Idea Production and Team Structure

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Abstract

In the modern world, technological and scientific innovation rely heavily on teamwork. The "lone genius" is mostly a story of the past, as innovating teams have become larger, more diverse, and more geographically dispersed. This paper builds a framework that links idea production and matching in teams to aggregate innovation. With this framework, the paper quantifies the forces behind rising team size and shifts in team composition and produces a method for evaluating the relative importance of specific expertise for aggregate innovation. To do so, I build a matching-in-teams model which I combine with novel empirical measures of inventor expertise from the US Patent and Trademark Office (USPTO). In the model, inventors with heterogeneous expertise decide to form teams by weighing the output produced, governed by an idea production function, against the costs of communicating within the team. I use measures of inventor and team *depth* (expertise within the focal patent technology) and *breadth* (expertise outside the focal patent technology) to estimate the idea production function. I use geographical dispersion to quantify the costs of communicating within teams. I find that both changes in the returns to depth and breadth in addition to falling communication costs explain a large share of the increase in team size from 1980-2000, with changing returns to depth and breadth explaining over half of this increase. The higher returns to depth and breadth suggest an increasing importance for fostering skills well-suited to teams. To quantify the overall impact of specific expertise, I ask how aggregate innovation responds to exogenous shifts in the distribution of inventor expertise in the economy. The results have natural implications for high-skilled immigration policies that shift a country's distribution of inventor expertise. I use these results to study the impact of the fall of the Soviet Union and resulting influx of Russian immigrant inventors to the US on American innovation.

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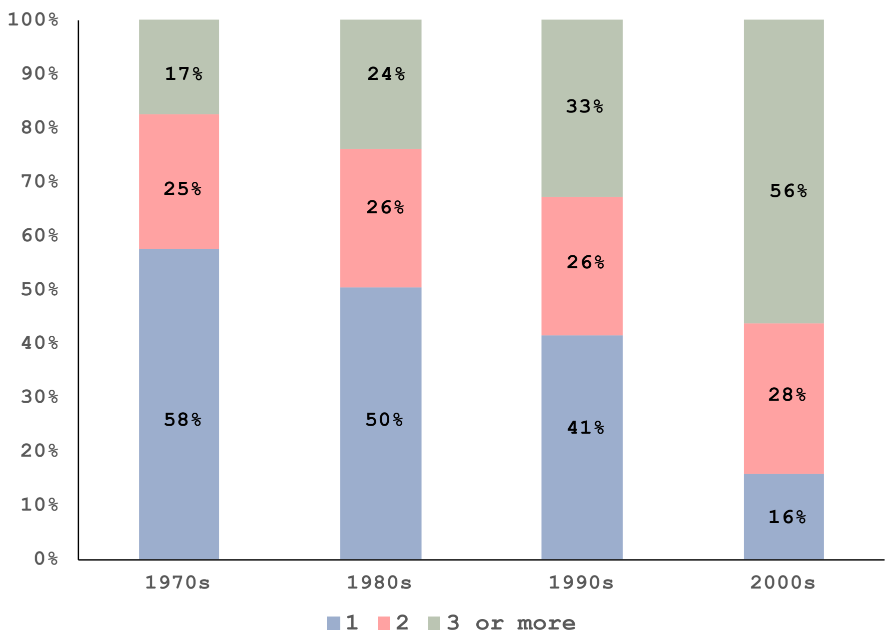
1 Introduction

Technological and scientific innovation today rely heavily on teamwork, even in traditionally individualistic fields. According to mathematician Terence Tao, "Nowadays, most problems in mathematics are interdisciplinary. You need expertise from many different fields of mathematics or subfields outside mathematics. More and more, you need to collaborate".[1] Figure 1 illustrates this increasing reliance on teamwork over time; among patents registered with the USPTO, the fraction of sole-inventor patents has fallen from around 60% of patents in the 1970s to less than 20% of patents this century.[2] Meanwhile, teams have also become more diverse across ethnicity, gender, and field background.[3] Teams of inventors with heterogeneous backgrounds and expertise have become central to innovation.

Figure 1: Team size by decade on USPTO patents



Source: USPTO

The rise of teams and their connection to idea production has immediate consequences for economic growth. Idea production is the central determinant of long-run economic growth and, in the modern world, the role and structure of teams is a crucial component of idea production. Further, different explanations for the rise of teams have different implications for immigration, R&D, and education policies. However, there is not a comprehensive framework to analyze links between teams of differentiated skills and aggregate innovation.

[1] Tao (2017)

[2] This fact has been documented in detail in Wuchty et al. (2007)

[3] See Appendix Figures B.8, B.9, and B.10 for these three facts.

This paper investigates and quantifies the sources and implications of the increasing importance of teams. In quantifying the sources, I focus on three classic economic concepts: *returns*, *costs*, and the *supply*. I map these three concepts into data-relevant counterparts: changes in the idea production function, which takes as inputs the *depth* and *breadth* of team expertise and outputs an idea of a given quality (*returns*); reductions in communication costs (*costs*); and changes in the aggregate composition of inventor expertise (*supply*). The quantification procedure allows for interesting policy counterfactuals. In particular, I focus on immigration policy with innovative teams, but also discuss R&D and education policies.

I find that changes in the idea production function are the most significant driving force behind the rise in team size, explaining 55% of this rise. The quality of ideas responds more significantly to team expertise today than in the past, which is driving inventors to match into larger, and sometimes more diverse, teams. Falling communication costs that foster team production explain 13% of the rise. The change in the composition of inventor expertise is important for understanding the composition of teams and technologies, but explains only 10% of the rise in team size. This analysis compares the 1990s to the 1980s but is part of a larger trend.

The changes in the idea production function induce a rise in the productivity of certain types of expertise that are highly complementary to teams. This expertise would be improperly undervalued in a framework without teams where agents are ranked according to their innovative productivity when alone. For instance, without taking into account the contribution of those with expertise in organic chemistry to teams across chemical (e.g., macromolecular compounds), biological (e.g., pharmaceuticals) and computational (e.g., electric techniques) fields, their expertise would be significantly undervalued. Organic chemists, while playing an important role in teams, are often less productive than other inventors when working alone. This paper develops a new lens for valuing the contribution of varieties of expertise.

This paper proceeds in four steps. First, I build a matching-in-teams model that links a team's return and communication cost to how often that team forms. The model allows for a quantitative decomposition of the roles of the three forces (return, cost, supply) in determining the changing pattern of teams. This model follows the spirit of Becker (1973) but allows for matching across teams of multiple sizes of any type (i.e., not restricted to two-sided markets).

Second, I use USPTO patent data to characterize the idea production function (which governs the returns to teams), communication costs, and inventor expertise. I build measures of individual skill expertise that are based on the distribution of an individual's work across patent classes and her quality of production within each class. I parameterize the idea production function in terms of the team's depth and breadth on the patent. Depth summarizes the team's expertise within the technology area of the patent of interest, whereas breadth summarizes the team's expertise in other technologies. I find that depth and breadth are important forces in idea production, and their importance has risen over time.

Third, I quantify the three forces by embedding the estimated production function and communication costs into the matching model. I compare the 1980s to the 1990s when evaluating the rise in team size. I find that changes in the idea production function explain the largest share of the increase in team size (55%). Falling communication costs across geographical distance also explains a significant portion of the rise in teams (13%). Falling communication costs more generally have enabled specific teams to form that would not have produced together in the past.

Fourth, I explore policy applications within the team production and matching framework: immigration, education, and R&D policies. I focus in particular on immigration policy and the response of innovative output to different expertise shocks. I find that without a model of teams, there would be significant underestimation of the positive impact of specific skills. Furthermore, the model delivers a reduced form statistic that relies on only two parameters to value heterogeneous expertise across technologies. This statistic has a very high rank correlation (0.80 unweighted, 0.93 weighted) with the outcome of increasing the given expertise by 1% of the total inventors in the economy. This provides a simple summary for characterizing the value of integrating different types of expertise to the economy.

In an application of this policy exercise, I make use of quasi-experimental evidence from the breakup of the Soviet Union. Russians[4] comprised consistently around 0.6% of inventors in the US in until the early 1990s, but around 1.4% of inventors by 2005. Leveraging this differential expertise, I shock the set of skills in the United States with the Soviet skills. To do this, I make use of patent records from the Soviet Union that illustrate how Russian skills across technologies differed from US skills across technologies before the breakup in 1991. This inventor expertise shock exercise finds that the patterns of Russians across teams qualitatively matches the results of the model and policy exercise. I also use this evidence to evaluate the contribution of Russians after the fall of the Soviet Union and find that their production in teams explains a significant component of their impact on US innovation. Without understanding the process behind their production and matching in teams, it would be challenging to make any statement on the quantity of their overall contribution.

Related Literature

Economists have long known that ideas are the building blocks of long-run economic growth (Lucas, 1988; Romer, 1990; Aghion and Howitt, 1992). Economists have also noted and hypothesized about the rise of teams in idea production (Wuchty et al., 2007). While there are competing theories on this rise (Jones, 2009; Bloom et al., 2017), the connection between specific expertise in teams and aggregate innovation has had less exploration. Given the rise of teams, the importance of heterogeneous expertise in innovation is a natural issue to command the attention of researchers in innovation and economic growth.

Two common hypotheses exist in explaining the rise of teams: changes in the expertise required to produce new ideas (the returns to teams – Falk-Krzesinski et al., 2010; Bennett and Gadlin, 2012) and changes in communication costs (the cost of forming teams – Jones et al.,

[4] I use ethnicities associated with the former Soviet Union as in Kerr (2007)

2008). Although there is a rising interest among macroeconomists in team mechanics (Akcigit et al., 2018; Herkenhoff et al., 2018; Jarosch et al., 2019), the role of horizontally differentiated expertise has not been incorporated into the growth literature. Given that human capital is the key component of idea production (Waldinger, 2016), understanding this horizontal expertise will provide an important lens for thinking about how the distribution of innovative skills affects innovation.

To incorporate these forces, I build on two literatures: i) endogenous economic growth and ii) matching and production in teams. In the case of empirical endogenous growth, economists have demonstrated the connection between patents and innovation. Akcigit et al. (2017b) find that regions and economies that produce more patents grow faster over the long-run. Bloom and Van Reenen (2002), Akcigit and Kerr (2018), and Kogan et al. (2017) show patents have a significant impact on firm-level productivity, employment, and stock market value. In addition to creative destruction and economic growth, patents also transfer knowledge. Jaffe and Trajtenberg (1999) illustrate the role that patents play in facilitating flows of international knowledge and Bloom et al. (2013) focus on the interaction of technology spillovers and the product market. Henderson et al. (1998) and Ahmadpoor and Jones (2017) further show that patents channel basic academic research to applied research and then to products.

There is a significant theoretical literature addressing the connection between the production of ideas and aggregate innovation (e.g., Jones, 2005; Lucas, 2009). Much of this literature focuses on the interactions of individuals and firms. Lucas and Moll (2014) and Perla and Tonetti (2014) focus on the choice to invest in learning versus production in driving aggregate innovation. Benhabib et al. (2014) and Konig et al. (2016) focus on the role of imitation versus innovation at firms, which also happens at the individual level. Eeckhout and Jovanovic (2002) address how imitation and knowledge spillovers can interact with the determinants of inequality. Luttmer (2015) focuses on this inequality and the endogenous matching process that creates students and teachers. Many of these mechanisms generate ideas of varying quality. Akcigit and Kerr (2018) provide a quantitative framework that links idea quality heterogeneity to aggregate innovation. Further, there have been empirical and theoretical frameworks providing evidence that skill breadth and depth and locational distribution matter and interact (e.g. Jones, 2009; Berkes and Gaetani, 2019). While idea heterogeneity serves an important role in the endogenous growth literature, its roots in domain specific human capital and individual interaction has received less attention.

I build this idea quality heterogeneity into a matching model wherein individuals with different expertise contribute in different ways to patent output depending on what team they join. This enables a more textured analysis of expertise in teams. The theoretical framework links the distribution of expertise to matching patterns in teams, building on the framework of Becker (1973). The paper additionally includes insights on the division of labor with skill heterogeneity (Becker and Murphy, 1992; Stokey, 2018). I stress the importance of skill both being multidimensional (Lindenlaub, 2017) and specific to certain knowledge domains (Hayek, 1945). The organization of an economy and the teams within it depends on this distribution of knowledge (Garicano and Rossi-Hansberg, 2006). In building the interaction of heterogeneous knowledge in teams into the matching process, the model most closely follows Choo and Siow (2006), who estimate a marriage model with transferable utility and heterogeneous preferences following Becker (1973) and McFadden (1974).

I unite this theoretical literature with an empirical literature on team production. I direct significant focus in this paper to horizontally differentiated expertise which economists have discussed as increasing patent impact (Singh and Fleming, 2010). Certain studies find positive (Jehn, 1995, 1997) and negative effects (De Dreu and Weingart, 2003) of horizontal differentiation on team production. Many papers find a nonmonotonic effect (e.g. Guimera et al., 2005; De Dreu, 2006; de Wit et al., 2011) I argue in this paper that there is a key connection between the underlying expertise and the technology that will determine the payoffs to different expertises working together. Furthermore, results from surprise deaths indicate that team complementarities are significant (Azoulay et al., 2010; Jaravel et al., 2018) and inventors do indeed benefit from working with others. Previous work has also suggested this element of team complementarities is carrying increasing importance over time (Freeman et al., 2014). This current paper explores the matching process underlying innovation to understand these forces.

The rise of sorting into teams has motivated academics to further investigate the foundational reasons for forming teams (Bikard et al., 2015; Wu et al., 2019). Even though the split of the rents to innovation may have significant heterogeneity across inventors within a team (Kline et al., 2019), the rise of teamwork suggests the benefits are strong.[5] Indeed, there is microeconomic theory on the benefits of teams with horizontally differentiated expertise (Hong and Page, 2004; Page, 2007) and how the response of teams to skill determines wages (Davis, 1997). The goal of this paper is to bring the intuition of these studies in a macroeconomic environment.

Discipling the role of expertise in teams and innovation has many potential applications. This paper directs its attention primarily to immigration policy. I build on a literature that delivers conflicting accounts of the role played by immigration in idea production. In one study, Borjas and Doran (2012) find that Russian mathematician immigrants substituted for US mathematicians crowded American mathematicians in idea production in mathematics. In another study, Moser et al. (2014) find positive spillovers from German-Jewish chemist emigres. Historically, immigrants have had a significant impact on American technology (Akcigit et al., 2017a). Immigrants are more concentrated in patent-heavy fields, leading them to be more innovative than natives (Hunt and Gauthier-Loiselle, 2010). Given the significant costs of moving, it makes sense that immigrants who move self-select into productive fields (Borjas, 1987). In regards to policy, Kerr and Lincoln (2010) find that increases in the H1B cap spurred innovation. I stress in this

[5] While this paper focuses primarily on team patent impact abstracting away from time, Figure C.23 illustrates that the speed at which teams produce patents is faster than sole-authors.

paper an important channel through which immigrants interact with the countries' "innovative capacity" (Furman et al., 2002) to shape overall output – the team production effect. I find that the distribution of individuals geographically still has first-order implications on the prevailing team structure and economic growth (Porter and Stern, 2001), which suggests policies related to immigration continue to have first order effects on innovation. Addressing horizontal skill differentiation is crucial to explore the interaction between immigrant and native expertise.

The paper is organized as follows. Section 2 describes the three empirical facts that motivate a quantitative model. Section 3 builds a team production model that links the value of teams to their observed frequency. Section 4 introduces the USPTO data and the construction of measures I use in the empirical analysis. Section 5 produces the empirical results and their implications for the quantitative framework. Section 6 discusses the quantitative decomposition of the role of the three forces in driving the change in team size. Section 7 illustrates the ability of these results to elucidate policy questions and focuses on immigration policy. Section 8 concludes.

2 Three Facts on Team Production in R&D

This section presents three forces related to changes in idea production: *return*, *cost*, and *supply*. These three forces will serve as an important reference point for the analysis for this paper. I use USPTO patent data that has the listed inventors and their residences.

Fact 1: The impact of team patents has been rising over time.

Teams increasingly produce higher cited patents than individuals. Figure 2 plots log lifetime citations on a patent by team size during the 1980s versus the 1990s.[6] The blue circle points indicate 1980s patents and red triangles indicate 1990 patents. During the 1990s, teams of size 2-5, relative to individuals working alone, created higher impact patents than did teams of similar size during the 1980s. This illustrates that *returns* to teams have risen. This is also true for output measured by stock market value as in Kogan et al. (2017).[7] Over time, combining individuals together has led to relatively more productive innovations.[8] The relatively higher output of teams suggests that in order to make use of this increased output, inventors would respond by working relatively more often in teams than they work alone.

Fact 2: Team collaborations are becoming more geographically dispersed over time.

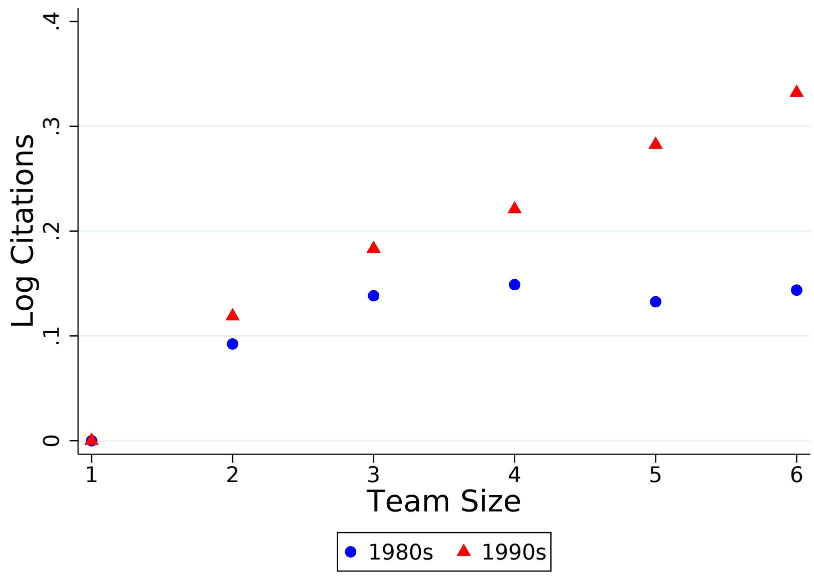
In addition to the output effect, a cost effect could explain the rise of teams. Individuals may form teams more often because team members can more easily communicate across geographical

[6] Log lifetime citations takes the current number of citations on a patent and adjusts by patent age and class. This adjustment accounts for the fact that older patents tend to have more citations. These citations proxy for the overall social value of a patent.

[7] See Appendix Figure B.6

[8] This has been noted in scientific publications by **?**

Figure 2: Log Lifetime Citations by Team Size

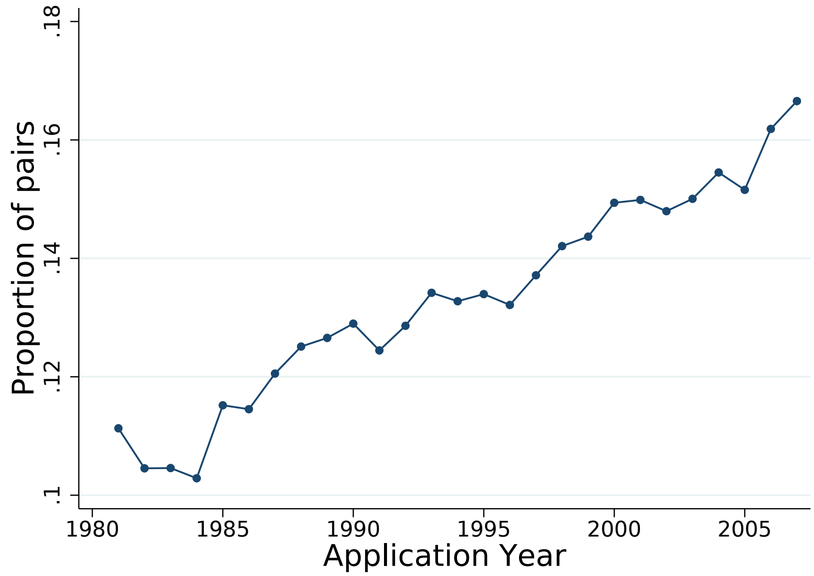


distances (e.g., email, internet, phone, travel costs). This has been studied from a theoretical angle as a key feature of the division of labor (e.g. Grossman and Rossi-Hansberg, 2008; Costinot, 2009). Figure 3 plots the proportion of 2-person teams that worked together while at least 100 miles apart from 1980-2005. I focus in Figure 3 on two-person teams to avoid the mechanical effect of team size. If I include teams that have more than two inventors, the pattern becomes stronger. This figure illustrates the shifting communication across distances from the reported residential address of patent inventors in two-person teams.

Fact 3: Immigrant inventors have made significant contributions to computer technology. Finally, I examine how the composition of inventors across technologies can shift the distribution of teams. Many immigrants arrived in the US and produced patents in the 1990s in technical fields. Figure 4 focuses specifically on Russians. Figure 4a shows, using ethnicity name probability matches, that ethnic Russians represented consistently around 0.6-0.7% of inventors in the 1980s. In the 1990s, this percentage increased sharply, rising to around 1.4% of US inventors by 2005.

Figure 4b illustrates that inventors from the former Soviet Union brought with them expertise that differed from the US population. Russians were especially strong in physics (with computing-related fields a subset) and construction. This figure uses patent records from the former Soviet Union in order to characterize the expertise of the population before the fall of the Soviet Union in 1991. When Russians came to the US they were more concentrated in patent classes that were heavily represented in the Soviet Union. This can be seen in the ethnic composition of inventors on patents in the US.

Figure 3: Proportion of co-authors in a different location



Source: USPTO

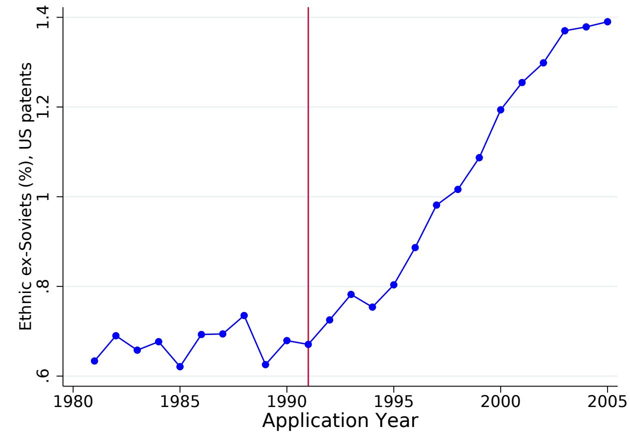
This immigration inflow was part of a larger phenomena of immigrant influx from Russia, China, and India that shifted the expertise distribution of US inventors over time. If these inventors were more concentrated in team-intensive technologies, the composition of inventors alone could shift the average observed team size.

3 A Model of Idea Production and Team Formation

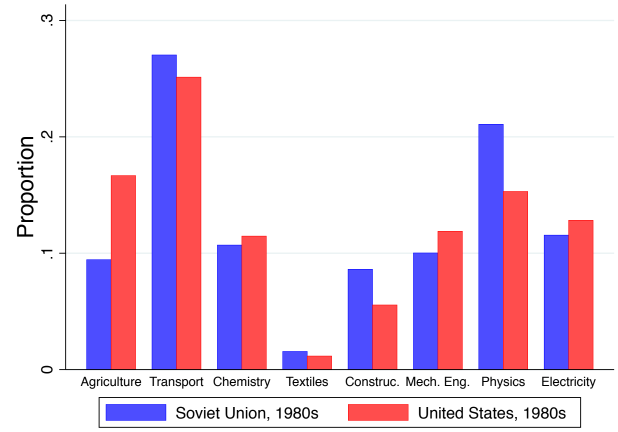
To understand the market for invention, I build a static model of inventors who match in teams. Inventors observe the set of possible teams they can join and make a decision to join based on the payoffs. This model embeds the intuition of the three forces discussed in the previous section that determine the matching pattern: the returns to teams, the costs of team formation and communication, and the supply of expertise.

