Resource Curse or Blessing? Sovereign Risk in Resource-Rich Emerging Economies

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Abstract

Business cycles in oil-exporting emerging economies are known to be tied to fluctuations in oil prices. These economies also have large external debt ratios and most of them have defaulted since the 1980s. We show that, in addition, higher oil prices and/or oil production are associated with lower sovereign risk in the long-run while the opposite is true for higher oil reserves. We propose a model of external sovereign default and oil extraction consistent with these observations. The default payoff is endogenous and depends on oil reserves. Higher oil production or prices reduce country risk by increasing debt repayment capacity but larger reserves can increase it by making autarky more valuable. Without default risk, the model is akin to an RBC model with terms-of-trade shocks: High oil prices incentivize increasing oil production and reducing reserves so that the gross return of oil equals the world interest rate plus a standard "equity premium." In contrast, with default risk, the sovereign internalizes that higher reserves reduce the price of its debt because of the higher option value of default and this increases the sovereign's rate of return on oil.

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1 Introduction

Fluctuations in commodity prices are a key determinant of macroeconomic performance in resource-rich economies. This is particularly the case in oil-exporting countries. Well-known transmission mechanisms connect world oil-price fluctuations with macroeconomic outcomes via their effects on incentives to extract oil and adjust oil reserves and on incentives to consume, invest in physical capital, and borrow or lend in international financial assets.[1] In contrast, the relationship between oil prices, default risk, and macroeconomic dynamics has remained relatively unstudied, despite the fact that country risk itself is also a well-known determinant of business cycles in emerging markets (see Uribe and Yue (2006), Mendoza and Yue (2010)) and that oil revenues are a significant driver of government solvency and country risk in oil-exporting countries.

In this paper we fill in this gap, by studying the empirical regularities connecting oil prices and country risk, and by proposing a new model that can help us rationalize those regularities. We start by showing the main stylized facts about the relationship between oil prices, sovereign risk and macroeconomic performance of oil-exporting economies. We show that, even though being a bigger oil producer decreases sovereign risk, having more oil reserves increases it. As we document in the next section using a dynamic fixed effects regression, the short-run elasticity of country risk with respect to changes in oil production is 0.003% and the long-run elasticity with respect to oil reserves is -0.17%. That is, when oil production increases by 1% country risk decreases by 0.003%, and when oil reserves increase by 1%, country risk increases by 0.17%.

We then develop a model of sovereign default on external debt in which optimal plans regarding oil extraction, debt, oil reserves, and default interact. We derive analytic results relating oil-price shocks to default incentives and default risk and conduct a quantitative analysis to assess the model's ability to explain the empirical facts.

Examining data for the 30 largest emerging market oil exporters over the period 1979-2014, we found that these countries hold an average external public debt[2] to GDP ratio of 39%

[1]These issues have been widely studied in the international business cycle literature, particularly the branch focusing on terms-of-trade shocks (e.g. Backus et al. (1994), Mendoza (1995), Ben Zeev et al. (2017), Schmitt-Grohe and Uribe (2018).

[2]We use external public debt data from the World Bank where public and publicly guaranteed debt comprises long-term external obligations of public debtors, including the national government, political subdivisions (or an

and sixteen countries in the sample have experienced between one and five default episodes. We highlight three features of the relationship between country risk and the size of the oil sector: First, as is natural to expect, a given oil exporting country is perceived by investors as less risky, the higher their oil production and the higher oil prices, allowing its public sector to support higher levels of public debt. Second, and perhaps less natural to expect, in the long run, country risk perception increases the higher the level of oil reserves of the country. This may reflect the fact that having a large stock of oil increases a country's outside option (the value of autarky), making default more appealing. Third, the data also shows that during default episodes, the median oil exporting country increases net oil exports. This evidence suggests that a country in default and excluded from international financial markets, increases its oil exports to withstand the consequences of financial autarky.

When we explore the relationship between oil price changes and macro performance, we find that increases in oil prices are associated with higher oil extraction and higher GDP growth rates, trade balance and current account improvement, lower sovereign risk perception and lower number of default events. Likewise, oil price decreases are associated with lower oil extraction and lower GDP growth rates, trade balance and current account deterioration, higher sovereign risk perception and a higher number of default events.

We build a small open economy model with two types of goods: a tradable and non-storable consumption good and oil. The sovereign government owns all oil reserves—and makes all decisions regarding its extraction—and can trade non-state contingent bonds with risk neutral competitive foreign lenders in international financial markets but cannot commit to repaying its debt. The relative price of oil and the consumption good are exogenously given.

We find that theory predicts that long-run reserves of the resource have two opposing effects in determining the long-term sovereign risk premium. Higher stock of reserves allow the country to have a higher extraction rate to support debt repayments, lowering default risk. However, they also allow the country to use the resource during default times, making the value of default more attractive. The tension between these two forces determines the long run default risk premium.

Our work links two large strands of the literature, one related to sovereign risk and an-

agency of either), and autonomous public bodies, and external obligations of private debtors that are guaranteed for repayment by a public entity. Data are in current local currency units.

other related to commodity markets. In our model, a sovereign facing incomplete financial markets may find optimal to default, as in Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008), and most of the literature thereafter. The sovereign however, owns a stock of reserves of a commodity that can (in addition to foreign borrowing) be used to smooth consumption affecting in turn the default risk premium. Bouri, de Boyrie and Pavlova (2017) document the transmission of volatility from commodity markets to credit default swaps (CDS) spreads of emerging markets. They find significant volatility spillovers specially coming from energy and precious metals. Reinhart, Reinhart and Trebesch (2016) document how major spikes in sovereign defaults occur when capital inflows surge and are followed by busts in capital and commodity markets. Fernandez, Schmitt-Grohe and Uribe (2017) present an empirical framework in which multiple commodity prices transmit to domestic business cycles, explaining up to 33% of output fluctuations of individual countries.

The paper proceeds as follows. Section 2 presents the empirical evidence, Section 3 presents the model, Section 4 presents the calibration, Section 5 the quantitative analysis of the model, and Section 6 concludes.

2 Empirical Evidence

This section documents important empirical regularities linking oil production, oil reserves and sovereign risk for the 1979-2010 period. We start by describing the data and then move on to document the stylized facts.

2.1 Data

We collected data for oil GDP, non-oil GDP, oil reserves, oil consumption, oil net exports, oil prices, total public debt, total external public debt, net foreign assets, default episodes and country risk for the thirty largest oil producing emerging economies in 2010.

The data on oil reserves, oil production, oil net exports (thousands of barrels per day), and oil prices (Brent crude oil, USD per barrel) is from the US Energy Information Administration (EIA). For reserves, we used proved reserves.[3]

[3] Reserves are difficult to measure given uncertainties about the quantity and quality of oil in the ground. Available measures include ultimately recoverable resources, proved, probable and possible reserves, and oil in

As an indicator of country risk, we use the Institutional Investor's Index for Country Credit Ratings (III from now on). The III is an index of country risk published biannually in the March and September issues of the *Institutional Investor*. These credit ratings are based on information gathered from the Institutional Investor's Country Credit Survey, which reflects information provided by senior economists and sovereign-risk analysts at leading global banks and money management and securities firms. The respondents grade each country on a scale of zero to 100, with 100 representing the smallest probability of default, and their responses are weighted according to their institutions' global exposure. The III is an indicator intended to capture a collection of risks related to investing in a particular country, including political risk, exchange rate risk, economic risk, and sovereign risk. We have biannual III data for the 1979-2010 period. The literature on sovereign risk typically uses spreads on sovereign debt measured with the Emerging Markets Bond Index (EMBI) as the measure of country risk. EMBI spreads, however, are only available since 1994 and for a small number of countries, which imposes limitations on the scope of the empirical analysis that we can conduct. For this reason we use the III. In Appendix A we show that the III is positively correlated with EMBI spreads, Moody's, and Fitch credit ratings.

Total public external debt is from the World Bank Global Development Finance database (GDF), and net foreign assets from the updated and extended version of the "External Wealth of Nations" dataset, constructed by Lane and Milesi-Ferreti (2007). Default data is from Borensztein and Panizza (2006) for the 1979-2004 period and from Reinhart and Rogoff (2010) for the 2005-2010 period. A sovereign default is defined as the failure to meet a principal or interest payment on the due date (or within the specified grace period) contained in the original terms of the debt issue, or an exchange offer of new debt that contains terms less favorable than the original issue (a restructuring).

2.2 Stylized Facts

The data we collected yields the following five key observations:

1. Oil exporters have high external sovereign debt ratios and many have defaulted. Figure 1 shows the average external public debt to GDP ratio (in blue) and total public debt to GDP

place. See Appendix A for details.

ratio (in red) for the twenty eight countries for which data is available in our sample.[4] The lowest average external debt ratio is around 5% (Iran) and the largest is around 110% (Syria). Across all countries, the average external public debt to GDP ratio is roughly 39%. Figure 2 shows the number of default episodes—which ranges between zero and 5—for our full set of countries. [5]

Figure 1: Average External Public Debt of Net Oil Exporters (1979-2014)

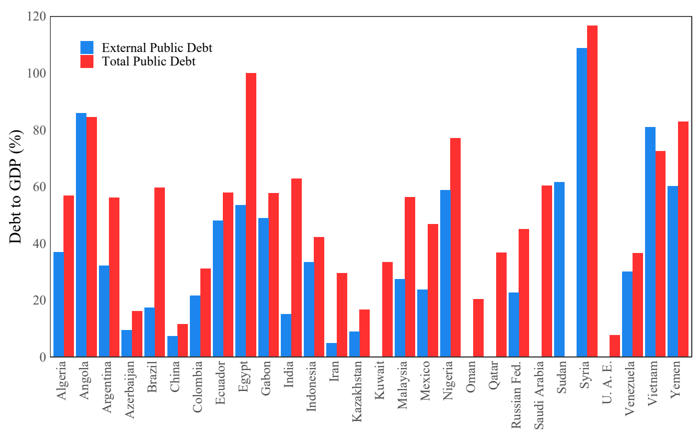
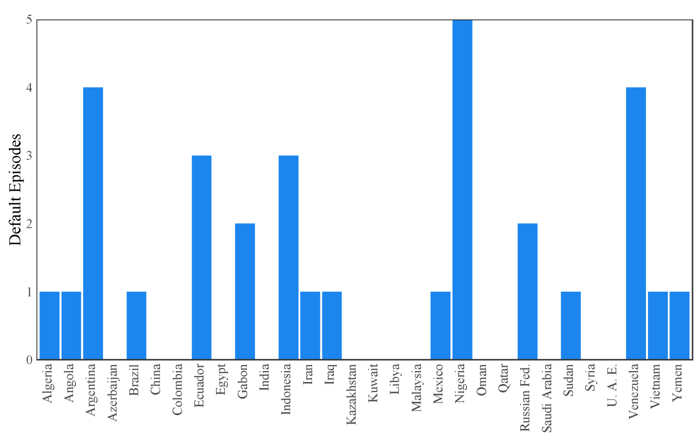


Figure 2: Net Oil Exporters Number of Default Episodes (1979-2010)



2. ***Higher external and total public debt are associated with higher country risk.*** Table 1 shows the unconditional correlation between the III and oil reserves, oil prices, external

[4]Libya and Iraq are droped from the graph due to lack of debt data.

[5]Note that in Figure 2 a value of zero means that the country has not defaulted, it is not a lack of data.

public debt to GDP, and total public debt to GDP for all the countries in our sample. Specifically, both external and total public debt are negatively correlated with the III, implying that they are positively correlated with sovereign risk (see columns IV and V of Table 1). This relationship is statistically significant in nearly all cases.

3. Country risk tends to decrease with oil prices but it has an ambiguous unconditional relationship with oil reserves. Oil prices and the III are positively correlated, (see column (III) of Table 1), while reserves and the III are positively correlated in some countries and negatively correlated in others (see column (II) of Table 1).

4. Higher oil output is associated with lower country risk but higher reserves are not. We established these empirical results by examining the data in two ways. First, by estimating unconditional between-means panel regressions of the III on oil production and on the ratio of oil reserves to production. These are simple OLS regressions estimated with the country full-sample averages of each variable.[6]

Figure 3 shows results for regressing the III on average oil production indicating that, in the long-run, countries that maintained higher mean oil production had a slightly higher average credit rating, with a regression coefficient of 0.28. On the other hand, the regression of the III on the oil reserves to production ratio, shown in Figure 4, indicates that the two variables are not correlated as the regression coefficient is -0.02. However, non of the two regression coefficients are statistically significant.

These between-means regressions have the limitation that they do not separate short- from long-run effects in the dynamic relationship between country risk and oil variables and do not condition for any relevant control variables. Hence, the second approach we followed to study the comovement between country risk and oil variables is based on full dynamic panel regressions.

Notice that our analysis has two dimensions. We want to study the effect of oil production (the flow) versus the effect of oil reserves (the stock) both in the short and the long-run. Thus far, these unconditional correlations point towards two mechanisms. First, extracting more oil (production) increases a country's ability to repay its debt, decreasing country risk.

[6]Reserves to production (i.e. oil extraction) represents the number of years that it would take a country to deplete its reserves assuming that they keep extracting at the same rate and there are no new discoveries.

