiosyncratic shocks are initially parametrized as AR(1) processes in logs with autocorrelations ρ_z and ρ_s and standard deviations of innovations $\sigma_{inn,z}$ and $\sigma_{inn,s}$. Then they are discretized following the Rouwenhorst method with two values for each shock.

In particular, aggregate productivity z_t is parametrized as in Khan and Thomas (2013), who estimate a process for the Solow residual in US data. This gives a standard deviation of innovations of .014 and an autocorrelation coefficient of .909.

The standard deviation of the process for the idiosyncratic shock s is calibrated to match the standard deviation of the distribution of investment ratios computed by Cooper and Haltiwanger (2006), which is .33. The autocorrelation of the process is parametrized as in Khan and Thomas (2013) to be equal to .65. This implies a standard deviation of innovations of .084.[11]

The investment technology is defined by two parameters: η and ϵ. The first parameter is calibrated to match the steady-state ratio of used capital to total capital purchased by investing firms. The target chosen is a ratio of 30%, which is an upper bound of the estimates found by Eisfeldt and Rampini (2007), in order to take into account that smaller firms out of their sample are likely to buy a higher ratio of used capital, as this ratio appears to be decreasing in firms' size in their empirical evidence.

The elasticity of substitution between new and used investment goods ϵ is a key parameter of the model. Edgerton (2011) estimates this elasticity using data from construction equipment, aircraft and farming equipment and exploiting a tax-credit reform that affected only new investment. He finds values in a range between 1 and 10. I set $\epsilon = 5$ as my benchmark value. Beside being an intermediate value in this range of estimates, it allows to match the standard deviation of the ratio between used and new capital expenditure from ACES data, which is around 1%. I show the results for different value of ϵ in the robustness section. Note that the baseline choice implies that the relative price of used capital will move less in the model than in the data shown in the empirical section, so that this parametrization is quite conservative. Finally, I set $\gamma = .01$, which implies an average level of irreversibility of .96, close to the constant resale price in Khan and Thomas (2013). Hence, in the stationary equilibrium the baseline economy is a very close match to a version of the Khan and Thomas (2013) economy without financial frictions.

5.5 Computation

I solve the model using an extension of the method of Krusell and Smith (1998) and Khan and Thomas (2008, 2013) that takes care of non-trivial market clearing in the market for used capital.[12] I approximate the distribution m with its first moment, aggregate capital. Agents perceive a law of motion $log\left(K'\right) = \hat{\phi}_0 + \hat{\phi}_1 log\left(K\right) + \hat{\phi}_2 z + \eta$

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[11]The discretization with a two-state Markov chain implies that the average autocorrelation of investment rates and the frequency of large adjustments (lumpiness) cannot be matched at the same time as the standard deviation of investment rate, differently from Khan and Thomas (2013).

[12]By non-trivial market clearing, Krusell and Smith (2006) mean that a price has to be solved for at each period during the simulation equating total supply and total demand (in this case for used capital), differently from what happens for instance in Krusell and Smith (1998), where the rental rate of capital can be easily solved for analytically given the predetermined level of aggregate capital.

Parameter	Value
β	.96
δ	.065
ψ	2.15
α	.27
ν	.6
$\sigma_{inn,z}$	.014
ρ_z	.909
$\sigma_{inn,s}$	.084
ρ_s	.65
η	.7
ϵ	5
γ	.01

Table 2: Parameter values in the baseline model

and price functions $\hat{q}(K, z)$, $\hat{w}(K, z)$. Given these perceptions, I solve the individual firm's problem by value function iteration and obtain the policy functions, making them dependent on the current resale price q_t. Then, I simulate a continuum of firms using the simulation method of Young (2010) and update the price functions by explicitly imposing market clearing in the used capital market (and in the labor market) along the simulation. Finally, I update the laws of motion using standard regression methods up to convergence. The accuracy of the solution is illustrated in Appendix C.

6   Results

This section presents the numerical results from the full model, starting from firm dynamics in a stationary equilibrium and then moving on to the business-cycle properties of the model.

6.1   Stationary equilibrium: no aggregate uncertainty

This subsection describes the equilibrium of the model when there is no aggregate uncertainty and z is always equal to its mean. Consider first the investment policy function obtained by solving the firms' optimization problem. As in the simpler models presented in the previous sections, the wedge between the price of investment goods Q_t and the resale price q_t generates inaction areas, where firms optimally let their capital depreciate without taking any action. As $q_t < Q_t$, it is always the case

that the capital level that solves (19) without the inequality constraint of positive investment, call it $k^i(k, s, z, m)$, is strictly less than the capital level that solves (20) without the inequality constraint of positive disinvestment, call it $k^d(k, s, z, m)$. It follows that the policy function for future capital will be:

$$k'(k, s, z, m) = \begin{cases} k^i(k, s, z, m), & k \leq k^i(k, s, z, m)/(1 - \delta) \\ (1 - \delta)k, & k^i(k, s, z, m)/(1 - \delta) < k \leq k^d(k, s, z, m)/(1 - \delta) \\ k^d(k, s, z, m), & k > k^d(k, s, z, m)/(1 - \delta). \end{cases}$$

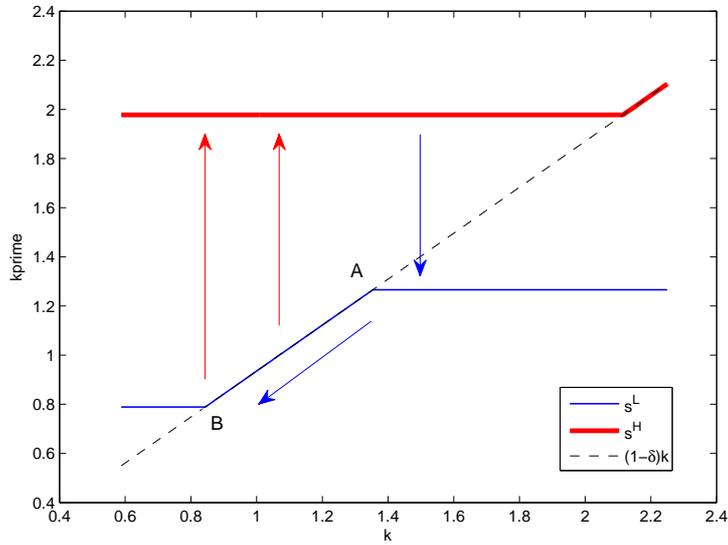Figure 6 illustrates the policy function for future capital for firms with low productiv-

Figure 6: Policy function for future capital level

ity (thin blue line) and high productivity (thick red line) under the parametrization reported in Table 1. The variable on the x-axis is the current capital level, while on the y-axis I plot next period capital. In a world without resale frictions, this picture would consist of only two horizontal lines, one at higher level for s^H and one at a lower level for s^L and firms would jump from one level to another depending on the current realization of s and regardless of their size k, given that there would be no adjustment costs. However, partial irreversibility induces disinvesting firms not to sell the whole amount of capital needed to jump to the bottom part of the blue line (point B). This is because they expect to need to reinvest in the future if they receive a positive idiosyncratic shock in the following period and they would clearly incur a loss due to the fact they would repurchase capital at a higher price than the one obtained for their disinvestment. In other words, the wedge between the price paid

for investment and the price received for disinvestment creates an option value from waiting and hence an inaction region where firms optimally wait before taking any action and just let their capital depreciate in the hope for a high productivity shock. The inaction region for low productivity firms is the upward sloping part of the thin blue line, between points A and B, which coincides with the depreciation line (dashed black line). Note that there is an inaction region also for high productivity firms (top right in the figure), but it turns out that firms never invest enough to enter this area in equilibrium.