The model follows a similar structure to that of Choo and Siow (2006), who use matching pairs in a two-sided framework to infer the value of the marriage. In this paper, the model is extended to a situation in which agents can take any role in a team; that is, agents can match with any other inventor and in multi-inventor teams. This fits the market for invention, as inventors often match in large teams and match with inventors of the same "type" as themselves (e.g. chemists work with chemists).

Figure 4: Immigrants and Technologies



(a) Russians on US Patents



(b) Soviet and US Tech Dist (IPC1)

3.1 Environment

A mass of inventors $M_{x,\ell}$ are indexed by their skill type $x \in \mathbb{X}$ and location $\ell \in \mathbb{L}$. There are a finite number of skill types x and locations ℓ. $x \in \mathbb{R}^S$ is a vector of expertise across S technologies that can be utilized to produce patents. ℓ is the single geographical location of each agent with the set $\mathbb{L}$ of total locations. The skills of team members matter for the quality of patent production. Geography matters for the costs of communicating in teams.

Risk-neutral inventors maximize linear utility in their wage and an idiosyncratic preference shock in a static setting. Inventors can either join a team up to some size $\overline{T}$ or work alone. Due to the finite team size $\overline{T}$ and finite number of types (x, ℓ), there are a finite number of total team types.

A team is indexed by $k = \{T, (x^1, \ell^1), ...(x^T, \ell^T)\}$. Team k generates output that is a function of team size T and the vector of skills of each type on the patent. The production functions for each team and individuals working alone are as follows:[9]

$$q_k = q(T, x^1, ..., x^T) \qquad ; \qquad q_{x0} = q(1, x) \tag{1}$$

Each operating team needs to pay a cost to communicate that depends on the team size and geographical dispersion of inventors, c_k. This cost can be understood as the cost of communicating on a project or the cost of forming the team. The communication cost for a single inventor working alone is zero; for multi-person teams, communication costs would likely increase with the geographical dispersion of the types and the number of team members:

[9]Teams also choose a patent technology class to work in. Given the team members, this optimal class would immediately follow. Because this immediately follows from the team composition, I leave this out of the model. I will be more specific on this problem when mapping this to its empirical counterparts in Section 5.

$$c_k = c(T, \ell^1, .., \ell^T) \qquad ; \qquad c_{\ell,0} = 0 \tag{2}$$

Thus, each team of inventor types has a corresponding total net output, $q_k - c_k$. Recall that the net output for individuals working alone is q_{x0}. Individual types are assigned in amount $N_k^{x,\ell}$ to team k. For instance, in a team with two of type (x, ℓ), $N_k^{x,\ell} = 2$. For each individual type (x, ℓ) on team k, the total output is shared such that there is no output left on the table. This means that the total wages paid out to team members is equal to the total team output:

$$\sum_{(x,\ell)\in k} N_k^{x,\ell} w_k^{x,\ell} = q_k - c_k \tag{3}$$

Having discussed the net team output, $q_k - c_k$, and a general condition on wages, I turn to the individual's problem of choosing her team.

The Individual's Problem

Individual i is an infinitesimal agent of type $(x, \ell) \in \mathbb{X} \times \mathbb{L}$ who derives systematic and idiosyncratic utility from working in team k. The systematic component, $w_k^{x,\ell}$, is a result of market forces and is the same across all types (x, ℓ) within a given team k. The second component, $\epsilon_k^{x,\ell}(i)$, is an iid preference shock across working in each team which is specific for a given individual i of type (x, ℓ). This idiosyncratic iid individual-by-team utility is drawn from an iid type-I extreme value. This shock represents heterogeneous and unobserved reasons for forming teams. The distribution of the shocks is not related to the systematic observable component of an agent's skills. These shocks follow a cumulative distribution function as in McFadden (1974):

$$F(\epsilon) = \exp\left(\exp\left(-\frac{\epsilon}{\phi}\right)\right)$$

Each inventor i observes a set of iid shocks across all teams they can join from $0, ..., \bar{k}$:

$$\boldsymbol{\epsilon}^{x,\ell}(i) = \{ \underbrace{\epsilon_{x0}^{x,\ell}(i)}_{\text{working alone}}, \underbrace{\epsilon_{x1}^{x,\ell}(i), \ldots, \epsilon_{\bar{k}}^{x,\ell}(i)}_{\text{shocks for each team}} \}$$

Thus, for each i of type (x, ℓ) there is a return equation for joining each team k:

$$\pi_k^{x,\ell}(i) = \underbrace{w_k^{x,\ell}}_{\substack{\text{systematic} \\ \text{component}}} + \underbrace{\epsilon_k^{x,\ell}(i)}_{\substack{\text{idiosyncratic} \\ \text{component}}} \tag{4}$$

Each individual i of type (x, ℓ) chooses her team k to maximize her return, $\pi_k^{x,\ell}(i)$. This is part of the equilibrium that is discussed next.

3.2 Equilibrium

The equilibrium is a set of wages across teams, the mass of individual types assigned to teams, and the mass of teams, $\{w_k^{x,\ell}, m_k^{x,\ell}, m_k\}$. The wages emerge as a result of the trading game to clear the market for each type and in each team. The endogenous assignment of types to teams, $m_k^{x,\ell}$, emerges from this process. The mass of a given team, m_k, is the mass of assigned types to the team divided by the number of unique team members.

The equilibrium is characterized by optimization of each agent i and market clearing within each team and within each type. The key equilibrium outcomes will be a set of wages for each type across each potential team they can join k and the mass of each team. The counterpart to each equilibrium object in the model will be explored in greater detail in the quantitative section when I map the model to patent data. I focus on the mass of each team m_k, the net return to each team $q_k - c_k$, and the expected value of being a given type (Proposition 3). I use these objects to evaluate the changing patterns across teams and the effects of subsidies and expertise shocks on the economy.

In equilibrium, agent i observes her vector of idiosyncratic shocks and the systematic return to working for each team. She then chooses the team k that delivers the maximum return. Wages are determined endogenously by market clearing in teams for all types. The equilibrium conditions are as follows:

Each $i \in \mathbb{X} \times \mathbb{L}$ chooses the team k^* to maximize the sum of their idiosyncratic and systematic income:

$$k^*(i) = \arg\max\{\pi_{x0}^{x,\ell}(i), \dots, \pi_k^{x,\ell}(i), \dots, \pi_k^{x,\ell}(i)\}$$

This maximization delivers a relationship between wages and allocations that is governed by the dispersion of the preference shock ϕ and the mass of a given type (x, ℓ), $M_{x,\ell}$:

$$m_k^{x,\ell} = M_{x,\ell} \frac{\exp(w_k^{x,\ell}/\phi)}{\sum_{\tilde{k}\in\mathcal{T}_{x,\ell}} \exp(w_{\tilde{k}}^{x,\ell}/\phi)} \tag{E1}$$

Wages for each agent on the team add up to total net output for each team k:

$$\sum_{(x,\ell)\in k} N_k^{x,\ell} w_k^{x,\ell} = q_k - c_k \tag{E2}$$

Markets clear for each inventor type (x, ℓ) (E3), the mass of each type assigned to each team is equal to the mass of the team multiplied by the number of this type on the team (E4), and there are no teams with negative mass (E5):

$$\sum_{k \in \mathcal{T}} m_k^{x,\ell} = M_{x,\ell} \qquad \forall \quad (x,\ell) \tag{E3}$$

$$m_k^{x,\ell} = N_k^{x,\ell} m_k \qquad \forall \quad (x,\ell) \in k \tag{E4}$$

$$m_k^{x,\ell} \geq 0 \qquad \forall \quad (x,\ell) \tag{E5}$$

With the conditions above satisfied, I proceed to Proposition 1.

Proposition 1. *There is an equilibrium where the (i) the matching pattern depends on (ii) the idea production function; and (iii) the communication costs as follows:*

$$\overbrace{\log m_k - \frac{1}{T_k} \textstyle\sum_{(x,\ell) \in k} \log \frac{m_{x0}^{x,\ell}}{N_k^{x,\ell}}}^{(i)} = \frac{\overbrace{q_k - \sum_{(x,\ell) \in k} N_k^{x,\ell} q_{x,0}}^{(ii)} - \overbrace{c_k}^{(iii)}}{\phi T_k} \tag{5}$$

Proof. See Appendix A.1. □

Proposition 1 links inventors' concentration in teams to the values of those teams; in particular, it relates the mass of a given team, m_k, relative to each type working alone to the value of the team q_k relative to working alone. Team frequency moves in a positive and log-linear way with q_k, which is the output of the team. As each agent becomes more productive alone (ii), or the communication costs increase (iii), the team will form less relative to each agent working alone. This framework provides the launching point of this paper as it will be key to quantifying the forces behind the rise of teams and enabling counterfactuals on the supply of types.

Next, I turn to a proposition that ensures the local identification and uniqueness of counterfactuals for small changes in the distribution of inventors. Let $\mathbf{M}$ be the exogenous distribution of each type in the economy and $\mathbf{m}$ be the equilibrium distribution of teams. I introduce a small perturbation to $\mathbf{M}$, $\mathbf{M}^*$. I ask the question, conditional on an observation of $\mathbf{m}$ in the initial equilibrium, whether there is a unique perturbed equilibrium distribution of teams $\mathbf{m}^*$, given the perturbation $\mathbf{M}^*$. By local identification, I mean a unique $\mathbf{m}^*$ corresponds to a unique $\mathbf{M}^*$. By uniqueness of counterfactuals, I mean a unique that for a unique $\mathbf{M}^*$, there is a unique $\mathbf{m}^*$.

Proposition 2. *Take a market with one- or two-person teams and the equilibrium from Proposition 1. Let* $\mathbf{M}$ *and* $\mathbf{m}$ *be the vector of the supply of each inventor type and vector of team masses, respectively. For* $\mathbf{M}^*$ *close to* $\mathbf{M}$, $\boldsymbol{m}^*$ *is uniquely determined.*

Proof. See Appendix A.1. □

Beyond this analytical proof, Appendix A.1 shows the conditions under which this identification condition holds for teams of any size up to $\bar{T}$. I find that this is true under the quantitative framework of this paper, which provides support for the counterfactual analysis in Section 7 when considering teams of size 1,2 or 3.

To fix ideas of what the model attempts to capture, I describe three pieces of the model that shift from the earlier period (e.g., 1980s) to the later period (e.g., 1990s). First, $q_k - \sum_{(x,\ell)\in k} q_{x0}$ may shift. Combinations of certain team types may yield different returns over time. At times, hardware producers see higher returns to working alone versus pairing together, or pairing up with a chemist.

Second, individual types may face changing communication costs over time. For instance, with the advent of email and file sharing, inventors in separate locations can more easily produce together. This ability could drive down the costs of forming certain teams over time.

Third, the composition of inventors may change. In particular, the 1990s saw a large movement toward fields related to computing, information storage, and hardware. This was driven by two forces beyond returns and costs. First, inventors selected into advanced degrees in these fields. Second, immigrant talent arrived with expertise in those fields, impacting the composition of inventors. Reductions in immigration restrictions in the US and corresponding outflows from Russia and China generated a large influx. As a result, the inventor composition in 2000 differed from the inventor composition in 1980.

The last result of this section characterizes the expected value of being an inventor of type (x,ℓ). This statistic for valuing types delivers a simultaneously intuitive and powerful predictor of the types that are the most productive in a large economy of teams. Proposition 3 discusses the properties of the ex ante value of being a specific type.

Proposition 3. *The expected value for an agent of type (x,ℓ) before her preference draws is as follows:*

$$\mathbb{E}[V_{x,\ell}] = cons + \underbrace{q_{x,0}/\phi}_{\textit{output alone}} + \underbrace{\log\left(\frac{M_{x,\ell}}{m_{x0}^{x,\ell}}\right)}_{\textit{concentration in teams}} \tag{6}$$

Proof. See Appendix A.1. □

Types that produce significant impact patents alone have higher value ex ante. Further, conditional on production alone, a high concentration in teams mean this type is receiving higher wages and thus will have higher expected value. For types who are very productive alone and are frequently in teams, the wages in teams are sufficiently high to draw them into the team. The adjustment comes from the dispersion on the shock ϕ. This result is important for evaluating how self-selection impacts inventor migration in Section 7.

Section 6 uses the model framework to quantify the contribution of three key forces on the changes in the matching pattern. These three forces, *returns*, *costs*, and *supply*, exemplified by $q_k - \sum_{(x,\ell)\in k} q_{x0}$, c_k, and $M_{x,\ell}$, respectively, impact the allocation to teams, m_k. Section 7 applies this framework to policy counterfactuals with a particular focus on immigration policy. First, Section 4 and Section 5 discuss the data and build the empirical framework to illustrate patterns important for the quantitative analysis.

4 Data and Measurement

4.1 Data Sources

USPTO Patent Data and Inventor Disambiguation Algorithm

Although this paper uses several distinct data sources, it primarily relies on USPTO patent data for patents granted from 1975 to 2010. According to the USPTO, "US patent applications must list the 'true and only' inventors." Thus, I characterize a patent p by a USPTO-assigned technology class s[10], a team of inventors who jointly produce the patent, and forward citations (which I use as the primary proxy for patent value) at subsequent dates.[11] I adjust for patent truncation using IPC1 patent class and the date of application in order to make comparable citations across different classes and time periods. This is common in the literature (Hall et al., 2001). The USPTO patent classes are harmonized to the 2010 USPTO classification system. For international patents, I utilize the International Patent Classification (IPC) framework when necessary. The patent technology class captures the area of production in technology space.

For inventor identification, I use a dataset from Li et al. (2014) that links the entire career of an inventor to her history. Li et al. (2014) use a Bayesian disambiguation algorithm that employs patent class, location of inventors, firms where an inventor works, and her corresponding co-authors to track the full history of individuals on patents. The two main problems these algorithms deal with are cases of misreportings (e.g., misspellings such as "Jonh Smith") and common names (e.g., "John Smith"). The ability to identify inventors over time is crucial for building expertise measures that can speak to how inventors contribute to a team.

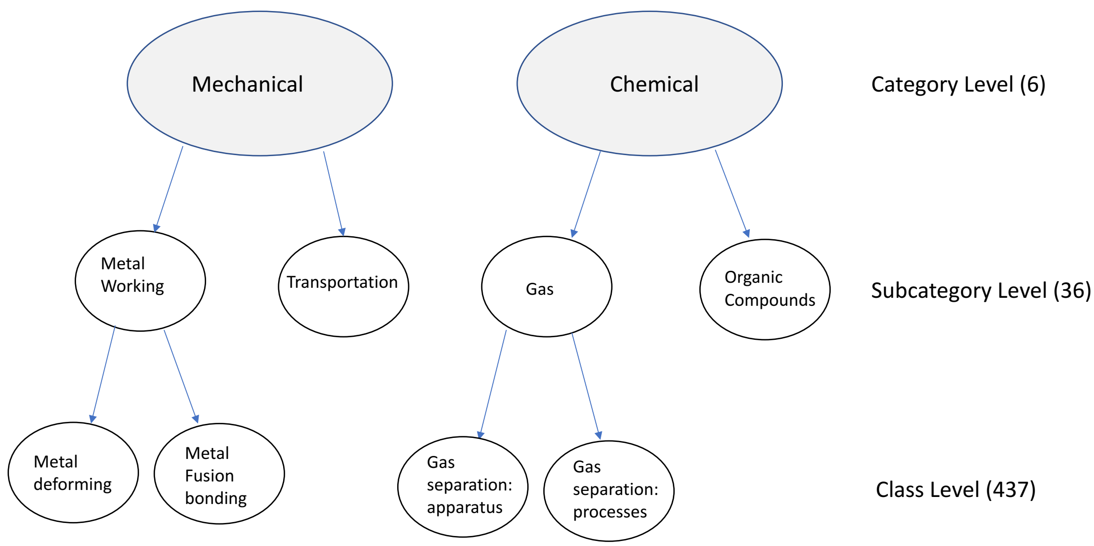
When focusing on the assignee, I include all types: firm, international, university, and government. I truncate the data on both ends to capture experience vectors of individuals and clean citation data. For the quantitative and empirical analyses, I focus on years 1980-2000, but I build expertise vectors based that are based on the entire sample. The resulting sample includes 2.2 million unique patents, 1.5 million unique inventors – 1.1 million unique inventors who have more than 1 patent, and 4.5 million patent $\times$ inventor observations.

The class system and citation network admit identification of technological areas where inven-

[10]USPTO assigns a primary technology class in USPTO and WIPO assigns a primary IPC classification

[11]Patent stock market value and renewals are also used as measurement of patent value in robustness checks.

Figure 5: Technology Classifications



tors operate. Figure 5 illustrates the potential levels of aggregation of classifications for analysis using USPTO classifications.[12]

For my measurement of skills, I take a stand on which level of classification to use to evaluate skill-sets. This paper explores three potential levels of classifications. The primary empirical analysis is done at the class level, but aspects of the quantitative analysis are done at the IPC2 and IPC3 level in order to speak to international datasets.[13] Figure 5 illustrates the differences across these three levels of classification. Note that at the broad level of categories, significant distinctions exist between the types of classes (e.g., chemical and mechanical). The class-level delivers a much finer distinction (e.g., subsets within gas and metal).

In addition to being a measure of patent quality, citations measure the connections between different ideas and technologies. Patent A will cite patents B and C to signify that A builds on B and C. Thus, patent citations indicate a flow of ideas. Inventors cite past patents as an indication of building on prior art, but patent examiners also contribute to citations. Citation data provide insights about a patent's impact and how patent classes are linked through knowledge flows.

Records from the Soviet Union

To apply the model to immigration counterfactuals and verify its qualitative results, I make use of a unique dataset from the former Soviet Union. This dataset contains patent records from all inventions from the Soviet Union from 1924-1993. These documents are provided by Rospatent

[12]See Appendix Figure B.13 for a similar characterization that uses international classifications.
[13]The qualitative results do not shift significantly depending on the level of classification.

(Russian Federal Institute of Intellectual Property) and the Federal Institute of Industrial Property (FIPS).

FIPS provides all the data in the form of DVDs that contain complete scans of patent documents granted in the Soviet Union. Because the Soviet Union was essentially uninvolved in global technology production, this dataset provides a unique insight into the technologies the Soviet Union was producing in. The DVDs record a total of 1.4 million unique documents.

There were two main types of patent documents in the Soviet Union: a patent and a certificate of authorship. A certificate of ownership was the most common patent document. These documents did not give an inventor the exclusive right to an invention, but the government did award prizes for inventions. Over the time period of the sample, there were a significant number of inventions, indicating that some incentives remained for inventors to innovate.

For the shock to be treated as quasi-experimental, it is necessary that inventors in the Soviet Union did not choose to innovate technologically with the expectation that their expertise as inventors would be integrated into the US market. This seems sensible given the unexpected nature of the fall of the Soviet Union and the fact that specialized human capital takes significant time to build. If this is true, the integration of Russian inventors into the global idea economy constituted an expertise shock. This is discussed further in Section 7.

Auxiliary Merged Patent Data

In addition to the standard patent measures and the disambiguation algorithm, I merge other datasets in order to speak to both robustness and ethnicity name matches. First, to ensure that patent citations are picking up similar outcomes to patent value, I merge in the stock market value measure of a patent from Kogan et al. (2017). This measure delivers the projected value of a patent based on the change in stock market value of the day the patent was granted. This data is available on a limited set of patents, since it requires the firm be publicly listed. Nevertheless, it serves as a good verification exercise for observing the response of patent value to depth and breadth.

To match the ethnicities of inventors on patents for the purposes of both tracing out the ethnic diversity of inventors and matching ethnic Russians, I adopt the probabilistic ethnicity matching algorithm from Kerr (2007). This algorithm exploits the fact that certain names are common for certain ethnicities (e.g., "Wu" as Chinese or "Rodriguez" as Hispanic). The match rate to an ethnicity on this dataset is 80%. I turn next to the measurement of inventor and team skills, which will be important for the empirical analysis and quantitative results.

4.2 Data Measurement

This section discusses the measures that are the bedrock of the empirical analysis. I start by building the measures of individual and team expertise and then proceed to discuss how these measures contribute to the idea production function.

Inventor and Team Expertise

I introduce a benchmark measure of an individual's skill across patent technology classes. For a given inventor, I compute her patent skill within each class as the average quality patent that an inventor produces in class s, adjusted by team size and concentration in the class; no patent means zero skill. I omit the individual's skill from the focal patent for any patent-level analysis. I denote the share of the patents of individual x in class s as $n_{x,s}$ and her team-size adjusted quality of patents as $q_{x,s}$.

- **Definition 1** *inventor (x) skill set*:

$$x_s = n_{x,s}\bar{q}_{x,s}$$

$$\text{Where } n_{x,s} = \frac{\#\text{ patents of } x \text{ in class } s}{\#\text{ patents of } x} \quad \text{and } \bar{q}_{x,s} = \frac{\sum_{s(p)=s} q_{x,p}}{\sum_{s(p)=s} T_{x,p}},$$

where each patent p lies in a technology class s. With an interest in measuring skills appropriate to a specific individual, I down-weight individuals who are on larger teams. Given $n_{x,s}$ and $q_{x,s}$ are require specific assumptions, I evaluate further potential measures and the associated results in the Appendix. Turning to the patent team, the skill of $k = (T, x^1, ..., x^T)$ in class s is characterized by the number of team members T and the aggregate skills of its members:

$$X_{k,s} = \sum_{x \in k} x_{k,s}.$$

The Idea Production Function

A team's skill vector contains 437 different skill metrics, the number of unique patent classes. Given production in class s, a team will bring 437 skills of varying strength to the project. Estimating the coefficient on each skill $\times$ skill $\times$ time requires a high-dimensional set of coefficients that will be highly noisy in estimation:

$$q_{k,s} = q_s\left(X_{k,1}, X_{k,2}, ..., X_{k,S}\right).$$

To explore the changing idea production function in a way that can be estimated and is more intuitive, I make some assumptions about the production process. I assume that for each s' there is a production function $q_b(\rho(s,s')X_{s'})$ that maps skill in s' to s on the basis of the contribution of class s' on s, $\rho(s,s')$. For $s' = s$, I allow for a different functional form, $q_d(\rho(s,s)X_s)$. Allowing for two distinct functional forms illustrates how patent quality might respond differently to skills in the area of production, on the one hand, and skills in other technology classes, on the other hand. Thus, the production function for ideas in class s can be written as follows:

$$q_{k,s} = \underbrace{q_d(\rho(s,s)X_{k,s})}_{\text{connection to own technology}} + \underbrace{\sum_{s' \neq s} q_b(\rho(s,s')X_{k,s})}_{\text{connection to other technologies}} \tag{7}$$

To estimate an approximation of the underlying production function, I apply a Taylor expansion at skill 0 to more fully understand the forces at play:

$$q_{k,s} \approx q_d'(0)\rho(s,s)X_{k,s} + q_b'(0)\sum_{s' \neq s} \rho(s,s')X_{k,s'}$$

This delivers two components of interest: depth, expertise within the focal patent class, and breadth, expertise in classes on which the focal patent is dependent for knowledge transmission. Ideas benefit from extensive knowledge within the domain of the idea being produced and from an array of related fields.

$$q_{k,s} \approx \alpha \underbrace{\rho(s,s)X_{k,s}}_{\text{depth}} + \zeta \underbrace{\sum_{s' \neq s} \rho(s,s')X_{k,s'}}_{\text{breadth}}$$

Using the citation pattern in the data, I evaluate the importance of knowledge in one class s', for another class s. I define the contribution of knowledge from class s' to s as the ratio of the number of times a patent class s cites patent class s' relative to the amount of times s cites itself.

