

Financial Regulation in a Quantitative Model of the Modern Banking System*

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Abstract

How does the shadow banking system respond to changes in the capital regulation of commercial banks? We propose a tractable, quantitative general equilibrium model with regulated and unregulated banks to study the unintended consequences of regulatory policy. Tightening the capital requirement from the status quo creates a safer banking system despite more shadow banking activity. A reduction in aggregate liquidity provision decreases the funding costs of all banks, raising profits and reducing risk-taking incentives. Calibrating the model to data on financial institutions in the U.S., we find the optimal capital requirement is around 15%.

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1 Introduction

The regulation of a complex system, such as the financial sector, may have unintended consequences that can jeopardize the goals of government policies. If regulated financial firms are competing against unregulated firms that provide similar services or products, tighter regulation can cause a shift to the unregulated sector and thus potentially can produce more financial instability. For example, Regulation Q, a cap on deposit rates introduced after the Great Depression that curbed excessive competition for deposit funds, was thought to have weakened the banking system. As long as interest rates remained low, savers had little incentive to pull their funds out of the traditional banking system. But as soon interest rates rose, depositors looked for alternatives and the competition for their savings generated one: money market mutual funds (Adrian and Ashcraft (2016)). This and numerous other examples¹ highlight the potential side-effects of regulatory policies.²

In this paper, we study and quantify the effects of capital requirements in a general equilibrium model featuring regulated (commercial) and unregulated (shadow) banks. Tightening the capital requirement on commercial banks can shift activity to shadow banks, potentially increasing the fragility of the entire financial system. Calibrating the model to aggregate data from the Flow of Funds as well as the Federal Reserve, Moody's, and the FDIC, we find that higher capital requirements indeed shift activity away from traditional banks. However, instead of becoming more fragile, the aggregate banking system becomes safer. Moreover, overall production is not harmed by tighter regulation. These last two effects are robust to different specifications. The optimal requirement achieves the welfare maximizing balance between a reduction in liq-

¹Asset-backed commercial paper conduits are another example for entities that emerged arguably as a response to regulation, more precisely capital regulation (see Acharya, Schnabl, and Suarez (2013)).

²We define shadow banks as financial institutions that share features of depository institutions, either by providing liquidity services such as money market mutual funds or by providing credit either directly (e.g. finance companies) or indirectly (e.g. security-broker and dealers). At the same time, they are not subject to the same regulatory supervision as traditional banks. Importantly, we adopt a consolidated view of the shadow banking sector, i.e. money market mutual funds invest in commercial paper that fund security brokers-dealers that provide credit. We view this intermediation chain as essentially being carried out by one intermediary. In our definition of shadow banks, we abstract from shadow banks as a form of commercial banks' off-balance sheet vehicles as our focus is on liquidity provision of banks.

uidity services and the increase in the safety of the financial sector and consumption. Welfare is maximized at a capital requirement of roughly 15%.

Our study considers a production economy consisting of households, commercial banks, shadow banks, and a regulator.³ The main features of our model are heterogeneous banks that make productive investments, have the option to default, and differ in their ability to guarantee the safety of their liabilities. At the same time, risk-averse households have preferences for safe and liquid assets in the form of bank liabilities whose liquidity value depends on their safety. The consumption good is produced with two technologies: one that households access directly through their ownership of a tree that stochastically produces an endowment of the good, and a second stochastic production technology, operated by banks that uses capital owned by these intermediaries and labor rented from households. Banks also possess an investment technology and compete over capital shares. Their assets are funded by issuing equity and debt to households. When either type of bank defaults on its debt, its equity becomes worthless and a fraction of the remaining bank value is destroyed in bankruptcy.

Since we assume the government only randomly bails out shadow banking debt, this debt is risky for households. Moreover, our model postulates that a fraction of depositors may cause a run on shadow bank debt⁴ forcing these banks to sell off a fraction of their assets in a fire-sale. This fraction is endogenous and depends negatively on the fire sale discount and positively on shadow bank leverage. Thus, shadow bank leverage increases not only the default probability of shadow banks but also the severity of bank runs during which their assets are less productive. Shadow bank debt is, thus, risky for investors, which they take into account. That is, the price at which shadow banks issue debt reflects the default probability implied by their leverage choice, up to the random bailout chance. Shadow banks internalize this trade-off and limit their leverage endogenously.

In contrast, commercial bank debt is insured and therefore always safe. However, because

³Figure 1 gives an overview of the model.

⁴We model runs as episodes of early withdrawals of funds following Diamond and Dybvig (1983) and Allen and Gale (1994).

commercial banks do not internalize their higher leverage, costly bankruptcies are more likely. Therefore, they do not account for the social costs of higher leverage in the form of greater bankruptcy losses that the government imposes on households through lump sum taxation.

In an economy without shadow banks, the main welfare trade-off of higher capital requirements weighs a reduction in liquidity provision against an increase in consumption due to lower deadweight losses and improved funding conditions as well as less fragility.⁵ In an economy with shadow banks, higher capital requirements, in principle, could raise financial fragility due to a shift of intermediation from regulated to unregulated banks.

So what are the effects in the economy with unregulated banks? Our findings suggest that the reduced liquidity production by commercial banks following tighter regulation increases the attractiveness of all types of bank liabilities and, hence, reduces the funding costs for all banks. While commercial banks are constrained from issuing more money-like assets, shadow banks are not. Instead, nonregulated banks have two options to meet the heightened liquidity demand: (i) increase leverage or (ii) increase their size, keeping leverage constant. If the unregulated sector were to choose (i), the financial sector would become more fragile. However, we find that shadow banks choose the second option mainly because the increase in profitability of lower funding costs lowers risk-taking incentives. Thus, instead of becoming riskier, the unregulated sector responds to higher capital requirements by increasing its size. For this reason, both the share of shadow intermediation as well as the amount of intermediated physical capital increases.⁶

Our research demonstrates that as long as households price bank debt rationally, higher capital requirements do not lead to more fragility by shifting activity to the unregulated sector. Moreover, we find that both the increased demand for collateral and, therefore, physical capital from shadow banks as well as improved funding conditions for all banks prevent the economy from suffering a decline in lending. These two key effects of higher capital requirements are

⁵Commercial banks reduce liquidity provision because higher capital requirements increase their cost of funds since at the margin banks prefer debt financing over equity financing due to the liquidity premium.

⁶We find that in this sense higher capital requirements move the economy closer to the constrained efficient equilibrium in which the planner chooses a 65% higher capital stock. This reduces the risk of the financial sector as banks are better collateralized. Additionally, it encourages more liquidity services.

qualitatively robust to different specifications and parametrization. Quantitatively, our results depend on household preferences for liquidity and the relative riskiness of shadow banks and commercial banks. Liquidity is affected by three key parameters: the share of liquidity services relative to consumption, the elasticity of substitution between shadow and commercial bank debt, and the sensitivity of the liquidity quality of shadow banking debt with regard to the default rate in the shadow banking sector. We infer these parameters from an estimate for the convenience yield of government debt from Krishnamurthy and Vissing-Jorgensen (2012) that we apply to commercial banking debt, the share of shadow bank activity (1/3) estimated by Gallin (2015), and the rate on shadow bank debt for which we use the 3-month rate on AA-rated financial commercial paper.

In our model, the relative riskiness depends on the random bailout probability and the riskiness of banks' investment opportunities. Since the bailout probability lowers the incentives of shadow banks to internalize the risk of their leverage choice, we infer the first parameter from the value of the weighted-average leverage of shadow banks using Compustat data, where the shadow bank definition includes security broker dealers, GSEs, finance- and investment companies. We target a roughly 1% annual charge-off rates on commercial bank loans and the cross-sectional volatilities of shadow and commercial banks' market-to-book ratios from Compustat data.

The precise welfare gain of higher capital requirements is decreasing as the bailout probability of shadow banks increases, because a higher bailout chance makes shadow banks relatively more risky, resulting in only a limited gain from shifting intermediation activity to unregulated banks.

Besides studying the consequences of a static requirement, we analyze the effects of a fair deposit insurance fee as well as time-varying capital requirements. We find that a greater deposit insurance fee for commercial banks does not lead to a safer financial system. While a higher fee shifts activity to shadow banks, it does not – unlike the capital requirement – make commercial banks sufficiently safer. Finally, our model demonstrates that the time-varying capital requirement is as successful as the static requirement. It is set such that the expected

default rate of commercial banks is lower than 10 basis point per quarter. It has a mean of 13% and is tighter during booms, in particular during financial sector booms.

Related Literature

This paper contributes to the growing literature at the intersection of macroeconomics and banking that investigates the optimal regulation of banks in a quantitative general equilibrium framework.⁷ Our modeling approach draws on the recent literature on the role of financial intermediaries in the macroeconomy.⁸ Like this research, our paper studies economies with assets that investors can access only through an intermediary. The wealth of the intermediary then emerges as an additional state variable driving asset prices and the dynamics of the economy. By introducing limited liability and deposit insurance and defining the role of banks as liquidity producers, we bridge the gap to a long-standing microeconomic literature on the function of banks in the economy. Several recent papers in this literature study the interaction of various types of banks subject to different degrees of regulation and different bailout guarantees.⁹

Our paper is most closely related to Moreira and Savov (2017), Huang (2016), and Gertler, Kiyotaki, and Prestipino (2016). Moreira and Savov (2017) study an intermediary asset pricing economy with two types of assets that differ in suitability as collateral for issuing safe and liquid liabilities (money). They demonstrate that the presence of shadow banks can lead to increased economic volatility as rational investors try to determine the liquidity of the debt issued by the financial sector. In contrast, Huang (2016) models shadow banks as an off-balance-sheet financing option for regular banks within the Brunnermeier and Sannikov (2014) framework. His study suggests financial stability is a U-shaped function of financial regulation (i.e. very tight regulation generates more off-balance activities). Gertler, Kiyotaki, and Prestipino (2016) construct a quantitative macro-finance framework with a role for both regular banks (retail)

⁷E.g. Begeau (2016), Christiano and Ikeda (2014), Elenev, Landvoigt, and Van Nieuwerburgh (2016), Gertler, Kiyotaki, and Prestipino (2016). See Nguyen (2014) and Corbae and D’Erasmus (2017) for quantitative models in partial equilibrium.

⁸E.g. Brunnermeier and Sannikov (2014), He and Krishnamurthy (2013), Garleanu and Pedersen (2011), Moreira and Savov (2017).

⁹E.g. Goodhart, Kashyap, Tsomocos, and Vardoulakis (2012), Gennaioli, Shleifer, and Vishny (2013), Plantin (2015), Harris, Opp, and Opp (2014), Klimenko, Pfeil, Rochet, and De Nicolò (2016)

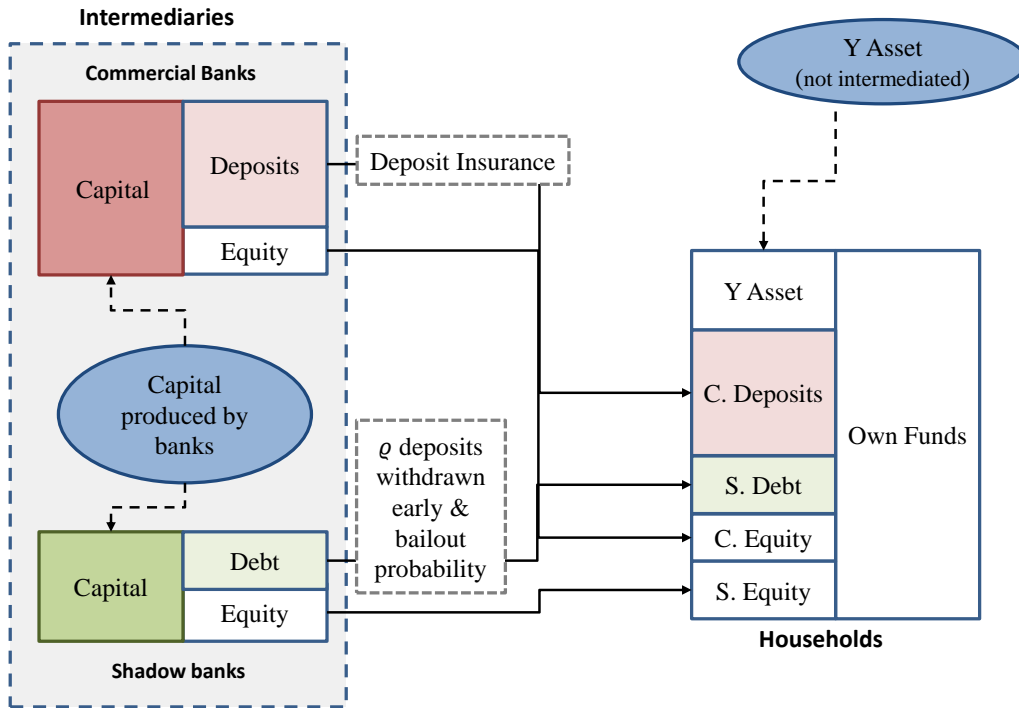


Figure 1: Model Overview

and shadow banks (wholesale). Since bank runs are endogenous in their model, it captures important features of the recent financial crisis. Our definition of banks is closely related to Gertler, Kiyotaki, and Prestipino (2016). However, a key difference from other work is our focus on liquidity provision as a fundamental role of banking and moral hazard arising endogenously from deposit insurance and limited liability.

The paper is structured as follows. Section 2 describes the model and Section 3 the main mechanism. Section 4 presents the calibration and the policy experiments.

2 A Model of the Banking System

First, we present the basic structure of the general equilibrium model economy (Figure 1 provides a quick overview of the model). We discuss the key assumptions of the model in Section 2.1.

Agents and Environment Time is discrete and infinite in the model. The economy possesses one consumption good produced via two different production technologies : (1) a tree that delivers stochastic income Y_t directly to households, and (2) a Cobb-Douglas production technology operated by banks in which labor and capital are combined to produce the consumption good. That is, both types of bank compete in terms of the consumption good. At the start of each period, bank production is exposed to an aggregate productivity shock Z_t . Capital is produced by financial intermediaries. We structure the model so that the claims to financial intermediaries are owned by households who, otherwise, have no direct access to the economy's capital stock, in order to capture the idea that households use banks to finance durable goods, such as housing and cars, and many non-financial firms rely on bank finance.

S-Banks There is a unit mass of S -banks indexed by i and each S -bank i holds $K_{t,i}^S$ units of capital at the beginning of period t . Capital trades at a market price of p_t . To fund their investment, S -banks can issue short term debt where the debt of S -bank i trades at price $q_{t,i}^S$. At the beginning of the period, S -bank i has $B_{t,i}^S$ bonds outstanding.

Banks operate a Cobb-Douglas production technology to produce output

$$Z_t (K_{t,i}^S)^{1-\eta} (N_{t,i}^S)^\eta,$$

using labor supplied by households at market wage w_t and capital. In addition, banks operate an investment technology and sell and buy capital in a frictionless market. They buy the consumption good and turn it into units of capital that are sold at price p_t . To generate $I_{t,i}^S$ units of capital, banks need to spend $I_{t,i}^S + \Phi(I_{t,i}^S, K_{t,i}^S)$ units of the consumption good, with

$$\Phi(I_{t,i}^S, K_{t,i}^S) = \frac{\phi}{2} \left(\frac{I_{t,i}^S}{K_{t,i}^S} - \delta_K \right)^2 K_{t,i}^S.$$

At the beginning of each period, the aggregate productivity shock Z_t is realized and all banks produce and invest. The profits generated by the two lines of S -bank business (production and investment) are therefore

$$\hat{D}_{t,i}^S = Z_t (K_{t,i}^S)^{1-\eta} (N_{t,i}^S)^\eta + (1 - \delta_K)p_t K_{t,i}^S - w_t N_{t,i}^S + I_{t,i}^S (p_t - 1) - \frac{\phi}{2} \left(\frac{I_{t,i}^S}{K_{t,i}^S} - \delta_K \right)^2 K_{t,i}^S. \quad (1)$$

Given capital $K_{t,i}^S$, S-banks choose labor input $N_{t,i}^S$ and investment $I_{t,i}^S$ to maximize (1). Note that the profit also includes the proceeds from selling depreciated capital after production, $(1 - \delta_K)p_t K_{t,i}^S$.

Using the usual properties of the constant returns production technology, we write the profit in Equation (1) as

$$\hat{D}_{t,i}^S = \Pi_{t,i}^S K_{t,i}^S,$$

where the marginal value of a unit of capital held by bank i at the beginning of the period (derived in Appendix Section A.1) is

$$\Pi_{t,i}^S = (1 - \eta)Z_t(n_{t,i}^S)^\eta + p_t - \delta_K + \frac{(p_t - 1)^2}{2\phi}, \quad (2)$$

with $n_{t,i}^S = N_{t,i}^S / K_{t,i}^S$. The expression for $\Pi_{t,i}^S$ already includes the optimal labor and investment choice of banks.

After production, S -banks face idiosyncratic payoff shocks $\rho_{t,i}^S$ that are proportional to the gross value of S -bank assets $\hat{D}_{t,i}^S$, with $\rho_{t,i}^S \sim F_\rho^S$ and $E(\rho_{t,i}^S) = 1$, i.i.d. across banks and time. After observing $\rho_{t,i}^S$, S -banks can decide whether to declare bankruptcy. In case of a bankruptcy, banks' equity is wiped out, and their assets are seized by their creditors.

Furthermore, bank managers incur an utility penalty $P(K_{t+i,i}^S)$ which is a function of the banks' assets.¹⁰

If the bank does not default, the total dividend paid to equity owners in period t is:

$$D_{t,i}^S = \rho_{t,i}^S \Pi_{t,i}^S K_{t,i}^S - B_{t,i}^S + q_{t,i}^S B_{t+1,i}^S - p_t K_{t+1,i}^S. \quad (3)$$

If banks decided against default, the present value of dividends paid to equity owners (households) is maximized by choosing new capital purchases $K_{t+1,i}^S$ as well as short term debt issuance

¹⁰The utility penalty is without loss of generality. For example, setting $P(K_{t+i,i}^S) = 0$ is the standard limited liability representation. When $P(K_{t+i,i}^S) > 0$, managers may act against the interest of shareholders and default at relatively low valuations.

