An Equilibrium Model of the International Price System*

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Abstract

The currency in which international prices are set is a factor of fundamental importance in international economics: it determines the benefits of floating versus pegged exchange rates and the spillover effects of national monetary policy on other economies. However, the standard assumption in existing models — that all prices are set in a currency of either the producer or the consumer — is inconsistent with two basic facts: the dominant status of the dollar in global trade and the radical transformation of the price system over history. In this paper, I develop a general equilibrium multi-country framework with *endogenous* currency choice that is consistent with these stylized facts and show that despite small costs for exporters, the aggregate effects of currency choice are large. First, I identify a novel source of positive U.S. monetary spillovers on foreign output that can outweigh the standard "beggar-thy-neighbor" effect. Second, I show that an optimal monetary policy implies a partial peg to the dollar, which is consistent with the "fear of floating" and the widespread use of the dollar as an anchor currency seen in the data.

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1 Introduction

The currency in which international prices are set is crucial for the transmission of monetary shocks across countries. In a world with sticky nominal prices and large fluctuations in exchange rates, the exporters' currency choice determines which relative prices in the global economy remain stable in the medium run and which ones fluctuate one-to-one with exchange rates. The answers to the fundamental questions in international economics can change drastically, depending on what assumptions are made about the firms' currency choices. In particular, while the classical argument in favor of floating exchange rates (Friedman 1953) holds when the prices are set in the currency of the producer (producer currency pricing, PCP), pegging the exchange rate can be optimal when prices are set in the currency of the consumer (local currency pricing, LCP) (Devereux and Engel 2003). Similarly, the spillover effects of monetary policy on foreign output, which have been at the center of public debates during the global recession (Bernanke 2017), are negative under PCP and positive under LCP (Betts and Devereux 2000).

The standard assumptions in the existing models are, however, inconsistent with two basic empirical facts about the "International Price System" (Gopinath 2016). First, while most of the theoretical literature has focused on the case of PCP and, to a lesser extent LCP, the empirical evidence shows that, for the bulk of international trade, prices are set in just a few currencies, with dollar being the dominant one (see Figures A1) (Goldberg and Tille 2008).[1] This suggests that the transmission of shocks across countries might be more asymmetric than predicted by the existing models. Second, the robust relationship between currency choice and the characterisitcs of the specific firm, industry and country (Gopinath, Itskhoki, and Rigobon 2010), as well as the radical transformation of the international price system over history (Eichengreen 2011) do not support the standard assumption of exogenous time-invariant invoicing. The models with exogenous currency choice are, therefore, subject to the Lucas critique and can potentially lead to poor policy implications.

This paper develops a tractable general equilibrium framework with *endogenous* currency choice that is consistent with the key stylized facts about international invoicing and shows these facts have important positive and normative implications. To this end, I augment a conventional New-Keynesian open-economy model a la Gali and Monacelli (2005) with two additional ingredients. First, rather than taken as exogenous, the currency of invoicing is optimally chosen by the individual exporters to minimize the deviation of the preset price from the optimal level (Engel 2006).[2] The currency choice is therefore determined by price stickiness — the same friction that makes this choice consequential for the aggregate economy in the first place — and is well-defined despite the complete asset markets and the zero transaction costs, because it allows the firm to increase average profits rather than hedge against risk. Second, I add input-output linkages and complementarities in price setting. These price

[1] This fact holds even if one excludes commodities and considers only manufactured goods.

[2] For example, if an optimal price of \$100 holds for the exporter regardless of shocks, invoicing is best done in dollars. Meanwhile, setting the price in euros makes the ex-post price deviate from the optimal level and causes the *average* profits of the firm to drop.

linkages are strong in the data, especially for the large firms that account for most of the international trade (see e.g. Amiti, Itskhoki, and Konings 2014, De Loecker, Goldberg, Khandelwal, and Pavcnik 2016). They have also been used to explain several puzzles in international economics (Itskhoki and Mukhin 2017, Casas, Díez, Gopinath, and Gourinchas 2017, Atkeson and Burstein 2008, Rodnyansky 2017), and as I show, are crucial to understand firms' currency choice.

By combining endogenous currency choice with price linkages across firms, I show that, depending on the parameter values, the model can sustain equilibria with producer, local or vehicle currency pricing. In the limiting case when marginal costs are stable and the markups are constant, the firms prefer to set prices in producer currency, which validates the standard assumption of PCP in most of the open economy models (see e.g. Obstfeld and Rogoff 1995, Clarida, Gali, and Gertler 2001, Gali and Monacelli 2005). Despite its prevalence in the theoretical literature, this knife-edge case with no links across firms provides a poor approximation to the data. Allowing for realistic complementarities in price setting, on the other hand, means exporters might choose LCP in order to align their prices with the prices of local competitors. Furthermore, allowing for multiple countries means that the exporters must deal with competitors and suppliers coming from different economies; thus, using a vehicle currency can be an optimal way to synchronize prices across firms (*cf.* Devereux and Engel 2001, Bacchetta and van Wincoop 2005, Bhattarai 2009).

The use of the dollar as a vehicle currency is driven by three factors in the model: the large share of dollarized economies in global trade, the relatively low volatility of U.S. exchange rate, and the path-dependence in currency choice. Intuitively, the large size of the U.S. market implies that foreign suppliers prefer to use dollars to align their prices with the local competitors. The U.S. exporters then find prices of their intermediate inputs stable in dollars and are more likely to use dollar currency pricing (DCP) in other markets. This increases the share of dollar-denominated inputs and competing products for the non-U.S. exporters, who then become more inclined to use DCP as well. In addition, other currencies are less suitable for synchronizing prices across exporters because of the relatively high volatility of exchange rates in the respective countries. As more firms switch to the dollar, the incentives for other exporters to use DCP both in the U.S. and other countries become even greater. While the endogenous complementarities in currency choice can potentially generate multiple equilibria for some values of the fundamentals, they also imply a possible inertia in currency choice. This explains the late transition from the pound to the dollar in the first half of the twentieth century and the dominant status of the dollar since then: the initial vehicle currency can retain its international position despite losing its advantage in terms of economic stability and size.[3]

Armed with this model of the international price system, I then re-examine the classical positive and normative policy questions. In the spirit of Mankiw (1985), I show that, despite only *second-order private* gains, the currency choice has *first-order aggregate* implications; because of multiple equilibria, a small

[3]Following the previous literature (e.g. Matsuyama, Kiyotaki, and Matsui 1993), I focus on the evolution of steady states and abstract from the dynamics between them.

perturbation of the fundamentals that makes firms switch from one invoicing regime to another leads to discontinuous changes in how prices, output, consumption, and trade balance respond to exogenous shocks.

First, I identify a novel source of U.S. monetary spillovers on foreign output that has largely been ignored in the previous debates (see e.g. Bernanke 2017). The stimulating monetary policy in the U.S. increases the aggregate demand and in particular, the demand for imported goods, but also makes U.S. goods cheaper relative to foreign ones because of the exchange rate depreciation. The classical conclusion is that the latter effect is stronger when the prices are set in producer currency and, as a result, the net spillovers are negative under PCP and positive under LCP (see e.g. Betts and Devereux 2000, Corsetti and Pesenti 2005). There is, however, an additional effect under DCP: a depreciation of the U.S. dollar decreases the prices of *all* internationally traded goods, which translates into lower producer and consumer price indices. As long as the aggregate nominal demand remains unchanged, the fall in prices drives the world consumption upwards (Goldberg and Tille 2009), stimulating production in the global economy. This channel has an unambiguously positive effect on foreign output and outweighs the standard expenditure switching towards U.S. goods under the baseline calibration.[4] At the same time, the depreciation of non-vehicle currencies has no additional positive spillover effects on the other economies and is also less effective in stimulating local output.

Second, I show that the currency choice per se does not invalidate the classical argument in favor of floating exchange rates (Friedman 1953). As has been demonstrated by Devereux and Engel (2003) in the context of a standard New-Keynesian open economy model, the optimal monetary policy implies floating exchange rates under PCP and pegging under LCP. I show, however, that, in a standard model, PCP is the only type of equilibrium invoicing that can arise under the optimal policy, when currency choice is endogenous. The decentralized invoicing decisions are, therefore, efficient in the sense that the first-best allocation can always be implemented by the monetary policy that stabilizes the producer price index (PPI). Though standard in the literature, the assumptions underlying this result are restrictive and are inconsistent with the data.

Third, I argue that, in a more realistic environment, there are complementarities between the firms' currency choices and monetary policy: the optimal policy under DCP involves a partial peg to the dollar, which, in turn, makes dollar invoicing more appealing to the firms. In particular, when the international prices are set in dollars, the U.S. monetary shocks tend to distort the terms of trade between third countries, and the monetary authorities lean against the wind by partially smoothing out movements in exchange rates against the dollar. The DCP can, therefore, contribute to the "fear of floating" and the widespread use of the dollar as an anchor currency seen in the data (Calvo and Reinhart 2002, Ilzetzki, Reinhart, and Rogoff 2017). At the same time, the resulting lower volatility of the U.S. exchange rate

[4] In contrast to the effect of dollar depreciation on global trade in Boz, Gopinath, and Plagborg-Møller (2017), the response of global output comes from the general equilibrium effects rather than partial equilibrium expenditure switching and does not depend on substitution between goods.

makes the dollar more attractive as a vehicle currency and helps to sustain the DCP equilibrium. The partial peg to the dollar also implies that the monetary policy is positively correlated across countries, which can contribute to the global financial cycle (Rey 2015). Importantly, however, despite worsening the trade-off that the policymakers face, the dollar pricing does not transform the "trilemma" into a "dilemma": the flexible exchange rates are effective in managing local shocks and, in comparison to fixed ones, allow to achieve higher welfare (see Gopinath 2017).

The last section of the paper discusses some additional mechanisms that can amplify the private benefits of dollar invoicing. I show that a volatile monetary policy makes the prices of inputs and competing products less stable in producer and local currencies respectively, and further encourages the exporters to set their prices in dollars. I then allow domestic firms to choose optimally the currency of invoicing and show that, while they are less likely to set prices in foreign currency than the exporters, an equilibrium with all firms using DCP can emerge in response to large fundamental shocks, e.g. volatile monetary policy. The complementarities in currency choice imply that the dollarization of emerging economies persists even after inflation is stabilized and contributes to the widespread use of the U.S. dollar in international trade.

There are three main strands in the literature that use different types of frictions to explain the dominant status of the dollar in international trade. First, there is a long tradition in economics, going back at least as far as Krugman (1980), that emphasizes the *transaction costs* in exchange markets: coordination on a single currency raises the chances of a "double coincidence of wants" (Matsuyama, Kiyotaki, and Matsui 1993) and increases the "thickness" of markets (Rey 2001, Devereux and Shi 2013, Chahrour and Valchev 2017). These theories, therefore, explain the widespread use of the dollar as a medium of exchange but have little to say about its role as an invoicing currency. Second, the use of the dollar as a unit of account can be due to *financial frictions*, as the firms try to synchronize the risks on their contracts (Doepke and Schneider 2013) and borrow in a cheaper currency (Gopinath and Stein 2017). While this is a promising direction for future research, there is so far little empirical evidence that financial frictions are significant for the large firms that account for most of the international trade.

This paper belongs to the third strand in the literature, the one that emphasizes the role of *nominal frictions* (see e.g. Devereux and Engel 2001, Bacchetta and van Wincoop 2005, Bhattarai 2009, Cravino 2014, Goldberg and Tille 2008, Drenik and Perez 2017) and has two advantages over the alternatives discussed in the previous paragraph. First, there exists direct empirical evidence in favor of this mechanism that allows to discriminate it against alternative theories (see Gopinath, Itskhoki, and Rigobon 2010). Second, sticky prices lie at the heart of the New-Keynesian open economy models. It is, arguably, preferable to have a theory where the currency choice is determined by the same type of friction that makes it relevant at the aggregate level.[5]

[5]That said, all three types of frictions are likely to be important in practice. It is therefore reassuring that these models broadly agree on the set of fundamentals that determine the firms' currency choice and imply similar comparative statics.

2 Baseline Model

2.1 Environment

I start with a simple framework that relies on conventional assumptions in the international macro literature and attains closed-form characterization. Since more than two countries are required for a vehicle currency to be well-defined, I assume there is a continuum of symmetric regions $i \in [0,1]$ as in Gali and Monacelli (2005). There is potentially one large economy (the U.S.) that includes regions $i \in [0,n]$, $n < 1$, and can also be interpreted as a currency union or a set of dollarized countries. The other regions $i \in (n,1]$ are small open economies, each with its own nominal unit of account, in which local wages W_{it} and prices are expressed. Denote the bilateral nominal exchange rate between regions i and j with $\mathcal{E}_{ijt}$, which goes up when currency i devalues relative to currency j.

In each country, there is a representative household, a local government and a continuum of firms producing different varieties of tradable and non-tradable goods. The tradable sector is characterized by intermediate goods in production, strategic complementarities in price setting and the home bias towards domestically produced goods. The prices are set before the realization of shocks and stay rigid for one period with a given probability. While the structure of the tradable sector is crucial, the other details of the model are less important. I make specific assumptions about preferences, the structure of asset markets and monetary policy to simplify exposition, and discuss below how they can be relaxed. The set of exogenous shocks includes changes in productivity, money supply, government spendings, preferences for imported goods and shocks in financial markets.

Households A representative household in region i chooses consumption bundle C_{it}, supplies labor L_{it}, invests in local risk-free nominal bond B_{it+1} and in complete set of internationally traded Arrow securities D^s_{it+1} to maximize expected utility[6]

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big(\log C_{it} - L_{it}\Big) \tag{1}$$

subject to a sequence of budget constraints:

$$P^C_{it}C_{it} + \Big(\frac{B_{it+1}}{R_{it}} - B_{it}\Big) + e^{\psi_{it}}\mathcal{E}_{i0t}\Big(\sum_{s\in S_{t+1}} Q^s_t D^s_{it+1} - D_{it}\Big) = W_{it}L_{it} + \Pi_{it} - T_{it} + \Omega_{it}, \tag{2}$$

where P^C_{it} is the price index of consumption bundle in country i, Π_{it} are profits of local firms, T_{it} is the lump-sum tax, R_{it} is the nominal interest rate and Q^s_t is the price of Arrow security that pays one dollar

[6]This functional form has widely been used in macroeconomic literature in a context of both closed and open economy (see e.g. Ball and Romer 1990, Golosov and Lucas 2007, Kehoe and Midrigan 2007) and arises naturally when labor is indivisible (Rogerson 1988, Hansen 1985).

in state $s \in S_{t+1}$ in the next period. I suppress state index s in other variables to simplify the notation. Both prices and returns on the Arrow securities are in dollars — which is without loss of generality because of complete markets – and are converted into local currency with the nominal exchange rate $\mathcal{E}_{i0t}$. I allow for cross-country wedge in asset prices and returns ψ_{it}, which can be interpreted as a shock in the local financial markets and might be an important source of exchange rate volatility.[7] The resulting profits (or losses) of the financial sector Ω_{it} are reimbursed lump-sum to local households.[8] The assumption of complete asset markets is to simplify exposition and the same results can be obtained in case of one internationally traded bond, as shown in Appendix A.4.

Consumption bundle consists of tradable and non-tradable goods combined with Cobb-Douglas aggregator:

$$C_{it} = \left(\frac{C_{Nit}}{1-\eta}\right)^{1-\eta}\left(\frac{C_{Tit}}{\eta}\right)^{\eta}. \tag{3}$$

Non-tradable sector In each country, there is a continuum of monopolistically competitive firms producing different varieties $\omega \in [0,1]$ of non-tradable goods using the same production technology:

$$Y_{Nit}(\omega) = e^{a_{Nit}} L_{Nit}(\omega). \tag{4}$$

The individual products are then combined into consumption basket C_{Nit} with a CES aggregator:

$$C_{Nit} = \left(\int_0^1 C_{Nit}(\omega)^{\frac{\theta-1}{\theta}} \mathrm{d}\omega\right)^{\frac{\theta}{\theta-1}}, \tag{5}$$

Firms preset prices in local currency before the realization of shocks and update them afterwards with a probability $\lambda < 1$.

Tradable sector The tradable sector differs from the non-tradable one in three dimensions. First, production of a continuum of unique tradable products $\omega \in [0,1]$ in country i requires both labor L_{Tit} and tradable intermediate goods X_{it}:

$$Y_{it}(\omega) = e^{a_{Tit}}\left(\frac{L_{Tit}(\omega)}{1-\phi}\right)^{1-\phi}\left(\frac{X_{it}(\omega)}{\phi}\right)^{\phi}, \qquad \phi < 1. \tag{6}$$

Second, the bundle of tradables used in consumption and production includes both local and foreign varieties, which are combined with a homothetic aggregator:

$$\Phi\left(\left\{\frac{C_{jit}(\omega)}{C_{Tit}}\right\}_{j,\omega}, \xi_{it}, \gamma\right) = 1, \tag{7}$$

[7] See e.g. Itskhoki and Mukhin (2017), Lustig and Verdelhan (2016), Devereux and Engel (2002), Kollmann (2005).

[8] The profits of financial sector are $\Omega_{it} = (e^{\psi_{it}} - 1)\mathcal{E}_{i0t}\left(\sum_{s\in S_{t+1}} Q_t^s D_{it+1}^s - D_{it}\right)$.

where $C_{jit}(\omega)$ denotes consumption of product ω from country j exported to country i, ξ_{it} is a relative demand shock for foreign versus domestic goods, and the home bias $1-\gamma$ reflects either trade costs or home bias in preferences, $\gamma \in (0,1)$. Note that when $n > 0$, the home bias is effectively higher for large country: in addition to locally produced goods, a positive fraction of expenditures in $i \in [0,n]$ is spent on goods produced in other regions of the U.S. The bundle of intermediate goods X_{it} is defined similarly. I use Kimball (1995) aggregator to specify $\Phi(\cdot)$ (see (A1) in Appendix A.2), which implies that equilibrium prices depend not only on marginal costs of production, but also on prices of competitors. I show this deviation from the CES benchmark is important for firms' currency choice.

Third, for each country of destination, firms choose the currency of invoicing and preset price in it before the realization of shocks. With a probability λ, the price can be updated after the uncertainty is resolved. While any currency can be used for invoicing in the international trade, for legal reasons local firms can set prices only in domestic currency. In Section 6, I relax this assumption and derive additional results when domestic firms optimally choose the currency of invoicing.

Government The regional government collects lump-sum taxes T_{it} from households to finance expenditures $G_{it} \equiv e^{g_{it}}$, which for simplicity have the same composition of products as the consumption bundle. The government runs a balanced budget, which is without loss of generality since Ricardian equivalence holds in the model:

$$T_{it} = P_{it}^{C} G_{it}. \tag{8}$$

The monetary policy is implemented with the nominal interest rates R_{it}.[9] To simplify the analysis, I assume in the baseline case that monetary policy rule is such that nominal wages $W_{it} = e^{w_{it}}$ follow exogenous stochastic process. In particular, the special case of fully stable nominal wages $w_{it} = 0$ discussed below approximates closely inflation targeting when non-tradable goods account for most of the consumer basket.[10] I discuss the optimal monetary policy and its interactions with firms' currency choice in detail in Section 5.

Equilibrium conditions In equilibrium, labor supply equals total demand of non-tradable and tradable sectors:

$$L_{it} = \int_0^1 \big(L_{Nit}(\omega) + L_{Tit}(\omega)\big)\mathrm{d}\omega. \tag{9}$$

Non-tradable goods are sold locally to households and the government:

$$Y_{Nit}(\omega) = C_{Nit}(\omega) + G_{Nit}(\omega). \tag{10}$$

[9] As is standard in the literature, I focus on the cashless limit and abstract from the potential multiplicity of equilibria.