Assumption 1 *technological contribution of s' to s is as follows*:

$$\rho(s,s') = \frac{\#s \text{ cites } s'}{\#s \text{ cites } s}$$

I use this assumption to characterize the two inputs into the production function, which are as follows.

Definition 2 *patent depth for patent p in class s and team k*:

$$D_{k,s} = X_{k,s}$$

Definition 3 *patent breadth for patent p in class s*:

$$B_{k,s} = \sum_{s' \neq s}^{S} \underbrace{\rho(s,s')}_{\text{contribution of } s' \text{ on } s} \times \underbrace{X_{k,s'}}_{\text{skill } s' \text{ of team}}$$

Patent depth summarizes a team's collective expertise within the focal patent class. Patent breadth captures a collection of the team's expertise in classes that the focal class is dependent on. For instance, a large team with expertise in organic chemistry producing a novel idea in organic chemistry will have high patent depth. This same team producing in bio-drug manufacturing will have low depth but high breadth, given that drug manufacturing patents cite organic chemistry patents.[14] Having illustrated the empirical measures, I now turn to the empirical analysis.

5 Empirical Analysis

This section focuses on the three forces that govern team production – returns, costs and supply. It maps these objects to specific empirical counterparts: the idea production function which depends on depth and breadth (returns); communication costs from geographical dispersion (costs); and inventor expertise composition and technological composition more generally (supply). I start by characterizing the idea production function non-parametrically.

Next, I evaluate the role of communication costs in both the quality and frequency of patent production. Lastly, I illustrate that a pure compositional effect of technologies is not a large force in the pattern behind the increase in team size with a simple decomposition. This section produces four empirical results:

> *Empirical Result I.a:* Patent impact increases significantly with depth and breadth.
>
> *Empirical Result I.b:* Teams with higher depth and breadth produce even more impactful patents in the 1990s than in the 1980s.
>
> *Empirical Result II.a:* Geographical distance reduces the probability that a team will form, but its importance diminishes over time.
>
> *Empirical Result II.b:* Geographical distance does not have strong effects on patent quality once controlling for inventor expertise.
>
> *Empirical Result III:* The compositional effects behind changing team size are small.

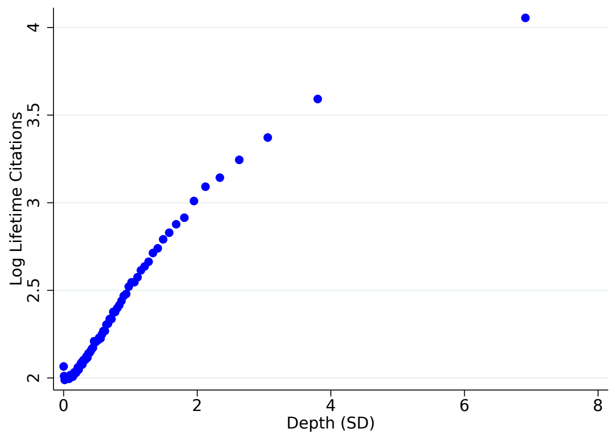
5.1 Estimation of the Idea Production Function

This section estimates the idea production function over time. Each team at each point in time has a depth and breadth for each class. When they work within a class, teams apply their depth and breadth to produce patents of quality q_p. In the following empirical exercises, the goal is to evaluate the relationship between patent value $\log q_p$. For q_p, I take the lifetime-adjusted citations of the focal patent. To ensure that the results are robust, I also use a measure of stock market value from Kogan et al. (2017).

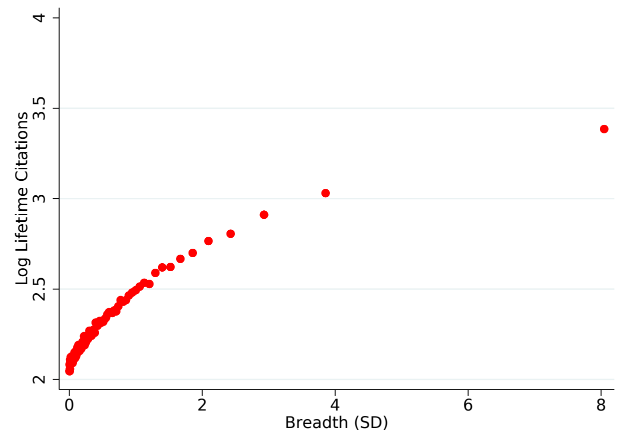
[14] The distribution of depth and breadth are in Appendix Figure B.11

Empirical Result I.a: Patent impact increases significantly with depth and breadth. Figure 6 plots the log patent quality against depth and breadth.[15] This result takes all patents from 1980-2000 and bins them in equal size bins based on their depth and breadth and plots the log citations in 100 bins, each of which contains approximately 22,000 patent observations. Note the positive relationship between citations and both depth and breadth:

Figure 6: Log Lifetime Citations and Depth/Breadth



(a) Depth



(b) Breadth

Many other forces aside from depth and breadth may be driving the strong correlation between team depth and breadth and patent quality. Individuals who form teams that have high depth and breadth may be different across a host of other dimensions. I use the following regression to evaluate the response of citations to depth and breadth, and how it has changed.

$$\log q_{p(k,s)} = \alpha_0 + \sum_{j=2}^{10} \alpha_j \mathbb{I}\{Decile(D_{k,s}) = j\} + \sum_{i=2}^{10} \zeta_j \mathbb{I}\{Decile(B_{k,s}) = j\} + \mathbf{Z}_{p(k,s)} + u \tag{8}$$

The left-hand side variable in Equation 8 is the log quality ($\log q_p$) of patent p in patent class s produced by team k. The coefficients on the deciles of the depth of team k in technology class s ($D_{k,s}$) and breadth ($B_{k,s}$) are the coefficients of interest, α_j, ζ_j. Due to the richness of the data and collapsing the right hand side into two variables of interest, I can perform these extensive non-parametric regressions throughout.[16] $\mathbf{Z}_{p(k,s)}$ contains a set of relevant controls: *patent class* $\times$ *application year*, *team size* (nonparametric), and *team experience* (nonparametric).

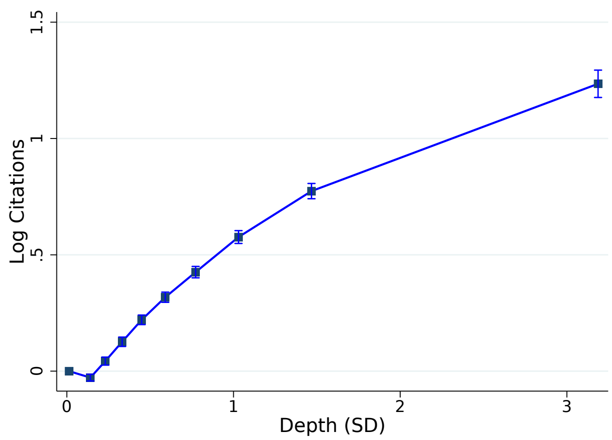
β_j is the marginal increase in log citations on patent p in depth decile j versus depth decile

[15] Each plot "log" citations is actually the inverse hyperbolic sine transformation: $x = \log(x + (x^2+1)^{1/2})$. The results are very similar if one takes log(citations) or log(citations+1).

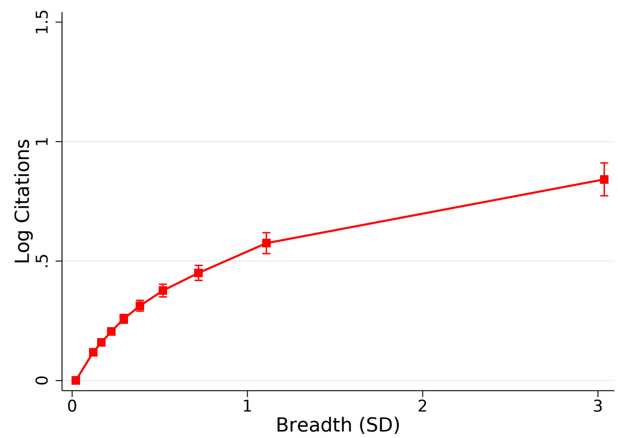
[16] In Appendix C, I explore variations on this regression, e.g. individual and firm fixed effects, and within different technology classes. I also explore variation in the skill measurement to ensure the patterns are robust to this.

1, holding fixed team size, experience, class×year, and breadth. ζ_j is the marginal increase in log citations on patent p in breadth decile j versus breadth decile 1, holding fixed team size, experience, class×year, and depth. I plot the coefficients and their clustered standard errors but detail the results further in the Appendix. In Figure 7a, holding team size, team experience, team breadth, and class×year fixed, the 9th decile of depth has approximately 75% more citations than the 1st decile of depth. Similarly, in Figure 7b, teams in the 9th decile of breadth have around 52% more citations than the 1st breadth decile.

Figure 7: Log Lifetime Citations and Depth/Breadth



(a) Depth



(b) Breadth

I discuss further results related to the general returns to depth and breadth in Appendix C. Figure 7 shows the strong association between patent quality and patent depth and breadth.

Empirical Result I.b: Teams with higher depth and breadth produce even more impactful patents in the 1990s than in the 1980s.

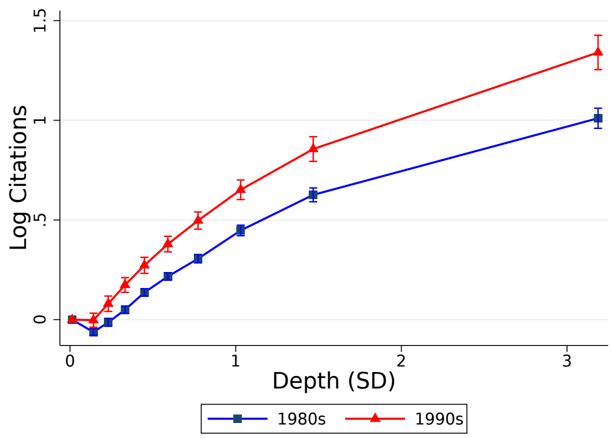
To evaluate the regression with depth and breadth as the driving force behind production, I perform the following regression:

$$\log q_{p(k,s)} = \alpha_0 + \sum_{j=2}^{10} \alpha_j \mathbb{I}\{Dec.(D_{k,s}) = j\} \times 1990s + \sum_{i=2}^{10} \zeta_j \mathbb{I}\{Dec.(B_{k,s}) = j\} \times 1990s + \mathbf{Z}_{p(k,s)} + u \tag{9}$$

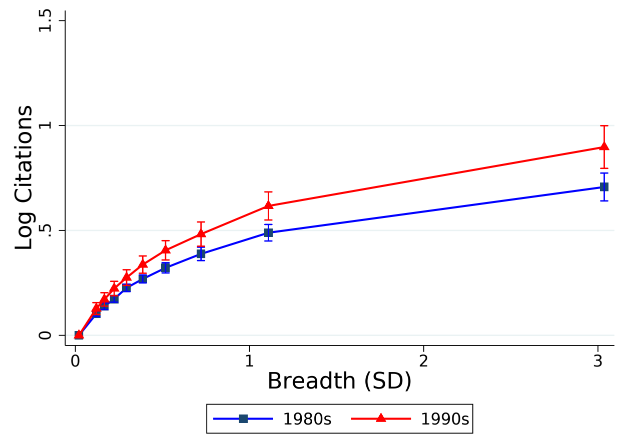
Equation 9 looks like Equation 8, but it includes an interaction term for the 1990s. The goal is to outline how log citations respond to depth and breadth across the two different time periods. If there were a change to the idea production function over time, then one would expect higher coefficients on depth and breadth in the 1990s relative to the 1980s.

Indeed, as indicated in Figure 8 by the relationship between the red and blue lines, higher-decile depth and breadth are linked to higher-quality patents in the later period. These results provide evidence that the relative returns to teams are increasing and that the increasing returns come from team skills, as expressed in depth and breadth.

Figure 8: Citations and Depth/Breadth, 1980s versus 1990s



(a) Depth



(b) Breadth

This section characterizes the gross output measures for various collections of skills of teams, q_k. In the next section, I explore the costs of forming teams, c_k, through the lens of inventor geography.

5.2 Communication Costs

This section explores how communication costs affect team formation and patent quality. Given the significant evidence that face-to-face interactions reduce frictions, I use geographical dispersion as a measure of communication frictions. I discuss here the effect of distance on team formation.

Empirical Result II.a: Teams are becoming more geographically dispersed

Geographical distance affects team formation, although over time this effect has become weaker. The following equation regresses the probability a two-person team is in the same location against time.There is a dummy for each decade from the 1970s to the 2000s (with 1970s omitted):

$$Diff\_Loc_p = \alpha_0 + \sum_{j=1980s}^{2000s} \beta_j \mathbb{I}\{Decade = j\} + u \tag{10}$$

While Table 1 indicates, via the constant term, that only a small proportion of collaborators resided in different locations (12.5%) during the 1970s, in the 2000's there were almost 1.5× as many two-person teams in a different location. If the analysis includes multi-person teams, the results are even more stark. This suggests forming teams at a distance has become easier.

Table 1: Different Location by Decade

	(1)	(2)
	Different Loc.	Different Loc.
1970s	0	0
	()	()
1980s	0.014	0.013
	(0.002)	(0.002)
1990s	0.036	0.037
	(0.002)	(0.002)
2000s	0.054	0.057
	(0.002)	(0.002)
Constant	0.126	-
	(0.002)	()
Observations	516182	516182
R^2	0.002	0.011
Class/Team Skill Controls	N	Y

Clustered standard errors at class-level in parentheses

Note from Column (2) in Table 1 that including a host of controls does not affect the overall pattern. This suggests that the increases in distance between co-authors is not a function of changes in the regional skill distribution or rising skill complementarities. If these increases were a result of rising skill complementarities, then the introduction of the controls would alter the changing pattern of allocations to cross-region teams over time, because it would be the strength of the complementarity driving some of the matching pattern. However, this is not observed. I turn next to technology composition effects.

Empirical Result II.b: Geographical distance and patent quality

In addition to showing the distance effect declining over time, I want to ensure the distance itself does not affect patent quality, which enables my primary focus on the link between patent quality and team expertise. Here, I show that locational distribution of inventors on a patent does not affect patent quality. In the following regression, I regress the value of an idea $\log \lambda_p$ on whether a team is in the same location or a different location. The goal of this regression is to isolate the effect of inventor distance on patent quality.

$$\log \lambda_p = \alpha_0 + \alpha_1 Diff\_Loc_p + \mathbf{Z}_{p(\tau,s)} + u \tag{11}$$

I find an *positive* effect of locational distance on patent quality, which is sensible through the lens of the model: teams that form more often despite high costs will have higher λ_τ. Once we control for the technology class, as well as depth and breadth, these forces go away. It appears locational distance does not have an effect on patent quality conditional on the same skill input.

Table 2: Patent Quality and Different Location

	(1) Log Patent Quality	(2) Log Patent Quality
Different Location	0.052**	0.017
	(0.019)	(0.009)
Class Controls		X
Depth Control		X
Breadth Control		X
Observations	525043	525043
R^2	0.005	0.212

Robust standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

5.3 Compositional Effects on Team Size

I present a simple decomposition exercise to evaluate the contribution of mechanical changes in the distribution of technologies to the increase in team size. Section 6 discusses the inventor skill composition more directly.

Empirical Result III: Compositional effects on changing team size are small.

The change in team size can be written out as follows, where ω_s is the weight of class s among all patent classes at time t and $T_{s,t}$ is the team size of patent class s at time t.

$$\sum_{s\in\mathcal{S}} \underbrace{\omega_{s,2000}T_{s,2000} - \omega_{s,1980}T_{s,1980}}_{\text{change in team size}} = \sum_{s\in\mathcal{S}} \underbrace{(\omega_{s,2000} - \omega_{s,1980})T_{s,1980}}_{\text{compositional change}} + \underbrace{\omega_{s,2000}(T_{s,2000} - T_{s,1980})}_{\text{change within class}}$$

I find that the compositional change explains 7% of the rise in team size, while the change within classes explains the rest. I turn next to the quantitative analysis which enables a simultaneous comparison of the three forces discussed here.

6 Quantitative Analysis: Forces Behind the Rise of Teams

To quantify the three forces driving the changes in team size and composition, I embed the empirical results into the matching model from Section 3. While the three forces (return, cost, supply) capture the broad economic elements at play, the quantitative framework must map these objects into data-relevant components. Section 6.1 discusses the classification procedure of types of agents, but this section starts with a discussion of the main equations for the estimation. The following equation links the (i) matching pattern to (ii) the idea production function, and (iii) communication costs. There is also a (iv) systematic unobserved component of matching that delivers the realized matching pattern:

$$\overbrace{\log m_k - \frac{1}{T_k}\sum_{(x,\ell)\in k} \log \frac{m_{x0}^{x,\ell}}{N_k^{x,\ell}}}^{\text{(i)}} = \frac{\overbrace{q_k - \sum_{(x,\ell)\in k} N_k^{x,\ell} q_{x,0}}^{\text{(ii)}} - \overbrace{\beta_c d_k}^{\text{(iii)}}}{\phi T_k} + \overbrace{\xi_k}^{\text{(iv)}}$$

The mass of each team k is given by m_k, where the team is of size T_k. The relative returns to teams in production comes from q_k and q_{x0}. The communication costs are characterized by the number of unique geographical locations of the team: $d_k \in \{1, 2, 3...\}$. β_c is a parameter that governs how much the locational dispersion affects the matching pattern. Lastly, the market clearing governs how the aggregate technological composition shapes the composition of teams:

$$\sum_{k \in \mathcal{T}_x} N_k^{x,\ell} m_k = M_{x,\ell}$$

Equations Q1 and Q2 are the main equations for the quantitative analysis. These equations relate the estimated production function and communication costs across teams to the frequency of matching (Q1), and estimate the underlying returns to teams on the skill-sets of inventors (Q2). I use the estimated production function to back out the parameters that relate team skills to q_k. The two main equations evaluated over time are as follows:

$$\tilde{m}_{k,t} = \tilde{\beta}_0 + \tilde{\beta}_1 \tilde{q}_{k,t} + \tilde{\beta}_{2,t} d_{k,t} + v_{k,t} \tag{Q1}$$

$$q_{p(k,s),t} = \alpha_{0,t} + \sum_{j=2}^{10} \alpha_{j,t} \mathbb{I}\{Decile(D_{k,s}) = j\} + \sum_{i=2}^{10} \zeta_{j,t} \mathbb{I}\{Decile(B_{k,s}) = j\} + u_p \tag{Q2}$$

The components of Equation Q1 are as follows:

$$\tilde{m}_{k,t} \equiv \log m_{k,t} - \frac{1}{T_k} \sum_{(x,\ell) \in k} \log \frac{m_{x0,t}^{x,\ell}}{N_k^{x,\ell}}$$

$$\tilde{q}_{k,t} \equiv \hat{q}_{k,t} - \sum_{(x,\ell) \in k} \hat{q}_{x0,t}$$

$\tilde{q}_{k,t}$ is defined as the predicted net return for team k at time t, which comes from Equation Q1. $d_{k,t}$ is the number of unique geographical locations of team k. Equation Q2 is estimated at the patent level. $\alpha_{j,s}$ and $\zeta_{j,s}$ are estimated within each IPC1 technology class. I fix the deciles to match the entire time period of 1980-2000. This captures the fact that different technology classes exhibit different returns to team skills. Equations Q1 and Q2 are estimated over two periods, the 1980s and the 1990s. To estimate Q2, I need to classify types into broad enough categories to leverage the observations of the types across teams. Section 6.1 discusses the classification procedure.

6.1 Classifying Inventor Types and Locations

Equation Q1 relies on regressions at the team level in order to make use of the structure of the model. To discuss the definition of the team, I turn first to the individual classification. Given the need to make use of the matching patterns across teams, I bin inventors into coarser types than in the initial estimation of the production function. I take an individual's most productive IPC3 classification (122 types) when producing *alone* as her skill type. For an individual who has never produced alone, I evaluate her most productive class in a team of two, and so on. I omit inventors who only have one patent.

Individual skills are multidimensional, and the quantitative exercise captures this. For a given skill type, I take the mean level of skill across all classes of all individuals within this skill type in each period. Thus, if computer engineers work on high-impact patents in semiconductors, the "computer engineer type" will have some skill in semiconductors. If computer engineers never work in basic chemistry, this type will have no skill in chemistry. After this exercise, each type has a vector of 122 skills during each period. The individual's skills will have a connection to her intention to form teams.

Each type also has a region ℓ. I split regions into five areas: East, West, North, South, and International. The first four regions are within the US while the last region is not. Allowing for only five regions will not capture the full intuition of how distance affects innovating patterns. However, it captures a significant component of cross-regional collaboration.

6.2 Sources of Shifting Teams

Before delving into the results, I discuss the intuition of how the quantitative framework elucidates the mechanisms discussed in this paper. Changes in the idea production function come from changes in $q_k - \sum_{x \in k} q_{x0}$ for different types x. As certain teams generate higher quality patents, individuals will form those teams more often. The strength of the response is governed by ϕ through the lens of the model, or the dispersion of the preference shock. In Equation Q1, $\tilde{\beta}_1$ governs this responsiveness.

There are two main sources that could be driving the changes in team size through the idea production function. Teams may exhibit larger returns to depth – ideas require deeper expertise and so inventors choose to form teams with those somewhat similar to themselves. Second, teams may exhibit larger returns to breadth – ideas require more expertise from different patent classes so inventors work with those not in the same class. This framework does not take a stand on whether inventors are on the whole less productive alone or more productive in teams. It only speaks to the relative value of producing in teams versus producing alone.

The pattern of teams also responds to the geographical distance between its members, $d_{k,t}$. $\tilde{\beta}_2$ in Equation Q1 governs the responsiveness of team frequency to communication costs, adjusted by team size T_k and the dispersion of the idiosyncratic shock, ϕ. If all inventors in the 1990s had been distributed across the map in the same way as they were in the 1980s, then a fall in $\tilde{\beta}_2$ would have generated teams with initially high d_k to form more often. The overall quantitative goal is to ask about the contribution of these forces to changes in the matching pattern. I discuss this next.

6.3 Quantitative Results

Using the machinery from the previous section, I proceed by estimating the parameters of interest: $\alpha_{j,s,t}$, $\zeta_{j,s,t}$ come from the production function equation. $\tilde{\beta}_{1,t}$ and $\tilde{\beta}_{2,t}$ come from the matching equation and identify ϕ and β_c in the underlying model.

Recall from Equation 7 the approximation to a production function that delivered depth and breadth as objects of interest for the idea production function. Using log lifetime citations as the outcome variable of interest, I perform regressions at the IPC1 level to retrieve depth and breadth coefficients in a nonparametric fashion separately for the 1980s and 1990s across technologies. Teams have a given depth and breadth across each class s.

Figure 9 plots the estimated production value of teams (y-axis) against the realized value for those teams observed in the data. This binned plot contains about 500 observations per point – the realized number of teams is around 25,000. The model fits the data well, but it misses outliers of high impact teams. This finding is expected because many teams are only realized a few times and so the realization will be noisy. This estimation procedure allows me to input values for noisy teams and teams not realized in the data in the pre- or post-period.

