

Market Power and Innovation in the Intangible Economy*

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Abstract

This paper offers a unified explanation for the slowdown of productivity growth, the decline in business dynamism and the rise of market power. I claim that the increasing use of intangible inputs – such as software – explains these trends because it causes a shift from variable costs towards fixed costs, which changes the way that firms produce and compete. Using data on the universe of French firms and U.S. publicly listed firms, I first show that the ratio of fixed over total costs rises when firms invest in intangibles. I then develop a quantitative framework with heterogeneous firms and endogenous productivity growth in which intangibles reduce marginal costs and raise fixed costs, which gives firms with high-intangible adoption a competitive advantage. This advantage deters other firms from entering new markets and lowers the overall rate of creative destruction. Economic activity reallocates disproportionately towards high-intangible firms, which have higher markups and lower labor shares. I structurally estimate the model and show that after initially boosting productivity for 11 years, the rise of intangibles causes a significant decline in productivity growth, consistent with the empirical trends observed since the mid-1990s. The model accounts for one-third of the productivity slowdown in the U.S., one-half of the rise of markups and most of the decline in dynamism.

Keywords: Productivity, Growth, Business Dynamism, Intangible Inputs, Market Power

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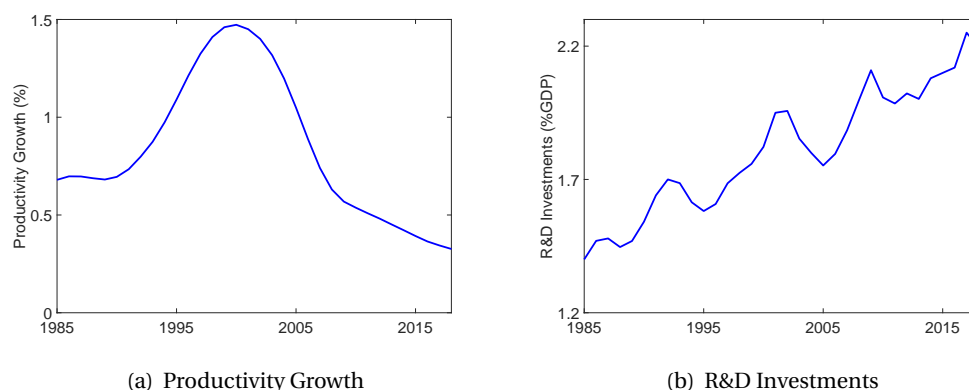
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1. Introduction

The decline of productivity growth has played a prominent role in recent academic and policy debate. Average productivity growth in the United States was less than 0.5% between 2005 and 2018, well below the long-term average of 1.3% (Figure 1a). A similar slowdown occurred across most of Europe, causing productivity in countries such as France and the United Kingdom to flatline (Adler et al. 2017). This followed after a decade of above-average growth in the 1990s, fueled by rapid improvements in information technologies (Fernald 2014). The slowdown occurred despite an increase in productivity-enhancing investments: U.S. investments in corporate research and development have increased by 61% as a fraction of national income over the last 30 years (Figure 1b). The slowdown therefore does not seem to be driven by a lack of effort to become more productive, but rather by a decline in the effect of innovative investments on productivity growth.¹

The initial surge and subsequent decline in productivity growth coincided with two other trends: the slowdown of business dynamism and the rise of markups. Signs that dynamism is weakening include the decline in the rate at which workers reallocate to different employers (e.g. Davis et al. 2006, Decker et al. 2014), the decline in skewness of the firm-growth distribution (e.g. Decker et al. 2016) and the decline in the start-up rate (e.g. Pugsley and Şahin 2018). The rise of markups has recently attracted academic attention (e.g. De Loecker et al. 2018) and has been linked to the decline of the labor share in GDP (e.g. Karabarbounis and Neiman 2013, Autor et al. 2017). Though the timing of these trends differs across countries, they are visible across most advanced economies (Calvino et al. 2016, Diez et al. 2018). Despite the growing body of evidence detailing these trends, there is thus far no consensus on what has caused them.

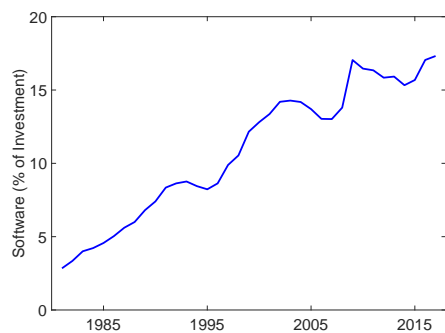
Figure 1. Trends in Productivity Growth and Research & Development Investments



Notes: Figure 1a plots annual productivity growth from the Fernald series (FRBSF). The plot is smoothed using an HP filter with an annual smoothing parameter of 100. 1b plots corporate R&D as a percentage of GDP. Data is from the BEA NIPA tables.

¹Bloom et al. (2017) provide an elaborate analysis of the decline in the effectiveness of research. They use multiple measures to show that the aggregate effect of innovative efforts is falling. They also show declines in the effectiveness of research in various case studies such as the research effort needed to double the power of computer chips (Moore's Law), agricultural productivity and pharmaceutical innovation. Using both Compustat and U.S. Census data, they further show that the effect of research on growth is also falling at the firm level.

Figure 2. Rise of Intangible Inputs: Software as a Percentage of Total Investments



Notes: The figure plots investment in software as a percentage of total private fixed investments, excluding residential investments, research and development and entertainment. Data is obtained from the BEA NIPA tables.

This paper claims that the trends in productivity growth, business dynamism and markups can jointly be explained by a secular shift in the way that firms produce. Specifically, I claim that an increase in the use of intangible inputs (in particular, the use of information technology and software) can drive these patterns. The rise of intangible inputs in the past decades has been dramatic: Figure 2 illustrates that software alone is now responsible for 17% of U.S. corporate investments.² Intangible inputs can explain the three trends because they have two features: they are scalable, and firms differ in the efficiency with which they deploy them.

Intangibles are scalable in the sense that they can be duplicated at close-to-zero marginal cost (e.g. [Haskel and Westlake 2017](#), [Hsieh and Rossi-Hansberg 2019](#)). This implies that when intangible inputs are used to produce a good, the cost structure of production changes. Firms invest in the development and maintenance of intangibles (which have depreciation rates upwards of 30%) but face minimal additional costs of using them when production is scaled up. An example of such an input is Enterprise Resource Planning (ERP), which allows firms to automate business processes such as supply chain and inventory management. ERP allows firms to automatically send invoices or order supplies, for example, which reduces the marginal cost of a sale. Alternatively, firms that sell products that include software (e.g. the operating system of a phone, the drive-by-wire-system of a car), face minimal marginal costs of reproducing that software in additional units. The rise of intangibles therefore shifts production away from variable costs towards fixed costs.

The ability to reduce costs through intangible inputs is not equal across firms. A considerable literature has found that for a given expenditure level on intangibles, some firms are able to reduce their marginal costs by a greater fraction than others. [Bloom et al. \(2012\)](#), for example, show that American-owned European establishments achieve greater productivity improvements from the use of information technology (IT). Their evidence suggests that intangible input productivity is a firm characteristic, especially because the IT productivity of European establishments increases

²This percentage understates expenditure on software by firms, as an increasing part of software expenses is incurred 'as a service' (SaaS). This is expensed and not counted as investment.

when they are *acquired* by an American multinational. [Schivardi and Schmitz \(2019\)](#) furthermore show that inefficient management practices can explain not only the low IT adoption by Italian firms but also why the productivity gains that these firms obtain from using IT are limited.³

I show that intangible inputs modeled along these lines can qualitatively and quantitatively explain the trends in productivity growth, business dynamism and markups. To do so, I develop and estimate an endogenous growth model that is both tractable and sufficiently rich to allow a quantification of the effect of intangibles. The model embeds intangibles as scalable production inputs in a framework with heterogeneous markups and endogenous entry/exit dynamics, in the spirit of [Klette and Kortum \(2004\)](#). Intangibles enable firms to reduce marginal costs, in exchange for a per-period cost to develop and maintain the intangible inputs. These costs do not depend on the quantity that firms sell. Firms produce one or multiple products that are added or lost through creative destruction. They invest in research and development (R&D) to produce higher quality versions of goods that are produced by other firms. Successful innovation causes the innovator to become the new producer, while the incumbent ceases to produce the good. Firm-level innovation along this process drives aggregate growth through the step-wise improvement of random goods.

Intangible inputs introduce a new trade-off between *quality* and *price* to this class of models. In most [Klette and Kortum \(2004\)](#)-models, firms that innovate become the sole producer of the good when they develop a higher quality version. Other firms may have the same marginal cost but are unable to produce the same quality, and hence cannot compete. Intangible inputs change this results, if some firms are able to reduce their marginal costs by a greater fraction than others. Heterogeneity in the efficiency with which a firm adopts intangible inputs, for example, will cause some firms to produce their output at lower costs, thereby allowing them to sell at lower prices. If a firm with a lower level of intangible-adoption develops a higher quality version of a good sold by one of these firms, the incumbent could undercut the innovator on price. Only if the quality difference is sufficiently large to offset the gap in marginal costs would the innovator become the new producer. The presence of firms with a high take-up of intangible inputs, therefore, deters other firms from entering new markets and from developing higher quality products. Paradoxically, the rise of firms with high intangible input productivity can therefore negatively affect growth.

To analyze whether this can explain the macroeconomic trends, I introduce a group of high-intangible firms to an economy where firms initially have similar levels of intangibles. These firms have lower marginal costs than their competitors, which they use to sell their goods at higher markups and, after paying the fixed costs of intangibles, greater profits. Over the transition path, the rise of high-intangible firms initially causes a boom in productivity growth. They ‘disrupt’ sectors across the economy, as they have a greater incentive to invest in R&D. This causes economic activity to concentrate disproportionately around these firms. Their entry raises productivity be-

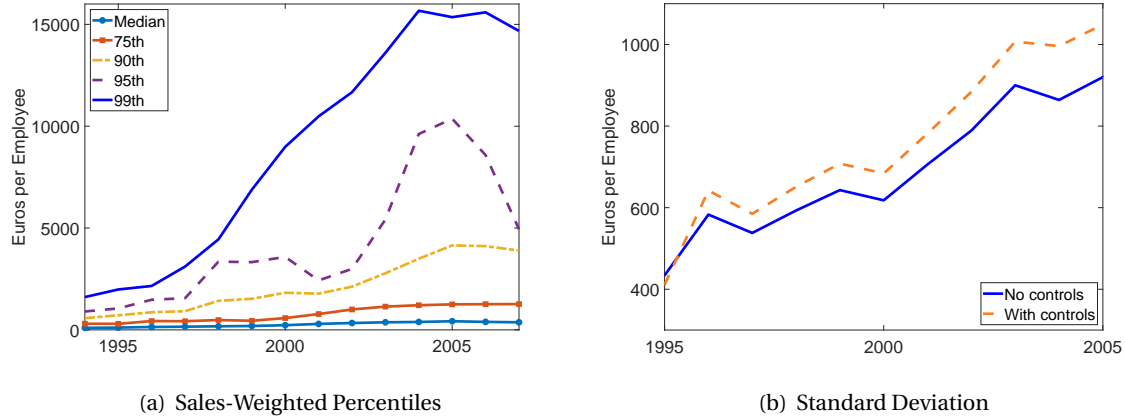
³[Bloom et al. \(2014\)](#) also find that structured management practices are closely related to IT adoption in American firms. Evidence also suggests that workplace organization and organization capital affect a firm’s IT productivity (e.g. [Crespi et al. 2007](#), [Bartel et al. 2007](#)). Changes to organization design come at the price of high adjustment costs, which makes IT productivity a persistent firm characteristic (e.g. [Bresnahan et al. 2002](#)).

cause they produce all their goods at lower marginal cost. The increase in aggregate productivity is not matched by a rise in wages, however, because high-intangible firms set proportionally higher markups. This causes average (and aggregate) markups to increase through reallocation, as found empirically by [Baqae and Farhi \(2018\)](#) and [Kehrig and Vincent \(2019\)](#). As the economy transitions to the new balanced growth path, there is a decline in entry as most start-ups do not have sufficiently low marginal costs to compete against high-intangible incumbents. Incumbents with low levels of intangibles similarly have fewer incentives to innovate. This causes a gradual decline in the growth rate of productivity, which falls below the initial steady state level around 20 years after the first high-intangible firms enter the market. Although overall R&D efforts increase, they concentrate around a smaller group of firms. Because the returns to these investments are concave (in line with evidence in, e.g., [Akcigit and Kerr 2018](#) or [Bloom et al. 2002](#)), the concentration of R&D lowers its effectiveness. This, combined with the fact that a fraction of innovations fail because high-intangible incumbents undercut innovators on price, explains how productivity growth can fall along the new balanced growth path while innovative investments increase.

I quantify the model using two calibrations, one for the United States and one for France. While evidence on the macroeconomic trends is stronger for the United States, I show that the trends are largely visible for France as well. Furthermore, I provide evidence on the mechanisms of the model using data from tax filings of the universe of French firms between 1994 and 2016. The advantage of using French data is that the full income statement and balance sheet are available for both public and private firms of all sizes, and that the data can be merged with surveys on innovation activities and investments in software or the adoption of IT systems. This allows a close inspection of the empirical validity of the model's mechanisms. Using a new measure of fixed costs, I show that the share of fixed costs in total costs has gradually increased from 9.5 to 14% in France between 1994 and 2016 and from 12 to 22.5% in the United States between 1980 and 2016. There is a positive within and across-firm correlation between fixed costs and investments in software, as well as the adoption of intangible inputs such as ERP. Firms with a high fixed-cost share also have higher markups, invest more in research and development and have higher average growth rates, in line with the model's predictions.

I then quantitatively analyse the effect of a rise in intangible inputs by structurally estimating the model. The French estimation relies on the administrative data for the universe of firms while that for the U.S. relies on data for listed firms. Matched moments include average growth, fixed costs, R&D investments and firm dynamics for 1994 in the French calibration and for 1980 in the U.S. calibration. A group of high-intangible firms is then introduced to match the empirical increase in the ratio of fixed costs over total costs along the new balanced growth path. The model predicts a slowdown in steady state growth of 0.21 percentage points in the French calibration and 0.37 percentage points in the U.S. calibration, on a base of 1.3%. Markups increase by 10.6 and 16.7 percentage points in the respective calibrations. The entry rate falls by 3.6 and 5.6 percentage points, respectively. Overall, the model is able to explain a significant part of the slowdown of productivity growth and the rise of markups, and most of the decline in business dynamism.

Figure 3. Divergence in Software Investments per Employee



Notes: Figure 3a plots sales-weighted percentiles of software investments per employee. Data on software comes from the *Enquête Annuelle d'Entreprises* (EAE), merged with employment data from FARE. Both are discussed in Section 2. Figure 3b plots the standard deviation after controlling for 5-digit industry fixed effects, a third degree polynomial of log sales and industry time-trends.

A central assumption behind these predictions is that firms are heterogeneous in their use of intangible inputs. In support of that assumption, Figure 3 uses the French administrative data to plot various measures of inequality in software investments (either developed in-house or purchased externally), expressed per employee. Figure 3a plots various sales-weighted percentiles of software investments and shows that the increase in software spending has been concentrated among the higher percentiles.⁴ While there was a modest increase in median spending from 100 to 300 euros per employee, spending at the 75th and 95th percentiles rose from 190 and 620 euros to 1240 and 5000 euros, respectively. Figure 3b shows that the rising inequality of intangible spending is also present within narrowly defined industries, also with controls for size.

Related literature The theoretical framework builds on Schumpeterian growth models of creative destruction in the tradition of Aghion and Howitt (1992) and Grossman and Helpman (1993). In particular, I build on the strand of Schumpeterian models where firms produce multiple products (Klette and Kortum 2004). This framework is attractive because it is analytically tractable, yet able to replicate many empirical features of firm dynamics when quantified (Lentz and Mortensen 2008). The framework was recently used to study the reallocation of innovative activity (Acemoglu et al. 2018), to discern the effect of innovation policy (Atkeson and Burstein 2018) and to compare different sources of innovation (Akcigit and Kerr 2018, Garcia-Macia et al. 2016). It has also been used to analyze imperfect competition in a setting with heterogeneous markups (Peters 2018).

I extend this framework with intangibles as scalable production inputs and, while preserving the framework's analytical flair and realistic firm dynamics, show that the model predicts the macroeconomic trends in productivity, business dynamism and market power. The extension fur-

⁴Unweighted percentiles show similar divergence over time.

thermore offers two theoretical insights. First, I show that the aggregate effect of research and development on productivity growth depends on how this expenditure is distributed across firms. Because firm-level returns are concave in expenditure, concentration of research and development negatively affects growth. Firm heterogeneity is therefore an important ingredient for this type of endogenous growth models. Second, I introduce a new distinction between quality and price to the framework. Quality accumulates step-wise through research and development and is the engine of long-term growth. Productive (high-intangible) firms are able to sell at lower prices, which can compensate for lower quality and can be used to undercut innovators. Differences in production efficiency across firms therefore reduce the effect of research and development on growth.

This paper is most closely related to recent work that jointly explains low productivity growth, the fall in business dynamism and the rise of market power. [Aghion et al. \(2019\)](#) build a model where a decline in the cost of IT increases a firm's span of control, thereby allowing firms with ex-ante higher productivity to grow larger. Because productive firms are likely to face productive competitors, this reduces their expected markups and lowers incentives to innovate. This reduces aggregate research and development and hence lowers growth. My mechanism is distinct: intangibles are a production technology that allows firms to reduce variable costs, at the expense of higher overhead fixed costs. This has a negative effect on innovation and business dynamism if firms differ in intangible productivity and adoption. High-intangible adopters can undercut other firms on price, which has a negative effect on the innovative activity of other firms. In line with the data there is an *increase* in aggregate research and development, but because it is concentrated among a smaller group of highly profitable, high-intangible firms, there is a negative effect on aggregate growth. This paper, furthermore, explains why entry rates have fallen, which is due to the fact that such high-intangible firms eventually produce a disproportionate number of goods. This reduces the probability of successful entry and raises effective entry costs. [Peters and Walsh \(2019\)](#) relate the decline of entry rates to the fall in labor force growth. The lack of entry stimulates expansion by incumbents, which raises firm concentration and markups and causes a slowdown of productivity growth. [Liu et al. \(2019\)](#) relate low productivity and business dynamism to a decline in interest rates in a model where two firms compete for leadership through R&D. A lower interest rate increases investment incentives most strongly for the market leader, discouraging investments by the follower and diminishing growth. [Akcigit and Ates \(2019\)](#) find that intellectual property rights are increasingly used anti-competitively, which also discourages entrants from investing. Both forms of discouragement differ from my framework, as my discouraging effect arises from the inability of low-intangible firms to compete on price. My framework is also different in that the rise of intangibles initially leads to a rise in productivity growth. It furthermore explains why markups could have increased rapidly at a time of low inflation: they were offset by a decline in marginal costs.

Other papers explain a subset of the macroeconomic trends. In close relation to my hypothesis, [Hsieh and Rossi-Hansberg \(2019\)](#) suggest that intangibles such as software explain the rise of firm concentration in services, wholesale and retail, as software can be deployed across markets after paying a fixed cost. This paper is complementary to their work as it shows that fixed-cost intangi-

bles can also explain the rise of markups and the slowdown of productivity growth. [Brynjolfsson et al. \(2018\)](#) claim that artificial intelligence is a new General Purpose Technology that initially requires firms to invest in unmeasured intangible capital, causing measured productivity growth to decline but eventually increase. This does not, however, explain the coincidental reduction in business dynamism and rise in concentration. [Hopenhayn et al. \(2018\)](#) add that the average age of firms has increased and link that to demographic aging. Their model predicts that the decline in the growth rate of the U.S. labor force is sufficient to explain most of the fall in business dynamism. [Korinek and Ng \(2019\)](#) show that intangible inputs can give rise to natural monopolies and may therefore explain the rise in industry concentration. [Martinez \(2019\)](#) shows that automation technology can drive a decline in the labor share, also when capital and labor are complimentary at the aggregate level.

This paper also contributes to the literature on the static costs of markups. In a model where large firms charge higher markups, [Edmond et al. \(2018\)](#) find that markups reduce welfare by 7.5%. [Baqae and Farhi \(2018\)](#) argue that eliminating markups would increase TFP by 20%, but also document that the upward trend of markups is driven by reallocation towards high-markup firms. This implies that allocative efficiency is improving, as high-markup firms produce inefficiently little. This is in line with the finding in [Autor et al. \(2017\)](#) and [Kehrig and Vincent \(2017\)](#) that the decline in the aggregate labor share is primarily driven by a reallocation of economic activity towards firms with a low labor share. My model similarly predicts that the rise of markups is driven by the reallocation of activity towards high-markup (high-intangible) firms, as these have a greater incentive to expand by investing in research and development.⁵

This paper also relates to the recent literature that studies the trends in productivity and market power from a disaggregated perspective. As summarized by [Van Reenen \(2018\)](#), there is substantial heterogeneity in the extent to which firms are subject to these trends, causing productivity and profitability to diverge across firms. [Andrews et al. \(2016\)](#) show that productivity growth at the most productive firms within 2-digit industries has not declined across the OECD.⁶ [Decker et al. \(2018\)](#) similarly find an increase in productivity dispersion within the U.S.⁷ The rise in markups in [De Loecker et al. \(2018\)](#) is also strongest in the highest deciles, a result that has been confirmed for several countries by [Diez et al. \(2018\)](#) and [Calligaris et al. \(2018\)](#).⁸ This paper contributes to this literature by showing that an increase in the ability to use intangibles by some firms can impose a negative externality on others, thereby driving the growing differences across firms as well as the aggregate trends in productivity growth, business dynamism and markups.

⁵This reallocation fades over time as the share of all products produced by these firms approaches its steady state level. The fading contribution of these high-intangible firms is somewhat in line with the finding by [Gutiérrez and Philippon \(2019a\)](#) that the contribution of the highest-valued firms on U.S. stock markets has fallen over time, though this depends on the extent to which market valuations reflect whether a firm has high intangibles in model's spirit.

⁶Dispersion in pay is also increasing across firms, as found by [Berlingieri et al. \(2017\)](#) and [Song et al. \(2018\)](#).

⁷[Kehrig and Vincent \(2019\)](#) note that an increase in productivity dispersion at the establishment level may reflect an improvement in factor allocation and a reduction of internal credit market frictions.

⁸Their methodology to estimate markups in these papers is criticized by [Traina \(2018\)](#). Recent summaries on the debate of markups and estimation methodologies are found in [Syverson \(2019\)](#) and [Basu \(2019\)](#).

This paper also relates to work on the rise of market power and corporate profits more broadly. [Barkai \(2016\)](#) finds that the share of excess profits increases over time because the sum of (estimated) payments to labor and capital has declined as a percentage of national income. [Caballero et al. \(2017\)](#) remark that this is partly offset by a rise in equity risk premia. [Karabarbounis and Neiman \(2019\)](#) add that unmeasured capital also explains the rise of excess profits, which they refer to as factorless income. [IMF \(2019\)](#) find that markups are increasing across advanced economies but not in emerging economies, and show that there is an inverse U-shaped relation between innovative activities and markups at the firm level. There is also a growing literature that relates the rise of industry concentration to an increase in entry costs - due for example to occupational licences or increases in regulation (e.g. [CEA 2016](#) and [Furman 2016](#)). [Gutiérrez and Philippon \(2019b\)](#) show that the response of entry to profitability of incumbents has declined over time, and that this correlates with anti-competitive lobbying efforts.⁹ [Gutiérrez and Philippon \(2017\)](#) further relate the lack of investments relative to Tobin's Q by U.S. listed companies to a decline in competition.

Whether the rise of market power is equally present across advanced economies remains a subject of debate. The slowdown of productivity growth and the decline of start-ups have been documented carefully in past work (see, e.g., [Adler et al. 2017](#) and [Calvino et al. 2016](#)). The rise of market power and firm concentration, however, seems to be larger in the United States. [Döttling et al. \(2017\)](#) and [Cavalleri et al. \(2019\)](#) find no increase in industry concentration in Europe between 2000 and 2013, using Bureau van Dijk's Orbis data. In contrast, [Bajgar et al. \(2019\)](#) document a rise in industry concentration for most European countries when accounting for ownership structures and improvements in the coverage of small firms in Orbis. [Aquilante et al. \(2019\)](#) also find an increase in firm concentration in the United Kingdom between 1998 and 2016. I find a modest rise in 5-digit industry concentration for France in the administrative data, especially between 1995 and 2005. In contrast to the United States, the labor share in European countries does not show a strong decline when controlling for fluctuations in residential housing income ([Gutiérrez and Piton 2019](#)). The rise of markups also seems to be stronger in the U.S. [De Loecker et al. \(2018\)](#) find an increase in U.S. markups of 40 percentage points, while markups across advanced economies on average increased by only 8 percentage points ([IMF 2019](#)). I show that markups in the French administrative data display a similarly modest upward trend.

I also contribute to recent work that links the trends in productivity growth, business dynamism and market power to the rise of intangibles. In empirical work, [Crouzet and Eberly \(2018\)](#) show that intangibles have caused an increase in market power and productivity for leading U.S. public firms. Similarly, [McKinsey \(2018\)](#) and [Ayyagari et al. \(2018\)](#) show that firms with high profitability and growth invest more in software and R&D. [Bessen and Righi \(2019\)](#) find that productivity of U.S. firms increases persistently after an increase in the stock of their IT-staff. [Farhi and Gourio \(2018\)](#)

⁹[Gutiérrez et al. \(2019\)](#) provide a theoretical exploration of this hypothesis in a structural model with entry cost shocks. While similar in terminology, these entry costs are distinct from the endogenous fixed costs of intangibles, as firms can choose to enter markets without facing these higher costs. This raises effective entry costs indirectly because low-intangible incumbents are less likely to successfully enter the market.

show that unmeasured intangibles can explain the rising wedge between the measured marginal product of capital and risk-free rates. In close connection to my results, [Baigar et al. \(2019\)](#) find that sectors with high intangible investments experienced a greater increase in industry concentration across the OECD. In related work, [Bessen \(2017\)](#) finds a positive relationship between the rise of firm-concentration and the use of IT systems in U.S sectors. In contrast to my approach, he stresses that the scalability of intangibles is advantageous to firms that are already large. Firm-level evidence on this is provided in [Lashkari and Bauer \(2018\)](#). Also at the sector level, [Calligaris et al. \(2018\)](#) find a positive correlation between the use of digital technologies and the rise of markups and concentration. [Bijmens and Konings \(2018\)](#), documenting a decline in Belgian business dynamism that resembles trends established for the U.S. by [Decker et al. \(2016\)](#), remark that the decline is strongest in industries with a higher IT intensity.