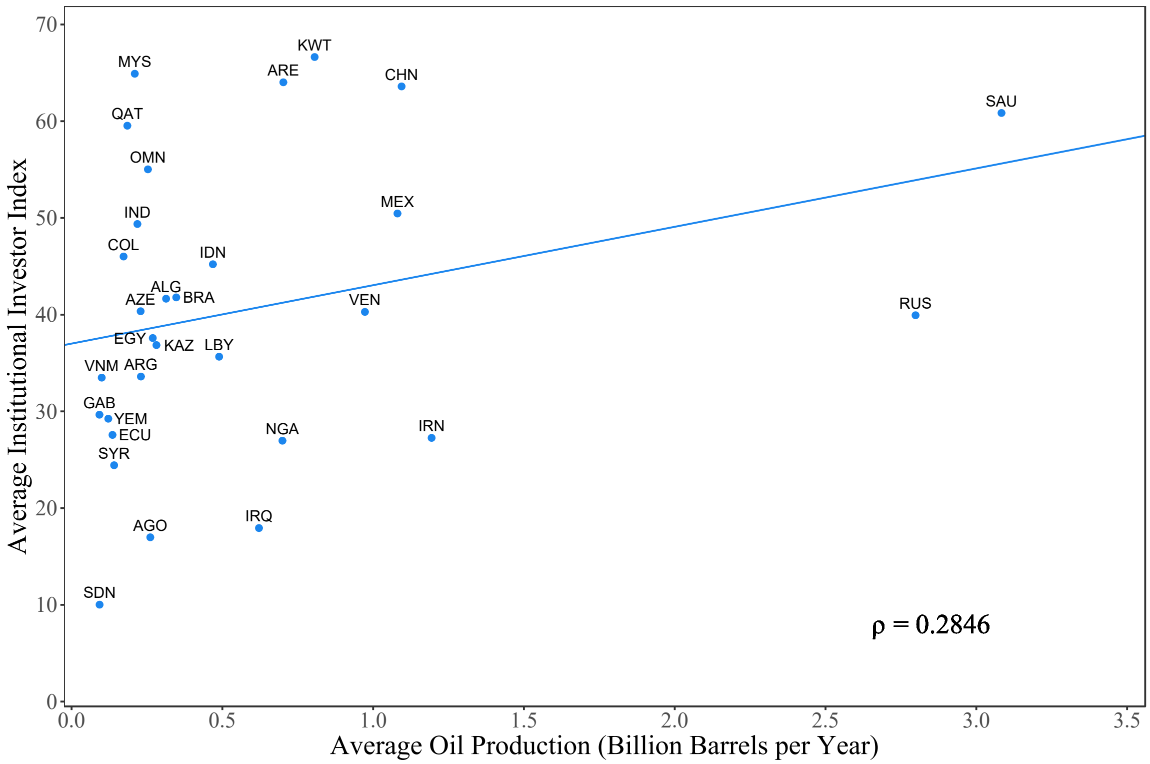
Table 1: Unconditional Correlations with the III

(I) Country	(II) Oil Reserves	(III) Real Oil Prices	(IV) External Public Debt to GDP	(V) Total Public Debt to GDP
Algeria	0.4098**	0.7744***	-0.7498***	-0.7413***
	(0.1588)	(0.0767)	(0.1135)	(0.1225)
Angola	0.8044***	0.8597***	-0.7438***	-0.7002***
	(0.1034)	(0.0619)	(0.1364)	(0.1908)
Argentina	-0.0059	0.3587***	-0.6159***	-0.6353***
	(0.1741)	(0.1132)	(0.1351)	(0.1434)
Azerbaijan	0.5799**	0.9004***	-0.8409***	-0.8994***
	(0.2456)	(0.0907)	(0.1632)	(0.1652)
Brazil	0.8174***	0.8856***	-0.7383***	-0.2157
	(0.1003)	(0.0563)	(0.1157)	(0.1783)
China	-0.5770***	0.8812***	-0.8191***	0.5051***
	(0.1422)	(0.0573)	(0.1014)	(0.1726)
Colombia	-0.1126	0.8565***	-0.8025***	-0.5751***
	(0.173)	(0.0626)	(0.1023)	(0.1519)
Ecuador	0.0083	0.5082***	-0.5903***	-0.6466***
	(0.1741)	(0.1044)	(0.1384)	(0.1393)
Egypt	-0.4703***	0.3351***	-0.6809***	-0.5912***
	(0.1536)	(0.1143)	(0.1256)	(0.1498)
Gabon	-0.3720**	0.6194***	-0.7295***	-0.6806***
	(0.1616)	(0.0952)	(0.1173)	(0.1338)
India	0.0708	0.8168***	-0.8436***	0.1667
	(0.1736)	(0.07)	(0.0921)	(0.1934)
Indonesia	0.2101	0.4709***	-0.5926***	-0.9226***
	(0.1702)	(0.107)	(0.1381)	(0.0704)
Iraq	-0.5215***	0.5946***		
	(0.1508)	(0.0975)		
Iran	0.7418***	0.0991	0.1229	-0.5640***
	(0.1186)	(0.1207)	(0.1782)	(0.1533)
Kazakhstan	0.8081***	0.8809***	-0.5395***	-0.7951***
	(0.1473)	(0.073)	(0.1837)	(0.1516)
Kuwait	0.0469	0.8257***		-0.9062***
	(0.1766)	(0.0684)		(0.113)
Libya	0.4660***	0.7069***		
	(0.154)	(0.0858)		
Malaysia	0.2187	0.7522***	-0.4409***	-0.3123*
	(0.1699)	(0.0799)	(0.1539)	(0.1764)
Mexico	-0.8381***	0.7638***	-0.7459***	-0.7085***
	(0.095)	(0.0783)	(0.1142)	(0.1311)
Nigeria	0.4223**	0.8194***	-0.7410***	-0.6734***
	(0.1578)	(0.0695)	(0.1152)	(0.135)
Oman	0.7066***	0.6547***		-0.4931***
	(0.1232)	(0.0917)		(0.1588)
Qatar	0.9091***	0.8442***		-0.4797**
	(0.0725)	(0.065)		(0.2013)
Russian Federation	0.7410***	0.6909***	-0.7539***	-0.8704***
	(0.1679)	(0.0877)	(0.1434)	(0.1194)
Saudi Arabia	-0.3566**	0.8461***		-0.9083***
	(0.1652)	(0.0646)		(0.1118)
Syria	0.4064**	0.3173***	0.8676**	-0.7369***
	(0.1591)	(0.115)	(0.2224)	(0.1234)
Sudan	0.5003***	0.6235***	-0.4986***	-0.8144***
	(0.1555)	(0.0948)	(0.1487)	(0.1407)
United Arab Emirates	0.3769**	0.7213***		0.3344*
	(0.1637)	(0.084)		(0.1721)
Venezuela	-0.2995*	0.3915***	-0.4065**	-0.7707***
	(0.1687)	(0.1116)	(0.159)	(0.1301)
Vietnam	0.3896*	0.8493***	-0.8134***	-0.7283***
	(0.201)	(0.0805)	(0.1269)	(0.1713)
Yemen	0.4929	-0.4010*	0.4628	-0.5306
	(0.2751)	(0.2048)	(0.2803)	(0.346)

Standard errors in parenthesis

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Figure 3: In between effects regression of the Institutional Investor Index (Y-Axis) on average oil production (X-Axis): 1979-2010.



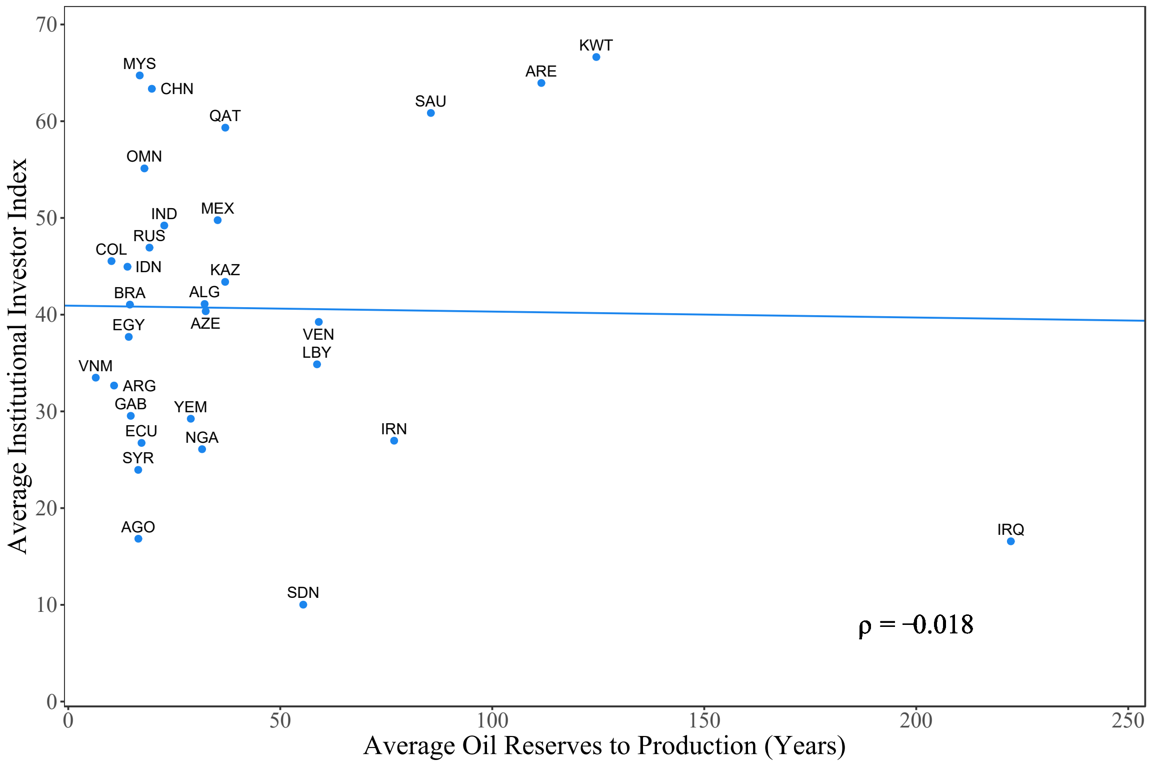
Second, owning a larger stock of oil (reserves) seems to be positively correlated with country risk, and this goes in line with the idea that if a country has a larger stock of a real asset, then financial autarky becomes a more attractive option.

In order to study more formally the presence of these two mechanisms in the data, and establish conditional correlations, we run a dynamic fixed effects estimation[7] of long-run, short-run and convergence coefficients. This allows us to put all the previous results together and be able to establish statistical significance of the relevant variables and timing. The results are shown in Table 2.[8]

[7]See Appendix C for the different methods used for the estimation and the Hausman test that determined that dynamic fixed effects was the dominant approach.

[8] Due to data limitations, Azerbaijan, Kazakhstan, Kuwait, Iraq, Libya, Oman, Qatar, Saudi Arabia, Syria, United Arab Emirates, and Yemen are dropped from the dynamic fixed effects regressions. Consequently, the estimation is performed taking into account 512, 509 and 509 observations in Model 1, 2, and 3 respectively.

Figure 4: In between effects regression of the Institutional Investor Index (Y-Axis) on average oil Reserves to production (X-Axis): 1979-2010.



Note that in each model the convergence coefficient has the expected sign and is statistically significant at the 1% level. Since the estimated coefficients take a value between -0.2 and -0.3, convergence in the III runs at an annual rate of about 0.25%, which means that each year the III covers about 0.25% of its distance to the "steady state." It should also be noted that convergence is slightly slower in Model (2), where the net foreign assets-to-GDP ratio is included and default is excluded.

If we focus on the short-run coefficients, we observe that an increase in oil GDP increases country risk, but this result is not statistically significant. An increase in non-oil GDP decreases country risk, and this result is significant to a 1% level. In the short run, a positive change in oil reserves (which can happen if extraction is lower than discoveries of oil in a given period), decreases country risk. In the second and third models this result is significant to a 10% level. As is expected, increases in external public debt increase country risk,

Table 2: Dynamic Fixed Effects Regression Results for Institutional Investor Index

	Δ Inst. Investor Index		
	Model (1)	Model (2)	Model (3)
Convergence coefficient			
Inst. Investor Index (-1)	-0.252***	-0.224***	-0.257***
	(0.023)	(0.023)	(0.023)
Short-run coefficients			
Δ Oil GDP	0.001	0.002	0.003
	(0.012)	(0.013)	(0.012)
Δ Non-Oil GDP	0.187***	0.204***	0.180***
	(0.061)	(0.062)	(0.061)
Δ Oil Reserves	0.043	0.047*	0.050*
	(0.027)	(0.027)	(0.027)
Δ Ext. pub. debt to GDP	-0.117***	-0.193***	-0.173***
	(0.044)	(0.068)	(0.066)
Δ Oil Discoveries	-0.019	-0.015	-0.019
	(0.020)	(0.020)	(0.019)
Δ NFA		-0.084*	-0.077*
		(0.043)	(0.042)
Long-run coefficients			
Oil GDP	0.089**	0.141***	0.100**
	(0.036)	(0.041)	(0.035)
Non-oil GDP	0.111*	0.126*	0.096*
	(0.059)	(0.068)	(0.057)
Oil Reserves	-0.154***	-0.140**	-0.168***
	(0.046)	(0.054)	(0.046)
Ext. pub. debt to GDP	-0.634***	-0.710***	-0.589***
	(0.092)	(0.176)	(0.151)
Default	-0.296***		-0.296***
	(0.054)		(0.052)
Oil Discoveries	0.184*	0.198*	0.188*
	(0.104)	(0.120)	(0.101)
NFA		0.146	0.095
		(0.125)	(0.104)
Constant	-0.395	-0.768*	-0.357
	(0.425)	(0.428)	(0.421)
Observations	512	509	509

Standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

and this result is statistically significant to a 1% level in all three models. Finally, a positive *change* in net-foreign assets increases country risk.