Figure 9: Realized and Model-implied Citations

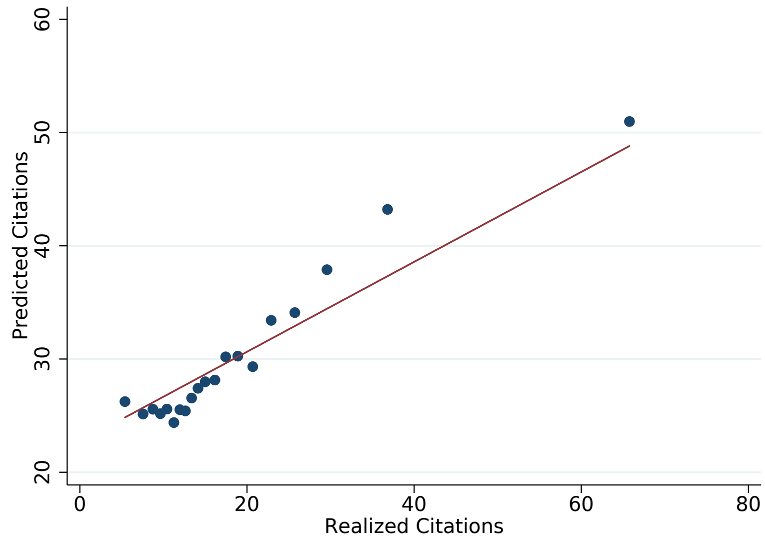
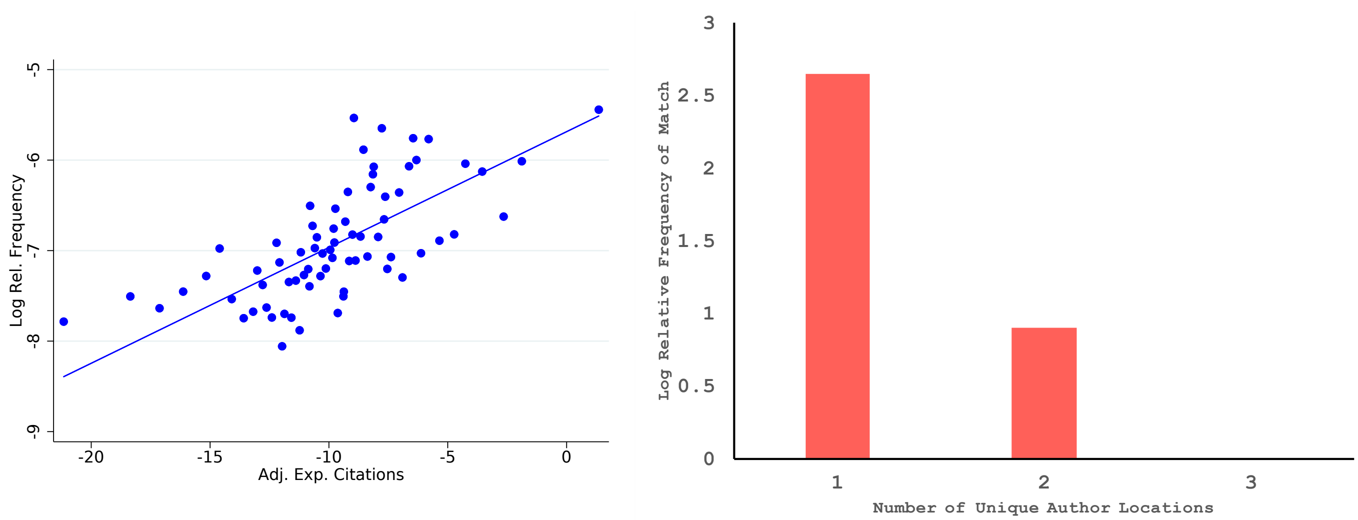


Figure 10 illustrates how the model qualitatively coheres with the data: The pattern of types across teams is strongly positively correlated with the net output of the team, $\tilde{q}$. Additionally, the geographical distance between types has a strong negative relationship to the matching pattern. If there is only one author location, this team is 2.5 log points more frequently paired.

Figure 10: Production, Cost, and Realized Frequency of Team Type



(a) Production $\tilde{m}$ and Allocation $\tilde{q}$

(b) Cost (num. loc.) and Allocation

Having illustrated that the qualitative features of the model match the data well, I turn to the quantitative properties of the model from Equations Q1 and Q2.

Table 3: Parameter Values

Parameter	Description	Value	Main Identification
— Panel A. From Production Function —			
$\alpha_{j,s,t}$	Depth share in idea production class s		Production Estimation (Q2)
$\zeta_{j,s,t}$	Breadth share in idea production class s		Production Estimation (Q2)
— Panel B. Internal Calibration —			
$\beta_{c,1980s}$	Coefficient on communication cost	0.187	$\tilde{\beta}_{2,t}$ from Q1
$\beta_{c,1990s}$	Coefficient on communication cost	0.177	$\tilde{\beta}_{2,t}$ from Q1
ϕ	Preference shock dispersion	8.13	$\tilde{\beta}_{1,t}$ from Q1

Note: Parameters in each panel are estimated jointly.

The key additional variables of interest in the quantitative procedure are the coefficient on coordination costs β_c and the dispersion of the preference shocks ϕ.

Results: Idea production function, communication costs, and expertise composition.
Once I fit the parameters that govern the production function, cost, and noise, I hold fixed a set of variables and allow each parameter to shift on its own. I start with a counterfactual analysis that allows for only a change in the idea production function holding other forces fixed. Then I proceed to do this for communication costs and inventor aggregate composition. This is enabled by the parameters of the two main identifying equations, Q1 and Q2.

Table 4 shows how much of the change in team size these three forces can explain when each is allowed to be the main force behind changing team size. Changes in the idea production function can explain 55% of the change in team size from the 1980s to the 1990s. Falling communication costs, as understood through regional dispersion, explains 13% of the change in team size. The composition of inventors has shifted the composition of teams, but the first two forces explain more of the change in team size. Composition explains 10% of the change according to

IPC2 classifications.

Table 4: Contribution of each force to changes in matching pattern

Causal Force	Proportion of Change in Team Size
Return (Idea Production Function)	55%
Cost (Communication across regions)	13%
Composition	10%
Unexplained	22%

This quantitative result finds that changes in the idea production function are the most salient force behind the changes in team size. This result illustrates that while communication costs are indeed falling, there is an increasing premium on understanding policies that foster complementary skills in teams. The next section focuses in particular on immigration policy, which is a natural extension of this framework.

7 Application: Immigration

The previous section illustrated that the rise of teams is connected in a significant manner to changes in the idea production function. Teams exhibit higher returns to depth and breadth, which encourages larger teams with varied expertise. Further, locational dispersion is still an important force determining the patterns of patenting activity (with less than 20% of 2-person team patents in the 2000s produced across different locations). What does this imply for economic policy? While there are a host of potential counterfactuals, I primarily discuss high-skilled immigration. Given the high mobility of inventors (Akcigit et al., 2016; Kerr, 2018), immigration policy can play a significant impact on the distribution of talent.

I focus on the contributions of an increase in the economy of specific expertise to aggregate innovation. Section 7.1 discusses model counterfactuals where I introduce a mass of 0.01 of each expertise to the pool of inventors. Section 7.2 explores an application of this framework using the fall of the Soviet Union and influx of Russian inventors as a real-world example. Section 7.3

discusses the implications for R&D and education policy in a more qualitative manner.

7.1 Model Counterfactuals

In the counterfactual exercise, I increase the supply of a specific expertise in the economy by a small amount and evaluate how this changes overall innovative output. [17] I define the aggregate innovation as Q, which is a function of the output across teams, q_k, and the matching across teams, m_k. I track the following equation:

$$Q = \sum_{k \in \mathcal{T}} m_k q_k$$

This equation takes the mass m_k of a given team k and multiplies this by the step size of k, q_k, which delivers aggregate innovation. For this equations, I take the estimated production function from the 1990s for each q_k, and the supply of each type x, M_x.[18] I use IPC3 classifications to characterize the expertise of each individual. I use these classifications to match the distribution of teams m_k and step size for aggregate output. I then introduce the increase in each type.

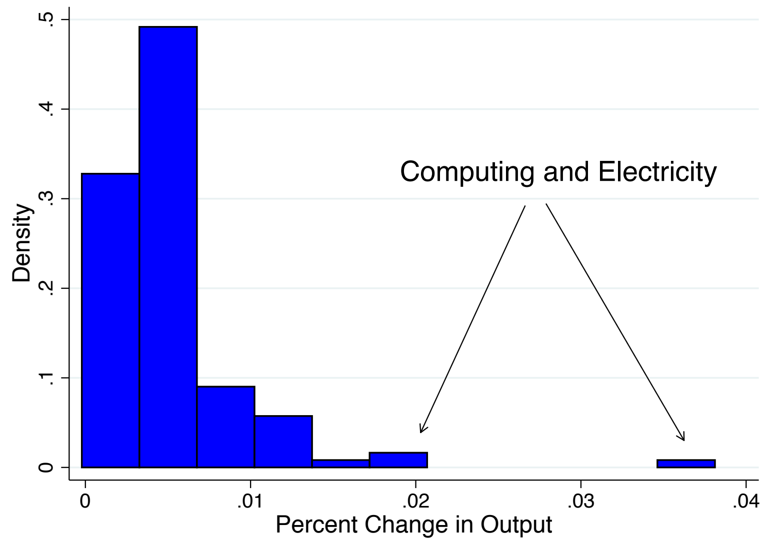
Due to the endogenous matching pattern, the model is necessary to explain how the matches change with an influx of a given type. Appendix A.2 discusses the computational procedure for ensuring the new equilibrium matches the necessary equilibrium conditions. Table 12 ranks 122 specializations as represented by the IPC3 patent category. Figure 11 plots the percentage change in output with the introduction of a new expertise type measured by the primary IPC3 category taking the 1990s production function across classes. Not surprisingly, there is a high contribution of expertise in computing to shifting aggregate output. The full composition of these ranks are presented in Table 12.

Self-selection is an important force determining who enters the destination country. Migration decisions are a choice based on how an agent's skills interacts with the country's innovative environment. In order to explore how this choice would impact overall innovation, I return to Proposition 3. Proposition 3 produced a framework for analyzing the marginal value of each given skill in the economy before matching in teams. I compare the rank of total output generated by 1% increase in inventor population of a specific expertise across 122 IPC3 categories to

[17] Because communication costs accrue to agents in the private market, I do not consider them a variable of interest for a policymaker interested in maximizing innovation.

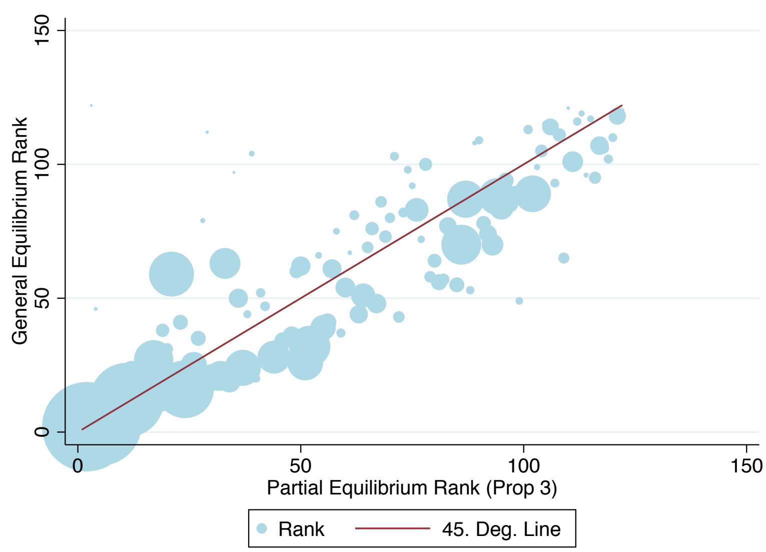
[18] For the case of immigration inflows, I abstract away from regional communication costs within the US, but further discuss this in Appendix D.3.

Figure 11: 1% Increase in IPC3 Expertise, 1990s



the rank from Proposition 3. Figure 12 plots the ranks from ex ante value and ex post contribution for the 1990s time period, with the bubble size indicating the quantity of each type. Notice the strong relationship between the value from Proposition 3 and the general equilibrium result of shocking the economy:

Figure 12: Relationship between PE values and GE result



Note the strong correlation between the expected value of types and their contribution to innovative output. To explore this more fully, I write the result of Proposition 3 with an unknown moving cost.

$$\mathbb{E}[V_x] = cons + \underbrace{q_{x0}/\phi}_{\text{value alone}} + \underbrace{\log\left(\frac{M_x}{m_{x0}}\right)}_{\text{value from teams}} - \underbrace{\psi}_{\text{moving cost}} \tag{12}$$

For each type, two forces – the value produced alone q_{x0} and the ratio of the mass of the type to the amount it works alone (capturing how often a type is in a team, adjusted by the dispersion of the preference parameter) are close to sufficient to characterize the outcome of increasing this specific expertise in the economy through immigration. This equation also provides a parsimonious way of summarize the ex-ante returns to specific types when there is no moving cost. The team element plays a key role in this value.

I stress two takeaways from this result. First, the self-selection of immigrants will amplify the aggregate output effect, as immigrants with a better fit of expertise for the society will be more likely to migrate. Second, for a policymaker whose goal is increasing aggregate innovation, there is only a limited set of information required to understand which types of expertise will make the largest contribution: the productivity of types when they work alone (q_{x0}), their concentration in teams, and the noise parameter, ϕ. Even though the general equilibrium forces don't make this a perfect predictor of overall contribution, its ability to approximate the outcome can be very informative for skills-based policies.

Indeed, policymakers without knowledge of the noise parameter ϕ may want two even easier pieces of information to collect: the productivity of types alone and specific types concentration in teams. Table 5 illustrates the top expertise in the 1990s with the estimated production function. Note that the value of types when ranked alone is different from in teams, and the framework of this paper provides a method to unite these two components.

This section explored the value of types within the model, addressing in particular their impact on aggregate output. Next, I turn to an example of a real-world immigration shock to explore the potential of the model to explain changes in the distribution of teams and their contribution to output in a more comprehensive manner. Further, I use this real-world shock to confirm the intuition of the model.

Table 5: Ranking Types across IPC3

Rank (IPC3)	(1) 1990s $E[V_x]$ rank	(2) 1990s, alone	(3) 1990s, in teams
1	Checking-Devices	Checking-Devices	Biochemistry
2	Medical Science	Medical Science	Organic Chemistry
3	Computing/Counting	Computing/Counting	Organic Macromolecular Compounds
4	Elec. Comm. Technique	Elec. Comm. Technique	Fatty Acids
5	Biochemistry	Signaling	Petroleum and Technical Gases

7.2 Real-World Immigration Shock

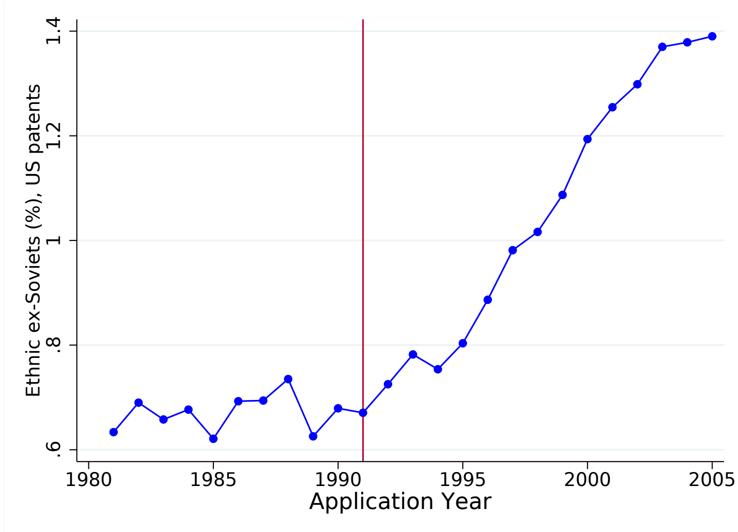
The model delivers sensible results on the pattern of allocations across teams. Further, the team production framework lends itself naturally to an application of studying immigration. However, the model does not rely on any quasi-experimental evidence in the structure of teams for identification. Since this has immediate applications in immigration policy, it seems sensible to use an observed shock on talent flows to see if integration to teams occurs as would be predicted. A real world immigration shock also enables a study of the process of self-selection as well. This section attempts to evaluate the predictions of the model with quasi-experimental evidence of talent flows and illustrates how this paper provides a lens to evaluate the contribution of immigrants to innovation.

The breakup of the Soviet Union created a mass movement of high-skilled workers from Russia to the United States. I use this large influx to explore a shift in the technological composition of inventors in the United States. I look at classes that were "exposed" to Russian inventors against classes that were "unexposed" to Russian inventors in the US.[19] This analysis proceeds in three steps. Step 1 verifies the contribution of Russian inventors to fields in the US that were prominent in the Soviet Union. This step confirms that the fall of the Soviet Union shifted the supply of inventor composition in the US. Step 2 evaluates the predicted and realized innovative output increase from Russian scientists. Step 3 compares the teams that formed with those predicted by the model in the 1990s.

To identify USPTO inventors who are Russian, I link ethnicity probabilities to inventors in the US using a procedure from Kerr (2007). I classify an inventor as of an ethnicity if they

[19] Appendix D.2 illustrates this heterogeneous exposure.

Figure 13: Percentage of ethnic Russians on patents in United States



have a more than a 0.5 probability of being this ethnicity through a first and last name match with the Melissa Database. In addressing how Russian expertise contributed to US innovation, I compare the distribution of the Soviet Union across IPC3 patent classes to the US distribution across IPC3 classes in the 1980s. I perform the same comparison with ethnic Russians in the US from 1995-2005 whose first patent was produced past 1991. Unfortunately, there is no way to match to specific names given the significant name changes that took place as Russians moved to America. Thus, I use the ethnicity of "new" patenters of Russian ethnicity to infer whether they are from the Soviet Union. While this may generate type-I errors, it picks up a significant share of the migrating Russians.

Figure 13 illustrates the growth of Russian inventors on US patents. Note the flat line in the proportion of Russians on US patents in the 1980s and the sudden uptick post-1991 when the Soviet Union fell. Figure 13 provides a promising example of a talent supply shock that can be explored both as a model verification and for its suggestive implications for immigration policies.

To explore the impact of this supply shock, I use the framework of the model to run through a shock of the given Russian expertise composition to the technological classes that had high Soviet expertise relative to the US. I start by illustrating that ethnic Russians produced in classes more exposed to patent classes from the former Soviet Union, as measured by IPC3 classification. I then use this shock of the fall of the Soviet Union to evaluate the contribution of Russian skills to the composition of teams with ethnic Russians in the US. For identification, I need to assume that the pre-1991 technological composition of inventors in the Soviet Union formed without expectations of contributing to US technology. This seems relatively reasonable given that the fall of the Soviet Union came unexpectedly to most of the world and human capital takes significant time to build.

As Borjas (1987) notes, immigrants to the United States exercise significant self-selection. I model this through a moving cost ψ which each immigrant faces. This cost will induce selection for immigrants associated with skills that have more value in the US. Since Proposition 3 delivered a rank order of the value of expertise, I leverage this to generate various cutoffs for expertise. Those with expertise in low value patent classes may not find it worth it to move given the small change in returns by moving to the US.

Due to the moving cost being unobserved, I need to take a stand on an approximate moving cost and then later evaluate robustness. When presenting the impact of Russian migrants, I focus on different implicit moving costs that would tend to draw high types as in Equation 12. Because of the flat return to skills in post Soviet Russia, it is sensible that types associated with higher value technologies will be more likely to migrate. Comparing the distribution in both countries, I calculate a measure of exposure in the US to scarce skill types that are provided by Russian inventors. I evaluate how this influx affected the patterns of teams in the US through the lens of the model and the ethnic classifications.

Table 6 illustrates the forces behind the migration pattern from the Soviet Union to the US. I take the concentration of the Soviet Union from 1975-1989 across 122 IPC3 classes and compare it to the concentration of the US in these classes. I then rank the 122 skills according to their relative exposure to the Soviet Union against the US in the 1980s. I evaluate three rank correlations. Row 1 suggests that classes relatively more exposed to the Soviet Union technologically is where the Russians in the US concentrated. Row 2 suggests that the classes the Soviet Union was exposed to were not particularly high value classes. Row 3 shows that the high value types were more likely to migrate, in line with the idea of a moving cost generating selection of types coming to the US.

I turn now to the model-implied impact of Russian scientists on aggregate innovation, understanding that self-selection of the migrating population will affect this impact. Table 7 shows the contribution of Russians to aggregate innovation split by value alone, model predicted innovative output change in teams (using US data), and predicted change in output (using ex-ante Soviet data). For value alone, I take ethnic Russians in the US and compute their proportion

Table 6: Predicted and Realized Inflow Results, 1995-2005

Measure	Value
Corr(Rank in SU, SU Rank in US)	0.26
Corr(Rank in SU, Val in US)	-0.043
Corr(SU Rank in US, Val in US)	0.21

Table 7: Russian Contribution to Aggregate Innovation, 1995-2005

Measure	Δ Agg. Innov (%)
— Panel A. Innovation in US —	
sole-authored innovation	0.3
incl. teams w/ model	0.9
— Panel B. Predicted from SU-US Match —	
predicted, no selection	0.6
predicted, selection at T50	0.8
predicted, selection at T20	1.1

of aggregate innovation. For Russians contributions in teams, I take the distribution of Russian inventors across types (as measured in IPC3 patent classes) to a counterfactual world without these Russians in the economy. For the contributions of Russians from the Soviet Union data, I shock the economy from 1995-2005 without Russians with Russians from the Soviet Union that represent 0.8% of the inventing population distributed across IPC3 classes as in the Soviet Union.

For the predicted change using Soviet data, I show how the output response changes depending on the degree of self-selection (no self-selection, and the top 50 and 20 IPC classes as cutoffs respectively). While I do not have a specific moving cost in the data, the interaction between self-selection and innovative output is an interesting avenue to explore.[20]

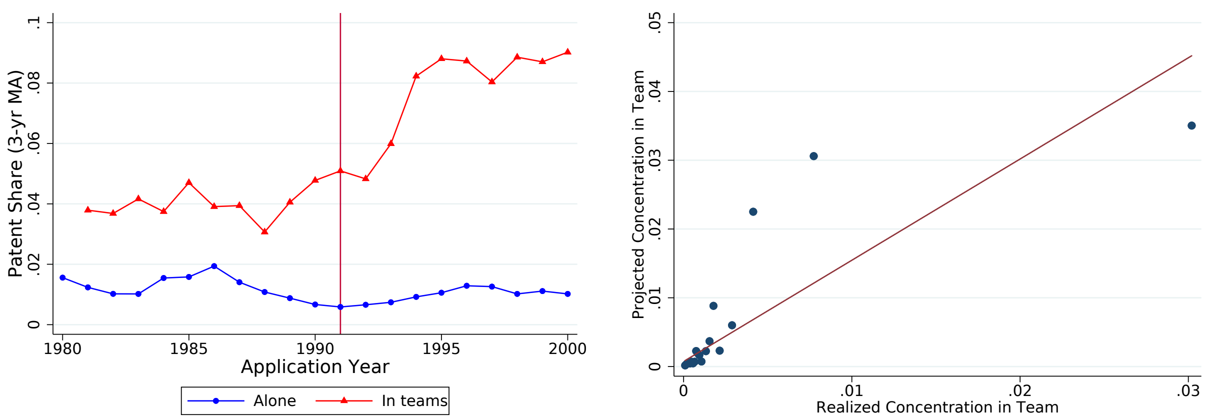
I stress two main results from Table 7. First, the Russian contribution to teams was a significant portion of their contribution to overall innovative output. This paper provides a method to evaluate and include this component in to research. Second, in order to match the overall contribution from Russians, the self-selection of migrating inventors must be modeled. If I shock the US distribution with the same mass of new inventors distributed as in the Soviet Union, I would undervalue the Russian contribution by over 30%. Thus, in order to understand the interaction

[20] Table 9 shows the results for stock market value of patents.

of the inventors in this economy, I must take into consideration the distribution in the Soviet Union, the degree of self-selection, and the team contribution channel.