Outline The remainder of this paper proceeds as follows. Section 2 introduces scalable intangible inputs empirically. Section 3 presents the growth model and discusses the main mechanism. The model is estimated in Section 4, and results are discussed in Section 5. Section 6.2 presents extensions, while Section 7 concludes.

2. Intangibles as Fixed Costs

This section introduces intangibles as inputs that cause a shift from marginal to fixed costs. To provide a foundation for the paper’s main analysis, I first introduce a simple framework where intangibles are modeled as such an input. I then introduce micro evidence on two facts that are consistent with this model: the share of fixed costs has increased over time, and there is a positive within and across-firm correlation between fixed costs and either software investments or measures of information technology adoption. I furthermore show that high-fixed cost firms have higher markups and research and development investments than their competitors, and subsequently grow faster. These results serve as the foundation for the full theoretical model in Section 3.

2.1. Framework

Consider a first-degree homogeneous production function $z(z_{it,1}, z_{it,2}, \dots, z_{it,k}) \cdot \omega_i$ with k traditional (tangible) production factors and Hicks-neutral productivity ω_{it} . Firm i ’s marginal cost function is $c(w_{1t}, w_{2t}, \dots, w_{kt}, \omega_{it})$, where w_{kt} denotes the factor price of tangible production factor k at time t . Intangible inputs are defined as inputs that allow firms to reduce their marginal costs by a desired fraction $s_i \in [0, 1]$.¹⁰ In the framework, the production function therefore reads

$$y_{it} = \frac{1}{1 - s_{it}} \cdot z(z_{it,1}, z_{it,2}, \dots, z_{it,k}) \cdot \omega_{it}, \quad (1)$$

¹⁰This definition applies to a subset of the total of possible intangible assets and inputs that firms may deploy. It might not apply, for example, to research and development expenses, which are treated separately in the model in Section 3. Throughout the text, the term ‘intangible inputs’ refers to inputs for which the definition applies.

which is associated with marginal costs $mc_{it} = (1 - s_{it}) \cdot c(w_{1t}, w_{2t}, \dots, w_{kt}, \omega_{it})$.¹¹ To reduce their marginal costs by s_{it} , firms must spend some amount on intangible inputs. The relationship between s_{it} and expenditure on intangibles is governed by a twice-differentiable function $f(s_{it}, \phi_i)$. ϕ_i is a firm-specific parameter that captures the efficiency with which firm i uses intangibles: firms with higher levels of ϕ_i are able to reduce their marginal costs by a greater fraction for a given expense on intangible inputs. $f(\phi_i, s_{it})$ is strictly convex on the domain $s_{it} \in [0, 1)$ and satisfies $\partial f(\phi_i, s_{it}) / \partial \phi_i < 0$, $f(\phi_i, 0) = 0$ and $\lim_{s_{it} \rightarrow 1} f(\phi_i, s_{it}) = \infty$. The latter implies that the cost of eliminating marginal costs completely is infinite, such that all firms have positive marginal costs in equilibrium. Firms pay $f(\phi_i, s_{it})$ before production occurs which, combined with the fact that $f(\phi_i, s_{it})$ does not directly depend on the amount that a firm sells, explains why they represent a fixed cost.¹² The term fixed here is different from usual, in the sense that firms choose the level of $f(\phi_i, s_{it})$ through a reduction in variable costs. Firms that do not increase their use of intangible inputs do not face an increase in fixed costs, and intangibles do not directly raise entry costs.

In this setup, total costs tc_{it} equal

$$tc_{it} = (1 - s_{it}) \cdot c(w_{1t}, w_{2t}, \dots, w_{kt}, \omega_{it}) \cdot y_{it} + f(s_{it}, \phi_i),$$

where the first term contains all variable costs while the second term contains fixed costs. It is straightforward to show that when firms increase their expenditure on intangibles there is a shift from variable to fixed costs, provided that a reduction in marginal costs does not lead to a large increase in demand. Formally, $\partial f(s_{it}) / tc_{it} / \partial s_{it} > 0$, provided that

$$\frac{\partial \ln z(z_{it,1}, z_{it,2}, \dots, z_{it,k})}{\partial s_{it}} < 1. \quad (2)$$

Under this condition, which I view as mild, the rise of intangible inputs is reflected by an increase in the average share of fixed in total costs, and this share should increase at the firm level when firms increase their use of intangible inputs.

2.2. Data

To test the empirical validity of the framework, I use administrative data on the universe of French firms and data from financial statements on U.S. publicly listed firms. Both can be used to analyze trends in the share of fixed in total costs over time. The French data additionally contains information on software investments and the adoption of IT systems, which allows a direct assessment of the link between intangible inputs and fixed costs. Appendix D replicates the macroeconomic

¹¹Instead of dividing by $1 - s_{it}$ one could multiply $z(\cdot)$ by some productivity term that depends on the use of intangibles. That approach is isomorphic to my approach, which I prefer because it leads to a convenient expression for marginal costs.

¹²This does not mean that there is no correlation between $f(\phi_i, s_{it})$ and output, as large firms have greater incentives to reduce marginal costs and choose a higher $f(\phi_i, s_{it})$. The empirical analysis therefore includes controls for size.

Table 1: Descriptive Statistics

	Mean	Std. Dev.	Median	10th Pct.	90th Pct.	Obs.
<i>French Firms in FICUS-FARE (1994-2016)</i>						
Sales (revenue)	4,684	103,285	617	149	4,996	9,913,058
Employment (headcount)	19	356	5	1	28	9,913,058
Wage bill	622	10,753	144	38	831	9,913,058
Capital stock	1,738	131,183	92	12	895	9,913,058
Intermediate inputs and raw materials	2,234	58,699	136	0	1,923	9,913,058
Other operating expenses	1,210	35,652	124	33	1168	9,913,058
<i>U.S. Compustat Firms (1980-2016)</i>						
Sales (revenue)	2,370,409	14,147,340	189,447	13,237	3,521,189	125,231
Operating expenses	2,011,057	12,196,450	164,736	12,767	2,990,300	125,231
Cost of good sold	1,596,605	10,677,900	113,106	7,007	2,290,489	125,231
Selling, General, and Adm. expenses	414,452	1,970,337	39,066	3,857	635,171	125,231
Capital stock	1,567,708	10,485,090	77,258	4,889	2,072,819	125,231

Notes: Nominal figures in thousands of Euros (France) and Dollars (U.S.). Sales, operating costs and materials are deflated with KLEMS sector deflators; the wage bill and capital are deflated with the GDP deflator.

trends that motivate this paper for France, and confirms that it has incurred a decline in productivity growth and business dynamism, as well as a modest increase in markups and concentration.

The French data comes from two administrative datasets (FICUS from 1994 to 2007 and FARE from 2008 to 2016, both are based on tax data from DGFIP). The data contains the full balance sheet and income statement, with detailed breakdowns of revenues and costs. I append FICUS with FARE using a firm identifier (the *siren code*) that consistently tracks firms over time. The unit of observation is a legal entity (*unité légale*), although subsidiaries of the largest companies are grouped as a single entity. I restrict the sample to private firms and drop contractors, state-owned enterprises and non-profit organisations, as well as companies that receive operating subsidies in excess of 5% of sales. Firms in financial industries and firms with missing or negative sales, assets, or employment are also excluded. Details on variable definitions are provided in Appendix B. The remaining sample contains data on 1,087,726 firms across 651 NACE industries between 1994 and 2016. Summary statistics for the main firm variables are provided in the upper panel of Table 1.¹³

Data for U.S. firms is obtained from S&P's Compustat. Compustat contains balance sheet and income statement data for all publicly listed firms in the U.S. I restrict the sample to firms outside of the financial, insurance and real estate industries between 1980 and 2016, and drop firms with missing or negative sales, assets and operating expenses. Following Baqaee and Farhi (2018), I also drop firms with ratios of sales to cost of good sold or of sales to selling, general, and administrative expenses outside of the 2.5-97.5 percentile range. The remaining sample covers 10,738 firms across 788 6-digit NAICS industries. Summary statistics are provided in the lower panel of Table 1.

¹³Access to the FICUS and FARE datasets was initially obtained for Burstein et al. (2019). The code to merge FICUS and FARE was developed for their project, and is partly provided by Isabelle Mejean. I thank them for their help in obtaining data access and for permission to use the data for this project.

2.3. Measurement

Testing the framework requires a measure of fixed costs. Past work typically infers fixed costs from the sensitivity of a firm's operating costs or profits to sales shocks, under the assumption that all variable costs are set freely.¹⁴ This is problematic when firms face adjustment costs for some variable inputs (when adjusting the size of their labor force, for example), and limits the analysis of changes to fixed costs over time. I therefore derive a new time-varying measure of fixed costs from the difference between the marginal cost markup and the profit rate, which equals operating profits over revenue. Under the first-degree homogeneity assumption of $z(z_{it,1}, \dots, z_{it,k})$, the accounting definition for the profit rate is

$$\frac{\pi_{it}}{p_{it} \cdot y_{it}} = \frac{(p_{it} - mc_{it}) \cdot y_{it}}{p_{it} \cdot y_{it}} - \frac{\tilde{f}_{it}}{p_{it} \cdot y_{it}},$$

where fixed costs \tilde{f}_{it} are the sum of expenditures on intangibles and other fixed costs (η_i), such that $\tilde{f}_{it} = f(s_{it}, \phi_i) + \eta_{it}$. Isolating fixed costs and defining the markup μ_{it} as the ratio of prices to marginal costs yields

$$\frac{\tilde{f}_{it}}{p_{it} \cdot y_{it}} = \left(1 - \frac{1}{\mu_{it}}\right) - \frac{\pi_{it}}{p_{it} \cdot y_{it}}. \quad (3)$$

I multiply the right-hand side of (3) with revenues and divide by total operating costs to obtain fixed costs as a share of total costs. The straightforward intuition behind (3) is that markups capture the firm's marginal profitability, while profits capture the firm's average profitability. Because fixed costs are incurred regardless of sales, a firm with positive fixed costs should have a profit rate below the markup. This also implies that rising markups do not necessarily reflect rising profitability.

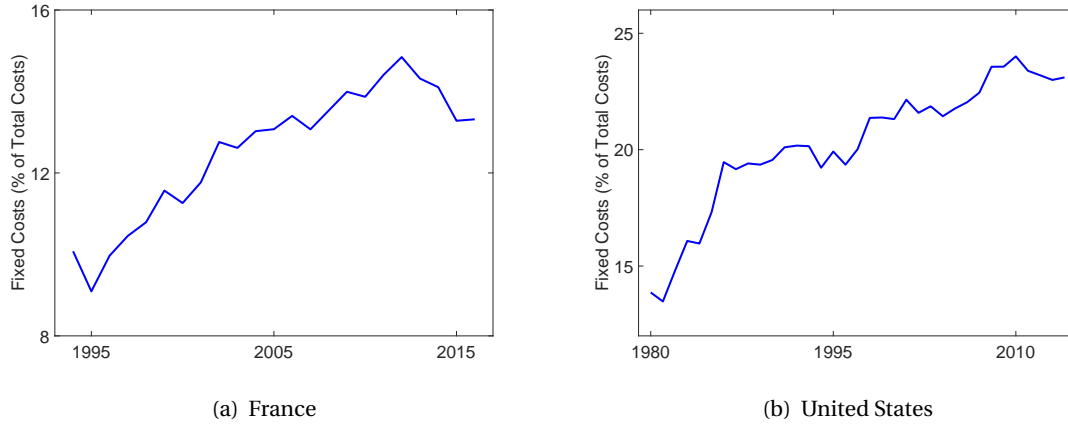
To implement the measure in equation (3), I require data on operating profits, revenues and markups. Operating profits and revenues are obtained from the income statement. Markups are not directly observed because income statement and balance sheet data lacks information on marginal costs and prices. Instead, I estimate markups using the method proposed by [Hall \(1988\)](#). He shows that markups are given by the product of the output elasticity β^m of a variable input m and the ratio of a firm's sales to its expenditure on that input. Formally:

$$\mu_{it} = \beta_{it}^m \cdot \left(\frac{p_{it} \cdot y_{it}}{w_t^m \cdot z_{it}^m} \right),$$

where the z_{it}^m denotes the quantity of m that firm i deploys in year t , and w^m denotes that input's unit cost. Revenue and expenditure on the input are observed on the income statement, while I

¹⁴Examples include [Lev \(1974\)](#) and [García-Feijóo and Jorgensen \(2010\)](#). Alternatively, [De Loecker et al. \(2018\)](#) assume that selling, general and administrative expenses on the income statement are fixed. Though appropriate for their purpose, it is likely that some of these costs (like shipping costs and sales commissions) are variable.

Figure 4. Weighted-Average Ratio of Fixed Costs to Total Costs over Time



Notes: Sales-weighted average of fixed costs as a percentage of total costs, universe of French firms (left) and U.S. listed firms (right). Fixed costs are inferred from the difference between profits as a percentage of sales and the marginal cost markup.

obtain the output elasticity by estimating a translog production function using the iterative GMM procedure proposed by [De Loecker and Warzynski \(2012\)](#).¹⁵

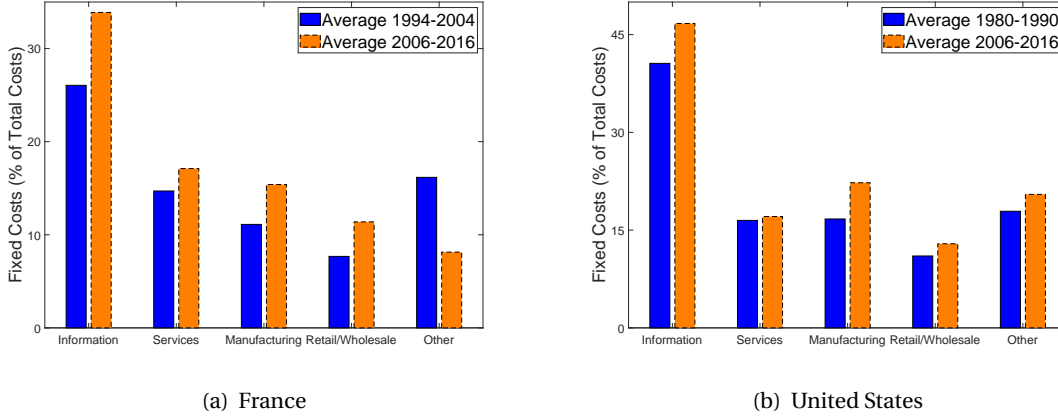
2.4. Empirical Analysis

Figure 4 depicts the sales-weighted average ratio of fixed to total costs as measured along equation (3). In line with trends in intangible such as software, the measure shows a persistent increase over the sample in both France and the United States. Fixed costs made up 9% (13%) of costs for French (American) firms at the lowest point over the sample, and close to 15% (24%) at the highest.¹⁶ Over the entire episode there is a greater increase in fixed costs for U.S. listed firms, but this seems to be due to the difference in time samples. Between 1995 and 2015, firms in both datasets have an average increase in the fixed costs share of approximately 5 percentage points. The increasing trend in fixed costs is robust to using alternative estimates for the markup, as shown in Appendix C. The appendix also contains a between-within decomposition which shows that 73% of the increase in fixed costs in France can be explained by a reallocation within 2-digit sectors, while the entirety of the increase in fixed costs of U.S. public firms occurs within sectors. An illustration of the sectoral composition of fixed costs is provided in Figure 5. It shows that fixed costs are especially high in the information sector, while variable costs are relatively important in retail and wholesale. Nearly all broad sectors have seen an increase in their share of fixed in total costs.

¹⁵Details are provided in Appendix C. The advantage of this approach to estimating markups is that it does not assume any form of market structure or competition, and is consistent with the framework in Section 2.1. Furthermore, markups are estimated based on a single variable input m . Other inputs may be fixed, variable or a combination of both: as long as one freely-set variable input is observed the markup is estimated consistently.

¹⁶The level of the fixed-cost measure mostly depends on the estimate of the supply elasticity that is used to calculate markups. Some estimations of these elasticities are consistently lower than the level used for fixed costs in Figure 4, and therefore imply a lower level of fixed costs. The trend was similar across estimations, however. Appendix C contains a full robustness check of all results in this section using different production function estimates.

Figure 5. Weighted-Average Ratio of Fixed Costs to Total Costs across Sectors



Notes: Sales-weighted average of fixed costs fraction by sector for universe of French firms (left) and U.S. listed firms (right). Sectors are ordered by the average fixed-cost share in the last ten years of the French sample. Industry definitions for France (NACE/ISIC): JB, JC for information, I, M, N for services, B, C, D, E for manufacturing, and G for wholesale and retail; for the United States (NAICS): 51 for information, 64 and above for services, 31, 32 for manufacturing, and 42, 44, 45 for wholesale and retail.

I next assess the relationship between the rise of fixed costs and the rise of intangible inputs. The framework in Section 2.1 implies that firms with higher intangible inputs should have greater fixed costs as a fraction of total costs, and that this fraction should increase when firms make additional investments in software. This can be tested using the French data, as it contains various measures of investments in software and information technology. The additional data comes from two surveys that are based on a (post-weighted) representative sample. The first is the *Enquête Annuelle d'Entreprises* (EAE), which is an annual survey of around 12,000 firms between 1994 and 2007. The survey provides a comprehensive panel of firms with more than 20 employees, and samples smaller firms in most sectors. I use this survey to obtain the amount that firms spend on software, either developed in-house or purchased externally.¹⁷ I estimate the following regression:

$$\frac{\tilde{f}_{it}}{tC_{it}} = \alpha_i + \psi_t + \gamma \cdot \frac{f_{it}}{p_{it} \cdot y_{it}} + \beta' g(p_{it} \cdot y_{it}) + \varepsilon_{it},$$

where f_{it} is observed software in Euro, g is a polynomial of size controls, while α_i and ψ_t respectively denote firm- and time fixed effects. Fixed effects are feasible because the full coverage of larger firms gives a sufficiently large panel. Results are presented in Table 2. Observations are weighted by their sample weights and variables are winsorized at their 1% tails. The table shows a consistently positive relationship between software investments and fixed costs, though the strength of the relationship depends on the inclusion of fixed effects. The latter may be due to the fact that only firms with more than 20 employees are sampled more than once. The fact that the positive relationship is also present when controlling for firm-fixed effects suggests that fixed

¹⁷The survey was previously used to measure firm-level software investments by Lashkari and Bauer (2018). I follow their steps when constructing the variables of interest. Details are provided in Appendix B.

Table 2: Relationship between Software Spending and Fixed-Cost Share (France)

<i>Fixed-Cost Share</i>	I	II	III	IV	V	VI
Software Investments	5.60*** (0.235)	5.19*** (0.235)	3.03*** (0.242)	2.69*** (0.242)	1.45*** (0.138)	0.55*** (0.127)
Year fixed effects	No	Yes	No	Yes	No	Yes
Firm fixed effects	No	No	No	No	Yes	Yes
Industry fixed effects	No	No	Yes	Yes	No	No
Size Poly.	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.125	0.132	0.289	0.295	0.073	0.196
Observations	136,208	136,208	136,208	136,208	136,208	136,208

Dependent variable is fixed costs as a percentage of total costs. Explanatory variable is software investments as a percentage of sales.

Sales is deflated with the sector-specific gross output deflator, software with the investment input deflator from EU-KLEMS.

Firm-clustered standard errors in parentheses. *, **, *** denote significance at the 10, 5 and 1% levels, respectively.

costs increase when firms increase their use of software. This supports the assumption to model intangible inputs as endogenous fixed costs in production. The coefficients in Table 2 are economically significant: a firm that moves from the median to the 95th percentile of software investments increases its fixed-cost share by 0.4 (column VI) to 4 (column I) percentage points.

I next show that there is also a positive relationship between fixed costs and the adoption of specific information technologies. Data on technology adoption comes from the *Enquête sur les Technologies de l'Information de la Communication* (TIC). This survey contains questions on the use of IT systems from 2008 to 2016 and covers an annual sample of around 10,000 firms with at least ten employees. The estimation equation for the TIC data reads

$$\frac{\tilde{f}_{ijt}}{tc_{ijt}} = \alpha_j^h + \psi_t^h + \gamma^h \cdot T_{ijt}^h + (\beta^h)' g(p_{ijt} \cdot y_{ijt}) + \varepsilon_{ijt}^h,$$

where T_{ijt}^h is a dummy that equals one if firm i in 5-digit industry j has adopted technology h . The TIC samples different firms each year and is therefore not a panel, except when firms have been sampled multiple times. This is mainly the case for large firms, which makes the sample unrepresentative as a panel. The specification therefore includes industry rather than firm effects. Though the TIC contains various measures of technology adoption, I focus on five production technologies that are available for a number of years and that are likely to capture s_{it} in the framework. Table 3 presents the results. The top of each column presents the technology used for T_{ijt}^h . ERP refers to enterprise resource planning, CRM to customer resource management software, CAO to collaborative design tools, SCM to supply chain management software and RFID to radio frequency identification. The explanatory variable in the final column is a dummy that equals one if the firm employs IT specialists. Observations are weighted by their sample weights for representativeness. Except for SCM they have a strong positive correlation with the share of fixed costs. The estimates are economically significant: a firm that uses ERP on average has a fixed cost ratio that is 1.5 percentage points higher than similarly-sized firms in the same 5-digit industry without ERP.

Table 3: Relationship between Technology Adoption and Fixed-Cost Share (France)

<i>Fixed-Cost Share</i>	Software Adoption					Skill
	ERP	CRM	CAO	SCM	RFID	
Adoption Dummy	0.015*** (0.002)	0.006*** (0.002)	0.020*** (0.006)	0.004 (0.003)	0.023*** (0.006)	0.045*** (0.004)
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Industry fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Size Poly.	Yes	Yes	Yes	Yes	Yes	Yes
<i>N</i>	63,928	69,200	30,415	45,685	16,847	46,806
<i>R</i> ²	0.320	0.319	0.317	0.346	0.385	0.355

Notes. Explanatory variable is a dummy for the adoption of the technology specified in the column header (details provided in main text). Industry fixed effects at the 5-digit NACE level. Firm-clustered standard errors in parentheses. *, **, *** denote significance at the 10, 5 and 1% levels, respectively. Observation counts differ as not every measure was included in each survey year.

The results so far support the notion that intangible inputs in the form of software investments or IT system adoption raise fixed costs. I next show that firms with higher fixed costs also have higher markups, investment more in research and development, and have higher rates of sales growth. These relationships are essential because this paper's theoretical analysis builds on the premise that high-intangible firms have higher markups and profits, causing them to have a greater incentive to invest in research and development and to subsequently grow faster. I test whether (conditional) correlations run in the appropriate direction in the French and the U.S. data, with the cautionary remark that this does not imply causality.

I start by estimating the following linear regression with OLS:

$$\mu_{it} = \alpha_i + \psi_t + \gamma \cdot \frac{\tilde{f}_{it}}{tc_{it}} + \beta' g(p_{it} \cdot y_{it}) + \varepsilon_{ijt}, \quad (4)$$

where μ_{it} denotes the markup of firm i in year t , estimated using De Loecker and Warzynski's (2012) iterative GMM implementation of the Hall (1988) markup equation, \tilde{f}_{it} denotes fixed costs as estimated in Section 2, $g(\cdot)$ is a polynomial of real sales to control for size, while α_i and ψ_t respectively denote firm- and time fixed effects. Results are presented in Table 4. The first four columns present OLS estimates for both the French and American samples. The results show a robustly positive correlation between fixed costs and markups, also when accounting for firm fixed effects. Column IV suggests that an increase of the fixed ratio by 10 percentage points is associated with a markup that is 12.8 percentage points (France) or 16.6 percentage points higher (United States).

The positive correlation in Table 4 may be spurious because measurement error in markups correlates positively with measurement error in fixed costs through (3). To address this, columns V and VI re-estimate this relationship with two-staged least squares. In the first stage I regress the fixed-cost share on a third-degree polynomial of software intensity, the dependent variable in Table 2. The fitted value of that regression is the explanatory variable in the second-stage regressions, such that the coefficient is only based on variation in fixed costs that is due to differences in software intensity. Column VI's estimate suggests that measurement error in OLS estimates indeed

Table 4: Relationship between Markups and Ratio of Fixed- to Total Costs

	OLS				2SLS	
	I	II	III	IV	V	VI
<i>French Firms in FICUS-FARE (1994-2016)</i>						
Fixed-Cost Share	1.74*** (0.002)	1.74*** (0.002)	1.26*** (0.002)	1.28*** (0.002)	1.85*** (0.449)	0.67*** (0.224)
R^2	0.71	0.71	0.49	0.52		
First Stage F-stat.					242	16.6
Observations	9,457,679	9,457,679	9,457,679	9,457,679	140,861	140,861
<i>U.S. Compustat Firms (1980-2016)</i>						
Fixed-Cost Share	2.54*** (0.025)	2.54*** (0.025)	1.63*** (0.030)	1.66*** (0.031)		
R^2	0.65	0.65	0.63	0.62		
Observations	125,231	125,231	125,231	125,231		
Year fixed effects	No	Yes	No	Yes	No	Yes
Firm fixed effects	No	No	Yes	Yes	No	Yes
Size polynomial	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Firm-clustered standard errors in parentheses. *** denotes significance at the 1% level. Size is controlled for through a third-degree polynomial in log real sales. 2SLS regressions instrument fixed costs through a third-degree polynomial in the ratio of software to sales. Variables are winsorized at 1% and 99% tails.

causes an overestimation on the correlation between markups and fixed costs, though an economically and statistically significant relationship remains.