Firm level dynamics in the stationary equilibrium are as follows. As soon as firms get a high idiosyncratic shock, they jump to the horizontal part of the thick red line. They stay there as long as they have high productivity. As soon as they get a bad shock that brings them to s^L, they sell part of their capital and jump down to the thin blue line, close to point A. Then, as long as they have productivity s^L they move down left along this line until they reach point B, where they stay until a further positive shock. Hence, on the market for used capital, supply comes from the firms that have a high level of capital and get a negative idiosyncratic shock, whereas demand comes from firms of all sizes that obtain a positive shock, plus the smallest firms with low productivity that invest to keep their size constant.

These firm-level dynamics give rise to the stationary distribution plotted in Figure 7, where the x-axis is again k and the y-axis is the mass of firms m. The thick red line with high mass on the right-hand side of the picture represents firms with productivity s^H. Moving towards the left, the thin blue lines with crosses represent the masses of firms with productivity s^L. Gradually, the mass decreases as some of the firms with those sizes receive a positive shock and only the remaining fraction let their capital depreciate for one more period. At the left end of the picture, there is a mass of low productivity firms that just rebuy their depreciated capital and keep the same small size until they get a positive idiosyncratic shock (point B).

6.2   Business cycles and capital reallocation

I will now turn to describe the properties of the economy when it is hit by aggregate productivity shocks. Table 3 shows standard business-cycle statistics as well as statistics for the resale price of capital and the reallocation series, taken from a simulation of the model economy. The first row presents the unconditional mean of the variables of interest. To construct the second and third rows, I HP-filter the data with a smoothing parameter of 6.25 and then compute relative standard deviations and correlations with output.[13] The standard deviation of output is in parentheses. It is instructive to compare these business-cycle statistics with those obtained in an economy with constant resale price (Table 4), which closely resembles a version of the Khan and Thomas (2013) economy without financial frictions.

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[13]Khan and Thomas (2013) use a smoothing parameter of 100. In the interest of a comparison with their results, I recompute these two tables with a smoothing parameter of 100 in Appendix C.
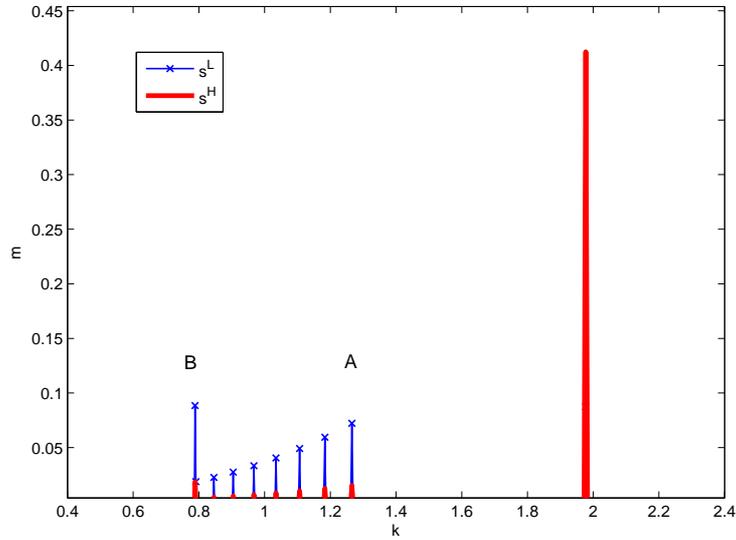
Figure 7: Stationary distribution of firms

As far as standard business-cycle variables are concerned, endogenous irreversibility reduces output and employment volatility slightly and investment volatility quite significantly (this result is discussed in more detail in a following subsection). By comparing the last columns of Tables 3 and 4, it can be seen that going from a constant q to a market-clearing resale price of capital, reallocation turns from being strongly countercyclical (the puzzle) to being strongly procyclical and approximately three times as volatile as output. In the data, the ratio between the standard deviation of (filtered) reallocation and output is 7.942 and their correlation is .562. Hence, the model cannot quite match the empirical volatility ratio and at the same time it overestimates the correlation with output. However, both statistics are significantly closer to the data than in the model with constant resale price.

In Appendix C, it can be seen that robustness exercises with respect to the elasticity of substitution ϵ lead to very similar qualitative results. The volatility of the reallocation series is decreasing in ϵ, as expected: the lower this elasticity, the more specific used capital, and hence the stronger the effects of aggregate shocks on reallocation. The high correlation with output is robust to different values of this elasticity.

A limitation of the model is that the volatility of the resale price q_t is small compared to the volatilities implied by the sectoral data presented in Section 2 and Appendix A. However, this suggests that even a small amount of volatility and procyclicality in this price can go a long way in explaining the empirical patterns of capital reallocation.

In Figures 8 and 9, I show the (unfiltered) paths of the resale price and the

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Statistic	Y	C	I	K	N	q	reall
mean	.587	.489	.097	1.490	.335	.9521	.048
$\sigma(.)/\sigma(Y)$	(1.39)	.474	4.102	.257	.546	.198	**3.276**
corr(.,Y)	1	.895	.936	-.271	.988	.963	**.959**

Table 3: Business-cycle statistics: baseline model

Statistic	Y	C	I	K	N	q	reall
mean	.587	.489	.097	1.493	.335	.9512	.0484
$\sigma(.)/\sigma(Y)$	(1.54)	.431	5.04	.291	.657	0	**1.184**
corr(.,Y)	1	.764	.914	-.212	.984	0	**-.7937**

Table 4: Business-cycle statistics: constant q

reallocation series when the economy goes from a long series of high realizations of the aggregate productivity shock to a long series of low realizations. It can be clearly seen that in the baseline model the initial fall in the resale price induces a large decrease in capital reallocation. This is in contrast with a model with constant resale price (black line with crosses), where reallocation actually increases in the first two years of the recession, and then falls gradually as the capital stock and the size of the whole economy decrease.