$B_{t+1,i}^S$, using the household pricing kernel $M_{t,t+1}$ as a discount factor.

$$V^S(K_{t,i}^S, B_{t,i}^S, Z_t) = \max_{K_{t+1,i}^S, B_{t+1,i}^S} D_{t,i}^S + E_t [M_{t,t+1} \max \{V^S(K_{t+1,i}^S, B_{t+1,i}^S, Z_{t+1}), -P(K_{t+1,i}^S)\}] \quad (4)$$

subject to Equation (3) and no-shorting constraints $K_{t+1}^S \geq 0$, $B_{t+1}^S \geq 0$. In appendix section A.1 we show that the optimization problem in Equation (4) can be recast as a sequence of static problems that only depend on the aggregate state of the economy. That is, individual bank net worth is not a state variable, because we have assumed no equity-related frictions that prevent banks from recapitalizing at the beginning of each period. Since the idiosyncratic payoff shocks are uncorrelated over time, all banks have the same value at the end of each period, i.e. identical problems allow us to drop the i -subscript. We choose a linear form for the default penalty function

$$P(K_t^S) = \delta_S \Pi_t^S K_t^S, \quad (5)$$

where the parameter $\delta_S \geq 0$ governs the magnitude of the penalty. Given Equation (5), the bank problem is homogenous of degree one in capital. Thus, we can simplify the problem further by defining debt per unit of capital purchased $b_{t+1}^S = B_{t+1}^S / K_{t+1}^S$, and the market value of bank leverage as

$$L_t^S = \frac{B_t^S}{\Pi_t^S K_t^S} = \frac{b_t^S}{\Pi_t^S}. \quad (6)$$

Using these definitions, we show in Appendix Section A.1 that the intertemporal problem of S-banks can be written in terms of the value function $v^S(Z_t)$ that is expressed in quantities per unit of capital, and only depends on the exogenous aggregate state

$$v^S(Z_t) = \max_{b_{t+1}^S \geq 0} q_t^S b_{t+1}^S - p_t + E_t [M_{t,t+1} \Pi_{t+1}^S \mathcal{F}^S(L_{t+1}^S)], \quad (7)$$

with

$$\mathcal{F}^S(L_{t+1}^S) = \left[(1 - F_\rho^S(L_{t+1}^S - \delta_S)) (\rho_{t+1}^{S,+} - L_{t+1}^S) - F_\rho^S(L_{t+1}^S - \delta_S) \delta_S \right]$$

where $\rho_t^{S,+} = E(\rho_t^S | \rho_t^S > L_t^S - \delta_S)$ is the expected idiosyncratic shock conditional on not defaulting, $F_\rho^S(L_t^S - \delta_S)$ is the probability of default, and $\mathcal{F}^S(L_t^S)$ is the leverage-adjusted payoff

of S-banks' portfolio, including the default option. The term before the expectation term represents the equity required to fund capital and investment, while the expectation term expresses the discounted net payoff of banks' portfolio (investment in capital) to equity holders. The choice of capital for the next period is then the solution to the problem

$$\max_{K_{t+1}^S} K_{t+1}^S v^S(Z_t), \quad (8)$$

subject to $K_{t+1}^S \geq 0$.

C-Banks The problem of C-banks is analogous to S-banks, but they differ from S-banks in three ways: (i) they issue short-term debt that is insured and risk free from the perspective of creditors, (ii) they are subject to regulatory capital requirements, and (iii) they pay an insurance fee of κ for each bond they issue. Since the problem of all surviving C-banks is identical to S-banks, using the same notation as for S-banks, we can write

$$v^C(Z_t) = \max_{b_{t+1}^C} (q_t^C - \kappa)b_{t+1}^C - p_t k_{t+1}^C + E_t [M_{t,t+1} \Pi_{t+1}^C \mathcal{F}^C(L_{t+1}^C)], \quad (9)$$

and

$$\max_{K_{t+1}^C} K_{t+1}^C v^C(Z_t), \quad (10)$$

subject to $K_{t+1}^C \geq 0$ and the equity requirement

$$(1 - \theta)p_t \geq b_{t+1}^C. \quad (11)$$

Bankruptcy and Government Budget Constraint If a bank declares bankruptcy, its equity becomes worthless and creditors seize and liquidate all the bank's assets. The recovery amount per bond issued is

$$r^j(L_t^j) = (1 - \xi^j) \frac{\rho_t^{j,-} \Pi_t^j K_t^j}{B_t^j} = (1 - \xi^j) \frac{\rho_t^{j,-}}{L_t^j}, \quad (12)$$

for $j = S, C$, with a fraction ξ^j lost in the bankruptcy proceedings, and with

$$\rho_t^{j,-} = E(\rho_t^j | \rho_t^j < L_t^j - \delta_j).$$

After the bankruptcy proceedings are completed, a new bank is set up to replace the failed one. This bank sells its equity to new owners, and is otherwise identical to a surviving bank after asset payoffs. Since bankruptcy occurs after production and investment, these decisions are not affected by bankruptcy liquidation¹¹.

Bankruptcy losses $\xi^j \rho_t^j - \Pi_t^j K_t^j$ are real losses to the economy. They reflect both greater capital depreciation of foreclosed banks, and real resources destroyed in the bankruptcy process that reduce bank profits.

If a S -bank defaults, the recovery value per bond is used to pay the claims of bondholders to the extent possible. If a C -bank declares bankruptcy, the bank is taken over by the government which distributes lump sum taxes and revenues from deposit insurance κB_{t+1}^C to pay out the bank's creditors in full. Summing up over defaulting C -banks that are bailed out, we define lump sum taxes are

$$T_t = F_{\rho,t}^C (1 - r^C (b_t^C)) B_t^C - \kappa B_{t+1}^C.$$

Households Households derive utility from the consumption C_t of the fruit of the Y -tree and the fraction of output $Y_t^{B,S}$ and $Y_t^{B,C}$ that is not invested. They supply labor to the bank-dependent sector inelastically. Households hold a portfolio of securities issued by either kind of intermediary. In particular, they buy equity shares S_t^j of both bank types that trade at price p_t^j for $j = S, C$, respectively. They also purchase short-term S - and C -bank bonds A_{t+1}^j trading at prices q_t^j for $j = S, C$, respectively.

Our model assumes households consume the liquidity services provided by the short-term debt they hold at the beginning of the period. That is, the liquidity services accrue at the time when the deposits from last period are redeemed. Let $A_t^j = \int_0^1 A_{t,i}^j di$, for $j = S, C$. Total liquidity services produced are $H(A_t^S, A_t^C)$. We specify utility as

$$U(C_t, H(A_t^S, A_t^C)) = \frac{(C_t^{1-\psi} H_t^\psi)^{1-\gamma}}{1-\gamma},$$

¹¹We also assume that wage payments to households are unaffected.

with

$$H(A_t^S, A_t^C) = [\Lambda_{S,t} (A_t^S)^\alpha + (A_t^C)^\alpha]^{1/\alpha}.$$

The elasticity of substitution between the two types of bank liabilities is $1/(1 - \alpha)$.

The weight on the liquidity services of shadow banks $\Lambda_{S,t}$ is

$$\Lambda_{S,t} = (1 - F_{\rho,t}^S)^\nu,$$

with $\nu > 0$. Note that the weight $\Lambda_{S,t}$, the quality of S -banks' liquidity services, (i) lowers the liquidity productivity of shadow banks relative to commercial banks and (ii) is endogenously time-varying with the fraction of surviving shadow banks.

Households receive the dividends on bank equity shares. The beginning-of-period dividend paid to households, conditional on survival, for S -banks is

$$D_t^{S,+} = \rho_t^{S,+} K_t^S \Pi_t^S - B_t^S + K_{t+1}^S (q_t^S b_{t+1}^S - p_t),$$

and for C -banks is

$$D_t^{C,+} = \rho_t^{C,+} K_t^C \Pi_t^C - B_t^C + K_{t+1}^C ((q_t^C - \kappa) b_{t+1}^C - p_t).$$

Households are endowed with one unit of labor that they supply inelastically to C -banks and S -banks¹², earning a competitive wage w_t . Denoting household wealth at the beginning of the period by W_t , we define the intertemporal problem of households:

$$V^H(A_t^S, A_t^C, W_t, Y_t) = \max_{C_t, A_{t+1}^S, A_{t+1}^C, S_t^S, S_t^C} U(C_t, H(A_t^S, A_t^C)) + \beta \mathbf{E}_t [V(A_{t+1}^S, A_{t+1}^C, W_{t+1}, Y_{t+1})]$$

subject to

$$W_t + Y_t - T_t + w_t = C_t + \sum_{j=S,C} p_t^j S_t^j + \sum_{j=S,C} q_t^j A_{t+1}^j \quad (13)$$

$$\begin{aligned} W_{t+1} = & \sum_{j=S,C} (1 - F_\rho^j(L_{t+1}^j)) (D_{t+1}^{j,+} + p_{t+1}^j) S_t^j \\ & + A_{t+1}^S [1 - F_\rho^S(L_{t+1}^S) + F_\rho^S(L_{t+1}^S) r_{t+1}^S] \\ & + A_{t+1}^C. \end{aligned} \quad (14)$$

¹²In equilibrium, households are indifferent between working in either bank type.

The budget constraint in Equation (13) shows that households spend their wealth and income on consumption and purchases of equity and debt of both types of intermediaries. The securities issued are the same for all banks, independent of the previous bankruptcy status. Purchases of equities of banks that have gone through bankruptcy at the beginning of period t can be understood as initial offerings for these banks equity; the purchases of equity of surviving banks are transacted in a secondary market. However, since both new and surviving banks hold identical portfolios, their securities have the same price and there is no need to distinguish primary and secondary markets.¹³

Market Clearing There is market clearing for asset markets

$$\begin{aligned}
K_{t+1}^S + K_{t+1}^C &= I_t^S + I_t^C + (1 - \delta_K) \sum_{j=S,C} (1 - \xi^j F_{\rho,t}^j \rho_t^{j,-}) K_t^j & (15) \\
B_t^S &= A_t^S \\
B_t^C &= A_t^C \\
S_t^S &= 1 \\
S_t^C &= 1,
\end{aligned}$$

the goods market

$$C_t + \sum_j (I_t^j + \Phi(I_t^j, K_t^j)) = Y_t + Z_t \sum_j (N_t^j)^\eta (K_t^j)^{1-\eta} - \sum_j \xi^j F_{\rho,t}^j \rho_t^{j,-} (\Pi_t^j - (1 - \delta_K) p_t) K_t^j, \quad (16)$$

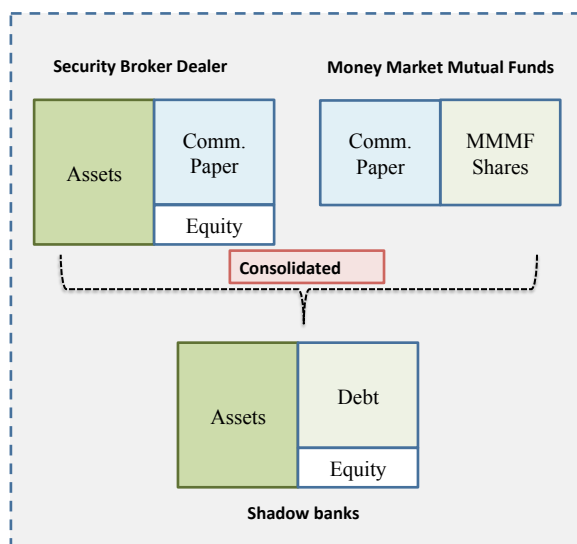
and the labor market

$$N_t^S + N_t^C = 1. \quad (17)$$

The market clearing condition for capital in Equation (15) is also the transition law for the aggregate capital stock. Note that bank failures lead to additional depreciation endogenously determined by the failure rates of banks $F_\rho^j(L_t^j)$. Similarly, condition (16) shows that bank failures also lead to a loss of resources in the goods market.

¹³It is easy to show that the price to an equity claim of bank types j , p_t^j , is equal to the value of that bank's security portfolio, $K_{t+1}^j p_t - q_t^j B_{t+1}^j$.

Figure 2: Shadow banks: sector consolidation



2.1 Discussion of key assumptions

Consolidated view of shadow banks We model shadow banks as consolidated entities (see Figure 2). In the data, many shadow banks have similarities to either the asset or the liability side of traditional banks. For instance, prime money market mutual funds typically hold commercial papers of financial institutions (e.g. security broker dealers, finance companies). Hence, in our consolidated view we generalize from frictions within the shadow banking sector.

Banks' Role as Intermediaries In the model, banks provide liquidity services (discussed below) help produce physical capital and a subset of the consumption good. The role of banks as intermediaries can be derived from first principles in various ways.¹⁴ Like Brunnermeier and Sannikov (2014), our banks own the production technology directly, that is, we abstract from any frictions between producers and banks.

¹⁴ For example, in models with asymmetric information between borrowers and lenders, lenders with access to a cheaper screening or monitoring technology than other lenders (regular households) become banks (see Freixas and Rochet (1998) for many other examples).

Bank Competition In our model, both bank types offer the same product (consumption good). They compete with each other in the product market, the capital market (i.e., producing and selling capital in the capital market can be interpreted as originating and selling loans), and in the deposit market. Our main result (see Section 4) is robust to other specifications, such as the assumption that only commercial banks can originate loans, as well as different degrees of substitutability in the deposit market.

The Role of Banks as Liquidity Providers Our model assumes households value and thus demand¹⁵ bank debt because it provides liquidity services¹⁶ and promises safe returns.¹⁷ Besides deposits, money-like assets include money market shares, commercial paper, repos, Treasuries, and high-grade financial sector corporate debt. For example, households, corporations, and investors hold deposits for transaction and liquidity reasons, and possess money market accounts for liquidity and safety. Aside from the government, financial intermediaries are the most important suppliers of these securities.¹⁸

Liquidity services in households preference We capture the idea that bank liabilities provide liquidity services with our utility specification. The households in our model represent a blend of different agents with demands for different types of safe and liquid assets (deposits, money market mutual fund shares, and so forth) provided by all financial institutions. Thus, our utility function aggregates the liquidity services of both bank types.

Commercial bank debt always provides liquidity services no matter their default probability.

In contrast, the value of a shadow bank's liquidity service depends on its probability of default.

¹⁵The savings glut hypothesis articulated in Bernanke (2005), first formalized by Caballero, Farhi, and Gourinchas (2008) and Mendoza, Quadrini, and Ríos-Rull (2009) and discussed in other recent work (e.g. Caballero and Krishnamurthy (2009), Gorton, Lewellen, and Metrick (2012), and Krishnamurthy and Vissing-Jorgensen (2012)) rests also on the notion that safe and liquid securities provide some economic functions agents demand.

¹⁶The idea to view banks as liquidity provider goes back to Diamond and Dybvig (1983). Other work has built upon this idea (e.g. Gorton and Pennacchi (1990)).

¹⁷Embedded in the notion of safety is the idea of deposits being not sensitive to information Gorton and Pennacchi (1990) and in Gorton, Lewellen, and Metrick (2012).

¹⁸Historically, money market mutual funds (a type of shadow bank) emerged precisely to satisfy demand for safe and liquid assets when Regulation Q imposed a ceiling on deposit rates.

This captures the idea that shadow bank debt is only safe as long as shadow banks are safe, that is, not too many of them go bankrupt.

We model the demand for liquidity services via a money-in-the-utility function specification. Since Sidrauski (1967), money-in-the-utility specifications have been used to represent the benefits of money-like securities for households in macro models. Feenstra (1986) proved the functional equivalence of models with money-in-the-utility and models with transaction or liquidity costs. Our functional form is a version of the money-in-the-utility-function specification of Poterba and Rotemberg (1986).

3 Bank Leverage and Relative Size of Shadow Banks

Banks' choice of their size (i.e. capital stock) and leverage are key to determining the effect of regulatory policies. This section characterizes the optimal leverage and scale choices of both types of banks. Appendix A.2 lists the full set of first-order conditions and their derivation.

C-bank Leverage Choice The leverage choice of C-banks is determined by their FOC for b_{t+1}^C and households' FOC for A_{t+1}^C , respectively:

$$\text{HH FOC: } q_t^C = E_t [M_{t+1} (1 + \text{MRS}_{t+1}^C)] \quad (18)$$

$$\text{C-bank FOC: } q_t^C = \lambda_t^C + \kappa + E_t [M_{t+1} (1 - F_{\rho,t+1}^C)], \quad (19)$$

where MRS_{t+1}^C is the marginal benefit of C-bank liquidity to households, defined in appendix A.2. Since households always can redeem C-bank deposits at face value, i.e. they are not directly affected by a C-bank default, the price of C-bank bonds q_t^C does not respond to changes in the default probability. Moreover, the liquidity benefit from C-bank deposits increases the value of deposits above their discounted payoff. These two features create a wedge in the pricing of C-bank bonds, leading C-banks to overproduce liquidity services, because banks do not internalize the effects of their capital structure choice on bankruptcy losses. In other words, with small enough κ , the capital requirement is always binding in the steady state, i.e. $\lambda^C > 0$. A binding

capital requirement means that C-banks fund a fraction $1 - \theta$ of each unit of capital with debt:

$$b_t^C = (1 - \theta)p_t.$$

S-bank Leverage Choice Analogously to C-banks, S-bank leverage is determined by banks' FOC for b_{t+1}^S and households' FOC for A_{t+1}^S , respectively:

$$\text{HH FOC: } q_t^S = \text{E}_t [M_{t+1} (1 - F_{\rho,t+1}^S + F_{\rho,t+1}^S r^S(L_{t+1}^S) + \text{MRS}_{t+1}^S)] \quad (20)$$

$$\text{S-bank FOC: } q_t^S = -b_{t+1}^S \frac{\partial q^S(b_{t+1}^S)}{\partial b_{t+1}^S} + \text{E}_t [M_{t+1} (1 - F_{\rho,t+1}^S)]. \quad (21)$$

Similarly to C-banks, the deposits of S-banks enjoy a liquidity benefit, the MRS_{t+1}^S term. However, unlike C-bank deposits, S-bank deposits are not redeemed at par when S-banks default but rather up to the recovery value of the bond. When households' expectation of S-bank default is high, the price of S-bank deposits decreases. S-banks internalize the effect of their capital structure choice on the default probability and, hence, on the price of debt. They issue debt until the marginal benefit of liquidity services is equal to the marginal cost of higher expected bankruptcy losses, a trade-off captured in Equation (21).¹⁹

While each shadow bank internalizes the trade-off between liquidity production and bankruptcy costs, it does not internalize how its leverage choice and default risk changes the *value* of liquidity services for households (the effect on Λ_t^S), a reasonable assumption if shadow bank bond prices are sensitive to the specific default risk of the issuer. Prices also move with changes in aggregate liquidity conditions, which are caused by actions of all shadow banks, but not by any individual bank. Thus, we can think of the changes in the value of aggregate liquidity services as an externality arising in general equilibrium.