I next assess the relationship between the ratio of fixed- to variable costs and a firm's research and development activities. Data on innovation activities for French firms is obtained from the *Enquête Communautaire sur L'Innovation* (CIS). The CIS was held in 1996 and 2000, and biannually since 2004. The main variable from this dataset is expenditures on research and development, including externally purchased research and development and expenditures on external knowledge or innovation-related capital expenditures. For Compustat firms, I use research and development expenditures from the income statement (*xrd*).¹⁸ The estimation equation reads

$$\frac{rd_{it}}{p_{it} \cdot y_{it}} = \alpha_i + \psi_t + \gamma \cdot \frac{\tilde{f}_{it}}{tc_{it}} + \beta' g(p_{it} \cdot y_{it}) + \varepsilon_{ijt}, \quad (5)$$

where R&D intensity is the dependent variable, as is standard in the literature (e.g. Hall et al. 2010). Results are presented in Table 5. The upper panel represents results for the French survey data, while the bottom panel presents results for the U.S. data. Upon adding firm fixed effects (columns III and IV), the tables present similar coefficients: firms with higher fixed shares are likely to invest more in research and development. The coefficients are reasonably large: average firms in Compustat invest 3.7% of their sales on R&D over the sample, and this number increases by 0.34 percentage points if the fraction of fixed in total costs increase by 10 percentage points.

¹⁸According to U.S. accounting standards research and development is expensed in Compustat and therefore negatively affects profit. I correct for this by adding *xrd* to the profitability measure in equation (3).

Table 5: Relationship between Research & Development and Ratio of Fixed- to Total Costs

	I	II	III	IV
<i>French Firms in FICUS-FARE (1996-2016)</i>				
Fixed-Cost Share	0.024*** (0.001)	0.023*** (0.001)	0.027*** (0.005)	0.019** (0.005)
R^2	0.007	0.012	0.003	0.016
Observations	92,536	92,536	92,536	92,536
<i>U.S. Compustat Firms (1980-2016)</i>				
Fixed-Cost Share	0.114*** (0.003)	0.106*** (0.003)	0.037*** (0.003)	0.034*** (0.003)
R^2	0.16	0.17	0.13	0.15
Observations	125,231	125,231	125,231	125,231
Year fixed effects	No	Yes	No	Yes
Firm fixed effects	No	No	Yes	Yes
Size polynomial	Yes	Yes	Yes	Yes

Notes: Firm-clustered standard errors in parentheses. ** and *** denote significance at the 5 and 1% level, respectively. Size is controlled for through a third degree polynomial in log real sales. Variables are winsorized at 1% and 99% tails.

Table 6 presents the regression coefficients from an estimation of equation (5) with the growth of sales as an alternative dependent variable. The explanatory variable is lagged to prevent a mechanically negative relationship through sales shocks, because fixed costs as a percentage of total costs fall inherently when sales increase unexpectedly.¹⁹ Though point estimates vary, there is a clear positive relationship between growth and fixed costs. Jointly, the correlations in this section provide preliminary evidence on favor of the mechanisms on which the model relies.

Table 6: Relationship between Sales Growth and Ratio of Fixed- to Total Costs

	I	II	III	IV
<i>French Firms in FICUS-FARE (1994-2016)</i>				
Lagged Fixed-Cost Share	0.155*** (0.001)	0.155*** (0.002)	0.455*** (0.002)	0.514*** (0.002)
R^2	0.082	0.084	0.057	0.049
Observations	8,670,007	8,670,007	8,670,007	8,670,007
<i>U.S. Compustat Firms (1980-2016)</i>				
Lagged Fixed-Cost Share	0.125*** (0.009)	0.132*** (0.009)	0.055*** (0.025)	0.107*** (0.025)
R^2	0.014	0.037	0.13	0.15
Observations	111,397	111,397	111,397	111,397
Year fixed effects	No	Yes	No	Yes
Firm fixed effects	No	No	Yes	Yes
Size polynomial	Yes	Yes	Yes	Yes

Notes: Firm-clustered standard errors in parentheses. *** denotes significance at the 1% level. Size is controlled for through a third-degree polynomial in log real sales. Variables are winsorized at 1% and 99% tails.

¹⁹A lag is furthermore appropriate because the effect of higher R&D investments by high- ϕ firms is unlikely to be immediate. Taking additional lags (e.g. the second or third) rather than the first lags also yields a significantly positive relationship between fixed costs and sales growth.

3. Intangibles, Firm Dynamics and Growth

I now embed scalable intangible inputs into a model of firm dynamics and endogenous growth in the spirit of [Klette and Kortum \(2004\)](#). The model features endogenous entry and exit, multi-product firms and heterogeneous markups in a general equilibrium setting.

3.1. Preferences and Market Structure

A continuum of identical households with unit mass choose the path of consumption that maximizes the following utility function:

$$U = \int_0^\infty \exp(-\rho \cdot t) \cdot \ln C_t dt,$$

where C_t is aggregate consumption and ρ is the discount factor.²⁰ Time is continuous and indexed by t , which is suppressed when convenient. The household is endowed with a single unit of labor which is supplied inelastically. The consumption good is composed of a continuum of intermediate goods, indexed by j . Each good can be produced by the set of firms I_{jt} that own the production technology, a patent, to produce good j at a certain level of quality $q_{ij} \geq 0$. The level of quality determines the value that each unit of a good produced by a firm $i \in I_{jt}$ contributes to aggregate consumption. The intermediate goods are competitively aggregated with the following Cobb-Douglas technology:

$$Y = \exp \int_0^1 \ln \left(\sum_{i \in I_j} q_{ij} \cdot y_{ij} \right) dj,$$

where Y denotes aggregate output, and $y_{ij} \geq 0$ is the amount of good j that is produced by firm i . All output is consumed such that $Y = C$.²¹

The firms that own the patent to produce good j compete à la Bertrand. This implies that, while multiple firms own the patent to produce good j at some level of quality, only one firm will produce the good in equilibrium. In a model where firms have identical production technologies this would always be the firm with the state-of-the-art patent that allows the firm to produce j at the highest quality level. In this paper's setup, intangibles create heterogeneity in production efficiency, which causes some firms to produce at lower marginal costs than others. It is optimal for the profit-maximizing aggregator to only demand good j from the firm that offers the highest combination of output and quality ($q_{ij} \cdot y_{ij}$) at a given expenditure. In other words, goods will be produced by the firm that is able to offer the lowest quality-adjusted price p_{ij}/q_{ij} .

²⁰It is straightforward to generalize the setup to feature a CRRA utility function. This would change the Euler equation and the relationship between discount factor ρ and interest rate r and hence require a different calibration of ρ .

²¹The Cobb-Douglas aggregator implies that the demand function has a unit elasticity such that prices of producers in the Bertrand-Nash equilibrium are bound by the marginal cost of the second-best firm. A generalization to CES would imply a similar bound on prices, up to the point that the wedge in marginal costs between the first- and second-best firms exceeds the monopolist markup (see, e.g., [Lentz and Mortensen 2008](#)). This gives rise to a kink in the profit function and puts a ceiling on the model's predicted markups. Given the absence of such a ceiling on markups in the data and to preserve tractability, I rely on the Cobb-Douglas technology instead.

3.2. Firms and Intangibles

There is a continuum of firms, indexed by i . In the spirit of [Klette and Kortum \(2004\)](#), firms are able to produce all goods for which they have a patent in their portfolio $J_{it} = \{q_{ij} : j \in \text{patents owned by } i\}$. This means that firms (potentially) produce more than one good. Given the market structure, firms produce the set of goods $\tilde{J}_{it} \in J_{it}$ for which they are able to offer the lowest quality-adjusted price p_{ij}/q_{ij} . A firm that does not produce any good in its patent portfolio exits the economy.

Following the general setup in Section 2, firms use tangible and intangible inputs. They choose the optimal fraction $s_{ij} \in [0, 1]$ by which they reduce their marginal costs through the use of intangibles. Firms optimize this fraction separately for each good that they produce and choose s_{ij} before production occurs each period. To preserve tractability, the only tangible input is production labor, such that intangibles allow firms to cut the amount of labor required to produce an additional unit of output. The production function reads

$$y_{ij} = \frac{1}{1 - s_{ij}} \cdot l_{ij}, \quad (6)$$

where l_{ij} denotes production labor dedicated by i to good j .²² The marginal cost of producing j equals $mc_{ij} = (1 - s_{ij}) \cdot w$, where w is the wage rate. The reduction in marginal costs through the use of intangibles comes at a cost $f(s_{ij}, \phi_i)$. This function satisfies the properties of the fixed-cost function in Section 2: fixed costs increase exponentially in s_{ij} , firms that do not reduce their marginal costs pay no fixed costs, and the costs of reducing marginal costs fully ($s_{ij} \rightarrow 1$) are infinite. To allow a quantification of the model I choose the following functional form:

$$f(s_{ij}, \phi_i) = (1 - \phi_i) \cdot \left(\left[\frac{1}{1 - s_{ij}} \right]^\psi - 1 \right), \quad (7)$$

where ψ is a curvature parameter and ϕ_i captures the efficiency with which firms are able to implement intangible technologies. Firms draw their type ϕ_i from a known discrete distribution $G(\phi)$ at birth and benefit from their level of intangible efficiency on each good that they produce. Note that fixed costs are not sunk as firms pay the fixed costs at each time t . The motivation for that is twofold. First, [Li and Hall \(2016\)](#) estimate depreciation rates of software investments to range between 30 and 40% per year. This implies that firms must spend considerable amounts each year to maintain a constant level of software. Second, an increasing share of enterprise software is sold *as a service* (SaaS), which means that firms pay periodic licence fees instead of an upfront cost for perpetual use.²³ Note that this does not mean that the model features no accumulation of intangible capital in the spirit of, e.g., [Corrado et al. \(2009\)](#). Firms also invest in research and development, and these investments have long-term effects on both firm size and national income.

²²In independent work, [Korinek and Ng \(2019\)](#) also model digitization as a shift from marginal to fixed costs. Rather than having heterogeneous fixed costs of marginal cost reduction, their model features heterogeneity in the maximum fraction of marginal costs that firms are able to cut.

²³For example, 35% of Microsoft's enterprise sales in Q2 of 2019 came from SaaS, at an annual growth of 48%.

3.3. Innovation

3.3.1. R&D Investments

Firms expand their portfolio of patents by investing in research and development (R&D). When investing, firms choose the Poisson flow rate $x_i \geq 0$ with which a new patent is added to their portfolio. In exchange for achieving x_i , firms employ rd^x scientists along

$$rd^x(x_i) = \eta^x \cdot x_i^{\psi^x} \cdot n_i^{-\sigma}, \quad (8)$$

where $\psi^x > 1$ and $\eta^x > 0$. The number of researchers that the firm employs is convex in the rate of innovation and declines in the number of goods that the firm produces, n_i . The former implies that the marginal return to R&D is diminishing within each time t . The latter is an assumption from [Klette and Kortum \(2004\)](#), and reflects the assumption that large firms have more in-house knowledge or organizational capital than small firms. Practically, the presence of $n_i^{-\sigma}$ governs the relationship between firm size and firm growth. For $\sigma = \psi^x - 1$, the model satisfies Gibrat's law of constant firm growth in size, while for $\sigma = 0$ a firm's growth declines rapidly with size. Following [Akcigit and Kerr \(2018\)](#), I allow for an intermediate case between these two extremes, and estimate $\sigma \in [0, \psi - 1]$ by targeting the empirical relationship between size and growth in the data.

A firm that innovates successfully becomes the owner of a state-of-the-art patent for a random good j . Innovation is not directed, in the sense that firms are equally likely to innovate on all products. As in [Aghion and Howitt \(1992\)](#), the state-of-the-art patent allows firm i to produce its new good at a quality level that is a multiple $(1 + \lambda_{ij})$ of the level of the current producer of the good:

$$q_{ij} = q_{-ij} \cdot (1 + \lambda_{ij}),$$

where $-i$ denotes the incumbent of good j while λ_{ij} denotes the realized innovation step size, which is drawn from an exponential distribution with mean $\bar{\lambda}$:

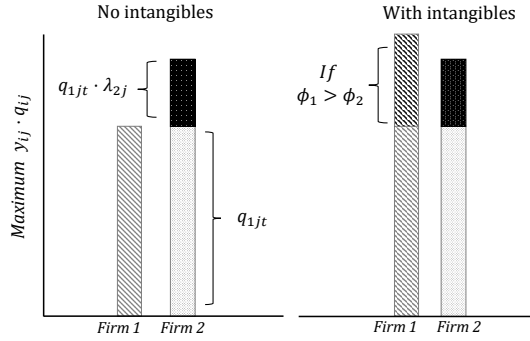
$$\lambda \sim \text{Exp}(\bar{\lambda}).$$

The level of firm i 's quality reflects the increase in quality from its innovation to product j , as well as the increases from all past successful innovations on that good. Because the latter is not internalized when firms choose their optimal level of R&D, the model is characterized by inefficiently low innovative investments, which is a standard feature of Schumpeterian growth models (see, e.g., [Lentz and Mortensen 2016](#)).

3.3.2. Innovation and Intangibles

Innovation in the model is different from the standard [Klette and Kortum \(2004\)](#) setup because the innovator of a certain good will not necessarily become its new producer. Innovators always be-

Figure 6. Innovation with and without Intangible Inputs



Notes: Illustration of the case where Firm 2 develops higher quality version of j , currently produced by Firm 1. The left-hand case is the model without intangibles, where Firm 2 always becomes the new producer. The right-hand case is the model with intangibles, where Firm 1 remains the producer if $\phi_1 > \phi_2$ causes the choke price to be sufficiently lower for Firm 1.

come the producer in other models because firms have identical marginal cost while the innovator owns the patent to produce at the highest quality level. Here, the owner of a lower-quality patent may still be the sole producer if it can offer the best combination of quality and price. The lowest price that the incumbent and the innovator are willing to set are their respective choke prices. The choke price $p^{choke}(\phi_i)$ is the price at which, after payment of the fixed costs, firm profits are zero.²⁴ If the incumbent has a lower choke price than the innovator does, the incumbent can undercut the innovator on price if the quality of the innovator is sufficiently close to that of the incumbent. Formally, the innovator only becomes the new producer of a good j that is initially produced by incumbent $-i$ if

$$\frac{q_{ij}}{p^{choke}(\phi_i)} \geq \frac{q_{-ij}}{p^{choke}(\phi_{-i})},$$

where the choke price is a decreasing function of ϕ_i because high- ϕ are able to reduce their marginal costs by a greater fraction for a given expenditure on intangibles. Rewriting yields

$$\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1, \quad (9)$$

The innovator is able to offer product j at a superior quality than the incumbent, but the incumbent can hold on to its product if it has a sufficiently low choke price. A greater difference between the choke prices is needed when the innovator has drawn a significant innovation (the realization of λ_{ij} is high), and the innovator will always become the new producer if its ϕ_i is the same or higher than that of the incumbent.

Figure 6 illustrates the hypothetical case where an innovator is unable to take over production. Firm 1 is the incumbent of the product on which firm 2 innovates. In other [Klette and Kortum](#)

²⁴This is with slight abuse of notation as $p^{choke}(\phi_i)$ also depends on output Y and wage w . It is expressed only in terms of ϕ_i because the ratio of choke prices between any two firms only depends on their relative ϕ s.

(2004) models, firm 2 becomes the new producer of the good because it is able to produce at greater quality. That is not necessarily the case in this model, however, because firm 1 may be of a higher ϕ -type than firm 2. Firm 1 could sell its product at a lower price, allowing consumers to compensate for the lower quality of the good by purchasing a greater quantity.

3.3.3. Quality and Intangibles

It is useful to highlight the difference between quality and price in this framework. In most models of growth through creative destruction the two are isomorphic. Prices reflect the ability of firms to produce at low marginal costs (that is; with high productivity). It may seem that this is equivalent to quality, in the sense that a firm can achieve a higher level of effective output $q_{ij} \cdot y_{ij}$ using the same tangible inputs through either selling at higher quality or deploying a greater share of intangibles.

The difference between the two lies in the extent to which they contribute to long-term growth. Innovation leads to an increase in the state-of-the-art level of quality q_{ij} with which good j can be produced. If an innovating firm successfully takes over production, this offers both a private benefit and an economy-wide benefit. The private benefit is the stream of profit that the firm earns while it produces j . The economy-wide benefit is the fact that all future innovations on j are step-wise improvements over q_{ij} : the innovation by firm i allows good j to be produced at a permanently higher quality. This positive externality makes the step-wise improvement of quality across products the source of long-term economic growth.

Intangibles do not come with a similar externality. They only improve production efficiency for the current producer. Intuitively, the fact that the incumbent is efficient at using software applications to reduce marginal costs does not benefit an innovating firm when it takes over production at some point in the future.

3.4. Entry and Exit

There is a mass of entrepreneurs that invest in R&D to obtain patents to produce goods that are currently owned by incumbents. The R&D cost function is analogous to the cost function for innovation by incumbents:

$$rd^e(e) = \eta^e \cdot e^{\psi^e}, \quad (10)$$

where $rd^e(e)$ denotes the number of researchers employed by potential entrants to achieve start-up rate e , and where $\eta^e > 0$, $\psi^e > 1$. Entrepreneurs that draw an innovation improve the quality of a random good that is currently produced by an incumbent. In similar spirit to models where firms draw idiosyncratic productivities at birth (e.g. [Hopenhayn 1992](#), [Melitz 2003](#)), entrants then draw their intangible productivity $\phi_e \in \Phi$ from the known distribution $G(\phi)$, and learn about the intangible efficiency of their incumbent. The entrant becomes the new producer of its good if it has drawn a sufficiently large step-size λ_{ej} to overcome any difference between its choke price and the choke price of its incumbent, along condition (9).

A firm exits the economy if it does not produce any good in its patent portfolio J_i . This happens when entrants or other incumbents develop higher-quality versions of the sole good that a firm produces, as explained in the next section.

3.5. Creative Destruction

Firms cease to produce a good if a different incumbent or an entrant successfully innovates on that product. The rate at which this happens is the rate of creative destruction, $\tau(\phi_i)$. The rate of creative destruction is endogenous, as it is determined by the respective efforts that incumbents and entrants put into innovation. It is a function of the firm's intangible efficiency ϕ_i , because a firm with a relatively high intangible efficiency is more likely to be able to undercut an innovative challenger on price. The rate of creative destruction for a firm with efficiency ϕ_i is given by

$$\tau(\phi_i) = \sum_{\phi_h \in \Phi} \text{Prob} \left(\lambda_{ih} \geq \frac{p^{choke}(\phi_h)}{p^{choke}(\phi_i)} - 1 \right) \cdot \left[\sum_{n=1}^{\infty} M(\phi_h, n) \cdot x(\phi_h, n) + e \cdot G(\phi_h) \right] \quad (11)$$

where $M(\phi_h, n)$ denotes the measure of firms with intangible efficiency ϕ_h that produce n products.²⁵ The outer-summation reflects that an incumbent with intangible efficiency ϕ_i faces innovative competitors from each intangible efficiency level $\phi_h \in \Phi$. Within the summation there are two terms: the probability that an innovation by a firm with efficiency ϕ_h is successful, multiplied by innovation efforts by firms with that level of efficiency. Under the exponential distribution, the probability that condition (9) is satisfied when i is the incumbent and h is the innovator equals

$$\text{Prob} \left(\lambda_{hj} \geq \frac{p^{choke}(\phi_h)}{p^{choke}(\phi_i)} - 1 \right) = \bar{\lambda}^{-1} \exp \left(-\bar{\lambda}^{-1} \cdot \left[\frac{p^{choke}(\phi_h)}{p^{choke}(\phi_i)} - 1 \right] \right), \quad (12)$$

where the right-hand side is the cumulative density function of the exponential distribution with mean $\bar{\lambda}$. This probability (and hence the creative destruction rate) is strictly lower when the incumbent is a high- ϕ firm, as these have a lower choke price. The term for innovation effort contains two parts. The first captures innovation efforts by incumbents of type ϕ_h . As is shown below, a firm's innovation effort is a function of its intangible efficiency as well as the number of products it currently produces, which explains the inclusion of the summation over n . The Poisson rate $x(\phi_h, n)$ is multiplied by the measure $M(\phi_h, n)$ to obtain total innovation effort. The second term measures innovative activities by entrants of type ϕ_h . It is equal to the entry rate e multiplied by the probability $G(\phi_h)$ that the entrant has that level of intangible efficiency.

3.6. Optimal Pricing and Intangibles

The firm maximizes operating profits for each good it produces by statically choosing the optimal price p_{ij} and the fraction by which it reduces marginal costs s_{ij} . The optimal price is determined

²⁵In a model with a discrete number of firms, the measure of firms $M(\phi_h, n)$ would simply be the number of firms with efficiency ϕ_h that produce n products. Here, it is the fraction of products produced by these firms, divided by n .

by the wedge between the firm that produces good j and the efficiency of the second-best firm for that good. The following timing assumption applies. At the start of each time t , all firms with a patent to produce good j observe the qualities and intangible efficiencies of all firms with a patent to produce good j . They then choose s_{ij} and commit to paying the associated fixed costs $f(s_{ij}, \phi_i)$ and subsequently post their prices and produce the goods demanded by consumers. In the Nash equilibrium of the associated simultaneous move game, firms that are unable to offer the lowest quality-adjusted price have no incentive to set $s_{ij} > 0$. Their marginal cost therefore equals the wage w . The demand for output from the firm with the lowest quality-adjusted choke price has a unit demand elasticity: $\frac{y_{ij}}{Y} = p_{ij}^{-1}$. The profit-maximizing price of the firm i with the lowest choke price is therefore bound by the marginal cost of the firm with the second-lowest choke price $-i$, adjusted for differences in quality between both firms:

$$p_{ij} = mc_{-ij} \cdot \frac{q_{ij}}{q_{-ij}},$$

where $-i$ identifies the second-best firm, $mc_{-ij} = w$, and $q_{ij}/q_{-ij} - 1$ is innovation realization λ_{ij} . The markup of firm i is found by dividing the profit-maximizing price by firm i 's marginal cost $w \cdot (1 - s_{ij})$ and by inserting the innovation step-size λ_{ij} for the ratio of qualities:

$$\mu_{ij} = \frac{1 + \lambda_{ij}}{1 - s_{ij}}, \quad (13)$$

which yields that markups increase in the difference in quality between the producer and the second-best firm, as well as the firm's use of intangibles. Note that while intangibles increase the markup, profits do not increase proportionally because the firm incurs an expense on intangibles. A part of the increase in markups is therefore a compensation for fixed costs.

To find the optimal intangible fraction s_{ij} , consider the definition of operating profits:

$$\pi_{ij} = (p_{ij} - mc_{ij}) \cdot y_{ij} - w \cdot f(s_{ij}, \phi_i),$$

where the fixed-cost function (7) is multiplied by w as costs are denominated in terms of labor. Inserting the demand function and markups (13) gives the following first-order condition:

$$s_{ij} = 1 - \left([1 + \lambda_{ij}] \cdot \frac{w}{Y} \cdot \psi \cdot (1 - \phi_i) \right)^{\frac{1}{\psi+1}}, \quad (14)$$

or $s_{ij} = 0$ when the right-hand side is negative. It follows that firms with lower intangible adoption costs are able to reduce their marginal costs by a greater fraction and consequently have higher markups. Note that the firm with the lowest quality-adjusted choke price sets s_{ij} along (14) irrespective of the level of intangible efficiency of the second-best firm, because that firm always sets $s_{-ij} = 0$ in the Nash equilibrium.

3.7. Equilibrium

I now characterize the economy's stationary equilibrium where productivity, output and wages grow at a constant rate g .

3.7.1. Optimal Innovation Decisions

Firms choose the level of spending on research and development that maximizes firm value. The associated value function, where notation is borrowed from [Akcigit and Kerr \(2018\)](#), reads as

$$rV_t(\phi_i, \tilde{J}_i) - \dot{V}_t(\phi_i, \tilde{J}_i) = \max_{x_i} \left\{ \begin{aligned} & \sum_{j \in \tilde{J}_i} \left[\frac{\pi_t(\phi_i, \lambda_{ij}) + \tau(\phi_i) \cdot [V_t(\phi_i, \tilde{J}_i \setminus \{\lambda_{ij}\}) - V_t(\phi_i, \tilde{J}_i)]}{p^{choke}(\phi_i)} \right] \\ & + x_i \cdot \text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \mathbb{E}_{\phi_i} [V_t(\phi_i, \tilde{J}_i \cup \lambda_{ij}) - V_t(\phi_i, \tilde{J}_i)] \\ & - w_t \eta_x(x_i)^{\psi_x} n_i^{-\sigma} - F(\phi_i, n_i) \end{aligned} \right\}$$

The value function is split into two parts. The first part on right-hand side contains the sum of all good-specific items. The first line gives the contemporaneous profits for a firm that sets prices along (13) and the fraction of marginal costs reduced through intangibles along (14). The second line gives the change in firm value if the firm would cease production of good j because of creative destruction by entrants or other incumbents. $V_t(\phi_i, \tilde{J}_i \setminus \{\lambda_{ij}\})$ denotes the value of producing the set of goods \tilde{J}_i except some good j with innovation realization λ_{ij} . The second part is not specific to product lines. The first line gives the expected increase in firm value from external innovation. $V(\phi_i, \tilde{J}_i \cup \lambda_{ij})$ denotes the firm's value if it successfully takes product j from firm $-i$. The change in firm value is multiplied by both the innovation rate as well as the probability that the firm is able to offer a sufficiently low quality-adjusted price. The final line gives the costs of research and innovation and a fixed term $F(\phi_i, n_i)$. Firms must pay the latter to operate, and it is assumed to equal the option value of research and development. This ad-hoc restriction, borrowed from [Akcigit and Kerr \(2018\)](#), ensures that the value function scales linearly in the number of goods that a firm produces, such that the model admits an analytical first-order condition. In Section 6.2 I remove this assumption and show that, though significantly reducing tractability, the results are qualitatively and quantitatively robust.