[10] Under log-linear preferences (1), this policy coincides with targeting nominal spendings, which is another common assumption in the literature (see e.g. Carvalho and Nechio 2011, Mankiw and Reis 2002).

Similarly, tradable goods are used for final consumption of local households and government, for production in the tradable sector and are exported to other regions:

$$Y_{it}(\omega) = Y_{iit}(\omega) + \int_0^1 Y_{ijt}(\omega)\mathrm{d}j, \qquad \text{where} \quad Y_{ijt}(\omega) = C_{ijt}(\omega) + G_{ijt}(\omega) + X_{ijt}(\omega) \tag{11}$$

for all $i, j, \omega \in [0, 1]$. Finally, the market clearing in local and international asset markets $s \in S_{t+1}$ implies

$$B_{it+1} = 0, \qquad \int_0^1 D^s_{it+1}\mathrm{d}i = 0. \tag{12}$$

Shocks I assume that each type of shock can be decomposed into a global component and the country-specific one, e.g. $g_{it} = \bar{g}_t + \tilde{g}_{it}$ for government spending shock, where $\tilde{g}_{it}$ are uncorrelated across i. In addition, the volatility of country-specific shocks in the U.S. is potentially lower than in other countries by the factor $\rho \leq 1$. This can be rationalized with a better diversification of regional risk in a large economy and weaker granularity forces a la Gabaix (2011), and results in a more stable exchange rate in the U.S. For simplicity, I do not impose any parametric relation between n and ρ and treat these parameters as exogenous.

Definition 1 *Given shocks $\{a_{Nit}, a_{Tit}, w_{it}, \xi_{it}, g_{it}, \psi_{it}\}$, a monopolistically competitive equilibrium is defined as follows: a) households maximize utility over consumption of products, labor supply and asset holdings, b) each firm maximizes expected profits over labor and intermediate inputs, currency of invoicing and prices in each market, taking the decisions of all other firms as given and setting domestic prices in local currency, c) the government collects taxes to satisfy budget constraint (8), d) all markets clear according to (9)-(12).*

2.2 Firm currency choice

This section describes the currency choice problem of an individual exporter. I derive a sufficient statistics for the optimal invoicing, which depends on both partial equilibrium and general equilibrium variables. The next sections discuss how the latter are determined. To obtain sharp analytical results, I approximate equilibrium conditions around the symmetric steady state (see Appendix A.2 for details). I denote log-deviations from the steady-state values with small letters and suppress time subscript for simplicity. The expectations and variances are therefore taken conditional on the information that agents have at the beginning of the period before the realization of shocks.

Let $\Pi_{ji}(p)$ denote the profit of exporter from j to i as a function of price p expressed in currency of

destination.[11,12] Define the optimal static price $\tilde{p}_{ji}$ that maximizes profits in a given state of the world:

$$\tilde{p}_{ji} = \underset{p}{\text{argmax}}\ \Pi_{ji}(p). \tag{13}$$

The firms that can adjust after the realization of shocks set price at $\tilde{p}_{ji}$. On the other hand, the optimal preset price replicates the *average* $\tilde{p}_{ji}$ expressed in currency of invoicing k:

$$\bar{p}^k_{ji} = \mathbb{E}\big[\tilde{p}_{ji} + e_{ki}\big]. \tag{14}$$

The expected value of ex post price $\bar{p}^k_{ji} + e_{ik}$ is therefore the same for all currencies and the currency of invoicing is not determined. It follows, to solve the currency choice problem, one needs to use the second-order approximation: while the preset price is chosen to replicate the mean value of the optimal price, the currency choice allows firms to target the second moment of $\tilde{p}_{ji}$ (see Engel 2006, Gopinath, Itskhoki, and Rigobon 2010, Cravino 2014). [13] Note this implies that expected movements in prices and exchange rates are fully absorbed by the preset price and have no effect on the currency choice.

Lemma 1 (Currency choice) *To the second-order approximation, the currency choice problem of exporter is equivalent to choosing the currency k, in which the optimal price $\tilde{p}_{ji} + e_{ki}$ is most stable:*

$$\max_{k\in[0,1]} \mathbb{E}\,\Pi_{ji}(\bar{p}^k_{ji} + e_{ik}) \quad \Leftrightarrow \quad \min_{k\in[0,1]} \mathbb{E}\Big[\bar{p}^k_{ji} + e_{ik} - \tilde{p}_{ji}\Big]^2 \quad \Leftrightarrow \quad \min_{k\in[0,1]} \mathbb{V}\Big[\tilde{p}_{ji} + e_{ki}\Big]. \tag{15}$$

As can be seen from the second expression, the optimal currency choice allows firms to mitigate the effect of sticky prices and to bring ex post price $\bar{p}^k_{ji} + e_{ik}$ closer to the optimal state-dependent value $\tilde{p}_{ji}$. This is achieved by choosing the currency, in which optimal price is most stable. For example, if the desired price is $\$100$ in all states of the world, then setting the price in dollars allows the firm to replicate the flexible-price allocation. Similarly, it is optimal to set price in pound sterling when the optimal price is $£100$ in all states.[14]

The choice is more nuanced when the optimal price is not fully stable in one currency, e.g. when $\tilde{p}_{ji}$ can be expressed as $\$50 + £50$. In this case, the firm would ideally like to set price in terms of a basket of currencies. As shown in Appendix A.6, under some restrictions on exogenous shocks, firms can

[11] Due to constant returns to scale in production, the marginal costs do not depend on quantity produced and the objective function of a firm is separable across markets. Therefore, exporters choose price and currency of invoicing independently for each destination.

[12] I assume that profits are expressed in real discounted units, i.e. $\Pi_{ji}(\cdot)$ includes stochastic discount factor (SDF). The variation in SDF, however, does not affect the results under the approximation used below.

[13] I use a classical result from portfolio theory established first by Samuelson (1970) and applied recently in a general equilibrium setup by Devereux and Sutherland (2011) to show that the second-order approximation to the profit function and the first-order approximation to all other equilibrium conditions are sufficient to get consistent solution.

[14] In other words, it is optimal to set price in currency k rather than in currency h if the pass-through of bilateral exchange rate shocks e_{kh} into the desired price $\tilde{p}_{ji} + e_{ki}$ is low: see e.g. Proposition 2 in Gopinath, Itskhoki, and Rigobon (2010).

perfectly replicate flexible prices when allowed to use currency baskets for invoicing.[15] The predictions of the model, however, are inconsistent with the key stylized facts about the international price system in this case. In particular, the share of dollar cannot exceed the share of the U.S. in global trade when invoicing is continuous. I therefore assume that currency choice is discrete and that individual firms find it suboptimal to use baskets of currencies for invoicing, presumably due to information frictions (see e.g. Sims 2003, Mankiw and Reis 2002).[16] In the spirit of Mankiw (1985), I show in Section 4 that small frictions are sufficient to rationalize discrete currency choice and can lead to large aggregate effects.

Notice that the firm's invoicing problem of choosing a basket of currencies that minimizes (15) resembles the classical portfolio problem a la Markowitz (1952). The assumption that currency choice is discrete is an analog of financial frictions that have been used to explain the global status of dollar in asset markets (see e.g. Bruno and Shin 2015, Rey 2015). It is worth emphasizing however, that despite these similarities, invoicing decisions of firms in the model are based on nominal frictions, not financial ones: exporters choose currency of invoicing to bring ex-post prices closer to the optimal level and increase *average* profits, not to redistribute profits across states to hedge against risk. Abstracting from financial frictions might be a reasonable assumption since most of the international trade is done by large firms, which arguably have a better access to financial markets.[17]

While the previous analysis is based on a one-period version of Calvo (1983) price setting, it also applies to other models of price rigidity. Appendix A.7 discusses four alternatives. In particular, I show that the baseline results about currency choice can be derived analytically for the case of multiperiod staggered pricing. Lemma 1 remains also valid under Rotemberg pricing with quadratic costs of price adjustment and in a menu cost model with fixed costs of adjustment and idiosyncratic productivity shocks. Finally, I relax the assumption that currency choice is made unilaterally by suppliers and show that the same results can be obtained in a model with bargaining between firms in the spirit of Hart and Moore (2008) (see Appendix A.7 for details).

2.3 Partial equilibrium

This section derives equilibrium conditions in the tradable sector that determine the optimal prices $\tilde{p}_{ji}$. A constant returns to scale technology ensures that equilibrium prices depend only on the supply side of the economy and can be analyzed separately from the quantities. In contrast to the CES case, Kimball demand implies there are strategic complementarities in price setting across firms, so that the optimal price of an exporter from j to i depends not only on its marginal costs but also on prices of competitors

[15] Note the currency basket is firm-specific and there is no one-size-fits-all solution like Special Drawing Rights (SDR).

[16] This however does not exclude mixed strategies when firms randomize across different currencies.

[17] At the same time, the model can be extended to incorporate effects of asset market imperfections on currency choice: e.g. when firms have to borrow in dollars to finance their inputs, the pass-through of dollar shocks into costs and optimal price $\tilde{p}_{ji}$ is high, which makes invoicing in dollars more appealing.

in market i:

$$\tilde{p}_{ji} = (1-\alpha)(mc_j + e_{ij}) + \alpha p_i, \tag{16}$$

where e_{ij} converts marginal costs into the destination currency. Parameter α depends on the curvature of $\Phi(\cdot)$ and is different from demand elasticity. In the limit $\alpha \to 0$, when Kimball aggregator converges to CES, the firms charge a constant markup and cost shocks are the only source of variation in the desired prices.

The cost minimization problem under constant returns to scale technology implies that marginal costs of production in country i are a weighted sum of local wages w_i and prices of intermediates p_i adjusted for productivity:

$$mc_i = (1-\phi)w_i + \phi p_i - a_{Ti}. \tag{17}$$

The first-order approximation to the ideal price index for Kimball aggregator is isomorphic to the CES index:

$$p_i = (1-\gamma)p_{ii} + \gamma p_i^I, \qquad \text{where} \qquad p_i^I = \int_0^1 p_{ji}\mathrm{d}j. \tag{18}$$

The aggregate index is therefore the sum of prices of locally produced goods p_{ii} and imported ones p_i^I with the weight of the former determined by the home bias $1-\gamma$. Lastly, the bilateral price index depends on prices of adjusting and non-adjusting firms:[18]

$$p_{ji} = \lambda\tilde{p}_{ji} + (1-\lambda)(\bar{p}_{ji}^k + e_{ik}). \tag{19}$$

A fraction λ of firms update prices after the realization of shocks and set them at the optimal level $\tilde{p}_{ji}$. The prices of other firms stay constant in the currency of invoicing k, which means they move one-to-one with exchange rate e_{ik} in the currency of the customers. The currency choice therefore has the first-order effect on ex-post prices. At the same time, Lemma 1 implies that invoicing decision of an individual firm is determined by optimal price $\tilde{p}_{ji}$, which depends on aggregate price indices p_i. Thus, the equilibrium price system can be defined as follows.

Definition 2 *Given $\{a_{Ti}, w_i, e_{ij}\}$, the equilibrium international price system consists of price indices $\{p_i\}$ and firms' currency choice $\{k_{ji}\}$ such that: (a) given invoicing regime, $\{p_i\}$ solve the system (16)-(19), (b) given prices, $\{k_{ji}\}$ solve problem (15).*

Out of the three variables that are exogenous to the tradable sector, two — nominal wages and exchange rates — are determined by the general equilibrium forces. The next section therefore solves the general equilibrium block for the second moments of w_i and e_{ij}.

[18] To simplify the notation, I assume that all exporters from j to i use the same currency of invoicing k. The results in Section 3 are however derived for the general case if not noted otherwise.

2.4 General equilibrium

Definition 2 implies that the only general equilibrium objects that matter for exporters' currency choice are the second moments of exchange rates, nominal wages, and productivity shocks. This section shows that under the assumptions made in the baseline model, these moments do not depend on the invoicing decisions of firms and therefore, the model attains the block-recursive structure: one can solve for equilibrium currency choice taking the relevant general equilibrium moments as given. Importantly, however, this result does *not* imply that invoicing decisions of firms have no general equilibrium effects. As Section 4 makes clear, the aggregate consumption, output, exports and imports do change with the currency of invoicing even though the equilibrium exchange rates do not.

Lemma 2 (Exchange rates) *The second moments of equilibrium exchange rates are independent from invoicing decisions of firms.*

The result follows from the combination of log-linear utility, complete asset markets and the monetary policy rule that targets nominal wages. While these assumptions are sufficient, they are not necessary for Lemma 2 to hold. In particular, Appendix A.4 shows the same result can be obtained under arbitrary isoelastic preferences, one internationally traded bond and exogenous interest rate shocks. It also shows that even less stringent assumptions are needed if one restricts the analysis to the equilibria with symmetric invoicing.

Lastly, note that the effect of monetary and productivity shocks on exporters' currency choice depends on their correlation with nominal exchange rates. Empirically, this correlation is close to zero (Meese and Rogoff 1983) and therefore, I abstract from monetary and productivity shocks in the benchmark model, i.e. $w_{it} = a_{Tit} = 0$. I discuss in detail both shocks in Section 5 when analyzing the optimal monetary policy. Section 6 provides additional results that emerge in the presence of large monetary shocks, Section A.10.3 discusses the case of inflation targeting and Section A.10.2 analyzes the role of productivity shocks.

3 Equilibrium Currency Choice

Throughout the history of modern capitalism, the overwhelming share of global trade has been priced in one currency — first in pound sterling and later in dollars. This section shows that the model is consistent with this observation. In particular, strategic complementarities in currency choice that arise naturally across firms due to input-output and price-setting linkages, imply that exporters are likely to share the same currency of invoicing. I show next there are two fundamental factors — the volatility of exchange rate and country's share in global trade — that make some currencies more attractive as vehicle ones. Finally, I combine these two results to analyze transition from one dominant currency to another: as fundamental advantages of pound sterling deteriorate, exporters become more likely to use

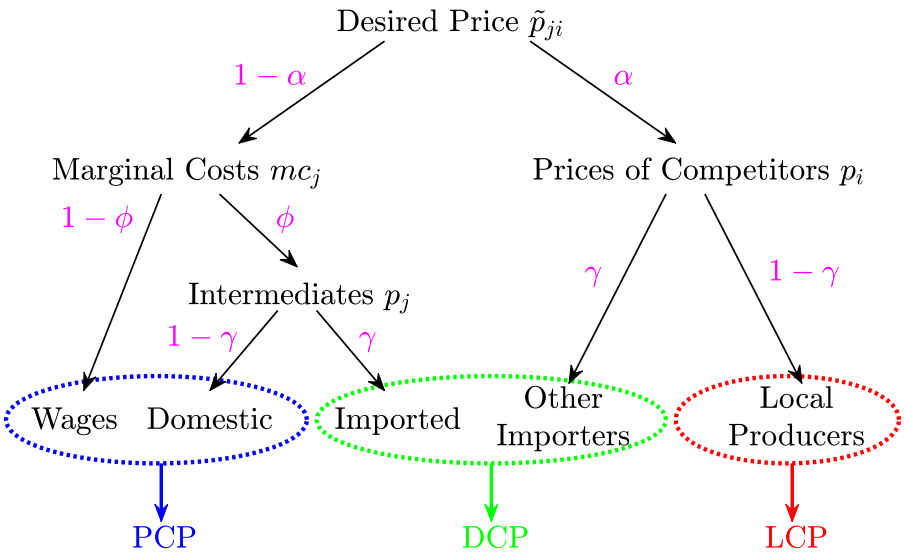


Figure 1: Optimal price and the currency choice of an individual firm

dollars instead. However, due to strategic complementarities, no firm wants to change the currency of invoicing before other ones do, generating path-dependence in currency choice. This result can account for the delayed transition from pound to dollar in the twentieth century and the wide use of dollar in modern economy despite increasing competition with euro and renminbi.

3.1 Why dominant currency?

While it is intuitive that firms might set prices in producer or customer currency, it is not immediately clear why invoicing in a third currency might be optimal. In this section, I show that a vehicle currency equilibrium (VCP) can arise naturally when price linkages across firms from different countries are strong enough. The question *which* currency is used as vehicle one is discussed in the next section.

According to Lemma 1, firms choose the currency of invoicing, in which their optimal price is more stable. The currency choice of individual exporter from j to i depends therefore on the properties of its desirable price $\tilde{p}_{ji}$, which is determined by the system of equilibrium conditions in tradable sector (16)-(19) summarized in Figure 1. The optimal price depends on marginal costs and the prices of competitors with the weight of the latter determined by strategic complementarities in price setting α. The marginal costs in turn consist of labor costs and the prices of intermediate goods with the weights $1-\phi$ and ϕ respectively. The fraction $1-\gamma$ of intermediate inputs is produced domestically, while the share γ is imported from other countries. Similarly, out of all competitors in the destination market, a fraction $1-\gamma$ are local producers, while importers from other countries account for the remaining share γ.[19]

Consider first the conventional case of CES aggregator and no intermediates in production. With

[19]There are three additional parameters that affect currency choice. The frequency of price adjustment λ affects the prices of inputs and competing products. The size of the large economy n determines the share of goods in global trade coming from the U.S. The relative volatility of exchange rates ρ affects the probability distribution of $\tilde{p}_{ji}$.

no complementarities in price setting under CES demand, the desired price is proportional to marginal costs (see Figure 1). The latter depends exclusively on nominal wages, which are by assumption stable in domestic currency. It follows, the optimal price of exporter $\tilde{p}_{ji}$ is constant in producer currency as well and therefore, PCP is always optimal.

Lemma 3 (No price linkages) *With no intermediates in production, $\phi = 0$, and CES aggregator, $\alpha = 0$, exporters always choose PCP, and no VCP equilibrium exists.*

Thus, the standard assumption of PCP in open economy models with $\phi = \alpha = 0$ and a stabilizing monetary policy (see e.g. Obstfeld and Rogoff 1995, Clarida, Gali, and Gertler 2001, Gali and Monacelli 2005) is internally consistent: the equilibrium would not change if firms were allowed to choose optimally the currency of invoicing. The proposition also implies that price linkages across firms are a necessary condition to rationalize the use of vehicle currencies in global trade.

The next result for autarky limit $\gamma \to 0$ clarifies that it is linkages with firms from the *third* countries that make vehicle currency more appealing. Notice that the currency choice is well defined for individual exporters of zero mass. Since countries of origin and destination are almost closed, the marginal costs of exporters are stable in producer currency and the prices of competitors are stable in local currency. As a result, depending on the value of α, firms choose either PCP or LCP.

Lemma 4 (Autarky limit) *Near the autarky limit $\gamma \to 0$, exporters choose PCP if $\alpha \leq 0.5$ and LCP if $\alpha \geq 0.5$, and no VCP equilibrium exists.*

Figure 2a shows equilibria in the autarky limit in the coordinates α and λ. The equilibrium is unique when tradable sector is almost closed since strategic interactions across exporters disappear as their share in the market converges to zero.[20] The figure also shows that for any values of other parameters, the existence of PCP (LCP) equilibrium can be guaranteed if economies are close to autarky and strategic complementarities in price setting are weak (strong).

On the other hand, when openness of economies γ is high, so that significant fraction of suppliers and competitors are coming from the third countries, the optimal price $\tilde{p}_{ji}$ of the exporter is no longer stable in either producer or local currency, and using vehicle currency might be optimal. The prices of inputs and competing products that individual exporter faces in this case depend on invoicing decisions of other firms: e.g. when prices of suppliers and competitors are sticky in dollars, the optimal price of exporter is more stable in dollars as well and DCP is more attractive.[21] Interestingly, both input-output and price-setting linkages play important role in generating complementarities in currency choice, there are important differences between the two. A higher share of intermediates in production ϕ

[20] Here and below I abstract from the knife-edge values of parameters, under which firms are indifferent between two invoicing options.