Using the deterministic version of the model we can state the following proposition about the endogenous leverage choice of S-banks:

¹⁹In Appendix A.2, we show that the derivative $\frac{\partial q^S(b_{t+1}^S)}{\partial b_{t+1}^S}$ is always negative and therefore imposes an endogenous bound on S-bank leverage. See Equation (33).

Proposition. *Leverage L^S of S-banks is*

$$L^S = \frac{MRS_S}{\xi_S f^S} - \frac{1 - \xi_S}{\xi_S} \delta_S. \quad (22)$$

To prove this proposition, combine households' FOC with regard to S-bank debt (Equation (20)) with shadow banks' FOC (Equation (21)) and substitute for $\frac{\partial q^S(b^S)}{\partial b^S}$ using the steady state version of Equation (33). Shadow bank leverage rises with the marginal benefit of shadow bank liquidity (MRS_S), while it falls with the default penalty δ_S . The bankruptcy cost ξ_S reduces leverage.

Size of the Banking Sectors We continue using the steady state version of the model to describe how the relative size of both sectors is determined.

The optimal choice of capital share purchases K^j for each bank type $j = C, S$ imply

$$v^j(Z) = 0, \quad (23)$$

for any $K^j > 0$.

We can think of the conditions (23) as determining the relative size of the j -bank sector:

$$p - (q^C - \kappa)b^C = \beta \Pi^C \mathcal{F}^C, \quad (24)$$

$$p - q^S b^S = \beta \Pi^S \mathcal{F}^S. \quad (25)$$

The left sides of these equations represent the funds (per unit of capital) required today to purchase the portfolio held by each bank. The right sides are the expected discounted payoff in the next period. Intuitively, the left side is the cost to equity holders of starting up a bank of type j today, and the right hand side is the value of this investment next period. The key factors determining the relative cost of each type of bank today are the bond prices q^C and q^S . These prices, in turn, depend on the marginal benefit of liquidity provided by each type of bank, represented by the MRS_t^j terms in the bond pricing Equations (18) and (20). Given imperfect substitutability between both kinds of liquidity, the marginal benefit of each type of liquidity depends on the relative supply of bank debt, N^C versus N^S .

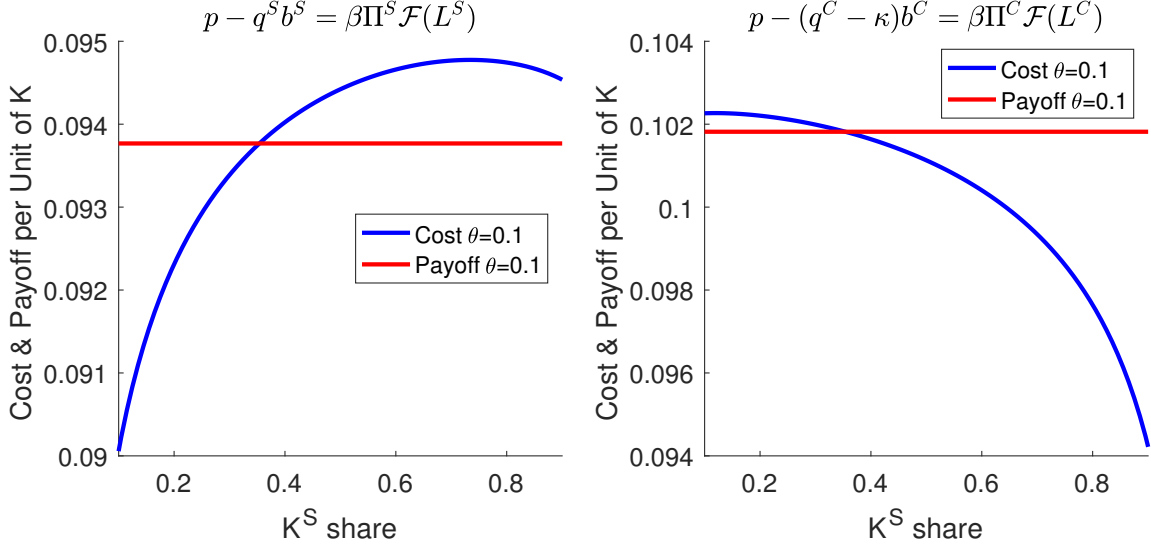


Figure 3: Equilibrium Determination of K^C and K^S

Left: marginal cost, i.e. required equity (blue curve) **Right:** marginal benefit (red curve) for asset holdings (K^j), while imposing market clearing $K = K^C + K^S$ and c.p.

Figure 3 illustrates how FOCs (24) and (25) effectively determine the scale of both banking sectors. The figure shows the LHS and RHS of both equations graphically for the calibrated model. We first numerically compute the equilibrium values of all variables, and then vary the size of S-banks and C-banks, K^S and K^C , while holding all other variables fixed (and imposing market clearing $K^S + K^C = K$). The blue curves (i.e. the equity funding costs) trace the value of the LHS of the first-order conditions (24) and (25) as we vary the relative size, $p - q^j b^j$, for $j = C, S$, respectively. For constant p and b^j , the only source of variation is through the bond price q^j . Since total debt issued by each bank type is the product of asset share times leverage per unit of assets, $N^j = b^j K^j$, the marginal benefit of deposits of each type changes as we vary the asset shares, which is reflected in the pricing equations for the q^j , Equations (18) and (20). In particular, as we increase the share of type j holding total provided liquidity constant, the marginal liquidity benefit of type j 's deposits will decline (for any $\alpha < 1$). Since type j 's bond price also will decline, the equity required to purchase the bank's initial asset position becomes larger for the same face amount of debt issued.

We also observe an opposite effect since the marginal benefit derived from each type's debt is also affected by the composition of the total debt as long as liquidity services provided by both

types of debt are imperfect substitutes. When we increase the share of C-banks, we decrease the share of S-banks due to market clearing. Consequently the composition of liquidity services becomes more unequal and the amount of services derived from total debt issued by both banks declines, which leads to a general increase in the marginal benefit of *both kinds* of liquidity. Both effects – the pure effect of an increase in the K^C share and the equilibrium effect through the implied decrease in the K^S share – can be seen in the left panel of Figure 3. Raising the K^S share to 0.7 causes a decrease in the marginal benefit of S-bank debt to households, which in turn lowers the bond price q^S and therefore raises the required initial equity of S-banks (the blue curve in the graph). If the S-bank share is increased any further, the composition of debt becomes so unequal that the marginal benefit of any liquidity rises again, causing both bond prices (also q^S) to increase again. Hence, required equity of S-banks decreases, yielding the overall non-monotonic shape.

The effect on the RHS of Equations (24) and (25), depicted by the red curves, is quantitatively much smaller. The discounted expected dividend per unit of assets is not affected by a change in the share of K^S , when leverage is constant in the steady state. In the dynamic model, there is an indirect effect through the households' discount factor which depends on consumption C .

The overall take-away from Figure 3 is that the relative sizes of both sectors are determined by the zero-expected-profit conditions (24) and (25). The key factor controlling the relative sizes is the marginal liquidity benefit to households of each type of bank debt. To get an interior split between both sectors, the model requires less than perfect substitutability in household preferences.²⁰ While some degree of imperfect substitutability is required, our calibration reveals a high elasticity of substitution of around 5 between liquidity services provided by the two types. Quantitatively, given any elasticity of substitution, the exact split between the two sectors, of course, depends on many other model parameters, such as the default risk in each sector σ^j , the sensitivity of S-bank liquidity to default risk ν , and the overall demand for liquidity ψ .

²⁰In the dynamic stochastic model, even with perfect substitutability an equilibrium with positive capital holdings of both types may exist, since households face a portfolio choice between both types of debt, which have imperfectly correlated returns. However, this force by itself will generally not be enough to prevent a corner solution given the low risk and low risk aversion in the calibrated model.

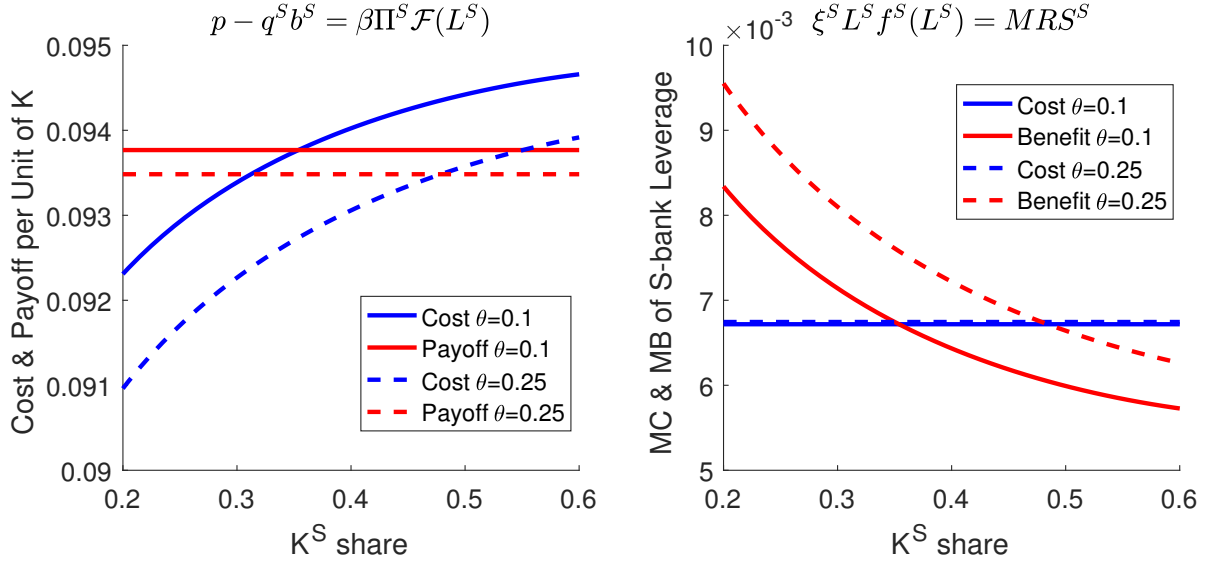


Figure 4: S-bank Scale for Increased Capital Requirement

Left: Required equity (blue curve) marginal benefit (red curve) of S-banks' first-order condition for asset holdings (K^S), while imposing market clearing $K = K^C + K^S$ and holding fixed all other variables. **Right:** LHS and RHS of optimality condition for S-bank leverage in equation (22) ($\delta^S = 0$).

Response of S-bank Leverage and Scale to Changes in Capital Requirement We also use the steady-state model to understand how the relative scale of both types of banks and the leverage (i.e. riskiness) of S-banks respond to an increase in the capital requirement.

Raising θ reduces the amount of debt issued and liquidity produced by C-banks. This higher requirement increases the marginal benefit of liquidity and, therefore, the bond prices for both types of banks. Generally the S-bank could satisfy this increased demand for liquidity in two ways: it could increase leverage to produce more debt for each unit of capital, or it could increase its scale at given leverage. Figure 4 demonstrates that an increase in S-bank scale (K^S) reduces the marginal benefit of S-bank liquidity until both the scale (left plot, Equation (25)) and leverage (right plot, Equation (22)) conditions are satisfied again. For the scale condition, increasing K^S lowers the bond price q^S which raises the required equity per unit of capital. For the leverage condition, higher K^S reduces the marginal benefit of more debt (MRS^S) until it is equal to the marginal cost of debt given by bankruptcy losses ($\xi^S L^S f^S(L^S)$).

Figure 5 again plots both sides of the scale and leverage conditions for S-banks, varying

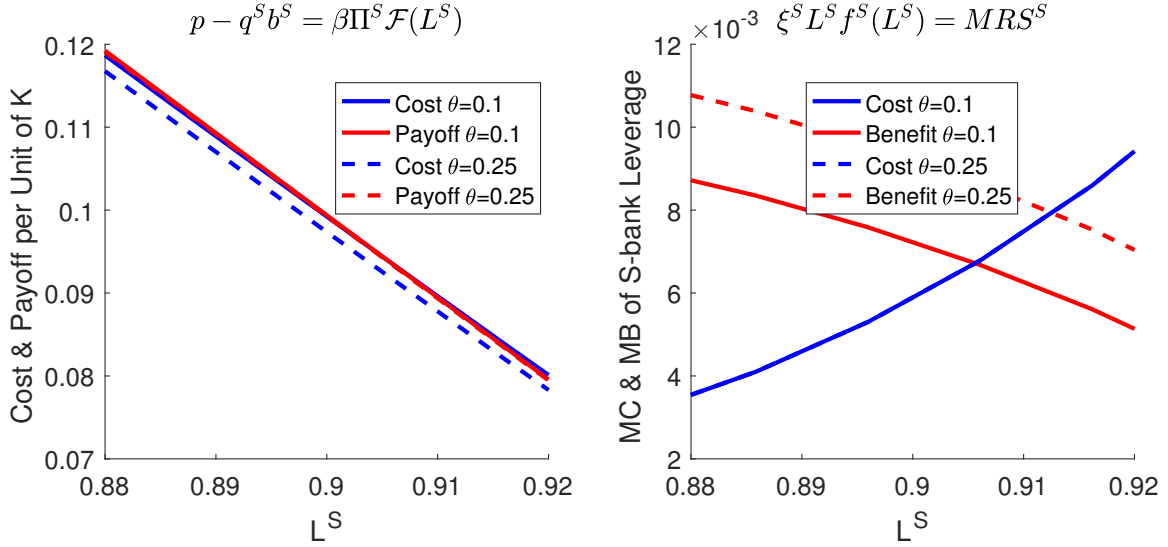


Figure 5: S-bank Leverage for Increased Capital Requirement

Left: Required equity (blue curve) marginal benefit (red curve) of S-banks' first-order condition for asset holdings, varying leverage L^S and holding fixed all other variables. **Right:** LHS and RHS of optimality condition for S-bank leverage in equation (22) ($\delta^S = 0$).

S-bank leverage L^S locally around the optimal leverage in the calibrated model and holding constant scale K^S . The left plot shows that increasing leverage at constant scale affects both the LHS and the RHS of the scale condition in a nearly linear fashion. Higher leverage reduces the payoff to equity holders (RHS) per unit of capital as well as the amount of equity required per unit of capital (LHS). Note that for very high levels of leverage, the amount of funds raised through debt issuance would decrease in leverage since the bond price would fall due to bankruptcy costs.²¹ However, the high leverage required to make the scale equation hold is not compatible with the optimality condition for leverage in the right plot.

To summarize, equilibrium requires an increase in the scale of the shadow banking sector in response to an increase in the C-bank capital requirement. Moreover, the increased demand for liquidity makes each S-bank more profitable and households expand the S-bank sector until its zero expected profit condition is satisfied again.

Welfare-maximizing Capital Requirement The deterministic model delivers a qualitative welfare result with respect to the optimal capital requirement for C-banks, θ . For this result to

²¹At leverage L^S of 1.3, the condition would hold for $\theta = 25\%$ at the optimal scale of the $\theta = 10\%$ economy.

obtain, the general conditions must be

- C1. C-banks are sufficiently risky such that there exists a range for low values of θ for which some C-banks default, i.e. $F^C < 1$.
- C2. S-banks are at least as risky as C-banks; in other words, the standard deviation of their idiosyncratic shocks σ^S is at least as large as that of C-banks.
- C3. Households derive a strictly positive utility benefit from liquidity services ($\psi > 0$), and the liquidity services provided by C-banks are at least as good as those of S-banks.

Under these fairly general conditions, trade-offs exist in the model that lead to a unique utility maximum in θ , illustrated in Figure 6. Increasing the capital requirement θ steadily reduces C-bank leverage and, therefore, also diminishes C-banks defaults and bankruptcy losses. Raising θ above the status quo 10% further causes the capital stock of the economy to increase (top left graph), as capital becomes a more valuable input for liquidity production. Both factors lead to an increase in consumption (top right graph). At the same time, decreasing C-bank leverage through tighter capital requirements lowers the total amount of liquidity services provided to households (top middle), as a greater share of the intermediated asset is shifted to S-banks (bottom left) and S-banks keep leverage roughly constant (bottom middle). Given total utility is a weighted sum of both components, there exists a unique welfare maximum that trades off the reduction in liquidity provision against the increase in consumption. The bottom right graph shows the consumption equivalent variation²² relative to the benchmark value of $\theta = 10\%$.