Proposition 1. *The value function of a type ϕ in the stationary equilibrium firm can be written as*

$$V(\phi_i, \tilde{J}_i) = \frac{\sum_{j \in \tilde{J}_i} \pi(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)},$$

which is increasing in ϕ_i . The optimal rate of innovation reads as

$$x(\phi_i, n_i) = \left(\text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \frac{\mathbb{E}_{\phi_i} \left[\frac{\pi_t(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)} \right]}{\eta^x \cdot \psi^x \cdot w_t} \right)^{\frac{1}{\psi^x - 1}} \cdot n_i^{\frac{\sigma}{\psi^x - 1}}. \quad (15)$$

The optimal entry rate is given by

$$e = \left(\sum_{\phi_e \in \Phi} G(\phi_e) \cdot \text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_h)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \frac{\mathbb{E}_{\phi_h} \left[\frac{\pi(\phi_e, \lambda_{ij})}{r-g+\tau(\phi_e)} \right]}{\eta^e \psi^e w_t} \right)^{\frac{1}{\psi^e - 1}}. \quad (16)$$

Proof: Appendix A.

The first-order condition in (15) is intuitive. Firms engage in more innovation when the expected increase in the value function is larger, and invest less when the innovation cost-parameters are high. Innovation increases in the number of product lines n_i , though if $\sigma < \psi^x - 1$ the firm's expected growth rate will decline with size. Firms that are better at adopting intangible technologies (higher ϕ_i) choose a higher innovation rate because their ability to reduce marginal costs and raise markups increases contemporaneous profits. They furthermore experience a lower rate of creative destruction, which decreases the effective discount factor. Firms with higher ϕ_i s also have a higher probability of becoming the new producer on products that they innovate on, which increases the expected profitability of research and development. Jointly, these effects cause a positive relationship between ϕ_i and the rate of innovation.

Innovation by entrants is such that the marginal cost of increasing the entry rate e is equal to the expected value of producing a single good, adjusted for the probability that the entrant is able to take over production from the incumbent by offering a sufficiently low quality-adjusted price. Because entrants only learn about their type after they have drawn an innovation, the expectation of the value of producing a good is taken over the distribution of firm types at entry $G(\phi)$.

Equation (15) predicts a positive relationship between ϕ_i and innovation efforts at the firm level. In line with this, the empirical analysis in Section 2 finds a significantly positive correlation between a firm's intangibles and its innovative activities and subsequent growth. How is this consistent with a slowdown of productivity growth?

A homogeneous increase of ϕ_i improves profitability for all firms and therefore raises innovation rates and productivity growth. That is not the case, however, when only a fraction of firms receive a higher intangible efficiency. High- ϕ_i firms would have a greater incentive to invest in research and development, which leads them to produce a disproportionate fraction of all goods. This has two negative externalities. First, the incentives to engage in research and development for lower- ϕ_i firms decline as some of their innovations are now unsuccessful. Second, there is a decline in the rate of entry; because high- ϕ_i firms expand, it is more likely that entrants face a high- ϕ_i incumbent than that they, themselves, are high- ϕ_i firms. In Section 5, I show that these externalities undo the positive effect of the high innovation rates by high- ϕ_i firms for a wide range of calibrations. Indeed, the increase in research and development by high- ϕ_i can be so large that aggregate research and development spending increases (in line with Figure 1), but growth declines because the spending is concentrated among a smaller group of firms.

3.7.2. Dynamic Optimization by Households

Maximizing life-time utility with respect to consumption and savings subject to the standard budget constraint gives rise to the standard Euler equation, combined with the standard transversality condition:

$$\frac{\dot{C}}{C} = r - \rho. \quad (17)$$

Along the balanced growth path, consumption grows at the same rate as output and productivity, such that

$$r - g = \rho.$$

3.7.3. Firm Measure and Size Distribution

The optimal innovation rate in (15) is a function of a firm's intangible input efficiency ϕ_i and the number of goods n_i it produces. The rate of creative destruction (and hence the growth rate of output and productivity) therefore depends on the equilibrium distribution of n and ϕ across firms. Along the balanced growth path, these distributions are stationary. To find the stationary distributions, consider the law of motion for the measure of firms that produce more than one product:

$$\begin{aligned} \dot{M}(\phi_i, n) &= (M(\phi_i, n-1) \cdot x(\phi_i, n-1) - M(\phi_i, n) \cdot x(\phi_i, n)) \cdot \\ &\text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) + (M(\phi_i, n+1) \cdot [n+1] - M(\phi_i, n) \cdot n) \cdot \tau(\phi_i), \end{aligned} \quad (18)$$

where the first term captures entry into and exit out of mass $M(\phi_i, n)$ through innovation by firms of type ϕ_i with $n-1$ products and n products, respectively. The second term captures entry and exit of firms with $n+1$ and n products that ceased producing one of their products through creative destruction. For the measure of single-product firms, the law of motion reads as

$$\dot{M}(\phi_i, 1) = (e \cdot G(\phi_i) - x(\phi_i, 1) \cdot M(\phi_i, 1)) \cdot \text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) + (2 \cdot M(\phi_i, 2) - M(\phi_i, 1)) \cdot \tau(\phi_i). \quad (19)$$

The stationary properties of the firm-size distribution follow from setting both equations to zero, which is done iteratively. The fraction of the unit measure of goods that is produced by firms with intangible efficiency ϕ_i is given by

$$K(\phi_i) = \frac{\sum_{n=1}^{\infty} n \cdot M(\phi_i, n)}{\sum_{\phi_h \in \Phi} \sum_{n=1}^{\infty} n \cdot M(\phi_h, n)}. \quad (20)$$

3.7.4. Labor Market Equilibrium

The solutions to the static and dynamic optimization problems of firms allow the labor market equilibrium conditions to be defined. Labor is supplied inelastically by households at a measure

standardized to 1. Equilibrium on the labor market requires that employment of workers on the various types of work in the economy satisfies

$$1 = L^p + L^f + L^{rd} + L^e,$$

where L^p is the labor used to produce intermediate goods. Inserting the unit-elastic demand function, markup (13) and intangible first-order condition (14) into $L^p = \int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} l_{ij} di dj$ yields

$$L^p = \int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \cdot \frac{Y}{w} \cdot \left[1 - \left([1 + \lambda_{ij}] \cdot \frac{w}{Y} \cdot \psi \cdot (1 - \phi_i) \right)^{\frac{1}{\psi+1}} \right] \cdot (1 + \lambda)^{-1} di dj,$$

where $\mathbf{1}_{j \in \tilde{J}_i}$ is the indicator function that equals one when firm i produces good j . L^f is the labor used to fulfill the intangible fixed costs:

$$L^f = \int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \cdot \left[\left([1 + \lambda_{ij}] \cdot \frac{w}{Y} \cdot \psi \cdot (1 - \phi_i) \right)^{-\frac{\psi}{\psi+1}} - 1 \right] \cdot (1 - \phi_i) di dj.$$

L^{rd} is the labor involved with research and development carried out by existing firms:

$$L^{rd} = \sum_{\phi_i \in \Phi} \sum_{n=1}^{\infty} \left[M_{\phi_i, n} \cdot \eta^x \cdot x(\phi_i, n)^{\psi^x} \right],$$

while L^e is the labor involved with research and development carried out by entrants $L^e = \eta^e \cdot e^{\psi^e}$, where innovation rates $x(\phi_h, n)$ and e are the dynamically optimized along (15) and (16).

3.7.5. Aggregate Variables

I can now characterize the economy's aggregate variables. The equilibrium wage is given by

$$w = \exp \left(\int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \cdot \ln \left[\frac{q_{ij}}{1 - s_{ij}} \right] di dj \right) \cdot \exp \left(\int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \cdot \ln \left[\frac{1 - s_{ij}}{1 + \lambda_{ij}} \right] di dj \right). \quad (21)$$

Proof: Appendix A.

The first term of (21) is the standard CES productivity term. The second term is the inverse of the expected markup. Note that a rise in the use of intangibles has no effect on the level of the wage because s_{ij} cancels out. While a firm that deploys more intangibles becomes productive, it is able to proportionally raise its markups. These have offsetting effects on the level of the wage.

Aggregate output is given by

$$Y = L^p \cdot \exp \left(\int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \cdot \ln \left[\frac{q_{ij}}{1 - s_{ij}} \right] di dj \right) \cdot \frac{\exp \int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \cdot \ln \mu_{ij}^{-1} di dj}{\int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \cdot \mu_{ij}^{-1} di dj}. \quad (22)$$

As in the model with heterogeneous markups and misallocation by [Peters \(2018\)](#), the last term captures the loss of efficiency due to the dispersion of markups. If all markups are equalized the term is equal to 1, while it declines as the variance of markups increases. Total factor productivity is the product of the second- and the last term in (22).

Equation (22) reveals that a rise in the use of intangibles has two counteractive effects on the level of output. The spread of markups increases when the average s_{ij} increases along (13), because s_{ij} amplifies the heterogeneity in markups caused by the heterogeneous innovation steps (the second term in 22). On the other hand, the increase in s_{ij} has a direct positive effect on total factor productivity because it increases the CES productivity index (the first term in 22). As will be clear below, the second effect dominates the first effect in feasible calibrations. That means that a rise in the use of intangibles initially has a positive effect on the level of output and on total factor productivity. The next proposition shows, however, that this may not be the case for growth along the balanced growth path.

3.7.6. Growth

The growth rate of total factor productivity and output is a function of creative destruction.

Proposition 2. *The constant growth rate of total factor productivity, consumption C , aggregate output Y and wages w is given by*

$$g = \sum_{\phi_i \in \Phi} K(\phi_i) \cdot \tau(\phi_i) \cdot \mathbb{E}_{-\phi_i}(\lambda_{hj}), \quad (23)$$

where $\mathbb{E}_{-\phi_i}(\lambda_{hj})$ is the expected realization of λ_{hj} when a firm with ϕ_i is the incumbent on a product line before a different firm h becomes the new producer due to successful innovation.

Proof: [Appendix A](#).

The proposition states that growth equals the product of the expected increase in quality if a good gets a new producer and the rate at which this happens, weighted by the fraction of product lines that firms of each intangible efficiency own.

Equation (23) shows the counteracting effects of an increase in ϕ at a subset of firms. On the one hand, firms with a higher ϕ have a greater incentive to invest in research and development, which causes the rate of creative destruction to increase. On the other hand, even at a constant innovation rate, the presence of high- ϕ firms has a negative effect on the rate of creative destruction because firms with lower productivities ϕ have a lower probability of successfully becoming the new producer. This has not only a direct effect on growth at given innovation rates, but also an indirect effect as these firms reduce their expenditure on research and development.

3.7.7. Equilibrium Definition

Definition 1. *The economy is in a balanced growth path equilibrium if for every t and for every intangible productivity $\phi_i \in \Phi$ the variables $\{r, e, L^p, g\}$ and functions $\{x(n_i, \phi_i), K_{\phi_i}, M_{\phi_i}, s(\phi_i, \lambda_{ij}), \tau(\phi_i)\}$ are constant, $\{Y, C, w, Q\}$ grow at a constant rate g that satisfies (23), aggregate output Y satisfies (22), innovation rates $x(n_i, \phi_i)$ satisfy (15), the entry rate e satisfies (16), firm distribution K_{ϕ_i} and measure M_{ϕ_i} are constant and satisfy (18) and (19), markups $\mu(\phi_i, \lambda_{ij})$ satisfy (13), the fraction of marginal costs reduced through intangibles $s(\phi_i, \lambda_{ij})$ satisfies (14) for all λ_{ij} , the rate of creative destruction $\tau(\phi_i)$ satisfies (11), and both the goods and labor market are in equilibrium such that $Y = C$ and $L^p = 1 - L^s + L^{rd} + L^e$.*

4. Quantification

In this section I set up the model for the analysis of the rise of intangible inputs. I first calibrate the model in Section 4.1 using a combination of parameters from the literature and results from a structural estimation, in order to match empirical characteristics of either the French or the United States economy. In Section 4.2 I then analyse the model's ability to replicate a set of targeted and untargeted moments along the original balanced growth path. The effects of the rise of intangibles on productivity growth, business dynamism and markups is analyzed in Section 5.

4.1. Calibration

In the baseline calibration all firms have the same intangible efficiency ϕ , which leaves nine parameters to be calibrated. Five parameters are calibrated using a structural estimation while four parameters are taken from the literature. The structural estimation is conducted separately for France and the United States, using the micro data from from Section 2.

4.1.1. Externally Calibrated Parameters

The model is calibrated at annual frequency. I calibrate the curvature of research and development for entrants (ψ^e) and incumbents (ψ^x) to 2. This is a key parameter because it determines the concavity of the return to research and development. If innovative activities concentrate among a small number of firms, the fact that $\psi^x > 1$ implies that the average effect of these investments on growth is lower. The literature that studies the elasticity of research and development with respect to the user costs of such activities typically finds elasticities around 0.5 for tax credit changes (see, e.g. Bloom et al. 2002 for a review, or Appelt et al. 2019 for recent evidence). The parameter ψ^x is the inverse of that elasticity. It is the same parameter value as the one used for corresponding parameters in Akcigit and Kerr (2018) and Acemoglu et al. (2018).

I calibrate the curvature parameter ψ of fixed cost function $f(\cdot)$ to match empirical estimates of the pass-through of marginal costs to markups. To see how these are related, note that the first-order conditions for markups (13) and for intangibles (14) imply an equilibrium log markup of

$$\ln \mu_{ijt} = \ln(1 + \lambda_{ij}) - \ln \left((1 + \lambda_{ij}) \cdot \frac{w_t}{Y_t} \cdot \psi \cdot (1 - \phi_i) \right) \cdot \frac{1}{\psi + 1}.$$

The elasticity of marginal costs with respect to wages is $(\psi + 1)/(\psi + 2)$, such that the elasticity of markups with respect to marginal costs at a given level of Y is

$$\frac{\partial \ln \mu_{ijt}}{\partial \ln mc_{ijt}} = -\frac{1}{\psi + 2}.$$

I set ψ to 2, which achieves a pass-through of marginal costs to markups of -25%. Empirical estimates of this elasticity vary. [Amiti et al. \(2019\)](#) find a pass-through of markups to marginal cost shocks of -35% in their main results. In robustness checks on the full sample they find values between -39% and -25%. For firms with fewer than 100 employees they find coefficients of -3%.

The discount rate ρ is set to 0.01, which gives rise to a 2.3% risk-free rate.

4.1.2. Structurally Estimated Parameters

The remaining five parameters are estimated using indirect inference by matching moments from either the French administrative data or the U.S. Compustat data on listed firms. The French calibration targets moments in the first year of the data (1994), or the first available year for variables based on surveys. The U.S. calibration targets moments for 1980, which is the first year that firm variables from Compustat can be complemented with administrative data on business dynamism.

The estimation proceeds as follows. I use the Genetic Algorithm to choose combinations of parameters within broad bounds on their possible values.²⁶ For a given combination of parameters I solve the equilibrium of the model as a fixed point in line with Definition 1 and obtain the equilibrium values for innovation and entry rates, the firm-size distribution, rates of creative destruction and aggregate quantities such as the efficiency wedge, wages and output. Details are provided in Appendix E. I then simulate the economy for 32,000 firms until the the distribution of s_{ij} has converged, and simulate data for five more years to collect moments on the simulated sample.²⁷ The Genetic Algorithm then updates the combinations of parameters based on a comparison of the theoretical and data moments along the following objective function:

$$\min \sum_{k=1}^5 \frac{|\text{model}_k - \text{data}_k|}{(|\text{model}_k| + |\text{data}_k|) \cdot 0.5} \cdot \Omega_k, \quad (24)$$

²⁶The Genetic Algorithm is a method for finding global minimums that is inspired by the process of natural selection. It involves taking convex combinations (children) of parameter vectors (parents). The performance of children on the optimization criteria determines their likely hood of becoming parents in the next generation of the algorithm. The algorithm was significantly better at finding global minimums than alternatives such as Simulated Annealing.

²⁷The firm simulation builds computationally on [Akcigit and Kerr \(2018\)](#) and [Acemoglu et al. \(2018\)](#).

Table 7: Overview of Parameters

Parameter	Description	Method	Value (France)	Value (U.S.)
ρ	Discount rate	External	.010	.010
ψ	Intangibles cost elasticity	External	2.00	2.00
ψ^x	Cost elasticity of innovation (incumbents)	External	2.00	2.00
ψ^e	Cost elasticity of innovation (entrants)	External	2.00	2.00
η^x	Cost scalar of innovation (incumbents)	Indirect inference	1.52	2.49
η^e	Cost scalar of innovation (entrants)	Indirect inference	1.93	2.10
$\bar{\lambda}$	Average innovation step size	Indirect inference	.061	.061
σ	Relationship firm-size and firm-growth	Indirect inference	.622	.587
ϕ	Intangible efficiency	Indirect inference	.752	.800

where model_i and data_i respectively refer to the simulation and data for moment i with weight Ω_i .

The following moments are used for the French calibration. I calibrate the initially homogeneous intangible efficiency parameter ϕ to match the 1994 ratio of fixed to variable costs of 9.5% in Section 2. The cost scalar of research and development by entrants (η^e) is estimated by targeting an entry rate of 10%. This is the fraction of firms that enter the FARE-FICUS dataset for the first time in 1995, the second year for which data is available and therefore the first year that entry is observed. The cost scalar of innovation by existing firms (η^x) is estimated by targeting the average ratio of research and development over sales in the CIS for 1996, which is 3.1%. Following [Akcigit and Kerr \(2018\)](#), I calibrate the parameter that governs the extent to which R&D scales with size (σ) by targeting a regression of size on growth. Specifically, I estimate the following OLS regression:

$$\Delta_i(p \cdot y) = \alpha_s + \beta \cdot \ln(p_i \cdot y_i) + \varepsilon_i, \quad (25)$$

where the left-hand side is the growth rate of sales using the measure of growth in [Davis et al. \(2006\)](#), α_s is a sector fixed effect, and data comes from 1994-1995. The estimated β is -0.035, which implies that a firm with 1% greater sales is expected to grow 0.035% less. The average innovation step-size $\bar{\lambda}$ is estimated by targeting a balanced growth path rate of 1.3%, which is the average growth rate of total factor productivity between 1969 and 1994 in the Penn World Tables.

The United States calibration relies on the American counterpart of the French moments. The intangible efficiency parameter is calibrated by ϕ matching the 1980 ratio of fixed to variable costs of 12% in Compustat. The cost scalar of research and development by entrants (η^e) is estimated by targeting an entry rate of 13.8% for 1980 in the Business Dynamics Statistics. The cost scalar of innovation by existing firms (η^x) is estimated by targeting the average ratio of research and development for firms with positive expenditures over sales in that year, at 2.5%. I calibrate σ to match the coefficient β in (25) in a regression on U.S. firms. [Akcigit and Kerr \(2018\)](#) run this regression on Census data and find a β of -0.035, coincidentally the same coefficient as I find for France. The average innovation step-size $\bar{\lambda}$ is estimated by targeting the average growth rate of total factor productivity between 1969 and 1980 in the Fernald series (1.3%) along the balanced growth path.

Table 7 presents an overview of the calibrated and estimated parameters. The lower R&D intensity of American firms gives rise to a slightly higher estimate of the innovation-cost scalar η^x ,

Table 8: Comparison of Empirical and Theoretical Moments (Targeted)

Parameter	Moment	Weight Ω	France		United States	
			Data	Model	Data	Model
λ	Long-term growth rate of productivity	1	1.3%	1.3%	1.3%	1.3%
ϕ	Fixed costs as a fraction of total costs	2	9.5%	9.5%	12.0%	12.0%
σ	Relation between firm growth and size	1	-.035	-.035	-.035	-.035
η^e	Entry rate (fraction of firms age 1 or less)	1	10%	10%	13.8%	12.2%
η^x	Ratio of research and development to sales	1	3.2%	2.6%	2.5%	2.5%

Notes: Data columns present the empirical moments while model columns present the theoretical moments. French moments are for 1994 or the first subsequent year for which the moment is present in the micro data. U.S. moments are for 1980 except for the regression coefficient of firm growth on firm size, which is taken from [Akcigit and Kerr \(2018\)](#).

while their higher ratio of fixed- to variable costs causes their estimated intangible efficiency ϕ to be higher than for France.

4.2. Model Properties

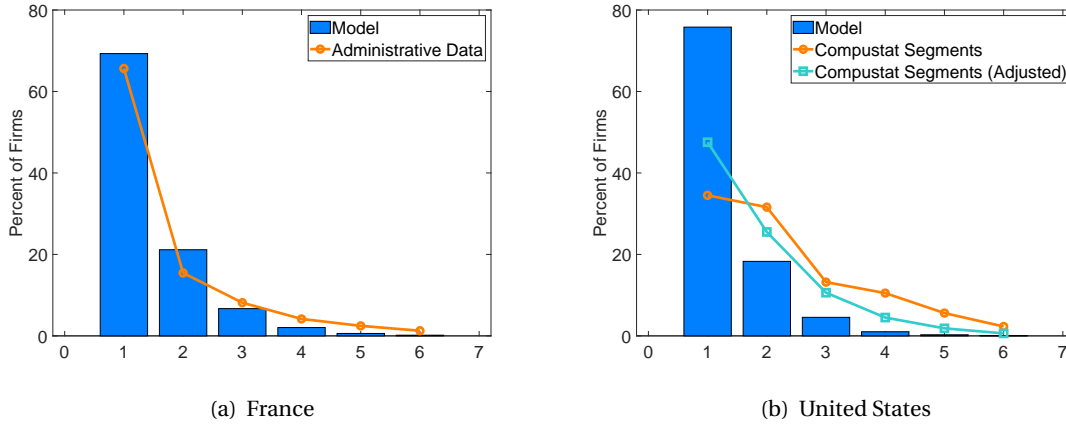
A comparison of theoretical and empirical targeted moments is provided in Table 8. The first column lists the parameter that corresponds most closely to the moment, the second column describes the moment, and the third column summarizes the moment's weight in the structural estimation. All moments receive the same weight except the share of fixed costs, which is most important for this paper's purpose. The model is able to match moments on growth, fixed costs and the relationship between firm growth and firm size precisely for both countries. The entry rate is also matched for France, while the estimated model implies an R&D intensity that is 0.6 percentage points below target. For the United States, the R&D intensity is matched precisely, but the entry rate is 1.6 percentage points below target.

The firm size distribution is untargeted. As in most [Klette and Kortum \(2004\)](#)-models, the Cobb-Douglas aggregator implies that a firm's revenue is determined by the number of goods that it produces. Figure 7 compares the distribution of the number of goods that a firm produces in the model to its counterpart in the data. Figure 7a plots the results for France. Data comes from the *Enquête Annuelle de Production dans L'Industrie* (EAP). This dataset is only available for firms in manufacturing, but contains product identifiers for each product that the firm sells.²⁸ The figure shows that distribution of the number of products that firms sell in the model is closely matched by the data. Figure 7b plots the same results for the United States. Because the Census counterpart of the EAP is not publicly available, I instead rely on the Compustat Segments data to count the number of NAICS industries that firms have segments in.²⁹ This is the orange-circled line in the figure. Results show that the sample of publicly listed U.S. firms are on average active in more sectors than the model predicts. Note that the Compustat segments are an imperfect measure of the number of products that firms produce because firms apply heterogeneous reporting standards on what a

²⁸The first year of the survey is 2009, which is plotted here. Further details are provided in Data Appendix B.

²⁹The first year with NAICS segment codes in the data is 1990, which is plotted here. Further details are provided in Data Appendix B.

Figure 7. Number of Products by Firm: Theory and Data



Notes: French data is taken from the *Enquête Annuelle de Production dans L'Industrie* (manufacturing only, 2009). U.S. data is taken from the Compustat Segments file and counts the number of primary 6-digit NAICS codes that firms report to operate in during 1990. Adjusted segments data assigns a segment count of 1 for firms that are not included in the segments file.

segment is. Further to that, 29.5% of firms do not report their segments at all. The green-squared line plots an alternative distribution of the product count setting the number of products to one for non-reporting firms. This brings the distribution closer to what is predicted. The difference between the fraction of firms with 2 and 3 (and 3 and 4) products is also accurately predicted.

Table 9 presents a set of additional untargeted moments. The left-hand columns present moments from the French administrative data while the right-hand columns present moments from U.S. firms in Compustat. The first panel analyzes the relationship between size and age. Size is measured as sector-deflated sales, while age is measured as years since creation in France and as years since entry into Compustat for the U.S. Both are transformed to within-year quartiles indexed from 1 to 4.³⁰ The model correctly predicts that young firms are on average smaller than older firms, as they have had less time to accumulate additional patents through research and development. The model also correctly predicts for France that young and small firms are more likely to exit and less likely to stop producing one of their products. For the U.S., the model accurately predicts that small firms are more likely to exit and less likely to stop producing a product, but cannot explain the relationship between exit and age. This could be because U.S. exit rates are calculated within Compustat, which can reflect that a firm was acquired or delisted. Exit rates for the U.S. in Table 9 are therefore not necessarily due to firm closure.

³⁰The first entry of the upper panel, for example, implies that firms in the first age quartile have a 1.21 average score on a 1-4 scale of the size quartiles.