Figure 10 illustrates the effect of endogenous irreversibility on the dispersion of marginal product of capital, as measured by the average marginal product for high productivity (s^H) firms and that for low productivity ones (s^L). Consistently with a growing body of empirical evidence (Bloom et al., 2012), the model implies that returns are more dispersed in recessions than in booms. This is clearly related to the lack of reallocation, because large unproductive firms downsize less than they would do in a model with constant irreversibility and this prevents an equalization of marginal returns. The previous literature has either taken the countercyclicality of dispersion of returns as fully exogenous (e.g. Bloom et al., 2012) or explained it as a consequence of financial frictions (e.g. Chen and Song, 2013). This paper suggests a different explanation, based on partial capital specificity, which bears important consequences for policy in the current recovery. If one interprets the high levels of observed dispersion of returns from capital as due to a worsening of credit frictions, it is possible that a credit expansion or unconventional monetary policy could facilitate reallocation and strengthen the recovery. If instead the high dispersion is fully efficient and due to capital specificity and equilibrium irreversibility, then no policy intervention is in order and credit expansions are not relevant.

The dynamics of the distribution of firms over the business cycle are illustrated in Figure 11. It can be seen that the distribution becomes more compressed when the

economy moves from a boom to a recession: large unproductive firms downsize by less than they do in booms (compare points A and A'), while highly productive units expand by less. Jointly, these facts explain the fall in reallocation and the increase in the dispersion of returns.
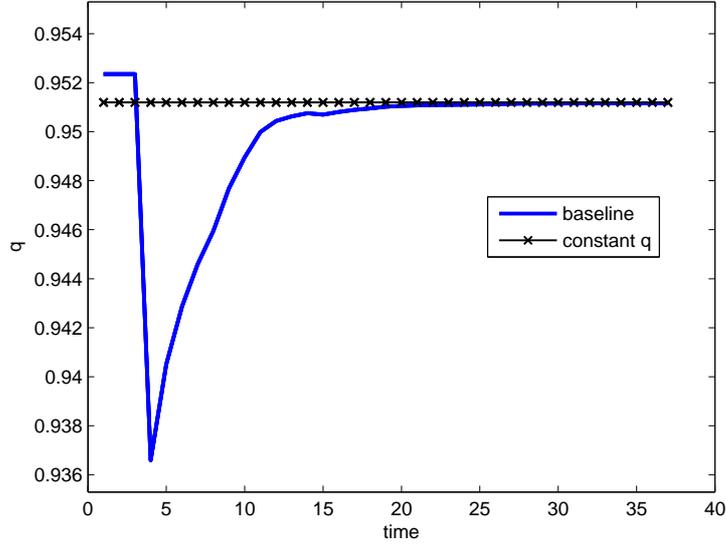

Figure 8: Price of used capital

Transition from long sequence $z_t = z^H$ to long sequence $z_t = z^L$. Response of the price of used capital. Comparison between the baseline model and a model with constant resale price. Unfiltered simulated data.

6.3 Amplification of aggregate TFP

The procyclicality of reallocation generates endogenous movements in aggregate TFP, amplifying the exogenous aggregate productivity shock. Measured TFP, call it Z_t, is the Solow residual that an econometrician would compute by assuming an aggregate production function $Y_t = Z_t K_t^\alpha N_t^\nu$. A large part of the variation in this variable is due to the exogenous component z_t, while the rest is due to how capital and labor are allocated across the heterogeneous productive units in the economy. In the model, this second component, $TFP_{end} \equiv log(Z_t) - log(z_t)$, is of second order, when compared to the exogenous one, so the absolute importance of the allocative component of TFP should not be overemphasized.

However, this component is magnified by endogenous irreversibility, as can be seen in Table 5, where both the ratio between its volatility and the volatility of the shock (σ_{TFPend}/σ_z) and the ratio between its volatility and that of output (σ_{TFPend}/σ_Y) in-
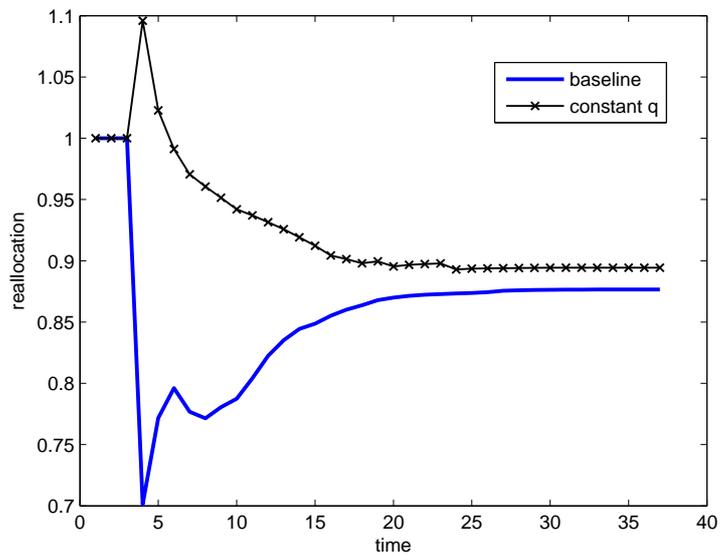
34

Figure 9: Capital reallocation

Transition from long sequence $z_t = z^H$ to long sequence $z_t = z^L$. Response of capital reallocation. Comparison between the baseline model and a model with constant resale price. Unfiltered simulated data.
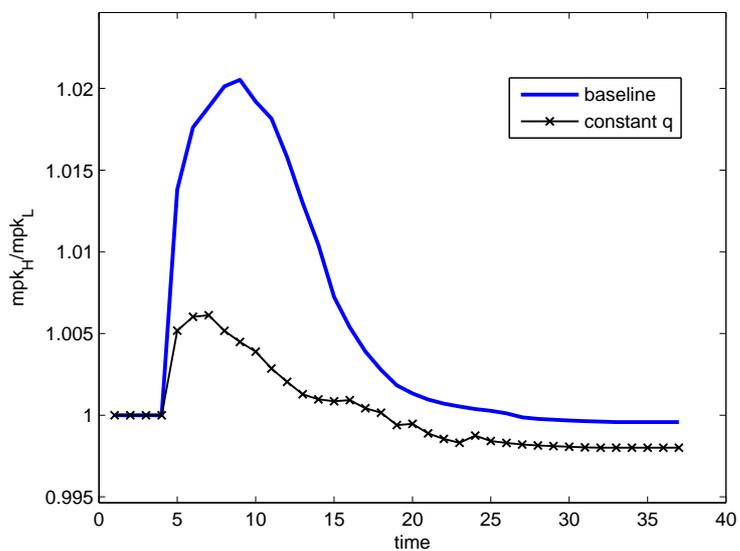
Figure 10: Dispersion of returns