If Russian inventors contributed to teams that would be predicted by the model, this would provide evidence that the model is appropriate for out of sample predictions. This would further provide support that this framework is applicable to study high-skilled immigration policy. Figure 14a illustrates the contribution of Russians to teams in technologies outside their direct scope. This plots the percentage of Russians with a patent in the closest ($\rho(s, s')$) downstream class that is not within their sphere from 1980-2000. The uptick in contribution to those classes after the event suggests the mechanics of the model are sensible in terms of the location of production. Further, Figure 14b suggests that there is a high correlation from the model in predicted allocation across teams (0.49) and realized allocation across teams. Figure 14 illustrates the contribution of Russians to teams that would be predicted by the model.

Figure 14: Russian Influx



(a) Teams in knowledge dependent technologies

(b) Allocation across Teams, Scatter

This can also be seen in the overall contribution of Russians to teams. Table 8 takes the Soviet distribution from the top 30 IPC3 patent classes (to about match the realized Russian impact on innovation, and projects some summary statistics of the Russian contributions to teams. The quantitative framework very closely predicts the proportion of Russian inventors who worked in teams, as seen in Table 8.

Table 8: Predicted and Realized Inflow Results, 1995-2005

Post-Soviet outcome	**Predicted**	**Realized**
Proportion in Teams	87.6%	88.1%
Modal Type	Computing/Counting	Computing/Counting
Modal Team-Pair	Computing/Counting $\times$ 2	Basic Electrical Elements $\times$ 2
Modal non-same pair	Basic Elec. Elements+ Elec. Comm. Tech.	Basic Elec. Elements+ Elec. Comm. Tech.

This section provides three main conclusions. First, there is a high correlation between migration patterns of Russians with specific expertise and its expected value from Proposition 3. This suggests that Proposition 3 is a suitable method for characterizing the value to an individual agent of having a specific expertise and modeling the self-selection process of migration. Second, the effect that Russians had on teams provides qualitative evidence the model is able to match real-world exogenous changes in the supply of talent. Third, in matching the effects of aggregate innovation ex ante with some degree of selection, the results suggest that this framework delivers a method for policy analysis of an increase in the supply of specific talents. On the whole, the team production and matching channel is crucial to characterize these forces.

7.3 Discussion: R&D Subsidies and Education Policy

I now turn to a qualitative discussion of R&D subsidies and education policy. I stress at the start that the team production channel induces a lot of potentially heterogeneous impact of R&D and education policy depending on its structure. The effect of R&D subsidies in this framework depend on how they target the idea market. For instance, if subsidies are directed to labs (teams), this naturally this will change team composition through inducing more collaboration. Agents would be more willing to overcome communication costs in order to join teams. If teams communication costs are borne privately while team innovative output is a public good, this could increase overall innovation. If R&D subsidies are directed towards all innovation, it will exacerbate the differences across productive and unproductive teams.

R&D subsidies would not directly hit communication costs. A subsidy on communication technology would induce more regional dispersion of teams. This is part of a natural progression, but a subsidy would induce more of this behavior. However, the R&D subsidy will hit differentially across the quality of teams q_k, whereas communication technology subsidies hit exclusively the high communication cost teams.

A policy of R&D subsidies seems more sensible when immigration policy becomes less feasible. Immigration policy does not require taxation that pulls resources away from other projects. Given that most patents that come from inventors across international borders are organized within firms, the presence of multinational firms would seem to be a useful mechanism for generating this subsidy to communication technology.

I also address the role of education policy in a qualitative manner. Through the lens of the model, education policy follows similar principles to immigration policy. If the cost of training is equal across domains of expertise, then education should be tilted towards classes that have the largest aggregate effects through both their own production channel and the team production channel. Further, fostering diverse expertise will be important for the economy given the rise of diverse expertise in teams.

However, it may be true that the high-value expertises are more costly to train. For instance, organic chemists are extremely valuable because they make contributions across classes. However, it is costly to train organic chemists. This paper has not built a framework to evaluate the mechanisms governing this tradeoff. Because the social value and private value of innovation can be misaligned, this seems like an exciting path for future research.[21]

A key result for education policy is that specific curricula should interact with the team structure of the economy. This is first-order for evaluating how different majors and fields contribute to aggregate innovation. Policy should be tilted towards building expertise that has large aggregate effects, and the policy must recognize the contribution that expertise makes to their productivity alone and to productivity in teams. This paper provides a benchmark for evaluating this issue from a general equilibrium framework.

8 Conclusion

Complex tasks in the economy increasingly require more varied skills and larger teams; this is particularly salient in the case of innovation. This paper addresses the forces that underlie the increasing importance of teams in innovation as well as their macroeconomic and policy

[21] Jones and Williams (1998) discuss this misalignment and find that there is significant underinvestment in R&D.

implications.

I proceed by building a novel team idea production and matching framework. With this framework and USPTO patent data, I quantify the role that three major forces play in driving these patterns: returns (the idea production function), costs (communication costs), and supply (inventor expertise composition). I find that all three forces are relevant for both the technological composition of the economy and the prevailing team size; changes in the idea production function, as understood through depth and breadth, explain the largest change in team size. Given the rising importance of fostering complementary skills, the results have relevant policy implications.

I focus on immigration policy through the lens of a team economy. I start by shocking the economy with different types of skills. I then compare the change in innovative output at these shocked economies to a statistic from the model for valuing individual skill types. I find that these two components have a high rank correlation (0.80 unweighted, 0.93 weighted). This suggests that for policy-makers interested in fostering high-valued skills, they only need a limited set of information on different skill types (i.e., the value of working alone and concentration in teams). Focusing on immigration policy with heterogeneous expertise allows a policymaker to be agnostic when it comes to training costs of various expertise. I use the fall of the Soviet Union as an example to how immigration can shift overall innovative output and find that the statistic from the value of types helps inform self-selection on migration to more ably quantify the impact of Russians from the Soviet Union on American innovation.

This framework suggests further avenues to explore. For instance, while this model delivers a distribution of expected values across skills, the cost of training each of these skill types is important to know for questions of skill investment and education policy. Understanding the interaction of the cost of training skills and the innovative output is a fruitful area for research. Further, understanding the role of the social value as measured in citations and the role of training expertise has the potential to shed light on multiple aspects of education and innovation policy. Lastly, this paper provides a method for thinking about how the incentives of firms to collect teams transmit to innovation lending insights to the endogenous growth literature which has mostly focused on firms and innovation. Thus, this paper can serve as a first step to addressing role of teams with heterogeneous domain expertise in aggregate innovation.

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A Theoretical Appendix

This theoretical appendix contains proofs of Propositions 1, 2, and 3 and further discussion on the effects of shocking the expertise distribution. I start with the proofs.

A.1 Proposition Proofs

Proof of Proposition 1

Proof. I ensure there exists an equilibrium that satisfies the 5 conditions set out in the equilibrium definition in Section 3. For notational purposes, I treat distinct types of skills x and discard location for notational purposes.

The mass of type x that is assigned to team k follows from the mass M_x multiplied by the probability a type x goes to team k. Following McFadden (1974), I show how I derive this probability given the set of sharing rules.

$$Pr\{k_x^*(i) = k\} = Pr(w_k^x + \epsilon_k^x(i) > w_{\tilde{k}}^x + \epsilon_{\tilde{k}}^x(i) \;\; \forall \;\; \tilde{k} \neq k)$$

I take individuals' shocks across all teams k as $F(\epsilon) = \exp(\exp(-\epsilon/\phi))$. Then, with utility from team k as follows:

$$\pi_k^x(i) = w_k^x + \epsilon_k^x(i)$$

The probability individual $i \in x$ chooses team k is:

$$\mathbb{P}\{\epsilon_{\tilde{k}}^x(i) < w_k^x - w_{\tilde{k}}^x + \epsilon_k^x(i) \; \forall \; \tilde{k} \neq k\}$$

$$= \int_{-\infty}^{\infty} \prod_{k \neq k} F(w_k^n - w_{\tilde{k}}^n + \epsilon_k^n) f(\epsilon_k^n) d\epsilon_k^n$$

Plug in the distribution of the shocks

$$= \int_{-\infty}^{\infty} \prod_{\tilde{k} \neq k} \exp\{-\exp[-(w_k^x - w_{\tilde{k}}^x + \epsilon_k^x)]\} \exp[-\epsilon_k^x - \exp(\epsilon_k^x)] d\epsilon_k^x$$

I perform a change of variable to generate $\psi = \exp(-\epsilon_k^x)$ and $z_{\tilde{k}} = \exp[-(w_k^x - w_{\tilde{k}}^x)]$. then:

$$\mathbb{P}(k^* = k) = \int_0^\infty \exp\left[\psi\left(1+\sum_{\tilde{k}\neq k} z_{\tilde{k}}\right)\right] d\psi = \frac{1}{1+\sum_{\tilde{k}\neq k} z_{\tilde{k}}} = \frac{\exp(w_k^n/\phi)}{\sum_{\tilde{k}\in\mathcal{T}_n}\exp(w_{\tilde{k}}^n/\phi)}$$

Thus, optimization leads us to the assignment to team k as follows:

$$m_k^x = M_x \cdot \frac{\exp M_{x,\ell}(w_k^x/\phi)}{\sum_{\tilde{k}\in\mathcal{T}_x}\exp(w_{\tilde{k}}^x/\phi)} \tag{13}$$

I use the knowledge of the value of working alone (q_{x0}):

$$m_{x0} = M_x \frac{\exp(q_{x0}/\phi)}{\sum_{\tilde{k}\in\mathcal{T}_x}\exp(w_{\tilde{k}}^x/\phi)}$$

as well as the market clearing condition in teams,

$$m_k^x = N_k^x m_k$$

To simplify Equation 13 as follows:

$$\log m_k - \log\frac{m_{x0}}{N_k^x} = \frac{w_k^x - q_{x0}}{\phi}$$

Finally, I use Equation E2 ($\sum_k w_k^{x'} = q_k - c_k$) to sum up this equation across each agent in the team, to get:

$$\log m_k - \frac{1}{T}\sum_{x\in k}\log\frac{m_{x0}}{N_k^x} = \frac{q_k - \sum_{x\in k} N_k^x q_{x0} - c_k}{\phi T_k}$$

Thus, satisfying (E1)-(E5) delivers an allocation and set of sharing rules that confirms the proposition.

□

Proof of Proposition 2

Proof. Define $\tilde{q}_k = \frac{q_k - \sum_{x\in k} N_k^x q_{x0} - c_k}{\phi T_k}$. For each type, there is the market clearing condition:

$$M_x = (1+\exp\tilde{q}_{xx})m_{x0} + \sum_{y\neq x}\exp\tilde{q}_{xy}\sqrt{m_{x0}m_{y0}}$$

By the method of *displacement*. Take the derivative with respect to m_{x0} and then plug in for $\exp\tilde{q}$ and m. This delivers the matrix as follows:

$$\begin{bmatrix} \frac{m_{x0}+m_{xx}+M_x}{2m_{x0}} & \dots & \frac{m_{xy}}{2m_{x0}} & \dots & \frac{m_{xX}}{2m_{x0}} \\ \vdots & \dots & \dots & \dots & \vdots \\ \frac{m_{xX}}{2m_{X0}} & \dots & \frac{m_{Xy}}{2m_{X0}} & \dots & \frac{m_{X0}+m_{XX}+M_x}{2m_{X0}} \end{bmatrix} d\boldsymbol{m} = d\boldsymbol{M}$$

The key result to show is that the matrix is invertible. If it is invertible, there is a unique distribution of teams that would be reached for a small change in the distribution of types. This follows from the diagonally dominant matrix theorem and the fact that if a matrix is invertible its transpose is invertible. All that is needed is that $m_{x0} + m_{xx} + M_x > \sum_{y \neq x} m_{xy}$. This immediately follows from the market clearing condition ($\sum_{y \neq x} m_{xy} + m_{x0} + m_{xx} = M_x$) and the fact that all possible teams are realized ($m_{x0} > 0 \Rightarrow \sum_{y \neq x} m_{xy} < M_x$) □

Addendum: Identification condition for multi-person teams

$$M_x = m_{x0} + \sum_{\mathcal{T}_x} N_k^x m_k \quad ; \quad m_k = \exp \tilde{q}_k \prod_{\tilde{x} \in k} \left(\frac{m_{\tilde{x}0}}{N_k^{\tilde{x}}} \right)^{N_k^{\tilde{x}}/T}$$

Goal is to get the components of m_{x0} that contribute to M_x, and as such:

$$M_x = m_{x0} + \sum_{\mathcal{T}_x} N_k^x \exp \tilde{q}_k \prod_{\tilde{x} \in k} \left(\frac{m_{\tilde{x}0}}{N_k^{\tilde{x}}} \right)^{N_k^{\tilde{x}}/T}$$

Take derivative:

$$\frac{\partial M_x}{\partial m_{x0}} = 1 + \sum_{\mathcal{T}_x} \exp \tilde{q}_k \frac{N_k^x}{T} \left(\frac{m_{x0}}{N_k^x} \right)^{\frac{N_k^x - T_k}{T}} \prod_{\tilde{x} \neq x \in k} \left(\frac{m_{\tilde{x}0}}{N_k^{\tilde{x}}} \right)^{N_k^{\tilde{x}}/T}$$

Plug back in the endogenous components:

$$\frac{\partial M_x}{\partial m_{x0}} = 1 + \sum_{\mathcal{T}_x} \frac{m_k}{\prod_{\tilde{x} \in k} \left(\frac{m_{\tilde{x}0}}{N_k^{\tilde{x}}} \right)^{N_k^{\tilde{x}}/T}} \frac{N_k^x}{T} \left(\frac{m_{x0}}{N_k^x} \right)^{\frac{N_k^x - T_k}{T}} \prod_{\tilde{x} \neq x \in k} \left(\frac{m_{\tilde{x}0}}{N_k^{\tilde{x}}} \right)^{N_k^{\tilde{x}}/T_k}$$

$$\frac{\partial M_x}{\partial m_{x0}} = 1 + \sum_{\mathcal{T}_x} m_k \frac{N_k^x}{T} \left(\frac{m_{x0}}{N_k^x} \right)^{\frac{N_k^x - T}{T}} \left(\frac{m_{x0}}{N_k^x} \right)^{-N_k^x/T}$$

$$\frac{\partial M_x}{\partial m_{x0}} = 1 + \sum_{\mathcal{T}_x} \frac{(N_k^x)^2}{T} \frac{m_k}{m_{x0}}$$

$$\frac{\partial m_{\tilde{x}}}{\partial m_{x0}} = \sum_{\mathcal{T}_{x\cup\tilde{x}}} \frac{N_k^x N_k^{\tilde{x}}}{T_k} \frac{m_k}{m_{x0}}$$

This delivers a condition under which the matrix of interest would be invertible due to diagonally dominant theorem:

$$1 + \sum_{\mathcal{T}_x} \frac{(N_k^x)^2}{T} \frac{m_k}{m_{x0}} > \sum_{\tilde{x}\neq x} \sum_{\mathcal{T}_{x\cup\tilde{x}}} \frac{N_k^x N_k^{\tilde{x}}}{T} \frac{m_k}{m_{x0}}$$

This identification condition is met in the quantitative exercise.

Proof of Proposition 3

Proof. Conditional on an agent $i \in (x, \ell)$ choosing a team k, the expected utility of this agent is:

$$\begin{aligned}
\mathbb{E}\left[\pi_k^{x,\ell} | k = \arg\max_{k'\in\mathcal{T}_{x,\ell}} \pi_{k'}^{x,\ell}\right] &= w_k^{x,\ell} + \mathbb{E}\left[\epsilon_k^{x,\ell}(i) \middle| w_k^{x,\ell} + \epsilon_k^{x,\ell}(i) > w_{\tilde{k}}^{x,\ell} + \epsilon_{\tilde{k}}^{x,\ell}(i)\ \forall\, \tilde{k} \neq k\right] \\
&= w_k^{x,\ell} + \mathbb{P}\left[\pi_k^{x,\ell} \middle| k = \arg\max_{k'\in\mathcal{T}_{x,\ell}} \pi_{k'}^{x,\ell}\right]^{-1} \times \\
&\int_{-\infty}^{\infty} \epsilon_k^{x,\ell}(i) \exp\left[-\epsilon_k^{x,\ell}(i) - e^{-\epsilon_k^{x,\ell}(i)} \left(1 + \sum_{\tilde{k}\neq k} \eta_{\tilde{k}}\right)\right] d\epsilon_k^{x,\ell}(i)
\end{aligned}$$

Where I define $\eta_{\tilde{k}} \equiv \exp\left[-\left(\epsilon_k^{x,\ell}(i) - \epsilon_{\tilde{k}}^{x,\ell}(i)\right)\right]$ The result above is standard and comes from the assumption on the distribution of the shocks across teams. I now use a standard result from math, which shows that $\int_{-\infty}^{\infty} x \exp(x - \eta e^x) dx = -(c + \log \eta)/\eta$ with c as Euler's constant. This delivers the following equation:

$$\mathbb{E}\left[\pi_k^{x,\ell} | k = \arg\max_{k'\in\mathcal{T}_{x,\ell}} \pi_{k'}^{x,\ell}\right] = c + \log\left(\sum_{\tilde{k}\in\mathcal{T}_{x,\ell}} \exp(w_{\tilde{k}}^{x,\ell}/\phi)\right) \tag{14}$$

Note this equation is independent of the specific team k, and only depends on the distribution of potential teams for the given inventor. I unite this equation with the equation that governs the demand equation for type (x, ℓ) in team (x, ℓ) as a sole inventor:

$$m_{x0}^{x,\ell} = M_{x,\ell} \frac{\exp(q_{x0}/\phi)}{\sum_{\tilde{k}\in\mathcal{T}_{x,\ell}} \exp(w_{\tilde{k}}^{x,\ell}/\phi)}$$

$$\mathbb{E}\left[\pi_k^{x,\ell} | k = \arg\max_{k'\in\mathcal{T}_{x,\ell}} \pi_{k'}^{x,\ell}\right] = c + \log\left(\frac{M_{x,\ell}}{m_{x0}^{x,\ell}} \exp(q_{x0}/\phi)\right) = c + \log \frac{M_{x,\ell}}{m_{x0}^{x,\ell}} + q_{x,0}/\phi \tag{15}$$

This delivers our result:

$$\mathbb{E}[V_{x,\ell}] \propto cons + \underbrace{q_{x,0}}_{\text{output alone}} + \underbrace{\phi \log\left(\frac{M_{x,\ell}}{m_{x0}^{x,\ell}}\right)}_{\text{concentration in teams}} \tag{16}$$

$\square$

A.2 Discussion on Theoretical Counterfactuals

In order to solve for a counterfactual scenario of an increase of the supply of type x given the existing distribution of skills, I need to solve a high dimensional nonlinear system of equations. However, this process can be simplified by an understanding of the Walrasian equilibrium and methods of tatonnement. The key element is to realize the excess demand function for each type is linked through the team formation equation as follows.

$$m_k = \exp\left(\frac{1}{T_k}\sum_{(x,\ell)\in k} \log \frac{m_{x0}^{x,\ell}}{N_k^{x,\ell}} + V_k\right)$$

V_k represents the net value of the team as estimated in the previous section and is known. It has been identified in the previous equiliibrium. m_k and $m_{x,\ell}$ are not observed in the counterfactual world. Given the identification of the model, there is a set of k equations for each team, and the market clearing for each type:

$$\sum_k N_k^{x,\ell} m_k = M_{x,\ell}$$

The overall process gets unwieldy. The following definition characterizes a vector of the mass of types alone:

$$\overrightarrow{m_0} = (m_{10}, ..., m_{X0})$$

And the excess demand equation for each type:

$$D_x^{\epsilon}(\overrightarrow{m_0}) = \sum m_k^x(\overrightarrow{m_0}) - M_x$$

Each excess demand function can be written out as a function of each type working alone, as each team equation is a function of the types working alone. The key condition is the following condition.

$$\begin{bmatrix} D_1^{\epsilon}(\overrightarrow{m_0}) = 0 \\ \vdots \\ D_x^{\epsilon}(\overrightarrow{m_0}) = 0 \\ \vdots \\ D_X^{\epsilon}(\overrightarrow{m_0}) = 0 \end{bmatrix}$$

The key result for counterfactuals is to find the tattonment equilibrium that satisfies these conditions. This enables the second quantitative exercise that explores changes in the supply of types. Instead of a high dimensional unwieldy equation, there are simply the same number of equations as number of types. These can be solved in standard packages, and require about 30 minutes per shock given a distribution of 122 types.

B Data Appendix

This section discusses some basic facts in the data related to teams and technologies.

B.1 Changing Team Size

This section discusses general patterns related to teams in the data, starting with the increases in team size across a wide array of categories. I start by discussing the team size distribution. Figure B.1 illustrates the team size distribution across the sample which has a nice shape and indicates the overall "small teams" model of innovators.

Figure B.1: Team Size Distribution

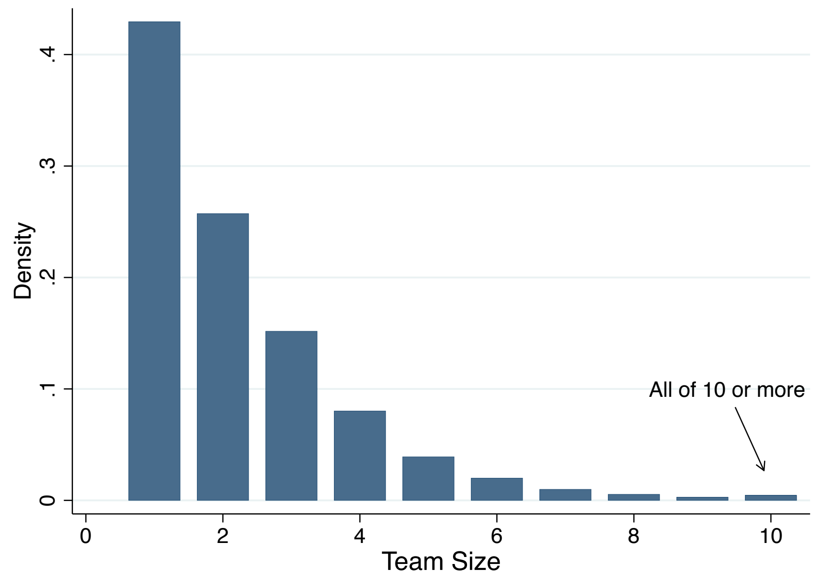


Figure B.2 splits teams by their major category in order to illustrate the pattern of strengthening teams is very common across all major technologies. Even splitting by subcategories, as in Figures B.3, B.4, and B.5.