When looking at the long-run coefficients, as shown in Pesaran et al. (1999), the usual interpretation—when series are in logs—is that of an elasticity. Then, the long-run oil GDP elasticity is 0.09 in the first model, 0.14 in the second, and 0.10 in the third, which means that when oil GDP increases by 1%, the III is between 0.09% and 0.14% higher in the long-run. With respect to non-oil GDP, long-run elasticities are positive, and country risk rating increases around 0.10% to 0.13% because of a 1% increment in non-oil GDP.

Moreover, a significant negative relationship between oil reserves and the III was found. A rise in oil reserves worsens our measure of country risk in the long term. Thus, an oil exporting economy is perceived as more risky in the future when it boosts its reserves today. This elasticity is statistically different from zero at a 5% level for the second model, at a 1% level for the first model, and third model where we control for net-foreign assets, default, and oil discoveries.

As expected, in the long-run, external public debt still has a negative effect on country risk and is again statistically significant to a 1% level for the three models. Different from the short run, in the long run the *level* of net-foreign assets does not matter for country risk. Finally, as expected, being in default increases country risk. When a country is in default, the III drops about 29%. This last result is statistically significant to a 1% level.

The effect of oil discoveries is only significant to a 10% in all three models, where an increase of 1% in oil discoveries, decreases country risk by about 0.19%.

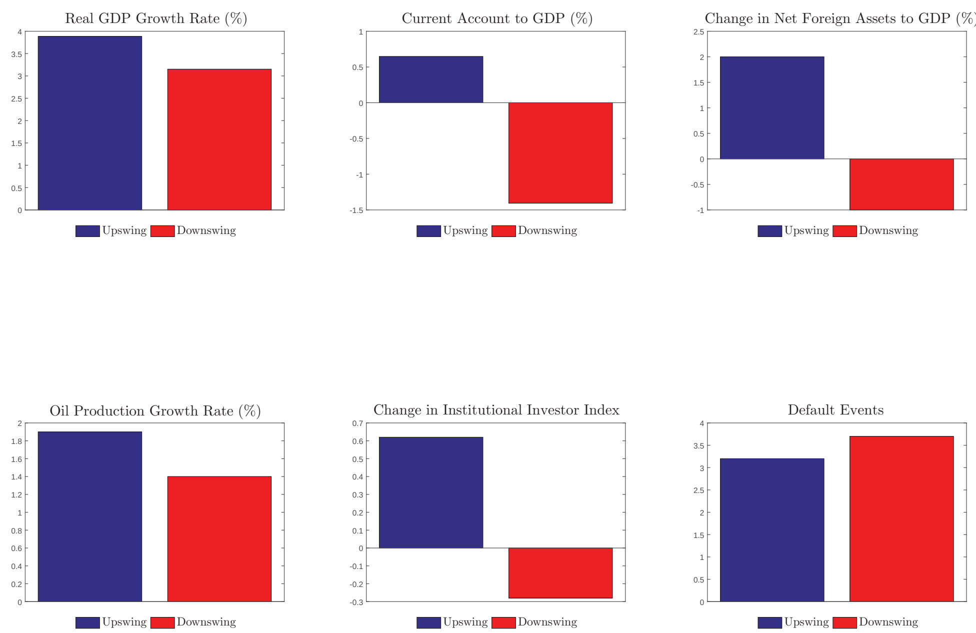
These results support the two mechanisms that we believe are behind the unconditional correlations presented above. Oil production decreases country risk by increasing a country's ability to repay, but greater oil reserves increase country risk by making autarky more attractive.

This results suggest that there is a trade of between the financial asset (public debt) and the real asset (oil). If I have a larger stock of oil, then I can increase production to smooth consumption, rather than borrowing in financial markets. This trade off should rely on the relative yield of the two assets, or in other words, on the price of oil and the price of sovereign debt.

5. Cycles in oil prices are associated with business cycles. This association is illustrated in Figure 5. We divided the panel dataset in two. In one group we have all years where oil prices were increasing (oil-price upswings), and in a second group we have all years where oil prices were decreasing (oil-price downswings). Table D.1 (see Appendix D) shows how each year corresponds to a downswing or an upswing. We then averaged the different macroeconomic variables over the upswings and downswings and Figure 5 shows the results for the relationship between the upswings, downswings, and different macro variables.

Oil price upswings are associated with higher oil extraction and higher GDP growth rates, trade balance and current account improvement, lower sovereign risk perception and lower number of default events. Likewise, oil price downswings are associated with lower oil extraction and lower GDP growth rates, trade balance and current account deterioration, higher sovereign risk perception and higher number of default events.

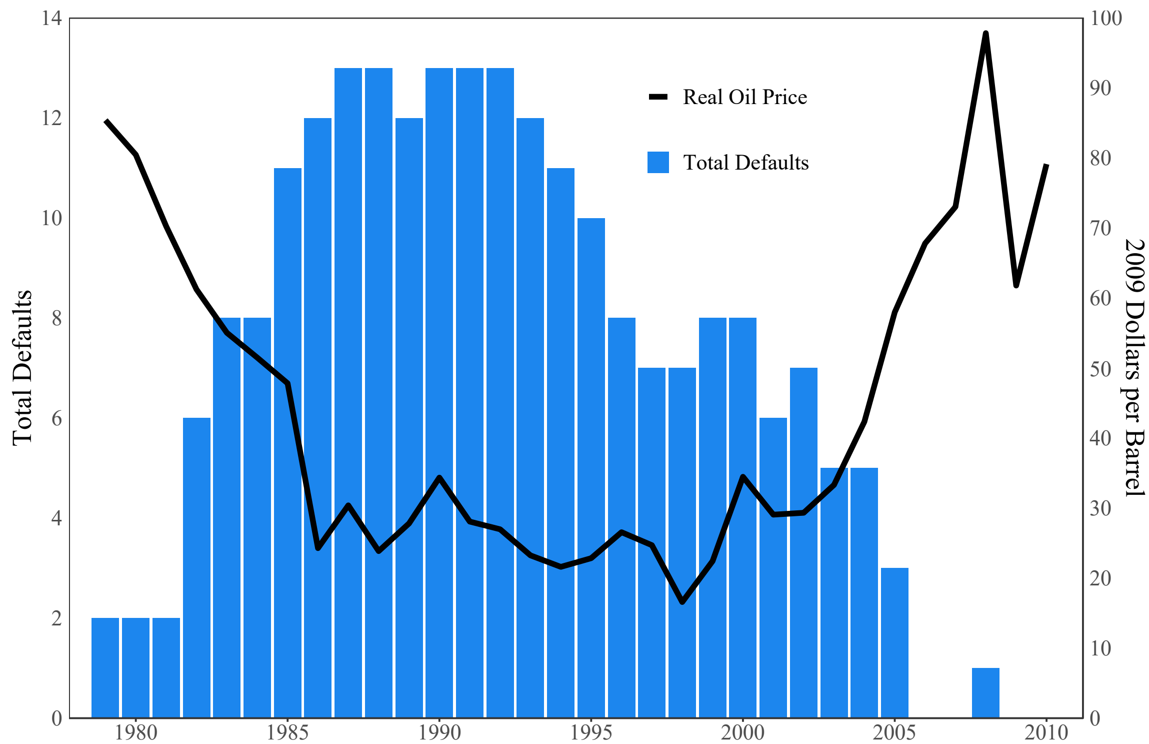
Figure 5: Oil Price Swings and Macro Performance



To expand on the relationship between the price of oil and default episodes, in Figure 6 we plot the total number of default episodes observed in the data by year (left axis) and the evolution of the real oil price over the same years (right axis). We can see that as the price of oil drops the number of default episodes increases. It is particularly interesting to see how between 2005 and 2007 there are no defaults—the price is high—and as soon as the price drops in 2008 a default is triggered. This supports the fact that for these countries, oil increases their ability to repay, and hence lower oil prices are related to more default episodes.

Facts 1, 2, and 5 are important because they highlight the relevance of studying the role of oil in business cycles and the sovereign debt and default dynamics of oil exporting emerg-

Figure 6: Oil Prices and Default Episodes



ing economies (and of commodity producers more broadly), but they are fairly well-known facts.

Facts 3 and 4 are particularly important new facts, because they indicate that oil has two opposing effects on country risk: Higher oil production and/or prices reduce country risk (maybe by increasing a country's ability to repay) but higher oil reserves do not reduce country risk (maybe because they indicate a higher default option value). The task for the remainder of the paper is to determine whether a model of sovereign default that endogenizes oil extraction and reserve accumulation can be consistent with all of these empirical regularities.

3 A Model of Sovereign Default and Oil Extraction

The model we propose is in the class of those based on the work of Eaton and Gersovitz (1981), in which a benevolent social planner cannot commit to repay external debt and chooses optimally whether to default or not. The key difference is that we introduce optimal oil extraction and reserves decisions. The planner owns the oil industry, and thus chooses oil extraction and reserves.[9] This is a nontrivial modification because it implies that the planner now has two vehicles for reallocating resources intertemporally (debt and oil reserves) and can affect the option value of default by altering oil reserves. In addition, the planner's income and repayment capacity are exposed to the risk of oil-price shocks.

3.1 Model structure

There are two types of goods in the model, oil and a tradable, non-storable consumption good. The price of oil relative to the consumption good, p, is stochastic and determined in world markets. Hence, the sovereign is a price-taker in the world oil market. The economy has an exogenous stochastic endowment of the tradable good (non-oil GDP), y, which has an exogenous world-determined price set to 1 without loss of generality. Oil prices and non-oil GDP follow a joint first-order, stationary Markov process with known realization vectors and a transition probability matrix denoted $\pi(p', y'|p, y)$.

Oil is extracted at a cost denominated in units of the consumption good. The cost of extracting x units of oil out of an existing stock of oil reserves s is determined by the extraction cost function $e\left(x, s\right)$, so that oil GDP is $y^O \equiv px - e(x, s)$. The extraction cost function has these properties: $e_s < 0$, $e_x > 0$ and $e_s\left(0, s\right) = 0$. Its functional form is as follows:

$$e(x, s) = \psi \left(\frac{x}{s}\right)^{\gamma} x. \tag{1}$$

Hence, the per-unit extraction cost ($\psi\left(\frac{x}{s}\right)^{\gamma}$) is homogeneous of degree zero in extraction and reserves.

Reserves follow the law of motion $s' = s - x + \kappa$, where κ denotes a constant amount of oil discoveries each period and s' denotes reserves carried over to the next period. Extraction cannot be negative ($x \geq 0$) and cannot exceed the sum of reserves plus discoveries ($x \leq s + \kappa$). Since oil is a form of capital with an endogenous return, it has an asset valuation that we label the "asset price of oil" defined as $q^O \equiv p - e_x(x, s) + \Delta\tilde{\psi}$, where $\Delta\tilde{\psi} \equiv [\psi^l - \psi^h]/u'(c)$ and ψ^l and ψ^h are the multipliers on the lower and upper bounds of x, respectively.[10]

[9] This assumption is in line with the dominant role of state-owned enterprises in commodity extraction and/or exports in many emerging and developing economies.

[10] Appendix F shows that, taking as given a bond pricing function, q^O equals the expected present discounted value (discounted with the sovereign's stochastic discount factor) of the income stream composed of oil "divi-

The world credit market is the same as in standard Eaton-Gersovitz (EG) models. The sovereign sells one-period, non-state contingent discount bonds denominated in units of the consumption good to risk-neutral foreign investors. The sovereign is indebted when its bond position is negative. The outstanding bond position is denoted b and newly issued bonds are denoted b'. The set of feasible bond positions is given by a discrete grid defined over the interval $B = [b_{min}, b_{max}]$where $b_{min} \leq b_{max} = 0$. The sovereign cannot commit to repay the debt. If it defaults, it does not repay b in the current period and is excluded from the credit market in the same period, so no b' can be issued. Next period, the sovereign can re-enter the credit market with probability λ. We also assume that the country can still participate in the oil market during the exclusion period. Hence, the sovereign can still export oil when it defaults. This is important because it implies that the sovereign's plans for the accumulation of oil reserves affect the value of default, since those reserves can be extracted and exported to generate income while access to credit markets remains closed. In contrast, in EG models the value of default is typically exogenous to the government's decisions.

The timing of decisions within a period is as follows: At the beginning of the period, s and b are known. The shocks p and y are realized. The sovereign then decides whether to repay or default on b by choosing the option that yields the highest value, as explained below. If the sovereign defaults, it makes oil extraction and reserves decisions, since the country is excluded from the world bond market but not from the oil market, and pays extraction costs. If the sovereign repays, it sells new bonds b' to foreign investors at the price q, makes extraction and reserves decisions, and pays extraction costs. The resources generated from debt and profits from oil exports are then transferred to households and used for consumption.