Transition from long sequence $z_t = z^H$ to long sequence $z_t = z^L$. Response of the ratio between the average marginal product of high productivity firms (s^H) and that of low productivity firms (s^L). Comparison between the baseline model and a model with constant resale price. Unfiltered simulated data.
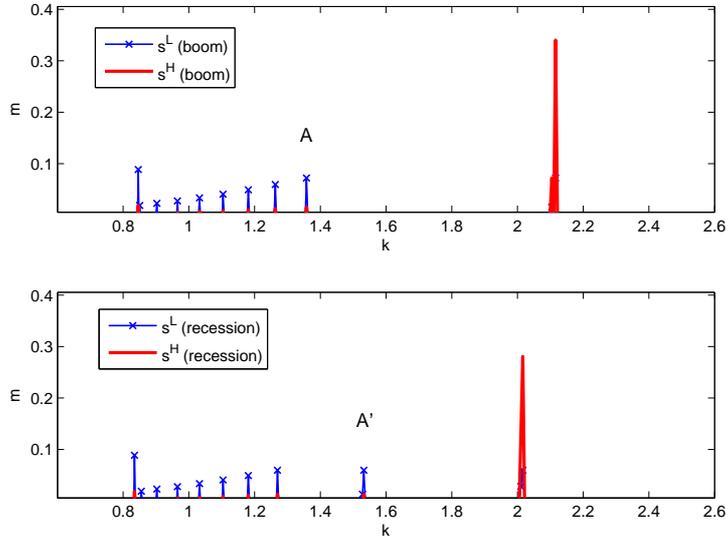
Figure 11: Distribution dynamics

Transition from long sequence $z_t = z^H$ to long sequence $z_t = z^L$. Cross-sectional distribution of firms before and immediately after the shock.

crease by more than four times when going from constant to endogenous irreversibility. Importantly, this should be seen as a lower bound for the importance of capital reallocation for aggregate TFP, because the model generates less volatility in both q_t and the reallocation series than we observe in the data.

The amplification mechanism for TFP works as follows: during recessions, reallocation decreases and firms with idiosyncratic productivity s^L are in a sense "too large", which not only implies that capital is less productively used, but also employment is "too high" in these relative less productive firms, as labor demand is an increasing function of a firm's capital stock. Hence the allocation of inputs gets further away from the one that would arise in a model where investment and disinvestment are fully flexible. However, the allocation is always efficient, in the sense that it would coincide with the choice of a planner that faced the same reallocation frictions.

Furthermore, to see how this mechanism is propagated over time, observe again the policy functions illustrated in Figure 6. When large firms are hit by a negative idiosyncratic shock, they sell part of their capital once and then they just let their capital depreciate until they become highly productive again. This means that if they sell a small amount of capital in the first period, they will remain "too large" (relative to a model with constant q) for several periods, until they get a positive idiosyncratic shock again. Hence, the negative effects of lower reallocation on aggregate productiv-

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ity in recessions are propagated over time trough these movements in the distribution of firms. This implication of the model seems consistent with the patterns observed in the current slow recovery of productivity in the UK, that Broadbent (2012) attributes precisely to insufficient capital reallocation.

It is worth emphasizing that the amplification of TFP in the model is an increasing function of the dispersion of idiosyncratic productivity across firms: the more dispersed productivity is, the larger the benefits from reallocation. Hence, the procyclicality of capital reallocation induces larger movements in aggregate productivity when the variance of s_t is higher. For instance, doubling the unconditional variance of s_t leads to $\sigma_{TFPend}/\sigma_z = 0.0278$ and $\sigma_{TFPend}/\sigma_Y = 0.0178$. In this paper, the volatility of idiosyncratic productivity is calibrated to match micro-level investment data following the methodology of Khan and Thomas (2008, 2013). However, in the literature there is no unanimous consensus on this parameter value and in general on the procedure to parametrize the idiosyncratic productivity process. For example, Bloom et al. (2012) estimate time-varying volatilities of firm-level productivity and get values for volatility of up to 12% quarterly in high uncertainty periods. This again suggests that the amplification of aggregate productivity delivered by the present paper is a lower bound for the empirical effect of procyclical capital reallocation on TFP.

Model	σ_{TFPend}/σ_z	σ_{TFPend}/σ_Y
Constant q	0.0034	0.0021
Baseline	0.0134	0.0093

Table 5: Amplification of endogenous TFP

6.4   Endogenous irreversibility smooths aggregate investment

The previous literature on investment irreversibility has debated whether the observed smoothness of the aggregate investment series can be attributed to irreversibility at the micro level. Bertola and Caballero (1994) affirm this point in a partial equilibrium model and suggest that the fear of not being able to disinvest may act as a smoothing device at the time of investing, making firms more cautious in their investment decisions and generating inaction regions. However, Veracierto (2002) introduces constant partial irreversibility in a general equilibrium model with heterogeneous plants and shows that the properties of aggregate investment are unchanged when moving from totally flexible to totally irreversible investment.[14] This is because the consumption smoothing force makes the interest rate adjust in such a way that

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[14]Of course, the properties of micro-level investment decisions are very different depending on the level of irreversibility.

aggregate investment has the same desired properties for the representative agent. Furthermore, aggregate shocks are just shifting the inaction region without affecting its size, so that the mass of firms in the inaction region is not changing significantly over time.

The present model reaffirms the original conjecture of Bertola and Caballero (1994) by making irreversibility an equilibrium outcome that moves over the business cycle. Investment becomes more irreversible in recessions, when unproductive firms would like to disinvest by more. This makes investment riskier for firms, hence making them more cautious at the time of investing. Furthermore, the endogenous prices for investment goods are acting in the direction of smoothing the investment decisions even more: in a recession, when investment falls, used capital is cheaper, which implies that total investment becomes slightly cheaper (Q_t falls) hence dampening the fall in aggregate investment. The opposite happens in booms, when firms want to invest more, but Q_t increases.

As can be seen comparing again Tables 3 and 4, the volatility of aggregate investment relative to output falls from 5 to 4. Following the previous literature on micro lumpiness and aggregate investment (e.g. Khan and Thomas, 2008), I also report the volatility and autocorrelation of the unfiltered investment/capital ratio. Table 6 compares the baseline model with (i) a fully flexible model without irreversibility and (ii) the model with constant q. With endogenous irreversibility, the investment/capital ratio is more persistent and its innovations are less volatile than in the two comparison models, and closer to the US data reported by Khan and Thomas (2008), presented in the last row.

Model	$\sigma_{inn,I/K}$	$\rho_{I/K}$
Frictionless	.010	.675
Constant q	.009	.582
Baseline	.008	.680
Data	.008	.695

Table 6: Volatility and autocorrelation of aggregate investment rates

Relatedly, another feature of the model is that following a bad aggregate shock the inaction region becomes larger. This is because the higher wedge between the price of new investment goods and the resale price increases the option value of inaction. This feature of the model resembles the behavior of a model with non-convex adjustment costs and uncertainty shocks (e.g. Bloom et al., 2007). In that case, the freezing of investment activity associated with a widening of the inaction region is driven by exogenous increases in uncertainty. In this model, the same behavior arises in response to first order productivity shocks, via the endogenous reaction of the resale price of capital, without resorting to changes in second order moments. This time-varying wedge may have important policy implications. For example, investing subsidies like

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those included in the US fiscal stimulus package of 2009 are likely to have procyclical multipliers in this setting, as more firms are in the inaction region in recessions and are thus likely to be less responsive to this kind of stimulus.