[21] The empirical evidence suggests that international prices are sticky with the frequency of adjustment of the same order of magnitude as producer and consumer prices (Gopinath and Rigobon 2008).

unambiguously increases the share of foreign suppliers and makes VCP more attractive. In contrast, the effect of complementarities in price setting α on VCP is non-monotonic: the optimal price is more stable in producer currency when α is low and in local currency when α is high. For intermediate values of α, however, neither of the two currencies dominates and VCP is more likely.[22] I summarize comparative static results in the next proposition.[23]

Proposition 1 (Vehicle currency pricing) *The region of the VCP equilibrium in parameter space is non-empty and is increasing in the openness of economies γ and the share of intermediates in production ϕ, and can be non-monotonic in complementarities in price setting α.*

Interpreting empirical evidence through the lens of the model, one can argue that globalization has contributed to the widespread use of the vehicle currency in the international trade. In particular, the high participation of several Asian countries in global value chains can be interpreted as a rise in $\gamma\phi$, which increases the chances of VCP relative to PCP and LCP. The higher openness γ of other countries, including the post-Soviet states, makes the use of vehicle currency in the international trade more appealing as well. Lastly, the model also suggests that the puzzling high share of dollar in imports and exports of such advanced economies as South Korea, Japan and Australia can be due to strategic complementarities in currency choice: with other countries in the region using DCP, it might be optimal for firms in these countries to set prices in dollars as well.

Complementarities in currency choice also imply that multiple equilibria can emerge despite unique currency choice of an individual firm. While the set of potential equilibria is very rich in a general case, the next proposition shows that uniqueness can be guaranteed when there is only one symmetric equilibrium, in which all exporters choose either PCP, LCP or DCP. Intuitively, the complementarities in currency choice imply that if a given regime is not chosen when *all* other firms are following it, then it cannot be optimal when only *some* firms are using it. The complementarities also imply that mixed-strategy equilibria are unstable: for example, if firms are indifferent between DCP and LCP in some market, a small exogenous increase in the share of importers pricing in dollars will make indifferent firms strictly prefer DCP to LCP.[24]

Definition 3 *An equilibrium is symmetric if all exporters in the world use either PCP, LCP or the same vehicle currency. The equilibrium is unstable if exogenous perturbation of currency choice of an arbitrarily small fraction of exporters makes a positive mass of other firms to change their invoicing decisions.*

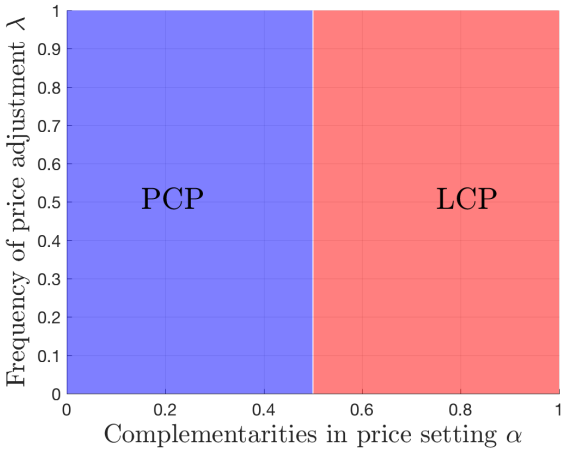
Proposition 2 *Assume that $n = 0$ and $\rho = 1$. Then*

[22] The VCP region can however be monotonic in α for some values of parameters.

[23] I use the following definition throughout the paper: the region of equilibrium Z in parameter space is said to be increasing in parameter x if for any $x_2 > x_1$ the set of parameters for which Z exists under $x = x_2$ includes the set for which Z exists under $x = x_1$.

[24] While complementarities in currency choice cannot be ensured for PCP and LCP in general case, they hold under the values of parameter, under which these equilibria can arise.

(a) Autarky limit



(b) Symmetric countries

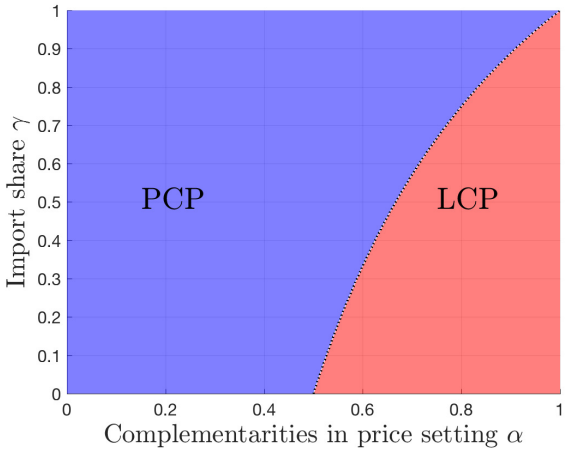


Figure 2: Two limiting cases with no DCP equilibrium

Note: the values of parameters are $\phi = 0.5$, $\rho = 1$, $n = 0$ in both plots and (a) $\gamma \to 0$, (b) $\lambda \to 1$. PCP and LCP denote the areas where producer and local currency pricing equilibria can be sustained respectively.

1. *at least one symmetric equilibrium always exists,*
2. *if symmetric equilibrium is unique, then no other equilibria exist,*
3. *all non-pure-strategy equilibria are unstable.*

3.2 Which currency is dominant?

While the previous section rationalizes the use of a vehicle currency in global trade, it does not tell us which currency plays this role. This section describes two fundamental advantages that can make dollar pricing more attractive than pricing in any other currency.

To separate fundamental factors from the complementarity motive, I focus on the flexible price limit $\lambda \to 1$, when almost all firms adjust prices ex-post and hence, invoicing decision of a given exporter does not depend on currency choice of other firms and the equilibrium price system is always unique (see Appendix A.2 for details). Notice that currency choice is well-defined in the presence of an arbitrary small price stickiness: exporter's invoicing decision depends only on the states of the world in which price remains unadjusted and has a solution even when the probability of these states converges to zero. This contrasts with the case of fully flexible prices $\lambda = 1$, when currency choice is completely inconsequential and therefore is not determined. I start with the case when no DCP equilibrium exists to outline necessary conditions for dollar invoicing.

Proposition 3 (No-DCP benchmark) *If prices are almost flexible, $\lambda \to 1$, and countries are symmetric, $n = 0$, $\rho = 1$, exporters choose PCP when $\alpha \leq \frac{1}{2-\gamma}$, LCP when $\alpha \geq \frac{1}{2-\gamma}$, and no DCP equilibrium exists.*

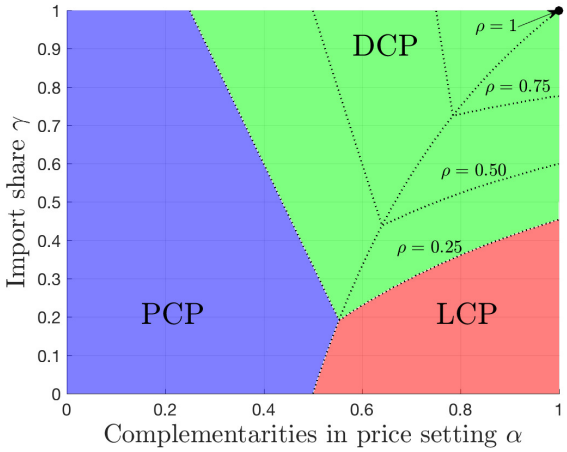
When countries are symmetric, $n = 0$, the fraction of U.S. products in other markets is trivial relative to domestic ones, and exporters find their marginal costs and competitors' prices more stable in producer and local currency respectively. Since the dollar exchange rate has also no advantage in terms of second moments, $\rho = 1$, DCP is strictly dominated by PCP and LCP. Figure 2b illustrates this result in the coordinates α and γ. The region of DCP is empty, while the choice between PCP and LCP depends on α and γ: using local currency is optimal only when complementarities in price setting are strong and the share of local firms in the destination market is sufficiently high.

I show next that any deviation from the benchmark described in Proposition 3 is sufficient to sustain DCP equilibrium for some values of other parameters. To this end, consider two points outside of the admissible range: $\phi = 1$ and $\gamma = \alpha = 1$. In the former case, no labor is used in production and as a result, there is no component in marginal costs of firms that is stable in producer currency. Both marginal costs and prices of competitors are equally stable in any currency in this case, and exporters are indifferent between PCP, LCP, DCP or any other currency of invoicing. Similarly, when $\gamma = \alpha = 1$ the optimal price of firms depends only on price index of other importers in the destination market, which is equally stable in all currencies and makes exporters indifferent between using any currency for invoicing. I argue next that any deviation from the conditions of Proposition 3 make exporters strictly prefer DCP in the neighborhood of these two points.

Volatility advantage Suppose first that countries are symmetric in terms of their size, $n = 0$, but the volatility of dollar exchange rate is lower relative to other currencies because of higher diversification of the U.S. economy and smaller fundamental shocks, i.e. $\rho < 1$. To see the benefits of the DCP in this case, consider two limiting cases from above: when $\phi = 1$ or $\gamma = \alpha = 1$, the prices of suppliers and competitors from all countries have a symmetric effect on the optimal price. Therefore, the exporter would like ideally to set price in terms of fully diversified basket of currencies. This is however not possible because of the discrete nature of the invoicing problem, and firms look for a currency with the lowest idiosyncratic volatility that can replicate most closely this diversified portfolio. If $\rho < 1$, DCP strictly dominates other alternatives.

Away from this limit, there is a trade-off between producer/local currency and dollar: the prices of domestic inputs and local competitors are more stable in the former, while dollar provides a better proxy for prices of goods coming from the third countries. At the same time, DCP strictly dominates any other potential vehicle currency. Figure 3a shows equilibria for different values of ρ in the coordinates α and γ. The line separating PCP and LCP equilibria remains the same as in Figure 2b as the value of ρ does not affect the trade-off between producer and local currencies. The region of DCP equilibrium is one point when $\rho = 1$ and increases continuously as dollar volatility goes down. Consistent with the discussion above, DCP equilibrium is more likely for higher import share γ and intermediate values of price complementarities α, while PCP and LCP are always optimal when import share γ is low.

(a) Low dollar volatility



(b) Large economy

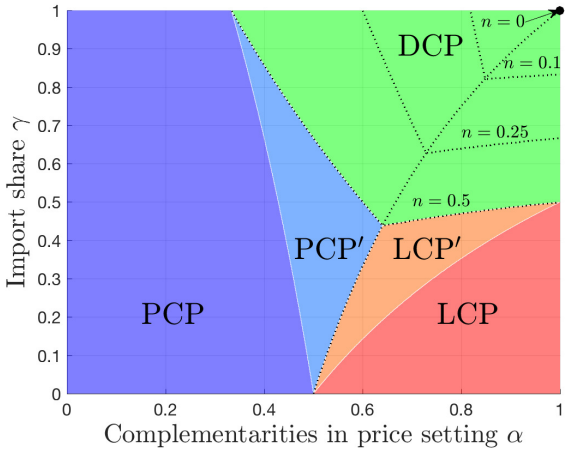


Figure 3: Currency choice: flexible price limit

Note: the figure shows equilibria for $\lambda \to 1$, $\phi = 0.5$ and (a) $n = 0$, different values of ρ, (b) $\rho = 1$, different values of n. PCP, LCP and DCP denote the regions of the corresponding symmetric equilibria. PCP′ (LCP′) denotes the region where small countries set prices in producer (local) currency when trading with each other and use dollars when trading with U.S.

Proposition 4 (Volatility advantage) *Assume $\lambda \to 1$ and $n = 0$. Then as long as dollar has lower volatility than other currencies, $\rho < 1$, the region in the parameter space with DCP as a unique equilibrium is non-empty and increases as ρ goes down.*

While this result alone is not sufficient to rationalize the global status of dollar, it explains why the use of currencies with volatile exchange rates in the international trade is very limited, e.g. almost all imports and exports of Latin American and Eastern European countries are invoiced in foreign currencies (Casas, Díez, Gopinath, and Gourinchas 2017). The model shows that the relative volatility is important even when exchange rate shocks are not associated with changes in nominal wages (*cf.* Devereux and Engel 2001, Bhattarai 2009). Section 6 shows on the other hand that the effect can be significantly amplified when differences in volatility are due to monetary shocks.

Large share in global trade Consider next the case when volatility of exchange rates is the same for all countries, $\rho = 1$, but the U.S. accounts for a non-trivial share of the global trade, i.e. $n > 0$.[25] This implies that a positive fraction of inputs used by firms in small economies are produced in the U.S. In addition, a positive mass of competitors in all markets are coming from the U.S. Both factors increase the chances of DCP equilibrium as n goes up. Figure 3b shows the region of DCP equilibrium for different values of n: while the set consists of only one point when $n = 0$, the region increases as n goes up. The currency of the large economy strongly dominates any other potential vehicle currency.

[25]Note that PCP, LCP and DCP coincide for trade flows between regions within the U.S.

Proposition 5 (Large economy advantage) *Assume $\lambda \to 1$. Then as long as the share of the U.S. economy in the international trade is positive, $n > 0$, the region in the parameter space with DCP as a unique equilibrium is non-empty and increases as n goes up.*

The figure also shows that an equilibrium with asymmetric invoicing can arise when $n > 0$.[26] In particular, firms might choose to use producer currency when trading between small economies, but set prices in dollars when exporting to the U.S. This is because the home bias is larger for the U.S. than for other economies when $n > 0$, and more competitors in the destination market have prices stable in local currency, i.e. in dollars. Similarly, exporters from the U.S. have a higher share of their marginal costs stable in dollars and can use DCP even when other firms prefer LCP.

3.3 Transition

The previous section argues that both fundamental factors, i.e. volatility and size advantage, and complementarities in currency choice contribute to the dominant status of dollar in today's world. What happens when these factors work in the opposite direction? This situation has happened to the pound sterling in the twentieth century and might be relevant for the dollar as China overgrows the U.S.

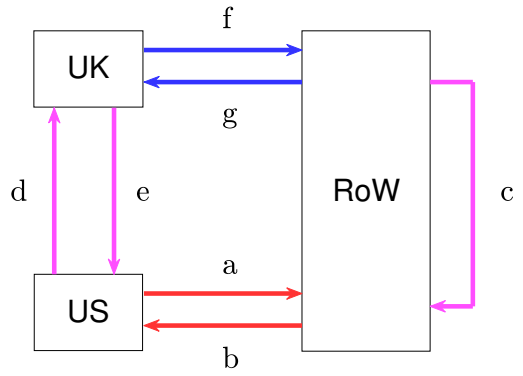
To answer the question, I allow for two large countries, the U.S. and the U.K. (see Figure 4a for illustration). The economy starts from the point when the U.K. has a fundamental advantage over the U.S. in terms of economy size or exchange rate volatility, which it gradually looses along the transition path. I make three simplifying assumptions as in Matsuyama, Kiyotaki, and Matsui (1993) and Rey (2001). First, all countries find it optimal to trade either in dollars or in pound sterling. Second, since I am interested in long-run changes in currency choice, the focus is on the evolution of steady state in response to changes in exogenous parameters, while transition between steady states is ignored. Third, with multiple equilibria in the model, there is a continuum of possible transition paths. For selection, I use the argument in the spirit of evolutionary game theory that most agents follow the rule of thumb that has been used before. This implies that as long as the old equilibrium exists, the firms do not coordinate to jump into the new equilibrium. Therefore, among all possible transition paths, the one with the highest hysteresis is chosen.[27] The next proposition characterizes transition driven by changes in one of the two fundamentals — relative volatility of shocks or relative size of the U.S. keeping the total share of two currency unions in the global economy constant.

Proposition 6 (Transition) *Let $T(x)$ denote the threshold of $\frac{\sigma^2_{UK}}{\sigma^2_{US}}$ or $\frac{n_{US}}{n_{US}+n_{UK}}$, at which trade flow x from Figure 4a switches from pound to dollar. Then*

[26] Strictly speaking, the same is true in a model with $n = 0$ and $\rho < 1$, but since U.S. economy has zero mass, that has no effects at the global level.

[27] While a dynamic model with staggered pricing can be used to select between "history" vs. "expectations", the equilibrium remains non-unique in general case (see e.g. Matsuyama 1991, Krugman 1991). Alternatively, one can use a global game approach in the spirit of Morris and Shin (2001), but its application in dynamic settings is complex and goes beyond the scope of this paper.

(a) Trade flows



(b) Transition path

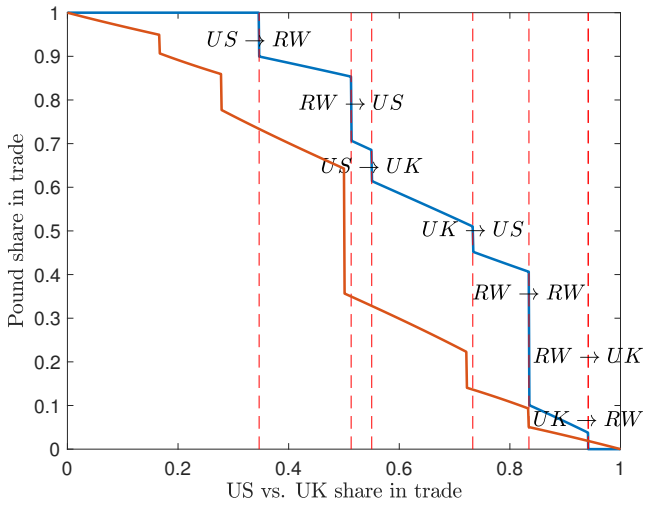


Figure 4: Transition from from pound to dollar

Note: plot (a) shows the structure of the economy with two currency unions — the U.K. and the U.S. — and the rest of the world (RoW) consisting of a continuum of small economies. The arrows are the trade flows between countries. Plot (b) shows transition from pound to dollar as the relative size of the U.S. increases assuming. The blue line is the benchmark transition under hysteresis and $\gamma = 0.6$, $\alpha = 0.5$, $\phi = 0.5$, $\lambda = 0.5$, $n_{US} + n_{UK} = 0.5$ and $\sigma^2_{UK} = \sigma^2_{US}$. The red line uses the same values except for $\lambda = 1$.

1. *the share of pound in the international trade is decreasing along the transition path,*
2. *the trade flows switch from pound to dollar in the following order:*
   - $T(a), T(b) \leq T(c) \leq T(f), T(g)$
   - $T(a) \leq T(d) \leq T(g)$
   - $T(b) \leq T(e) \leq T(f)$

Thus, as U.K. economy becomes smaller or/and more volatile, the share of pound in the international trade monotonically decreases. Figure 4b provides an example of a transition path for changes in union size, while Figure A2 in the Appendix shows transition driven by changes in volatilities. While the fundamental factors do change the equilibrium price system, there is also a path-dependence due to strategic complementarities in currency choice.[28] In particular, when the size of U.K. and U.S. is about the same, the share of pound in global trade remains as high as 85%. At the same time, the transition is much faster in the limit of flexible prices $\lambda \to 1$ with no complementarities in currency choice.

The model has also clear predictions about the order, in which trade flows in the global economy switch from pound to dollar. The trade between the U.S. and small economies is the first to become invoiced in dollars because of the prevalence of U.S. firms with costs stable in dollars. At the second stage, the small economies start using dollar as a vehicle currency when trading with each other, and

[28]The standard caveat that there are also equilibria with fast adjustment applies here as well. See Figure A2 for the lower and upper boundaries on the transition paths.