Table 9: Comparison of Theory and Data for Untargeted Moments

	Quartile	France			United States		
		Model	Data	St. Dev.	Model	Data	St. Dev.
Size and Age	1st (Age)	1.21	1.98	(1.01)	1.23	2.17	(1.04)
	2nd (Age)	1.61	2.39	(1.06)	1.62	2.28	(1.05)
	3rd (Age)	1.92	2.69	(1.07)	1.93	2.47	(1.09)
	4th (Age)	2.11	3.04	(1.03)	2.14	3.05	(1.08)
Exit Rate and Age	1st (Age)	.145	.060	(.238)	.149	.114	(.318)
	2nd (Age)	.121	.055	(.229)	.132	.122	(.317)
	3rd (Age)	.105	.038	(.190)	.118	.110	(.306)
	4th (Age)	.094	.036	(.189)	.106	.075	(.265)
Exit Rate and Size	1st (Size)	.159	.114	(.318)	.156	.127	(.333)
	2nd (Size)	.159	.040	(.196)	.156	.109	(.312)
	3rd (Size)	.029	.028	(.165)	.156	.091	(.287)
	4th (Size)	.004	.024	(.153)	.023	.067	(.251)
Product Loss Probability and Age	1st (Age)	.175	.105	(.306)	0.172	.045	(.208)
	2nd (Age)	.205	.127	(.333)	0.195	.048	(.213)
	3rd (Age)	.234	.152	(.359)	0.218	.055	(.228)
	4th (Age)	.252	.164	(.370)	0.233	.068	(.252)

Notes: French data is from the full FICUS-FARE dataset (1994-2016). U.S. data is from Compustat data from 1980 to 2016. Size is measured as sector-deflated sales, age as the number of years since creation or Compustat entry. Exit is a dummy equal to 1 if a firm no longer appears in FICUS-FARE/Compustat in subsequent years. Product loss is a dummy equal to 1 if a firm produces fewer goods the subsequent year. Items under ‘model’ and ‘data’ are the mean of the variable within the quartile considered.

5. Analysis

I now turn to the main exercise: an analysis of the effect of a rise of intangibles on productivity growth, business dynamism and markups. In Section 5.1 I first discuss how the original calibration, which reflects the economy before the rise of intangibles, is changed to achieve the increase in fixed costs discussed in Section 2. I then quantitatively analyse the model’s predicted slowdown of productivity growth, fall in business dynamism and rise of markups along the new balanced growth path in Section 5.2. The transition path to the new balanced growth path is described in Section 5.3, which shows that the rise of intangibles initially causes a boom in productivity growth, in line with empirical trends since the mid-1990s. In Section 5.4 I present comparative statics to show that these results are quantitatively robust to changes in parameter estimates.

5.1. Introducing Heterogeneous Intangible Efficiency

I analyze the effect of intangibles by comparing the calibration in Section 4 to a calibration where a fraction of all entrants have a higher intangible efficiency than other firms.³¹ This approach captures two empirical characteristics of the rise of intangibles. First, it matches that the average share of fixed costs in total costs increased by 4.5 percentage points in France and by 10.6 percentage

³¹The model is able to analyse the effect of any finite combinations of intangible efficiencies. Computational complexity is exponential in the number of different levels of ϕ , however, which is why this calibration sticks to two types.

points in the United States. Second, it matches that the increase in intangibles after 1994 was not homogeneous (see Figure 3), even for firms of similar sizes in narrowly defined industries.

To implement this approach I recalibrate the level of intangible efficiency $\bar{\phi}$ of entrants that incur higher efficiency and the fraction $G(\bar{\phi})$ of entrants to which this applies. Both parameters are related because when a greater fraction of entrants draws $\bar{\phi}$, a smaller reduction in $\bar{\phi}$ is sufficient to achieve the empirical increase in fixed costs along the new balanced growth path. I calibrate $\bar{\phi}$ and $G(\bar{\phi})$ by targeting the increase in the ratio of fixed to total costs and the decline in the rate of entry. The former corresponds directly to $\bar{\phi}$ because a higher average intangible efficiency leads to a greater use of intangibles and, hence, to higher fixed costs. The rate of entry depends on the share of firms with a higher intangible efficiency $G(\bar{\phi})$ because the latter determines what fraction of entrants benefit from the rise of intangibles. For low levels of $G(\bar{\phi})$ there is little chance that an entrant is highly efficient at intangibles. Because high-intangible firms expand strongly, however, entrants are likely to face a high-intangible incumbent when they attempt to enter. This raises effective entry costs and lowers the incentive to enter. In the French calibration, 4% of all new entrants benefit from the high-intangible efficiency, which is 25% higher than that of other firms. In the U.S. calibration, 6% of all new firms benefit from a 30% higher intangible efficiency.³²

5.2. Balanced Growth Path Comparison

The balanced growth path after the rise of intangibles is summarized in Table 10. It presents the main variables of interest in differences from the original balanced growth path. The upper panel presents the change in the fraction of total costs that is fixed as well as the change in the entry rate. Both are targeted, and matched well in the new calibration. The bottom panel of Table 10 presents results for untargeted objects in the new versus the previous steady state. These include the slowdown of productivity growth, the decline in business dynamism and the rise of markups. In the French calibration, the model is able to explain all of the decline in the reallocation rate and nearly all of the rise of markups. The model produces a 0.2 percentage-point decline in productivity growth. While this does not explain why productivity growth has fallen to zero in France, it does imply a 16% reduction in growth. For the United States, the model is able to explain about half of the rise of markups and more than one-third of the slowdown of productivity growth. The model overestimates the decline in business dynamism as measured through reallocation (though not for entry), because all growth in the model occurs through creative destruction.³³

³²An alternative experiment would be to allow incumbents to differ ex-ante in their ability to use intangibles, but to allow that ability to ‘activate’ at a certain moment (e.g. due to a technological change). That would imply the same steady state results as in Table 10. It would cause the transition to the new steady state to be faster because part of the high-intangible firms would already be active as incumbents. The difference is not quantitatively significant.

³³An empirically relevant additional source of innovation is the improvement of goods that firms already produce (e.g. Garcia-Macia et al. 2016, Akcigit and Kerr 2018). In the context of the model, internal innovation would be affected similarly by the rise of intangibles. The rate at which firms innovate depends on the rate at which they discount future profits. This rate is highest for low-intangible firms, which would therefore invest less. High-intangible firms do have a strong incentive to invest in internal innovation. In a model like Peters (2018), however, internal innovation primarily raises a firm’s market power, hence furthering the rise of markups and the decline in (relative) wages.

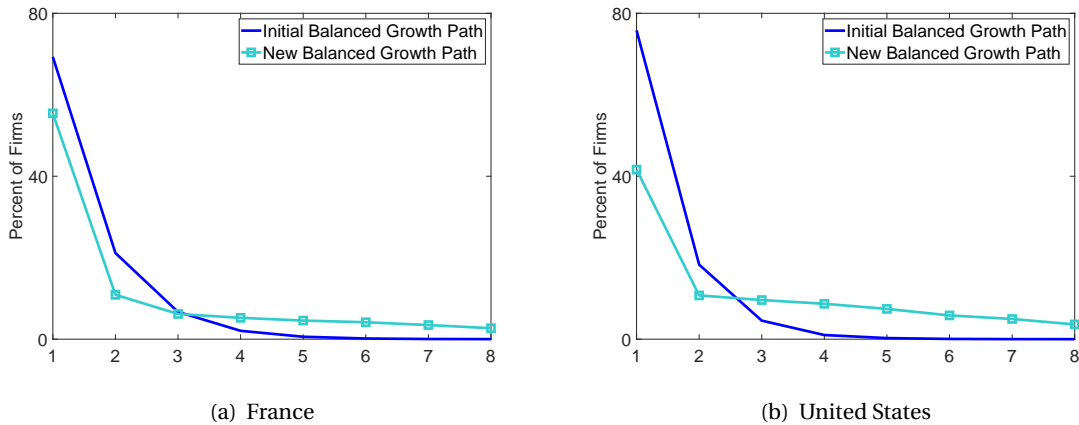
Table 10: Balanced Growth Path before and after Increase in Intangible Efficiency of Top Firms

		France		United States	
	Targeted	Δ Model	Δ Data	Δ Model	Δ Data
<i>Rise of Intangibles</i>					
Average Fixed-Cost Share	Yes	4.5 pp	4.5 pp	10.1 pp	10.6 pp
<i>Slowdown of Productivity Growth</i>					
Productivity Growth Rate	No	-0.21 pp	-1.3 pp	-0.36 pp	-0.9 pp
Aggregate R&D over Value Added	No	37.7%	2.1%	43.0%	64.5%
<i>Decline of Business Dynamism</i>					
Entry rate	Yes	-3.6 pp	-3.6 pp	-5.6 pp	-5.8 pp
Reallocation Rate	No	-27.6 %	-23 %	-36.7 %	-23 %
<i>Rise of Market Power</i>					
Average Markup	No	10.6 pt	11 pt	16.7 pt	30 pt

Notes: Data columns present the empirical moments, while model columns present the theoretical moments. The change in productivity growth is the difference between growth from 1969-1994 (France) or 1969-1979 (U.S.) to growth post 2005. Other French moments equal the difference between values in 1994 and in 2016. Other U.S. moments equal the difference between 1980 and 2016.

The model predicts a decline in productivity growth despite an *increase* in aggregate research and development, in line with the data in France and the United States.³⁴ In a model with homogeneous firms this would be paradoxical, because there is a direct relationship between aggregate research and development and growth. Higher investments and lower growth co-exist in this model because innovation activity is concentrated in a smaller group of high-intangible firms, and because some innovations by low-intangible entrants and incumbents fail to enter the market.

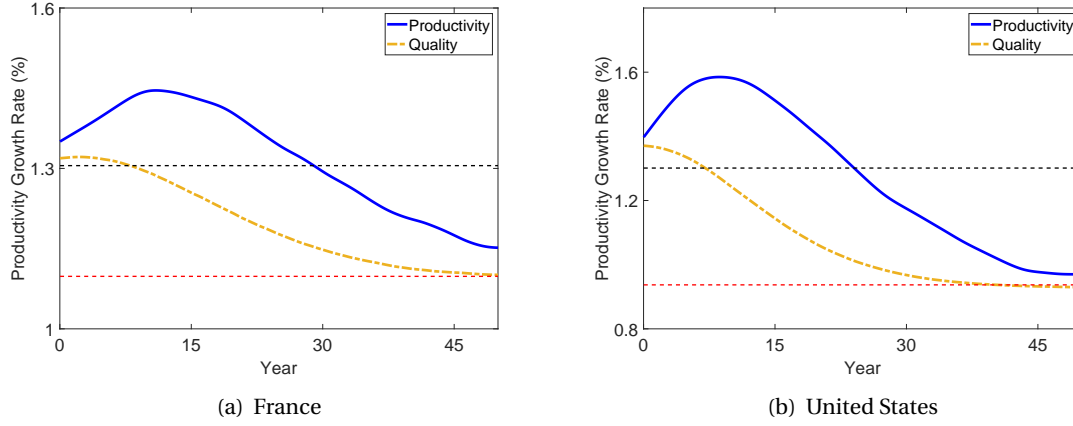
Figure 8. Number of Products before and after an Increase in Intangible Efficiency of Top Firms



Notes: Lines plot the fraction of firms that produce the number of products on the horizontal axis. Solid lines are from the original calibration in Section 4 and correspond to the bars in Figure 7. Squared lines present the counterpart for the balanced growth path after a group of high-intangible firms has been introduced.

³⁴The French increase in Table 10 is measured over 1994-2016 while the U.S. increase is measured over 1980-2016. France experienced a 28.7% increase in the ratio of R&D over national income between 1980-2016, which is closer to what the model predicts.

Figure 9. Transition: Growth Rate of Total Factor Productivity



Black- and red-dashed lines (respectively) indicate the original and the new steady state.

The increase in firm concentration is illustrated in Figure 8, which plots the distribution of firms over the number of products that they produce. This is the most direct measure of firm-concentration in the model. The original balanced growth path (solid-blue line) is characterized by a lower firm concentration as there are more firms that produce one or two goods than in the new balanced growth path (squared-green line). Conversely, the right tail of the firm-size distribution is fatter, indicating that there are more large firms along the new balanced growth path. Note that the increase in concentration is endogenous: high-intangible firms have higher markups and therefore have more incentives to invest in research and development. This causes them to produce a disproportionate fraction of all goods and to grow larger than other firms.

5.3. Transitional Dynamics

The analysis thus far has studied the effect of a rise in intangibles along the balanced growth path. The short-term dynamics, however, are substantially different. The birth of firms with higher ϕ s raises the average fraction of marginal costs that firms reduce through intangibles, causing a transitory boom in productivity growth. To quantify this and other transitory effects, I numerically solve for the path of productivity, research and development, markups and wages.³⁵

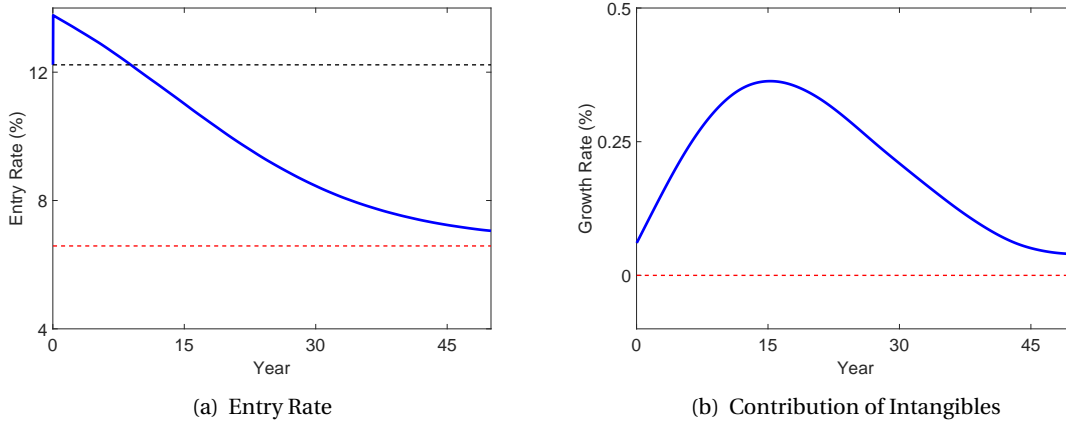
5.3.1. Transitory Boom in Productivity Growth

The path of productivity growth is presented in Figure 9. Figure 9a presents results for the French calibration, Figure 9b for the U.S. calibration.³⁶ The solid-blue line plots the path of growth in total factor productivity as defined in (22). The dash-dotted yellow line plots the increase in productivity due to the step-wise improvement of quality, which is the source of long-term growth.

³⁵The computational algorithm is described in Appendix E.

³⁶Figures in the remainder of this section only plot results for the U.S. calibration because the results are qualitatively similar in both calibrations. Full French results are provided in Appendix F.

Figure 10. Transition: Contributors to Productivity Growth

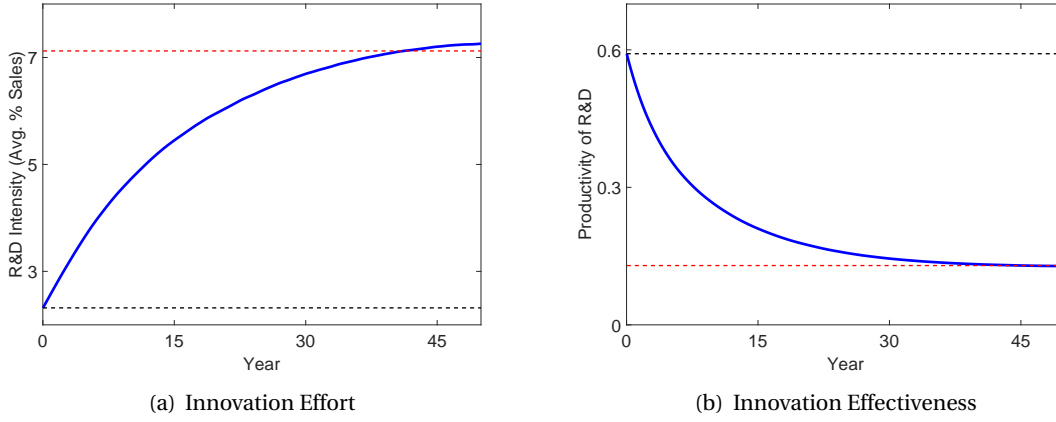


Black- and red-dashed lines (respectively) indicate the original and the new steady state. Calibration is for the U.S.

When high- ϕ firms start entering the economy in year 0 there is initially a slight increase in productivity growth compared to the original steady state (the black upper-dashed line). This is because of a slight rise in entry, driven by the fact that new firms now have a positive probability of being the profitable high- ϕ type, while the low- ϕ entrants do not face high- ϕ incumbents yet (Figure 10a). As the high-types enter the economy there is a further increase in productivity because they reduce the marginal costs of any good that they produce through the use of intangibles. This causes productivity growth to temporarily exceed the growth rate of quality. At peak growth, which happens 11 years after the introduction of high- ϕ entrants in the U.S. calibration, this boosts growth up to 1.6%. This is similar to the empirical rise of productivity in both magnitude and length, as plotted in Figure 1. The transitional boom evolves more slowly and is of a smaller magnitude in France, because a smaller fraction of start-ups benefit from the higher intangible efficiency (4% in the French calibration versus 6% in the U.S. calibration). The extraordinary growth is predominantly intangible-driven (Figure 10b), consistent with the finding that above-average productivity growth from the mid-1990s to the mid-2000s was primarily caused by IT (Fernald 2014). Note that neither the size of the transitory productivity growth spike nor the length of the transitional dynamics are targeted.

A slowdown of productivity occurs from year 12 onward. The entry rate is on a steady decline because high- ϕ incumbents produce an increasingly large share of all products in the economy. The probability that an entrant benefits from drawing a high- ϕ therefore falls below the probability that the entrant faces a high- ϕ incumbent, which increases the likelihood of a failed innovation. The additional transitory growth from software peaks 16 years after the high- ϕ firms first enter. Transitional growth declines as the fraction of all products that are produced by high- ϕ converges to its level along the balanced growth path. While the increase in productivity brought by intangible-adoption is permanent, the additional productivity growth is not.

Figure 11. Transition: Research and Development Intensity and Effectiveness



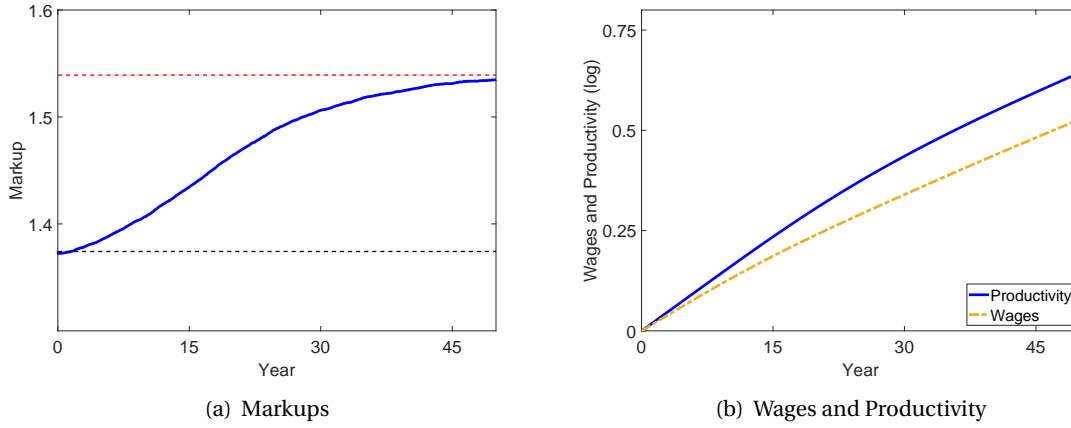
Black- and red-dashed lines (respectively) indicate the original and the new steady state. Calibration is for the U.S.

5.3.2. Innovative Investments and 'Ideas TFP'

Figure 11 plots the development of research and development (R&D) by incumbents. Figure 11a plots average R&D intensity, which is the ratio of expenditures on research and development to sales. One of the empirical facts discussed in Section 1 is that the decline in productivity growth occurred despite an increase in the intensity of research and development. This fact is matched by the model: average R&D intensity increases from 2.5 to 7%. This is quantitatively similar to the data. Among U.S. public firms with positive R&D, the average R&D intensity increased from 2.5 (the calibration target) to 8.7%.³⁷ R&D intensity among all public firms increased from 2.0 to 6.7%, again similar to the increase predicted by the model. 11b plots the ratio of productivity growth to the average intensity of R&D, which measures the effectiveness of innovative investments. A unit value implies that productivity grows by one percent when R&D intensity is 1%. The decline in the effectiveness of innovative expenditures is in line with empirical estimates of the 'ideas production function' in Bloom et al. (2017). They find that innovative investments have an increasingly small effect on innovation at both aggregate, sector, and micro levels, from which the authors conclude that ideas are getting harder to find. This paper provides an explanation for that result. Because high-intangible firms have higher markups, they have a greater incentive to innovate. Because the returns to R&D are concave these additional investments have limited effects on growth but increase average R&D intensity considerably, causing the decline in the productivity of research. The presence of high- ϕ incumbents further means that a fraction of the innovations fail to be introduced to the market, further diminishing the effect of research on growth.

³⁷ French R&D expenditure over sales increased from 3.1% among positive spenders (the calibration target) to 4.0%.

Figure 12. Transition: Markups and Wages



Black- and red-dashed lines in Figure 12(respectively) indicate the original and the new steady state. Calibration is for the U.S.

5.3.3. Markups and the Decoupling of Wages and Productivity

The transitional dynamics also shed light on two recent macroeconomic puzzles. The first is why wages did not keep up with productivity growth in the past 20 years (e.g. [Kehrig and Vincent 2017](#)), which has caused a decline in the labor share. While the entry of higher- ϕ firms leads to a reduction of marginal costs and an increase in productivity, there is no increase in wages because productivity is offset by higher markups. The rise of markups is plotted in Figure 12a. This leads to a decoupling of wages (dashed-yellow) and productivity (solid-blue), as plotted in Figure 12b. Wages continue to grow at the rate of the increase in quality, but do not benefit from the transitory increase in productivity growth from intangible adoption. A second puzzle addressed by this paper is how markups could have increased while inflation remained low. This is one common critique, for example, on [De Loecker et al. \(2018\)](#). In my framework, markups increase proportionally to a reduction of marginal costs through intangible adoption. As prices are the product of the markup and marginal cost, they are therefore unaffected.

5.3.4. Summary

The transitional dynamics show that the model has starkly different predictions for the short- and the long-run effect of the rise of intangibles. In the short-run, the entry of high-intangible firms causes a boom in productivity growth because these firms produce their goods at lower marginal costs than the previous producers. Their cost advantage allows them to charge a higher markup, which suppresses demand for labor and prevents wages from benefiting from the boom in productivity growth. Productivity growth in the long-run is lower than the original steady-state level because high-intangible firms eventually produce a disproportionate fraction of all goods. This leaves productivity at a permanently higher level, but the additional growth from intangible adoption wears off over time. High-intangible firms have high innovation rates, but because the firm-

level returns to research and development are concave, the higher investments are insufficient to compensate for the lack of research and development by other incumbents and by entrants.

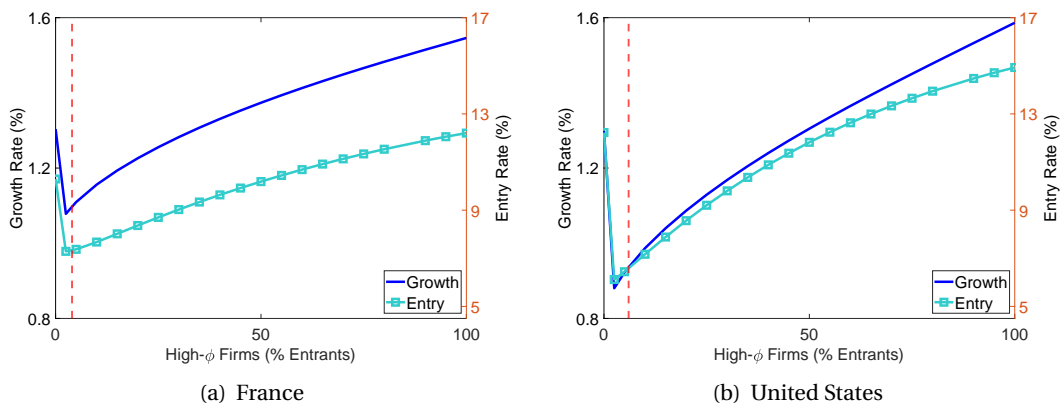
5.4. Comparative Statics

I now quantitatively analyze the modeling choices and parameter choices that drive the slowdown of productivity growth, the fall business dynamism, and the rise of markups along the new balanced growth path.