6.5   Investment-specific shocks

In the baseline version of the model, business cycles were driven by aggregate productivity shocks. However, a large literature emphasize the importance of shocks to the productivity of investment as important drivers of aggregate fluctuations (e.g. Greenwood et al., 2000).

In this subsection, I argue that the main mechanism of endogenous irreversibility and capital reallocation is robust to this different source of business cycles. To see why this is the case, let us first define the modified model. For simplicity, let the aggregate productivity parameter z be constant and equal to 1. Let p_t be the relative price of new investment goods in terms of consumption. Investment goods are produced using the output good as input by a competitive firm. Hence shocks to the marginal cost of production of new investment goods translate into shocks to p_t. Let this shock follow a Markov chain with two values $p_t \in \left\{p^L, p^H\right\}$.

The CES price index for a bundle of new and used investment is now

$$Q_t = [\eta p_t^{1-\epsilon} + (1-\eta)(q_t + \gamma)^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{21}$$

where q_t is the relative price of used capital in terms of the consumption good.

Consider a persistent 1% increase from p^L to p^H, illustrated in the first panel of Figure 12 (solid black line). When the shock hits the economy, new investment becomes more expensive, inducing a recession as is standard in the literature on investment-specific shock. Importantly, in the present model, the shocks also leads to a fall in capital reallocation (second panel).

To see why this is the case, consider first the buyers on the market for used capital. The increase in the price for new investment goods has two effects of opposite sign on their demand for used capital: on the one hand, total investment is now more expensive (Q_t is higher), leading to a fall in demand for all kinds of investment goods. On the other hand, for a given total investment, firms are willing to partially substitute from new to used investment goods, leading to an increase in demand for used. It turns out that for the calibrated elasticity of substitution between new and used investment, the first effect dominates and demand for used capital falls. Hence q_t (first panel, dashed blue line) increases only gradually with the result that q_t/p_t is below its average for several periods.

For disinvesting firms, given partial irreversibility, when the shock hits it is a bad time to sell assets: they know that the resale price is likely to increase and that they might have to rebuy some capital at a higher Q_t in case they receive a positive idiosyncratic shock in the near future. This implies that also investment-specific shocks induce a procyclical response of capital reallocation.
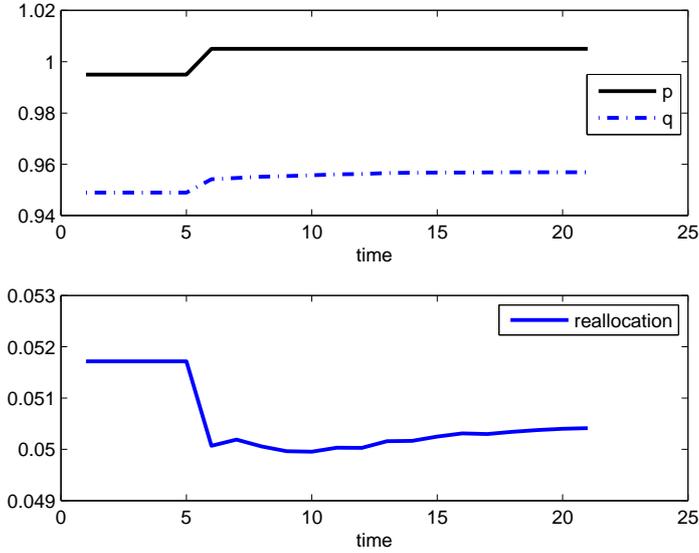
Figure 12: Investment-specific shock: investment prices and reallocation

Transition from long sequence $p_t = p^L$ to long sequence $p_t = p^H$. Response of the relative price of used capital (in terms of consumption) and capital reallocation (lower panel).

7   Conclusion

This paper shows that the procyclicality of capital reallocation can be rationalized in a model where the resale price of capital is endogenous. According to the sectoral data presented, this price is strongly procyclical, making it harder to reverse past investment decisions during recessions. The model generates this fact as an equilibrium outcome by assuming that new and used investment goods are imperfect substitutes because of partial firm-level capital specificity. Hence, in a recession higher supply of used capital and lower demand lead to a fall in the price, inducing both static and dynamic effects on investment and reallocation via the equilibrium response of real option values. This mechanism induces endogenous movements in aggregate TFP, because during expansions, when the resale price of capital increases, the allocation of capital and labor gets closer to the one that would arise in a flexible model and viceversa in downturns more capital is operated by unproductive firms.

Endogenous irreversibility is a plausible mechanism behind a smooth aggregate investment series. In this sense the paper provides an explicit microfoundation for what in the aggregate resembles a convex capital adjustment cost. Furthermore, the model generates a countercyclical dispersion of returns, consistently with a growing literature on firm-level uncertainty. Importantly, this result is fully efficient in a Pareto sense. Previous work has connected a high dispersion of returns in recessions with

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the malfunctioning of credit markets, hence providing one rationale for expansionary credit policies. This paper suggests that part of this increased dispersion, which has been emphasized both in academic and policy work during the recent recession, may be unrelated to credit conditions and attributable to partial capital specificity.

While this model assumes perfect capital markets, it is clear that financing constraints may play an important role in shaping investment dynamics. Introducing a collateral constraint that ties the borrowing capacity to the resale value of a firm's capital is likely to further amplify the mechanism described in the paper. This has important implications for the question on the source of business cycles. Previous work based on a constant value of collateral has suggested that procyclical capital reallocation is evidence in favor of exogenous credit shocks (Cui, 2014). However, the value of a firm's collateral depends on the resale price of its assets. Hence, an extension of the present model with collateral constraints could potentially generate an endogenous tightening of collateral constraints after a negative TFP shock, reconciling both the cyclicality of reallocation and that of credit availability with standard productivity shocks.

Furthermore, US plant level data suggest that while entry is strongly procyclical, exit is almost acyclical (Lee and Mukoyama, 2013). This evidence on exit is to some extent a puzzle for models with productivity shocks where the exit decision is driven by a fixed cost of production denominated in units of the output good. In such models, after a bad aggregate TFP shock, more firms optimally decide to liquidate their capital and exit. Endogenous irreversibility seems to be a promising explanation for this puzzle. In a recession, on the one hand the value of staying in business falls, so that more firms would like to exit, but on the other hand also the value of exit falls, as it depends of the resale price of capital, so that overall the incentive to liquidate is dampened.