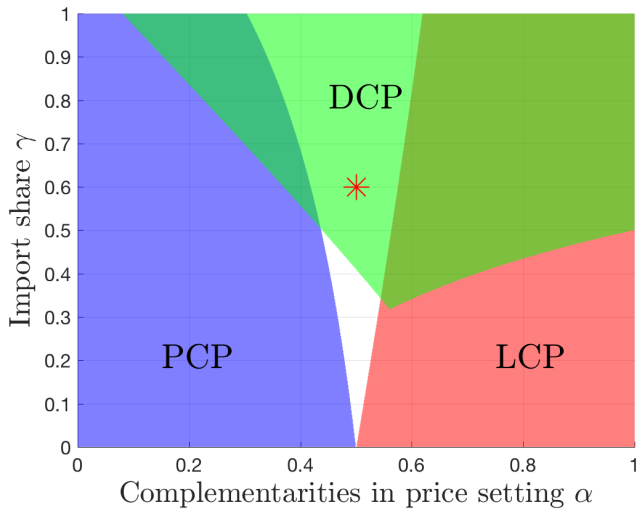


Figure 5: Currency choice under the baseline calibration

Note: the figures show the regions where symmetric PCP, LCP and DCP equilibria can be sustained (there is no symmetric equilibria in the white region). Parameter values are from the benchmark calibration: $\phi = 0.5$, $\lambda = 0.5$, $\rho = 0.5$, $n = 0.3$ and the red star shows the baseline calibration for $\gamma = 0.6$ and $\alpha = 0.5$.

the trade flows between two unions also change the currency of invoicing. Finally, the trade between the U.K. and small economies switches to DCP as well. Note if complementarities in currency choice are strong enough, some flows might remain invoiced in pound even as $n_{UK} \to 0$.

These predictions are broadly consistent with the historical evidence — the transition from pound to dollar was sluggish, followed with the lag after the U.S. overtook the U.K. as the largest economy, and was accelerated by large jumps in pound exchange rate after World War I and in 1931 (Eichengreen 2011). While the invoicing data is scarce for the beginning of the twentieth century, the experience of the Eurozone also fits predictions of the model. In particular, the euro is more commonly used in Eurozone trade with developing countries, much less so in trade with the U.S. and even more rarely as a vehicle currency (Kamps 2006).

Summary I next calibrate the model to the data and check whether DCP equilibrium can be supported under reasonable values of the parameters. Despite large differences between countries, industries and firms, I argue that the standard calibration with $\alpha = \phi = 0$ and γ close to U.S. import-to-GDP ratio of 0.15 does not provide a good approximation to the real world. In particular, a large fraction of non-tradable goods in GDP masks high import share in tradable (manufacturing) sector, which is about 0.6 for small economies and 0.4 for the U.S. Both firm-level data and the aggregate input-output tables imply that intermediate share in production is around $\phi = 0.5$, while the recent empirical estimates of complementarities in price setting suggest $\alpha = 0.5$ (Amiti, Itskhoki, and Konings 2016).[29]

[29] Both α and ϕ are higher for large firms that account for most of the global trade (Amiti, Itskhoki, and Konings 2014).

Assuming that one period corresponds to a year, I calibrate $\lambda = 0.5$, so that half of firms update prices by the end of the first year and the remaining ones adjust by the end of the second year. Assuming that the volatility of bilateral exchange rate between developing countries is higher than the volatility of exchange rate between a developing country and the U.S. by 33%, I get $\rho = 0.5$. Finally, I use $n = 0.3$, which is a conservative value relative to the large share of dollarized economies in the world (see Ilzetzki, Reinhart, and Rogoff 2017). Figure 5 shows that DCP equilibrium can be sustained under the baseline calibration.

Combining the mechanisms outlined above, this result can be interpreted as follows. Given that the U.S. is the largest economy in the world, the foreign firms selling in the U.S. market compete with a high number of local producers, which set prices in dollars. To avoid losing the market share because of unexpected movements in exchange rates, foreign firms synchronize their prices with the competitors by using dollar invoicing. The U.S. exporters then find the costs of both labor and intermediate inputs stable in dollars and are more inclined to use DCP in other markets. This increases the share of intermediate inputs and competing products invoiced in dollars that exporters in other economies face. Moreover, the firms that export from one developing country to another often find exchange rates of both countries too volatile to be used for invoicing and hence, are looking for a stable vehicle currency. With both the U.S. and emerging economies using dollars, the firms in developed countries might also find it optimal to switch to DCP. The exporters to the U.S. are then even more likely to set prices in dollars, which further strengthens the initial argument. Finally, while there might be also other equilibria with different dominant currencies, the path dependence in currency choice implies exporters might still use DCP despite the loss of its fundamental advantages relative to the middle of the twentieth century.

4 Transmission of Monetary Shocks

This section shows that despite only *second-order* effect on firm's profits, the currency choice has *first-order* general equilibrium implications. In particular, a small perturbation of fundamentals that makes firms switch from one invoicing regime to another leads to discontinuous changes in how prices, output, consumption, and trade balance react to monetary shocks. I argue below that in the empirically relevant case of DCP, the stimulating effect of exchange rate depreciation on local output is higher in the U.S. and lower in other economies, and that spillover effects of dollar depreciation on foreign output are more positive than predicted by the standard models with PCP and LCP.[30] I use a simple calibration of the model to show that small private costs of currency choice can lead to large differences in business cycles (*cf.* Mankiw 1985).

A tractable general equilibrium block of the model allows me not only to formalize several conjectures about the trade balance adjustment from Gopinath (2016) and Goldberg and Tille (2006), but also

[30]The words "positive" and "negative" in this section refer to signs of the effects and not to their welfare implications.

to analyze the response of output and consumption to monetary shocks and to identify parameters that determine the sign and the magnitudes of the effects. I show in particular there are significant general equilibrium effects of dollar depreciation on foreign output even in the limiting cases when partial equilibrium ones cancel out. For simplicity of exposition, I focus exclusively on equilibria with symmetric invoicing and unexpected shocks, suppressing the time subscript below.

Local effects The effect of exchange rate depreciation on trade balance, consumption and output depends on how import and export prices respond to these shocks. As emphasized by the previous literature, the pass-through of exchange rates into customer prices is high under PCP and low under LCP, which implies that quantities respond much less under LCP than PCP (see e.g. Betts and Devereux 2000). Relative to this benchmark, invoicing in dollars introduces two types of asymmetries — between export and import prices, and between the U.S. and other economies. In particular, the price response resembles PCP on export side and LCP on import side for the U.S. and the other way around for other countries. Thus, in response to positive monetary shock, the trade balance adjusts more through higher exports in the U.S. and lower imports elsewhere.[31]

The differences in trade balance adjustment across countries under DCP translate into the asymmetric response of consumption and output. The depreciation of exchange rate stimulates production more in the U.S. than in other countries because of larger expenditure switching towards exported goods and lower increase in prices of foreign intermediates. At the same time, the lower pass-through of exchange rate shocks into CPI implies that the U.S. enjoys smaller fall in consumption.

Proposition 7 (Transmission of monetary shocks) *Assume $n = 0$ and DCP. Then relative to the effects of monetary shock in other economies, an expansionary monetary policy in the U.S. implies*

1. *higher exports and imports,*
2. *lower inflation and higher output,*
3. *the same net export.*

Interestingly, despite these asymmetries across countries, the elasticity of *net export* with respect to the trade-weighted exchange rate is the same for all economies including the U.S. — the higher elasticity of exports and the lower elasticity of imports in the U.S. exactly offset each other — which has two important implications. First, even under asymmetric currency choice, the trade-weighted rather than invoicing-weighted exchange rate remains sufficient statistics for net exports. Second, consider the case of incomplete international asset markets when exchange rate adjusts to ensure that trade balance holds. The same elasticity of net exports implies then that response of exchange rate to exogenous

[31] The total effect is however more than just a convex combination of the two due to input-output linkages. Consider non-U.S. economy. Relative to LCP case, imported intermediates are more expensive and therefore prices of adjusting exporters fall less, depressing exports even further. Relative to PCP case, a weaker growth in exports implies lower demand for foreign intermediates, which amplifies contraction in imports.

shocks is symmetric across countries. Therefore, DCP does not necessarily generate lower (or higher) volatility of U.S. exchange rate.

International spillovers The last decade has witnessed a lively debate about the spillover effects of the Fed's monetary policy on other countries (see e.g. Bernanke 2017). On the one hand, easy monetary policy increases demand for both domestic and imported goods, stimulating production in all economies. On the other hand, such policy also leads to depreciation of the national currency, which can potentially make local goods cheaper relative to foreign ones and have negative spillovers on other economies. The classical result in the literature is that the net effect is negative under PCP and positive under LCP: while the former effect does not depend on currency of invoicing, the latter one is large under PCP and mild under LCP (see e.g. Betts and Devereux 2000, Corsetti and Pesenti 2005). I next show that additional channel with unambiguously positive spillovers arises under DCP that has been largely ignored in the previous literature.[32] To this end, consider the effect of U.S. monetary shock that increases nominal spendings in the U.S. I discuss the difference in spillovers that arises under DCP vs. PCP/LCP. Since the aggregate demand effect is independent from currency of invoicing, I focus below exclusively on the pass-through of dollar shocks into global prices, trade and output.

For any variable x_i, which can denote prices or quantities in country i, define the global counterpart as $x = \int_0^1 x_i \mathrm{d}i$.[33] Aggregating the import price index (in destination currency) across countries, one obtains:

$$p^I = -\frac{(1-\lambda)(1-n)\mu^D}{1-\lambda(\alpha+(1-\alpha)\phi)} e_0, \tag{20}$$

where μ^D equals one if equilibrium invoicing is DCP and zero otherwise. Thus, even when U.S. accounts for a positive share of the world economy $n > 0$, the pass-through of dollar exchange rate into aggregate import price index is zero if prices are set not in dollars: depreciation of dollar simultaneously decreases prices of U.S. export and increases U.S. import prices, leaving the global price index unchanged. On the other hand, when prices are sticky in dollars, depreciation of e_0 decreases international prices in currency of destination for all importers except the U.S., hence $(1-n)$ term. A fall in international prices in turn translates into lower price index for tradable goods p and lower CPI p^C:

$$p = \gamma p^I, \qquad p^C = \eta p + (1-\eta) p^N,$$

where p^N is the price index in non-tradable sector and η is the share of tradable sector.[34]

The movements in international and domestic prices translate into changes in the volume of global

[32] The important exception is the paper by Goldberg and Tille (2009), which shows in a context of a three-country model that U.S. shocks have larger effect on global consumption under DCP.

[33] Note that all variables including prices are expressed in same units — log-deviations from the steady state values — and can therefore be integrated across countries with different units of account.

[34] Despite its global implications, the dollar exchange rate e_0 is determined solely by U.S. shocks and is independent from invoicing regime according to Lemma 2.

trade y^I. The pass-through of prices into quantities can be decomposed into four channels:

$$y^I = -\theta\underbrace{(p^I - p)}_{\text{ES}} + \phi\underbrace{(w - p)}_{\text{Int. vs. L}} + (1-\eta)\underbrace{(p^N - p)}_{\text{T vs. NT}} + \underbrace{(w - p^C)}_{\text{Real Wage}}. \tag{21}$$

The first term corresponds to expenditure switching: a fall in relative price of imported goods $p^I - p$ implies that buyers switch from domestic goods towards internationally traded ones with the effect increasing in elasticity of substitution θ. The second term in (21) shows that firms substitute labor with cheaper intermediates in production. Similarly, consumers switch from non-tradables to tradables with the effect proportional to the share of non-tradables in consumption basket $1-\eta$. Finally, lower prices for tradables decrease CPI, which stimulates labor supply through higher real wages. All these effects work in the same direction and increase global trade in response to dollar depreciation under DCP even when U.S. share in trade is zero. The prediction of the model is therefore consistent with the growing empirical evidence about the effect of dollar shocks on global trade (see Boz, Gopinath, and Plagborg-Møller 2017, Casas, Díez, Gopinath, and Gourinchas 2017).

The increase in global demand for imported products translates into higher output y and consumption c worldwide:[35]

$$y = \eta\phi\underbrace{(w - p)}_{\text{Int. vs. L}} + \underbrace{(w - p^C)}_{\text{Real Wage}} \qquad \text{and} \qquad c = \underbrace{(w - p^C)}_{\text{Real Wage}}. \tag{22}$$

The terms $p^N - p$ and $p^I - p$ cancel out due to aggregation between sectors and countries. In particular, substitution from non-tradable goods for tradables does not affect total output. Similarly, expenditure switching effect increases both exports and imports, with the latter crowding out local production. The net effect is therefore, zero and does not depend on the elasticity of substitution θ. The two remaining effects — firm substitution towards intermediate goods and labor supply effect — however boost global production in response to dollar depreciation under DCP. The effect is stronger when the pass-through of dollar shocks into producer price index in tradable sector p and consumer price index p^C is high.[36]

Proposition 8 (International spillovers) *Relative to PCP/LCP benchmark, the dollar invoicing implies that expansionary monetary policy in the U.S.*

1. *increases the volume of international trade y^I,*
2. *increases the global output y and consumption c, with the effect independent from elasticity θ,*
3. *decreases CPI of other economies and boosts consumption and production if θn is low.*

[35] Under more general preferences, the pass-through of e_0 into global consumption depends on all four channels.

[36] In particular, it can be close to zero if one assumes that both firms and households buy products from a wholesale/retail sector with very sticky prices.

How is the global output y divided between the U.S. and the rest of the world? Under both PCP and LCP, the depreciation of e_0 leaves y unchanged and decreases the relative prices for U.S. goods. It follows that expenditure switching towards U.S. output shrinks production in other economies (see Appendix A.8 for details). The negative spillovers are therefore larger when U.S. share in world trade n and demand elasticity θ are higher. Under DCP, on the other hand, depreciation of dollar increases global output y. In the limit $n = 0$, the whole "pie" goes to the RoW and spillover effects are unambiguously positive. Intuitively, expenditure switching towards U.S. goods has zero effect when $n = 0$. At the same time, lower international prices boost trade between non-U.S. countries and stimulate production through general equilibrium effects. When $n > 0$, there are both positive spillovers from trade between third countries and negative ones from trade with the U.S., so that the sign of the net effect depends on parameter values.[37]

Thus, the spillover effects of dollar depreciation on foreign output can be positive even when monetary authorities are constrained by the zero lower bound and cannot stimulate the aggregate demand. This contrasts with the conclusions of the previous literature that depreciation of exchange rate in this case is a zero-sum beggar-thy-neighbor policy that exports recession to other countries and can potentially lead to "currency wars" (see e.g. Caballero, Farhi, and Gourinchas 2016).[38] On the other hand, the appreciation of dollar can have negative effect on other economies if their output is already inefficiently low. At the same time, the devaluation of non-vehicle currencies leads to standard expenditure-switching effect and is closer to the beggar-thy-neighbor benchmark.

Private costs vs. aggregate effects While the model is intrinsically stylized and abstracts from both cross-country heterogeneity and several ingredients from the DSGE literature (e.g. capital, habit formation, wage rigidity, etc.), it might still be informative to put some numbers on the effects outlined above and to compare private costs with aggregate effects. I use the same values of α, γ, ϕ, λ, n and ρ as in Section 3.2. In addition, the share of tradable sector $\eta = 0.15$ is calibrated to the share of manufacturing in global GDP and the elasticity of substitution between goods $\theta = 2$ is close to the numbers used in the previous literature (see e.g. Chari, Kehoe, and McGrattan 2002, Backus, Kehoe, and Kydland 1994, Feenstra, Luck, Obstfeld, and Russ 2014).

Table 1 shows the medium-run effects of a monetary shock that increases nominal spendings by 10%. The first three columns correspond to the U.S. monetary shock, while the next ones show effect of monetary expansion in another country. Despite large share of the U.S. in global economy, $n = 0.3$, the results from Proposition 7 hold: under DCP, the stimulating monetary policy is significantly more efficient in the U.S. than in other countries: the GDP increases by around 5.5% in the U.S. and 4.8% in

[37] The small previous literature that studied transmission of shocks under DCP has mostly assumed only two countries (see e.g. Canzoneri, Cumby, Diba, and López-Salido 2013, Corsetti and Pesenti 2007). In this case, all imports of the RoW come from the U.S., so that effectively $n = 1$ and as expression (20) shows, there are no positive spillovers: depreciation of dollar generates expenditure switching exclusively towards U.S. goods instead of exports of other countries.

[38] Dollar depreciation also implies lower inflation, giving more room for stimulating monetary policy in other countries.

Table 1: Local and spillover effects of monetary shocks

		U.S. shock			Non-U.S. shock		
		DCP	PCP	LCP	DCP	PCP	LCP
U.S.	y_T	11.06	11.27	5.83	–	–	–
	gdp	5.52	5.17	5.46	–	–	–
	c	5.41	4.91	5.36	–	–	–
Non-U.S.	y_T	6.24	−1.05	1.28	4.82	12.32	4.55
	gdp	0.69	0.12	0.00	4.82	5.05	5.46
	c	0.74	0.23	0.04	4.68	4.68	5.32

Note: the table shows the percentage change in production of tradables, GDP and consumption of U.S. and other countries in response to a local and a foreign monetary shock that increases domestic nominal spendings by 10%.

other economies. Consistent with the results from the previous literature, the spillovers of U.S. shock on foreign production in tradable sector is negative under PCP and positive under LCP. The positive effect is however 5 times higher under dollar invoicing.[39] As a result, foreign GDP and consumption increase respectively by 0.69% and 0.74% when prices are set in dollars.

Lastly, I compare these effects with the private costs of currency choice. To this end, I calibrate the standard deviation of the bilateral exchange rate between non-U.S. countries to 0.15 and assume that it is driven by financial shocks. I then calculate losses for an individual exporter of using dollar pricing instead of the optimal basket of currencies keeping the aggregate DCP equilibrium constant. The aggregate costs across all exporters are only 0.02% of the global GDP, which is more than one magnitude lower than the spillover effects discussed above. The result resembles the classical argument of Mankiw (1985) and Ball and Romer (1990) that small menu costs can lead to large business cycles. In case of open economy, there is however an additional dimension as exporters choose in which currency to set their prices. These decisions are based on the second-order effects on firm's profits (Lemma 1), but have first-order implications for the transmission of monetary shocks within and across countries. The complementarities in currency choice play the same role as real rigidities in the context of price adjustment decisions and amplify the difference between private and aggregate effects.

5 Optimal Monetary Policy

The optimal monetary and exchange rate policy is one of the central questions in the international economics. Should the policy focus on inflation targeting and output stabilization as in the closed economy, or movements in exchange rates can be a separate concern for policymakers? Under which

[39] In contrast to the conventional model, the spillovers on foreign GDP are positive under PCP because of high share of intermediate goods: dollar depreciation decreases costs of inputs in other countries and stimulates production and consumption. See Rodnyansky (2017) for the empirical evidence in favor of this mechanism.

conditions is it optimal to peg exchange rate rather than let it float? Which price index is the relevant policy target — consumer prices (CPI) or producer prices (PPI)? While the previous literature has shown that the answers to these questions depend crucially on invoicing of international trade, firms' currency choice has predominantly been taken as exogenous.[40] The results are therefore potentially subject to Lucas critique: the models ignore the fact that firms might change their invoicing decisions in response to monetary policy. In addition, the literature has predominantly focused on PCP and LCP rather than a more empirically relevant case of DCP.

This section fills in this gap. I first augment a conventional open-economy model from the previous normative literature with the endogenous currency choice and show that the first-best allocation can be implemented by the optimal policy that targets PPI. The individual invoicing decisions generate no inefficiencies in this case and do not alter the classical argument for floating exchange rates. While standard in this literature, the assumptions underlying this result are restrictive and are inconsistent with the data. I then relax them and show that all three types of currency regimes can emerge under the optimal policy depending on parameter values. I argue there are complementarities between exporters' invoicing decisions and the optimal monetary policy, which can explain the dominant status of dollar in trade and as an anchor currency in the exchange rate policy.