5.4.1. Intangible Inequality

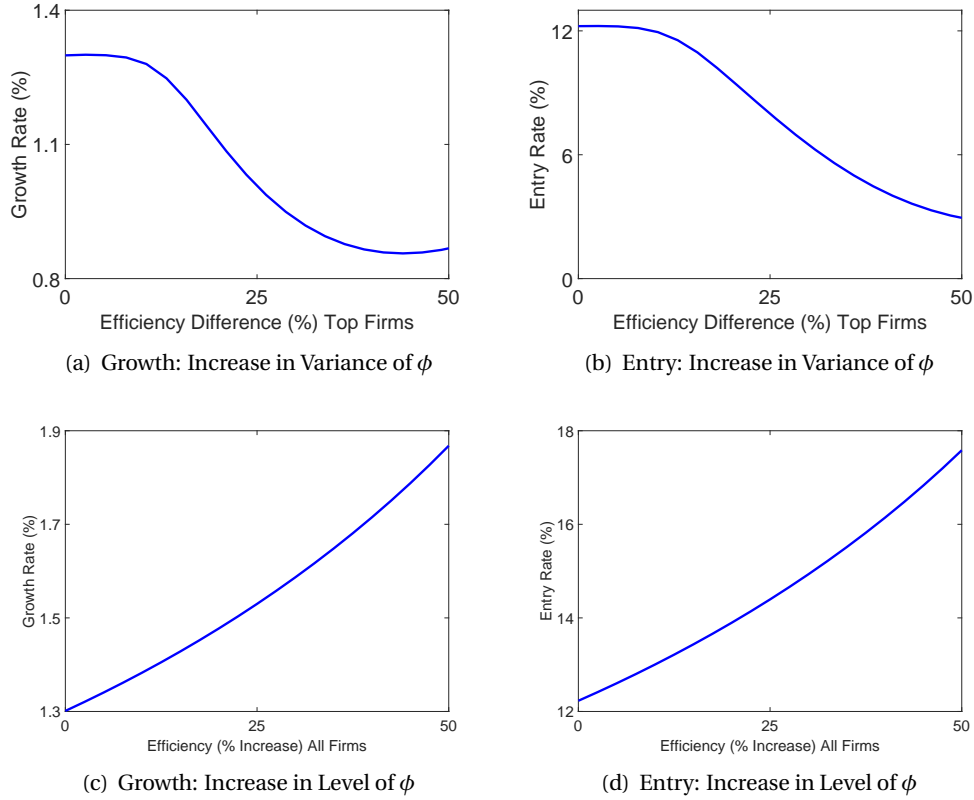
Productivity growth falls along the new balanced growth path because the rise of intangibles is unequal: only a fraction of entrants benefits from the higher $\bar{\phi}$. This means that the new balanced growth path is characterized by both a higher expected level of intangible efficiency for entrants and a higher variance of intangible efficiency. The more *widespread* the increase in intangible efficiency, the higher productivity growth is along the new balanced growth path. Figure 13 illustrates this. It plots the effect of an increase in intangible efficiency of 0 to 100% of firms. In line with the calibration behind Table 10, high-intangible firms in the French calibration (Figure 13a) have a 25% higher ϕ , while high-intangible firms in the U.S. calibration (Figure 13b) have a 30% higher ϕ than other firms. The horizontal axis denotes the fraction of all entrants that benefit from the higher intangible efficiency $G(\bar{\phi})$. At $G(\bar{\phi}) = 0$ the economy is in the original steady state. As the share of entrants with high intangible-efficiency becomes positive there is a large decline in growth and entry. This is because the smaller $G(\bar{\phi}) > 0$, the greater the increase in variance and the smaller the increase of the expected intangible efficiency. If all firms would see an increase in ϕ , average

Figure 13. Balanced Growth Path Effects of an Increase in Intangible Efficiency for Top Firms



Notes: The figures present the balanced growth path levels of the growth and entry rate for various levels of $G(\bar{\phi})$. 13a plots results for the French calibration, in which $\bar{\phi}$ exceeds the ϕ of other firms by 25%. Figure 13b plots the U.S. calibration, in which $\bar{\phi}$ exceeds the ϕ of other firms by 30%. Red-dashed lines present $G(\bar{\phi})$ in the calibration of Table 10 which equals 4% for the French calibration and 6% for the U.S. calibration. The lowest $G(\bar{\phi}) > 0$ plotted is 2.5%.

Figure 14. Balanced Growth Path Effects of Increase in the Variance versus the Level of ϕ



Notes: Figures 14a and 14b plot growth and entry at different variances of ϕ , implemented through a mean preserving spread (MPS). The MPS is such that 6% (94%) of entrants have various levels of high (low) intangible ability while the expected ϕ remains 0.8. Figures (c) and (d) plot growth and entry at different levels of ϕ . The horizontal axis expresses the % difference from $\phi = 0.8$.

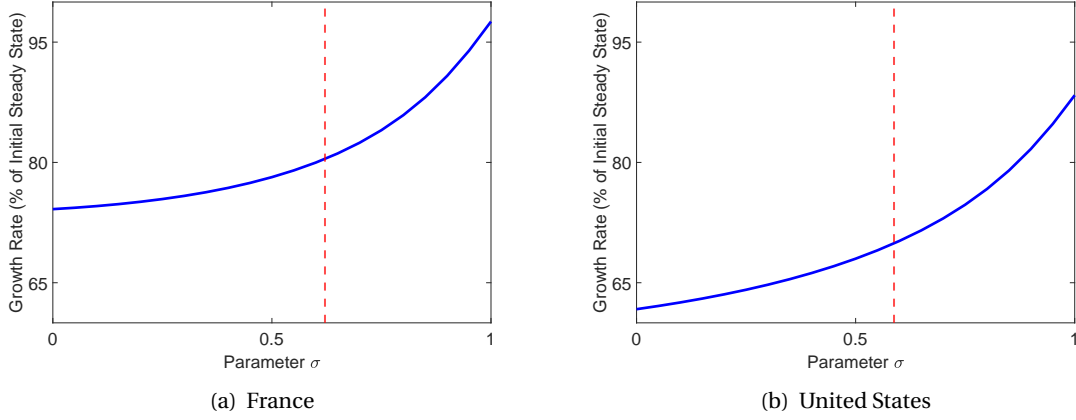
markups increase and the incentive to innovate by investing in research and development would increase. A sufficiently homogeneous increase in intangible efficiency therefore raises entry and growth above the old steady state level.³⁸ Conversely, a mean-preserving spread of ϕ has a negative effect on growth because it reduces incentives to enter.

Figure 14 illustrates the difference between an increase in the variance of intangible efficiency and an increase in the level of intangible efficiency. Figures 14a and 14b respectively plot the steady state growth and entry rate (vertical axis) for mean-preserving spreads (horizontal axis) of intangible efficiency, while Figures (c) and (d) plot the effect of an equal increase in ϕ for all firms. The mean-preserving spread is such that the expected level of ϕ remains 0.8, in line with the U.S. calibration.³⁹ As the variance of ϕ increases there is a persistent decline in the growth and entry rate.

³⁸In Figure 13 this happens when around 45% of entrants receive the higher efficiency in both calibrations. Note that this is an exaggeration because the figure does not correct for the fact that the increase in the model's steady state fixed-cost share would exceed the empirical increase when a larger fraction of entrants receive $\bar{\phi}$. In the French calibration, for example, fixed costs increase by 14 percentage points when $G(\bar{\phi}) = 1$, compared to 4.5 percentage points in the data.

³⁹Because of the qualitative nature of this exercise, results are only presented for the U.S. calibration. Corresponding graphs based on the French calibration are provided in Appendix F.

Figure 15. Comparative Statics: New Balanced Growth Rate at Calibrations of σ



Notes: Figures plot the growth rate of productivity in the new steady state for varying levels of σ , holding all other parameters constant.

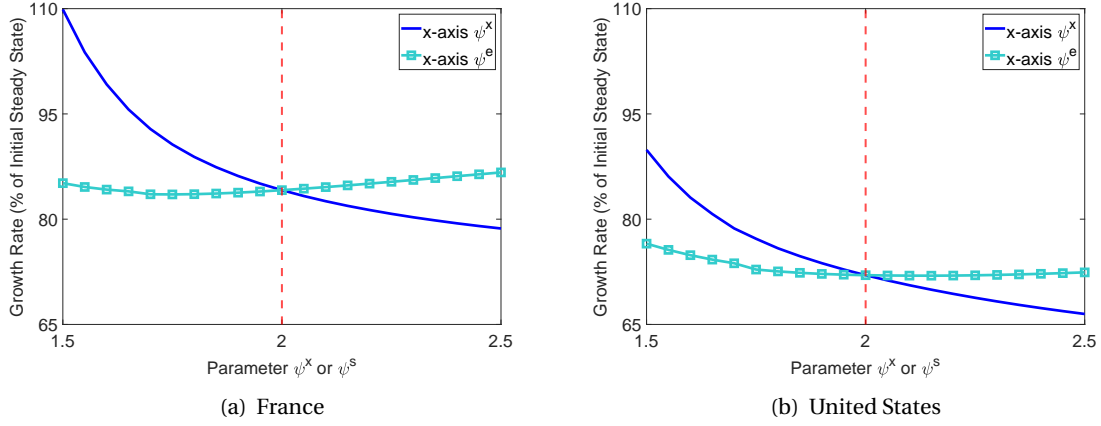
Dashed-red lines present the value of σ in the baseline calibration (0.62 for France, 0.58 for the United States). σ governs the relationship between firm growth and firm size. Firm growth is unaffected by size if $\sigma = 1$. The Vertical axes are expressed in percent of the original steady states where firms have a homogeneous intangible efficiency (see Table 7).

Firms with high intangible efficiencies have a greater incentive to expand through research and development and therefore produce a disproportionate fraction of their goods. For example, if high- ϕ firms have a 25% higher intangible efficiency than low- ϕ firms, the former produce 84% of goods in the steady state while they represent only 6% of entrants. The probability that an entrant benefits from a higher intangible efficiency is therefore much lower than the probability that it faces such a firm as an incumbent, which drives the negative effect on entry in Figure 14b. The effect of a higher level of intangible ability for all firms as plotted in in Figures 14c and 14d is uniformly positive because it raises profitability of production without preventing a subset of firms from successfully entering.

5.4.2. Parameter Dependence

I next show how the effects in Section 5.2 depend on the model's key parameters. The first is σ , which governs the degree to which innovation efforts scale with size. For $\sigma = 1$, the growth rate is constant with size, while for $\sigma < 1$ small firms on average grow faster than large firms. Figure 15 plots the relationship between the calibration of σ and the growth rate in the new steady state. The Red-dashed lines present the estimated value of σ . The figure shows that the effect of introducing a subset of high-intangible firms is higher for low values of σ . This is because high-intangible firms invest more in research and development than other firms, and subsequently grow larger. For $\sigma < 1$, their innovation intensity falls because the costs of choosing a higher innovation rate are convex within periods. At higher levels of σ this is offset because the costs of a given innovation rate fall mechanically with size, allowing the growth rate of large firms to remain high. When σ is

Figure 16. Comparative Statics: New Balanced Growth Rate at Calibrations of ψ^x , ψ^s



Notes: Figures plot the growth rate of productivity in the new steady state for varying levels of either ψ^x or ψ^e , holding other parameters constant. Dashed-red lines present the value of ψ^x and ψ^e in the baseline calibration (2 in both calibrations). Vertical axes are expressed in percent of the original steady states, where firms have a homogeneous intangible efficiency (see Table 7).

calibrated such that the model matches empirical firm dynamics, however, growth is lower when industry concentration is higher.

Figure 16 provides similar plots for the cost elasticities of research and development by incumbents (ψ^x) and entrants (ψ^e). Solid-blue lines plot the effect of changing ψ^x , holding ψ^e constant, while squared-green lines plot the effect of the converse. The model's results are robust to changes in ψ^e , but depend critically on the assumed level of ψ^x . The parameter ψ^x determines the degree of concavity of the returns to research and development. For any $\psi^x > 1$, the productivity of research and development is maximized when all same-sized firms spend an equal amount on research and development. The introduction of inequality in intangible efficiency causes some firms to invest more than others in research and development as high- ϕ firms have a greater incentive to invest. This lowers the average rate of return of these investments. For low values of ψ^x , high investments by high-intangible firms are sufficient to offset the lack of entry and investments by other firms in the French calibration. A value $\psi^x = 2$ is standard in the literature, however, and comes with a significant decline in growth compared to the original steady state.

6. Extensions

6.1. Welfare and Intangible Diffusion

I next assess how the rise of high-intangible firms analyzed in Section 5 affects welfare. The model predicts that intangibles have two counteracting effects. There is an initial boost of growth which permanently raises the level of productivity, which is positive for welfare. There is a subsequent slowdown of growth that lowers the growth rate beyond the previous steady state, which reduces welfare. The increase in R&D expenditures as a percentage of GDP furthermore implies that a

Table 11: Welfare Comparison of Various Levels of Intangible Adoption

$G(\bar{\phi})$:	France					United States				
	0.04	0.10	0.25	0.50	1.0	0.04	0.10	0.25	0.50	1.0
Δ Welfare	-2.20%	-2.05%	-0.86%	0.77%	3.86%	-3.65%	-3.36%	-1.74%	0.94%	5.95%
Δ Markup	8.29%	8.50%	8.78%	9.16%	9.71%	12.16%	12.17%	12.23%	12.44%	12.82%

Notes: Welfare is expressed in percent change from the original balanced growth path. $G(\bar{\phi})$ is the share of entrants with the 25% higher ϕ in the French calibration and the share share of entrants with the 30% higher intangible efficiency in the U.S. calibration. $G(\bar{\phi}) = 0.04$ for France and $G(\bar{\phi}) = 0.06$ for the U.S. in the main analysis. The change in markup is the new balanced growth path average markup as a percentage of the average markup along the original balanced growth path.

smaller fraction of the labor force is dedicated to the production of consumption good, which has a further negative effect on welfare.

To assess which of the two effect dominates I compare utility of the original path of consumption to utility of the new path. To do so, I calculate the value of the discretized expression for utility

$$U = \sum_{t=0}^{\infty} (1 + \rho)^{t \cdot \Delta_t} \cdot \log C_t,$$

where Δ_t denotes the number of periods within a year, which I set to 50. Results are presented in Table 11. Section 5 analyzed the effect of assigning a higher intangible efficiency to 4% of entrants in the French calibration and to 6% of entrants in the U.S. calibration. The table shows that, despite the transitory increase in growth, the overall effect of these entrants on welfare is negative. The decline is economically significant: welfare falls by 2.20% in the French calibration and by 3.65% in the calibration for the United States.

Table 11 also highlights how important inequality is for the welfare effect of the rise of intangibles. If a quarter of entrants are of the high-intangible type, for example, the decline in welfare is less than half as large as the decline when 4% (France) or 6% (U.S.) of entrants are of that type. When the majority of firms benefit from higher efficiency the welfare effect is strictly positive. The potential increase in welfare is large: if all firms are of the high- ϕ type there is a 3.86% and 5.95% improvement of welfare compared to the original steady state in the French and U.S. calibration, respectively. Any policy that improves the diffusion of intangibles would reduce the variance of ϕ and raise it's mean, and therefore have positive effect on entry, growth and welfare. The bottom row of Table 11 shows that such policies lead to a further increase in steady state markups, because a greater share of products is produced by high-intangible firms. The rise (and the associated decline in the labor share) is modest compared to the increase in welfare, however, because high-intangible firms always produce a disproportionate number of products in the steady state.

6.2. Value Function Specification

The preceding analysis has relied on a simplified dynamic optimization problem where firms did not internalize the change in their innovation capacity when they added a new product to their portfolio. This assumption significantly improves tractability, as it allows for a closed-form expres-

sion of the first-order conditions for innovation. This section shows that results are qualitatively and quantitatively robust to removing this assumption. The new value function is characterized by

$$rV_t(\phi_i, \tilde{J}_i) - \dot{V}_t(\phi_i, \tilde{J}_i) = \max_{x_i} \left\{ \begin{aligned} & \sum_{j \in \tilde{J}_i} \left[\pi_t(\phi_i, \lambda_{ij}) + \right. \\ & \quad \left. \tau(\phi_i) \cdot [V_t(\phi_i, \tilde{J}_i \setminus \{\lambda_{ij}\}) - V_t(\phi_i, \tilde{J}_i)] \right] \\ & \quad + x_i \cdot \text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \\ & \quad \cdot \mathbb{E}_{\phi_i} [V_t(\phi_i, \tilde{J}_i \cup \lambda_{ij}) - V_t(\phi_i, \tilde{J}_i)] - w_t \eta_x(x_i)^{\psi_x} n_i^{-\sigma} \end{aligned} \right\}$$

The solution of this function is considerably less tractable than the solution in Section 3 because the function no longer scales linearly in firm size. As the firm gets larger the option value of investing in research and development increases, causing it to choose a higher innovation rate. The innovation rate does not fully scale with size, however, because the estimated value of the scaling parameter σ is such that the model matches the negative empirical relationship between firm size and growth. The following proposition summarizes the new solution:

Proposition 3. *The value function of a firm with intangible-ability ϕ_i that produces a portfolio of goods \tilde{J}_i with cardinality n_i grows at rate g along the balanced growth path and is given by*

$$V_t(\phi_i, \tilde{J}_i) = \sum_{j \in \tilde{J}_i} \Upsilon_t^1(\phi_i, \lambda_{ij}) + \Upsilon_{t, n_i}^2(\phi_i),$$

where Υ_1 is the present value of the profit flow from producing good j . Matching coefficients gives

$$\Upsilon^1(\phi_i, \lambda_{ij}) = \frac{\pi_t(\phi_i, \lambda_{ij})}{r - g + \tau(\phi)},$$

while Υ_{2, n_i} is the option value of research and development which evolves along this sequence:

$$\begin{aligned} \Upsilon_{t, n_i+1}^2(\phi_i) + \Upsilon_t^1(\phi_i, \lambda_{ij}) &= \left[\frac{(r - g) \cdot \Upsilon_{t, n_i}^2(\phi_i) + n_i \cdot \tau(\phi_i) \cdot [\Upsilon_{t, n_i}^2(\phi_i) - \Upsilon_{t, n_i-1}^2(\phi_i)]}{\psi^x - 1} \right]^{\frac{\psi^x - 1}{\psi^x}} \\ &\quad \cdot \text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right)^{-1} \cdot \psi^x \cdot (\eta \cdot w_t)^{\psi^{x-1}} \cdot n_i^{-\frac{\sigma}{\psi^x}} + \Upsilon_{t, n_i}^2(\phi_i), \end{aligned}$$

such that the first-order condition for optimal research and development and read

$$\begin{aligned} x(\phi_i, n_i) &= \left(\text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \frac{\mathbb{E}_{\phi_i} [\Upsilon_t^1(\phi_i, \lambda_{ij}) + \Upsilon_{t, n_i+1}^2(\phi_i) - \Upsilon_{t, n_i}^2(\phi_i)]}{\eta^x \cdot \psi^x \cdot w_t} \right)^{\frac{1}{\psi^x - 1}} \cdot n_i^{\frac{\sigma}{\psi^x - 1}}, \\ e &= \left(\sum_{\phi_h \in \Phi} G(\phi_h) \cdot \text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_h)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \frac{\mathbb{E}_{\phi_h} [\Upsilon_t^1(\phi_i, \lambda_{ij}) + \Upsilon_{t, 1}^2(\phi_i)]}{\eta^e \cdot \psi^e \cdot w_t} \right)^{\frac{1}{\psi^e - 1}}. \end{aligned} \quad (26)$$

Table 12: Structural Estimation - Alternative Value Function Specification

Parameter	Moment	France			United States		
		Par. Value	Data	Model	Par. Value	Data	Model
λ	Productivity Growth	0.07	1.3%	1.3%	0.07	1.3%	1.3%
ϕ	Fixed Costs (%)	0.76	9.5%	9.5%	0.82	12.0%	12.0%
σ	Gibrat's Law	0.59	-0.035	-0.035	0.46	-0.035	-0.035
η^e	Entry Rate	2.99	10.0%	9.9%	2.86	13.8%	12.2%
η^x	R&D Intensity	2.64	3.1%	2.6%	3.95	2.5%	2.5%

Notes: Data columns present the empirical moments while model columns present the theoretical moments. French moments are for 1994 or the first subsequent year for which the moment is present in the micro data. U.S. moments are for 1980 except for the regression coefficient of firm growth on firm size, which is taken from [Akçigit and Kerr \(2018\)](#).

Proof: Appendix A.

To structurally estimate the alternative specification of the model, I match the same moments as in Section 4. I solve for the sequence of option values numerically as an inner loop within the main solution routine and apply the Genetic Algorithm to minimize loss function (24). Estimation results are presented in Table 12. The first panel presents results based on moments for France while the second panel presents results for the United States. Compared to the calibration in Table 7, the estimated values for innovation step-size λ and the initially homogeneous intangible-efficiency ϕ are largely unchanged. The value of σ , which governs the firm-size and growth relationship, falls by 0.03 in the French calibration and by 0.13 in the United States calibration. In both calibrations there is an increase in the innovation cost-shifters η^x and η^e , which ensures that innovation intensities remain at their target level while the option value of future research and development provides a greater incentive to invest.

Table 13 compares the effect of introducing a group of high-intangible firms in the model with the new value function specification compared to the previous model. Conditional on calibration, both models predict a decline in productivity by 0.2 percentage points annually in France and 0.4 percentage points in the United States. Because growth is driven by creative destruction, this implies that the models also have very similar decline in the reallocation rates. The increase in average markups is slightly larger in the United States in the new specification, which is due to the high-intangible firms occupying a slightly greater fraction of all products in equilibrium. Intuitively, the σ in equation (8) governs the degree to which an expansion of n increases innovation capacity.

Table 13: Comparison of Steady States - Alternative Value Function Specification

	France		United States	
	Δ Old Model (Section 4)	Δ New Model (Section 6.2)	Δ Old Model (Section 4)	Δ New Model (Section 6.2)
Avg. Fixed-Cost Share	4.5 pp	4.6 pp	10.1 pp	10.6 p.p.
Entry Rate	-3.1 pp	-2.6 pp	-5.6 pp	-5.0 p.p.
Productivity Growth	-.21 pp	-.21 pp	-.36 pp	-0.37 pp
Reallocation Rate	-27.6 %	-28.8 %	-36.7 %	-38.4 %
Avg. Markup	10.6 pp	11.3 pp	16.7 pp	19.3 pp

Given that $\sigma < \psi - 1$, the change in innovation capacity diminishes in n . As before, it further implies that research and development costs do not fall proportional to firm size. Both channels create a negative relationship between firm size and growth. The parameter σ was previously calibrated such that empirical deviation from Gibrat's Law was matched by the model through the second channel. Adding the option-value channel has limited effects because the change in the relation between firm size and growth is largely offset by the new calibration of σ . This ensures that the model-implied deviation from Gibrat's Law is in line with the deviation in the data.

7. Conclusion

This paper proposes a unified explanation for the decline of productivity growth, the fall in business dynamism and the rise of markups. I hypothesize that the rise of intangible inputs, in particular information technology and software, can explain these trends. Central to the theory is that intangible inputs change the way that firms compete and produce, as they cause a shift towards fixed costs. Using income statement and balance sheet data on the universe of French firms and U.S. publicly listed firms, I calculate a new measure of fixed costs and show that the share of fixed costs in total costs has been steadily rising over time. Fixed costs have a positive correlation, within and between firms, with software expenses and IT system adoption, suggesting that intangibles can be modeled as scalable inputs to production. I also find that firms with higher fixed costs invest more in research and development, and subsequently grow more than other firms.

I rationalize these findings in an endogenous growth model with heterogeneous multi-product firms, variable markups and realistic entry and exit dynamics. The model suggests that when a subset of new firms becomes more efficient at using intangible inputs, the aggregate rise of intangibles is accompanied by a decline in both entry and long-term growth. I structurally estimate the model to match administrative micro data on the universe of French firms and U.S. listed firms, and find that intangibles cause a decline of long-term productivity growth of 0.2 percentage points in the calibration based on French firms and 0.4 percentage points in the U.S. calibration. Despite the decline of growth, there is an increase in R&D expenditures, in line with empirical evidence. Research and development becomes less effective because it is concentrated among a small number of firms and because a fraction of innovators are unable to beat high-intangible incumbents.

While negatively affecting growth in the long term, the short-run effect of the rise of intangibles is positive. By numerically solving the transition path between the original and the new balanced growth path, I show that growth initially increases for 11 years. This is because the entry of firms with high intangible efficiency causes a decline in marginal costs. This is mirrored by an increase in markups, however, which means that wages do not increase in response to the boom. Markups also offset the effect of the decline in marginal costs on prices, which could explain why the last decade has been simultaneously characterized by rising markups and low inflation.

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Appendix A. Proofs and Derivations

Derivation of positive derivative in Section 2.1

The first order condition for intangibles implies that firms with lower adoption costs (higher ϕ) choose to reduce their marginal costs by a greater fraction s_i . To show that these firms also have a higher share of fixed (intangible) costs in total costs I prove that the latter increases in the fraction of marginal costs automated (s_{it}). Define b_{it} as the log of the share and take the derivative with respect to s_{it} :

$$\frac{\partial b_{it}}{\partial s_{it}} = \frac{\partial f(s_{it}, \phi_i) / \partial s_{it}}{f(s_{it}, \phi_i)} - \frac{\partial f(s_{it}, \phi_i) / \partial s_{it} + (1 - s_{it}) \cdot c(..) \cdot (\partial y_{it} / \partial s_{it}) - y_{it} \cdot c(..)}{f(s_{it}, \phi_i) + (1 - s_{it}) \cdot c(..) \cdot y_{it}}$$

Grouping terms yields:

$$\frac{\partial b_{it}}{\partial s_{it}} = \frac{\partial f(s_{it}, \phi_i)}{\partial s_{it}} \cdot (f(s_{it}, \phi_i)^{-1} - (f(s_{it}, \phi_i) + (1 - s_{it}) \cdot c(..) \cdot y_{it})^{-1}) + \frac{c(..) \cdot [y_{it} - (1 - s_{it}) \cdot (\partial y_{it} / \partial s_{it})]}{f(s_{it}, \phi_i) + (1 - s_{it}) \cdot c(..) \cdot y_{it}}$$

All terms on the right hand side of this expression are positive, provided that $y_i \geq (1 - s_{it}) \cdot (\partial y_{it} / \partial s_{it})$. Given that $y_{it} = (1 - s_{it})^{-1} \cdot z(z_{it,1}, z_{it,2}, \dots, z_{it,k}) \cdot \omega_i^{-1}$, this condition can be written as:

$$z(z_{it,1}, z_{it,2}, \dots, z_{it,k}) \geq \frac{\partial z(z_{it,1}, z_{it,2}, \dots, z_{it,k})}{\partial s_{it}}$$

which is the condition set out in equation (2).