The planner's payoff at beginning of the period is:

$$V(b, s, y, p) = \max \left\{ v^{nd}(b, s, y, p), v^{d}(s, y, p) \right\},$$

where $v^{nd}(b, s, y, p)$ is the value of no-default and $v^{d}(s, y, p)$ is the value of default.

The value of no-default is characterized by the following constrained maximization problem:

$$v^{nd}(b, s, y, p) = \max_{\{c, x, b', s'\}} \left\{ u(c) + \beta E \left[V \left(b', s', y', p' \right) \right] \right\} \tag{2}$$

dends", $d^O \equiv -e_s(t) + \psi^h_{t+1}/u'(c_t)$, and the marginal revenue resulting from the effect of accumulating higher oil reserves on the price of debt.

subject to the following constraints:

$$c = y - A + px - e\left(x, s\right) + b - q\left(b', s', y, p\right) b', \tag{3}$$

$$s' = s - x + \kappa, \tag{4}$$

$$0 \leq x \leq s + \kappa. \tag{5}$$

Constraint (3) is the resource constraint, (4) is the law of motion of oil reserves, and (5) represents the feasibility constraints on extraction. In the resource constraint, $q\left(b', s', y, p\right)$ is the pricing function for the risky sovereign bond, which will depend at equilibrium on the choices of bonds and reserves and the realizations of (p, y), and A represents autonomous (exogenous) spending allocated to investment expenditures so that the consumption-GDP ratio can be calibrated later to match the data (consumption will include private and public consumption). Note that we are assuming that extraction costs are factor payments abroad.[11]

The value of default is characterized by the following constrained optimization problem:

$$v^{d}(s, y, p) = \max_{\{c,x,s'\}} \left\{ u(c) + \beta\left(1 - \lambda\right) E v^{d}\left(s', y', p'\right) + \beta \lambda E V\left(0, s', y', p'\right) \right\} \tag{6}$$

subject to the same law of motion of reserves and feasibility constraint as in the no-default case and the following resource constraint:

$$c = y - A + \theta p x - e\left(x, s\right). \tag{7}$$

This resource constraint includes the assumption that there could be an exogenous default cost equal to a fraction $(1 - \theta)$ of oil export revenues. This is a trade penalty that can capture the empirical observation that international trade is negatively affected by sovereign default. We discuss this issue in more detail in Section 4. In the right-hand-side of the value of default (6), the sovereign re-enters credit markets with probability λ and a clean slate of debt ($b' = 0$), and it retains its oil reserves s'. It remains in default with probability $(1 - \lambda)$ but again it retains its oil reserves s'.

For given (b, s), default is optimal for the pairs $\{y, p\}$ for which $v^{d}(s, y, p) \geq v^{nd}(b, s, y, p)$. Hence, the default set is given by:

$$D\left(b, s\right) = \left\{ \{y, p\} : v^{d}(s, y, p) \geq v^{nd}(b, s, y, p) \right\}. \tag{8}$$

[11]This assumption can be relaxed by assuming that a fraction ϕ of extraction costs are domestic factor income. In which case $e(x, s)$ is replaced with $(1 - \phi)e(x, s)$ in the resource constraint.

The default decision rule associated with this default set is given by the function $d(b, s, y, p)$, which takes the value of 1 for $(y, p) \in D(b, s)$ and 0 otherwise (i.e. it equals 1 if the government defaults).

The probability of default one-period ahead conditional on current-period information, $P^d(b', s', y, p)$, can then be induced from the default decision rule and the Markov process of the shocks as follows:

$$P^d(b', s', y, p) = \sum_{y'} \sum_{p'} d(b', s', y', p')\pi(y', p'|y, p). \tag{9}$$

Since foreign investors are risk neutral, sovereign bond prices are determined by the standard no-arbitrage condition:

$$q(b', s', y, p) = q^* \left(1 - P^d(b', s', y, p)\right),$$

where q^* is the price of a risk-free bond such that $q^* \equiv 1/R^*$ where R^* is the world's risk-free gross real interest rate that represents the opportunity cost of funds for foreign investors.

3.2 Model properties

Appendix G includes six propositions that show useful features of the asset price of oil and oil profits, demonstrate that some of the properties of the standard EG model hold in this setup, and characterize the effects of oil reserves and oil-price shocks. Relative to the standard model, obtaining analytic results is more difficult because of the endogeneity of the default payoff on the choice of oil reserves (whereas in most EG models the default payoff is exogenous). As we explain below, this is particularly the case for deriving results related to how reserves affect default risk, what contracts are feasible under repayment when default is possible, and how default incentives respond to y and p shocks.

The propositions rely on three conjectures: 1) Asset prices of oil are non-negative under repayment and default; 2) optimal consumption under repayment is nondecreasing in s; and 3) for (y, p) pairs in the default set when this set is non-empty, the available contracts for new debt and choices of oil reserves under repayment yield a trade balance at least as large as the difference in oil profits between repayment and default.

Since the propositions rely on these conjectures and some of them impose also parameter restrictions (i.i.d shocks, permanent exclusion after default, and no oil-extraction default costs) and provide only sufficiency conditions, we evaluated numerically both the conjectures and the propositions using the calibration specified in the next Section. They all hold in 100 percent of the possible model evaluations that apply to each, except for Conjecture 2 and Proposition 3 for the default payoff which hold 97 and 99 percent of the corresponding evaluations, respectively (see Appendix G for details). We also evaluated the non-negativity of profits included in Conjecture 1 and the trade balance conditions that are part of Propositions 5 and 6.[12] Profits are strictly positive for all optimal decision rules of s' under repayment and default. The trade balance conditions of Propositions 5 and 6 hold only 26 and 32 percent of all model evaluations, respectively. Removing the trade balance conditions, the main results of those propositions, namely that default incentives strengthen at lower y (Proposition 5) or lower p (Proposition 6), hold at 68 and 100 percent, respectively. Thus, although the (sufficiency) trade balance condition of Proposition 6 fails frequently, it is still the case that in the numerical results, lower oil prices *always* strengthen default incentives. This is not the case, however, for lower non-oil-GDP, since even removing the trade balance condition, default incentives strengthen for lower y only in roughly 2/3rds of the evaluations. Hence, the model predicts that defaults do not need to coincide with negative non-oil-GDP shocks.

Proposition 1. The repayment payoff is non-decreasing in b and default sets shrink as b rises (i.e. grow as debt rises)

For all $b^1 \leq b^2$, $v^{nd}(b^2, s, y, p) \geq v^{nd}(b^1, s, y, p)$. Moreover, if default is optimal for b^2 ($d(b^2, s, y, p) = 1$) for some states (s, y, p) then default is optimal for b^1 for the same states (s, y, p) (i.e. $D(b^2, s) \subseteq D(b^1, s)$ and $d(b^1, s, y, p) = 1$) .

This is analogous to Proposition 1 in Arellano (2008). It implies that the country risk premium is non-decreasing in the amount of new debt issued ($q(\cdot)$ is non-decreasing in b').

Proposition 2. If asset prices of oil are positive, oil profits are increasing in s, for given s', and decreasing in s', for given s.

Given Conjecture 1, oil profits under repayment and default are increasing in $s \in [\underline{s}, \overline{s}]$, namely $M_s^{nd}(\cdot), M_s^d(\cdot) > 0$, and decreasing in $s' \in [s+\kappa-s(p/\psi)^{(1/\gamma)}, s+\kappa]$, namely $M_{s'}^{nd}(\cdot), M_{s'}^d(\cdot) < 0$.[13]

This proposition shows that, if the asset prices of oil are positive under repayment and default, the corresponding profits from oil extraction are higher if reserves carried over from the previous period are higher, for a given value of s', and lower if new reserves are higher

[12] We also checked whether the boundary conditions for x (or s') bind and found that they are never binding.

[13] The lower bound of s' follows from assuming oil profits are non-negative. The upper bound is at the point where extraction is set to zero. See Appendix G for details.

(i.e. extraction falls) for a given value of s. We show in Appendix F that positive asset prices of oil are equilibrium outcomes in three variants of the model without default risk (financial autarky and an exogenous bond pricing function set equal to q^* or to a function with the same properties as that of a model with default). The result under financial autarky implies also that $q^{Od}(\cdot) > 0$ in the model with default and $\lambda = 0$.

Proposition 3. The default and repayment payoffs are non-decreasing in s.
For all $s^1, s^2 \in [\underline{s}, \overline{s}]$ and $s^1 \leq s^2$, $v^{nd}(b, s^2, y, p) \geq v^{nd}(b, s^1, y, p)$ and $v^d(s^2, y, p) \geq v^d(s^1, y, p)$.
This result follows from Proposition 2, and demonstrates that one of the conditions needed for the default sets to shrink in b in Proposition 1 (namely that the default and repayment payoffs are non-decreasing in b) also applies with respect to s. This is not sufficient, however, to yield the result that default sets shrink in s, as the next proposition shows.

Proposition 4. Default sets shrink as s rises (i.e. grow as reserves fall).
Assume $\theta = 1$ and $\lambda = 0$ for simplicity. For all $s^1, s^2 \in [\underline{s}, \overline{s}]$ and $s^1 \leq s^2$, if default is optimal for s^2 ($d(b, s^2, y, p) = 1$) for some states (b, y, p), then default is optimal for s^1 for the same states (b, y, p) (i.e. $D(b, s^2) \subseteq D(b, s^1)$ and $d(b, s^1, y, p) = 1$).
This proposition establishes sufficiency conditions under which the result about country risk with respect to the bond position established in Proposition 1 extends to oil reserves. It relies on the three conjectures and Propositions 2 and 3 and establishes that the country risk premium is non-decreasing in the choice of s' (i.e., $q(\cdot)$ is non-decreasing in s'). This result does not follow just from analogy to Proposition 1 (and Proposition 3), because both the repayment and default payoffs vary with s, whereas in the case of b the default payoff does not vary with b. The key to this Proposition is Conjecture 3, which states that, when the default set is non-empty for a given (b, s), the available debt contracts and reserves choices associated with any (y, p) in the default set imply trade surpluses at least as large as the excess of oil profits under repayment over those under default. Intuitively, the net resources that all available debt contracts and reserves choices can generate for consumption under repayment are at most the same as those obtained with the optimal reserves chosen under default.

Proposition 5. If the trade balance is sufficiently large, default incentives strengthen as non-oil GDP falls.
Assuming i.i.d shocks, $\lambda = 0$ and $\theta = 1$, for all $y_1 < y_2$, if $y_2 \in D(b, s)$ and $tb(b^1, s^1, b) \geq M(s^1, s, p) - M(\tilde{s}^2, s, p)$ (where $b^1 \equiv b'(b, s, y_1, p)$, $s^1 \equiv s'(b, s, y_1, p)$ are the optimal choices of bonds and reserves under repayment with y_1 and $\tilde{s}^2 \equiv s^d(s, y_2, p)$ is the optimal reserves choice under

default with y_2), then $y_1 \in D(b, s)$.

This proposition shows conditions under which Proposition 3 in Arellano (2008) holds in this model. It shows that the sovereign has stronger default incentives at lower levels of non-oil GDP when the optimal trade balance under repayment with low y is larger than the difference in optimal oil profits under repayment at that same low y relative to those under default at a higher y. As noted earlier, this trade balance condition holds infrequently in the numerical solution but still the default incentives strengthen as y falls in 94 percent of the state space. In the remainder 6 percent, defaults can occur even if y does not fall.

Proposition 6. If the trade balance is sufficiently large and reserves chosen under default at high oil prices exceed those chosen under repayment at low prices, default incentives strengthen as oil prices fall.

Assuming i.i.d shocks, $\lambda = 0$ and $\theta = 1$, for all $p_1 < p_2$ and $p_2 \in D(b, s)$, if $tb(b^1, s^1, b) \geq M(s^1, s, p_2) - M(\tilde{s}^2, s, p_2)$ and $s^1 \leq \tilde{s}^2$ (where b^1, s^1 are the optimal bonds and reserves choices under repayment in state (b, s, y, p_1) and $\tilde{s}^2$ is the optimal reserves choice under default in state (s, y, p_2), then $p_1 \in D(b, s)$.

This proposition shows sufficiency conditions under which the result in Proposition 5 with respect to non-oil GDP also applies to oil prices (namely that the sovereign has stronger default incentives when p is lower). This Proposition assumes not only a sufficiently large trade balance but also that the oil reserves the sovereign chooses under default at a high p are larger than those it chooses under repayment at a low p. This property holds in all of the state space of the calibrated model. The trade balance condition holds infrequently, but still in the numerical solution we found that default incentives strengthen as oil prices fall in all of the state space.

Summing up, the above theoretical findings indicate that the model preserves the standard properties of EG models with respect to debt and that these extend to oil reserves. In particular, repayment payoffs are nondecreasing in b or s, the bond pricing function is increasing in (b, s) and default incentives are generally stronger at lower y or lower p. The theory also predicts that the default payoff is non-decreasing in s and that optimal oil profits are increasing in existing reserves and decreasing in new reserves (i.e., increasing in extraction). Next, we use these results and the findings from the analysis in Appendix F for the model without default risk to provide an economic intuition of how oil extraction and debt compare in their effects on resources disposable for consumption under default and repayment, and to examine how the dependency of bond prices on debt, reserves and oil prices interact in the formulation of optimal extraction plans.