5.1 Efficient benchmark

In general case, when nominal prices are sticky, the relative international prices get distorted and the equilibrium allocation is not efficient. However, as has been famously argued by Milton Friedman, the floating exchange rates can mitigate these distortions as they allow relative prices across countries to adjust even if nominal prices remain fully rigid: "It is far simpler to allow one price to change, namely, the price of foreign exchange, than to rely upon changes in the multitude of prices that together constitute the internal price structure" (Friedman 1953). This argument has been formalized fifty years later by Devereux and Engel (2003), who showed that the *first-best* allocation can be implemented with the *optimal* monetary policy under floating exchange rates if firms set prices in *producer currency*. Under LCP, on the other hand, the efficient allocation cannot be achieved and keeping exchange rates constant might be optimal.

I use the model with endogenous currency choice to reexamine conclusions of this literature. To identify the externalities coming from invoicing decisions of firms and make results directly comparable to the previous literature, I start with the case when price rigidity is the only source of distortions in the economy. In particular, the international asset markets are complete, there is only one sector in the economy, the steady-state markup arising from monopolistic power of firms is eliminated with a fixed subsidy, monetary policy is cooperative across countries and there is no lack of commitment (see

[40] To mention just a few: Clarida, Gali, and Gertler (2001), Benigno and Benigno (2003), Corsetti and Pesenti (2005), Monacelli (2005), Gali and Monacelli (2005), Devereux, Shi, and Xu (2007), Corsetti, Dedola, and Leduc (2010), Engel (2011).

Appendix A.9 for details).[41]

Proposition 9 *Assume (i) no complementarities in pricing, $\alpha = 0$, (ii) full commitment, (iii) cooperative policy across countries. Then efficient allocation can be implemented by the optimal monetary policy that allows for floating exchange rates and stabilizes producer prices (PPI). The equilibrium invoicing is PCP.*

One way to interpret the optimal policy is to note that implementation of the first-best allocation requires that the planner replicates the corresponding relative prices. Since nominal prices of goods are sticky, the optimal monetary policy keeps PPI fully stable and makes sure that other prices — nominal wages, interest rates and exchange rates — adjust to replicate optimal relative prices. Under PCP, movements in exchange rates guarantee that product prices in customers' currency adjust optimally in response to shocks even though prices remain fully stable in currency of producer. This summarizes the logic behind the result from Devereux and Engel (2003).

In contrast to their setup, however, the model with endogenous currency choice generates an additional constraint on the planner's problem. The key insight of Proposition 9 is that this constraint is not binding at the optimum: the firms always choose PCP under the optimal monetary policy. The assumption $\alpha = 0$ implies that producer prices are proportional to their marginal costs. As a result, the monetary policy that targets PPI also stabilizes marginal costs and the optimal price of exporters in producer currency, which means that firms unambiguously prefer PCP. Importantly, however, while PCP constraint is not binding under the *optimal* policy, the same statement does not hold globally. In other words, condition $\alpha = 0$ alone is not sufficient to guarantee PCP equilibrium — depending on parameter values, DCP or multiple equilibria can arise. The fact that planner can commit to target PPI even in off-equilibrium states of the world, in which firms set prices in dollars, is important to implement the first best.

Proposition 9 implies therefore that decentralized currency choice per se does not generate additional inefficiencies. This contrasts with the conclusion of the previous literature that LCP is an important source of distortions in the global economy. In particular, the proposition shows that the analysis of the optimal policy under exogenous LCP and DCP is subject to Lucas critique: it is not possible to sustain such equilibria under the optimal policy without some additional assumptions.

5.2 Discretionary policy

While Proposition 9 provides an important benchmark that clarifies the effect of endogenous currency choice on the optimal policy, the underlying assumptions are hardly realistic. In particular, as has been discussed above, the complementarities in price setting are strong in the data and play the key role in firms' currency choice. The previous normative literature has largely ignored price complementarities

[41] The result holds for general isoelastic preferences, an arbitrary frequency of price adjustment λ and all exogenous shocks except for financial ones and markup shocks. The assumption that prices are flexible after one period is not important: the same result can be derived under Calvo pricing.

and provides little guidance about their effect on the optimal policy even when currency choice is exogenous. This section fills in this gap. In addition, I assume that monetary policy is discretionary, i.e. it is chosen after the realization of shocks and takes the ex-ante currency choice of exporters as given. In other words, the planner cannot make a credible threat to punish firms if they deviate from a given invoicing. I therefore solve for the Nash equilibrium, in which firms simultaneously choose the currency of invoicing, taking into account that the monetary policy is determined by the aggregate currency regime.[42]

To simplify the analysis and obtain sharp analytical results, I follow Devereux and Engel (2003) and assume fully sticky prices, $\lambda = 0$, symmetric countries, $n = 0$, symmetric invoicing and only productivity shocks a_i (see Appendix A.9 for details).

Proposition 10 (Discretionary policy) *Under the optimal discretionary policy,*

1. *exchange rates are more flexible under PCP than under LCP, and are the same under DCP as PCP except for fully stabilized U.S. exchange rate:*

$$e_{ij}^{LCP} = \frac{1-\gamma}{1-(1-\gamma)\phi}(a_i - a_j), \quad e_{ij}^{PCP/DCP} = \frac{1}{1-(1-\gamma)\phi}(a_i - a_j), \quad e_{i0}^{DCP} = \frac{1}{1-(1-\gamma)\phi}a_i,$$

2. *the regions of PCP and LCP do not overlap and cover the whole parameter space and the region of DCP is non-empty:* $\alpha \leq \frac{1}{2-\gamma}$ *for PCP,* $\alpha \geq \frac{1}{2-\gamma}$ *for LCP, and* $\frac{1}{2} \leq \alpha \leq \frac{1}{2(1-\gamma)}$ *for DCP,*

3. *when multiple equilibria coexist for given parameter values, the welfare can be ordered as follows:* $\mathbb{W}^{DCP} \geq \mathbb{W}^{LCP}$ *and* $\mathbb{W}^{DCP} \leq \mathbb{W}^{PCP}$.

Consider first the classical case of PCP and LCP. When nominal prices are rigid, the relative prices do not adjust in response to productivity shocks and the equilibrium allocation is inefficient without government intervention. The monetary policy then stimulates local demand and depreciates exchange rate in response to positive productivity shock. Under PCP, this makes both local and foreign consumers switch to goods produced in country i increasing the efficiency of the allocation. When prices are sticky in local currency, on the other hand, there is no expenditure switching and the monetary policy can only affect local demand. Since monetary expansion increases demand for all goods, including imported ones, the global planner has to trade off local benefits with negative spillovers on other countries. The optimal response is therefore proportional to the share of local goods $1-\gamma$ and is zero in the limit with no home bias $\gamma \to 1$. The implied exchange rates are fully fixed in this case, $e_i \to 0$, which resembles the second key result from Devereux and Engel (2003).[43]

[42] The efficient steady state and cooperative monetary policy ensure there is no inflationary bias or terms-of-trade manipulation, and the currency choice is the only source of time inconsistency.

[43] Importantly, I show in Appendix A.9 that the second-order approximation to the planner's objective function that is derived from market clearing and risk-sharing conditions, does not depend on α and is the same as for CES aggregator. The effect of α on optimal policy comes therefore only from currency choice of firms.

Interestingly, the regions of PCP and LCP equilibria do not overlap and cover the whole parameter space under the optimal policy (see Figure 6a) and are exactly the same as in the flexible-price limit with no productivity shocks in Figure 2b. Intuitively, this is because in all three cases the labor wedge is equal zero: in Section 3.2 this comes from stable productivity and nominal wages, under PCP implementing the optimal real wage is sufficient to eliminate other wedges as well due to flexible exchange rates, while under LCP the planner cannot affect other distortions in any case. As a result, the relative volatility of prices and marginal costs expressed in the same currency is constant across three regimes. Since this is a sufficient statistic for exporters' choice between producer and local currency, the regions of equilibria are the same under flexible prices, PCP and LCP. Notice that LCP equilibrium disappears as the home bias converges to zero, $\gamma \to 1$, which implies that the region where fixed exchange rates are optimal consists of only one point $\alpha = \gamma = 1$.

Turing to the case of dollar pricing, the optimal monetary policy is much closer to the one under PCP than under LCP. When sticky in dollars, the prices of all imported goods move together and it is impossible to generate expenditure switching towards products with lower costs of production. In contrast to LCP case, however, the relative demand for home vs. foreign goods does depend on exchange rates, and the planner finds it optimal to follow the same policy as under PCP. In addition, the planner fully stabilizes dollar exchange rate because the losses from the suboptimal demand for U.S. goods are infinitely small when $n = 0$, while distortions coming from fluctuations in prices invoiced in dollars are large. Because of the intermediate degree of expenditure switching, the implemented allocation under DCP is less efficient than under PCP, but is more efficient than under LCP.

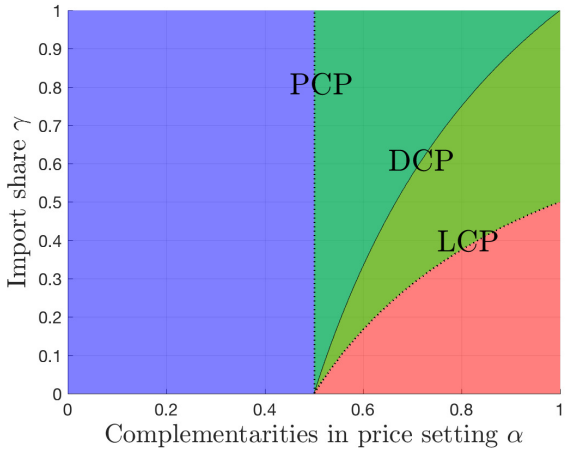
In contrast to PCP and LCP, dollar pricing generates strategic complementarities between firms' invoicing decisions and the monetary policy. When firms choose PCP/LCP, the policy is symmetric across countries and dollar has no volatility advantage given the same volatility of productivity shocks in the U.S. as in other economies. This policy therefore provides no incentives for firms to set prices in dollars. On the other hand, the planner optimally sets $e_0 = 0$ when firms choose DCP no matter how volatile the productivity shocks in the U.S. are. The lower volatility of dollar makes it a more appealing vehicle currency and stimulates firms to choose DCP.[44]

5.3 Non-cooperative policy

As Section 4 suggests, there might be significant spillover effects from U.S. policy on other countries when international prices are set in dollars. These spillovers are fully offset by the global planner, which minimizes the volatility of dollar exchange rate under DCP. This section relaxes the assumption of a cooperative monetary policy and derives the optimal response of other countries to U.S. mone-

[44]Figure A3 in Appendix shows the optimal policy and equilibrium invoicing are well approximated by Proposition 10 away from the limit with fully rigid prices. In addition, panels (c) and (d) show that the insight from Engel (2011) is approximately true despite additional complementarities in price setting: the optimal policy is close to CPI targeting under LCP and targeting marginal costs under PCP. The implied CPI and marginal costs under DCP are close to PCP case.

(a) Cooperative policy



(b) Non-cooperative policy

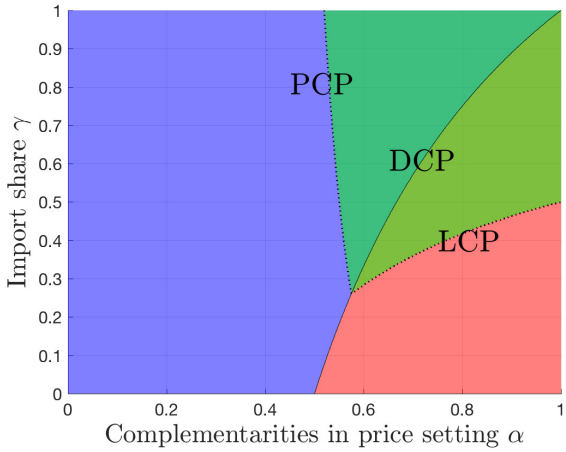


Figure 6: Currency choice under the optimal discretionary monetary policy

Note: the figure shows the equilibrium invoicing under the optimal discretionary monetary policy given the following values of parameters: $\lambda = 0$, $\phi = 0.5$, $n = 0$, $\theta = 2$. Plot (a) assumes cooperative policy, while plot (b) assumes that U.S. economy is closed and the monetary policy there is chosen independently from other countries with $\sigma^2_{a_0} = \sigma^2_a$. The solid line shows the boundary between PCP and LCP, while the dashed one shows the boundary of DCP region.

tary shocks. Since solving for non-cooperative policy is analytically challenging and usually requires restrictive assumptions on parameter values (see e.g. Benigno and Benigno 2003, Corsetti and Pesenti 2001, Farhi and Werning 2012), I consider the limiting case when U.S. economy is closed, $\eta \to 0$, and the monetary policy of all other countries remains cooperative. This captures the fact that the optimal policy in the U.S. is more inward-looking than in small economies: in the closed economy limit, it is independent from the global trade invoicing and the policy in other countries.

Proposition 11 (Non-cooperative policy) *Under the optimal non-cooperative discretionary policy,*

1. *bilateral exchange rates between non-U.S. countries are freely floating under both DCP and PCP, and depend only on relative productivities between countries:* $e_{ij} = \frac{1}{1-(1-\gamma)\phi}(a_i - a_j)$,
2. *under DCP, monetary policy in all countries comoves positively with the U.S. one and partially smooths out exchange rates against dollar relative to PCP case,*
3. *DCP region is non-empty even when volatility of productivity shocks is the same across countries.*

The optimal policy in the U.S. adjusts aggregate demand in response to local productivity shocks and achieves efficient allocation within the country. When international prices are set in producer or local currency, the U.S. policy has zero effect on other countries and the equilibrium exchange rates are the same as under full cooperation. Thus, both PCP and LCP equilibria remain the same as described in Proposition 10. In contrast, under DCP, there are two types of shocks that policymakers face — local changes in productivity and external movements in terms-of-trade driven by fluctuations

in dollar exchange rate. The optimal response to the former is the same as before: while incomplete, the expenditure switching between domestic and imported goods allows to reallocate global demand towards products with lower costs. As a result, the bilateral exchange rates between non-U.S. countries remain the same as under cooperative policy.

The fluctuations in terms-of-trade between third countries that come from movements in dollar exchange rate, on the other hand, are distortionary as they do not reflect relative productivities of the economies. The optimal policy therefore "leans against the wind" and partially offsets movements in e_0, which implies that bilateral exchange rates against dollar are less volatile under DCP than under PCP.[45] While exchange rate stabilization allows bringing relative prices *across* countries closer to the efficient level, such policy is costly as it distorts relative prices *within* countries. As a result, U.S. shocks are only partially offset under the optimal policy, and the equilibrium exchange rate is neither floating nor fixed. This prediction of the model is consistent with the empirical fact that more than 70% of countries follow managed float regime ("crawling peg", "dirty float") and use dollar as an anchor currency in their exchange rate policy (Ilzetzki, Reinhart, and Rogoff 2017, Calvo and Reinhart 2002). Proposition 11 also contributes to the recent debate about implications of dollar invoicing for the "trilemma": while the trade-off is worsened by DCP relative to PCP benchmark, the flexible exchange rates still allow to achieve higher welfare than the fixed ones (see Bernanke 2017, Gopinath 2017).

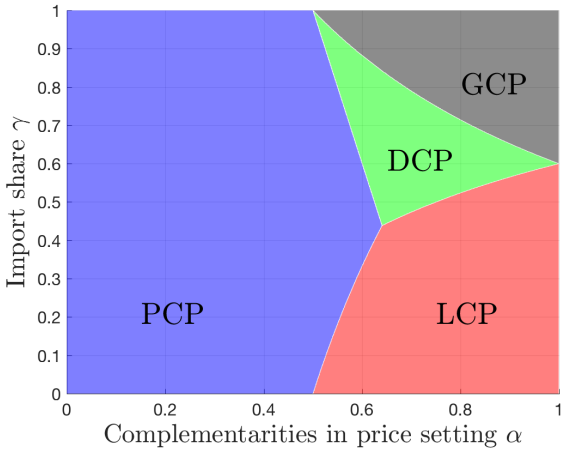
The fact that all economies respond to movements in U.S. exchange rate also implies that monetary policy is correlated across countries despite the assumption that fundamental shocks are purely idiosyncratic. An expansionary monetary policy in the U.S. leads to depreciation of dollar exchange rate and makes central banks in other countries to ease their policy as well. This is consistent with the evidence on the global financial cycles (Rey 2015) and shows that a positive comovement of monetary policy across countries can arise not only due to financial linkages, but also because of the dominant status of dollar in international trade (*cf.* Aoki, Benigno, and Kiyotaki 2016).

Finally, the monetary policy feeds back into firms' currency choice. Even when volatility of fundamental shocks is the same for the U.S. as for other countries, the optimal policy of pegging exchange rates to dollar implies that dollar is more stable than other currencies and hence, exporters are more likely to use DCP. Thus, the model predicts strategic complementarities between firms' invoicing decisions and the monetary policy: DCP makes it optimal to peg exchange rates to dollar, which in turn increases incentives of exporters to set prices in dollars. Figure 6b shows that the resulting region of DCP equilibrium can be large even when the U.S. has no fundamental advantage.[46]

[45] This result contrasts with the conclusion of Goldberg and Tille (2009) that monetary policy of periphery countries should focus exclusively on local shocks as the latter model does not take into the account losses from price dispersion. The analysis also complements the general result from Casas, Díez, Gopinath, and Gourinchas (2017) that the optimal policy targets price misalignments under DCP.

[46] While equilibrium exchange rates and welfare implications depend on the type of exogenous shock, the results about partial peg to dollar, global cycles in monetary policy and non-empty DCP region are robust and hold in particular for financial shocks.

(a) Flexible price limit



(b) Sticky prices

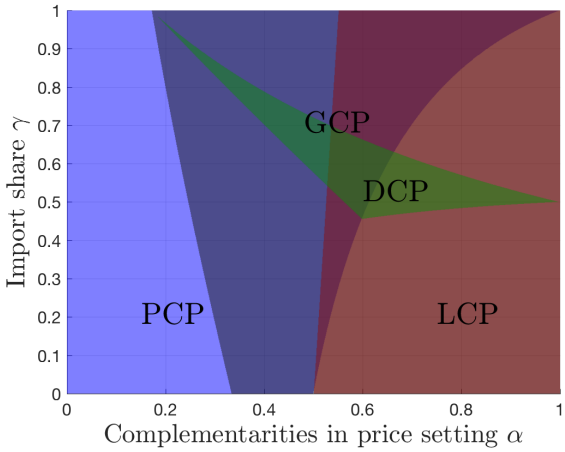


Figure 7: The optimal invoicing of domestic firms

Note: figure (a) shows equilibria in the flexible price limit $\lambda \to 1$ and $\rho = 0.5$, while figure (b) shows symmetric equilibria under sticky prices $\lambda = 0.5$ and $\rho = 1$. The grey area is the region of global currency pricing (GCP) equilibrium with all firms including domestic ones using dollar for invoicing. Other parameters: $\phi = 0.5$, $n = 0$.

6 Extensions

This section relaxes two assumptions from the baseline model and provides new mechanisms that can significantly amplify the benefits of dollar invoicing and increase the region of DCP equilibrium. I first allow domestic firms to make optimal currency choice and show that while they are less likely to set prices in dollars than exporters, a persistent DCP equilibrium can emerge once local firms switch to dollar invoicing. I then argue that exporters are more likely to use DCP when monetary shocks account for a significant fraction of exchange rate volatility. The section discusses the intuition behind these results, while the formal results can be found in Appendix A.10.

Dollarization In contrast to the assumption in the baseline model, it is not uncommon for local firms in developing countries to set prices in dollars (see e.g. Drenik and Perez 2017). I therefore extend the model allowing domestic producers in the tradable sector to choose optimally the currency of invoicing and define the global currency pricing (GCP) equilibrium, in which all firms in tradable sector including domestic ones set prices in dollars.