Finally, this paper provides a natural framework to analyze movements in the utilization rate of capital both in the aggregate and at the micro-level. A large firm hit by a negative profitability shock can choose whether to reallocate its assets or to keep them idle for some time, hoping for an improvement in business conditions. The previous literature on heterogeneous firms and capacity utilization has imposed restrictions on the possibility to sell assets after the realization of idiosyncratic productivity shocks in order to justify the choice by unproductive firms to keep some idle capital (e.g. Hansen and Prescott, 2005, Sustek, 2011).

In the context of a model with endogenous irreversibility, no such assumptions are necessary. A version of the model that includes endogenous capacity choice (Lanteri, 2014) yields a natural solution to the question of whether to sell assets or keep them idle. Depending on aggregate conditions, the resale price may be high enough to induce reallocation or low enough to make it optimal to keep capital idle. The key mechanism works through equilibrium changes in two option values: the put option value to resell and the value to keep capital idle and save on production costs. The price of used capital responds to aggregate shocks and makes one or the other option

more valuable at different points in the business cycle. Hence cyclical movements in output, reallocation and utilization can be jointly explained in a model of endogenous irreversibility.

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Appendix A: Additional empirical evidence and data sources

Further evidence on capital reallocation: UK sales of used equipment (Figure 13) and global sales of used commercial ships (Figure 14).
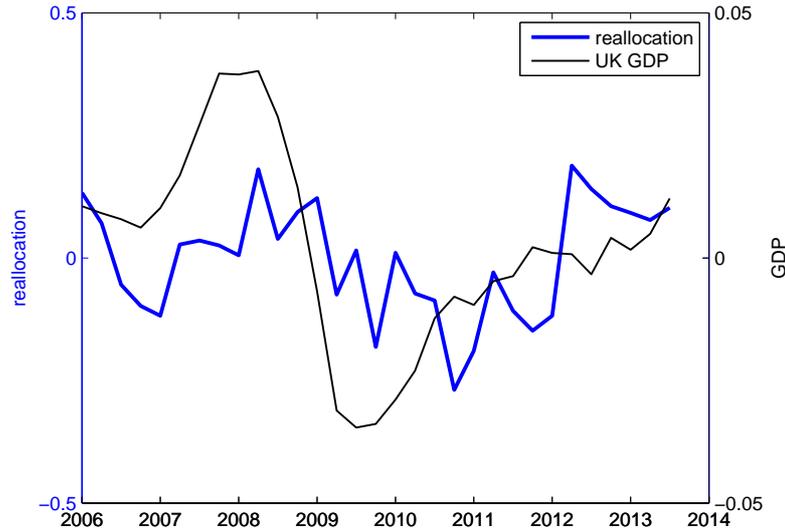

Figure 13: Capital reallocation in UK during the Great Recession

Cyclical components of quarterly sales of used equipment and UK real GDP, deflated using the GDP deflator.

Further evidence on the price of used capital over the business cycle:

Ships I gather price series for new and used ships from the mid-90's onwards. It is interesting to observe that prices and quantities traded fall contemporaneously in 2008, and that the price index of used ships is more volatile than the price index of new ships (Figures 14 and 15). Similarly to what discussed in Section 2 for the aircraft sector, also in the case of ships the resale price of more specific models in terms of possible routes (e.g. the very heavy and large Capesize bulk carrier) grows more strongly in the period 2006-2008 and then falls by a larger fraction towards the end of 2008 than that of less specific ones (e.g. the more flexible and small Handysize bulk carrier). This is shown in Figure 16.

Vehicles In the case of vehicles and trucks one can compare two separate separate CPI series, one for new (CPI new vehicles) and one for used (CPI used cars and trucks). Figure 17 shows the cyclical components (HP-filtered) of the CPI for used cars and trucks and the CPI for new vehicles, both relative to the total CPI including all items. It emerges that the price of used vehicles is much more volatile and more
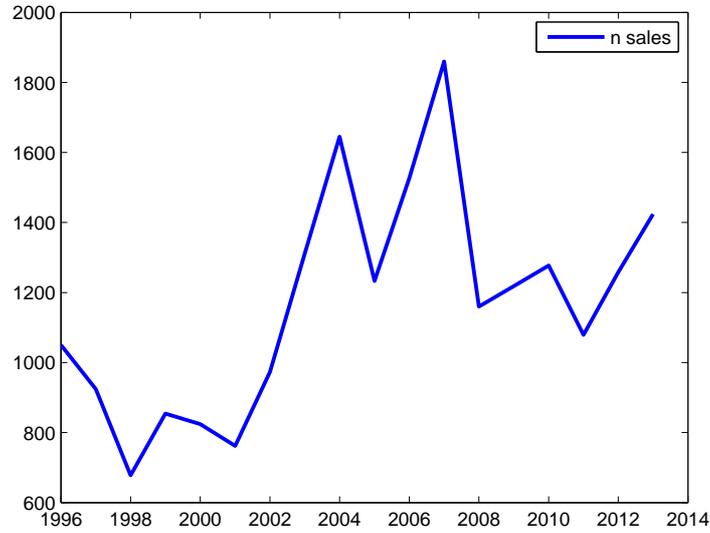
48

Figure 14: Ships: number of second-hand sales

Global yearly sales of second-hand commercial ships. Source: Clarkson

procyclical than that of new ones, which is actually acyclical (their correlations with GDP are 0.41 and -0.09 respectively). The volatility of prices of used vehicles is smaller than that of aircraft and ships, arguably because vehicles are a less specific type of asset. Hence, this difference in volatilities is broadly consistent with a theory based on capital specificity.

Construction equipment Edgerton (2011) constructs an index of the price of used construction machinery by collecting data on auctions where this equipment is reallocated across US construction firms. These data are illustrated in Figure 18 and show that the price of used construction equipment fell by more than the corresponding PPI (construction machinery) both in the 2001 and in the 2009 recession, and is in general significantly more volatile. In 2009 the index of used construction equipment is more than 15% below trend, while the corresponding PPI of new construction machinery is slightly above trend.
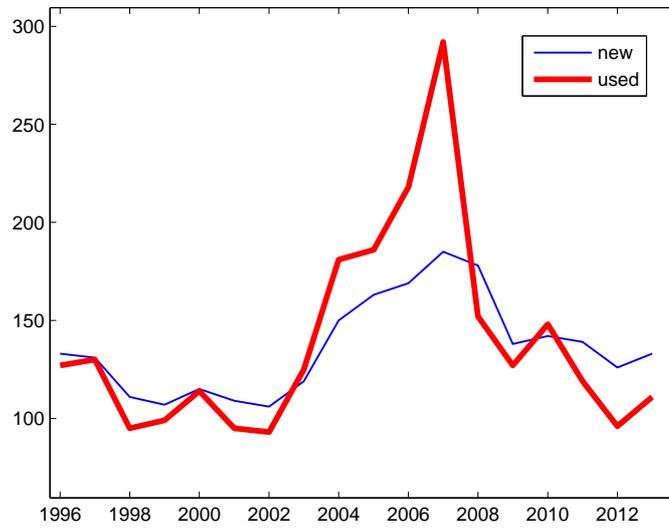
Figure 15: Ships: price indices of new and used