Constrained Efficient Allocation Like most models with capital under collateral constraints, in our economy competitive equilibria are not efficient if agents do not fully internalize

²²To be precise, let the steady state of household utility at the benchmark value of $\theta_0 = 10\%$ be $\frac{1}{1-\beta} \frac{(C_0^{1-\psi} H_0^\psi)^{1-\gamma}}{1-\gamma}$. Then the consumption equivalent variation at θ_1 is given by χ_1 such that

$$\frac{1}{1-\beta} \frac{(\chi_1 C_0^{1-\psi} H_0^\psi)^{1-\gamma}}{1-\gamma} = \frac{1}{1-\beta} \frac{(C_1^{1-\psi} H_1^\psi)^{1-\gamma}}{1-\gamma}.$$

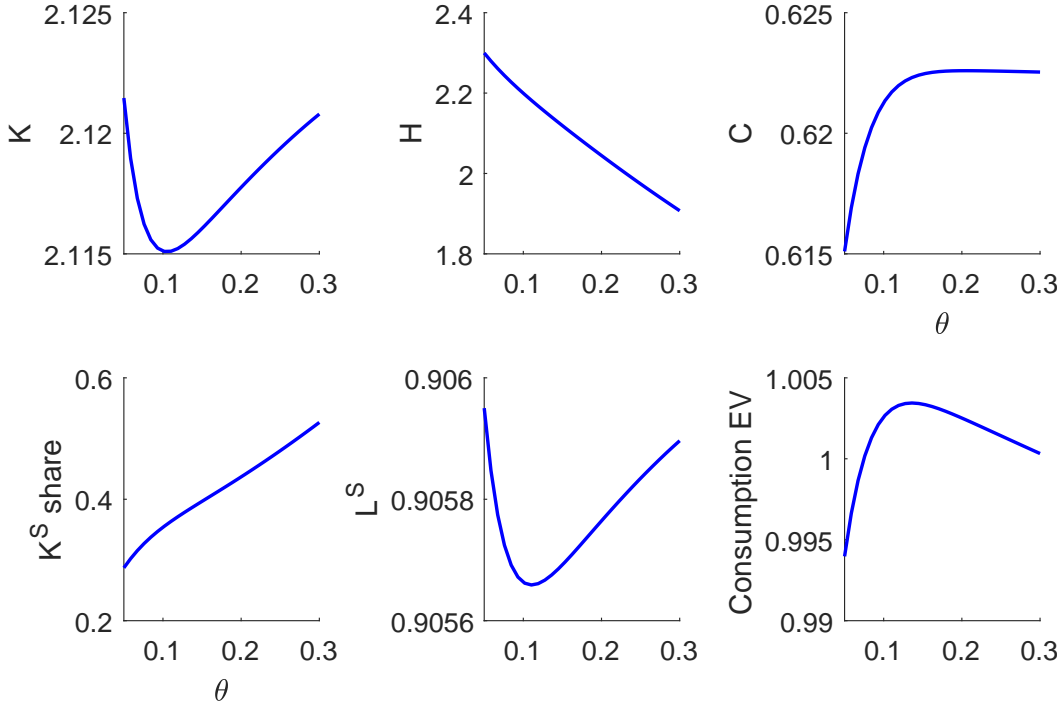


Figure 6: Comparative Statics in Capital Requirement θ

Top row: capital stock, liquidity services, consumption, bottom row: capital share of S-banks, S-bank leverage, household utility.

the value of capital as collateral. To get an idea of the magnitude of the externality, we solve for the constrained efficient allocation in the steady-state model. The social planner maximizes household utility by choosing the capital stock and the leverage of each type of bank, subject to the same technology used in the decentralized economy. In particular, both types have the same investment and production technology, and can only produce liquidity services by issuing debt subject to limited liability. In other words, the social planner cannot simply issue safe debt but must utilize the same risky bank technology to produce liquidity like the competitive equilibrium. Further, leverage of C-banks is subject to the same regulatory constraint.²³

Table 1 compares several aspects of the decentralized economy to the constrained efficient social planner solution. The choice of parameter values is described in detail in Section 4.1 below. The benchmark economy is the steady state of the model with a 10% C-bank capital

²³The capital price appears in the definition of market leverage L^S and L^C , as well as in the C-bank's collateral constraint. However, prices are generally not defined as part of the social planner allocation. To preserve the same liquidity production technology for the social planner, we define capital prices to be $p_K^j = 1 + \phi(I^j/K^j - \delta_K)$ for the planner problem. Note that we do not constrain the prices to be identical across sectors.

Table 1: Comparison of competitive equilibrium to constrained efficient allocation

	K	GDP	C	H	K^S/K	L^C
Benchmark	2.12	0.68	0.62	2.06	0.31	0.90
Social Planner	3.50	0.71	0.62	3.38	0.51	0.84
	L^S	DWL/GDP	I/GDP	C/GDP	Debt/GDP	C EV
Benchmark	0.92	0.31%	0.08	0.92	2.84	
Social Planner	0.85	0.04%	0.12	0.88	4.16	1.4%

requirement. To obtain the social planner (SP) solution, we maximize steady-state household utility numerically for the same parameter values. The SP chooses a 65% greater capital stock for the intermediated sector, which produces a 4.4% higher GDP given decreasing returns to scale in capital (note that 0.5 of the 0.68 GDP in the benchmark economy are the endogenous endowment Y). Even though the capital stock is greater in the SP economy, consumption in both economies is identical, since the investment-to-GDP ratio is significantly higher in the SP economy. However, the much larger capital stock means that the economy has more collateral for liquidity production. While the benchmark economy produces a bank debt to GDP ratio of 2.84, this ratio is 4.16 in the SP economy. Abundant collateral produces greater liquidity at lower leverage. Thus, the regulatory constraint of the C-bank is not binding in the SP economy, and S-bank leverage also is lower, resulting in almost zero bankruptcy losses. Since the SP generates 64% more liquidity services at the same level of consumption, aggregate welfare is 1.4% higher in the SP economy (in consumption-equivalent units). Moreover, absent any bankruptcies, both kinds of banks are producing liquidity services of the same quality, and therefore the SP chooses roughly equal sizes of both sectors.

4 Dynamic Results

In this section, we examine the properties of the model and their implication for the regulation of the financial system. To investigate the degree to which the economy is prone to shadow bank runs, we augment our previous model with bank runs (modeled like Diamond and Dybvig (1983)) and random bailouts of shadow banks. The full version of the model is described in the

Appendix Section A.4. The main trade-offs do not differ from the model presented in Section 2.

4.1 Parametrization

We match the model to quarterly data from the Flow of Funds,²⁴ Compustat, FRED, and NIPA for the period from 1999 (after the passage of the Gramm-Leach-Bliley Act that revoked parts of the Glass-Steagall Act) until the last quarter of 2015. All quantities are expressed per capita in 2009 dollars.

We choose depository institutions as data counterparts for C -banks and shadow bank institutions as data counterparts for S -banks' defining shadow banks in accordance with the available data in the Flow of Funds, i.e. security broker and dealers, finance companies, GSEs, and asset-backed security issuers. We use data on profits from Compustat for publicly traded unregulated financial institutions. (Our shadow bank definition based on Compustat data mirrors the Flow of Funds designation.)²⁵ Appendix B lists the other data series we use to compute empirical counterparts to model objects.

Table 2 lists the parameters in our model organizing them into groups: parameters based on normalizations; parameters directly set to a data counterparts; shadow bank run related parameters; those matched to unconditional first moments; and those matched to unconditional second moments.

The stochastic process for the Y-tree (not intermediated by banks) is a AR(1) in logs

$$\log(Y_{t+1}) = (1 - \rho_Y)\log(\mu^Y) + \rho_Y\log(Y_t) + \epsilon_{t+1}^Y,$$

where ϵ_t^Y is i.i.d. with mean zero and volatility σ^Y . To capture the correlation of asset payoffs

²⁴The Flow of Funds tables are organized according to institutions and instruments. We focus on the balance sheet information on institutions. This is important, as we want to take into account all positions of traditional banks and shadow-banks when we quantify the model.

²⁵For our Compustat based data, we define shadow banks as financial institutions with the following SIC codes 6111-6299, 6798, 6799, 6722, 6726, excluding 6200, 6282, 6022.

with fundamental income shocks, we model the payoff of the intermediated asset as

$$Z_t = \phi^Z Y_t \exp(\epsilon_t^Z),$$

where ϵ_t^Z is i.i.d. with mean zero and volatility σ^Z , independent of ϵ_t^Y . This structure of the shocks implies that Z_t inherits all stochastic properties of aggregate income Y_t and is subject to a temporary shock reflecting risks specific to intermediated assets, such as credit risk.

To parametrize the shock processes, we use the time series of bank-dependent²⁶ and non-bank-dependent GDP (defined as the difference between real per capital NIPA GDP and bank dependent output). We normalize the mean of the non-bank-dependent GDP growth process to $\mu^Y = 0.5$ and derive σ^Y and ρ_Y from the observed volatility and autocorrelation. The bank dependent GDP share is on average 26% over our sample period. We choose the scale ϕ^Z to match this share and set σ^Z to match the series' observed volatility.

The parameters for our quantitative results have been chosen to match unconditional first and second moments. We intuitively infer the size of the capital stock is mainly affected by β , because β determines the saving propensity of agents. Since capital is held by banks only, we target the ratio of financial assets to GDP using Flow of Funds data.

The value of liquidity provision is mainly determined by three key preference parameters: (1) the elasticity of substitution between shadow bank and commercial bank liquidity services ($\alpha = 1 - 1/\text{elasticity}$), (2) the weight on liquidity services ψ in households' preferences, and (3) the parameter ν that governs the sensitivity of the liquidity quality of shadow bank debt. We choose these three parameters for the following reasons.

Liabilities from regulated and unregulated banks are similar in terms of risk (absent bailout guarantees or deposit insurance) and liquidity; although they produce liquidity services, they are not perfectly substitutable. For example, money market fund shares cannot be used directly for transactions unlike most deposits. In the model, the degree of substitutability between these two securities is governed by α . Since the two types of banks provide differentiated liquidity

²⁶Appendix B describes in detail how we define this data series.

services, α helps quantify the relative size of commercial and shadow banks. Our target is the estimated 33% share of shadow banks in total financial intermediation from Gallin (2015).²⁷ The utility weight on liquidity services ψ affects how valuable liquidity services are relative to consumption. We target Krishnamurthy and Vissing-Jorgensen (2012) estimate of government debt liquidity services as a proxy for the liquidity of commercial banks ψ . The parameter ν for shadow bank debt liquidity quality is intended to match the yield on 3-month AA financial commercial paper.

The utility penalty parameters δ^S and δ^C for bank managers determine the default threshold level of bank leverage. We choose δ^C to target a 1% annual net loan charge off rate.²⁸ The parameter δ^S targets the spread between the non-jumbo 3-month deposit rate and the 3-month financial commercial paper rate from FRED.

We choose the bailout probability π^B to match the market value leverage of shadow banks in Compustat. That is, we compute the value weighted leverage of publicly traded shadow banks²⁹ as debt over assets weighted by the relative market value of each institution, and average across time and banks. The result is a value-weighted leverage ratio of 94%. The volatility of the idiosyncratic valuations shocks σ^S and σ^C are proxies for the relative riskiness of investment opportunities for banks. Our target for these two parameters is the cross-sectional volatility of Tobin's q (market value of assets relative to book value) for depository institutions and shadow banks in Compustat. The remaining parameters are described in Appendix B.

²⁷More precisely, Gallin (2015) used data from the Flow of Funds to carefully trace back how much shadow banking sector funding the real economy received. Since many shadow banks fund each other and not necessarily real activity the actual share of shadow activity is much lower (around 33%) than what one would expect given the total asset size of the financial sector.

²⁸Choosing the right data target for δ^C is not trivial. Targeting commercial bank defaults using FDIC data suggests a higher rate (4%). However, a 4% default rate speaks mainly to smaller less important bank. Moody reports typically suggest a lower rate (0.6% for the entire banking sector). Previous experiments with targets of 4% of C-bank default did not meaningfully change the optimal requirement number.

²⁹We define shadow banks as all institutions with SIC codes 6111-6299, 6798, 6799, 6722, 6726, excluding SIC codes 6200, 6282, 6022, 6199.

Table 2: Parametrization

	Values	Target	Data	Model (determ.)
μ_Y	0.5	normalization		
μ_ρ^C	1	normalization		
μ_ρ^S	1	normalization		
set directly				
η	0.67	NIPA labor share		
δ_K	0.025	10% annual depreciation		
γ	2	Risk aversion		
θ	0.1	capital requirement		
κ	0.000617	insurance fee (FDIC)		
σ^Y	0.011	non-bank dependent sector vol		
δ_Y	0.366	non-bank dependent sector AC		
μ^Z	0.13	$\mu^Y \times \phi^Z$		
σ^Z	0.0168	bank dependent output vol		
calibrated				
ϕ^Z	0.274	Val add. FinSec/GDP	26%	26%
β	0.989	fin assets relative to GDP	3.66	3.14
α	0.745	shadow banking share Gallin	33%	30%
ψ	0.0293	match liquidity premium C-banks KV2012	73 bp	98 bp
ν	85	match rate on AA Financial CP	0.23%	0.19%
δ_S	0.335	S-C bank debt spread	0.03%	0.08%
δ_C	0.18	quarterly net loan charge-offs	0.25%	0.29%
ξ_C	0.51	recovery rate Moody's	37%	37%
ξ_S	0.37	recovery rate Moody's	37%	37%
π_B	0.63	Sbank leverage Compustat	93%	91%
2nd moments				
σ_{ρ^C}	0.115	vol(mkt assets/assets)	0.07	0.10
σ_{ρ^S}	0.175	vol(mkt assets/assets)	0.49	0.40
ϕ	2	vol(financial sector asset growth)	0.021	0.007
run				
$\underline{\delta}_K$	0.05	haircut (discount) x_t	75%	77%
ϱ	[0, 0.3]	fraction of households run (Covitz et al 2013)		
Prob ϱ	$\begin{bmatrix} 0.93 & 0.07 \\ 0.40 & 0.60 \end{bmatrix}$	uncond. run prob.	3%	3%
\underline{Z}	0.9	Z in run state		

4.2 Solution Method and Aggregate State Variables

We solve the dynamic model using nonlinear projection methods. The equilibrium of the economy is described by a system of nonlinear equations of the state variables whose unknown functions represent the agents' choices, the asset prices, and the Lagrange multiplier on the C -bank's leverage constraint. We parametrize these variables as unknown functions of the discretized state variables and iterate on the system until convergence. To insure the approximations of the unknown functions are accurate, we check the relative Euler equation errors at the solution we obtain. We then simulate the model for many periods and compute moments of the simulated series. Appendix C lists all equations that characterize the equilibrium of the model economy and contains details on the computational solution procedure.

The model has three exogenous state variables, the stochastic endowment Y_t , productivity Z_t , and the run shock ϱ_t . These shocks are jointly discretized as a first-order Markov chain with three nodes for Y_t and three nodes for Z_t . We assume that runs only occur in low productivity states, yielding a total of 12 different discrete states.

In addition, the model has four endogenous aggregate state variables: (1) the aggregate capital stock $K_t = K_t^S + K_t^C$, the outstanding amount of banks debt (2) B_t^S and (3) B_t^C , and (4) the share of capital held by one type of bank, e.g. $\frac{K_t^S}{K_t}$.³⁰ Appendix Section D.1 discusses the moments of the benchmark economy under a capital requirement of 10%.

4.3 Shadow Bank Runs

Shadow banks are fragile in our model because they experience occasional large-deposit outflows, forcing them to sell their assets at fire-sale prices. These infrequent episodes of shadow bank fragility quantitatively capture a possible downside of too much shadow-banking activity that

³⁰Inspecting the optimization problem for S-banks and C-banks, one can see that the value functions v^S and v^C do not depend on past portfolio choices of the bank. Why then are the debt of both types of banks and the share of capital held by each type aggregate state variables? The reason is that the level of debt of each type determines household utility from liquidity, and these quantities are therefore aggregate state variables for households. Furthermore, in combination with each type's capital share, the outstanding debt determines beginning-of-period leverage L_t^j , which in turn determines household wealth and consumption (alternatively, we could choose household wealth as state variable instead of the capital share).

is absent in the deterministic version of the model.

Runs on the deposits of shadow banks ($\varrho > 0$) shrink their balance sheets, erode their capital share and diminish their ability to issue debt. At the same time, their default rate spikes and the quality of the liquidity services they provide (Λ^S) drops. Interestingly, we find shadow bank deposit rates fall during these periods. Due to the high bailout probability (63%), interest rates only reflect credit risk partially, but the drop in shadow bank liquidity raises its marginal benefit. This quantity effect dominates the compensation for increased bankruptcies.

Figure 7 compares the impulse response functions to a low realization of Z (low productivity in the financial sector) (blue curve) to a shock with both a low Z realization and a run on the shadow banking sector (red curve). The benchmark unconditional economy is in black. A run on shadow bank deposits leads to a higher default rate in the shadow banking sector (panel (a) of 7 top left), causing shadow banking debt to be less liquid (bottom left), and inducing a lower overall production of liquidity services in this economy (bottom right). The default rate in the commercial banking sector is higher during a crisis without a run, while the opposite is true for shadow banks.

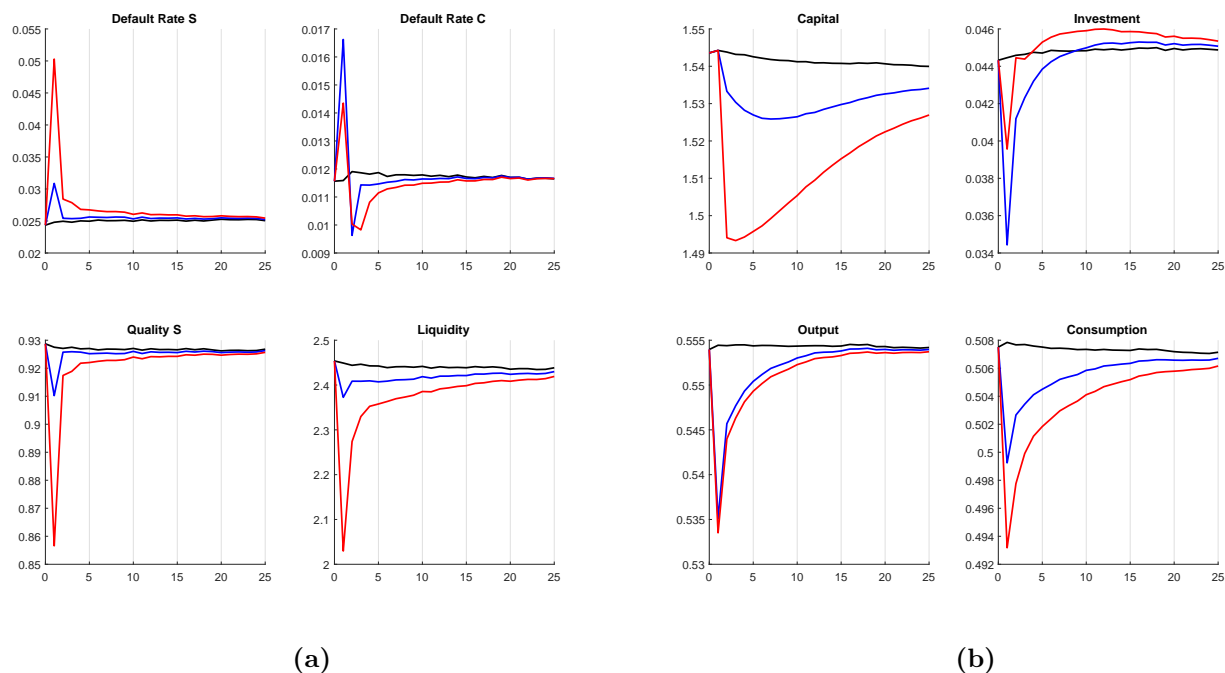


Figure 7: Dynamic Response to Low Z : The graphs show the dynamic responses of model variables to a regular low payoff shock for the intermediated asset (blue curve) and a low payoff shock combined with a run on shadow banks (red curve), compared to the unconditional evolution of all variables (black curve). Responses are the mean paths of 1000 model simulations for 25 periods.