Consider first the flexible price limit $\lambda \to 1$ (see Figure 7a). With almost all firms adjusting prices after the realization of shocks, the currency choice of domestic producers has no effect on invoicing decisions of exporters, which remain the same as in the baseline model. Since producer and local currencies coincide for domestic firms, they are less likely to use dollar invoicing. The GCP equilibrium is therefore a subset of the DCP equilibrium from the benchmark model. The equilibrium invoicing

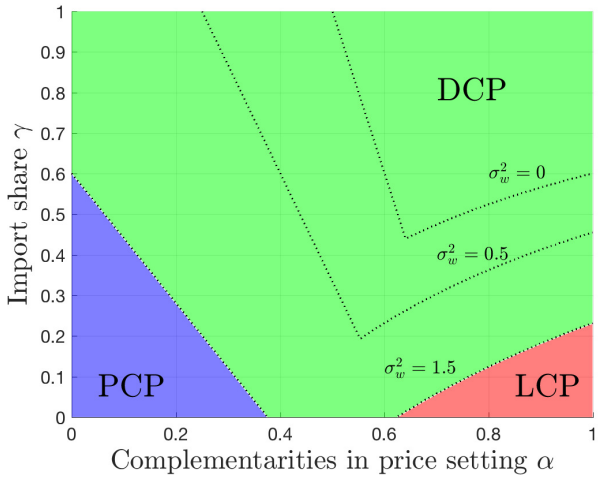


Figure 8: Currency choice under exogenous monetary shocks

Note: the figure shows equilibria in a model with exogenous monetary and financial shocks. The volatility of financial shocks is normalized to one, while the volatility of nominal shocks is shown in the figure, $\lambda \to 1$, $\phi = 0.5$, $n = 0$, $\rho = 0.5$.

looks very different when prices are sticky: in the limiting case of fully rigid prices, the DCP region is always a subset of the GCP one. Intuitively, strategic complementarities in currency choice that arise under sticky prices imply it is easier to support equilibrium with all firms invoicing in dollars than the one with only exporters using dollars and domestic firms setting prices in local currency. As Figure 7b shows, even incomplete price rigidity is sufficient for GCP region to dominate both DCP and LCP ones.

Thus, the model predicts that while domestic firms are less likely to switch to dollar invoicing than exporters, once they do so — e.g. because of unstable monetary policy discussed below — the DCP equilibrium can be sustained more easily and can persist even after fundamentals turn against dollar. The wide use of dollar in Latin American and some East European countries contributes therefore to the status of dollar in the international trade.

Monetary shocks While movements in exchange rates are largely disconnected from monetary shocks for most economies (Meese and Rogoff 1983), the correlation is much higher for countries with unstable inflation. I therefore relax assumption $w_i = 0$ and allow for exogenous stochastic shocks in nominal wages.[47]

Consider first the limiting case when w_i is the only shock in the economy and prices are almost flexible $\lambda \to 1$. The labor costs are no longer stable in producer currency and as a result, neither are the prices of domestic intermediate goods. At the same time, a positive monetary shock is associated with a one-to-one depreciation of local exchange rate, which implies that nominal wages can actually be more stable in foreign currency than in local one. In particular, as long as $\rho < 1$, the volatility

[47] I focus on the second rather than first moments of monetary shocks, which complements the effect of inflation rate on currency choice emphasized by the previous literature (see e.g. Drenik and Perez 2017).

of nominal wages in dollars $w_i + e_{0i}$ is lower than the volatility in producer currency w_i, and firms unambiguously prefer DCP to PCP. A symmetric argument applies to LCP. The DCP is therefore a unique equilibrium for arbitrary values of other parameters and in particular, can be sustained even in the limit of closed economy $\gamma \to 0$. This prediction of the model is consistent with the wide use DCP during the episodes with high and unstable inflation in Latin American countries in 1980s and in Eastern Europe in 1990s.

More generally, in the presence of other shocks, the higher volatility of monetary shocks increases the correlation between wages and exchange rates and extends the region of DCP (see Figure 8). Importantly, this result holds even when volatility of U.S. monetary shocks increases proportionately with nominal shocks in other countries. In contrast to mechanism outlined in Devereux, Engel, and Storgaard (2004), a higher volatility of monetary shocks makes DCP more appealing to firms not because of increasing volatility of other currencies relative to dollar, i.e. falling ρ, but because of lower stability of input and competitor prices in producer and local currencies respectively. The model thus suggests that periods of high *global* inflation — as the one observed in 1970s — can actually increase the use dollar in international trade despite higher volatility of U.S. exchange rate.

7 Conclusion

In this paper, I propose a tractable framework with endogenous currency choice for examining the determinants and the implications of the international price system. The model is broadly consistent with the key stylized facts, including the dominant status of dollar as a vehicle currency in global trade and the delayed transition from pound to dollar in the twentieth century. Despite small private costs, the currency choice of exporters has large aggregate effects. In particular, the spillover effects of dollar depreciation on foreign output are more positive when international prices are set in dollars than predicted by the standard models with producer/local currency pricing. The optimal policy analysis, on the other hand, shows a close relation between the dominant status of dollar in the international trade and the wide use of dollar as an anchor currency in exchange rate policy.

The tractability of the baseline model allows for several other extensions and applications, which is a part of my research agenda. First, augmenting the model with a more realistic financial sector would allow analyzing the interactions between the dominant status of dollar as a vehicle currency in the international trade and as a reserve currency in global asset markets. Second, a quantitative version of the model can be obtained by introducing heterogeneity across countries and industries. That would allow to test the cross-sectional predictions of the model about the currency of invoicing, perform counterfactuals about future changes in the international price system and quantify the spillover effects separately for individual countries. Finally, a simple extension of the model with heterogenous firms can be used as a basis for the micro-level empirical analysis of exporters' currency choice.

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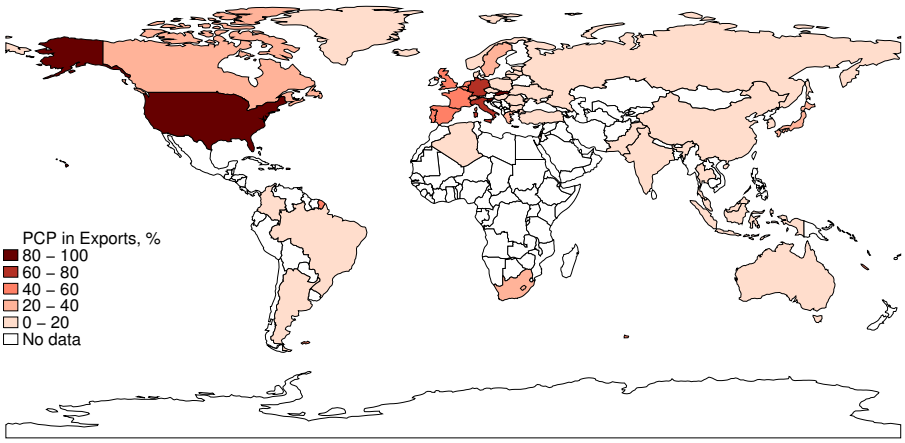
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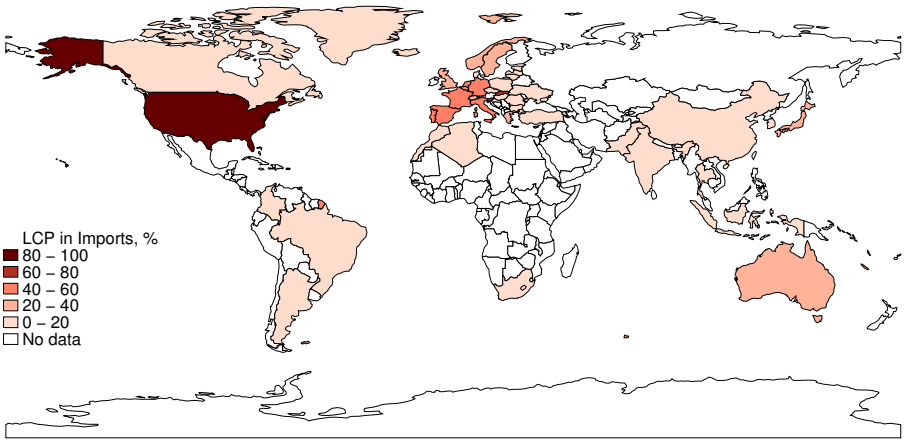
A Appendix

A.1 Additional figures and tables

(a) Share of country's exports priced in producer currency (PCP)



(b) Share of country's imports priced in local currency (LCP)



(c) Share of country's exports priced in dollar (DCP)

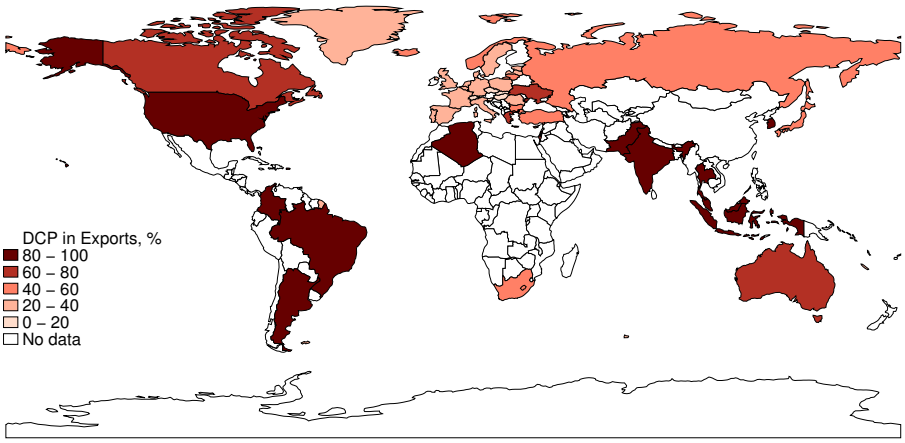
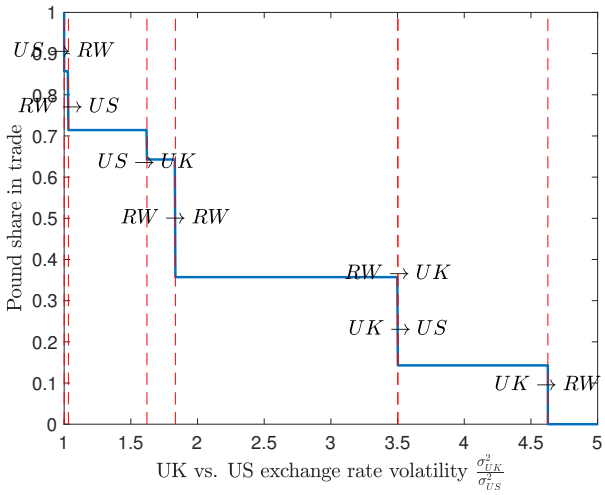


Figure A1: The use of producer currency, local currency and dollar in global trade

Data sources: Gopinath (2016), Kamps (2006), Lai and Yu (2015), Sokolova (2015).

(a) Changes in volatility



(b) Bounds on transition path

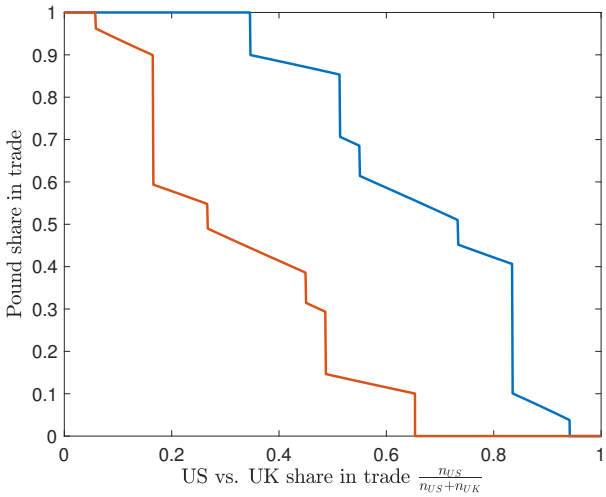
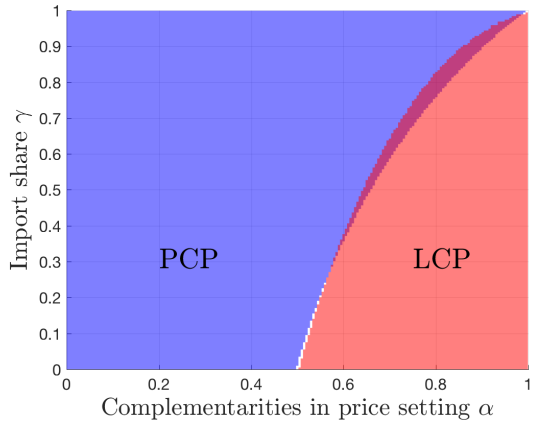


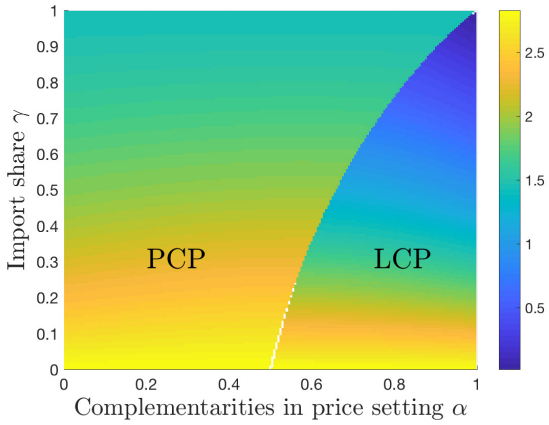
Figure A2: Transition from pound to dollar

Note: figure (a) shows transition from pound to dollar as the relative volatility of shocks in the U.K. goes up, while figure (b) shows lower and upper bounds for transition paths, i.e. the slowest and the fastest transition from pound to dollar. The parameter values are $\gamma = 0.6$, $\alpha = 0.5$, $\phi = 0.5$, $\lambda = 0.5$ and $n_{US} = n_{UK} = 0.25$.

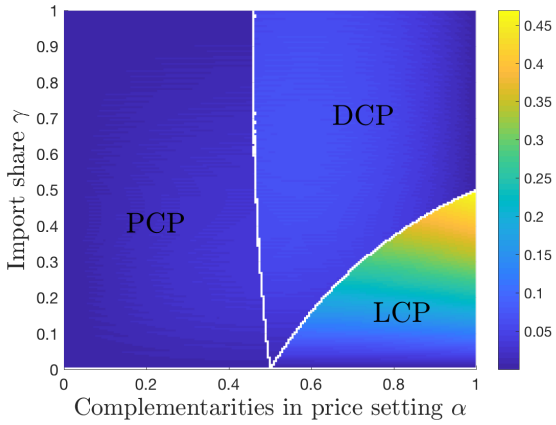
(a) Regions of PCP and LCP



(b) Exchange rate volatility



(c) Volatility of marginal costs



(d) Volatility of CPI

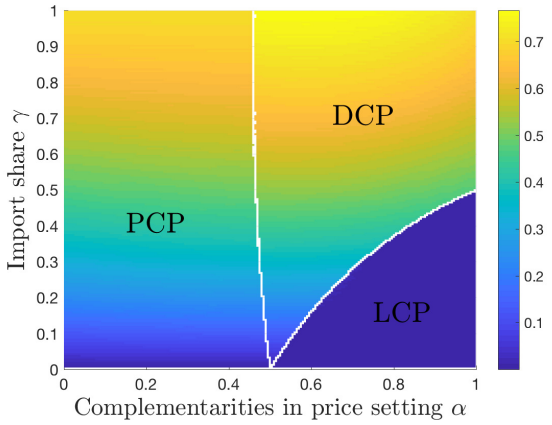


Figure A3: Optimal policy with partially flexible prices

Note: the plots show the resulting equilibria under the optimal cooperative policy without commitment when the share of price adjustments is $\lambda = 0.5$. In plots (c) and (d), only volatility under DCP is shown in regions with multiple equilibria. All volatilities are the standard deviations relative to exogenous productivity shocks, $\phi = 0.5$, $\theta = 2$, $\eta = 1$, $n = 0$.

A.2 Equilibrium system

The Kimball aggregator for consumption bundle of tradable goods in region i is defined as

$$(1-\gamma)e^{-\gamma\xi_i}\int_0^1 \Upsilon\left(\frac{C_{ii}(\omega)}{(1-\gamma)e^{-\gamma\xi_i}C_{Ti}}\right)\mathrm{d}\omega + \gamma e^{(1-\gamma)\xi_i}\int_0^1\int_0^1 \Upsilon\left(\frac{C_{ji}(\omega)}{\gamma e^{(1-\gamma)\xi_i}C_{Ti}}\right)\mathrm{d}\omega\mathrm{d}j = 1, \quad \text{(A1)}$$

where $\Upsilon(1) = 1$, $\Upsilon'(\cdot) > 0$ and $\Upsilon''(\cdot) < 0$. I borrow expressions for price index and demand for individual goods under Kimball aggregator from Itskhoki and Mukhin (2017) and Amiti, Itskhoki, and Konings (2016). The equilibrium system of the model consists of the following blocks:

1. Labor supply and labor demand:

$$C_{it} = \frac{W_{it}}{P_{it}^C}, \tag{A2}$$

$$L_{it} = (1-\phi)\left(\frac{P_{it}}{W_{it}}\right)^{\phi} \frac{Y_{it}}{A_{Tit}} + \frac{Y_{Nit}}{A_{Nit}}. \tag{A3}$$

2. Demand for non-tradables:

$$Y_{Nit} = \int_0^1 \left(\frac{P_{Nit}(\omega)}{P_{Nit}}\right)^{-\theta} \mathrm{d}\omega \left(C_{Nit} + G_{Nit}\right),$$

where

$$C_{Nit} + G_{Nit} = (1-\eta)\frac{P_{it}^C}{P_{it}^N}\left(C_{it} + G_{it}\right).$$

3. Price setting in non-tradable sector:

$$P_{it}^N(\omega) = \begin{cases} \bar{P}_{it}^N, \text{ w/p } 1-\lambda \\ \tilde{P}_{it}^N, \text{ w/p } \lambda \end{cases},$$

where

$$\tilde{P}_{it}^N = \arg\max_P \left(P - (1-\tau)\frac{W_{it}}{A_{Nit}}\right)\left(\frac{P}{P_{it}^N}\right)^{-\theta}\left(C_{Nit} + G_{Nit}\right),$$

$$\bar{P}_{it}^N = \arg\max_P \mathbb{E}_{t-1}\left(P - (1-\tau)\frac{W_{it}}{A_{Nit}}\right)\left(\frac{P}{P_{it}^N}\right)^{-\theta}\left(C_{Nit} + G_{Nit}\right).$$