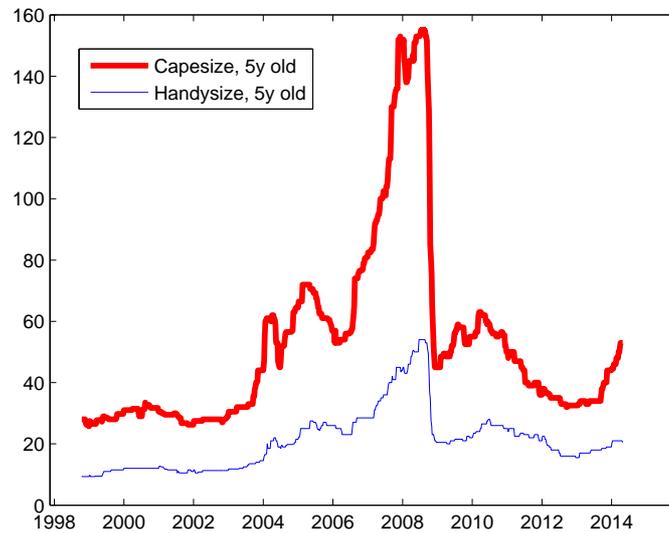Price indices of new and second-hand commercial ships. Yearly frequency.

Figure 16: Ships: price of used Capesize and used Handysize

Prices in million $ of second-hand 5 year-old Capesize (more specific) and Handysize (less specific). Weekly frequency: estimated values based on actual transactions and shipping market information.
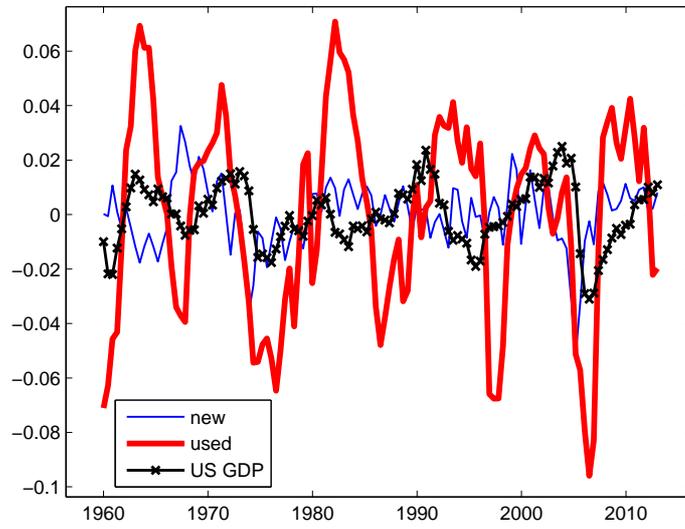
Figure 17: Vehicle prices and GDP

Cyclical components of CPI new vehicles, CPI used cars and trucks and US real GDP. Quarterly frequency. Both CPI series are divided by CPI All items.
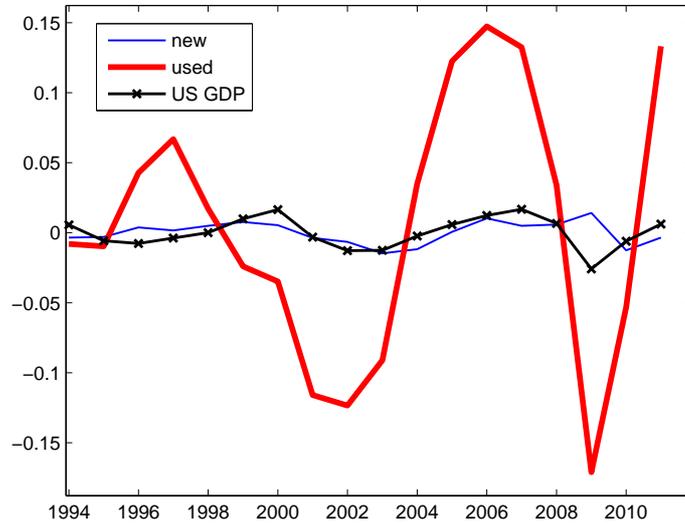

Figure 18: Construction equipment prices and GDP

Cyclical components of construction equipment PPI, price index of used construction equipment (Edgerton, 2011) and US real GDP. Yearly frequency.

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Data sources Data on capital reallocation in the US come from the Compustat dataset and have been kindly made available by Andrea Eisfeldt on her personal webpage. Data on sales of used equipment in the UK come from the Office for National Statistics (ONS) Survey of Capital Expenditures. Data on aircraft prices are compiled by Aircraft Values. Data on commercial ships are compiled by Clarkson and VesselsValue. The price index for used commercial equipment has been constructed by Edgerton (2011) using auction prices. Data on US GDP, GDP Deflator, CPI, CPI for new and used vehicles, as well as PPI for the construction sector come from the US Bureau of Economic Analysis and the US Bureau of Labor Statistics. Data on UK GDP and GDP deflator come from the ONS.

Appendix B: Proof of Proposition 1

(i) By equating (5) and (6), the market-clearing condition for used capital can be written as follows:

$$G(q, z, \epsilon) \equiv \theta(q) \int_{s^I} \left[\left(\frac{\alpha z s}{Q(q)} \right)^{\frac{1}{1-\alpha}} - k_0 \right] dF(s) - \int^{s^D} \left[k_0 - \left(\frac{\alpha z s}{q} \right)^{\frac{1}{1-\alpha}} \right] dF(s) = 0.$$

(22)

where $s^I = \frac{Q(q)}{\alpha z k_0^{\alpha-1}}$, $s^D = \frac{q}{\alpha z k_0^{\alpha-1}}$, $Q(q) = [\eta + (1-\eta)(q+\gamma)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}$, $\theta(q) = (\frac{q+\gamma}{Q(q)})^{-\epsilon}(1-\eta)$ is the ratio of used investment to total investment for investing firms and I have left implicit the dependence of θ, q and Q on ϵ. Equation (22) defines the market-clearing price q as an implicit function of the aggregate productivity parameter z and the elasticity of substitution between new and used capital ϵ. We can obtain the derivative of q with respect to z by applying the Implicit Function Theorem to function G and we get[15]

$$\frac{dq}{dz} = -\frac{G_z}{G_q}$$

(23)

with

$$G_z = \frac{\theta}{(1-\alpha)z} \int_{s^I} \left(\frac{\alpha z s}{Q} \right)^{\frac{1}{1-\alpha}} dF(s) + \frac{1}{(1-\alpha)z} \int^{s^D} \left(\frac{\alpha z s}{q} \right)^{\frac{1}{1-\alpha}} dF(s)$$

and

$$G_q = \theta_q \int_{s^I} \left(\frac{\alpha z s}{Q} \right)^{\frac{1}{1-\alpha}} dF(s) - \frac{\theta Q_q}{(1-\alpha)Q} \int_{s^I} \left(\frac{\alpha z s}{Q} \right)^{\frac{1}{1-\alpha}} dF(s) - \frac{1}{(1-\alpha)q} \int^{s^D} \left(\frac{\alpha z s}{q} \right)^{\frac{1}{1-\alpha}} dF(s)$$

Note that in applying Leibniz rule to derive this expression we do not need to worry about the derivatives of the end points s^D and s^I because by their definition, the respective integrands are equal to zero when evaluated at these points.