Our results suggest the aggregate consequences of a crisis with a run on the shadow banking sector is significantly worse. Even though the difference in output is negligible, the drop in the aggregate capital stock, investment and consumption is more dramatic. The larger negative response in consumption is related to the need for higher savings to make up for the shortfall in investment and capital stock (panel (b) of figure 7). Since investment is subject to adjustment costs, the large initial impact in the run-crisis has persistent effects over many years (x-axis is in quarters).

4.4 Welfare

We find there is a unique maximum aggregate welfare derived from increasing the capital requirement θ . To study the effect of a change in the capital requirement, we consider two cases. First, we compare economies with different θ for the benchmark calibration with a bailout probability of $\pi^B = 63\%$. In these economies, a capital requirement of around 15 percent maximizes

welfare. The main forces producing this result are the same as the ones we discussed for the steady state. Increasing θ makes the debt of commercial banks safer, thus reducing bankruptcy losses and increasing aggregate consumption. At the same time, a higher θ restricts the amount of liabilities and, thus, liquidity commercial banks can produce for each unit of assets.

We also consider several variations of this experiment; in particular, we explore the effect of an increase in the deposit insurance fee and a time-varying capital requirement. In Appendix D.2, we report results for an increase in θ when the probability of a S-bank bailout is lower than is, at $\pi^B = 20\%$.

Capital requirement experiment under benchmark calibration The middle panel of Table 3 shows that an increase in the capital requirement shifts a greater fraction of the intermediated asset to shadow banks. Deposit insurance and a convenience yield on C-bank debt imply that the capital requirement for C-banks is binding. That is, the bank has issued as much debt as regulations permit. A higher requirement tightens the constraint further, making each unit of deposits effectively more costly to issue and causing commercial banks to delever. Restricting the liquidity production by commercial banks increases any kind of marginal benefit of liquidity to households. Shadow banks take advantage of the lower funding costs by holding more of the intermediated assets.

In response to the scaling back of commercial banks, shadow banks do not increase their leverage. Rather, they keep their leverage roughly constant. Figures 4 and 5 explain why. A higher capital requirement reduces the funding costs of shadow banks which gives them incentives to issue more debt. Nonregulated banks can produce more debt by increasing leverage and, therefore, becoming riskier, or by increasing the size of their balance sheet (i.e. issuing equity). However, since a reduction in the cost of debt makes all banks more profitable, only an increase in the size of the balance sheet, not leverage (which does not change banks' profitability on the margin) can be an equilibrium. Overall liquidity production falls monotonically as long as the liquidity produced by S-banks is less valuable to households than that produced by

Table 3: Model Moments for Different Capital Requirements

	10%		15%		20%	
	mean	stdev	mean	stdev	mean	stdev
Prices						
Capital Price	1.002	0.006	1.001	0.006	1.001	0.006
Deposit Rate S	0.16%	0.42%	0.16%	0.43%	0.16%	0.43%
Deposit Rate C	0.14%	0.42%	0.08%	0.43%	0.02%	0.43%
EER Equity S	0.98%	0.01%	0.98%	0.01%	0.98%	0.01%
EER Equity C	1.00%	0.06%	1.05%	0.07%	1.11%	0.07%
Liquidity						
Λ^S	0.842	0.022	0.842	0.022	0.842	0.022
Convenience Yield S	1.01%	0.06%	1.01%	0.06%	1.01%	0.06%
Convenience Yield C	0.98%	0.01%	1.04%	0.01%	1.10%	0.01%
Quantities						
Capital stock	2.100	0.027	2.099	0.030	2.097	0.026
Capital Share S	0.310	0.032	0.350	0.037	0.388	0.040
Debt S	0.308	0.035	0.348	0.040	0.386	0.044
Debt C	0.692	0.035	0.652	0.040	0.614	0.044
Asset Value S	0.652	0.069	0.735	0.079	0.815	0.086
Asset Value C	1.451	0.069	1.365	0.078	1.284	0.083
Leverage S	0.907	0.005	0.907	0.005	0.907	0.005
Leverage C	0.900	0.006	0.850	0.006	0.800	0.006
Early Liquidation Share	0.009	0.053	0.010	0.055	0.010	0.055
Utility						
Liquidity	2.183	0.043	2.101	0.046	2.024	0.043
Consumption	0.621	0.003	0.622	0.003	0.622	0.003
GDP	0.675	0.008	0.675	0.009	0.675	0.008
Δ Welfare in CE			0.08%		-0.01%	
Bankruptcy Losses						
DWL S	0.026	0.005	0.030	0.006	0.033	0.006
DWL C	0.152	0.035	0.020	0.005	0.002	0.001
Default Rate S	0.202%	0.031%	0.202%	0.031%	0.203%	0.031%
Default Rate C	0.296%	0.065%	0.045%	0.012%	0.004%	0.001%

^a: Deadweight losses are in units of the consumption good, multiplied by factor 100.

^b: Welfare is the percentage change of the mean household value function relative to the benchmark model with $\theta = 0.1$ expressed in consumption equivalent units.

C -banks (which is the case as long as $\Lambda^S < 1$).

While the capital stock increases with the capital requirement in the static model, it remains more or less constant in the dynamic model. The increase in the capital stock is caused by a sufficiently high demand of shadow banks that offsets the reduction in the demand of commercial banks. This demand arises because higher capital requirements increase shadow bank profits. In the dynamic model, expected profits are lower compared to profits in the static model since every so often, a run leads to large losses in the shadow banking sector, and depresses the shadow bank share, muting its response to an increase in the capital requirement.

Commercial banks become safer as the capital requirement is raised, generating fewer overall deadweight losses and higher consumption. However, in the dynamic model increasing the requirement creates greater liquidity volatility, which produces a small offsetting effect for the risk-averse household, making the overall welfare gain smaller than in the steady state. Thus, we find that when the capital requirement is raised total welfare already is decreasing even before all bankruptcy losses are eliminated; in other words, our calibrated model shows that making commercial banks completely safe is not optimal.

In the Appendix Section D.2 we present variations of the main experiment to reinforce our principal results that (i) no increase in fragility and (ii) no production losses are robust to different parametrizations and model specifications. In an unreported experiment, we have also tested the robustness of our main findings when the unregulated shadow banking sector does not own an investment technology. This version of the model simulates an economy in which shadow banks cannot originate loans themselves but must buy loans from commercial banks. The main conclusion of this modification is a lower shadow bank share, but welfare effects of higher capital requirements are unaltered.

Fair Deposit Insurance Fee & Time Varying Capital Requirement Table 4 compares the benchmark economy to an environment with a time-varying insurance fee κ_t and to an economy with a time-varying capital requirement.

Table 4: Effect of Fair Insurance Fee & Time Varying Capital Requirement

	baseline			fair kappa			time varying theta		
	mean	stdev	Corr Y	mean	stdev	Corr Y	mean	stdev	Corr Y
	Liquidity								
Λ^S	0.884	0.035	0.531	0.881	0.036	0.538	0.883	0.036	0.531
Conv. Yield S	0.013	0.000	-0.163	0.013	0.000	-0.232	0.013	0.000	-0.186
Conv. Yield C	0.013	0.000	-0.520	0.013	0.000	-0.412	0.013	0.000	-0.258
	Quantities								
Capital Stock	2.100	0.027	0.125	2.050	0.026	0.088	2.100	0.028	0.107
Debt S	0.592	0.063	-0.009	0.794	0.095	0.153	0.640	0.066	0.043
Debt C	1.306	0.062	0.102	1.059	0.084	-0.121	1.215	0.061	0.015
Capital Share S	0.310	0.032	-0.030	0.427	0.049	0.145	0.335	0.034	0.025
Asset Value S	0.652	0.069	0.032	0.876	0.105	0.192	0.705	0.073	0.086
Asset Value C	1.451	0.069	0.167	1.177	0.096	-0.078	1.397	0.071	0.101
Leverage S	0.907	0.005	-0.821	0.905	0.005	-0.842	0.907	0.005	-0.817
Leverage C	0.900	0.006	-0.456	0.900	0.006	-0.457	0.870	0.005	-0.820
	Utility								
Liquidity	2.183	0.043	0.575	2.132	0.041	0.642	2.135	0.041	0.572
Consumption	0.621	0.003	0.809	0.621	0.003	0.814	0.622	0.003	0.795
GDP	0.675	0.008	0.894	0.674	0.008	0.889	0.675	0.008	0.890
Welfare in CE	-139.911			-0.066%			0.078%		
	Bankruptcy Losses								
DWL S	0.03	0.00	-0.615	0.03	0.01	-0.533	0.03	0.01	-0.586
DWL C	0.15	0.04	-0.425	0.12	0.03	-0.543	0.05	0.01	-0.758
Default Rate S	0.20%	0.03%	-0.740	0.19%	0.03%	-0.765	0.20%	0.03%	-0.736
Default Rate C	0.30%	0.07%	-0.463	0.29%	0.07%	-0.464	0.10%	0.02%	-0.794
θ_t							13%	0.37%	0.792

^a: Deadweight losses are in units of the consumption good, multiplied by factor 100.

^b: Welfare is the percentage change of the mean household value function relative to the benchmark model with $\theta = 0.1$ in consumption equivalent units.

We implement the fair deposit insurance fee experiment in the following manner: we set κ each period assuming that all the bankruptcy losses from defaulting commercial banks can be fully covered. The fair κ is on average 18bps, three times the benchmark value. In an economy without shadow banks, commercial banks would internalize deadweight losses caused by their high leverage. In this model, DWL caused by commercial banks are indeed slightly reduced. However, the main effect of such a policy in our model is to shift intermediation activity to shadow banks. While both types of banks become slightly safer, i.e. lower default rates, the net effect is still a shift from the relatively safer sector to the relatively less safe sector. Similar to a higher capital requirement, the fair insurance fee reduces the size of the commercial banking sector. But unlike an increased capital requirement, the higher insurance fee does not limit the leverage of commercial banks and fails to reduce the bulk of DWL. Hence, setting a fair κ_t causes a welfare loss and produces a reduction in liquidity but no gain in consumption.

In our next experiment that investigates a time-varying capital requirement, we set the capital requirement such that commercial banks' expected default rate is less than 40 basis points p.a. (10 bp per quarter). The resulting time-varying requirement has a mean of 13%, a standard deviation of 0.40% and co-varies strongly with the business cycle. Leverage and default rates of commercial banks becomes more countercyclical, while the overall levels are lower, reducing fragility. When the capital requirement allows commercial banks to increase leverage temporarily during bad times, they become more vulnerable to adverse economic shocks. This tends to increase the co-movements of default rates and deadweight losses from bankruptcy with bad states.

In contrast to the fair insurance premium scenario, the intermediation activity of shadow banks does not pick up, because subsidized commercial banks can offer competitive rates. At the same time, a higher capital requirement (13 percent compared to the 10 percent benchmark) forces commercial banks to lower their leverage. Altogether the economy produces less liquidity. Moreover, since commercial bank deadweight losses are more countercyclical, consumption becomes more procyclical, which risk-averse agents dislike. However, the reduction

in deadweight losses is significant enough to improve welfare compared to the benchmark static capital requirement of $\theta = 10\%$. Compared to the benchmark, the welfare gain is almost as large as under a static requirement of $\theta = 15\%$.

5 Conclusion

This paper proposes a novel, quantitative model to study the consequences for the economy of capital requirement rules on commercial banks. Our research shows the optimal level of capital regulation trades off a reduction in liquidity services against an increase in the safety of the banking system and consumption. Increasing the capital requirement for regulated banks leads to more intermediation activity by the shadow banking system. However, this shift in market share does not imply that unregulated banks become riskier. Instead we find that higher capital requirements increase profitability through lower funding costs due to an increased scarcity of coveted, money-like assets. As a result, shadow banks expand their operations and increase their size rather than becoming riskier by choosing higher leverage. Our findings are robust across different modeling assumptions concerning the baseline riskiness of unregulated institutions.

Since a higher capital requirement shifts intermediated assets to unregulated banks, the net benefit of such a policy depends on the baseline fragility of unregulated banks. Our qualitative finding that a higher capital requirement increases welfare is robust across different policy experiments. However, our quantitative results highlight the impact that additional policy measures, such as implicit bailout guarantees, have on unregulated banks. Our findings further suggest that a higher deposit insurance fee does not make commercial banks safer. Rather, rising fees shift intermediation activity to the more fragile shadow banks without limiting the leverage of commercial banks. Finally, when the capital requirement is procyclical, the average capital requirement can be lower compared to a static rule.

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A Model Appendix

A.1 Bank Optimization Problem

This section describes in detail how the bank optimization problem in Equation (7) in the main text is derived. Starting from the problem in Equation (4), the first-order condition for labor demand is the usual intratemporal condition equating the wage to the marginal product of labor

$$w_t = Z_t \eta \left(\frac{N_{t,i}^S}{K_{t,i}^S} \right)^{\eta-1} = Z_t \eta (n_{t,i}^S)^{\eta-1}.$$

Similarly, the first-order condition for investment yields the usual relationship between the capital price and the marginal value of a unit of capital

$$p_t = 1 + \phi \left(\frac{I_{t,i}^S}{K_{t,i}^S} - \delta_K \right) = 1 + \phi (i_{t,i}^S - \delta_K).$$

We can substitute both conditions back into the definition of the dividend in (3) to eliminate the wage and investment

$$D_{t,i}^S = \rho_{t,i}^S \Pi_{t,i}^S K_{t,i}^S - B_{t,i}^S + q_{t,i}^S B_{t+1,i}^S - p_t K_{t+1,i}^S,$$

where

$$\begin{aligned} \Pi_{t,i}^S &= Z_t (n_{t,i}^S)^\eta + (1 - \delta_K) p_t - w_t n_{t,i}^S + i_{t,i}^S (p_t - 1) - \frac{\phi}{2} (i_{t,i}^S - \delta_K)^2 \\ &= (1 - \eta) Z_t (n_{t,i}^S)^\eta + p_t - \delta_K + \frac{(p_t - 1)^2}{2\phi} \end{aligned}$$

is the gross marginal value (including output and revenue and cost of investment) of a unit of capital at the beginning of the period as defined in Equation (2) in the main text.

After the bankruptcy decision, non-bankrupt banks choose their portfolio for next period, and households set up new banks to replace defaulted banks. With respect to the portfolio choice for period $t + 1$, the optimization problem of all banks is identical because the idiosyncratic valuation shocks are uncorrelated over time. We can exploit this feature of the problem by dropping the i subscripts and defining a value function $\hat{V}^S(Z_t)$ that only depends on the aggregate state:

$$V(K_t^S, B_t^S, Z_t) = \rho_t^S \Pi_t^S K_t^S - B_t^S + \hat{V}^S(Z_t),$$

and

$$\hat{V}^S(Z_t) = \max_{K_{t+1}^S, B_{t+1}^S} q_t^S B_{t+1}^S - p_t K_{t+1}^S + \text{E}_t \left[M_{t,t+1} \max \left\{ \rho_{t+1}^S \Pi_{t+1}^S K_{t+1}^S - B_{t+1}^S + \hat{V}^S(Z_{t+1}), -\delta_S \Pi_{t+1}^S K_{t+1}^S \right\} \right].$$

We conjecture that $\hat{V}^S(Z_t) = 0$ always. Then we can define bonds issued per unit of capital $b_{t+1}^S = B_{t+1}^S / K_{t+1}^S$ and the scaled value function

$$v^S(Z_t) = \frac{\hat{V}^S(Z_t)}{K_{t+1}^S} = \max_{b_{t+1}^S} q_t^S b_{t+1}^S - p_t + \text{E}_t \left[M_{t,t+1} \max \left\{ \rho_{t+1}^S \Pi_{t+1}^S - b_{t+1}^S, -\delta_S \Pi_{t+1}^S \right\} \right], \quad (26)$$

and further divide the optimization problem

$$\hat{V}^S(Z_t) = \max_{K_{t+1}^S \geq 0} K_{t+1}^S v^S(Z_t).$$

Optimization requires that either $K_{t+1}^S = 0$ or $v^S(Z_t) = 0$, which confirms the conjecture that $\hat{V}^S(Z_t) = 0$.

Using the definition of market leverage $L_t^S = B_t^S / (\Pi_t^S K_t^S)$, we rewrite the maximum operator, i.e. the continuation value, as

$$\begin{aligned} & K_t^S \max \{ \rho_t^S \Pi_t^S - b_t^S, -\delta_S \Pi_t^S \} \\ & = K_t^S \Pi_t^S (1 [\rho_t^S \geq L_t^S - \delta_S] (\rho_t^S - L_t^S) - 1 [\rho_t^S < L_t^S - \delta_S] \delta_S). \end{aligned}$$

Taking the expectation of this expression with respect to ρ_t^S results in

$$K_t^S \Pi_t^S \times \underbrace{\left[(1 - F_\rho^S(L_t^S - \delta_S)) (\rho_t^{S,+} - L_t^S) - F_\rho^S(L_t^S - \delta_S) \delta_S \right]}_{\equiv \mathcal{F}^S(L_t^S)}, \quad (27)$$

where $\rho_t^{S,+} = \mathbb{E}(\rho_t^S | \rho_t^S > L_t^S - \delta_S)$ is the expected idiosyncratic shock conditional on not defaulting, $F_\rho^S(L_t^S - \delta_S)$ is the probability of default, and $\mathcal{F}^S(L_t^S)$ is the leverage-adjusted payoff of S-banks' portfolio, including the default option. Substituting the expression in (27) into (26) yields the value function given in Equation (7) in the main text.

A.2 Equilibrium Characterization

This section characterizes the equilibrium via banks' and households' optimality conditions.