4. Demand for tradables:

$$\begin{aligned} Y_{it} = {} & (1-\gamma)\, e^{-\gamma\xi_{it}} \int_0^1 h\left(\frac{D_{it}P_{iit}(\omega)}{P_{it}}\right)\mathrm{d}\omega\left(C_{Tit} + X_{it} + G_{Tit}\right) \\ & + \gamma \int_0^1 e^{(1-\gamma)\xi_{jt}} \int_0^1 h\left(\frac{D_{jt}P_{ijt}(\omega)}{P_{jt}}\right)\mathrm{d}\omega\left(C_{Tjt} + X_{jt} + G_{Tjt}\right)\mathrm{d}j, \end{aligned} \tag{A4}$$

with intermediate and final demand given by

$$X_{it} = \phi\left(\frac{W_{it}}{P_{it}}\right)^{1-\phi}\frac{Y_{it}}{A_{Tit}}, \tag{A5}$$

$$C_{Tit} + G_{Tit} = \eta\frac{P_{it}^C}{P_{it}}\left(C_{it} + G_{it}\right).$$

5. Price setting and currency choice in tradable sector:

$$P_{jit}(\omega) = \begin{cases} \bar{P}_{jit}^k, \text{ w/p } 1-\lambda \\ \tilde{P}_{jit}, \text{ w/p } \lambda \end{cases},$$

where

$$\tilde{P}_{jit} = \arg\max_{P} \left(P\mathcal{E}_{jit} - (1-\tau) MC_{jt}\right) \gamma e^{(1-\gamma)\xi_{it}} h\left(\frac{D_{it}P}{P_{it}}\right)\left(C_{Tit} + X_{it} + G_{Tit}\right),$$

$$\bar{P}^{k}_{jit} = \mathcal{E}_{ikt} \cdot \arg\max_{P,k} \mathbb{E}_{t-1}\left(P\mathcal{E}_{jkt} - (1-\tau) MC_{jt}\right) \gamma e^{(1-\gamma)\xi_{it}} h\left(\frac{D_{it}P\mathcal{E}_{ikt}}{P_{it}}\right)\left(C_{Tit} + X_{it} + G_{Tit}\right),$$

and marginal costs of production are

$$MC_{jt} = \frac{1}{A_{Tjt}} W_{jt}^{1-\phi} P_{jt}^{\phi}. \tag{A6}$$

6. Definition of price indices

$$P_{it}^{C} = \left(P_{it}^{N}\right)^{1-\eta} P_{it}^{\eta},$$

$$\int_0^1 \left(\frac{P_{it}^{N}(\omega)}{P_{it}^{N}}\right)^{1-\theta} \mathrm{d}\omega = 1,$$

$$(1-\gamma) e^{-\gamma\xi_{it}} \int_0^1 \Upsilon\left(h\left(\frac{D_{it}P_{iit}(\omega)}{P_{it}}\right)\right)\mathrm{d}\omega + \gamma e^{(1-\gamma)\xi_{it}} \int_0^1\int_0^1 \Upsilon\left(h\left(\frac{D_{it}P_{jit}(\omega)}{P_{it}}\right)\right)\mathrm{d}\omega\mathrm{d}j = 1,$$

$$(1-\gamma) e^{-\gamma\xi_{it}} \int_0^1 h\left(\frac{D_{it}P_{iit}(\omega)}{P_{it}}\right)\frac{P_{iit}(\omega)}{P_{it}}\mathrm{d}\omega + \gamma e^{(1-\gamma)\xi_{it}} \int_0^1\int_0^1 h\left(\frac{D_{it}P_{jit}(\omega)}{P_{it}}\right)\frac{P_{jit}(\omega)}{P_{it}}\mathrm{d}\omega\mathrm{d}j = 1.$$

7. Asset demand / risk-sharing:

$$e^{\Delta\psi_{it+1}}\Theta_{it+1}\frac{\mathcal{E}_{i0t+1}}{\mathcal{E}_{i0t}} - e^{\Delta\psi_{0t+1}}\Theta_{0t+1} = 0, \tag{A7}$$

where the stochastic discount factor is defined as $\Theta_{it+1} = \frac{C_{it}P_{it}^{C}}{C_{it+1}P_{it+1}^{C}}$.

8. Country budget constraint is a side equation under complete markets. The net export expressed in dollar terms is

$$\begin{aligned} NX_{it} = \int_0^1\int_0^1 \Big\{ & \gamma e^{(1-\gamma)\xi_{jt}} \mathcal{E}_{0jt} P_{ijt}(\omega)\, h\left(\frac{D_{jt}P_{ijt}(\omega)}{P_{jt}}\right)\left(C_{Tjt} + X_{jt} + G_{Tjt}\right) \\ & - \gamma e^{(1-\gamma)\xi_{it}} \mathcal{E}_{0it} P_{jit}(\omega)\, h\left(\frac{D_{it}P_{jit}(\omega)}{P_{it}}\right)\left(C_{Tit} + X_{it} + G_{Tit}\right)\Big\}\mathrm{d}\omega\mathrm{d}j. \end{aligned} \tag{A8}$$

Symmetric steady state Consider symmetric steady state with zero net foreign asset positions and all shocks equal zero:

$$a_{Ni} = a_{Ti} = w_i = \xi_i = \psi_i = G_i = 0.$$

I assume that production subsidy eliminates monopolistic distortion $\tau = \frac{1}{\theta}$. This assumption has no effect on the first-order approximation of the equilibrium system discussed below, but is important for the welfare analysis.

The symmetry implies that bilateral exchange rate between any countries is one, $\mathcal{E}_{ij} = 1$, and therefore, the prices for all products equal one as well:

$$P_i = P_{ii} = P_{ji} = P_i^N = P_i^C = 1.$$

Steady-state consumption can then be found from labor supply condition:

$$C_i = 1.$$

Combining market clearing in non-tradable sector

$$Y_{Ni} = C_{Ni} = (1 - \eta) C_i$$

and tradable one

$$Y_i = C_{Ti} + X_i = \eta C_i + \phi Y_i,$$

one can solve for steady state level of labor and output:

$$L_i = (1 - \phi) Y_i + Y_{Ni} = 1,$$

$$Y_i = \frac{\eta}{1 - \phi}.$$

A.3 Log-linearized system

I next log-linearize the equilibrium system around the symmetric steady state. It is convenient to split the system into four blocks — prices, quantities, dynamic equations and currency choice, and solve them recursively. The time index is suppressed in static blocks to simplify the notation. Small letters denote log-deviations from the steady state, while small letters without subscript i denote the global means, i.e. $x \equiv \int_0^1 x_i \mathrm{d}i$.

I decompose bilateral exchange rates into country-specific components: $e_{ijt} = e_{it} - e_{jt}$. Such decomposition is non-unique: intuitively, in a world with N countries, there are only $N-1$ independent bilateral exchange rates. I therefore normalize the mean of exchange rates across countries to zero, i.e. $\int_n^1 e_{it} \mathrm{d}i = 0$. The country-specific exchange rate e_{it} can then be interpreted as an average of bilateral exchange rates against other countries.

To get consistent solution, I use a classical result from portfolio theory established first by Samuelson (1970) and applied recently in a general equilibrium setup by Devereux and Sutherland (2011). In a context of my model, the argument consists of two parts. First, the second-order approximation to the profit function is required to determine the zero-order component of currency choice. From Lemma 1, it follows then the first-order approximation to other variables is sufficient to solve for currency choice. Second, the zero-order component of the currency choice from Lemma 1 is sufficient to get an accurate first-order solution for other variables. Thus, to get consistent solution, one needs to take the second-

order approximation to the profit function and the first-order approximation to all other equilibrium conditions.

A.3.1 Prices

The price index for non-tradable goods and consumer price index are

$$p_i^N = \lambda \left(w_i - a_{Ni} \right), \tag{A9}$$

$$p_i^C = \eta p_i + (1 - \eta) \, p_i^N. \tag{A10}$$

The price block in tradable sector includes marginal costs of production

$$mc_i = \phi p_i + (1 - \phi) \, w_i - a_i, \tag{A11}$$

the optimal static price

$$\tilde{p}_{ji} = (1 - \alpha) \left(mc_j + e_i - e_j \right) + \alpha p_i, \tag{A12}$$

the import price index and the aggregate price index

$$p_i^I = \int_0^1 p_{ji} \mathrm{d}j, \tag{A13}$$

$$p_i = (1 - \gamma) \, p_{ii} + \gamma p_i^I, \tag{A14}$$

and the biletaral price index:

$$p_{ji} = \lambda \tilde{p}_{ji} + (1 - \lambda) \left(e_i - e_{k_{ji}} \right), \tag{A15}$$

where k_{ji} denotes the currency choice of exporters from country j to i. For future use, define also the export price index as

$$p_i^E = \int_0^1 p_{ij} \mathrm{d}j, \tag{A16}$$

Assume that domestic firms set prices in local currency and invoicing is symmetric across countries. Combine next equations (A11)-(A15) to solve for p_i:

$$p_i = \chi e_i - \chi_0 e_0 + \chi_w w_i + \chi_{\bar{w}} w - \chi_a a_i - \chi_{\bar{a}} a, \tag{A17}$$

where

$$
\begin{aligned}
\chi &= \frac{\gamma\left[\lambda\left(1-\alpha\right)+\left(1-\lambda\right)\left(\mu^{P}+\mu^{D}\right)\right]}{1-\lambda\left(\alpha+\left(1-\gamma\right)\left(1-\alpha\right)\phi\right)},\\
\chi_0 &= \frac{\gamma}{1-\lambda\left(\alpha+\left(1-\gamma\right)\left(1-\alpha\right)\phi\right)}\left[\lambda\left(1-\alpha\right)n+\left(1-\lambda\right)\left(n\mu^{P}+\mu^{D}\right)+\frac{\lambda\left(1-\lambda\right)\left(1-\alpha\right)\gamma\phi\mu^{D}\left(1-n\right)}{1-\lambda\left(\alpha+\left(1-\alpha\right)\phi\right)}\right],\\
\chi_w &= \frac{\lambda\left(1-\gamma\right)\left(1-\alpha\right)\left(1-\phi\right)}{1-\lambda\left(\alpha+\left(1-\gamma\right)\left(1-\alpha\right)\phi\right)},\\
\chi_{\bar{w}} &= \frac{\lambda\gamma\left(1-\alpha\right)\left(1-\lambda\alpha\right)\left(1-\phi\right)}{\left[1-\lambda\left(\alpha+\left(1-\gamma\right)\left(1-\alpha\right)\phi\right)\right]\left[1-\lambda\left(\alpha+\left(1-\alpha\right)\phi\right)\right]},\\
\chi_a &= \frac{\lambda\left(1-\gamma\right)\left(1-\alpha\right)}{1-\lambda\left(\alpha+\left(1-\gamma\right)\left(1-\alpha\right)\phi\right)},\\
\chi_{\bar{a}} &= \frac{\lambda\gamma\left(1-\alpha\right)\left(1-\lambda\alpha\right)}{\left[1-\lambda\left(\alpha+\left(1-\gamma\right)\left(1-\alpha\right)\phi\right)\right]\left[1-\lambda\left(\alpha+\left(1-\alpha\right)\phi\right)\right]}.
\end{aligned}
\tag{A18}
$$

Integrate across countries to obtain the global price index

$$
p = (\chi n - \chi_0)e_0 + (\chi_w + \chi_{\bar{w}})w - (\chi_a + \chi_{\bar{a}})a, \tag{A19}
$$

Finally, solve for import price index

$$
\begin{aligned}
p_i^I =& -\lambda\left[\left(1-\alpha\right)\left(1-\phi\chi\right)n+\left(\left(1-\alpha\right)\phi+\alpha\right)\chi_0\right]e_0-\left(1-\lambda\right)\left(n\mu^{P}+\mu^{D}\right)e_0\\
&+\lambda\left(1-\alpha+\alpha\chi\right)e_i+\left(1-\lambda\right)\left(\mu^{P}+\mu^{D}\right)e_i\\
&+\lambda\alpha\chi_1 w_i+\lambda\left[\left(1-\alpha\right)\left(1-\phi+\phi\chi_1\right)+\left(\left(1-\alpha\right)\phi+\alpha\right)\chi_2\right]w\\
&-\lambda\alpha\chi_a a_i-\lambda\left[(1-\alpha)(\phi\chi_a+1)+\chi_{\bar{a}}(\alpha+(1-\alpha)\phi)\right]a
\end{aligned}
\tag{A20}
$$

and export price index

$$
\begin{aligned}
p_i^E =& \lambda\left[\left(1-\alpha+\alpha\chi\right)n-\left(\left(1-\alpha\right)\phi+\alpha\right)\chi_0\right]e_0+\left(1-\lambda\right)\left(n\mu^{P}-\left(1-n\right)\mu^{D}\right)e_0\\
&-\lambda\left(1-\alpha\right)\left(1-\phi\chi\right)e_i-\left(1-\lambda\right)\mu^{P}e_i\\
&+\lambda\left(1-\alpha\right)\left(1-\phi+\phi\chi_w\right)w_i+\lambda\left[\alpha\chi_w+\left(\left(1-\alpha\right)\phi+\alpha\right)\chi_{\bar{w}}\right]w\\
&-\lambda(1-\alpha)(\phi\chi_a+1)a_i-\lambda\left[((1-\alpha)\phi+\alpha)\chi_{\bar{a}}+\alpha\chi_a\right]a
\end{aligned}
\tag{A21}
$$

A.3.2 Quantities

The market clearing conditions for labor and goods allow to express consumption, labor and output as functions of prices and shocks. First, labor supply condition determines consumption

$$
c_i = w_i - p_i^C. \tag{A22}
$$

Second, substitute final demand for tradables

$$c_{Ti} = p_i^C - p_i + c_i + g_i \tag{A23}$$

and intermediate demand for tradables

$$x_i = mc_i + y_i - p_i \tag{A24}$$

into the market clearing condition

$$y_i = (1-\gamma)\, y_{ii} + \gamma y_i^E, \tag{A25}$$

where the volume of exports is

$$y_i^E = \int_0^1 y_{ij} \mathrm{d}j \tag{A26}$$

and bilateral trade flows are

$$y_{ii} = -\gamma\xi_i - \theta\,(p_{ii} - p_i) + (1-\phi)\, c_{Ti} + \phi x_i, \tag{A27}$$

$$y_{ij} = (1-\gamma)\,\xi_j - \theta\,(p_{ij} - p_j) + (1-\phi)\, c_{Tj} + \phi x_j. \tag{A28}$$

Integrate across countries, use equation (A22) for consumption as well as equations (A11) and (A10) from price block to solve for global production of tradable goods:

$$y = (1+\phi)\,(w-p) + g - \frac{\phi}{1-\phi} a. \tag{A29}$$

Substitute this expression back into the market clearing condition of a given country to solve for output:

$$\begin{aligned} y_i = \frac{\gamma\theta}{1-(1-\gamma)\,\phi}\left[\left(p_i^I - p_i^E\right) - (p_i - p)\right] + \frac{(1-\gamma)\,(1-\phi^2)}{1-(1-\gamma)\,\phi}\,(w_i - p_i) + \frac{\gamma\,(1+\phi)}{1-(1-\gamma)\,\phi}\,(w-p) \\ -\frac{\gamma\,(1-\gamma)}{1-(1-\gamma)\,\phi}\,(\xi_i - \xi) + \frac{(1-\gamma)\,(1-\phi)}{1-(1-\gamma)\,\phi} g_i + \frac{\gamma}{1-(1-\gamma)\,\phi} g \end{aligned} \tag{A30}$$

Third, total labor demand

$$l_i = \eta l_{Ti} + (1-\eta)\, l_{Ni}$$

is the sum of demand from tradable sector

$$l_{Ti} = mc_i + y_i - w_i$$

and non-tradable sector

$$l_{Ni} = y_{Ni} - a_{Ni},$$

where market clearing for non-tradable goods implies

$$y_{Ni} = c_{Ni} = p_i^C - p_i^N + c_i + g_i. \tag{A31}$$

Combine these equations together with tradable output (A30) to solve for labor in terms of prices and shocks:

$$l_i = (1-\eta)\left(p_i^C - (1-\eta)p_i^N\right) - (1-\eta)\,\eta p_i. \tag{A32}$$

Fourth, to the first-order approximation, the aggregate imports and exports of country i are

$$im_i = p_i^I + y_i^I, \qquad ex_i = p_i^E + y_i^E,$$

where volume of imports is defined as

$$y_i^I = \int_0^1 y_{ji}\mathrm{d}j. \tag{A33}$$

Use expressions for output (A30), consumption (A22) and bilateral trade flows (A28), (A26) and (A33) to solve for exports

$$y_i^E = -\theta\left(p_i^E - p\right) + (1-\eta)\left(p^N - p\right) + \phi\,(w-p) + \left(w - p^C\right) + g + (1-\gamma)\,\xi - \frac{\phi}{1-\phi}a \tag{A34}$$

and imports

$$\begin{aligned} y_i^I = &\frac{1-\phi}{1-(1-\gamma)\,\phi}\Big\{-\theta\left(p_i^I - p_i\right) + (1-\eta)\left(p_i^N - p_i\right) + \phi\,(w_i - p_i) + \left(w_i - p_i^C\right) \\ &+ g_i + (1-\gamma)\,\xi_i - \frac{\phi}{1-\phi}a_i\Big\} + \frac{\gamma\phi}{1-(1-\gamma)\,\phi}y_i^E. \end{aligned} \tag{A35}$$

The linearized equation for net exports is

$$nx_i = ex_i - im_i + (e_i - ne_0)\,.$$

Substitute in expressions for exports (A34) and imports (A35) to get

$$\begin{aligned} nx_i = &(e_i - ne_0) - (p_i - p) + \left[\frac{(1-\phi)\,\theta}{1-(1-\gamma)\,\phi} - 1\right]\left[\left(p_i^I - p_i\right) - \left(p_i^E - p\right)\right] \\ &- \frac{(1-\phi)}{1-(1-\gamma)\,\phi}\Big\{\phi\left[(w_i - w) - (p_i - p)\right] + (1-\eta)\left[\left(p_i^N - p^N\right) - (p_i - p)\right] + \left[(w_i - w) - \left(p_i^C - p^C\right)\right]\Big\} \\ &- \frac{(1-\phi)\,(1-\gamma)}{1-(1-\gamma)\,\phi}\,(\xi_i - \xi) - \frac{(1-\phi)}{1-(1-\gamma)\,\phi}\,(g_i - g) + \frac{\phi}{1-(1-\gamma)\,\phi}\,(a_i - a)\,. \end{aligned} \tag{A36}$$

A.4 Equilibrium exchange rates

There are two types of dynamic equations in the model that pin down equilibrium exchange rates — the Euler equations and countries' budget constraints. I show in this section that result from Lemma 2 can be derived under several alternative sets of assumptions about the structure of asset markets, preferences and monetary policy rule. In all cases, exchange rate shocks are uncorrelated $\text{corr}(e_i, e_j) = 0$ for $\forall i \neq j$ and the relative volatility of exchange rates depends only on volatility of exogenous shocks $\frac{\mathbb{V}(e_0)}{\mathbb{V}(e_i)} = \rho$ for $\forall i \in (n, 1]$.

A.4.1 Baseline case

Proof of Lemma 2 When asset markets are complete, the countries achieve full risk-sharing:

$$\Delta e_{it} - \Delta e_{0t} = (\Delta c_{it} - \Delta c_{0t}) + \left(\Delta p_{it}^C - \Delta p_{0t}^C\right) + (\Delta \psi_{it} - \Delta \psi_{0t}) .$$

Since countries are symmetric ex ante, the budget constraint implies that the same condition holds not only in changes, but also state by state:

$$e_{it} - e_{0t} = (c_{it} - c_{0t}) + \left(p_{it}^C - p_{0t}^C\right) + (\psi_{it} - \psi_{0t}) . \tag{A37}$$

Substitute in expressions for consumption (A22) to obtain

$$e_{it} - e_{0t} = (w_{it} - w_{0t}) + (\psi_{it} - \psi_{0t}) .$$

Integrate the risk-sharing condition across countries from n to 1, apply the law of large numbers for uncorrelated shocks and use normalization of exchange rates to get

$$e_{0t} = \tilde{w}_{0t} + \tilde{\psi}_{0t},$$

where $\tilde{s}_{it}$ denotes the country-specific component of shock s_{it}. Substitute this condition back into the previous expression to get for any $i \in [0, 1]$

$$e_{it} = \tilde{w}_{it} + \tilde{\psi}_{it}. \tag{A38}$$

Thus, the second moments of exchange rates are independent from firms' currency choice. ■

A.4.2 Alternative assumptions

Proposition A1 (Exchange rates) *Assume that*

1. *the only internationally traded asset is a risk-free nominal bond denominated in arbitrary currency and that all shocks are integrated of the first order,*
2. *one of the following conditions is satisfied:*

- *preferences are log-linear and monetary policy is set in terms of exogenous shocks in W_{it},*
- *arbitrary isoelastic preferences and monetary policy is set in terms of exogenous shocks in R_{it},*

3. *$\beta \to 1$ or all exporters in the world use either PCP, LCP or DCP.*

Then correlation and relative volatility of exchange rates are independent from firms' currency choice.