Figure B.2: Mean Team Size by Category

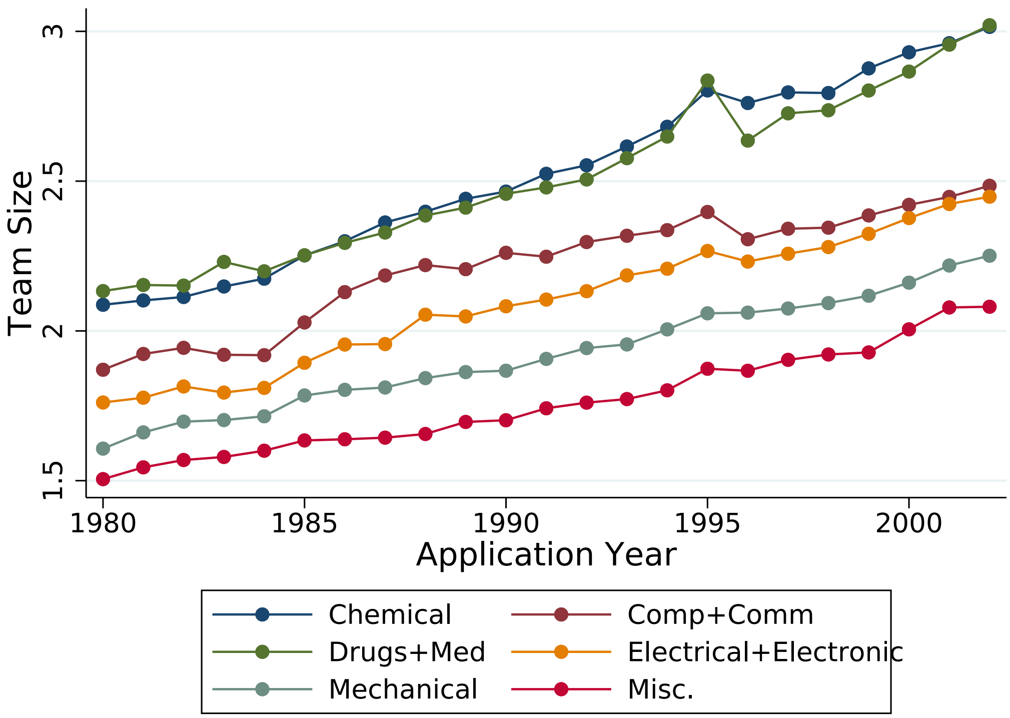
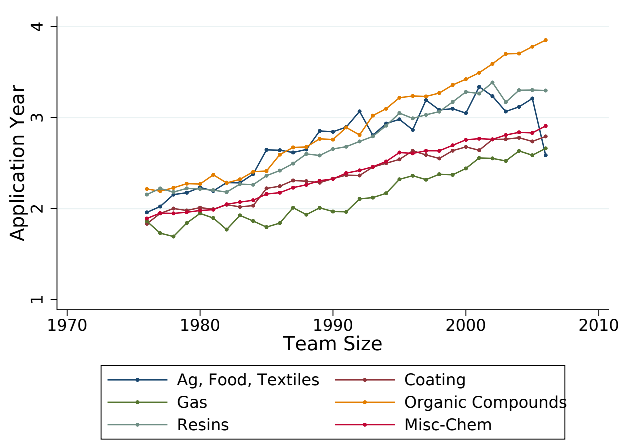
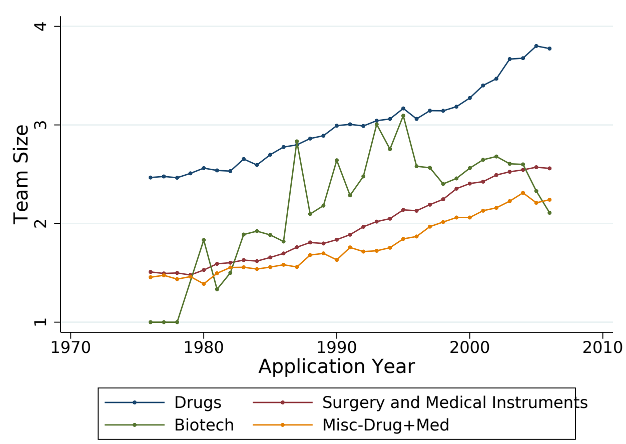


Figure B.3: Team Size Growth by Subcategory

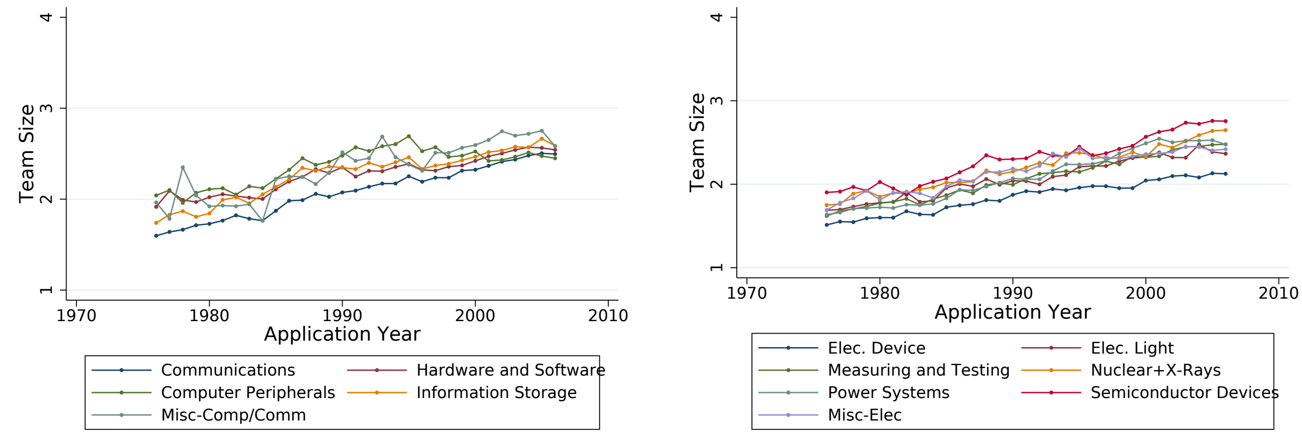


(a) Chemical



(b) Drugs/Medical

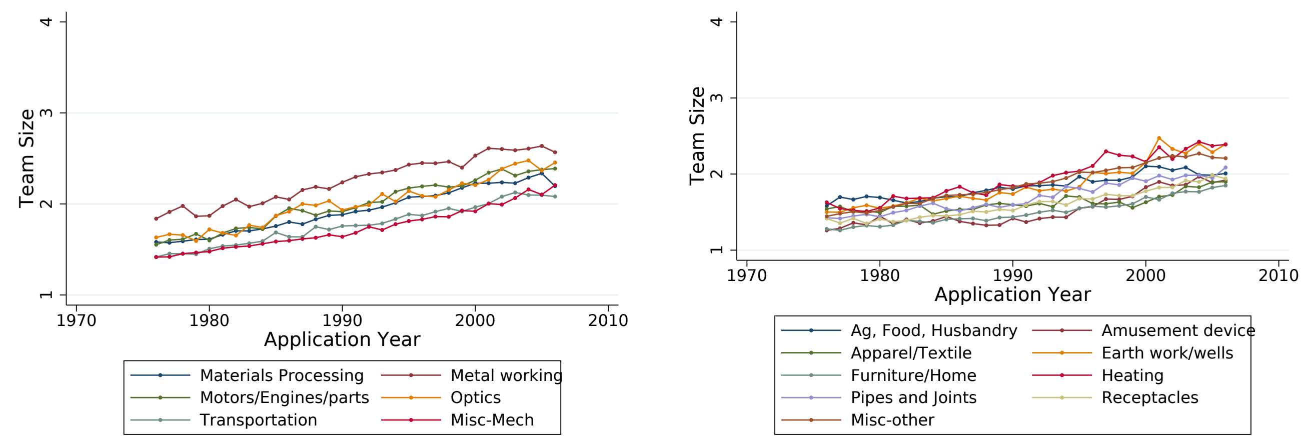
Figure B.4: Team Size Growth by Subcategory



(a) Computers/Communications

(b) Electrical/Electronics

Figure B.5: Team Size Growth by Subcategory



(a) Mechanical

(b) Other

These facts motivated the initial question, and the responsiveness of patent quality output to skills motivated the second–that there is a link between the structure of individual skills and the rise of teams. I attempted to do this by

B.2 Rising Returns to Teams

This has the figure that illustrates Fact 1 about the rising returns to teams but focuses on stock market value instead of citations. Both forces are pointing in the same direction. Figure B.6 illustrates this for stock market value:

Figure B.6: Citations and Stock Market Value, Team to no Team

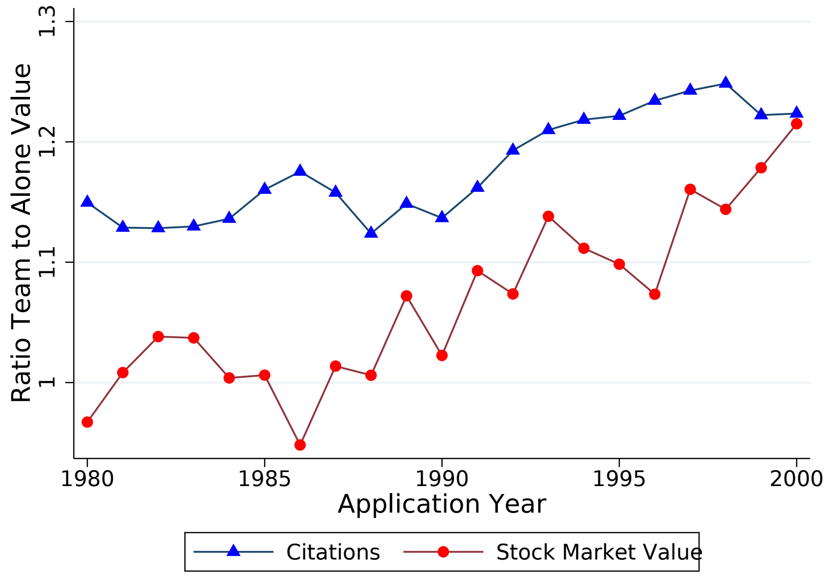
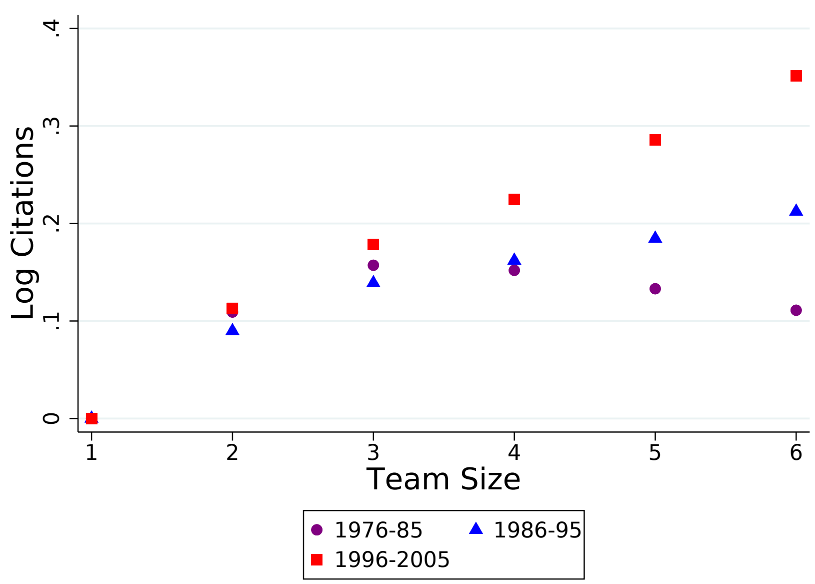


Figure B.7 illustrates that the fact comparing the 1980s to the 1990s can be extended to three subperiods, 1976-1985, 1986-1995, and 1996-2005. This plots the mean inverse hyperbolic sine of citations by team size up to teams of size 6. The rising returns to teams appears to be consistent over time:

Figure B.7: Split According to Three Periods

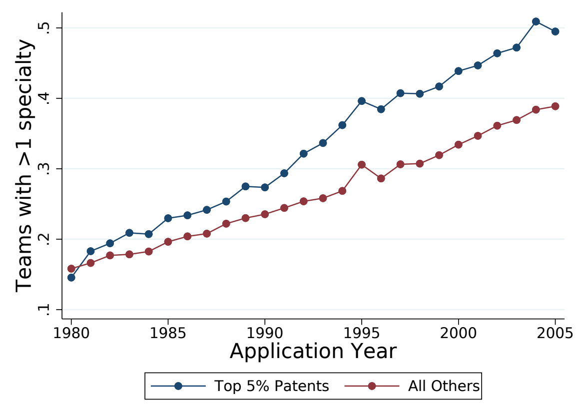


B.3 Rising Diversity

I discuss in the introduction the rise in team diversity. Here I document three facts that are indicative of this rising diversity. While the first two (male, female & ethnic diversity) are not relevant for economic output, scholars have noted the link between background diversity and cognitive diversity (Hoever et al., 2012; Jang, 2017), and found that more diverse teams ethnically produce higher impact patents (see Freeman and Huang, 2015).

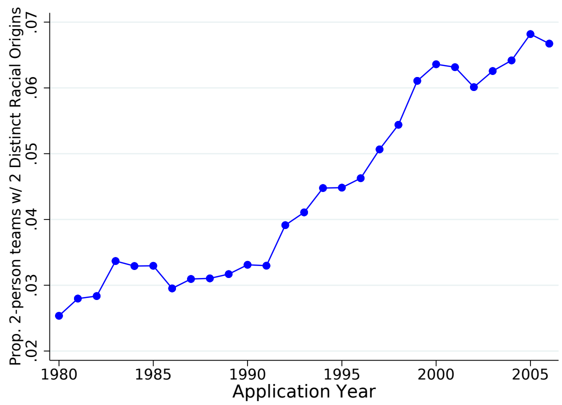
I show three graphs that depict the rise in diversity in teams. First, note the rising trend of inventors collaborating together who started in different fields. Figure B.8 illustrates the proportion of patents that have at least two inventors whose initial patent was different from the other inventor. Further, it suggests these are the higher impact patents especially that are being assigned to the diverse teams:

Figure B.8: Prop. patents with ≥ 2 unique technological backgrounds



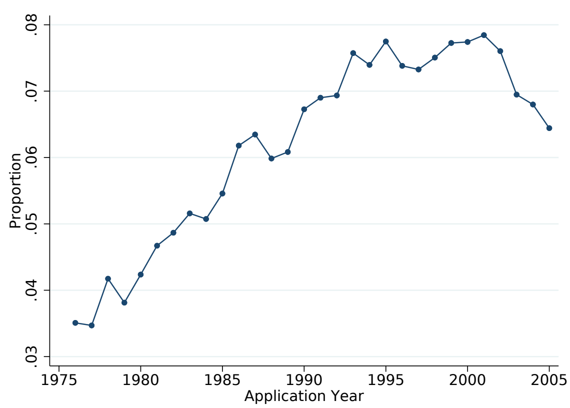
Next, I turn to ethnic diversity. Figure B.9 takes the same ethnic measures as Kerr (2007) and asks how many patents have two distinct ethnicites:

Figure B.9: Prop. 2-person team patents with $\geq$ 2 unique ethnic backgrounds



Lastly, I note that teams of males and females working together is on the rise, with names probabilistically matched to genders:

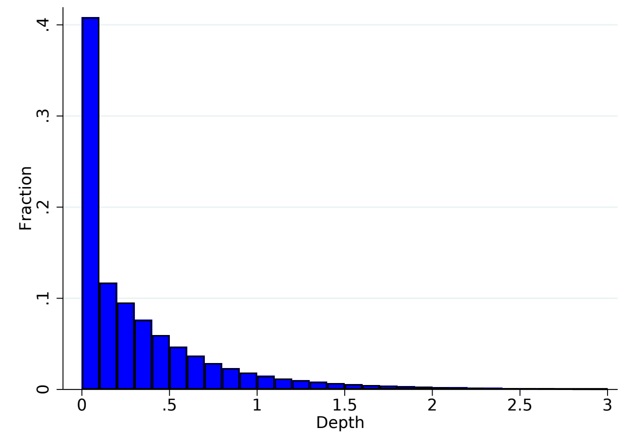
Figure B.10: Proportion of teams with male+female



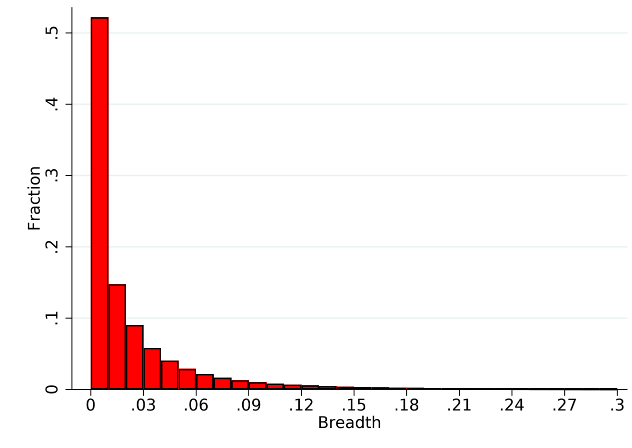
B.4 The Distribution of Breadth and Depth on Patents

Figure B.11 shows the distribution of depth and breadth on patents. This takes the total set of patents and plots the histogram across each unique patent the total depth and total breadth. Note the fat right tail in both cases, as some inventors bring a lot of expertise from other areas or are especially strong within the focal area.

Figure B.11: The Distribution of Depth and Breadth



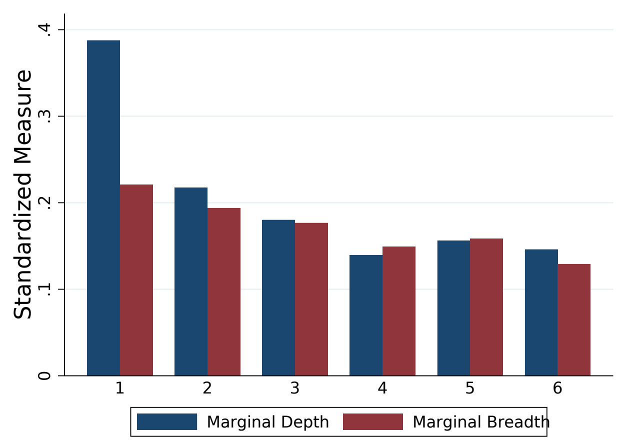
(a) Depth



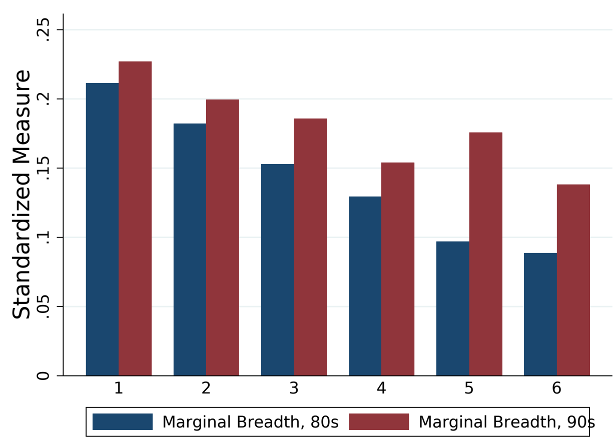
(b) Breadth

Further, Figure B.12 the incremental increase in both depth and breadth as the size of the team increases. Given this result along with the fact that controlling for depth and breadth kills the team size effect through the production function, it is suggestive depth and breadth have strength in determining the sorting pattern:

Figure B.12: Breadth/Depth and Team Size, 1980s and 1990s



(a) Depth and Breadth

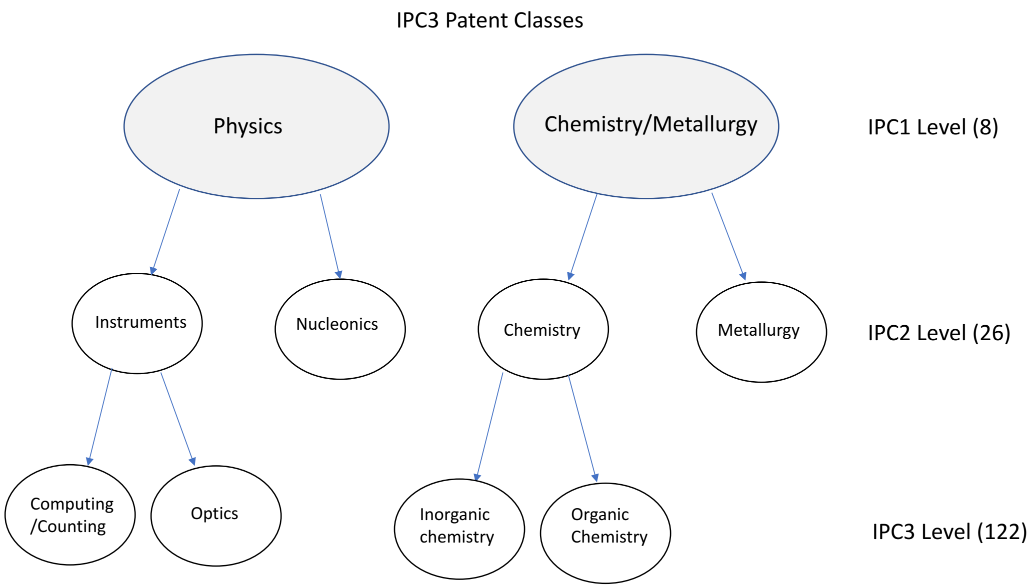


(b) Breadth, 80s v. 90s

B.5 Classification Example: IPC3

This paper uses both USPTO and IPC classifications. Both are standard in the patent literature. In order to match the data to records from the Soviet Union, I exploit IPC classification for the quantitative exercise, both at the 2-digit and 3-digit IPC level.

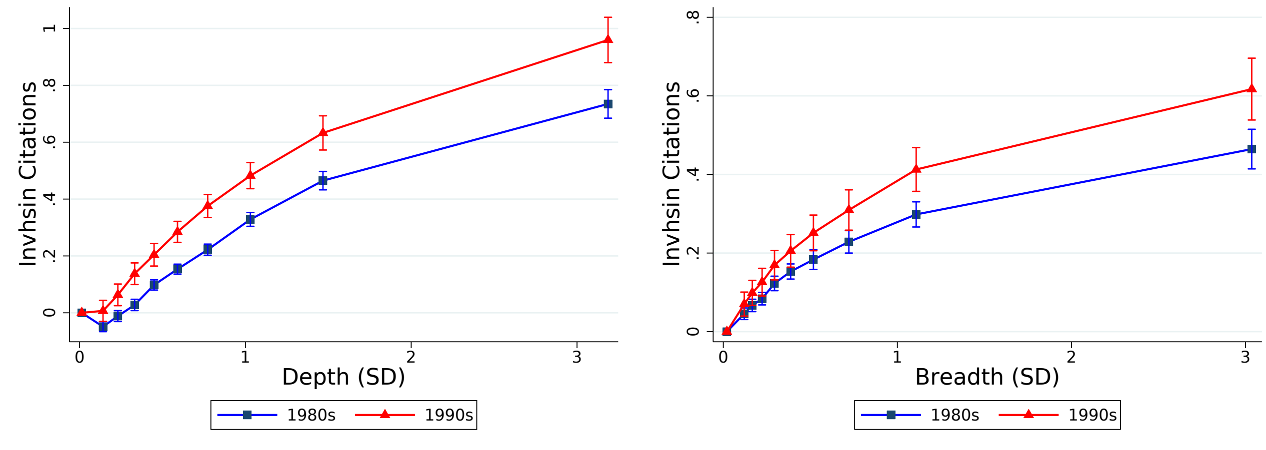
Figure B.13: Technology classifications (IPC)



C Empirical Appendix

The empirical section stressed 6 facts. I discuss them more in depth here. Figure C.14 shows that even when controlling for firm fixed effects the 1990s shows higher slope response to depth and breadth.