Household Households' first-order conditions for purchases of bank equity are, for $j = S, C$,

$$p_t^j = \mathbb{E}_t \left[M_{t,t+1} \left(1 - F_\rho^j(L_{t+1}^j) \right) \left(D_{t+1}^{j,+} + p_{t+1}^j \right) \right],$$

where we have defined the stochastic discount factor

$$M_{t,t+1} = \beta \frac{U_C(C_{t+1}, H_{t+1})}{U_C(C_t, H_t)} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left(\frac{C_{t+1}^{1-\psi} H_{t+1}^\psi}{C_t^{1-\psi} H_t^\psi} \right)^{1-\gamma}.$$

The intratemporal marginal rate of substitution between consumption and liquidity services is defined as

$$Q_t = \frac{\psi}{1-\psi} \frac{C_t}{H_t}.$$

The marginal rate of substitution between consumption and liquidity services of bank type j is

$$\text{MRS}_{j,t} = Q_{t+1} \Lambda_{j,t+1} \left(\frac{H_{t+1}}{A_{t+1}^j} \right)^{1-\alpha}.$$

Then the first-order conditions for purchasing bonds of either type of bank are

$$q_t^C = \mathbb{E}_t \{ M_{t,t+1} [1 + \text{MRS}_{C,t+1}] \}, \quad (28)$$

$$q_t^S = \mathbb{E}_t \{ M_{t,t+1} [1 - F_\rho^S(L_{t+1}^S) + F_\rho^S(L_{t+1}^S) r^S(L_{t+1}^S) + \text{MRS}_{S,t+1}] \}. \quad (29)$$

The payoff of commercial bank bonds is 1, whereas the payoff of shadow bank bonds depends on their default probability and recovery value. The last terms in each expectation expression represent the marginal benefit of liquidity services to households as the product of the intratemporal marginal rate of substitution (between consumption and liquidity services) and the marginal value of deposits for aggregate liquidity services.

Investment Each bank produces its own capital that can be sold in a frictionless capital market. The first-order condition with respect to investment relates the price of capital to the marginal cost of investment.

$$p_t = 1 + \phi \left(i_t^j - \delta_K \right), \quad \forall j \in (S, C).$$

Labor Frictionless labor markets require that workers are paid the marginal product of labor, which must be the same across different bank types, i.e.

$$w_t = \eta Z_t (n_t^S)^{\eta-1} = \eta Z_t (n_t^C)^{\eta-1}.$$

This immediately implies

$$n_t^S = n_t^C. \quad (30)$$

The split of labor across C - and S -banks follows from Equations (17) and (30) according to which

$$\begin{aligned} N_t^S &= \frac{K_t^S}{K_t^C + K_t^S} \\ N_t^C &= 1 - N_t^S. \end{aligned} \quad (31)$$

Banks S -banks are subject to an endogenous borrowing constraint. Each S -bank is effectively a monopolist for its own debt, as it internalizes the effect of supplying additional bonds on the bond price.

Specifically, each S -bank views the price of its debt as a function of its supply of bonds $q_t^S = q(b_{t+1}^S)$ that is determined by households' first-order condition in Equation (29).

Differentiating Equation (7), we obtain the FOC of S -banks for leverage is³¹

$$q(b_{t+1}^S) + b_{t+1}^S q'(b_{t+1}^S) = E_t [M_{t+1}(1 - F_{t+1}^S)]. \quad (32)$$

The partial derivative $q'(b_{t+1}^S)$ can be obtained directly from households' FOC for purchases of shadow bank debt. Differentiating Equation (29) yields

$$\frac{\partial q^S(b_{t+1}^S)}{\partial b_{t+1}^S} = -E_t \left\{ M_{t,t+1} \left[\frac{F_{t+1}^S r_{t+1}^S}{b_{t+1}^S} + \frac{f_{t+1}^S}{b_{t+1}^S} ((1 - \xi_S)\delta_S + \xi_S L_{t+1}^S) \right] \right\}. \quad (33)$$

The RHS of Equation (33) is strictly negative, implying that the price of shadow bank debt is decreasing in shadow bank leverage b_{t+1}^S . The first term reflects that the recovery value per bond in case of bankruptcy decreases if the shadow bank issues more debt. The second term is the loss for lenders from a marginal increase in the probability of default.

S -bank leverage L_{t+1}^S is determined by combining Equations (32) and (33), and substituting for the bond price from households' first-order condition (29).

³¹Appendix A.2.1 contains details of the derivations in this section.

The debt price of commercial banks is independent of their leverage choice. Therefore the FOC of C -banks for leverage is

$$q_t^C - \kappa = \lambda_t^C + \text{E}_t [M_{t,t+1}(1 - F_{\rho,t+1}^C)]. \quad (34)$$

with λ_t^C being the Lagrange multiplier on the leverage constraint in Equation (11).

The condition for a binding leverage requirement is³²

$$\lambda_t^C = \text{E}_t [M_{t,t+1} (F_{\rho,t+1}^C + \text{MRS}_{C,t+1})] - \kappa > 0.$$

The first term is the marginal value of liquidity services derived from commercial bank deposits. As long as households are not satiated with liquidity, the marginal value is positive. The second term is the expected discounted value in case of default of carrying one unit of resources into the next period, which is positive as long as commercial banks default with some probability $F_{\rho,t+1}^C > 0$. The last term is the deposit insurance fee. Taken together, this condition shows a small enough κ implies that the leverage requirement will always bind. This is the case for all parametrizations we consider³³.

A binding leverage constraint implies

$$(1 - \theta)p_t = b_t^C.$$

The first-order conditions for asset purchases K_{t+1}^j follow from the constant returns to scale (i.e. zero-profit) condition of each type's problem, requiring $v^S(Z_t) = 0$ and $v^C(Z_t) = 0$, respectively:

$$\begin{aligned} p_t - q_t^S b_{t+1}^S &= \text{E}_t [M_{t,t+1} \Pi_{t+1}^S \mathcal{F}^S(L_{t+1}^S)], \\ p_t - (q_t^C - \kappa) b_{t+1}^C &= \text{E}_t [M_{t,t+1} \Pi_{t+1}^C \mathcal{F}^C(L_{t+1}^C)]. \end{aligned}$$

A.2.1 First-order conditions with respect to b_t^S

Differentiating Equation (7) with respect to b_t^S (b_t^C is identical) gives

$$0 = q(b_{t+1}^S) + b_{t+1}^S q'(b_{t+1}^S) + \text{E}_t \left[M_{t,t+1} \Pi_{t+1} \frac{\partial \mathcal{F}^S(L_{t+1}^S)}{\partial b_{t+1}^S} \right].$$

First, rewrite

$$\mathcal{F}^S(L_{t+1}^S) = -(1 - F_{\rho}^S(L_{t+1}^S - \delta^S))L_{t+1}^S - F_{\rho}^S(L_{t+1}^S - \delta^S)\delta^S + \int_{L_{t+1}^S - \delta^S}^{\infty} \rho f_{\rho}^S(\rho) d\rho.$$

Now compute

$$\begin{aligned} \frac{\partial \mathcal{F}^S(L_{t+1}^S)}{\partial L_{t+1}^S} &= f_{\rho}^S(L_{t+1}^S - \delta^S)L_{t+1}^S - (1 - F_{\rho}^S(L_{t+1}^S - \delta^S)) \\ &\quad - f_{\rho}^S(L_{t+1}^S - \delta^S)\delta^S - f_{\rho}^S(L_{t+1}^S - \delta^S)(L_{t+1}^S - \delta^S) \\ &= - (1 - F_{\rho}^S(L_{t+1}^S - \delta^S)), \end{aligned}$$

which combined with the fact that

$$\frac{\partial L_t^S}{\partial b_t^S} = \frac{1}{\Pi_t}.$$

yields the result in Equation (32).

³²This condition is derived from combining the FOC of commercial banks with respect to leverage with household's FOC with respect to debt (28).

³³Conversely, if a regulator could set

$$\kappa > \text{E}_t [M_{t,t+1} (F_{\rho,t+1}^C + \text{MRS}_{C,t+1})] \quad \forall t,$$

commercial banks may never choose leverage at the regulatory limit.

A.2.2 Derivation of $(q^S)'(b_t^S)$

Recall the definition of the recovery value for S-banks is

$$r^S(b_t^S) = (1 - \xi^S) \frac{\rho^{S,-}}{L_t^S},$$

with the conditional expectation

$$\rho_t^{S,-} = \mathbb{E}_{\rho,S}[\rho \mid \rho < L_t^S - \delta].$$

We can therefore rewrite the recovery value times the probability of default as

$$F_t^S r_t^S = \frac{(1 - \xi_S)}{L_t^S} \int_{-\infty}^{L_t^S - \delta} \rho dF_{\rho,S}(\rho).$$

Differentiating this expression with respect to L_t^S gives

$$\begin{aligned} \frac{\partial F_t^S r_t^S}{\partial L_t^S} &= -\frac{(1 - \xi_S)}{(L_t^S)^2} F_t^S \rho_t^{S,-} + \frac{f_t^S}{L_t^S} (1 - \xi_S) (L_t^S - \delta) \\ &= -\frac{F_t^S r_t^S}{L_t^S} - \frac{f_t^S}{L_t^S} (1 - \xi_S) (\delta - L_t^S). \end{aligned}$$

Using the fact that

$$\frac{\partial L_t^S}{\partial b_t^S} = \frac{1}{\Pi_t^S},$$

we get

$$\frac{\partial F_t^S r_t^S}{\partial b_t^S} = -\frac{F_t^S r_t^S}{b_t^S} - \frac{f_t^S}{b_t^S} (1 - \xi_S) (\delta - L_t^S).$$

The complete partial derivative of the household's bond pricing equation (see Equation 33) for q^S (29) with respect to b_{t+1}^S is therefore

$$\begin{aligned} \frac{\partial q^S(b_{t+1}^S)}{\partial b_{t+1}^S} &= -\mathbb{E}_t \left\{ M_{t,t+1} \left[\frac{f_{t+1}^S}{\Pi_t^S} + \frac{F_{t+1}^S r_{t+1}^S}{b_{t+1}^S} + \frac{f_{t+1}^S}{b_{t+1}^S} (1 - \xi_S) (\delta - L_{t+1}^S) \right] \right\} \\ &= -\mathbb{E}_t \left\{ M_{t,t+1} \left[\frac{f_{t+1}^S}{b_{t+1}^S} L_{t+1}^S + \frac{F_{t+1}^S r_{t+1}^S}{b_{t+1}^S} + \frac{f_{t+1}^S}{b_{t+1}^S} (1 - \xi_S) (\delta - L_{t+1}^S) \right] \right\} \\ &= -\mathbb{E}_t \left\{ M_{t,t+1} \left[\frac{F_{t+1}^S r_{t+1}^S}{b_{t+1}^S} + \frac{f_{t+1}^S}{b_{t+1}^S} ((1 - \xi_S) \delta + \xi_S L_{t+1}^S) \right] \right\}. \end{aligned}$$

A.3 Gamma Distribution

The gamma cumulative distribution function is $\Gamma(\rho; \chi_0, \chi_1)$ with parameters (χ_0, χ_1) . The parameters map into mean and variance of the distribution is

$$\begin{aligned} \chi_1 &= \sigma_\rho^2 / \mu_\rho \\ \chi_0 &= \mu_\rho / \chi_1. \end{aligned}$$

Denote the pdf as $\gamma(\rho; \chi_0, \chi_1)$.

A standard result in statistics states that the conditional expectations are

$$\begin{aligned} \mathbb{E}(\rho | \rho < x) &= \mu_\rho \frac{\Gamma(x; \chi_0 + 1, \chi_1)}{\Gamma(x; \chi_0, \chi_1)}, \\ \mathbb{E}(\rho | \rho > x) &= \mu_\rho \frac{1 - \Gamma(x; \chi_0 + 1, \chi_1)}{1 - \Gamma(x; \chi_0, \chi_1)}. \end{aligned}$$

This means the expressions for payoffs and recovery values are

$$\begin{aligned} F_\rho^j \rho^{j,-} &= \mu_\rho \Gamma\left(\frac{L_t^j - \delta_j}{1 - \ell_t^j(1 - x_t)}; \chi_0 + 1, \chi_1\right), \\ (1 - F_\rho^j) \rho^{j,+} &= \mu_\rho \left(1 - \Gamma\left(\frac{L_t^j - \delta_S}{1 - \ell_t^j(1 - x_t)}; \chi_0 + 1, \chi_1\right)\right). \end{aligned}$$

A.4 Model with Shadow Bank Run and Stochastic Bailout

This section presents the details of the model we sketch out in Section 4.3 and use to compute our dynamic results. We introduce a shadow bank run similar to Allen and Gale (1994). Shadow banks are exposed to aggregate run-risk. That is, a fraction of shadow bank deposits ϱ is withdrawn early within a given period (affecting all shadow banks equally), where $\varrho_t^S \in \{0, \varrho^* > 0\}$, following a two-state Markov chain. When deposits are withdrawn, shadow banks need to liquidate a fraction $\ell_{t,i}^S$ of their assets. Assets that are liquidated early are sold at price Π_t^H in a separate market at the beginning of the period and do not yield any payoff to the bank. The buyers of these assets are households, which have an inferior ability to operate the asset, leading to a lower payoff $\underline{Z}_t < Z_t$ and a higher depreciation rate $\underline{\delta}_K > \delta_K$. They further do not have access to an investment technology. Households sell the assets again in the regular capital market later in the same period. Household optimization implies that the fire-sale price is³⁴

$$\Pi_t^H = (1 - \eta) \underline{Z}_t (n_{t,i}^H)^\eta + p_t (1 - \underline{\delta}_K), \quad (35)$$

where household labor demand per unit of household capital

$$n_t^H = \frac{N_t^H}{\ell_t^S K_t^S}$$

is implied by the usual first-order condition

$$w_t = \eta \underline{Z}_t (n_{t,i}^H)^{\eta-1}.$$

The timing of decisions within each period is now as follows:

1. Aggregate productivity shocks Z_t , \underline{Z}_t and the early withdrawal shock ϱ_t are realized.
2. If $\varrho_t = \varrho^*$, S-banks sell capital worth $\varrho^* B_t^S$ to households at price Π_t^H .
3. Production of all banks and households and investment decisions of banks ensue.
4. Idiosyncratic payoff shocks of banks are realized. Default decisions.

³⁴Since both transactions take place within the same period and households are unconstrained, it immediately follows that Π_t^H must be equal to the marginal product of capital to households, which is equivalent to Equation (35). Also, note that the marginal product of capital to households is always lower than that of firms, so households never optimally own any capital at the end of the period.

5. Trade occurs in asset markets. Surviving banks pay dividends and new banks are set up to replace liquidated bankrupt banks.
6. Households consume.

We define the fire-sale discount

$$x_t = \frac{\Pi_t^H}{\Pi_t^S} < 1,$$

with Π_t^S defined as in the main text. To pay out its depositors in case of a withdrawal shock ($\varrho_t^S = \varrho^*$), the bank must sell $\varrho^* B_t^S$ worth of assets. Hence the fraction of assets that needs to be liquidated is

$$\ell_{t,i}^S = \frac{\varrho_t^S B_{t,i}^S}{K_{t,i}^S \Pi_t^H} = \frac{\varrho_t^S L_{t,i}^S}{x_t},$$

using the definition of leverage and the fire-sale discount.

Since the idiosyncratic valuation shocks are uncorrelated over time, it is convenient to write the optimization problem of surviving banks after the bankruptcy decision and asset payoffs. All banks have the same value and face identical problems:

$$\begin{aligned} \max_{K_{t+1}^S, B_{t+1}^S} \quad & q_t^S B_{t+1}^S - p_t K_{t+1}^S \\ & + \text{E}_t [M_{t,t+1} \max \{ \rho_{t+1}^S K_{t+1}^S (\ell_{t+1}^S \Pi_{t+1}^H + (1 - \ell_{t+1}^S) \Pi_{t+1}^S) - B_{t+1}^S, \\ & - \delta_S K_{t+1}^S \Pi_{t+1}^S \}]. \end{aligned}$$

Using the definitions for book and market leverage we rewrite the maximum operator, i.e. the continuation value, as

$$\begin{aligned} & K_t^S \Pi_t^S \max \{ \rho_t^S (1 - \ell_t^S (1 - x_t)) - L_t^S, -\delta_S \} \\ & = K_t^S \Pi_t^S \left(\mathbb{1} \left[\rho_t^S \geq \frac{L_t^S - \delta_S}{1 - \ell_t^S (1 - x_t)} \right] (\rho_t^S (1 - \ell_t^S (1 - x_t)) - L_t^S) \right. \\ & \quad \left. - \mathbb{1} \left[\rho_t^S < \frac{L_t^S - \delta_S}{1 - \ell_t^S (1 - x_t)} \right] \delta_S \right). \end{aligned}$$

Taking the expectation of this expression with respect to ρ_t^S , we obtain

$$\begin{aligned} & \underbrace{\left[\left(1 - F_\rho^S \left(\frac{L_t^S - \delta_S}{1 - \ell_t^S (1 - x_t)} \right) \right) (\rho_t^{S,+} (1 - \ell_t^S (1 - x_t)) - L_t^S) - F_\rho^S \left(\frac{L_t^S - \delta_S}{1 - \ell_t^S (1 - x_t)} \right) \delta_S \right]}_{\equiv \mathcal{F}^S(L_t^S)} \times K_t^S \Pi_t^S \end{aligned}$$

where $\rho_t^{S,+} = \text{E} \left(\rho_t^S \mid \rho_t^S > \frac{L_t^S - \delta_S}{1 - \ell_t^S (1 - x_t)} \right)$, $F_\rho^S \left(\frac{L_t^S - \delta_S}{1 - \ell_t^S (1 - x_t)} \right)$ is the probability of default, and $\mathcal{F}^S(L_t^S)$ describes the leverage- and run-adjusted payoff of S-banks' portfolio, including the default option. The payoff is higher the lower banks' leverage, the lower the fraction of liquidated assets ℓ_t , the closer fire-sale prices are to regular market prices x_t , and the lower the utility penalty.

We can now define the per-asset value function

$$v^S(Z_t) = \max_{b_{t+1}^S} q_t^S b_{t+1}^S - p_t + \text{E}_t [M_{t,t+1} \Pi_{t+1}^S \mathcal{F}^S(L_{t+1}^S)], \quad (36)$$

such that the full optimization problem of the S-bank is given by

$$\max_{K_{t+1}^S} K_{t+1}^S v^S(Z_t),$$

subject to $K_{t+1}^S \geq 0$.

C-Banks There is a unit mass of C -banks which are different from S -banks in three ways: (i) they issue short-term debt that is insured and risk free from the perspective of creditors, (ii) they are subject to regulatory capital requirements, and (iii) they are not subject to run risk, i.e. $\varrho_t^C = 0$ and thus $\ell_t^C = 0 \forall t$. They pay an insurance fee of κ for each bond they issue. Using the same notation as for S -banks, the problem of all surviving C -banks is identical to the main text.