Consider first the effects of newly issued debt b' and reserves choice s' on resources available for consumption. Combining the constraints for the optimization problem under repayment yields:

$$c = y - A + p(s+\kappa) - ps' - e\left(s', s\right) + b - q\left(b', s', y, p\right) b', \tag{10}$$

where we replaced x with s' as an argument of $e(.)$. Note that, since $e(.)$ is increasing in x and x decreases with s', $e(.)$ is decreasing in s'. The above expression shows key similarities and differences faced by the sovereign in the choice of b' v. s' for reallocating consumption intertemporally. By borrowing more (reducing b'), the government alters resources for current consumption according to the familiar debt Laffer curve of EG models.[14] Reducing s' is akin to borrowing in that it increases resources for consumption by the amount by which $-(ps' + e(s', s))$ rises. In contrast with debt, however, there is no Laffer curve when "borrowing with reserves." Conditional on not hitting the feasibility boundaries of extraction, lower s' always increases resources available for consumption.[15] Borrowing with s' also differs from b' in that it alters resources in the default state, by increasing them by the amount $-(\theta ps' + e(s', s))$ as s' falls (i.e., at a lower rate than under repayment).[16]

Debt and reserves also have similarities and differences in how outstanding debt b and existing reserves s affect resources for current consumption. They are similar in that arriving at the repayment state with more debt (lower b) reduces resources by the amount b, while arriving with fewer reserves reduces resources by the amount ps. But they differ in that the debt repayment is non-state-contingent while the resources provided by s vary with p. It is often noted in the sovereign debt literature that debt has poor hedging properties because it does not reduce the burden of repayment in "bad" states of nature (i.e., the repayment is uncorrelated with total income), but oil reserves are *worst* in this regard because the resources

[14]When b' is low so that default risk is low or zero, additional debt always gains resources for consumption, because bond prices fall little or stay at q^*, but as debt rises enough for default risk to reduce $q(\cdot)$ sufficiently, additional debt results in fewer resources for consumption.

[15]Note that $\partial c/\partial s' = -p - e_{s'}\left(s', s\right) = -(p - e_x\left(x, s\right)) < 0$ because $q^{Ond} > 0$ implies that $p - e_x\left(x, s\right) > 0$ for an interior solution of x (see Appendices F and G). Hence, borrowing with reserves always increases resources for consumption because the asset price of oil is positive.

[16]In the default state, $\partial c/\partial s' = -\theta p - e_{s'}\left(s', s\right) = -(\theta p - e_x\left(x, s\right)) < 0$ because $q^{Od} > 0$ implies that $\theta p - e_x\left(x, s\right) > 0$ for an interior solution of x (see Appendices F and G).

they provide correlate positively with oil prices (i.e., they provide fewer resources at lower p). Hence, viewing b and s as assets for hedging income fluctuations, reserves are inferior to debt. Moreover, the sovereign can default on b to reduce the debt burden ex-post.

Qualitatively, debt and oil reserves have similar effects on conditional default probabilities and default risk. With regard to debt, Proposition 1 established that, as in EG models, default sets shrink with b and as a result the conditional probability of default and default risk are non-decreasing in debt. Thus, $q(.)$ is non-decreasing in b'. On the side of oil reserves, Proposition 4 showed that default sets also shrink with s and thus the conditional probability of default and default risk are non-decreasing in reserves. Thus, $q(.)$ is non-decreasing in s'. The rationale is that, although in the case of oil reserves the default payoff is increasing in s instead of constant, the repayment payoff grows more than the default payoff as s rises. Notice these are short-term or contemporaneous effects that refer to how country risk at date t responds to the sovereign choosing to increase debt or reduce reserves at t.

Next we examine the interaction between sovereign risk and the sovereign's optimal oil extraction plans. For simplicity, so that we can conduct the analysis with familiar no-arbitrage conditions in sequential form, assume that we give to a sovereign who is committed to repay the model's equilibrium bond pricing function, $q\left(s_{t+1}, b_{t+1}, y_t, p_t\right)$, assuming that it is differentiable and satisfies other regularity properties.[17] The optimality conditions of the sovereign's problem yield the following no-arbitrage condition between the expected return on oil and the return on sovereign bonds (see Appendix F):

$$E_t\left[\tilde{R}^o_{t+1}\right] = R^b_{t+1}\left(s_{t+1}, b_{t+1}\right) - \frac{\operatorname{cov}_t\left(u'\left(c_{t+1}\right), \tilde{R}^o_{t+1}\right)}{E_t\left[u'\left(c_{t+1}\right)\right]}. \tag{11}$$

In this expression, $R^b\left(s_{t+1}, b_{t+1}\right) \equiv \frac{1}{q(t+1)+q_b(t+1)b_{t+1}}$ is the sovereign's gross return on bonds. Since we are assuming commitment, there is no default risk, but because $q(\cdot)$ is the equilibrium pricing function of the model with default, the planner internalizes that higher debt carries a higher interest rate than R^* (since $q_b(\cdot) > 0$). Also, since debt is non-state-contingent,

[17] We assume that $q\left(\cdot\right)$ is strictly concave and increasing in b_{t+1} for $b_{t+1} \in [-\overline{b}(s_{t+1}), 0]$, where $-\overline{b}(s_{t+1})$ is the threshold debt above which default is certain for a given s_{t+1} (i.e., $D(\overline{b}(s_{t+1}), s_{t+1})$ includes all (y_{t+1}, p_{t+1}) pairs, which exists because of Proposition 1), with $q\left(\cdot\right) = q^*$ for $b_{t+1} \geq 0$ and $q\left(\cdot\right) = 0$ for $b_{t+1} \leq \overline{b}(s_{t+1})$. $q\left(\cdot\right)$ is also increasing and concave in s_{t+1} for $s_{t+1} \in [\tilde{s}(b_{t+1}), s_t + \kappa]$, where $\tilde{s}(b_{t+1}) = max[s_t + \kappa - s_t(p_t/\psi)^{(1/\gamma)}, \overline{s}(b_{t+1})]$ and $\overline{s}(b_{t+1})$ is the threshold oil reserves below which default is certain for a given b_{t+1} (i.e., $D(b_{t+1}, \overline{s}(b_{t+1}))$ includes all (y_{t+1}, p_{t+1}) pairs, which exists because of Proposition 4). We also assume that $\overline{b}(s_{t+1})$ is increasing in s_{t+1} and $\overline{s}(b_{t+1})$ is increasing in b_{t+1}.

the Euler equation for bonds implies that at equilibrium $R^b_{t+1}(s_{t+1}, b_{t+1}) = \frac{u'(c_t)}{\beta E[u'(c_{t+1})]}$. The term $\tilde{R}^o_{t+1} \equiv \frac{q^O_{t+1}+d^O_{t+1}}{\left[q^O_t + q_s(t+1)b_{t+1}\right]}$ is the sovereign's gross return on oil inclusive of the financial benefit of higher reserves increasing resources available for consumption by rising the price of newly-issued debt. This rate of return can be rewritten as $\tilde{R}^o_{t+1} \equiv R^o_{t+1}\left[\frac{1}{1+q_s(s_{t+1},b_{t+1})b_{t+1}/q^O_t}\right]$, where $R^o_{t+1} \equiv \frac{q^O_{t+1}+d^O_{t+1}}{q^O_t}$ is the "physical" return on oil and $\left[\frac{1}{1+q_s(s_{t+1},b_{t+1})b_{t+1}/q^O_t}\right]$ is the financial return from higher reserves increasing $q(\cdot)$.

Condition (11) implies that the sovereign's optimal extraction and reserves plans are set so that the total marginal gross return on the oil it extracts exceeds the full marginal cost of its liabilities by a premium equal to $-\frac{\text{cov}_t\left(u'(c_{t+1}),\tilde{R}^o_{t+1}\right)}{E_t[u'(c_{t+1})]}$. This is akin to a standard equity premium, with the caveat that both the return on oil and the return on bonds include financial components. The former (latter) because of the effect of lower oil reserves (higher debt) reducing the price of sovereign debt (increasing the interest rate).

Appendix F examines the implications of condition (11) in two other scenarios: (i) permanent financial autarky (which is the same as the solution of the default payoff if $\lambda = 0$) and (ii) a constant bond price set at $q = q^*$ (which renders the model akin to a small-open-economy RBC model).

Under financial autarky, the model resembles a canonical closed-economy RBC model, in which condition (11) reduces to $E_t[u'(c_{t+1})\,R^O_t] = u'(c_t)$. Hence, the planner uses oil reserves in a manner akin to capital accumulation in the closed-economy RBC model. Markets are incomplete because there are no assets to insure away the risk of the shocks to p and y. Thus, the planner self-insures with reserves so as to facilitate consumption smoothing. There is also an implicit endogenous domestic real interest rate represented by the stochastic marginal rate of substitution in consumption. In the model with default, the planner has a similar incentive in the default state: being excluded from credit markets, it will use reserves to facilitate consumption smoothing, except that, because $\lambda > 0$ it assigns some probability to being able to re-enter the credit market.

In the case with $q = q^*$, condition (11) reduces to $E_t\left[R^o_{t+1}\right] = R^* - \frac{\text{cov}_t\left(u'(c_{t+1}),R^o_{t+1}\right)}{E_t[u'(c_{t+1})]}$ which is analogous to the one obtained in small-open-economy RBC models for the excess return on physical capital. Markets are again incomplete, but here the sovereign has access to no-state-contingent bonds for self-insurance and consumption smoothing. Oil is a risky asset and carries a risk premium, but the returns on oil and bonds and the risk premium do not include the financial terms due to the effects of debt and reserves on the price of bonds. Moreover, since the risk premium is small (as is typical in RBC models), the model is close to yielding the Fisherian separation of the extraction and reserves plans from the savings and consumption plans that holds strictly without uncertainty. We show in Appendix F that the no-arbitrage condition without uncertainty becomes $R^o_{t+1} = R^*$ and yields a second-order difference equation in s that determines the extraction and reserves decision rules independently of the bonds and consumption decision rules.

In the model with default, since default is infrequent quantitatively, when debt and/or reserves (and the history of oil-price and non-oil GDP shocks) are such that the probability of default becomes positive only in the distant future, the dynamics of oil extraction and reserves will display similar features. The model will behave in a manner similar to a canonical small-open-economy RBC model. One important prediction of this model is that, when oil prices are low, and therefore expected to rise due to mean-reversion, the planner has the incentive to cut extraction and increase reserves. To see this, use the definitions of the asset price of oil and oil dividends to rewrite the no-arbitrage condition $R^o_{t+1} = R^*$ as follows (assuming an internal solution for x_t for simplicity):

$$\frac{p_{t+1} - e_x\left(x_{t+1}, s_{t+1}\right) - e_s\left(x_{t+1}, s_{t+1}\right)}{p_t - e_x\left(x_t, s_t\right)} = R^*. \tag{12}$$

Since $e(\cdot)$ is increasing in x_t and decreasing in s_t, when p_t falls relative to p_{t+1}, the planner reallocates extraction from t to $t+1$ by increasing s_{t+1}. This is a key incentive that is also a work in the model with default, but there it interacts with the planner's incentives to default and to affect the price of issuing new debt by adjusting reserves. As Propositions 4 and 6 show, the incentives to default at date t are stronger when p_t is low but, if the sovereign chooses not to default, the incentive to increase s_{t+1} in response to lower p_t *reduces* the default risk premium paid on bonds sold at t (i.e., increases the price of newly issued bonds) because default sets shrink with s.

4 Quantitative Analysis

4.1 Calibration

We calibrate the model to Venezuela using annual data. The model has nine structural parameters: the discount factor, β, the coefficient of relative risk aversion, μ,[18] the price of the risk-free bond, q^*, the trade penalty incurred when in default, θ, the probability of reentering financial markets when in the default state, λ, discoveries, κ, a constant amount of autonomous spending, A (that accounts for investment to make the model's national accounts to be consistent with the data), and the parameters γ and φ which determine extraction costs.

The values of μ and q^* are set to standard values in the literature: $\mu = 2$ and $\bar{r} = 0.00775$. The latter corresponds to the average ex-post, US-CPI deflated yield on a 3-month U.S. Treasury bill for the 1955-2014 period (see Bianchi, Liu and Mendoza (2016)).

We set the probability of reentry at $\lambda = 0.332$ for a yearly frequency, according to Dias and Richmond (2007) who using a sample of 128 episodes of sovereign default on foreign currency bank debt and foreign currency bonds during the period 1980-2005, find that the median period of exclusion for partial financial market of a country is three years after default.

Rose (2005) estimates the effect of sovereign debt renegotiations on international trade, and finds that debt renegotiation is associated with an economically and statistically significant decline in bilateral trade between a debtor and its creditors of 8% of GDP per year, and that persists for about fifteen years. Based on this finding, we set the fraction of oil that can be exported while the economy is in default to $\theta = 0.47$. Because oil GDP represents 15% of total GDP for Venezuela, imposing a $\theta = 0.47$, it means that the trade penalty costs the country 80% of its oil output, 8% of total output.