Proof of Proposition 1

The value function is given by the following Bellman equation:

$$r V_t(\phi_i, \tilde{J}_i) - \dot{V}_t(\phi_i, \tilde{J}_i) = \max_{x_i} \left\{ \begin{aligned} & \sum_{j \in \tilde{J}_i} \left[\pi_t(\phi_i, \lambda_{ij}) + \right. \\ & \left. \tau(\phi_i) \cdot [V_t(\phi_i, \tilde{J}_i \setminus \{\lambda_{ij}\}) - V_t(\phi_i, \tilde{J}_i)] \right] \\ & + x_i \cdot \text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \mathbb{E}_{\phi_i} [V_t(\phi_i, \tilde{J}_i \cup \lambda_{ij}) - V_t(\phi_i, \tilde{J}_i)] \\ & - w_t \cdot \eta_x \cdot (x_i)^{\eta_x} \cdot n_i^\sigma - F(\phi_i, n_i) \end{aligned} \right\}$$

Guess that the solution takes the following form:

$$V_t(\phi_i, \tilde{J}_i) = \sum_{j \in \tilde{J}_i} v_t(\phi_i, \lambda_{ij})$$

where $v_t(\cdot)$ (and hence V_t) grows at a constant rate g in the balanced growth equilibrium. Then $v_t(\phi_i, \lambda_{ij})$ can be written as:

$$[r - g + \tau(\phi_i)] \cdot v_t(\phi_i, \lambda_{ij}) = \pi_t(\phi_i, \lambda_{ij}) + \Gamma$$

where Γ is the option value of innovation adjusted for the fixed term $F(\phi_i, n_i)$:

$$\Gamma = \max_{x_i} \left[\frac{x_i}{n_i} \cdot \text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \mathbb{E}_{\phi_i} [v_t(\phi_i, \lambda_{ih})] - w_t \cdot \eta_x \cdot (x_i)^{\psi_x} \cdot n_i^{\sigma-1} \right] - \frac{F(\phi_i, n_i)}{n_i} \quad (1)$$

which is a function Γ . In order for the value function to scale with size along the guess (a simplification that is removed in Section 6.2), Γ must not change with the number of goods that the firm produces. I achieve that by choosing $F(\phi_i, n_i)$ such that $\Gamma = 0$. To find the $F(\phi_i, n_i)$ that achieves this, use that the first order condition satisfies:

$$\text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \mathbb{E}_{\phi_i} [v_t(\phi_i, \lambda_{ih})] = \psi^x \cdot w_t \cdot \eta_x \cdot (x_i)^{\psi_x-1} \cdot n_i^\sigma$$

such that if $\Gamma = 0$, the fixed term satisfies:

$$F(\phi_i, n_i) = (\psi^x - 1) \cdot w_t \cdot \eta_x \cdot [x(\phi_i, n_i)]^{\psi_x} \cdot n_i^\sigma$$

With this constraint, optimal research and development expenditures satisfy the equation in Proposition 1:

$$x(\phi_i, n_i) = \left(\text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \frac{\mathbb{E}_{\phi_i} \left[\frac{\pi_t(\phi_i, \lambda_{ij})}{r-g+\tau(\phi_i)} \right]}{\eta^x \cdot \psi^x \cdot w_t} \right)^{\frac{1}{\psi^x-1}} \cdot n_i^{\frac{\sigma}{\psi^x-1}}$$

It follows that

$$V_t(\phi_i, \tilde{f}_i) = \frac{\sum_{j \in \tilde{f}_i} \pi_t(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)}$$

where operating profits satisfy:

$$\pi_t(\phi_i, \lambda_{ij}) = \left[1 - \frac{(\lambda_{ij} \cdot \frac{w}{Y} \cdot (1 - \phi_i))^{\frac{1}{\psi+1}}}{\lambda_{ij}} \right] \cdot Y - w \cdot (1 - \phi_i) \cdot \left(\left[\lambda_{ij} \cdot \frac{w}{Y} \cdot (1 - \phi_i) \right]^{\frac{-\psi}{\psi+1}} - 1 \right)$$

which increases at rate g along the balanced growth path, confirming the initial guess.

Derivation of Aggregate Quantities and Proof of Proposition 2

The equilibrium wage is derived as follows. Start with the definition of aggregate output when each sector is in a betrand equilibrium:

$$\ln Y = \int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \ln (q_{ij} \cdot y_{ij}) di dj$$

Inserting the firm's production function $y_{ij} = l_{ij}/(1 - s_{ij})$ and demand function $y_{ij} = Y/p_{ij}$ yields:

$$\ln Y = \ln Y + \int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \ln \left(q_{ij} \cdot (w \cdot [1 - s_{ij}])^{-1} \cdot \mu_{ij}^{-1} \right) di dj$$

Isolating wage on the left hand side gives:

$$\ln w = \int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \ln \left[\frac{q_{ij}}{1 - s_{ij}} \right] di dj + \int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \ln \left[\frac{1 - s_{ij}}{1 + \lambda_{ij}} \right] di dj$$

The derivation of GDP is as follows. Labor market equilibrium requires:

$$L^p = \int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} l_{ij} di dj$$

Inserting the firm's production function $y_{ij} = l_{ij}/(1 - s_{ij})$ and demand function $y_{ij} = Y/p_{ij}$ yields:

$$L^p = \int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} Y \cdot p_{ij}^{-1} \cdot (1 - s_{ij}) di dj$$

Isolate Y on the left hand side, insert the first order condition for pricing, and insert the equilibrium wage to obtain:

$$Y = L^p \cdot \exp \left(\int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \ln \left[\frac{q_{ij}}{1 - s_{ij}} \right] di dj \right) \cdot \frac{\exp \int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \ln \mu_{ij}^{-1} di dj}{\int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \mu_{ij}^{-1} di dj} \quad (2)$$

Define total factor productivity Q_t as the terms to the right of L^p in expression (2). A balanced growth path equilibrium is characterized by constant type-shares $K(\phi_i)$. Given that markups equation $\lambda_{ij}/(1 - s_{ij})$ where s_{ij} is given by equation (14), the law of large numbers assures that the third term in (2) is constant. Hence $g \equiv \partial \ln Q / \partial t$ is given by:

$$g = \int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \frac{\partial \ln q_{ij}}{\partial t} di dj = \sum_{\phi_i \in \Phi} K(\phi_i) \cdot \tau(\phi_i) \cdot \mathbb{E}_{-\phi_i}(\lambda_{hj})$$

which uses that $K(\phi_i) \cdot \tau(\phi_i)$ is the fraction of goods that changes producer each instance and where initially produced by ϕ_i -type firms.

Proof of Proposition 3

The value function is given by the following Bellman equation:

$$r V_t(\phi_i, \tilde{J}_i) - \dot{V}_t(\phi_i, \tilde{J}_i) = \max_{x_i} \left\{ \begin{aligned} & \sum_{j \in \tilde{J}_i} \left[\pi_t(\phi_i, \lambda_{ij}) + \right. \\ & \quad \left. \tau(\phi_i) \cdot [V_t(\phi_i, \tilde{J}_i \setminus \{\lambda_{ij}\}) - V_t(\phi_i, \tilde{J}_i)] \right] \\ & \quad + x_i \cdot \text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \\ & \quad \cdot \mathbb{E}_{\phi_i} [V_t(\phi_i, \tilde{J}_i \cup \lambda_{ij}) - V_t(\phi_i, \tilde{J}_i)] - w_t \cdot \eta_x \cdot (x_i)^{\psi_x} \cdot n_i^{-\sigma} \end{aligned} \right\}$$

Guess that the solution takes the following form:

$$V_t(\phi_i, \tilde{J}_i) = \sum_{j \in \tilde{J}_i} \Upsilon_t^1(\phi_i, \lambda_{ij}) + \Upsilon_{t,n_i}^2(\phi_i)$$

where firm i produces a portfolio of goods \tilde{J}_i with cardinality n_i , and where $\Upsilon_t^1(\cdot)$ and $\Upsilon_{t,n_i}^2(\cdot)$ (and hence V_t) grow at a constant rate g in the balanced growth equilibrium. Grouping terms yields:

$$(r - g + \tau(\phi_i)) \cdot \Upsilon_t^1(\phi_i, \lambda_{ij}) = \pi_t(\phi_i, \lambda_{ij}) \Rightarrow \Upsilon_t^1(\phi_i, \lambda_{ij}) = \frac{\pi_t(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)}$$

The proof of proposition 1 showed that profits grow at rate g , confirming the guess. Furthermore:

$$(r - g) \cdot \Upsilon_{t,n_i}^2(\phi_i) = \max_{x_i} \left\{ \begin{aligned} & n_i \cdot \tau(\phi_i) \cdot [\Upsilon_{t,n_i-1}^2(\phi_i) - \Upsilon_{t,n_i}^2(\phi_i)] + x_i \cdot \text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \\ & \cdot \mathbb{E}_{\phi_i} [\Upsilon_{t,n_i+1}^2(\phi_i) - \Upsilon_{t,n_i}^2(\phi_i) + \Upsilon_{t,n_i}^1(\phi_i, \lambda_{ij})] - w_t \cdot \eta_x \cdot (x_i)^{\psi_x} \cdot n_i^{-\sigma} \end{aligned} \right\}$$

The first order condition of the maximization reads:

$$\text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \mathbb{E}_{\phi_i} [\Upsilon_{t,n_i+1}^2(\phi_i) - \Upsilon_{t,n_i}^2(\phi_i) + \Upsilon_{t,n_i}^1(\phi_i, \lambda_{ij})] = w_t \cdot \psi_x \cdot \eta_x (x_i)^{\psi_x-1} n_i^{-\sigma}$$

Inserting the first order condition and isolating $\Upsilon_{t,n_i+1}^2(\phi_i)$ and $\Upsilon_t^1(\phi_i, \lambda_{ij})$ on the left hand side gives the sequence for Υ_{t,n_i+1}^2 along:

$$\begin{aligned} \Upsilon_{t,n_i+1}^2(\phi_i) + \Upsilon_t^1(\phi_i, \lambda_{ij}) &= \left[\frac{(r - g) \cdot \Upsilon_{t,n_i}^2(\phi_i) + n_i \cdot \tau(\phi_i) \cdot [\Upsilon_{t,n_i}^2(\phi_i) - \Upsilon_{t,n_i-1}^2(\phi_i)]}{\psi^x - 1} \right]^{\frac{\psi^x-1}{\psi^x}} \\ &\quad \cdot \text{Prob} \left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right)^{-1} \cdot \psi^x \cdot (\eta \cdot w_t)^{\psi^x-1} \cdot n_i^{-\frac{\sigma}{\psi^x}} + \Upsilon_{t,n_i}^2(\phi_i). \end{aligned}$$

Appendix B. Data

B1. Construction of French Administrative Dataset

Balance Sheet and Income Statement The main firm-level datasets are FICUS from 1994 to 2007 and FARE from 2008 to 2016. I keep all firms in legal category 5, which means all non-profit firms and private contractors are excluded from the sample. I also drop firms with operating subsidies in excess of 10% of revenues. From 2004, INSEE starts to group firms that are owned by the same company in single *siren* codes. This treatment has been gradually extended over time, which means that data on groups in later years of the data contain more consolidated firms. From 2009 onwards, data is provided separately for the underlying firms (legal entities) and for the group. To have a consistent panel (and prevent an artificial increase in firm concentration), I group firms along the pre-2009 definitions and extend that treatment backwards and forwards.

Software and IT Data on software comes from the Annual Enterprise Survey (*Enquête Annuelle d'Entreprises*, EAE), which is an annual survey of around 12,000 firms between 1994 and 2007. There are separate surveys for major industries (agriculture, construction, manufacturing, services, transportation) which differ in variables and coverage. The survey is comprehensive for firms with at least 20 employees, and smaller firms are sampled for all sectors except manufacturing. The survey is merged to FARE-FICUS using the SIREN firm identifier. The level of observation is the legal unit, for firms that are aggregated prior to 2009 by INSEE as discussed in the main text. From 2008 onwards I use data from the E-Commerce Survey (*Enquête sur les Technologies de l'Information de la Communication* - TIC). This survey contains questions on the use of IT systems annually from 2008 to 2016. This dataset contains dummies on the adoption of specific IT systems such as Enterprise Resource Planning and Customer Resource Management.

Research and Development Data on R&D comes from the Community Innovation Survey (*Enquête Communautaire sur L'Innovation* - CIS). The CIS is carried out by national statistical offices throughout the European Union, and is coordinated by Eurostat. The survey is voluntary, but sample weights are adjusted for non-response to create nationally representative data. The French survey is carried out by INSEE, and contains consistent variables on research and development expenditures in 1996, 2000, 2004, 2006, 2008, 2010, 2012, 2014 and 2016.

Product Count The number of products by firm comes from the Annual Production Survey (*Enquête Annuelle de Production*, EAP). This survey is used for annual data on industrial production for the EU's PRODCOM statistics. The survey is available for manufacturing only, from 2009 to 2016. I count the number of unique products each year by firm, excluding products on which the firm acts as outsourcer, or was only involved in product design (M1 and M5).

B2. Variable Definitions

French Administrative Data

Revenue is total sales, including exports. In FICUS years this is CATOTAL, in FARE years this is REDI_R310. In regressions, firm-size is controlled for by a third degree polynomial of log revenue.

Employment Employment is the full-time equivalent of the number of directly employed workers by the firm averaged over each accounting quarter. In FICUS, the data is based on tax records for small firms, and on a combination of survey and tax data for large firms (variable name: EFFSALM). In FARE the variable is REDI_E200, which is based on the administrative DADS dataset.

Wage bill The wage bill is defined as the sum of wage payments (SALTRAI in FICUS, REDI_R216 in FARE) and social security contributions (CHARSOC in FICUS, REDI_R217 in FARE).

Direct production inputs are calculated as the sum of merchandise purchases (goods intended for resale) and the purchase of raw materials, corrected for fluctuations in inventory. In FICUS, the respective variables are ACHAMAR, ACHAMPR, VARSTMA, and VARSTMP. The corresponding variables in FARE are REDI_R210, REDI_R212, REDI_R211, and REDI_213.

Other purchases Other purchases are defined as purchases of services from other firms. This includes outsourcing costs, lease payments, rental charges for equipment and furniture, maintenance expenses, insurance premiums, and costs for external market research, advertising, transportation, and external consultants (AUTACHA in FICUS, REDI_R214 in FARE).

Operating profits is defined as revenue minus the wage bill, expenditure on direct production inputs, other purchases, import duties and similar taxes (IMPOTAX in FICUS, REDI_R215 in FARE) capital depreciation (DOTAMOR in FICUS), provisions (DOTPROV in FICUS), and other charges (AUTCHEX in FICUS). The sum of the wage bill, material input expenses, capital depreciation, provisions, and other charges is REDI_R201 in FARE.

Capital stock Capital is measured as the stock of fixed tangible assets. This includes land, buildings, machinery, and other installations. The associated variable is IMMOCOR in FICUS, and IMMO_CORP in FARE. The capital stock is not calculated using the perpetual inventory method because investment data is unavailable for 2008.

Industry codes Industry codes are converted to NACE Rev. 2 codes using official nomenclatures. Firms that are observed before and after changes to industry classifications are assigned their NACE Rev. 2 code for all years, while other firms are assigned a code from official nomenclatures. Firms in industries without a 1-to-1 match in nomenclatures are assigned the NACE Rev. 2 that is observed most frequently for firms with their industry codes. Firms that switch industry codes are assigned their modal code for all years.

Research and Development R&D investments are measured as all innovative expenditures by firms as reported in the CIS. Subcategories of expenditures fluctuate with each version of the survey, but total expenditures seems consistently defined. In 2012 total expenditures are found in RALLX. In some year I add up underlying variables to create a similar variable. Details for each year are available upon request.

Software Investments The variable for software investments closely follows the definition in [Lashkari and Bauer \(2018\)](#). The underlying variables are observed from 1994 to 2007 in the EAE. The main variable for software is I460. This variable contains all software investments and is available for all sectors. Because missing observations are coded as 0, I drop these firm-years when analysing software. An additional sub-division into externally purchased and internally developed software is available for a subset of firms (I461, I462, I463, I464, I465). Where available, I use this to clean cases where I460 is smaller than I461-I465, and verify that summary statistics match [Lashkari and Bauer \(2018\)](#).

Compustat Data

Revenue is total sales. The Compustat Fundamentals variable is SALE.

Cost of good sold involves all direct costs involved with producing a good. This includes the cost of materials and other intermediate inputs, as well as the labor directly used to produce a good. It is observed on the income statement. The Compustat variable is COGS.

Selling, general and administrative expense are all direct and indirect selling, general and administrative expenses. They include overhead costs and costs such as advertisement or packaging and distribution. It is observed on the income statement. The Compustat variable is XSGA.

Operating expenses are the sum of cost of good sold and selling, general, and administrative expenses. The Compustat variable is XOPR.

Capital stock The firm's production capital is defined as the contemporaneous balance sheet value of gross property, plants and equipment (tangible fixed assets). The Compustat variable is PPEGT.

Operating profits are measured as income before extraordinary items. I add expenditures on research and development because these are expensed in the American data yet not in the French data. This furthermore prevents a spuriously positive correlation between the fixed cost measure (which declines in profits) and research and development. The Compustat variable is IB.

Research and development expenditures include all the costs incurred for the development of new products and services. They also include R&D activities undertaken by others for which the firm paid. They are observed on the income statement. The Compustat variable is XRD.

Product count is obtained from the Compustat Historical Segments File. I count the number of products that firms produce as the number of unique primary 6-digit NAICS codes of business segments that firms report. In the adjusted product count I assign a product count of 1 for firms that are not present in the segments file.

Appendix C. Fixed Costs Estimation

This appendix summarizes the implementation of the iterative GMM approach by [De Loecker and Warzynski \(2012\)](#) that is used to estimate the output elasticity of variable input m in Section 2.2. The production function estimation relies on codes developed for [Burstein et al. \(2019\)](#) who analyse the cyclical properties of French markups, and I thank the authors for permission to use the code for this project.

C1. Estimation Procedure

France

Because equation (1) contains both tangible (through $z(\cdot)$) and intangible inputs (through s_i), the framework in Section 2.1 implies a production function along $\tilde{z}(z_{it,1}, \dots, z_{it,k}; u_{it,1}, \dots, u_{it,h}) \cdot \omega_{it}$ with k tangible and h intangible inputs, Hicks neutral productivity ω_{it} , and potentially increasing returns to scale. I approximate this general production function by estimating a flexible translog function that contains the (squared) log of all observed inputs. I first estimate a production function with capital k , labor l and materials m for each 2-digit industry with at least 12 firms in the data, along:

$$y_{it} = \beta^l \cdot l_{it} + \beta^{ll} \cdot l_{it}^2 + \beta^k \cdot k_{it} + \beta^{kk} \cdot k_{it}^2 + \beta^m \cdot m_{it} + \beta^{mm} \cdot m_{it}^2 + \omega_{it} + \epsilon_t \quad (3)$$

where cross-terms are omitted to prevent measurement error in one of the inputs to directly affect the estimated elasticity of other inputs.⁴⁰ Capital is measured through fixed tangible assets, labor is the number of employees and materials equal firm purchases. In contrast to (i.e.) U.S. Census data, data on materials is available annually for firms in all industries.

The three-factor production function is commonly used in the literature and is therefore the basis of estimates in the main text. To assess the robustness of these estimates, I also estimate a more extensive production function with four production factors. The FARE-FICUS dataset allows materials to be divided into direct production inputs v (intermediate goods for resale and expenses on primary commodities) and other purchases o , which include the purchase of external services like advertising. I estimate an additional production function that separates these logged factors along:

$$y_{it} = \beta^l \cdot l_{it} + \beta^{ll} \cdot l_{it}^2 + \beta^k \cdot k_{it} + \beta^{kk} \cdot k_{it}^2 + \beta^v \cdot v_{it} + \beta^{vv} \cdot v_{it}^2 + \beta^o \cdot o_{it} + \beta^{oo} \cdot o_{it}^2 + \omega_{it} + \epsilon_t \quad (4)$$

Because of the large number of firms in the data, I estimate this more extensive production function separately for each 4-digit industry.

⁴⁰This follows [De Loecker and Eeckhout \(2017\)](#) in their treatment of capital.

All inputs but material are likely to be a combination of tangible and intangible inputs in the context of Section 2.1's model, with the exception of direct production inputs.⁴¹ Direct production inputs are tangible, as they only include expenses on intermediate goods for resale or expenses on primary commodities. An output elasticity can only be used to estimate markups when the factor is freely set each period, which seems most likely to hold for v . That is why I use the elasticity of output with respect to v to estimate markups from the four-factor production function.

Both production functions are estimated under the assumption that a firm's demand for material is an invertible function $m(\cdot)$ (or $v(\cdot)$) of the firm's productivity ω_{it} and capital and labor inputs. As a consequence, the production functions can be written as:

$$y_{it} = \beta^l \cdot l_{it} + \beta^{ll} \cdot l_{it}^2 + \beta^k \cdot k_{it} + \beta^{kk} \cdot k_{it}^2 + \beta^m \cdot m_{it} + \beta^{mm} \cdot m_{it}^2 + m^{-1}(\omega_{it}, l_{it}, k_{it}) + \epsilon_t \text{ and}$$

$$y_{it} = \beta^l \cdot l_{it} + \beta^{ll} \cdot l_{it}^2 + \beta^k \cdot k_{it} + \beta^{kk} \cdot k_{it}^2 + \beta^v \cdot v_{it} + \beta^{vv} \cdot v_{it}^2 + \beta^o \cdot o_{it} + \beta^{oo} \cdot o_{it}^2 + v^{-1}(\omega_{it}, l_{it}, k_{it}) + \epsilon_t$$

respectively. Under this assumption, I purge log gross output y_{it} from measurement error by estimating:

$$y_{it} = h(l_{it}, k_{it}, m_{it}) + \epsilon_{it} \text{ and } y_{it} = h(l_{it}, k_{it}, v_{it}, o_{it}) + \epsilon_{it}$$

where h is a non-parametric function approximated by a third degree polynomial in the inputs.

After purging gross output, the production function is estimated iteratively. The algorithm is as follows. First, I guess the coefficients of the production function using OLS estimates. Given (purged) output, inputs, and the production function, I calculate ω_{it} . The algorithm then estimates the autoregressive process of productivity along:

$$\omega_{i,t} = g' \left[1 \ \omega_{i,t-1} \ \omega_{i,t-1}^2 \right]' + \xi_{i,t}$$

where residual ξ_{it} captures shocks to productivity not explained by (squared) lagged values of productivity, while g is a vector of coefficients obtained by minimizing the sum of squared residuals $\xi_{i,t}$:

$$g = \left(\begin{bmatrix} 1 & \omega_{t-1} & \omega_{t-1}^2 \end{bmatrix} \begin{bmatrix} 1 \\ \omega_{t-1} \\ \omega_{t-1}^2 \end{bmatrix} \right)' \left(\begin{bmatrix} 1 & \omega_{t-1} & \omega_{t-1}^2 \end{bmatrix} \omega_t \right) \quad (5)$$

The algorithm iterates the production function coefficients until the errors of the AR(1) process for productivity satisfy:

$$\mathbb{E}(\xi_{it} Z_{i,t}) = 0 \quad (6)$$

where $Z_{i,t}$ is a vector of instruments:

$$Z_{i,t} = [l_{it-1} \ l_{it-1}^2 \ k_{it} \ k_{it}^2 \ m_{it-1} \ m_{it-1}^2]'$$

⁴¹Labor may seem a tangible input, but if labor is used to develop or deploy software for production then the intangible input labor appears on the income statement through the wage bill.

or for the four-factor production function:

$$Z_{i,t} = [l_{it-1} \ l_{it-1}^2 \ k_{it} \ k_{it}^2 \ v_{it-1} \ v_{it-1}^2 \ o_{it-1} \ o_{it-1}^2]'$$

By instrumenting k with its current value, I assume that firms cannot increase capital in response to a contemporaneous productivity shock. By instrumenting l , m , v and o by their lagged value I assume that they are set freely each period, but require autocorrelation in factor prices.⁴²

United States

To estimate markups for the calculation of fixed costs of U.S. publicly listed firms I deploy the same procedure. A constraint of the analysis of markups for these firms is that data on materials and the wage bill is not available from the income statement. Instead, there is a broad category of operating expenses (cost of good sold) that captures all expenditures that are directly related to the cost of production. This is the variable used for flexible inputs in [De Loecker et al. \(2018\)](#), whose procedure I follow closely. Results in the main text are based on a fixed cost measure that uses these markup estimates.

One critique on using a production function estimation with capital and cost of good sold is that it does not account for selling, general, and administrative expenses (SG&A), which have become more important over time. Adding SG&A to cost of good sold to form a single input in a production function is evenly problematic because 1) a large part of SG&A are fixed overhead costs as well as expenditures on intangible inputs,⁴³ and 2) this assumes that all types of operating expenses are perfect substitutes. Instead, I test the robustness of my main results by adding SG&A as a separate input in a production function along (3).

C2. Robustness of Fixed Cost Trends

France

The results in the main text are robust to using the more extensive four-factor production function. After estimating the industry-level production function coefficients, I calculate the firm-level markup along equation (2.3) and calculate the fixed cost share along (3). Markups at the firm-level are summarized in Table A1. The table shows that the extensive production function estimates a very similar average markup to the markup from the standard three-factor production function. The variance of markups, however, is significantly greater when using the four-factor production function. This is likely due to the additional parameters that need to be estimated at the 4-digit level, or because firms have some flexibility in what costs fall under direct production inputs v versus other purchases o . The firm-level correlation coefficient between both markups is 0.35.

⁴²For France it is reasonable to assume that labor is, in fact, not set freely and could therefore be instrumented by contemporaneously. This turns out to have no significant effect on the estimated production function.

⁴³Heterogeneity in fixed costs across firms will then cause an underestimation of the input elasticities and markups.