Figure C.14: Citations and Depth/Breadth, Firm Fixed Effects, 1980s versus 1990s

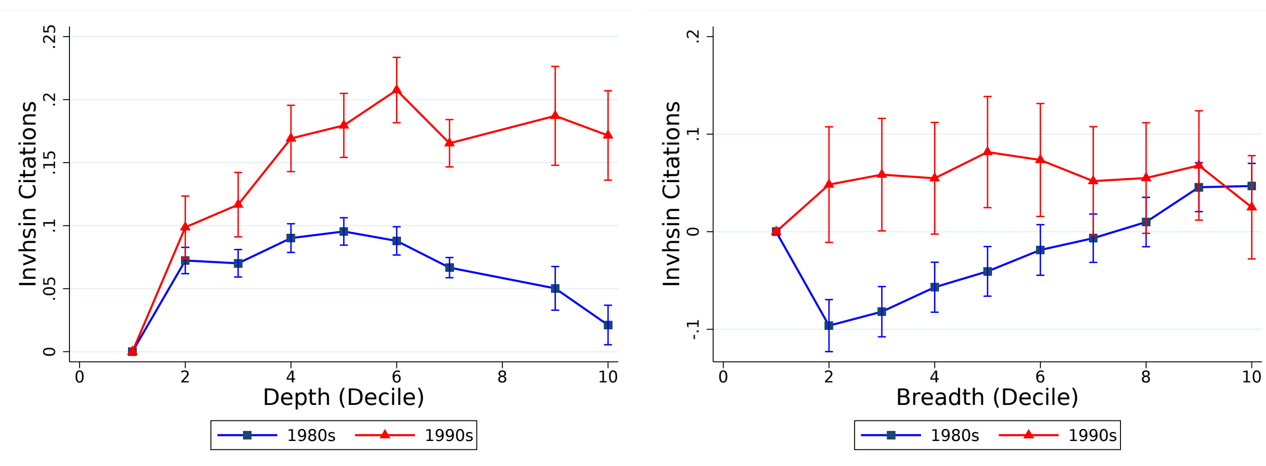


(a) Depth

(b) Breadth

Figure C.15 shows that even when controlling for individual fixed effects, the returns to depth and breadth are higher in the 1990s than the 1980s. This is true even when exploring different definitions of breadth.

Figure C.15: Citations and Depth/Breadth, Individual Fixed Effects, 1980s versus 1990s

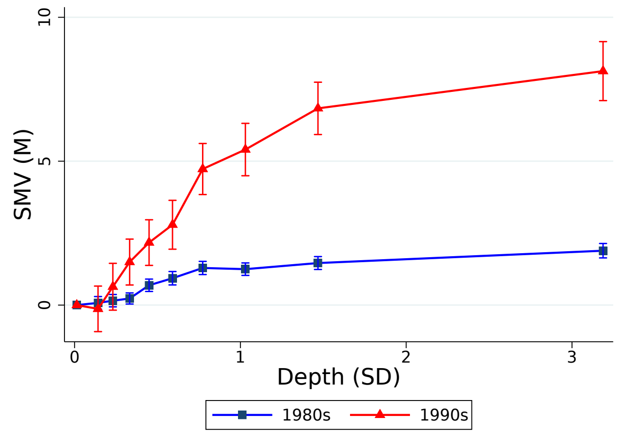


(a) Depth

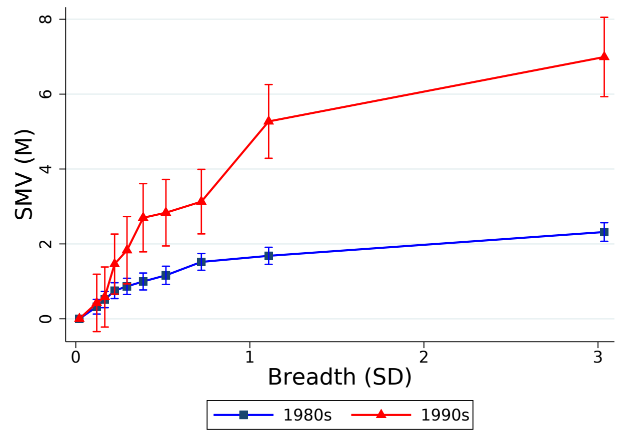
(b) Breadth

Figure C.16 illustrates that, if anything, the stock market value responsiveness measured by Kogan et al. (2017) has an even stronger response to depth and breadth in the 1990s period than the 1980s.

Figure C.16: Stock Market Val. and Depth/Breadth, 1980s versus 1990s



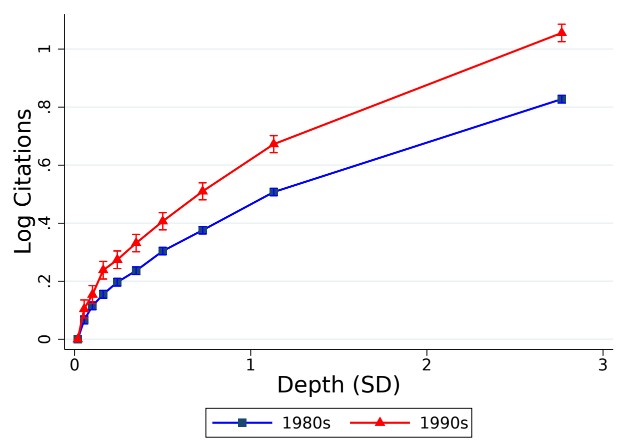
(a) Depth



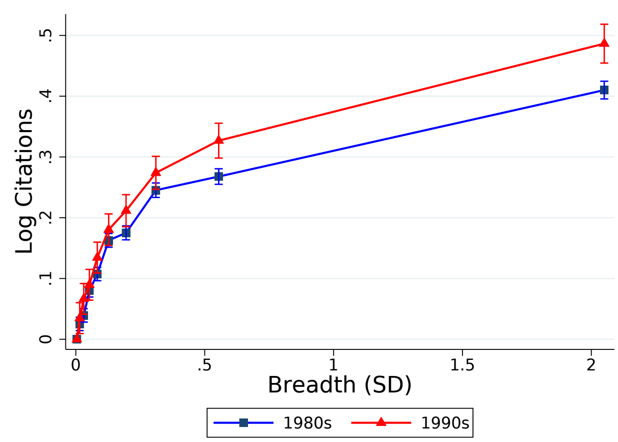
(b) Breadth

Figure C.17 illustrates that, if anything, the stock market value responsiveness measured by Kogan et al. (2017) has an even stronger response to depth and breadth in the 1990s period than the 1980s.

Figure C.17: Use Only Working Alone for Skill, 1980s and 1990s



(a) Depth

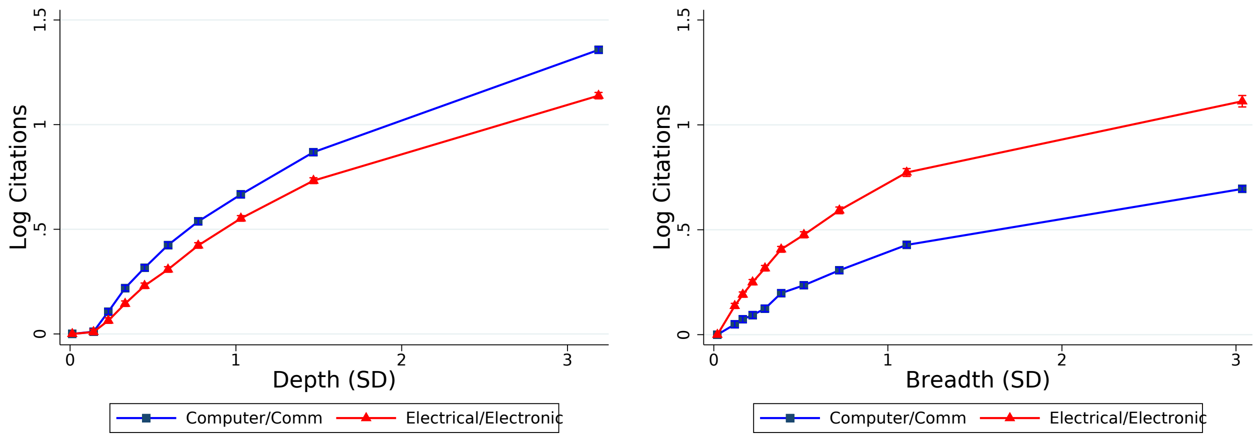


(b) Breadth

C.1 Breadth and Depth as Forces in Idea Production

This section discusses some additional evidence on the responsiveness to depth and breadth at the patent level. One might expect the returns to depth and breadth to vary due to the underlying technology. I do a split by technologies in order to determine which types of technologies get larger returns to depth and breadth. Figure C.18 demonstrates that electrical processes, which are often more linked to basic research and a broader array of classes, exhibit higher returns to breadth versus Computing and communications. Conversely, Computing and communications, which tap into expertise within the focal class, exhibits a larger response to depth than to breadth.

Figure C.18: Variation across Technologies

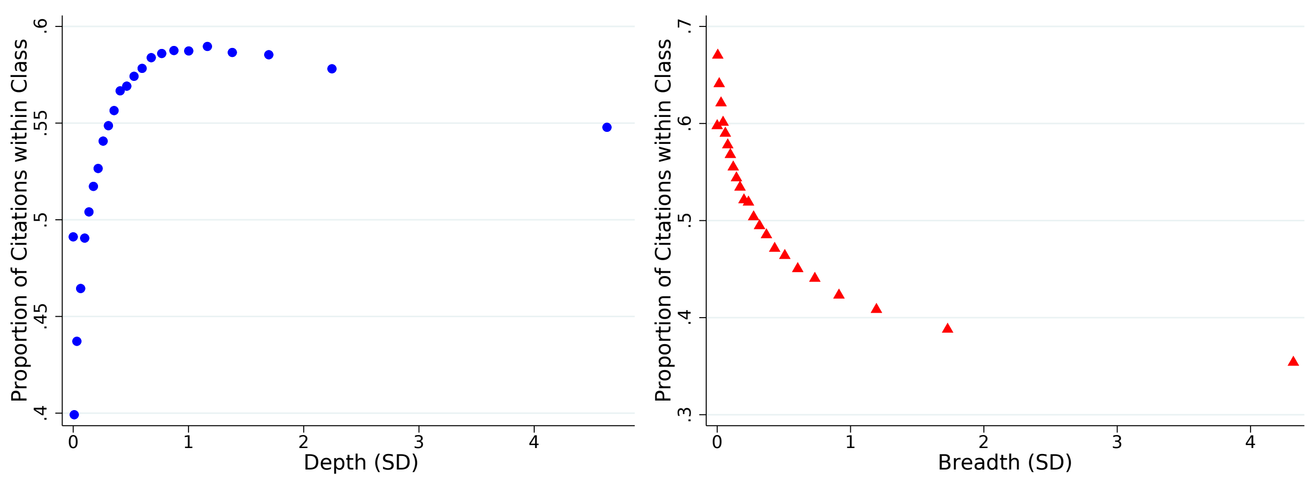


(a) Depth, Electric v. Comp

(b) Breadth, Electric v. Comp

In addition to the heterogeneous responsiveness to depth and breadth, high depth patents are more likely to provide knowledge to the focal technology class through forward citations. High breadth patents are more likely to provide knowledge to other technology classes via forward citations. This is true when controlling for a host of other factors like the focal patent class, year, and team size. Figure C.19 illustrates this result:

Figure C.19: Destination Citations



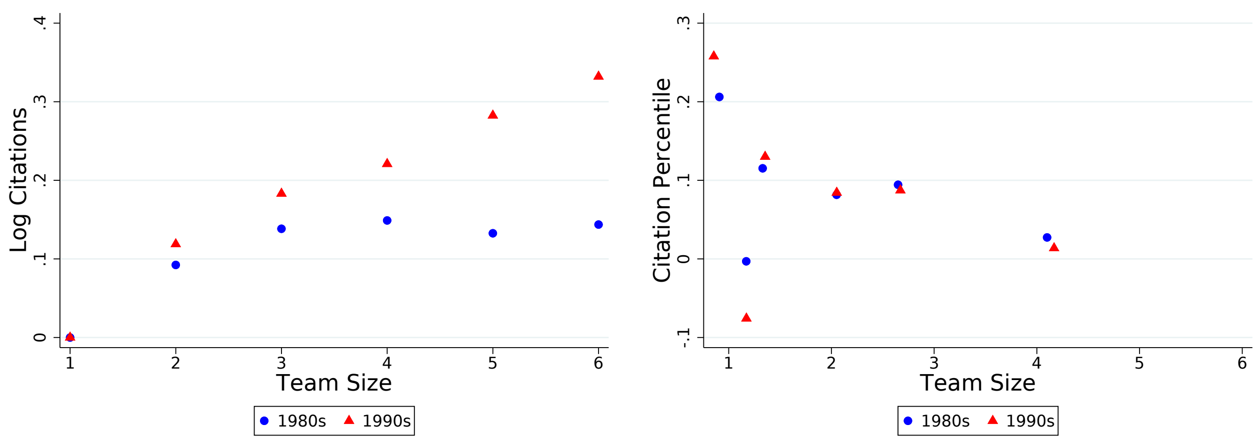
(a) Depth and Forward Citations

(b) Breadth and Forward Citations

Figures C.20 shows the increasing returns to teams are better understood through the skills that individuals bring rather than simply team size. Figure C.20 shows that teams produce patents with higher citations in general, and that this is more pronounced in the 1990s than the 1980s.

Once I control for depth and breadth on the patent and plot the residuals from these regressions, the relationship between patent quality and team size goes away and does not shift from the 1980s to the 1990s. The fact that controlling for depth and breadth kills the returns to teams indicates the importance of understanding *expertise* for evaluating the changing patent value.

Figure C.20: Citation value and team size, 1980s versus 1990s

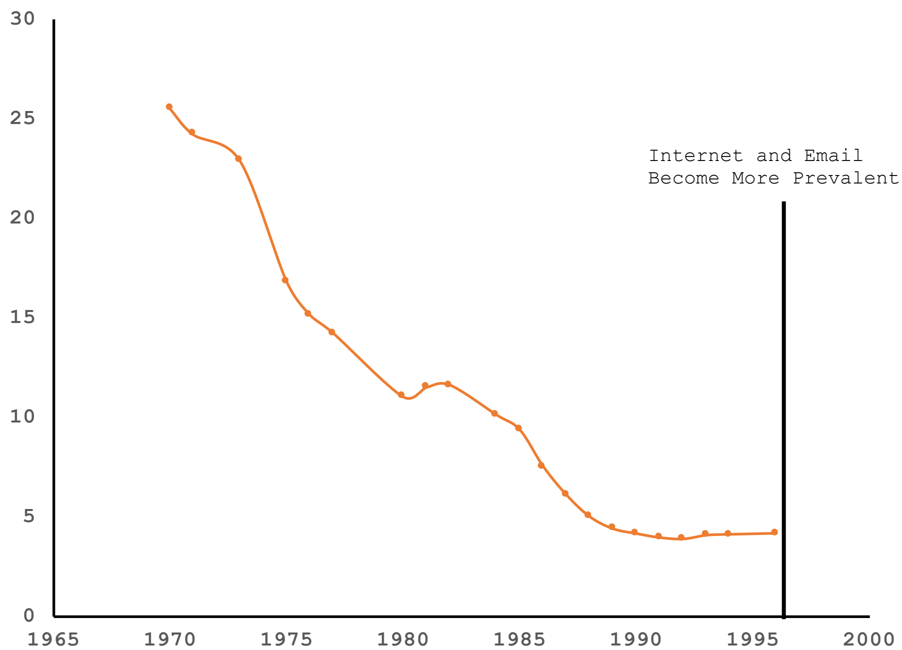


(a) Raw Means (b) +controls for depth and breadth

C.2 Communication Costs

Figure 3 illustrates how more inventors are collaborating across the country. I provide further evidence on the falling communication costs driving this phenomena. Figure C.21 shows the cost of a 10-minute phone call in 2010 US dollars over time. The cost flattens out around \$4.00 when email and the internet become more prevalent:

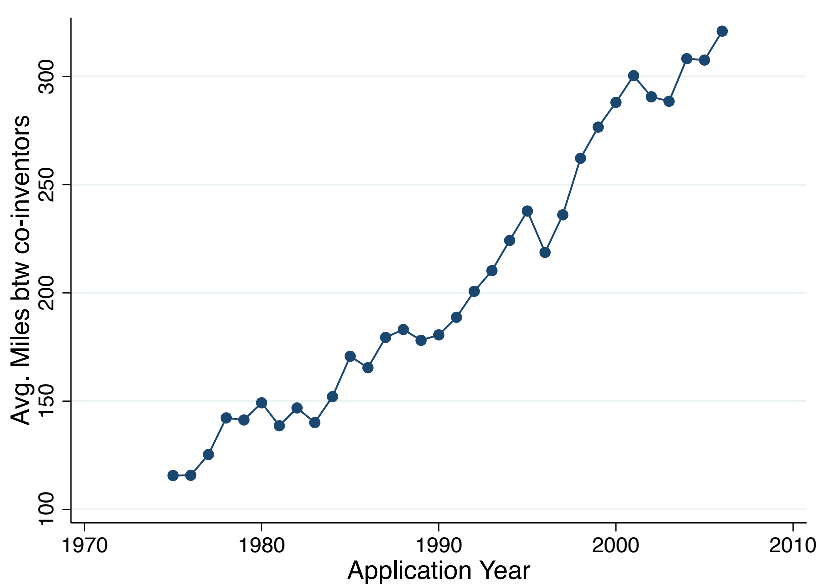
Figure C.21: Cost of a 10-minute cross-country phone call, by year



Source: FCC on AT&T Charges

Further, while Figure 3 showed the frequency of two co-authors being in different locations, the relationship is even more stark when I look at distance in miles. Figure C.22 plots the mean miles distance between co-inventors from 1976-2006:

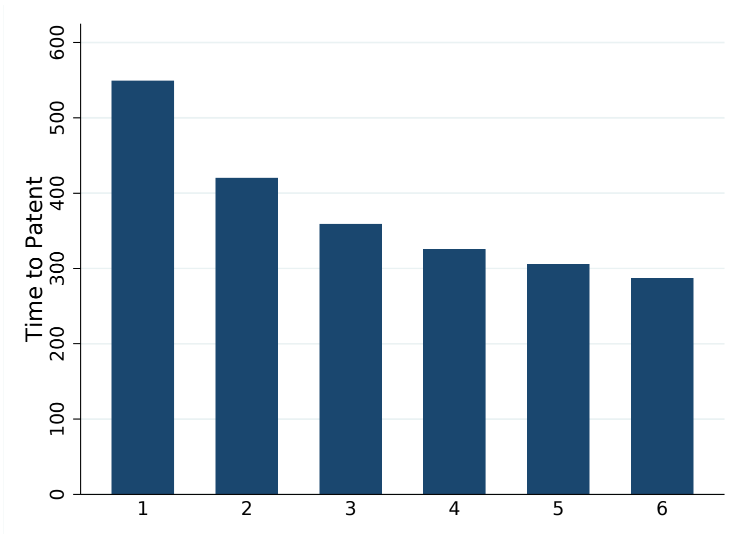
Figure C.22: Geographical Distance on Co-Authored Patents Over Time



C.3 Further Benefits to Teams

While it did not play a main part in the quantitative or empirical section of this paper, teams produce patents much faster than individual inventors. Figure C.23 illustrates this benefit to teams. This figure takes the lead author (taking the average across authors provides a similar result) on a patent, and looks at the average number of days since their most recent patent by team size.

Figure C.23: Time Between Patents $\times$ Team Size



D Quantitative Appendix

In the quantitative section, I classify individuals according to IPC2 types (26) and five regions. This delivers 130 unique types, with an average of 7,500 observations per type per period. In order to leverage the mechanics of the model, I need to bin inventors into wider bins. When not using regional information (e.g. talent supply shocks to US), I use a more narrow classification to speak to a finer level of expertise.

D.1 Classification of Types: Regions and Quantitative

The empirical section used the entire dataset which provided a rich set of information on agents and how teams combine their skills. The restriction for the quantitative section required some judgment calls on classifying inventors. Inventors that produce alone get assigned their class alone. For inventors that don't produce alone, they are assigned their most common class (adjusted by team size) as their type. For agents with only one patent, they do not get a type.

Due to the fact that the estimation is across the entire 1990s, there will be repeated information used in the estimation. Thus, for types observed multiple types they will show up as multiple observations across teams.

D.1.1 Classification for Application: Immigration Shocks

When modeling talent supply shocks, I use a more granular definition of skills by mapping individual expertise to IPC3 classes with sufficient observations (122 unique categories). I take the individual's top category when working alone, then on teams of 2, etc. Inventors with only one patent observation are not classified according to a type.

D.2 Soviet Union Shock

Figure D.24 plots the concentration of the Soviet Union and the United States across patent classes according to the IPC3 patent classifications. This figure illustrates the heterogeneous exposure across classes that enable use of the fall as a shock to the supply of talent.

Figure D.24: Concentration of US and Soviet Union across IPC3, 1980-1990

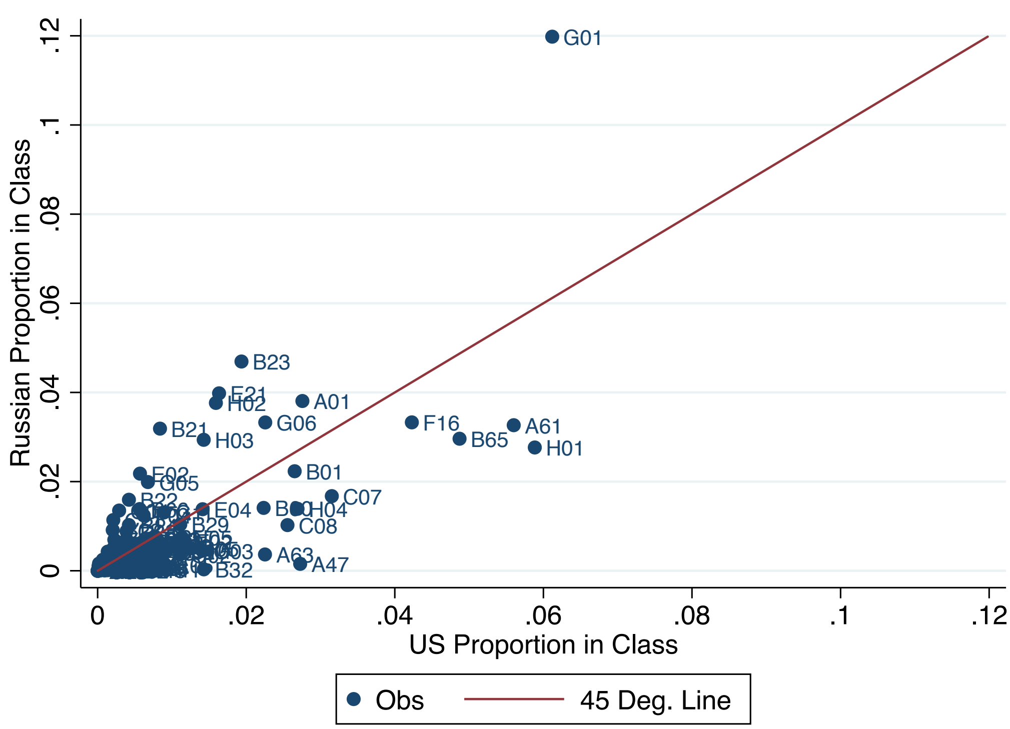
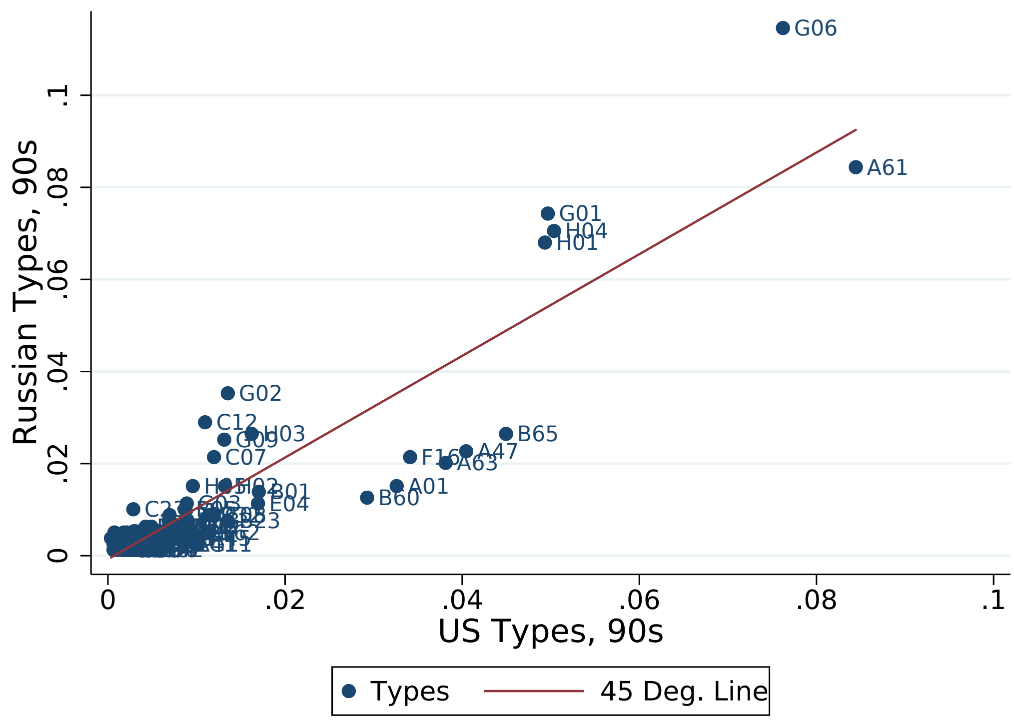


Figure D.25 illustrates that the concentration of Russians across types in the IPC3 categories has some resemblance of matching the pre period. However, there is selection in the migration pattern, as discussed in Section 7.