Bankruptcy If a bank declares bankruptcy, its equity becomes worthless, and creditors seize all of the banks assets, which are liquidated. The recovery amount per bond issued is hence

$$\begin{aligned} r^j(L_t^j) &= (1 - \xi^j) \frac{\rho_t^{j,-} \left(\ell_t^j \Pi_t^H + (1 - \ell_t^j) \Pi_t^S \right) K_t^j}{B_t^j} \\ &= (1 - \xi^j) \frac{\rho_t^{j,-} \left(1 - \ell_t^j (1 - x_t) \right)}{L_t^j}, \end{aligned} \quad (37)$$

for $j = S, C$, with a fraction ξ^j lost in the bankruptcy proceedings, and with

$\rho_t^{j,-} \equiv \mathbb{E} \left(\rho_t^j \mid \rho_t^j < \left(\frac{L_t^j - \delta_j}{1 - \ell_t^j (1 - x_t)} \right) \right)$. For shadow banks the recovery value, that is, the amount of assets needed to be liquidated as well as the discount (smaller x_t) is also decreasing. After the bankruptcy proceedings are completed, a new bank is set up to replace the failed one. This bank sells its equity to new owners, and is otherwise identical to a surviving bank after asset payoffs.

If a S -bank defaults, the recovery value per bond is used to pay the claims of bondholders to the extent possible. We further consider the possibility that the government bails out the bond holders of the defaulting S -bank with a probability π_B , known to all agents ex ante. If a C -bank declares bankruptcy, the bank is taken over by the government that uses lump sum taxes and revenues from deposit insurance, κB_{t+1}^C , to pay out the bank's creditors in full. Summing over defaulting C -banks and S -banks that are bailed out, we define lump sum taxes as

$$T_t = F_{\rho,t}^C (1 - r^C(b_t^C)) B_t^C - \kappa B_{t+1}^C + \pi_B F_{\rho,t}^S (1 - r^S(b_t^S)) B_t^S.$$

The beginning-of-period dividend paid by banks to households, conditional on survival, for S -banks is

$$D_t^{S,+} = \rho_t^{S,+} K_t^S (\ell_t^S \Pi_t^H + (1 - \ell_t^S) \Pi_t^S) - B_t^S + K_{t+1}^S (q_t^S b_{t+1}^S - p_t),$$

and for C -banks

$$D_t^{C,+} = \rho_t^{C,+} K_t^C \Pi_t^C - B_t^C + K_{t+1}^C ((q_t^C - \kappa) b_{t+1}^C - p_t).$$

Households The set-up of the household problem is identical to the one presented in the main text up to random shadow bank bailout. Debt they hold at the beginning of the period affects the household problem only through the budget constraint. Denoting household wealth at the beginning of the period by W_t , we obtain the following expression for the complete intertemporal problem of households:

$$V^H(A_t^S, A_t^C, W_t, Y_t) = \max_{C_t, A_{t+1}^S, A_{t+1}^C, S_t^S, S_t^C} U(C_t, H(A_t^S, A_t^C)) + \beta \mathbb{E}_t [V(A_{t+1}^S, A_{t+1}^C, W_{t+1}, Y_{t+1})]$$

subject to

$$W_t + Y_t + w_t - T_t = C_t + \sum_{j=S,C} p_t^j S_t^j + \sum_{j=S,C} q_t^j A_{t+1}^j \quad (38)$$

$$\begin{aligned} W_{t+1} = & \sum_{j=S,C} (1 - F_\rho^j(L_{t+1}^j)) (D_{t+1}^{j,+} + p_{t+1}^j) S_t^j \\ & + A_{t+1}^S [1 - F_\rho^S(L_{t+1}^S) + F_\rho^S(L_{t+1}^S) (\pi_B + (1 - \pi_B) r_{t+1}^S)] \\ & + A_{t+1}^C. \end{aligned} \quad (39)$$

Market Clearing The market clearing conditions are identical to the main text except for the goods market, capital market clearing, and labor market clearing conditions. Capital market clearing is

$$\sum_j K_{t+1}^j = \sum_j I_t^j + (1 - \delta_K) \sum_j \left(1 - \xi^j F_{\rho,t}^j \rho_t^{j,-}\right) (1 - \ell_t^j) K_t^j + (1 - \underline{\delta}_K) \sum_j \ell_t^j K_t^j. \quad (40)$$

Note that the additional depreciation of foreclosed S-banks only affects the capital held by S-banks at the time of bankruptcy, $(1 - \ell_t^S) K_t^S$, but capital held by households at the time of production has a higher baseline rate of depreciation $\underline{\delta}_K$.

Goods market clearing is

$$\begin{aligned} C_t + \sum_j \left(I_t^j + \Phi((1 - \ell_t^j) K_t^j, I_t^j) \right) = & Y_t + Z_t \sum_j (N_t^j)^\eta \left((1 - \ell_t^j) K_t^j \right)^{1-\eta} + \underline{Z}_t (N_t^H)^\eta \left(\sum_j \ell_t^j K_t^j \right)^{1-\eta} \\ & - \sum_j \xi^j F_{\rho,t}^j \rho_t^{j,-} \left(\ell_t^j \Pi_t^H + (1 - \ell_t^j) (\Pi_t^j - (1 - \delta_K) p_t) \right) K_t^j. \end{aligned} \quad (41)$$

This condition³⁵ also indicates that at the time of production and investment, banks only have the capital that they did not have to sell due to early withdrawals, $(1 - \ell_t^j) K_t^j$. Labor market clearing now also includes labor demand of households

$$1 = N_t^S + N_t^C + N_t^H.$$

Household Optimal Choice of S-bank Debt The first-order condition for households' choice of S-bank debt now takes into account the probability of a bailout

$$q_t^S = \text{E}_t \left\{ M_{t,t+1} [1 - F_\rho^S(L_{t+1}^S) + F_\rho^S(L_{t+1}^S) (\pi_B + (1 - \pi_B) r_{t+1}^S(L_{t+1}^S)) + \text{MRS}_{S,t+1}] \right\}. \quad (42)$$

S-banks Optimal Leverage Choice with Runs Differentiating Equation (36) with respect to L_{t+1}^S results in the FOC of S-banks for leverage

$$q(b_{t+1}^S) + b_{t+1}^S q'(b_{t+1}^S) = \text{E}_t \left[M_{t+1} (1 - F_{t+1}^S) \left(1 + \ell_{t+1}^S (1 - x_{t+1}) \frac{\rho_{t+1}^{S,+}}{L_{t+1}^S} \right) \right]. \quad (43)$$

The partial derivative $q'(b_{t+1}^S)$ can be obtained directly from households' FOC for purchases of shadow bank debt. Differentiating Equation (42) yields

³⁵For notational symmetry, both conditions allow some C-bank capital to be held by households. Given our assumptions, we have $\ell_t^C = 0$ everywhere.

$$\frac{\partial q^S(b_{t+1}^S)}{\partial b_{t+1}^S} = -\mathbb{E}_t \left\{ (1 - \pi_B) M_{t,t+1} \left[\frac{F_{t+1}^S r_{t+1}^S}{b_{t+1}^S (1 - \ell_{t+1}(1 - x_{t+1}))} + \frac{f_{t+1}^S}{b_{t+1}^S} \mathcal{L}^S(L_{t+1}^S) ((1 - \xi_S)\delta_S + \xi_S L_{t+1}^S) \right] \right\}, \quad (44)$$

where

$$\mathcal{L}^S(L_{t+1}^S) = \frac{1 - \delta_S \ell_{t+1} \frac{1-x_{t+1}}{L_{t+1}^S}}{(1 - \ell_{t+1}(1 - x_{t+1}))^2}.$$

A.4.1 FOC for b_t^S

Rewrite $\mathcal{F}^S(L_t^S)$ as

$$\begin{aligned} \mathcal{F}^S(L_t^S) &= (1 - F_t^S) \left(\rho_t^{S,+} (1 - \ell_t(1 - x_t)) - L_t^S \right) - F_t^S \delta_S \\ &= (1 - \ell_t(1 - x_t)) \int_{\frac{L_t^S - \delta_S}{1 - \ell_t(1 - x_t)}}^{\infty} \rho f_t^S(\rho) d\rho - (1 - F_t^S) L_t^S - F_t^S \delta_S. \end{aligned}$$

Further compute

$$\frac{\partial(1 - \ell_t(1 - x_t))}{\partial L_t^S} = -\rho_t^S \frac{1 - x_t}{x_t} = -\ell_t \frac{1 - x_t}{L_t^S}.$$

and define

$$\mathcal{L}_t^S(L_t^S) = \frac{\partial}{\partial L_t^S} \frac{L_t^S - \delta_S}{1 - \ell_t(1 - x_t)} = \frac{1 - \delta_S \rho_t^S \frac{1-x_t}{x_t}}{(1 - \ell_t(1 - x_t))^2} = \frac{1 - \delta_S \ell_t \frac{1-x_t}{L_t^S}}{(1 - \ell_t(1 - x_t))^2}.$$

Now we can calculate

$$\begin{aligned} \frac{\partial \mathcal{F}^S(L_t^S)}{\partial L_t^S} &= -\ell_t \frac{1 - x_t}{L_t^S} (1 - F_t^S) \rho_t^{S,+} - f_t^S \mathcal{L}_t^S(L_t^S) (L_t^S - \delta_S) - (1 - F_t^S) + f_t^S \mathcal{L}_t^S(L_t^S) L_t^S - f_t^S \mathcal{L}_t^S(L_t^S) \delta_S \\ &= -(1 - F_t^S) \left(1 + \ell_t(1 - x_t) \frac{\rho_t^{S,+}}{L_t^S} \right). \end{aligned}$$

Therefore the FOC for b_t^S is

$$q_t^S = -b_t^S q'(b_t^S) + \mathbb{E}_t \left[M_{t+1} (1 - F_t^S) \left(1 + \ell_t(1 - x_t) \frac{\rho_t^{S,+}}{L_t^S} \right) \right].$$

A.4.2 Derivation of $(q^S)'(b_t^S)$

Recall the definition of the recovery value for S-banks is

$$r^S(L_t^S) = (1 - \xi^S) \frac{\rho_t^{S,-} (1 - \ell_t(1 - x_t))}{L_t^S},$$

with the conditional expectation

$$\rho_t^{S,-} = \mathbb{E}_{\rho,S} \left[\rho \mid \rho < \frac{L_t^S - \delta_S}{1 - \ell_t(1 - x_t)} \right].$$

First compute

$$\frac{\partial}{\partial L_t^S} \frac{1 - \ell_t(1 - x_t)}{L_t^S} = \frac{-\varrho_t^S \frac{1-x_t}{x_t} L_t^S - (1 - \ell_t(1 - x_t))}{(L_t^S)^2} = -\frac{1}{(L_t^S)^2}.$$

We can rewrite the recovery value times the probability of default as

$$F_t^S r_t^S = \frac{(1 - \xi_S)(1 - \ell_t(1 - x_t))}{L_t^S} \int_{-\infty}^{\frac{L_t^S - \delta_S}{1 - \ell_t(1 - x_t)}} \rho dF_{\rho, S}(\rho).$$

Differentiating this expression with respect to b_t^S gives

$$\begin{aligned} \frac{\partial F_t^S r_t^S}{L_t^S} &= -\frac{(1 - \xi_S)}{(L_t^S)^2} F_t^S \rho_t^{S,-} + \frac{f_t^S}{L_t^S} (1 - \xi_S) \mathcal{L}_t^S(L_t^S)(L_t^S - \delta) \\ &= -\frac{F_t^S r_t^S}{L_t^S(1 - \ell_t(1 - x_t))} - \frac{f_t^S}{L_t^S} \mathcal{L}_t^S(L_t^S)(1 - \xi_S)(\delta - L_t^S). \end{aligned}$$

Using $\partial L_t^S / \partial b_t^S = 1 / \Pi_t^S$, the complete partial derivative of the household's bond pricing equation for q^S in Equation (29) with respect to b_{t+1}^S is therefore

$$\frac{\partial q^S(b_{t+1}^S)}{\partial b_{t+1}^S} = -E_t \left\{ (1 - \pi_B) M_{t,t+1} \left[\frac{F_{t+1}^S r_{t+1}^S}{b_{t+1}^S(1 - \ell_{t+1}(1 - x_{t+1}))} + \frac{f_{t+1}^S}{b_{t+1}^S} \mathcal{L}_{t+1}^S(L_{t+1}^S) ((1 - \xi_S)\delta_S + \xi_S L_{t+1}^S) \right] \right\}.$$

B Calibration Appendix

The data counterparts to key model objects are listed in Table 5.

Choosing the appropriate data targets is not trivial, e.g. the size of the bank-dependent sector. In the U.S. a large part of the economy is not bank-dependent, i.e. most firms (value-weighted) can access public bond markets and issue public equity. A sizeable share (equal weighted) of the economy, however, does depend on bank funding. We follow Kashyap, Lamont, and Stein (1994) and define bank-dependent firms as firms that do not have a S&P long-term credit rating.³⁶ Mortgages make up the largest share of bank loan portfolios. For this reason, we also add construction and real estate firms to the set of bank-dependent firms.³⁷ The idea is that households finance their purchases of housing with the help of the banking sector. Bank-dependent output is defined as the sum of the quarterly sales of bank-dependent firms. Table 5 summarizes our other data target choices. There has been a fairly persistent downward trend in interest rates over our sample period (particularly since 2008). For this reason, we target average rates since the crisis, i.e. 2009-2015.

The bankruptcy cost parameters ξ_S and ξ_C are relevant for the deadweight loss from bank default. We choose a bankruptcy cost of 37% to target a recovery rate of 63% from Moody reports on financial sector bond recoveries.³⁸ We set κ , the deposit insurance fee to 0.0006168. This is in the range of quarterly FDIC assessment rates.

In the full quantitative version of the model described in Appendix A.4, each period a fraction of households runs on the shadow banking sector.³⁹ This fraction is denoted as ϱ_t and follows a

³⁶We use data from Compustat quarterly fundamentals (compm/fundq /) as well as Compustat's credit rating database (compm/rating/).

³⁷That is, we include firms with sic code 6500-6599 (real estate), 1500-1599 (construction), and 1700-1799 (construction contractors, special trades).

³⁸We use Moody's 1984-2004 reports. Exhibit 9 in the report presents the recovery rates of defaulted bonds for financial institutions. We use the mean for financial institutions over all bonds and preferred stocks.

³⁹An alternative interpretation of the run states in the model are episodes of large money market mutual fund withdrawals.

Table 5: Mapping Model to Data

	Description	Mapping to Data	Data Source	Period
A^S	Money-like liab. of S-banks	Money market shares, CP, Repo	Flow of Funds	1999-2015
A^C	Money-like liab. of S-banks	Deposits + currency	Flow of Funds	1999-2015
$p \times K$	Financial Sector Assets -FDI	Assets fin. sector -misc. assets - interbank liab.	Flow of Funds	1999-2015
F^C	Default rate of C-banks	Charge-off rates	FRB/FDIC	1999-2015
C	Consumption	real p.c. consumption	NIPA	1999-2015
Y^B	Bank-dependent output	Real estate, Construction & no LT Debt Rating	Compustat	1999-2015
Y	Non-bank-dependent output	GDP (2009\$) p.c. $-Y^B$	NIPA	1999-2015
L^C	Leverage	value weighted book leverage SIC $\in \{[6000, 6089], 6712\}$	Compustat	1999-2015
L^S	Leverage	value weighted book leverage SIC $\in \{[6111, 6299]\}$	Compustat	1999-2015
q^S	C-bank debt price	Financial AA commercial paper rate 3 month	FRED	2009-2015
q^C	S-bank debt price	National Rate on Non-Jumbo Deposits 3 month	FRED	2009-2015

Markov process that takes two values: zero in the first state (no run) and 0.3 in the second state (run state), i.e. 30% of depositors run on shadow banks. In the simulation, bank runs occur only in bad economic states. We interpret the run on shadow banks in the model as akin to a run on ABCP programs and repurchase agreements (e.g. Covitz, Liang, and Suarez (2013) and Gorton and Metrick (2012)). Commercial paper and repurchase agreements are common funding sources for dealer banks and finance companies. We pick a household run fraction of 30% based on Covitz, Liang, and Suarez (2013)'s observation that 30% of ABCP programs experienced a run at the beginning of the crisis. We set the transition probability between bank run and non-bank run states to target an unconditional probability of a bank-run of 3%, i.e. there is one bank run every 33 years, roughly consistent with the past 100 years of U.S. banking history.

C Computation Appendix

C.1 System of Equations Characterizing Equilibrium

The solution to the model including bank runs as specified in Appendix A.4 can be written as a system of 17 nonlinear functional equations in equally many unknown functions of the state variables. The model's state variables are $\mathcal{S}_t = (Y_t, Z_t, \varrho_t, K_t^C, K_t^S, A_t^C, A_t^S)$.

The functions are aggregate consumption $C(\mathcal{S}_t)$, prices of C-bank and S-bank equity ($p^C(\mathcal{S}_t), p^S(\mathcal{S}_t)$), prices of C-bank and S-bank deposits ($q^C(\mathcal{S}_t), q^S(\mathcal{S}_t)$), C-bank and S-bank deposit issuance per unit of capital ($b^C(\mathcal{S}_t), b^S(\mathcal{S}_t)$), the Lagrange multiplier on C-bank leverage $\lambda^C(\mathcal{S}_t)$, C-bank and S-bank capital purchases ($K^C(\mathcal{S}_t), K^S(\mathcal{S}_t)$), the capital price $p(\mathcal{S}_t)$, C-bank and S-bank investment ($I^C(\mathcal{S}_t), I^S(\mathcal{S}_t)$), labor demand of C-bank, S-bank and households ($N^C(\mathcal{S}_t), N^S(\mathcal{S}_t), N^H(\mathcal{S}_t)$), and the wage $w(\mathcal{S}_t)$. Collect these functions in the vector $\vec{X}(\mathcal{S}_t)$.