Table A1: Summary Statistics on Estimated Markups

	Mean	Std. Dev.	Median	10th Pct.	90th Pct.	Observations
<i>France</i>						
Basic production function	1.38	0.43	1.26	0.96	1.91	9,913,058
Extended production function	1.42	1.25	1.01	0.53	2.59	8,477,467
<i>United States</i>						
COGS production function	1.52	.620	1.33	1.01	2.27	125,231
COGS and SG&A production function	1.33	.589	1.15	0.86	2.02	125,231

The trends of aggregate fixed costs are plotted in Figure A1. The solid-blue line is replicated from the main text and is for the three-factor standard production function, while the squared-green line uses the four-factor extensive production function. Both figures show that the sales-weighted average fixed cost share has increased strongly over the 1994 to 2016 sample, with the largest increase occurring between 1994 and 2010, after which the increase moderates.

United States

Markups from the two-factor and three-factor production functions are highly correlated. The bottom panel of Table A1 presents summary statistics for both and shows that they mainly differ in terms of their level. When adding SG&A, over 30% of all firms have markups below 1 and the median markup is 1.15. Though the 2-factor admits markups around 15 percentage points above that at most percentiles, both series co-move strongly. The firm-level correlation is 0.92. While the correlation of the markup series is close, the difference in levels between the series have a large effect on the predicted level of fixed costs. The right plot in Figure A1 shows that the 3-factor production function predicts *negative* average fixed costs as a percentage of total costs between 1980 and 2004. This is likely to be driven by an underestimation of the markup; of the firms with a

Figure A1. Robustness of Trends in Aggregate Fixed Cost Share

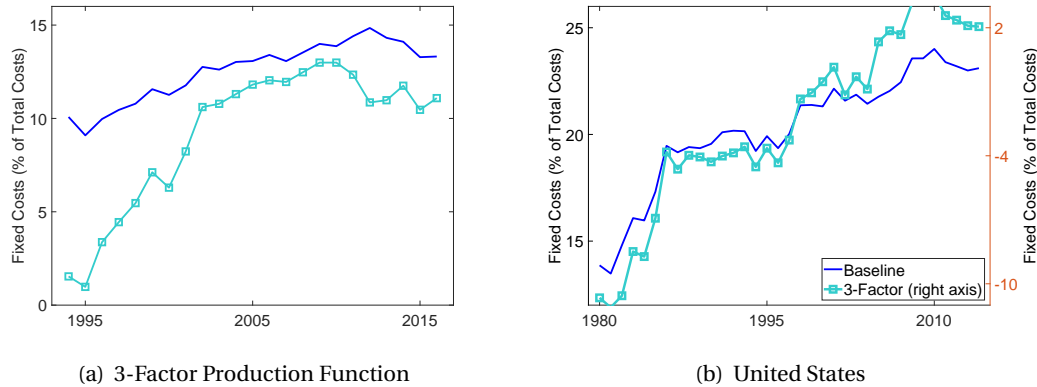
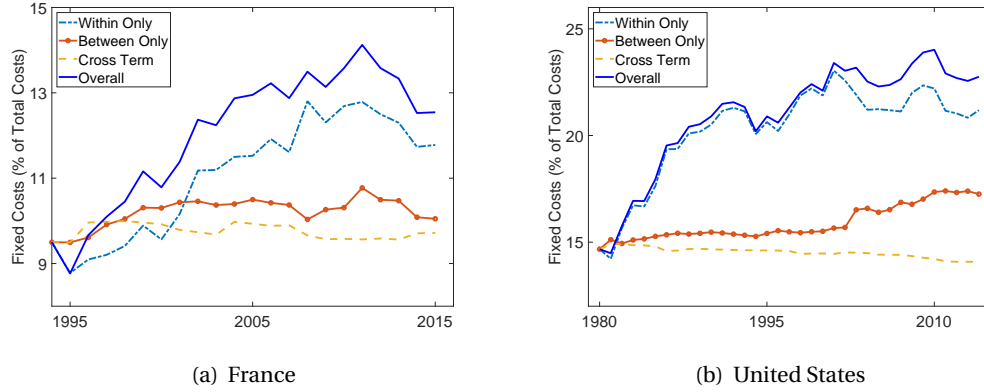


Figure A2. Within-Between Decomposition of the Rise of Fixed Costs



3-factor markup below unity, 63% report positive profits. The predicted increase in fixed costs over the sample is 13 percentage points, which is similar to the predicted increase in the main text.⁴⁴

C3. Within versus Between Sector Changes in Rise of Fixed Costs

I perform the following within-between decomposition:

$$\Delta \frac{\tilde{F}_t}{TC_t} = \sum_{j \in J} s_{jt-1} \cdot \Delta \frac{\tilde{F}_{jt}}{TC_{jt}} + \sum_{j \in J} \Delta s_{jt} \cdot \frac{\tilde{F}_{jt-1}}{TC_{jt-1}} + \sum_{j \in J} \Delta s_{jt-1} \cdot \Delta \frac{\tilde{F}_{jt}}{TC_{jt}}$$

where \tilde{F}_t/TC_t is the aggregate fixed cost share, \tilde{F}_{jt}/TC_{jt} the sector-level counterpart, and s_j the fraction of sales by sector j . The first term captures changes due to increases in fixed costs within sectors. The second term captures the ‘between’ share: changes because of changes in the relative size of sectors. The last term is the interaction of both. I perform the decomposition annually and regress each term on the change in the aggregate fixed cost share. The resulting coefficients are presented in Table A2. Figure A2 illustrates the contribution of the within and between share over time, by plotting the development of fixed costs holding other contributors constant.

Table A2: Decomposition of Changes in Aggregate Fixed Cost Share

	Within Sectors	Between Sectors	Cross Term	Total
<i>France</i>	0.73*** (0.003)	0.21*** (0.003)	0.06*** (0.003)	1
<i>United States</i>	1.02*** (0.053)	0.00 (0.050)	-0.02 (0.015)	1

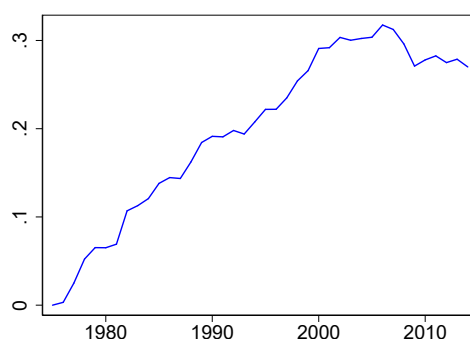
Standard errors in brackets. *** denotes significance at the 1% level.

⁴⁴Figure A1 does raise concerns about the correct calibration target for the initial level of fixed costs. The baseline calibration uses 12%. De Loecker et al. (2018) assume that SG&A find that the of fixed costs has increased from 18% to 24% for Compustat firms. In unpublished work, Saibene (2017) finds that the share of fixed costs and total costs from 10% to 20% for Compustat firms, based on the sensitivity of costs to sales shocks. I conclude that the 12% calibration target for 1980 is within the plausible range of estimates.

Appendix D. Macroeconomic Trends in France

The introduction summarizes three recent trends: the slowdown of productivity growth, the fall in business dynamism and the rise of corporate profits. This appendix gives an overview of the macroeconomics trends discussed in the introduction for France. The slowdown of productivity growth is depicted in Figure A3. It plots an index of the log of total factor productivity at constant prices, standardized to 0 in 1975. The figure shows that total factor productivity was growing at a rate close to 2% for most years between 1975 and 2000, and has not increased (and even modestly decreased) since.

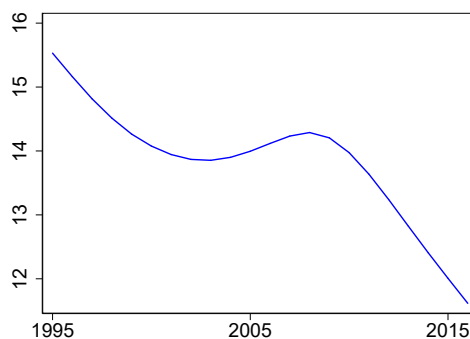
Figure A3. Total Factor Productivity in France



Log TFP at constant prices, 1975=0. Data: Penn World Tables.

The slowdown of business dynamism is summarized with three statistics, following the literature. The first is the reallocation rate in Figure A4, which is the sum of job destruction and creation rates. I calculate the reallocation rate across French firms using the FARE-FICUS dataset for 1994-2016. Because this sample coincides with the Great Recession, which brought a strong transitory increase in reallocation due to job destruction, I plot the HP trend.

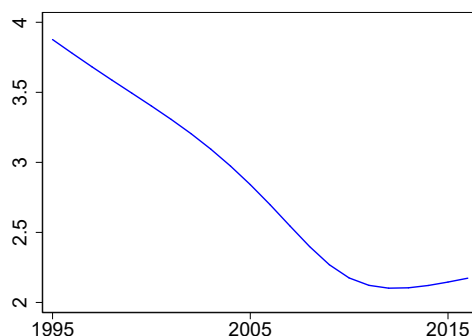
Figure A4. Reallocation Rate in France



Sum of job creation and job destruction rates across companies. HP Trend.
Source: own calculations based for universe of French firms (FARE-FICUS)

The second fact is the decline of entry of new firms. Figure A5 captures this trend by plotting the fraction of employees that work for a firm that enters the FARE-FICUS dataset in a given year. Note that this may include firms that have undergone significant organizational changes that have caused their firm identifier to change. The figure shows that employment by entrants has declined by almost half within the 1994-2016 sample.

Figure A5. Fraction of Employment by Entrants in France

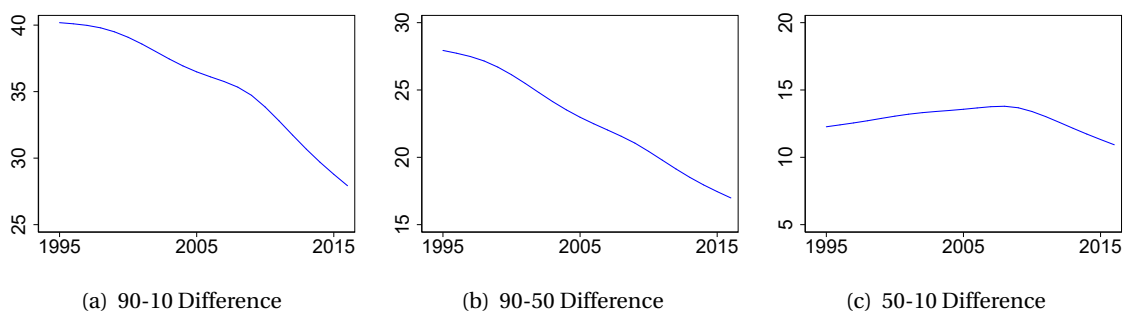


Percentage of employment by new firms (≤ 1 yr) in private sector employment. HP trend

Source: own calculations based for universe of French firms (FARE-FICUS)

The third fact is the decline of skewness of the firm growth distribution. As discussed by [Decker et al. \(2017\)](#), small (young) high-growth firms have historically been an important contributor to productivity growth. They infer the decline in skewness of the growth distribution from the decline between the 90th and 10th, and between the 90th and 50th percentile of the growth distribution. Figure A6 shows that both have declined by around 40% between 1994-2016. The difference between the 50th and 10th percentile has remained flat, in line with evidence for the U.S.

Figure A6. Skewness of the Employment-Growth Distribution

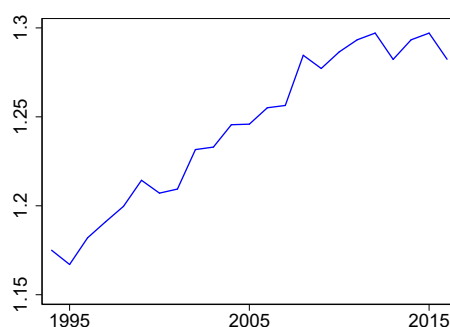


Difference (perc. point) in growth between percentiles of the employment-growth distribution. HP trend.

Data: universe of French firms (FARE-FICUS)

The rise of corporate profits is measured through the marginal cost markup. This is a measure of marginal rather than average profits, a distinction that is key in Section 2. Figure A7 plots the average sales-weighted markups for French firms between 1994 and 2016. The markups has increased modestly, in line with previous evidence (e.g. IMF 2019).

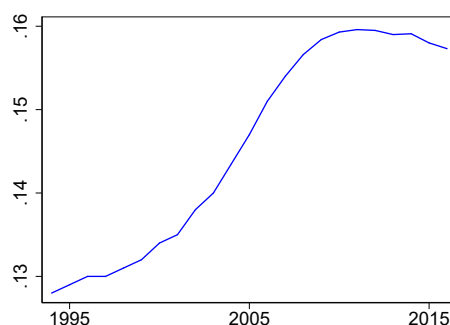
Figure A7. Marginal Cost Markups for France



Sales-weighted marginal cost markups using the Hall (1988) equation with production function elasticities estimated with iterative GMM as in De Loecker and Warzynski (2012). Details in Appendix C. Data: universe of French firms (FARE-FICUS)

Concentration displays a modestly positive trend over the sample. This is shown in Figure A8, which depicts the average Herfindahl Index across 5-digit industries. The rise of concentration has been linked to the decline in the labor share by Autor et al. (2017) through the reallocation of activity to firms with low labor shares. This result has been replicated for France for 1994-2007 by Lashkari and Bauer (2018). Note that the increase in concentration depends on measurement. The graph below presents an average of the Herfindahl across sectors. Weighing sectors by value added gives an increase in the Herfindahl index from 0.087 in 1994 up to 0.122 in 2008, but a modest decline to 0.117 afterwards. Further issues with measures of concentration include the fact that geographical segments (both regional and internal) are not well defined and that industry codes may not accurately capture product markets.

Figure A8. Firm Concentration



Average Herfindahl index across 5-digit NACE industries

Source: own calculations based for universe of French firms (FARE-FICUS)

Appendix E. Computational Algorithm

The balanced growth path equilibrium along definition 1 is found by solving the system of de-trended equilibrium equations as a fixed point. The algorithm works as follows:

1. Solve the fixed point:

- (a) Guess a level of Y/Q , w/Q , $\tau(\phi)$, and $K(\phi)$.
- (b) Collect choke prices by solving:

$$\left(p^{choke}(\phi_i) - w \cdot [1 - s^*(\phi_i)] \right) \cdot Y - w \cdot (1 - \phi_i) \cdot ([1 - s(\phi_i)]^{-\psi} - 1) = 0 \text{ where } \phi_i \in \Phi$$

- (c) Given the vector of choke prices and the guess for $K(\phi)$, calculate the following objects:

- a $|\Phi| \times |\Phi|$ matrix \mathbf{P} with probabilities that a firm of type $\phi_i \in \Phi$ successfully innovates when facing $\phi_{-i} \in \Phi$ along (12) and a vector with the weighted average over this probability $\sum_{\phi_{-i} \in \Phi} K(\phi_{-i}) \cdot \mathbf{P}(\phi_i, \phi_{-i})$ with the probabilities that a type's innovation is successful in general.
- the set of distributions of $\lambda_{ij} \sim \text{Exp}(\bar{\lambda})$ for each combination of $\phi_i \in \Phi$ and $\phi_{-i} \in \Phi$ truncated at $p^{choke}(\phi_i) / p^{choke}(\phi_{-i})$.
- the expectation of markups along (13) given the truncated distributions and the guess for $K(\phi)$.
- the optimal innovation efforts by incumbents and entrants given markups, \mathbf{P} , Y , w , $\tau(\phi)$, and $K(\phi)$.

- (d) Calculate Y along (22) and w along (21). Use the innovation effort by incumbents and entrants to calculate $\tau(\phi)$ along (11) and (18), (19) and (20) to find $K(\phi)$.
- (e) Repeat from step (b) until the model has converged.

2. Perform the firm simulation:

- (a) Collect the equilibrium Y , w , $\tau(\phi)$, $K(\phi)$, $x(\phi, n)$, e for all n and all $\phi_i \in \Phi$.
- (b) Discretize time by introducing a sufficiently large number of instances per year such that $x(\phi, n) < 1$ and $e < 1$.
- (c) Initialize the firm-size distribution along (18) and (19).
- (d) Simulate firms until the markup distribution has converged, then collect moments.

The transitional dynamics are numerically solved using the following algorithm:

1. Create a fine grid with a T -year horizon in order to discretize time, allowing each year to consist of \tilde{T} instances.
2. Guess an initial value function of innovation activities $V(\cdot)$ equal to the new steady-state level for each type in $\phi_i \in \Phi$ at each point of the grid. Similarly guess the paths of wages w/Q and output Y/Q at their new steady-state level.
3. Initialize the firm-size and type distribution variables $K(\phi)$ and $M(\phi, n)$ to their original steady state values.
4. Iterate over the path of the value function as follows:
 - (a) Solve the static optimization problem and the dynamic innovation decisions for incumbents and entrants for each point on the grid using the initial guess for $V(\cdot)$.
 - (b) Given the innovation and static decisions, simulate the development for a large (N) number of products and track the innovation step-sizes λ in $N \times (T \cdot \tilde{T})$ matrix Λ and similarly a matrix of ownership types using a forward loop over the grid.⁴⁵
 - (c) Update the value function for the next iteration using the new sequences for Y , w , the firm-type and -size distribution, and distributions for markups and λ s implied by Λ . This involves calculating:
 - i. the expectation of profits $\pi_{kt}(\phi_i, \lambda_{ij})$ at each instance t on the grid $t = 1, \dots, T$ separately for each cohort of patents k .
 - ii. the value of obtaining the patent to produce an additional product for incumbents of type ϕ at time k as follows:

$$V^k(\phi_i) = \mathbb{E}_{\phi_i}^k \left[\sum_{t=k+1}^{\epsilon \cdot T} \prod_{h=k+1}^t \left(\frac{1 - \tau_h(\phi_i)}{1 + \rho} \right) \cdot \pi_{kt}(\phi_i, \lambda_{ij}) \right]$$

which is a discretization of the original value function, where ϵ is set sufficiently high such that the present value of profits in instances exceeding $\epsilon \cdot T$ approaches zero.⁴⁶

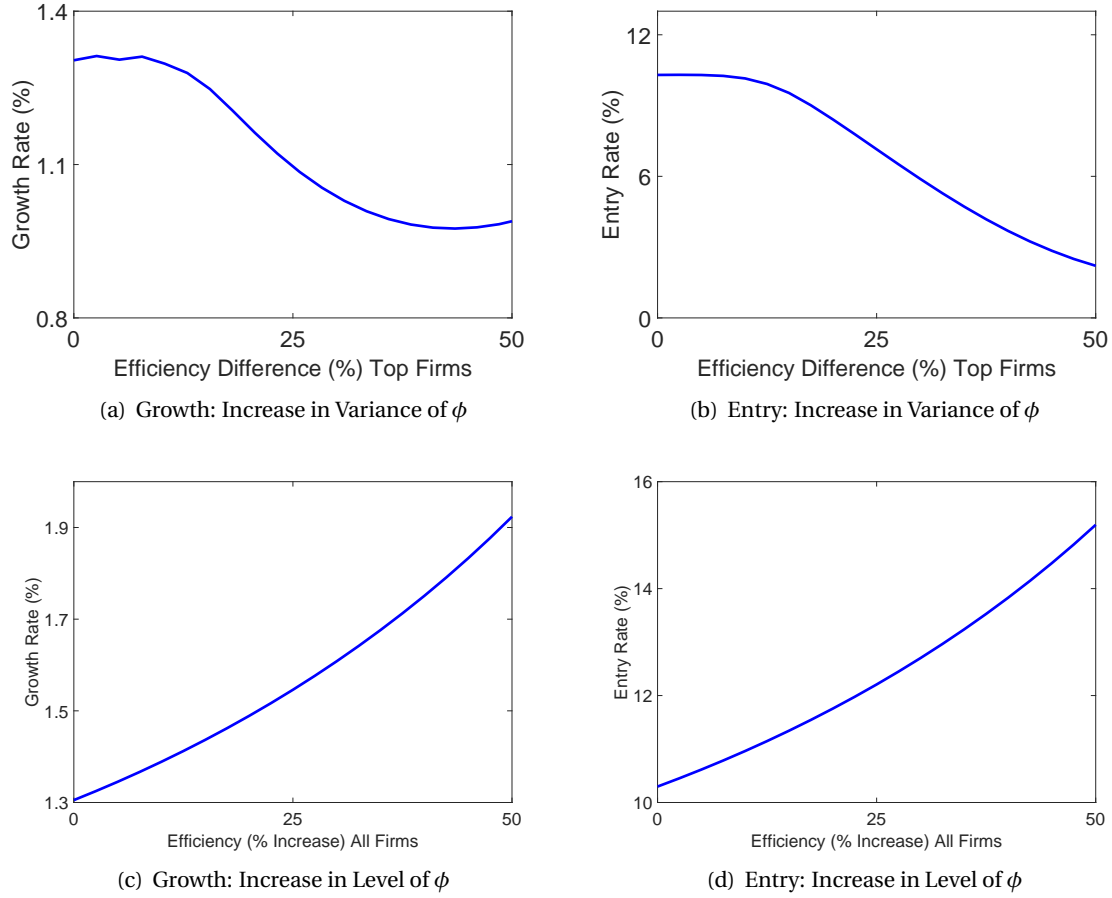
- (d) Use the resulting value for each type on each point of the grid as the guess for $V(\cdot)$ in step (a) in the next iteration. Continue until the path of the value function has converged.

⁴⁵This simulation is needed because the changing composition of firm types means the distribution of realized λ s has no analytical representation. I then use the resulting distribution of markups to calculate the efficiency wedge along (22), as well as a path for Y and w . These serve as the basis for the algorithm's next iteration.

⁴⁶I set $T = 3000$ (corresponding to 60 years), $\epsilon = 11$ (allowing for a profit horizon of 10 times the studied transition, 600 years), and set $N = 10000$.

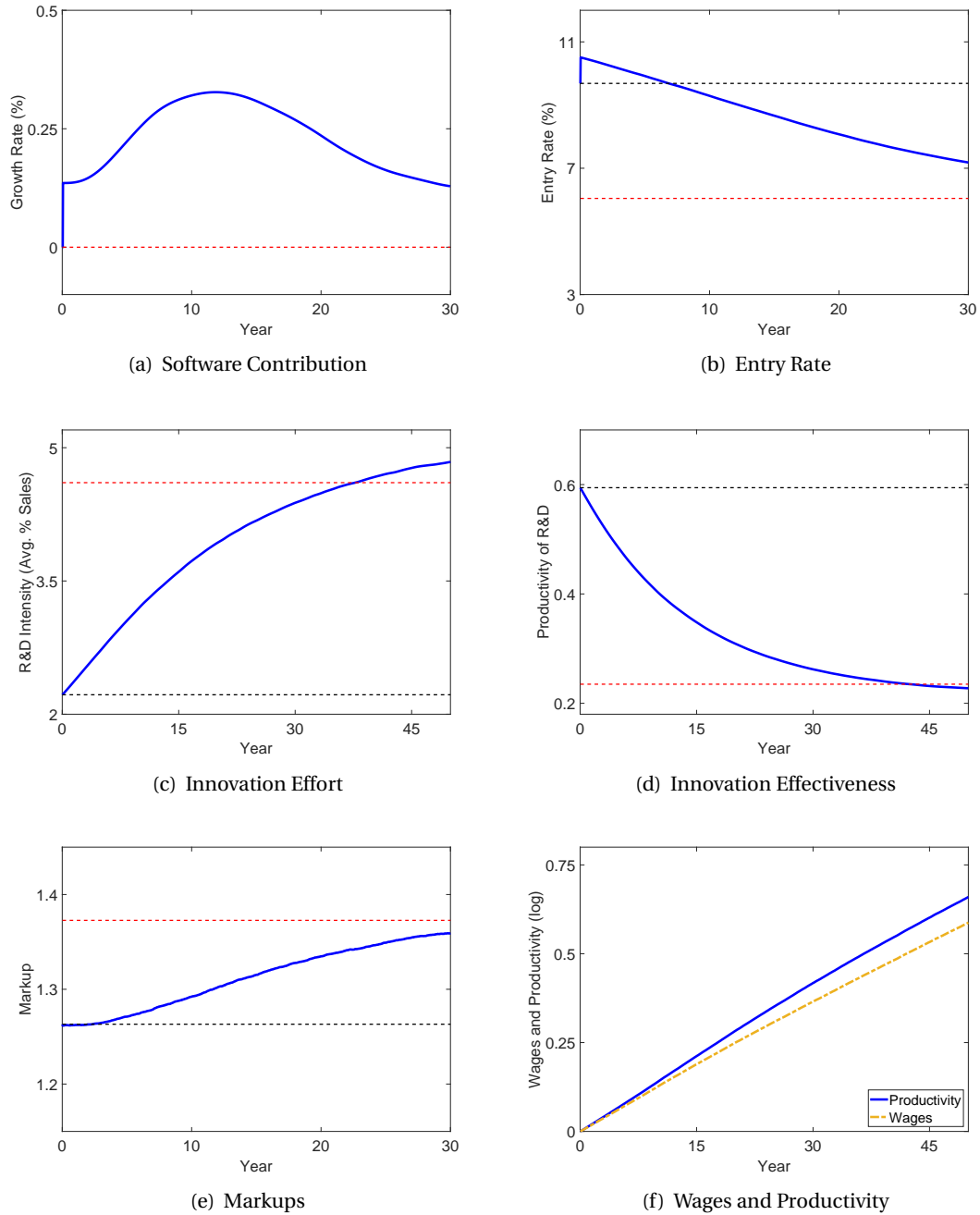
Appendix F. Additional Figures

Figure A9. Increase in the Variance versus the Level of ϕ : French Calibration



Notes: Figures (a) and (b) plot growth and entry at different variances of ϕ , implemented through a mean preserving spread (MPS). The MPS is such that 4% (96%) of entrants have various levels of high (low) intangible ability while the expected ϕ remains 0.8. Figures (c) and (d) plot growth and entry at different levels of ϕ . X-axis expresses the % difference from $\phi = 0.75$.

Figure A10. Transitional Dynamics: French Calibration



Solid blue line indicates growth rate of total factor productivity in percent over the transition path.

Black- and red-dashed lines (respectively) indicate the original and the new steady state.