Figure D.25: Concentration of US and Soviet Union across IPC3, 1991-2000



Taking into account selection changes the Russian overall contribution to output. Table 9 follows from Table 7 which used citations, but restricts attention to patents that have an associated stock market value (Kogan et al., 2017). Note again that there is a fairly close match to the predicted and realized in-sample exercise, where the selection on the types arriving to the US shapes the overall contribution.

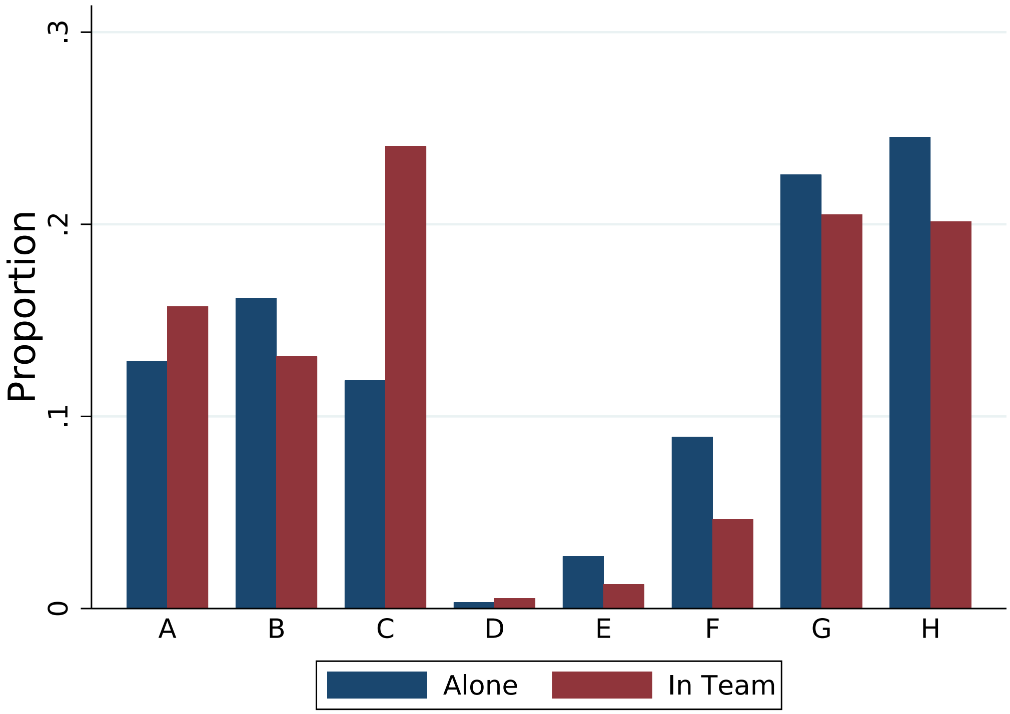
Table 9: Stock market value of migrating Russian output, 1995-2005

Measure	Δ Agg. Innov ($)
— Panel A. Innovation in US —	
sole-authored innovation (Q)	10.8B
model predicted (incl. teams) (Q)	33.4B
— Panel B. Predicted innovation from SU-US Match —	
predicted, from SU data, no selection (Q)	26.2B
predicted, from SU data, selection at T50 (Q)	32.5B

*Market value of patents (in sample) from 1995-2005 is 3.92T

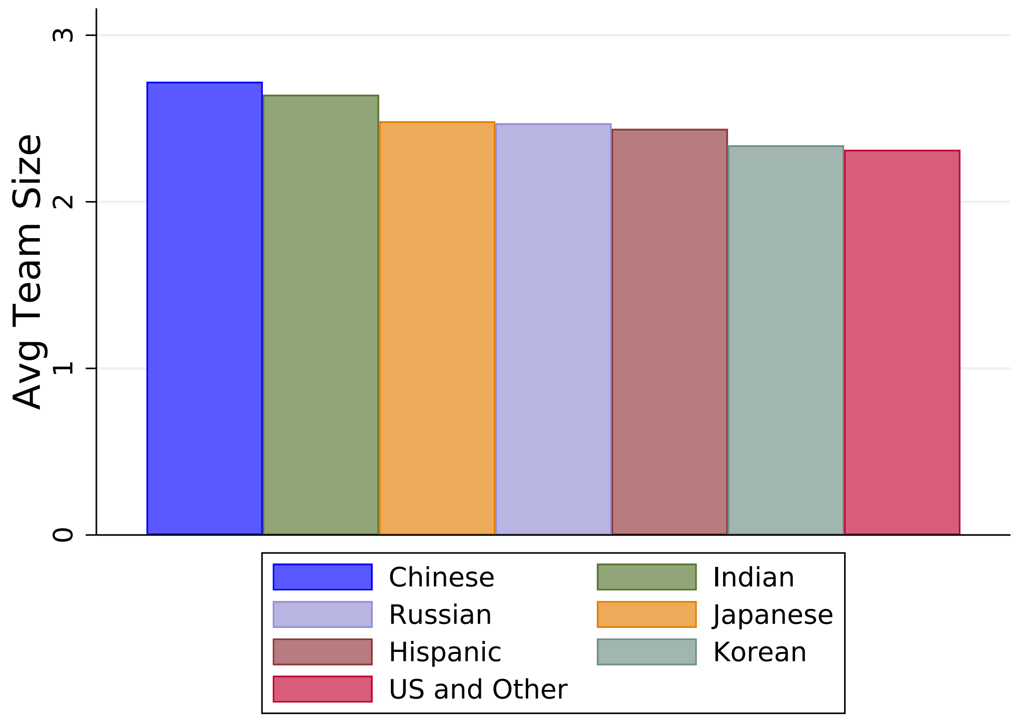
Figure D.26 illustrates that Russians contributed to certain classes in teams, while they contributed to other classes more often alone. In particular, Russians were not heavily concentrated in chemistry (C) alone, while they were concentrated in classes related to physics and electricity (G,H). Given the fact that chemistry is often pulling in large teams with varied expertise and the Soviet Union had low representation in this field, this is not surprising:

Figure D.26: Russian Innovations Alone and in Teams



Lastly, I note the high contributions to teams from Russians and other ethnicities that came to the US during the 1990s. Figure D.27 suggests that noting the importance of immigrants in teams is especially salient:

Figure D.27: Avg Team Size by Ethnicity in the US, 1990-2005

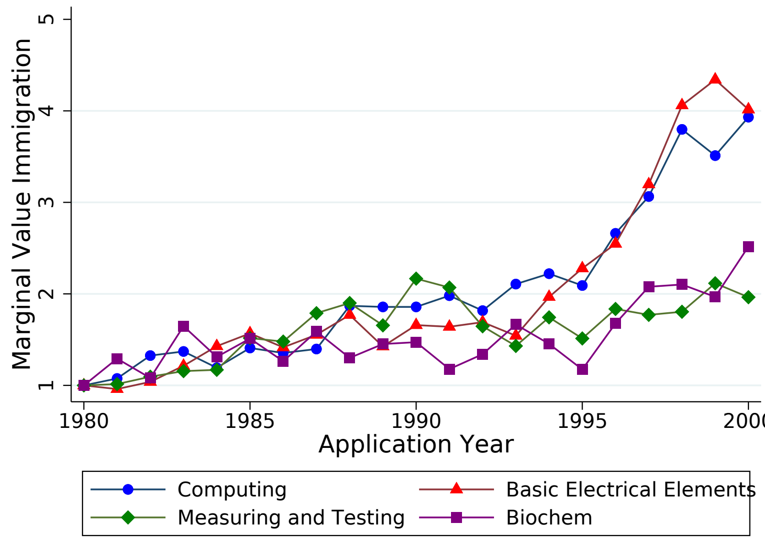


D.3 Changes in Teams and Immigration Policy: Further Exploration

The idea production function and communication costs push in opposite directions to impact change in aggregate innovation that results from immigrant inflows. First, an addition of a worker to a country is of higher value because of their ability to contribute to teams. This would suggest immigration policy is becoming more important and it is crucial to link immigrants into the global market. Second, because international collaboration is increasing, it is less important to bring immigrants directly into the home country. Thus, it is a quantitative question of what is the dominant force to consider when designing immigration policy.

Figure D.28 expresses the intuition on the rising importance of specifically valuable skills:

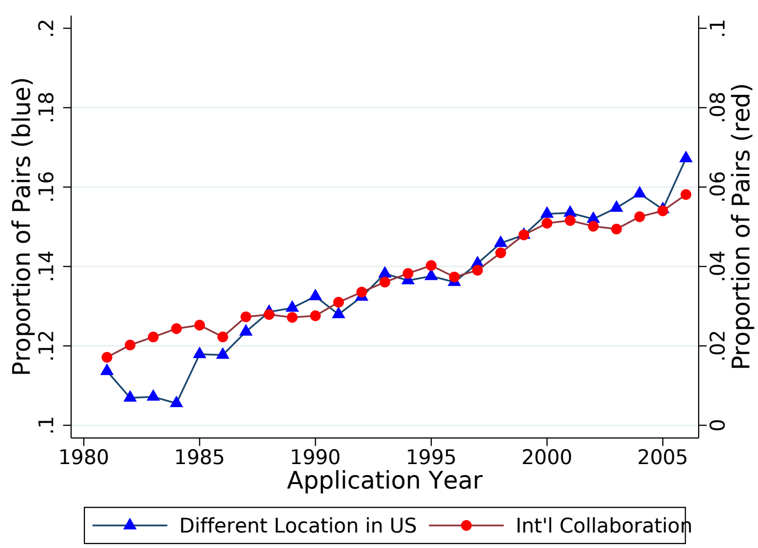
Figure D.28: Normalized Marginal Val. Immigration



Note: Falling coordination costs *not* included

Note that inventors of various skills are seeing changes in their value because of the rise of teams. With communication costs, this is more salient across all groups, as international collaboration expands. This can be seen in Figure D.29:

Figure D.29: Co-Inventors across locations (2-person teams)



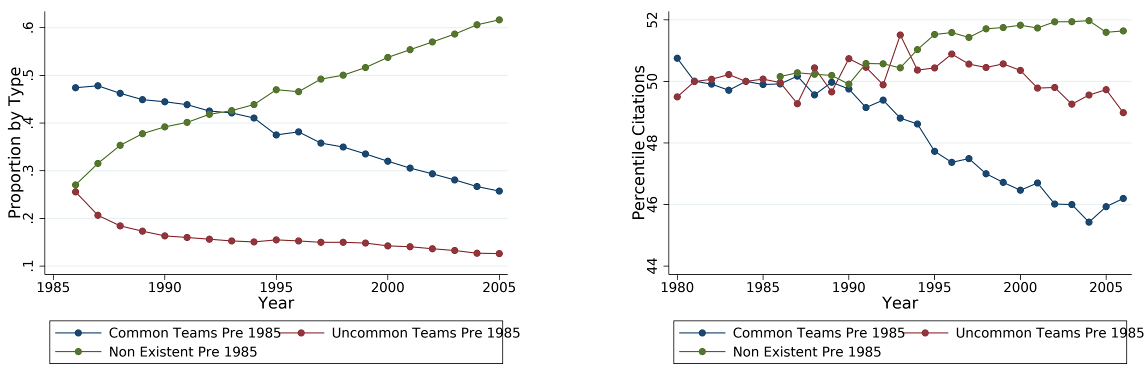
Different location: not within 100 miles by straight line

The decomposition enables us to stress that while communication costs are making cross-national collaboration easier, the changing idea production function keeps the premium on having the right immigration policy high.

D.4 Horizontal Differentiation in Teams

Figure D.30 plots the amount of "new" teams in as in new combinations of specific expertise since 1986. These teams are becoming much more common, as skill combinations are changing over time. Further, the impact of their patents are higher than teams in the past, suggesting a shifting value of expertise across various domains.

Figure D.30: Team Types Evolution



(a) Proportion of New + Old teams

(b) Citation percentile by New v. Old

Table 10 notes the most common types within the USPTO classification, and their most common partner.

Table 10: Most common types in the data: USPTO

	(1)	(2)
specialization:	m_n	$x^*_m(i) \neq x_n$
drug, applied bio-treatment	0.037	organic compounds
semiconductors: process	0.024	solid-state device/transistors
chem: molecular and micro-bio	0.023	drug, bio-treatment
stock material	0.020	synthetic resins
solid-state device/transistors	0.019	semiconductors
all	1	

Table 11 plots the largest growth in team size with classifications according to USPTO patent classes. I find the largest growth in what were initially more distant types

Table 11: Growth in most common matches in the data: USPTO

	(1)	(2)
specialization:	$x_m^*(i) \neq x_n$	explanation
semiconductors	abrading	*semiconductor advances*
pulse or digital communications	image analysis	*rise of communication by image*
computer graphics	digital processing: memory	*graphics integrated with memory*
semiconductors	measuring and testing	*semiconductor advances*
all	1	

D.5 Rank of Expertise

Here, I rank expertise by their contribution to aggregate innovation and notes their partial equilibrium rank. The general equilibrium rank is based on increasing the supply of the given type by 0.01 and observing the corresponding patterns across teams. The partial equilibrium rank relies on Proposition 3. Table 12 details the overall ranking, using data from 1991-2000:

Table 12: A Rank of Expertise Across IPC3 Categories

Expertise Type	**GE Rank**	**PE Rank**	**Proportion of Pop.**
CHECKING-DEVICES	1	1	0.0048663
MEDICAL OR VETERINARY SCIENCE;	2	2	0.1254494

COMPUTING; CALCULATING; COUNTI	3	5	0.1081146
ELECTRIC COMMUNICATION TECHNIQ	4	6	0.0671355
SIGNALLING	5	8	0.0042086
BASIC ELECTRIC ELEMENTS	6	11	0.0856206
MUSICAL INSTRUMENTS; ACOUSTICS	7	14	0.0054672
CONTROLLING; REGULATING	8	13	0.005233
BIOCHEMISTRY; BEER; SPIRITS; W	9	7	0.0121443
EARTH OR ROCK DRILLING; MINING	10	15	0.0095658
COMBINATORIAL TECHNOLOGY [2006	11	9	0.0001055
INFORMATION STORAGE	12	18	0.0239818
OPTICS	13	10	0.0167979
NANOTECHNOLOGY [7]	14	22	0.0032794
ELECTRIC TECHNIQUES NOT OTHERW	15	25	0.0059509
PAPER-MAKING; PRODUCTION OF CE	16	31	0.0022594
DYES; PAINTS; POLISHES; NATURA	17	16	0.0059914
MEASURING; TESTING	18	24	0.0511502
GENERATION, CONVERSION, OR DIS	19	32	0.0129695
EDUCATING; CRYPTOGRAPHY; DISPL	20	34	0.0070125
BASIC ELECTRONIC CIRCUITRY	21	37	0.0192122
PRINTING; LINING MACHINES; TYP	22	26	0.0098581

VEHICLES IN GENERAL	23	52	0.0255578
CRYSTAL GROWTH [3]	24	40	0.0006626
GRINDING; POLISHING	25	48	0.0045533
LAYERED PRODUCTS	26	30	0.0043826
COATING METALLIC MATERIAL; COA	27	12	0.0049353
ORGANIC MACROMOLECULAR COMPOUN	28	17	0.0230767
ELECTROLYTIC OR ELECTROPHORETI	29	20	0.00171
ANIMAL OR VEGETABLE OILS, FATS	30	23	0.0028558
SPORTS; GAMES; AMUSEMENTS	31	51	0.0189342
BRAIDING; LACE-MAKING; KNITTIN	32	43	0.0013564
PHYSICAL OR CHEMICAL PROCESSES	33	44	0.0159891
FOOTWEAR	34	45	0.0020525
LIGHTING	35	46	0.0039711
DISPOSAL OF SOLID WASTE; RECLA	36	47	0.0006133
MACHINES OR ENGINES IN GENERAL	37	67	0.0051312
CEMENTS; CONCRETE; ARTIFICIAL	38	27	0.0028613
WORKING OF PLASTICS; WORKING O	39	55	0.0090339
COMBUSTION ENGINES; HOT-GAS OR	40	64	0.008281
GLASS; MINERAL OR SLAG WOOL	41	49	0.0022911
SEPARATING SOLIDS FROM SOLIDS;	42	59	0.0009044

INORGANIC CHEMISTRY	43	19	0.0022462
EXPLOSIVES; MATCHES	44	38	0.0006276
POSITIVE-DISPLACEMENT MACHINES	45	63	0.0044701
NATURAL OR MAN-MADE THREADS OR	46	42	0.0010456
PHOTOGRAPHY; CINEMATOGRAPHY; A	47	33	0.0138867
NUCLEAR PHYSICS; NUCLEAR ENGIN	48	41	0.0009646
PETROLEUM, GAS OR COKE INDUSTR	49	36	0.0049342
SEPARATION OF SOLID MATERIALS	50	53	0.000911
TREATMENT OF WATER, WASTE WATE	51	56	0.0041637
ORGANIC CHEMISTRY [2]	52	21	0.030416
TOBACCO; CIGARS; CIGARETTES; S	53	88	0.0007315
OPENING OR CLOSING BOTTLES, JA	54	72	0.0015863
RAILWAYS	55	82	0.0016563
HOROLOGY	56	99	0.0005717
STORING OR DISTRIBUTING GASES	57	54	0.0004174
SPRAYING OR ATOMISING IN GENER	58	60	0.0053599
AIRCRAFT; AVIATION; COSMONAUTI	59	81	0.0033549
SUGAR INDUSTRY [4]	60	4	0.0000672
COMBUSTION APPARATUS; COMBUSTI	61	84	0.0026019
FOODS OR FOODSTUFFS; THEIR TRE	62	50	0.0050688

POSITIVE-DISPLACEMENT MACHINES	63	61	0.0001329
REFRIGERATION OR COOLING; COMB	64	57	0.0048784
HEATING; RANGES; VENTILATING	65	85	0.0028558
METALLURGY; FERROUS OR NON-FER	66	69	0.0019474
CONVEYING; PACKING; STORING; H	67	86	0.0237935
HABERDASHERY; JEWELLERY	68	109	0.0014232
MAKING ARTICLES OF PAPER, CARD	69	62	0.0010686
LIFE-SAVING; FIRE-FIGHTING	70	79	0.0015578
AMMUNITION; BLASTING	71	65	0.0016049
LAND VEHICLES FOR TRAVELLING O	72	93	0.0065091
BOOKBINDING; ALBUMS; FILES; SP	73	80	0.0023841
LOCKS; KEYS; WINDOW OR DOOR FI	74	92	0.0041538
CENTRIFUGAL APPARATUS OR MACHI	75	58	0.0004109
MACHINE TOOLS; METAL-WORKING N	76	76	0.007829
ENGINEERING ELEMENTS OR UNITS;	77	87	0.0204665
WORKING CEMENT, CLAY, OR STONE	78	77	0.0005291
DOORS, WINDOWS, SHUTTERS, OR R	79	91	0.0026742
GENERATING OR TRANSMITTING MEC	80	28	0.0001745
WEAPONS	81	83	0.0040893
AGRICULTURE; FORESTRY; ANIMAL	82	102	0.019893

MICROSTRUCTURAL TECHNOLOGY [7]	83	35	0.0000234
TREATMENT OF TEXTILES OR THE L	84	66	0.0022014
BRUSHWARE	85	70	0.0011934
HYDRAULIC ENGINEERING; FOUNDAT	86	96	0.0031185
BUILDING	87	95	0.0086575
CLEANING	88	68	0.0015983
HEAT EXCHANGE IN GENERAL	89	73	0.0010949
DRYING	90	74	0.00061
FURNITURE; DOMESTIC ARTICLES O	91	94	0.0200035
WEARING APPAREL	92	97	0.0029478
WORKING OR PRESERVING WOOD OR	93	98	0.0012459
CONSTRUCTION OF ROADS, RAILWAY	94	100	0.0023042
WATER SUPPLY; SEWERAGE	95	116	0.0017877
DECORATIVE ARTS	96	107	0.0008333
FERTILISERS; MANUFACTURE THERE	97	75	0.0004119
CASTING; POWDER METALLURGY	98	78	0.0020875
HAND TOOLS; PORTABLE POWER-DRI	99	111	0.0058863
METALLURGY OF IRON	100	71	0.0008432
WEAVING	101	103	0.0003222
STEAM GENERATION	102	39	0.0002565

YARNS; MECHANICAL FINISHING OF	103	114	0.0001843
BAKING; EQUIPMENT FOR MAKING O	104	90	0.0006965
HAND CUTTING TOOLS; CUTTING; S	105	104	0.0019912
HOISTING; LIFTING; HAULING	106	108	0.0021215
FLUID-PRESSURE ACTUATORS; HYDR	107	101	0.0009581
CRUSHING, PULVERISING, OR DISI	108	118	0.0012142
HEADWEAR	109	119	0.0009187
HAND OR TRAVELLING ARTICLES	110	117	0.0046036
MECHANICAL METAL-WORKING WITHO	111	106	0.0036701
BUTCHERING; MEAT TREATMENT; PR	112	120	0.0008246
MACHINES OR ENGINES FOR LIQUID	113	89	0.000169
SEWING; EMBROIDERING; TUFTING	114	105	0.0005597
SKINS; HIDES; PELTS; LEATHER	115	29	0.0000376
WRITING OR DRAWING IMPLEMENTS;	116	112	0.0007217
PRESSES	117	115	0.0004371
SHIPS OR OTHER WATERBORNE VESS	118	121	0.0038879
FURNACES; KILNS; OVENS; RETORT	119	113	0.0003299
SADDLERY; UPHOLSTERY	120	122	0.0002007
ROPES; CABLES OTHER THAN ELECT	121	110	0.0000464
ROPES; CABLES OTHER THAN ELECT	122	3	0.0000147