For the equations, we will use time subscripts to express dependence on state variables. All variables can be expressed as functions of current (\mathcal{S}_t) or one-period ahead (\mathcal{S}_{t+1}) state variables. Note that several of the equations contain auxiliary functions such as the household discount factor $M_{t,t+1}$ or the marginal product of capital Π_t^j . All these functions are explicit functions of the fundamental unknowns $\vec{X}(\mathcal{S}_t)$ and are defined in the main text of the paper and the model appendix.

$$p_t^C = \text{E}_t \left[M_{t,t+1} \left(D_{t+1}^{C,+} + p_{t+1}^C \right) \right] \quad (45)$$

$$p_t^S = \text{E}_t \left[M_{t,t+1} \left(D_{t+1}^{S,+} + p_{t+1}^S \right) \right] \quad (46)$$

$$q_t^C = \text{E}_t \left[M_{t,t+1} \left(1 + Q_{t+1} \left(\frac{H_{t+1}}{A_{t+1}^C} \right)^{1-\alpha} \right) \right] \quad (47)$$

$$q_t^S = \text{E}_t \left[M_{t,t+1} \left(1 - F_{t+1}^S (1 - (\pi_B + (1 - \pi_B)r_{t+1}^S)) + (1 - F_{t+1}^S)^\nu Q_{t+1} \left(\frac{H_{t+1}}{A_{t+1}^S} \right)^{1-\alpha} \right) \right] \quad (48)$$

$$\begin{aligned} C_t + I_t^C + I_t^S + \Phi(I_t^C, K_t^C) + \Phi(I_t^S, (1 - \ell_t^S)K_t^S) &= Y_t + Y_t^C + Y_t^S + Y_t^H \\ - \xi^C F_t^C \rho_t^{C,-} (\Pi_t^C - (1 - \delta_K)p_t) K_t^C & \\ - \xi^S F_t^S \rho_t^{S,-} (\ell_t^S \Pi_t^H + (1 - \ell_t^S)(\Pi_t^S - (1 - \delta_K)p_t)) K_t^S & \end{aligned} \quad (49)$$

$$q_t^S + b_{t+1}^S (q^S)'(b_{t+1}^S) = \text{E}_t \left[M_{t,t+1} (1 - F_{t+1}^S) \left(1 + \ell_{t+1}^S (1 - x_{t+1}) \frac{\rho_{t+1}^{S,+}}{L_{t+1}^S} \right) \right] \quad (50)$$

$$q_t^C - \kappa = \lambda_t^C + \text{E}_t [M_{t,t+1} (1 - F_{t+1}^C)] \quad (51)$$

$$\lambda_t^C (p_t - (1 - \theta)b_{t+1}^C) = 0 \quad (52)$$

$$p_t - q_t^S b_{t+1}^S = \text{E}_t [M_{t,t+1} \Pi_{t+1}^S \mathcal{F}^S(L_{t+1}^S)] \quad (53)$$

$$p_t - (q_t^C - \kappa) b_{t+1}^C = \text{E}_t [M_{t,t+1} \Pi_{t+1}^C \mathcal{F}^C(L_{t+1}^C)] \quad (54)$$

$$\begin{aligned} K_{t+1}^C + K_{t+1}^S &= I_t^C + I_t^S + (1 - \delta_K) \left(1 - \xi^C F_{\rho,t}^C \rho_t^{C,-} \right) K_t^C \\ &+ (1 - \delta_K) \left(1 - \xi^S F_{\rho,t}^S \rho_t^{S,-} \right) (1 - \ell_t^S) K_t^S + (1 - \delta_K) \ell_t^S K_t^S \end{aligned} \quad (55)$$

$$I_t^C = \left(\frac{p_t - 1}{\phi} + \delta_K \right) K_t^C \quad (56)$$

$$I_t^S = \left(\frac{p_t - 1}{\phi} + \delta_K \right) (1 - \ell_t^S) K_t^S \quad (57)$$

$$w_t = \eta Z_t (n_t^C)^{\eta-1} \quad (58)$$

$$N_t^C = \frac{K_t^C}{K_t^C + (1 - \ell_t^S) K_t^S + \ell_t^S K_t^S (Z_t/Z_t)^{1/(1-\eta)}} \quad (59)$$

$$N_t^S = \frac{(1 - \ell_t^S) K_t^S}{K_t^C} N_t^C \quad (60)$$

$$N_t^H = 1 - N_t^C - N_t^S \quad (61)$$

Equations (45) to (48) are the household first-order conditions for purchases of bank equity and debt, for C- and S-banks, respectively. Equation (49) is the market clearing condition for the consumption good. Equations (50) and (51) are the S-bank and C-bank first-order conditions for leverage, and equation (52) is the complementary slackness condition for the C-bank’s leverage constraint. Equations (53) and (54) are the zero expected profit conditions determining the size of the C- and S-bank sectors (equivalently the first-order conditions for capital purchases of C- and S-bank). Equation (55) is the market clearing condition for capital. Equations (56) and (57) are the first-order conditions for investment of each banking sector. Equation (58) is the first-order condition for labor demand of C-banks. Finally, equations (59) to (61) solve for labor demand in each sector by using the first-order conditions for labor demand for S-banks and the household sector (in case of a run), and imposing market clearing in the labor market.

Note that (45) – (48), (50), (51), (53), and (54) have expectations terms on the RHS (they are “Euler equations”). To simultaneously evaluate the left and right side of these equations, it is necessary to know the Markovian transition mapping of the state variables

$$\mathcal{S}_{t+1} = \mathcal{H}(\mathcal{S}_t).$$

For the exogenous state variables (Y_t, Z_t, ϱ_t) , this mapping is directly given by the parameters of the model. For the endogenous state variables $(K_t^C, K_t^S, A_t^C, A_t^S)$, the mapping trivially follows from the C- and S-bank choice variables $(b^C(\mathcal{S}_t), b^S(\mathcal{S}_t))$ and $(K^C(\mathcal{S}_t), K^S(\mathcal{S}_t))$, i.e.

$$\begin{aligned} K_{t+1}^C &= K^C(\mathcal{S}_t), \\ K_{t+1}^S &= K^S(\mathcal{S}_t), \\ A_{t+1}^C &= K^C(\mathcal{S}_t)b^C(\mathcal{S}_t), \\ A_{t+1}^S &= K^S(\mathcal{S}_t)b^S(\mathcal{S}_t). \end{aligned}$$

Given the set of unknown functions $\vec{X}(\mathcal{S}_t)$ and the transition mapping for the state variables $\mathcal{H}(\mathcal{S}_t)$, the solution procedure consists of the following steps

1. **Define approximating basis for the unknown functions.** There is an equal number of unknown functions $\vec{X}(\mathcal{S}_t)$ and nonlinear functional equations (45) – (61). To approximate the unknown functions in the space of the state variables, we discretize the state space and use multivariate linear interpolation (splines or polynomials of various orders achieved inferior results due to their lack of global shape preservation). Denote grid points in the discretized state space as \mathcal{S}_i for $i = 1, \dots, N_S$.
2. **Iteratively solve for the unknown functions.** Given an initial guess for the transition mapping $\mathcal{H}^0(\mathcal{S})$ and for the unknown functions $\vec{X}^0(\mathcal{S})$, first compute tomorrow’s states as function of today’s states at all points in the discretized state space, $\mathcal{S}'_i = \mathcal{H}^0(\mathcal{S}_i)$. Then use the guess $\vec{X}^0(\mathcal{S})$ to compute tomorrow’s optimal policies as functions of tomorrow’s states, $\vec{X}^0(\mathcal{S}'_i)$. Using these future optimal policies, compute the expectation terms using quadrature methods. Given the expectations, solve the system of nonlinear equations for the current optimal policies at each point in the discretized state space. Using the solution vector for current policies, compute the next iterate of the approximation $\vec{X}^1(\mathcal{S})$ and repeat until convergence. The system of nonlinear equations at each point in the state space is solved using a standard nonlinear equation solver.
3. **Simulate the model for many periods using approximated policy functions.** To obtain the quantitative results, we simulate the model for 10,000 periods after a “burn-in” phase of 500 periods. We verify that the simulated time path stays within the bounds of the state space for

which the policy functions were computed. We further check the accuracy of the solution by computing relative Euler equation errors at all simulation periods. Maximum errors are bounded below 0.5% for all equations, with average errors well below 0.01%.

D Results Appendix

D.1 Benchmark Economy

Table 6 reports means, standard deviations, business cycle correlations (correlations with Y), correlations with Z and ϱ , as well as autocorrelations, for the benchmark economy with $\theta = 10\%$.

The price for the intermediated asset moves together with Y and Z . Good times make intermediation more valuable. Co-movements with Z are not surprising, while co-movements with Y are induced by the complementarity between consumption and liquidity on the demand side and the loosening of the leverage constraint (regulatory and endogenous, respectively) on the supply side. The deposit rates for commercial and shadow banks are countercyclical in the Z shock. Time variation in the interest rate is caused by (i) the standard intertemporal consumption smoothing motive, and (ii) fluctuations in the marginal benefit of liquidity. While the first source of variation induces the usual procyclical movement with GDP shocks, the second source causes interest rates to be very countercyclical in the Z shock. The return on equity for both types of banks is of similar magnitude – the expected excess return for both types is around 1 percentage point. Both bank’s equity returns contain countercyclical risk premiums.

The liquidity quality factor Λ^S is decisively procyclical as shadow banks’ survive with higher likelihood in good times. During run states liquidity quality deteriorates. The convenience yields measure the marginal benefit of liquidity and are components of the bond prices q^C and q^S . These convenience yields are quantitatively large given our calibration. In states with low payoffs of the Z asset, banks tighten their supply of debt that yields liquidity services, leading to a high marginal benefit and high convenience yields during those times.

The model captures countercyclical market value leverage for both banks. The balance sheet of commercial banks moves with Z , a reflection of a relaxed leverage constraint in good states.⁴⁰

Overall, the financial sector is more stable in good economic times as default risk is lower. On the flipside, deadweight losses due to intermediary bankruptcy are higher in bad states of the economy, in particular those bad states that affect the financial sector the most (low Z shock).

D.2 Robustness of Main Effects

In this section, we test how robust our main results are to changes in the environment. We present the same capital requirement experiment as in the main text under three different parameter setting. The first experiment studies the effects of higher capital requirements when the bailout probability is lower than in the benchmark calibration. The second experiment studies the impact of larger haircuts in the run states for our main experiment. In the final variation of our main experiment, runs on shadow banks are not allowed.

⁴⁰We designed the leverage constraint to reflect a Basel II type of capital requirement that has been criticized for its procyclical effects.

Table 6: Moments of Key Variables of the Benchmark Economy ($\theta = 10\%$).

	Mean	SD	Corr Y	Corr Z	Corr ρ	AC
Prices						
Capital Price	1.002	0.006	0.871	0.804	-0.136	0.260
Deposit Rate S	0.16%	0.42%	-0.808	-0.901	0.054	0.247
Deposit Rate C	0.14%	0.42%	-0.796	-0.912	0.195	0.242
EER Equity S	0.98%	0.01%	-0.540	-0.434	-0.171	0.811
EER Equity C	1.00%	0.06%	-0.138	0.035	-0.973	0.213
Liquidity						
Λ^S	0.842	0.022	0.744	0.817	-0.399	-0.005
Convenience Yield S	1.01%	0.06%	-0.070	-0.213	0.986	0.232
Convenience Yield C	0.98%	0.01%	-0.558	-0.423	-0.390	0.682
Quantities						
Capital Stock	2.100	0.027	0.125	0.046	-0.010	0.971
Debt S	0.308	0.035	-0.020	0.061	-0.500	0.472
Debt C	0.692	0.035	0.020	-0.060	0.494	0.468
Capital Share S	0.310	0.032	-0.030	0.001	-0.198	0.204
Asset Value S	0.652	0.069	0.032	0.049	-0.202	0.227
Asset Value C	1.451	0.069	0.167	0.107	0.175	0.235
Leverage S	0.907	0.005	-0.821	-0.792	0.119	0.005
Leverage C	0.900	0.006	-0.456	-0.539	0.097	-0.366
Early Liquidation Share	0.009	0.053	-0.019	-0.188	1.000	0.197
Bankruptcy Losses						
DWL S	0.03%	0.00%	-0.615	-0.663	0.210	-0.077
DWL C	0.15%	0.04%	-0.425	-0.552	0.103	-0.328
Default Rate S	0.20%	0.03%	-0.740	-0.814	0.408	-0.004
Default Rate C	0.30%	0.07%	-0.463	-0.579	0.067	-0.354
Utility						
Liquidity	2.183	0.043	0.575	0.531	-0.280	0.766
Consumption	0.621	0.003	0.809	0.664	-0.061	0.593
Aggregate Welfare	-139.911	0.114	0.298	0.059	0.767	0.466

Table 7: Higher Capital Requirements at $\pi^B = 20\%$

	10%		15%		20%	
	mean	stdev	mean	stdev	mean	stdev
	Utility					
Liquidity	2.270	0.039	2.194	0.038	2.127	0.040
Consumption	0.621	0.003	0.622	0.003	0.622	0.003
Welfare CE units			0.079%		-0.072%	
	Bankruptcy Losses					
DWL S	0.012	0.002	0.013	0.003	0.014	0.003
DWL C	0.136	0.032	0.018	0.005	0.002	0.000
Default Rate S	0.078%	0.013%	0.079%	0.013%	0.078%	0.013%
Default Rate C	0.296%	0.065%	0.045%	0.011%	0.004%	0.001%

^a: Deadweight losses are in units of the consumption good, multiplied by factor 100.

^b: Welfare is the percentage change of the mean household value function relative to the benchmark model with $\theta = 0.1$ expressed in consumption equivalent units.

Lower Bailout Probability ($\pi^B = 20\%$) Shadow banks have more incentives to internalize the credit risk of their bonds when the bailout probability is lower. This leads to half the default rates of shadow banks compared with the benchmark calibration (see Table 7). The table further shows that the overall level of welfare is higher (more liquidity provision due to higher shadow banking liquidity quality at about the same level of consumption and less volatility) when shadow banks are safer. Since an increase in θ shifts intermediation activity towards shadow banks, the optimal capital requirement has to trade-off a reduction in deadweight losses due to commercial banks against an increase in deadweight losses due to shadow banks. If those banks have a lower failure rate and cause fewer bankruptcy losses, then for each dollar of intermediated assets shifted to shadow banks there is a greater reduction in deadweight losses. This provides more scope for ameliorating incentives for moral hazard in the commercial banking sector. The optimal capital requirement is in the interval between 15% and 20%.

This exercise shows that the potential benefit from higher capital requirements depends on the response of non-regulated banks in the economy. In our calibration, reducing implicit bailout guarantees for shadow banks at the same time turns out to increase the benefit of raising capital requirement.

Larger Haircuts in Run States ($\delta_K = 0.3$) So far our results do not suggest that a higher shadow banking intermediation share leads to more financial fragility. In this section we study the response to a higher capital requirement when average haircuts in run states amount to roughly 35% (see Table 8). The first-order effect of higher haircuts in our model is significantly lower shadow banking intermediation share in the baseline model (at $\theta = 10\%$). Households price shadow bank rationally, pricing the large expected losses during run states. As in our other results, higher capital requirements increase the intermediation share of the unregulated banking sector but at a significantly lower level.

The deadweight losses caused by the unregulated sector are lower primarily due to their reduced

Table 8: Higher Capital Requirements at $\bar{\delta}_K = 0.3$

	10%		15%		20%	
	mean	stdev	mean	stdev	mean	stdev
	Financial Intermediation Size					
Capital stock	2.077	0.033	2.089	0.036	2.067	0.035
Capital Share S	0.141	0.022	0.163	0.028	0.186	0.031
	Utility					
Liquidity	2.105	0.071	2.034	0.080	1.935	0.083
Consumption	0.620	0.003	0.622	0.003	0.621	0.003
	Bankruptcy Losses					
DWL S	0.011	0.011	0.013	0.013	0.015	0.015
DWL C	0.186	0.042	0.026	0.007	0.002	0.001
Default Rate S	0.194%	0.165%	0.194%	0.172%	0.194%	0.167%
Default Rate C	0.294%	0.065%	0.045%	0.012%	0.004%	0.001%

^a: Deadweight losses are in units of the consumption good, multiplied by factor 100.

share. The default rate is about the same as in the benchmark model. The results in this table suggests that the economy confines the riskiness of shadow banks by simply reducing their size.

No Runs on Shadow Banks Table 9 presents the moments of the model without bank runs in the shadow banking sector under three different capital requirements. This variation produces very similar effects as our capital requirement experiment in the steady-state economy discussed in Section 3. A higher capital requirement in the steady-state economy led to an increase in the capital stock. Moreover, as the unregulated sector is less risky, its intermediation share is higher compared to the benchmark experiment discussed in Section 4.4.

Table 9: Moments for Different Capital Requirements (No Bank Run Model)

	10%		15%		20%	
	mean	stdev	mean	stdev	mean	stdev
Prices						
Capital Price	1.002	0.006	1.001	0.006	1.000	0.006
Deposit Rate S	0.190%	0.425%	0.189%	0.430%	0.185%	0.429%
Deposit Rate C	0.121%	0.420%	0.058%	0.425%	-0.009%	0.423%
EER Equity S	0.941%	0.057%	0.941%	0.057%	0.941%	0.060%
EER Equity C	1.011%	0.057%	1.069%	0.057%	1.126%	0.060%
Liquidity						
Λ^S	0.840	0.020	0.840	0.020	0.840	0.021
Convenience Yield S	0.969%	0.009%	0.969%	0.009%	0.971%	0.009%
Convenience Yield C	0.990%	0.012%	1.054%	0.012%	1.118%	0.013%
Quantities						
Capital stock	2.112	0.027	2.112	0.027	2.115	0.027
Debt S	0.353	0.004	0.396	0.003	0.436	0.003
Debt C	0.647	0.004	0.604	0.003	0.564	0.003
Capital Share S	0.353	0.004	0.396	0.003	0.436	0.003
S-bank Capital Stock	0.745	0.013	0.835	0.014	0.922	0.015
Investment	0.054	0.006	0.053	0.006	0.053	0.006
Asset Value S	0.746	0.013	0.836	0.014	0.922	0.015
Asset Value C	1.369	0.02	1.277	0.017	1.193	0.016
Leverage S	0.908	0.005	0.908	0.005	0.908	0.005
Leverage C	0.9 00	0.006	0.850	0.006	0.800	0.005
Utility						
Liquidity	2.196	0.040	2.115	0.040	2.042	0.041
Consumption	0.621	0.003	0.623	0.003	0.623	0.003
GDP	0.676	0.008	0.676	0.008	0.676	0.008
Bankruptcy Losses						
DWL S	0.030	0.004	0.034	0.005	0.038	0.005
DWL C	0.144	0.033	0.019	0.005	0.002	0.001
Default Rate S	0.205%	0.028%	0.205%	0.029%	0.205%	0.029%
Default Rate C	0.297%	0.065%	0.045%	0.012%	0.005%	0.001%

^a: Deadweight losses are in units of the consumption good, multiplied by factor 100.