The autonomous spending parameter, A, was set such that $A = 1 + rb - c$, where b and c are the mean external debt to GDP ratio and the mean private plus public consumption share in GDP respectively, for the period 1989-2016. The value of r is set to $\bar{r} + spread$, where $spread$ corresponds to the average value of JP Morgan's EMBI+GSS spread for the period 1979-2016 (777 bp). Since the JP Morgan data starts in 1998 but we have the Institutional Investor Index for a longer sample, we estimate a linear regression between the Institutional

[18]The utility function is assumed to be $u(c) = \frac{c^{1-\mu}}{1-\mu}$

Investor Index and the EMBI for the period 1998-2016, and then use the regression and the observed values of the Institutional Investor Index to recover the Venezuelan EMBI spread for the period 1979-1997.

We set extraction equal to discoveries in the deterministic steady state such that $x = \kappa$, and normalize oil prices $p = 1$. We also normalize total GDP to one, $y + px = 1$, such that the share of oil production in GDP is $\frac{px}{y+px} = \kappa = 0.15$ to match the share of oil production in Venezuela's GDP which is 15%.

We calibrate β and γ to match as closely as possible two moments in the data: the standard deviation of oil GDP (37%) and a 28% ratio of external debt stocks (public and publicly guaranteed) to GDP for the period 1992-2015. We find $\beta = 0.75$ and $\gamma = 2.5$.

The remaining parameter, φ, is set such that in the deterministic steady-state $\varphi = n^{\gamma}/(1+\gamma-(1/(1+r))\gamma/n)$, where n is the ratio of Venezuelan oil reserves to production (in years) for the period 1980-2010. In accordance with the US Energy Information Administration (EIA) we set $n = 59$ years. Table 3 summarizes all the parameter values mentioned above.

Table 3: Summary of Parameters

Parameter	Description	Value
β	discount factor	0.75
μ	risk aversion	2
q^*	risk-free debt price	0.99
θ	trade penalty	0.47
k	discovery rate	0.15
λ	redemption probability	0.33
γ	curvature extraction cost	2.5
A	rescaling parameter	0.37

There are two exogenous processes. One for non-oil output and another one for the price of oil. We estimate a VAR for these two variables using annual data for the real Brent price of crude oil (average of the daily price) and the real GDP (excluding oil rents) as a proxy of our non-oil endowment for the period 1979-2016. The estimated VAR(1) model for p_t and y_t, imposing the restriction that Venezuelan non-oil GDP does not impact international oil prices, is:

$$\begin{bmatrix} p_t \\ y_t \end{bmatrix} = \begin{bmatrix} \rho_p & 0 \\ \rho_{py} & \rho_y \end{bmatrix} \begin{bmatrix} p_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma_p & \sigma_{yp} \\ \sigma_{py} & \sigma_y \end{bmatrix} \begin{bmatrix} \epsilon_{pt} \\ \epsilon_{yt} \end{bmatrix},$$

where ϵ_{pt} and ϵ_{yt} are mean-zero, i.i.d. random variables. The diagonal of the estimated covariance matrix of the innovations is a matrix with variances σ_p^2 and σ_y^2. Table 4 shows the results of the estimation.

Table 4: VAR Process for Non-Oil Output and Oil Prices

Parameter	Description	Value
ρ_p	oil price auto-correlation	0.5
ρ_y	non-oil output auto-correlation	0.94
ρ_{py}	oil price non-oil output correlation	0.17
σ_p^2	variance oil price	0.064
σ_y^2	variance non-oil output	0.007
σ_{py}, σ_{yp}	covariance non-oil output, oil price	-0.01

4.2 Quantitative Findings

We start the analysis of the quantitative results by comparing the behavior of the model generated data with that of the observed one, by comparing the paths followed by the two, given observed non-oil output and oil prices, and by comparing the correlation between country risk and other variables for the model generated data in the same way we did with the observed data in the empirical part. In Subsection 5.2 we show the relevance of having an endogenous extraction decision, and Subsection 5.3 discusses the default sets and studies the default costs generated by the model.

4.3 Model versus data

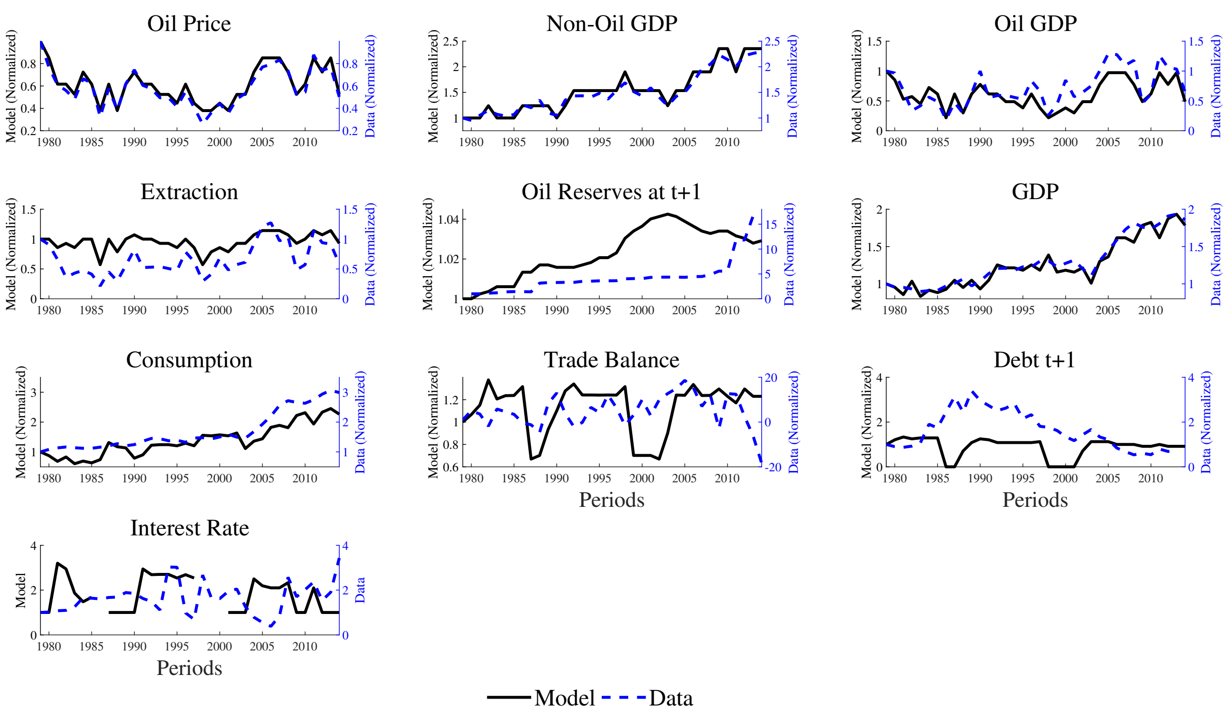
In this subsection we start by comparing the model generated data with the observed data. To do so, we take realized non-oil output for Venezuela and oil prices (BRENT) and feed them into the policy functions for the different variables. Figure 7 shows the results. The black lines (left axis) represent the model generated data, and the dashed blue lines (right axis) represent the observed data.

Note that the difference in the oil price and non-oil GDP plots between the model and the data is a result of discretizing the stochastic process for this two variables. It is important to note that the model does a good job at generating an extraction that is qualitatively very similar to that in the data, and because the oil price is as in the data, the oil GDP generated by the model is very close to the observed one. GDP is the sum of non-oil GDP and the oil price times extraction. As a results it is not surprising that it looks so similar to the data. All of these rely on the fact that the model does a great job at replicating oil extraction. Oil reserves are increasing both in the model and in the data, and the qualitative discrepancies are coming from the fact that in the model discoveries are constant while in the data they are not. The behavior of consumption is also very similar to the data.

In the model the behavior of debt will depend directly on the default episodes given that in the model there is full financial exclusion during default periods. But it can be seen from the figure that broadly speaking, debt in the model and in the data, increases in between default episodes.

The interest rate in the data is calculated as the EMBI (Emerging Markets Bond Index) spread. That is, the difference between the risk free rate—represented by the the interest rate on 90 day treasury bills—and the EMBI rate. We want to highlight that even though in some years, like around 1992, and 2005, the model doesn't seem to line up that well with the data, what is really remarkable is that Venezuela defaulted in 1983, 1990, 1995, and 2004, and the model generates an increase in interest rates—reflecting an increase in sovereign risk—in exactly those years.

Figure 7: Model versus Data



To tie down our theoretical results with the empirical ones presented in Subsection 2.2, we run the same regression we did before, but instead of being a panel, its run for a single country i.e. Venezuela, and another for the model generated data. To run the regression on the model generated data, we simulate data for 10,000 periods using the policy functions of the model, and bootstrap thirty year windows to run the same regression shown in Table 2. Remember that the measure of country risk used in the data is the Institutional Investor Index (III). Off course that in the model we do not have such an object, so for the III counterpart in the model we use the price of the risky bond. There is also the detail that in the model the price of the risky bond is zero when there is a default, so whenever the bond takes a value of zero we fix it to the value implied by the highest spread observed in Venezuela for the time period comprised by our data. In this way we can also control for the default dummy. Finally, in the model regression we cannot control for discoveries because they are constant. Everything else is the same as in Subsection 2.2.

The results comparing the regression coefficients for the model generated data, just Venezuela and the full panel dynamic fixed effects are shown in Table 5. Note that a positive coefficient implies a negative relationship with country risk. In other words, a positive coefficient means that the control variable increases either the price of the risky bond (in the case of the model) or increases the III (in the case of the data), implying a lower sovereign risk. Likewise a negative coefficient implies an increase in country risk.

What we are looking for with this exercise is to see if the correlations go in the same direction in both model and data. In other words we care about the signs of the coefficients. As was highlighted in Section 2, the interesting result about the data is that oil GDP is associated with a lower sovereign risk (because it increases a country's ability to repay their sovereign debt), but having a larger stock of oil reserves is associated with a higher sovereign risk, which we speculated before might be due to a limited commitment mechanism, where underground oil reserves increase the value of autarky, decreasing the need to access international financial markets. There is a substitution effect between the financial asset (debt) and the real asset (oil). Table 5 shows that indeed, the model generated data exhibits the same relationships between sovereign risk and oil production and oil reserves as observed in data. Oil GDP has a positive coefficient in the model, when the regression is run for only Venezuela, and for the entire DFE panel regression, and oil reserves have a negative coefficient in the three cases. This result implies that the mechanisms underlying the model, serve

Table 5: Model Short versus Long-run Coefficients

	Δ Inst. Investor Index					
	Model		Venezuela		Data	
	Model (1)	Model (2)	Model (1)	Model (2)	Model (1)	Model (2)
Convergence coefficient						
Inst. Investor Index (-1)	-0.996***	-1.030***	-0.624***	-0.634***	-0.249***	-0.251***
	(0.002)	(0.002)	(0.174)	(0.178)	(0.023)	(0.023)
Short-run coefficients						
Δ Oil GDP	-0.014***	-0.003**	-0.059	-0.078	-0.0001	0.00
	(0.003)	(0.001)	(0.073)	(0.089)	(0.012)	(0.012)
Δ Non-Oil GDP	-0.005***	-0.014***	0.167	-0.025	0.193***	0.190***
	(0.001)	(0.001)	(0.373)	(0.442)	(0.061)	(0.061)
Δ Oil Reserves	0.044***		0.144	0.180	0.039	0.042
	(0.009)		(0.112)	(0.119)	(0.027)	(0.027)
Δ Ext. pub. debt to GDP	-0.005**	0.017***	-0.337	-0.498	-0.114**	-0.115***
	(0.002)	(0.002)	(0.293)	(0.340)	(0.045)	(0.045)
Δ Oil Discoveries				-0.030		-0.018
				(0.108)		(0.020)
Long-run coefficients						
Oil GDP	0.039***	0.064***	0.317***	0.350***	0.096***	0.094**
	(0.003)	(0.001)	(0.092)	(0.101)	(0.037)	(0.037)
Non-oil GDP	0.006***	0.012***	0.676**	0.622*	0.112*	0.105*
	(0.001)	(0.001)	(0.265)	(0.370)	(0.060)	(0.059)
Oil Reserves	-0.0003		-0.553***	-0.546***	-0.146***	-0.152***
	(0.0003)		(0.123)	(0.154)	(0.047)	(0.047)
Ext. pub. debt to GDP	-0.022***	0.018***	0.278	0.235	-0.635***	-0.644***
	(0.001)	(0.001)	(0.253)	(0.302)	(0.093)	(0.093)
Default	-0.400***	-0.400***	-0.164***	-0.192***	-0.283***	-0.283***
	(0.001)	(0.001)	(0.051)	(0.059)	(0.054)	(0.054)
Oil Discoveries				0.190		0.179*
				(0.283)		(0.105)
Constant	-0.020***	-0.039***	-12.828*	-12.629	-0.447	-0.383
	(0.003)	(0.001)	(6.707)	(7.838)	(0.425)	(0.427)
Observations	8,816	8,912	30	30	515	515

Standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

to understand this pattern that is observed in the data.