Hence $\phi_{q,z}(\epsilon) \equiv \frac{dq}{dz}\frac{z}{q}$, the elasticity of q with respect to z, is

$$\phi_{q,z}(\epsilon) = -\frac{F_z}{F_q} = \frac{\frac{\theta}{(1-\alpha)} \int_{s^I} \left(\frac{\alpha z s}{Q} \right)^{\frac{1}{1-\alpha}} dF(s) + \frac{1}{(1-\alpha)} \int^{s^D} \left(\frac{\alpha z s}{q} \right)^{\frac{1}{1-\alpha}} dF(s)}{\theta_q q \int_{s^I} \left(\frac{\alpha z s}{Q} \right)^{\frac{1}{1-\alpha}} dF(s) + \frac{\theta Q_q q}{(1-\alpha)Q} \int_{s^I} \left(\frac{\alpha z s}{Q} \right)^{\frac{1}{1-\alpha}} dF(s) + \frac{1}{(1-\alpha)} \int^{s^D} \left(\frac{\alpha z s}{q} \right)^{\frac{1}{1-\alpha}} dF(s)}$$

(24)

Now, note that when $\epsilon = 0$ (Leontief investment technology), the share of used capital to total investment becomes $\theta = 1 - \eta$, so that $\theta_q = 0$, while the price index becomes $Q = \eta + (1-\eta)(q+\gamma)$, so that we get $Q_q = 1 - \eta$. Hence we can write

$$\phi_{q,z}(0) = \frac{(1-\eta) \int_{s^I} \left(\frac{\alpha z s}{Q} \right)^{\frac{1}{1-\alpha}} dF(s) + \int^{s^D} \left(\frac{\alpha z s}{q} \right)^{\frac{1}{1-\alpha}} dF(s)}{(1-\eta)^2 \frac{q}{\eta+(1-\eta)(q+\gamma)} \int_{s^I} \left(\frac{\alpha z s}{Q} \right)^{\frac{1}{1-\alpha}} dF(s) + \int^{s^D} \left(\frac{\alpha z s}{q} \right)^{\frac{1}{1-\alpha}} dF(s)}$$

(25)

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[15]Notation: Call f_x be the partial derivative of function f with respect to argument x.

and this establishes that $\phi_{q,z}(0) > 1$ as $q < 1 \Rightarrow (1 - \eta)\frac{q}{\eta+(1-\eta)(q+\gamma)} < 1$. Standard arguments can be used to show that $\phi_{q,z}$ is continuous.

(ii) It suffices to observe that the equilibrium supply of used capital S^*_{used}, i.e. is total reallocation, is a decreasing function of $\frac{z}{q}$ (as above, we can disregard the derivative of s^D as the integrand is zero when evaluated at s^D):

$$S^*_{used} = \int^{s^D} \left[k_0 - \left(\frac{\alpha z s}{q} \right)^{\frac{1}{1-\alpha}} \right] dF(s) \tag{26}$$

Hence, the sign of its derivative with respect to z is the sign of $\phi_{q,z} - 1$. This establishes that in the limit for sufficiently low elasticity of substitution between new and used capital, reallocation is increasing in z, i.e. "procyclical".

Appendix C: Accuracy and robustness

Figure 19 illustrates the accuracy of the solution by showing the simulated series of aggregate capital (solid red line) and a forecast series constructed using the estimated coefficients of the law of motion and iterating on the forecast (blue crosses), as suggested by den Haan (2010). The R^2 of the regression of log capital on constant, its lag and aggregate productivity is .9993.
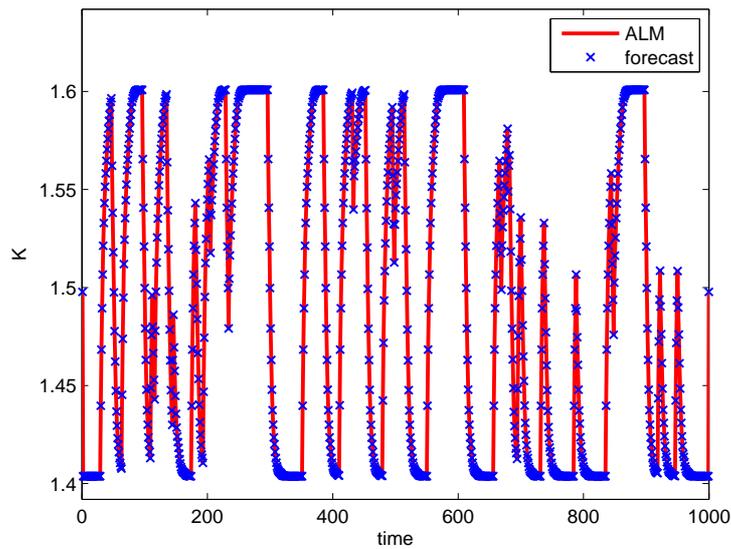

Figure 19: Actual law of motion and its forecast

Robustness exercises: business-cycle statistics for different values of ϵ and HP-filter smoothing parameter.

Statistic	Y	C	I	K	N	q	reall
mean	.595	.495	.099	1.518	.335	.948	.042
$\sigma(.)/\sigma(Y)$	1.34	.470	4.25	.257	.539	.192	4.25
corr(.,Y)	1	.838	.950	-.281	.989	.952	.956

Table 7: Business-cycle statistics: $\epsilon = 1$

Statistic	Y	C	I	K	N	q	reall
mean	.588	.489	.097	1.490	.336	.959	.055
$\sigma(.)/\sigma(Y)$	(1.40)	.442	4.242	.266	.558	.172	2.617
corr(.,Y)	1	.905	.936	-.277	.984	.970	.960

Table 8: Business-cycle statistics: $\epsilon = 10$

Statistic	Y	C	I	K	N	q	reall
$\sigma(.)/\sigma(Y)$	(2.05)	.539	3.786	.403	.532	.183	3.011
corr(.,Y)	1	.914	.930	.048	.963	.916	.9585

Table 9: Business-cycle statistics: baseline model, HP smoothing parameter = 100

Statistic	Y	C	I	K	N	q	reall
$\sigma(.)/\sigma(Y)$	(2.25)	.545	4.398	.407	.626	0	1.055
corr(.,Y)	1	.828	.887	.114	.966	0	-.592

Table 10: Business-cycle statistics: constant q, HP smoothing parameter = 100