We explore these mechanisms in the following subsections to understand the economic intuition behind our results.

4.4 Why does endogenous extraction matter?

To better understand the mechanisms behind the model and the data, we start by looking at the comparison between the moments of the observed data and those of the data generated by the model.

Table 6 compares the average external debt to GDP, the standard deviation of oil GDP, average reserves in years and the default rate in the data, the benchmark model (described in Section 3), a specification of the model with constant oil extraction, and a specification of the model where there is no default in equilibrium (i.e. the risk free case). We include the results for the latter two cases in order to isolate the effect of endogenous extraction, and the importance of allowing for default within the model.

As explained in Section 4, the mean debt ratio and the standard deviation of oil GDP[19] were used as calibration targets, so the model comes close to matching those data moments (a mean debt ratio of about 28% of GDP and a standard deviation of oil GDP of 37%). The rest of the moments shown in the table are not targeted, and as such, can be contrasted with those of the data to gauge the model's ability to replicate it.

The model generates average reserves in years of 62, while in the data this number is 59. This number represents how many more years the country has left extracting at their historical rate in the absence of any new discoveries. Finally, the default rate for Venezuela is 4% while in the model it is 1%.

If extraction is fixed constant, the model will underestimate the volatility of oil GDP, and if there is no default risk, the model will over estimate the equilibrium average external debt to GDP, as would be expected. Finally, and most importantly, the default rate under constant extraction is lower that with endogenous extraction. We elaborate on this point in the next subsection.

Table 6: Data vs Model Moments

Description	Data	Model		
		Benchmark	Constant Extraction	Risk Free
Average External Debt to GDP	0.28	0.26	0.28	0.36
Standard Deviation of Oil GDP	0.37	0.39	0.28	0.38
Average Reserves (in years)	59	62	62	61
Default Rate	0.040	0.009	0.006	NA

Estimates of the proven reserves for Venezuela correspond to those of the US Energy Information Administration.

Table 7 shows the comparison between other volatilities and correlations. The correlation between oil GDP and total GDP is 5% in the data and 47% in the model. The volatility of the overall GDP is 6% in the data and 8% in the model.

Total consumption (including government) has a correlation with total GDP of 83% in the data and in the model it is 95%, and its volatility in the data is 7% and 8% in the model.

[19]Oil GDP in the data is defined as the share of oil rents times real GDP, and in the model it is given by px. Both observed and model generated data are filtered using the Hodrick and Prescott filter with a 6.25 lambda.

Table 7: Summary of Comovements - Venezuela vs. Model

1979-2016. HP filter Log-Data*

	Std. Dev.				Correlation vs GDP			
	Data	Benchmark	Constant Extraction	Risk Free	Data	Benchmark	Constant Extraction	Risk Free
Oil GDP	0.37	0.39	0.28	0.38	0.05	0.46	0.31	0.46
Total GDP	0.06	0.08	0.07	0.08	1.00	1.00	1.00	1.00
Total Consumption	0.07	0.08	0.08	0.8	0.83	0.95	0.97	0.99
Trade Balance	0.07	0.08	0.08	0.08	-0.48	-0.39	-0.37	-0.45
Spread	359	61	51	0	-0.01	-0.34	-0.05	0.00

* Except TB which is relative to GDP and EMBI which is in levels (basis points).

The model does very well in terms of the cyclicality and volatility of the trade balance. It has a standard deviation of 7% in the data compared to 8% in the model, and in the data the correlation of the trade balance to GDP is -0.48 versus -0.39 in the model.

For the spread we use the EMBI spread calculated as described in the previous subsection. The model generates a spread that is less volatile than in the data and more countercyclical. This is not surprising as our model exhibits the usual feature of sovereign default models where the price of the risky bond exhibits a ladder like pattern.

When comparing the moments of the benchmark model generated data, with those of the data generated by the model with constant extraction or the risk free model, one can see that they are similar in some dimensions but different in others. The model seems to exhibit some sort of fisherian separation, where consumption decisions are independent of oil extraction decisions. In other words, the moments of total consumption, total GDP and trade balance are similar among the three specifications. However, note that having endogenous extraction and/or allowing for default makes considerable difference for the moments of oil GDP and the spread.

To explore further the importance of having endogenous extraction and risky bonds, we study the behavior of default episodes within the model. To do so, we simulate data over ten thousand periods using our benchmark specification and identify all the default episodes. We take ten periods before and after each of these episodes and take the median for each corresponding period. To find the path in the constant extraction and risk free models, we take the realization of the oil price and non-oil GDP in each default episode of the benchmark case (starting ten periods before and going up to ten periods after) and feed them into the policy functions of the model with constant extraction and the risk free model to recover the paths followed by the different variables in those two specifications. Figure 8 shows the comparison of the dynamics around default episodes for the three models. The solid line represents the benchmark, the dashed line the model with constant extraction, and the dotted line the risk free model. The default occurs in period zero.

Before the default there is a fairly stable debt to GDP ratio, a slightly increasing interest rate, and most importantly an increase in the size of oil reserves. Before the default, prices are around average and the default is triggered by a drop in oil prices of one standard deviation coupled with an increase in non-oil GDP.

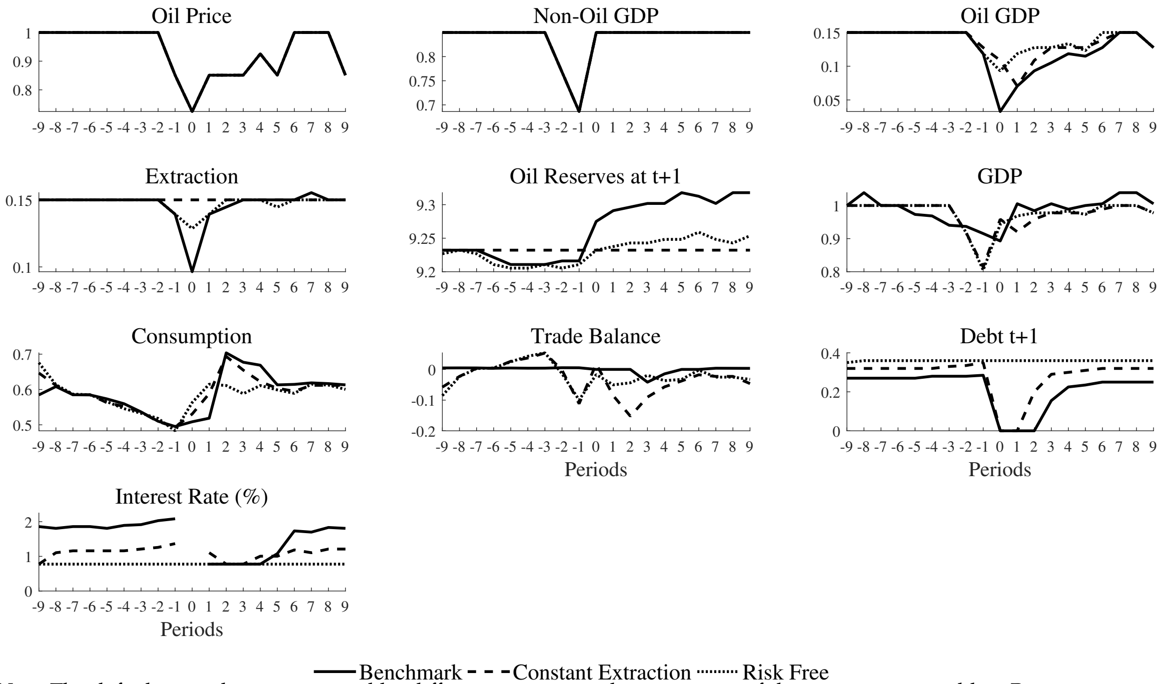
At the time of default, extraction falls as today's price is lower than expected future prices, generating a further increase in reserves, and consumption increases because no debt needs to be repaid, non-oil output increased, and although extraction goes down, so do extraction costs.

If we compare the path followed by the variables under the risk free case with the benchmark, we can see that the main difference is in terms of the behavior of debt, the interest rate, oil reserves and consumption. Because there is no default risk, debt remains high, as does the interest rate, oil extraction does not drop as much, there isn't such a large accumulation of reserves, and there is more consumption smoothing. Note that the difference between the two mechanisms exemplifies the mechanism behind extraction versus indebtedness decisions. The planner can use either the financial asset (debt) or the real asset (oil) to smooth out consumption. Whether he uses one or the other will depend on the relative yield. With interest rates expected to remain high and the oil price expected to go back up, then its better to not decrease oil extraction as much as in the case when they can default, interest rates go down and so after the default they rather smooth consumption with debt and not so much with oil, increasing oil reserves.

If we now compare the constant extraction model with the benchmark model, the "fisherian separation" that we mentioned earlier is clear. Under constant extraction, there is not too much difference in consumption, GDP or the trade balance, but there is a large difference in terms of extraction and interest rates.

We believe that an important result to highlight from Figure 8 is that oil reserves increase at the time of default. Remember that one of our main results from the empirical part is that oil reserves are positively correlated with sovereign risk. However, our results do not speak about causality. What this result about default episodes is saying is that a default episode

Figure 8: Before and after default episodes: benchmark, constant extraction and risk free model



Note: The default episodes are triggered by different natures on the movement of the exogenous variables. Between $t = -1$ and $t = 0$, oil prices increase in average 11% of the cases, do not vary in 17% and decrease 72%. Non-oil GDP increases in every default episode.

generates an increase in reserves because it is optimal to decrease extraction as present oil prices are lower than expected, saving oil for future sales. This mechanism can explain that positive relationship between oil reserves and sovereign risk that we observe in the data.

To further explore the importance of having endogenous extraction in the model, we split the default episodes of the benchmark model into two groups. In the first group we plot the median of those episodes in which the path followed by the oil price and non-oil output triggers a default in both the benchmark and the constant extraction model. This happens for 59% of the default episodes. In the second group we plot the median of those episodes in which the path followed by the oil price and non-oil output resulted in a default in the benchmark model, but it did not trigger a default in the model with constant extraction. This happens in 41% of the default episodes.

This exercise shows that endogenous extraction can affect in important ways the choice of whether to default or not, and how different the resulting path of the different variables can be. The results are shown in Figure 9, where panel (a) shows the results for the first group and panel (b) for the second group.

As can be seen from the first plot, conditional on there being a default, both models exhibit similar dynamics—except for extraction and the interest rate—as discussed before. However, what makes the difference between default happening or not in the constant extraction case, is the behavior of non-oil GDP. Note that, when there is a drop in oil prices coupled with an increase in non-oil output, there is default both in the benchmark and the constant extraction case, however, if there is a drop in the oil price accompanied by a drop in non-oil output, there is no default in the constant extraction case.

This means that our benchmark model is able to generate defaults that occur both in good and in bad times (see how GDP is increasing at the time of default in panel (a) and decreasing in panel (b)), while the model with constant extraction only generates defaults in good times. This is an important feature of our model as in the data sometimes defaults happen in good times and some others they happen in bad times (see Tomz and Wright (2007)).

This result also illustrates the intuition behind our main result in Subsection 5.1. The observation that a large stock of oil reserves is positively associated with sovereign risk is due to the fact that having a large stock of the real asset increases the value of autarky making default more appealing (note that oil reserves are increasing prior to the default in both panels (a) and (b)), and oil production is negatively associated with sovereign risk because it increases the ability of the sovereign to repay. Therefore the ability of our benchmark model generated data to replicate the empirical results (see Table 5).

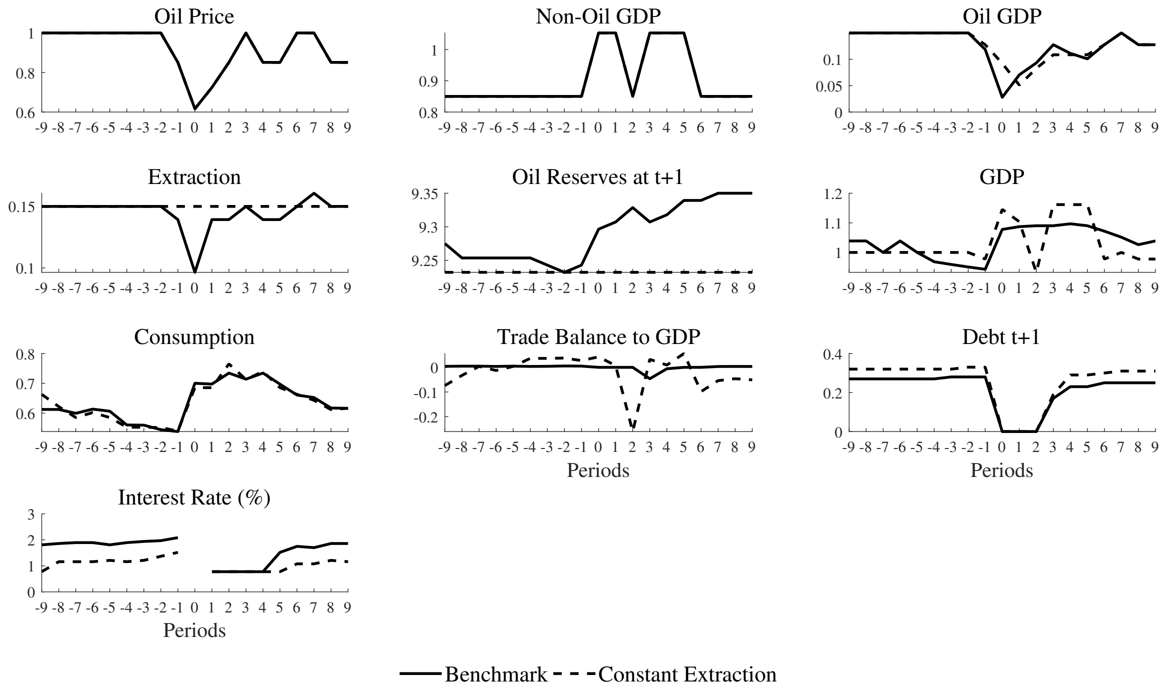
4.5 Default sets and default costs

Given that this model is different from the usual sovereign default model in the literature, in this subsection we look at default sets and default costs.

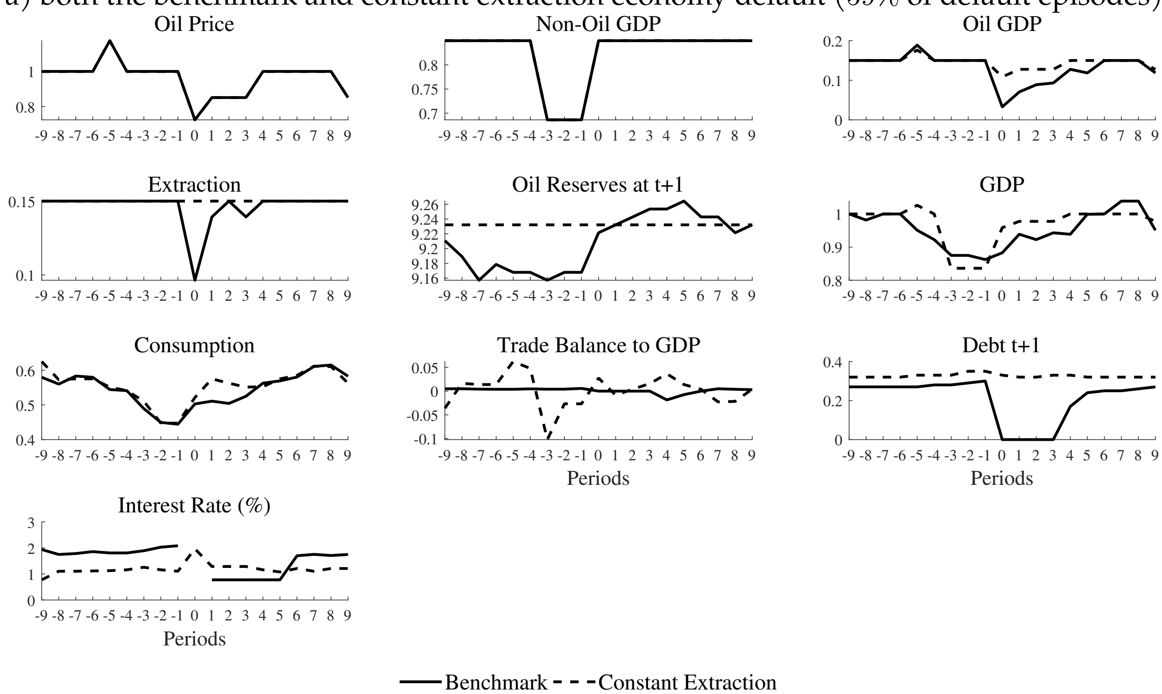
Default sets

Figure 10 shows the default sets. In the subplots prices are increasing to the right and non-oil GDP is increasing down. In the x axis we have debt increasing to the left, and in the y axis

Figure 9: Before and after default episodes: benchmark versus constant extraction



a) both the benchmark and constant extraction economy default (59% of default episodes)

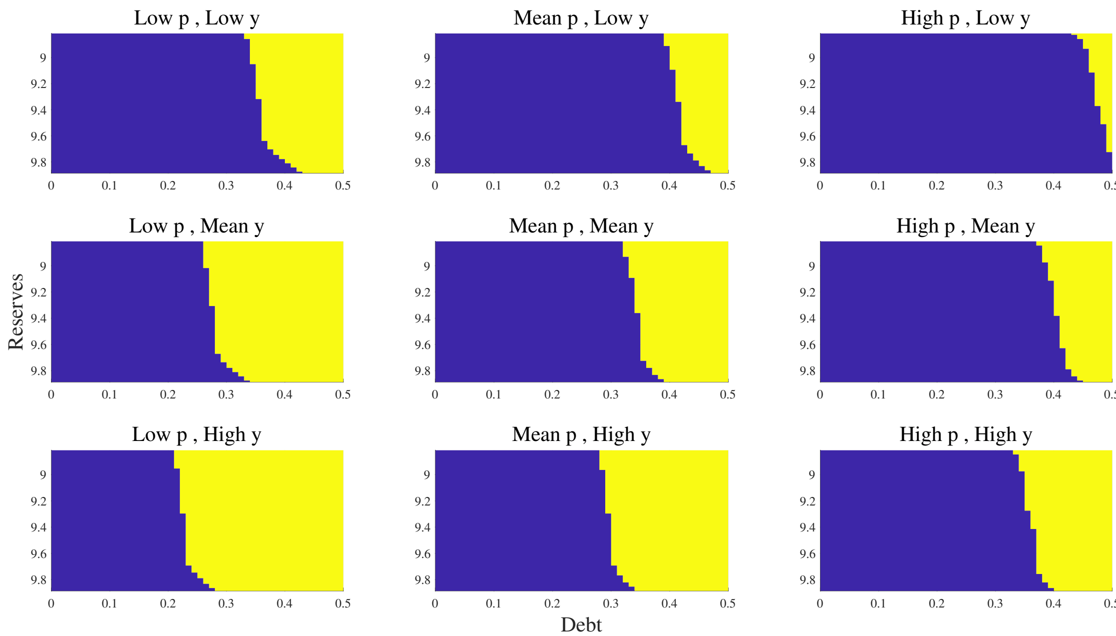


b) the benchmark economy defaults and the constant extraction economy does not (41% of default episodes)

Note:

we have reserves increasing downward. The light area is the region of default and the dark area is the repayment region. As can be seen from the figure, the default region is decreasing in prices and increasing in non-oil GDP. Higher oil prices represent a higher ability to repay, and higher non-oil output increases the value of financial autarky.

Figure 10: Default Sets



Default costs

In order to prevent default from happening too often and allowing models to sustain debt, the literature introduces an adhoc default cost on output. This cost ranges from just loosing a percentage of output, to more complicated linear or non-linear loss functions. In the more involved cases, this cost is zero for output values below a certain threshold and is increasing with output above that threshold as to induce default in bad times.

Recall that in our model total output is $y^T = y + px$, and we don't impose any default cost on y, we only introduce a trade penalty on oil exports such that in a state of default the oil revenue becomes θpx. We can define the default cost in terms of output or in terms of consumption. Let us denote with a d the allocation that would result under default, and with an f the allocation that would result in the risk free model.

Figure 11, left panel, depicts the default cost in terms of consumption $-\left(\frac{c^d}{c^f} - 1\right)$ for two different cases. The case when extraction is constant ($x = k$) and the case for our benchmark calibration. For our benchmark calibration the cost is increasing with the price of oil. This is intuitive as extraction under default is in general lower than under repayment so if on top the sovereign looses a fraction of it, then it will imply a higher default cost. The default cost is concave because as the price increases, so does extraction but there is an upper bound because the sovereign cannot extract more than the available reserves.

Figure 11: Default Costs (at s,b,y means)

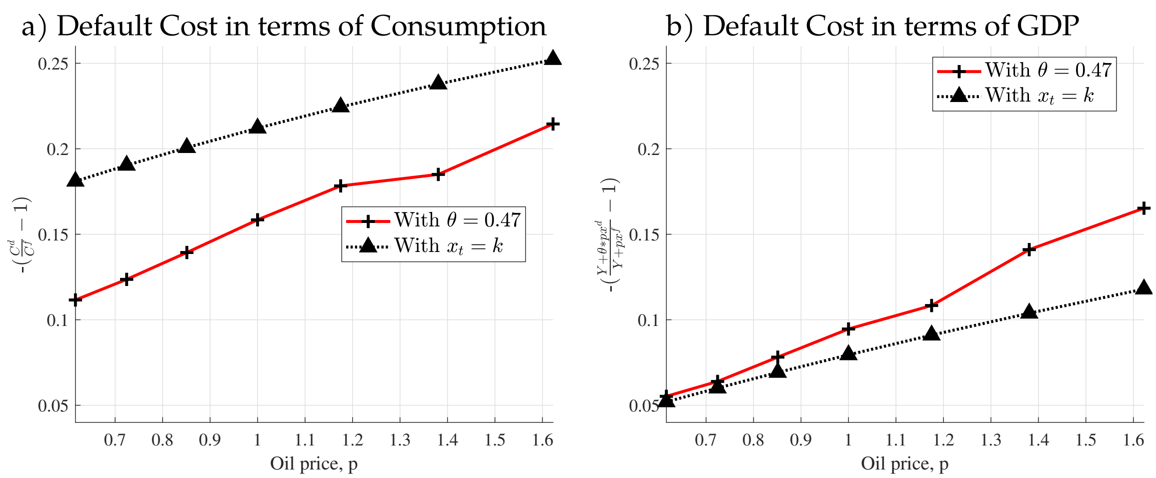


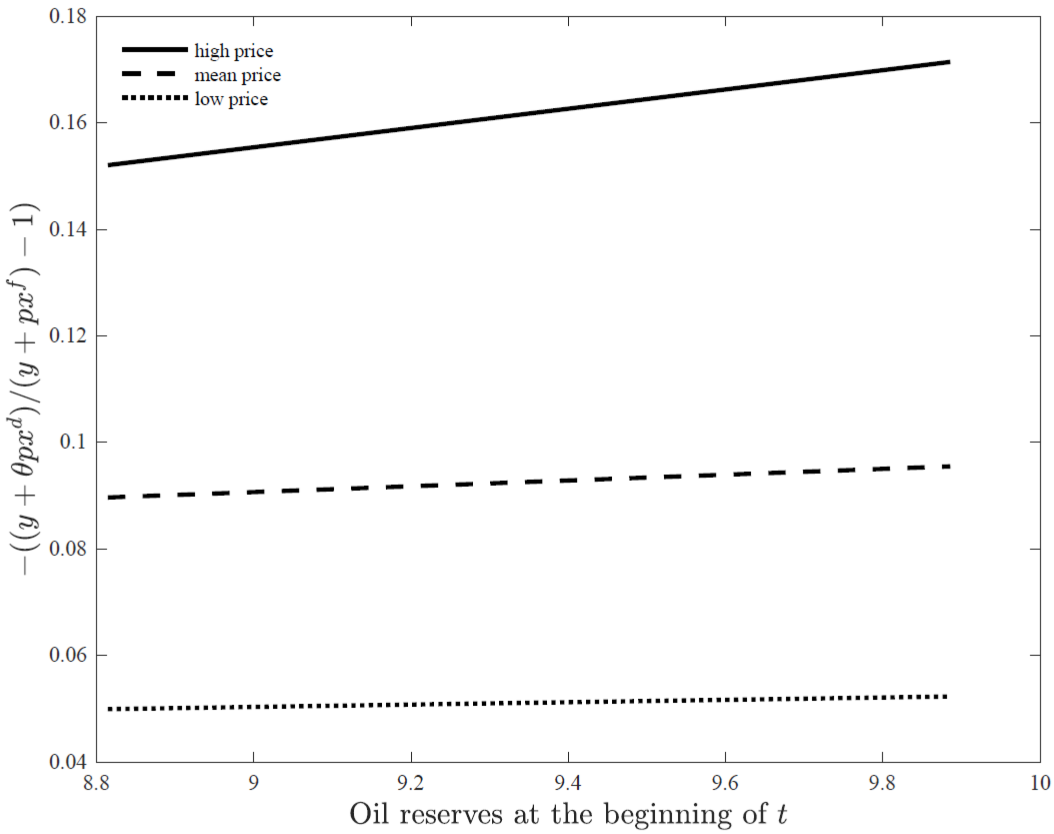
Figure 11, right panel, depicts the default costs in terms of total GDP $\left(-\left(\frac{y+\theta p x^d}{y+p x^f}-1\right)\right)$, it incorporates the effect of prices and non-oil output. In our benchmark calibration, there are costs of default that range between 5% and 17%, and the cost is increasing in p, and slightly convex. The second line in the plot corresponds to the case where oil production is fixed at a constant level. It shows that having endogenous oil production increases the cost of default.

5 Conclusion

We have shown, that being a resource rich country—and more specifically—having oil, has two different effects on country risk. First, in the short-run it decreases country-risk because it increases a countries ability to repay and second, in the long-run, having a large stock of oil reserves increases country risk as it increases the value of autarky for the country as it helps them withstand exclusion from international financial markets.

We develop an off-the-shelf sovereign default model with oil extraction and show that

Figure 12: Default Cost in terms of GDP as Reserves Change for different Price Levels



the model is capable of generating the same relationships that are present in the data.

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