Incomplete markets Consider the case of incomplete asset markets when only one nominal bond is traded internationally. I assume it pays one dollar in every state of the world, which is without loss of generality under the first-order approximation used to solve the model. The no-arbitrage conditions

$$\mathbb{E}_t \left\{ e^{\Delta\psi_{it+1}} \Theta_{it+1} \frac{\mathcal{E}_{i0t+1}}{\mathcal{E}_{i0t}} - e^{\Delta\psi_{0t+1}} \Theta_{0t+1} \right\} = 0$$

can be log-linearized to get the UIP condition with the risk premium $\varsigma_{it} \equiv \mathbb{E}_t \Delta\psi_{it+1}$:

$$\mathbb{E}_t \left[\Delta e_{it+1} - \Delta e_{0t+1} \right] = \mathbb{E}_t \left[\left(\Delta c_{it+1} - \Delta c_{0t+1} \right) + \left(\Delta p^C_{it+1} - \Delta p^C_{0t+1} \right) \right] - \left(\varsigma_{it} - \varsigma_{0t} \right).$$

Substitute the labor supply condition (A22) to get

$$\mathbb{E}_t \left[\Delta e_{it+1} - \Delta e_{0t+1} \right] = \mathbb{E}_t \left[\Delta w_{it+1} - \Delta w_{0t+1} \right] - \left(\varsigma_{it} - \varsigma_{0t} \right).$$

Integrate across countries from n to 1, apply the law of large numbers for uncorrelated shocks and use normalization of exchange rates to get

$$\mathbb{E}_t \Delta e_{it+1} = \mathbb{E}_t \Delta \tilde{w}_{it+1} - \tilde{\varsigma}_{it} \tag{A39}$$

for any $i \in [0, 1]$.

The intertemporal budget constraint is

$$\sum_{\tau=0}^{\infty} \beta^\tau NX_{it+\tau} + D_{it} = 0,$$

where D_{it} denotes country's debt in dollars. Rewrite it in log-linear form and assume that initial debt is zero, which is without loss of generality since we are interested in the conditional moments:

$$\sum_{t=0}^{\infty} \beta^t nx_{it} = 0$$

This can be decomposed into net export in the first period with sticky prices and in all other periods when prices are flexible:

$$\sum_{t=1}^{\infty} \beta^t nx_{it} + nx_{i0} = 0.$$

Expression (A36) together with price indices implies that under flexible prices the net export of country i can be written as

$$nx_{it}^{fp} = k_e \left(e_{it} - ne_{0t}\right) + k_s \left(s_{it} - ns_{0t}\right), \tag{A40}$$

where s_{it} is the vector of shocks and (k_e, k_s) is a vector of constants independent from firms' currency choice. Combining the last two expressions, one obtains

$$\sum_{t=1}^{\infty} \beta^t \left[k_e \left(e_{it} - ne_{0t}\right) + k_s \left(s_{it} - ns_{0t}\right)\right] + nx_{i0} = 0.$$

Integrate across countries from n to 1, apply the law of large numbers and exchange rate normalization to get for any $i \in [0, 1]$

$$\sum_{t=1}^{\infty} \beta^t \left[k_e e_{it} + k_s \tilde{s}_{it}\right] + \widehat{nx}_{i0} = 0,$$

where $\widehat{nx}_{i0} \equiv nx_{i0} - \int_n^1 nx_{i0} \mathrm{d}i$. Rewrite the last equation in terms of initial values and growth rates

$$\sum_{t=1}^{\infty} \beta^t \left[k_e e_{i0} + k_s \tilde{s}_{i0} + \sum_{\tau=1}^{t} \left(k_e \Delta e_{i\tau} + k_s \Delta \tilde{s}_{i\tau}\right)\right] + \widehat{nx}_{i0} = 0,$$

change the order of summation and substitute in the UIP condition (A39):

$$\beta \left(k_e e_{i0} + k_s \tilde{s}_{i0}\right) + \beta \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[k_e \Delta \tilde{w}_{it+1} - k_e \Delta \tilde{\psi}_{it+1} + k_s \Delta \tilde{s}_{it+1}\right] + (1 - \beta) \widehat{nx}_{i0} = 0.$$

Assume that all shocks are integrated of the first order and take the limit $\beta \to 1$ using the fact coefficients (k_e, k_s) do not depend on β:

$$e_{it} = -\frac{k_s}{k_e} \tilde{s}_{it} - \mathbb{E}_t \sum_{\tau=0}^{\infty} \left[\Delta \tilde{w}_{it+\tau+1} - \tilde{\varsigma}_{it+\tau} + \frac{k_s}{k_e} \Delta \tilde{s}_{it+\tau+1}\right]. \tag{A41}$$

Since invoicing decisions of exporters have no effect on the coefficients in this expression, the (conditional) second moments of exchange rate are independent from firms' currency choice.

Interest rate shocks Assume again one internationally traded bond and use the Euler equation for domestic bond to write the no-arbitrage condition as

$$\mathbb{E}_t \left\{\Theta_{it+1} \left[e^{\Delta \psi_{it+1}} R_{0t} \frac{\mathcal{E}_{i0t+1}}{\mathcal{E}_{i0t}} - R_{it}\right]\right\} = 0.$$

This implies the UIP condition with the risk premium shock:

$$\mathbb{E}_t \left[\Delta e_{it+1} - \Delta e_{0t+1}\right] = r_{it} - r_{0t} - \left(\varsigma_{it} - \varsigma_{0t}\right).$$

If interest rate shocks are exogenous and are the sum of global and country-specific components as other shocks, then using integration across countries and exchange rate normalization, we get

$$\mathbb{E}_t \Delta e_{it+1} = \tilde{r}_{it} - \tilde{\varsigma}_{it}.$$

I abstract from the issue of multiple equilibria and take the path of interest rates as given as in Farhi and Werning (2016). Following the same steps as before and taking the limit $\beta \to 1$, the budget constraint of country i together with the UIP condition imply

$$e_{it} = -\frac{k_s}{k_e}\tilde{s}_{it} - \mathbb{E}_t \sum_{\tau=0}^{\infty} \left[\tilde{r}_{it+\tau} - \tilde{\varsigma}_{it+\tau} + \frac{k_s}{k_e}\Delta \tilde{s}_{it+\tau+1} \right].$$

This, the second moments of exchange rates are independent from firms' currency choice. Note that this result holds for arbitrary preferences.

Symmetric invoicing I show that the elasticity of net export with respect to trade-weighted exchange rate is the same for all countries under symmetric invoicing and therefore, the expression similar to (A40) holds also in the short-run. As a result, the equilibrium response of exchange rates to local shocks is the same for all countries and the relative volatility of exchange rates depends only relative volatility of exogenous shocks.

Lemma A1 *When all exporters in the world use either PCP, LCP or DCP, the elasticity of net exports with respect to $e_i - ne_0$ is the same for all countries including the U.S.*

Proof From (A17)-(A21), it follows that $p_i - p$ and $p_i^I - p_i^E$ are proportional to $\chi(e_i - ne_0)$ and

$$\left[\lambda\left((1-\alpha)(2-\phi\chi)+\alpha\chi\right)+(1-\lambda)\left(2\mu^P+\mu^D\right)\right](e_i - ne_0)$$

respectively. The expression for net exports (A36) implies then that the elasticity of nx_i with respect to $e_i - ne_0$ is the same for all countries. ■

A.5 Currency choice

Proof of Lemma 1 Let s denote the aggregate state of the economy that individual firms take as exogenous. Suppress country indices and take the second-order approximation of the profit function at price p around the state-dependent optimal price $\tilde{p}_{ji}$:

$$\Pi(p,s) = \Pi(\tilde{p}_{ji},s) + \Pi_p(\tilde{p}_{ji},s)(p-\tilde{p}_{ji}) + \frac{1}{2}\Pi_{pp}(\tilde{p}_{ji},s)(p-\tilde{p}_{ji})^2 + \mathcal{O}(p-\tilde{p}_{ji})^3,$$

The first term on the right hand side does not depend on currency of invoicing. From the first-order condition for optimal price, $\Pi_p(\tilde{p}_{ji}, s) = 0$. Finally, the zero-order approximation,

$$\Pi_{pp}(\tilde{p}_{ji}, s) = \Pi_{pp}(0,0) + \mathcal{O}(s) < 0,$$

where $\Pi_{pp}(0,0)$ denotes the derivative in the deterministic steady state. Therefore, to the second-order approximation, the currency choice problem is equivalent to minimization of $\mathbb{E}(p - \tilde{p}_{ji})^2$. Note that only first-order approximation is required for p and $\tilde{p}_{ji}$. In particular, the optimal preset price in currency k is $\bar{p}^k_{ji} = \mathbb{E}(\tilde{p}_{ji} - e_{ik})$, so that ex post price is $p = \bar{p}^k_{ji} + e_{ik}$. Substitute this expression into the objective function to write the currency problem as

$$\min_k \mathbb{V}(\tilde{p}_{ji} + e_{ki})^2, \tag{A42}$$

which completes the proof of the lemma. ■

Combining equations (A11)-(A15) and suppressing monetary and productivity shocks, we get the optimal price in terms of currency k:

$$\tilde{p}_{ji} + e_{ki} = e_k - (1-\alpha)(1-\chi\phi)e_j - \alpha(1-\chi)e_i - (\alpha + (1-\alpha)\phi)\chi_0 e_0. \tag{A43}$$

It is easy to verify that the aggregate pass-through coefficients (A18) are positive and no greater than one, i.e. $0 \leq \chi, \chi_0 \leq 1$. It follows that the coefficients before e_j, e_i and e_0 are between 0 and 1 as well. Since exchange rates e_i are uncorrelated across countries, a firm is more likely to choose the currency with the higher weight in (A43). This result underlies the comparative statics analysis below.

Proof of Lemma 3 When $\alpha = \phi = 0$, we get $\tilde{p}_{ji} + e_{ki} = e_k - e_j$ and the minimum volatility is attained by setting $k = j$, i.e. exporters choose PCP. ■

Proof of Lemma 4 Expression (A18) implies that in the autarky limit $\gamma \to 0$, the pass-through coefficients are $\chi, \chi_0 \to 0$. Thus, $\tilde{p}_{ji} + e_{ki} \to e_k - (1-\alpha)e_j - \alpha e_i$ and $\mathbb{V}(\tilde{p}_{ji} + e_{ki})^2$ is equal $2\alpha^2\sigma_e^2$ under PCP, $2(1-\alpha)^2\sigma_e^2$ under LCP and $(\rho + \alpha^2 + (1-\alpha)^2)\sigma_e^2$ under DCP. Hence, exporters choose $k = j$ when $\alpha \leq 0.5$ and $k = i$ when $\alpha \geq 0.5$. ■

Proof of Proposition 1 Consider for example the limit $\gamma, \alpha \to 1$, so that $\chi \to \mu^P + \mu^D$, $\chi_0 \to \mu^D$ and $\tilde{p}_{ji} + e_{ki} \to e_k - (1-\chi)e_i - \chi_0 e_0$. Conjecture that other firms choose DCP, so that $\mu^D = 1$. Hence, $\tilde{p}_{ji} + e_{ki} \to e_k - e_0$ and the firm finds it optimal to choose $k = 0$. The DCP equilibrium can therefore be sustained in the neighbourhood of $\gamma = \alpha = 1$ when prices are sticky.

Note that both χ and χ_0 are increasing in γ and ϕ. In addition, given χ and χ_0, the coefficient before e_j is decreasing in ϕ, while the coefficient before e_0 is increasing in ϕ. It follows that higher γ and ϕ

decrease the weights of e_j and e_i and increase the weight of e_0 in (A43), which makes PCP and LCP less likely and raises the chances of DCP. The effect of α, on the other hand, is not monotonic. ■

Lemma A2 *In the flexible-price limit $\lambda \to 1$, the equilibrium exists and is generically unique. The invoicing is symmetric across small countries.*

Proof In the flexible-price limit $\lambda \to 1$, the pass-through coefficients from (A18) converge to $\chi \to \frac{\gamma}{1-(1-\gamma)\phi}$ and $\chi_0 \to \frac{\gamma n}{1-(1-\gamma)\phi}$ and do not depend on invoicing decisions of firms. The currency choice problem (A42)-(A43) then has unique solution except for some borderline values of parameters. Finally, since coefficients before exchange rates are the same for exporters from all small economies and the volatility of exchange rates is also the same, the equilibrium invoicing is symmetric across them. ■

Proof of Proposition 3 When $n = 0$, the desired price of exporters is

$$\tilde{p}_{ji} + e_{ki} = e_k - \frac{1-\phi}{1-(1-\gamma)\phi}\Big[(1-\alpha)e_j + \alpha(1-\gamma)e_i\Big]. \tag{A44}$$

Since volatility of all exchange rates is the same when $\rho = 1$, the exporter chooses between producer and local currency based on their weights in (A44): $k = j$ when $1 - \alpha \geq \alpha(1-\gamma) \Leftrightarrow \alpha \leq \frac{1}{2-\gamma}$ and $k = i$ otherwise. ■

Proof of Proposition 4 Rewrite for simplicity expression (A44) as $\tilde{p}_{ji} + e_{ki} = e_k - ae_j - be_i$. The volatility (A42) under DCP is then $(\rho + a^2 + b^2)\sigma_e^2$. Since ρ does not affect volatility under PCP and LCP, lower values of ρ unambiguously increase the chances of DCP. Note that in the limit $\phi \to 1$, we have $a = b = 0$ and under $\rho < 1$ DCP strictly dominates both PCP and LCP. ■

Proof of Proposition 5 The desired price in the flexible-price limit with $n > 0$ is

$$\tilde{p}_{ji} + e_{ki} = e_k - \frac{1-\phi}{1-(1-\gamma)\phi}\Big[(1-\alpha)e_j + \alpha(1-\gamma)e_i\Big] - \frac{\gamma(\alpha + (1-\alpha)\phi)}{1-(1-\gamma)\phi}ne_0.$$

As long as $n > 0$, choosing $k = 0$ is optimal for example in the limit $\phi \to 1$. Moreover, keeping the values of other parameters fixed, higher n increases the relative weight of e_0 in the optimal price and therefore, makes DCP more likely. ■

The proof of Proposition 2 requires a few additional lemmas. When $n = 0$ and $\rho = 1$, the currency choice of exporters is based on the following inequalities:

$$PCP \succ LCP \quad \Leftrightarrow \quad (1-\alpha)\,\phi\chi + \alpha\,(2-\chi) < 1, \tag{A45}$$

$$PCP \succ DCP \quad \Leftrightarrow \quad (1-\alpha)\,\phi\,(\chi + \chi_0) + \alpha\,(1+\chi_0) < 1, \tag{A46}$$

$$DCP \succ LCP \quad \Leftrightarrow \quad (1-\alpha)(1-\phi\chi_0) + \alpha[2-(\chi+\chi_0)] < 1. \tag{A47}$$

where $\succ$ stays for "prefered to". I also denote with χ^X and χ_0^X the values of the pass-through coefficients in (A18) under symmetric invoicing X.

Lemma A3 *If DCP is prefered to PCP (LCP) under PCP (LCP) price index, then this ordering holds under DCP price index as well. Symmetrically, if PCP (LCP) dominates DCP under DCP price index, then this ordering holds under PCP (LCP) price index as well.*

Proof Since condition (A46) gets tighter with χ and χ_0 and $\chi_0^P = \chi_0^D$, $\chi_0^P < \chi_0^D$, the relation $DCP \succ PCP$ for χ^P (χ_0^P) implies the same ordering for χ^D (χ_0^D). Since condition (A47) is relaxed by higher χ and χ_0 and $\chi^L < \chi^D$, $\chi_0^L < \chi_0^D$, the relation $DCP \succ LCP$ for χ^L & χ_0^L implies the same ordering for χ^D & χ_0^L. ■

Lemma A4 *It is impossible that exporter chooses PCP when all others choose LCP and simultaneously chooses LCP when all others choose PCP.*

Proof Suppose that were the case. Then from (3) $\frac{1-\phi\chi^P}{2-\chi^P(1+\phi)} < \alpha < \frac{1-\phi\chi^L}{2-\chi^L(1+\phi)}$. But this requires $\chi^L > \chi^P$, which is not the case. ■

Lemma A5 *Consider pure-strategy NE with a choice only between PCP and LCP. If symmetric LCP equilibrium does not exist, the only possible pure-strategy NE is symmetric PCP.*

Proof Pure-strategy equilibria can be parametrized by cdf $F(\cdot)$ for $\mu_i^P \in [0,1]$ across countries. PCP is chosen by exporter from country j to country i iff

$$(1-\alpha)\phi\chi_j + \alpha(2-\chi_i) < 1 \quad \Rightarrow \quad \mu_j < a + b\mu_i$$

for some positive constants a and b. Integrating across importers, we then derive the equilibrium condition: $\mu_i = \int_j \mathbb{I}\{\mu_j < a + b\mu_i\}\, dj$, or equivalently

$$\int_0^1 \mathbb{I}\{z < a + bx\}\, dF(z) = F(a+bx) = x$$

for any x with positive density. Suppose next that symmetric LCP equilibrium does not exist, i.e. $F(a) = 0$ is unattainable. This is possible only if $a > 1$. But then for any $x > 0$ with positive density we have $x = F(a+bx) \geq F(a) = 1$, i.e. symmetric PCP is the only PSE. ■

Proof of Proposition 2 (1) Suppose there are no symmetric equilibria for some combination of parameters. Note that since $\chi^P = \chi^D$, it follows from (A45) that the preferences between PCP and LCP should be the same under PCP and DCP price indices. First, suppose that $PCP \succ LCP$ under DCP and PCP. Since there is no PCP equilibrium, we must have $DCP \succ PCP$ under PCP price index. But by Lemma A3, we have $DCP \succ PCP$ under DCP price index as well and hence, DCP equilibrium exists. Second, suppose that $LCP \succ PCP$ under DCP and PCP. Then from Lemma A4, we have

$LCP \succ PCP$ under LCP price index. Non-existence of LCP equilibrium requires then $DCP \succ LCP$ under LCP price index. By Lemma A3, $DCP \succ LCP$ under DCP price index as well and hence, we obtain DCP equilibrium. In both cases we arrive to contradiction.

(2) First, suppose that DCP is a unique symmetric equilibrium. Then $DCP \succ LCP$ under LCP and $DCP \succ PCP$ under PCP price index. Since χ^i and χ_0^i can get higher as one deviates from symmetric LCP, constraint (A47) implies that DCP dominates LCP in any PSNE. But then χ^i stays the same and χ_0^i can only increase relative to symmetric PCP and constraint (A46) implies that DCP dominates LCP in any PSNE as well. Second, suppose that LCP is a unique symmetric equilibrium. Since χ^i and χ_0^i can only get lower as one deviates from symmetric DCP, constraint (A47) implies that LCP dominates DCP in any PSE as well. The existence of symmetric LCP requires according to constraint (A45) that $\alpha > \frac{1-\phi\chi^L}{2-\chi^L(1+\phi)} > \frac{1}{2}$. This implies $\alpha > (1-\alpha)\phi$, so that constraint (A45) relaxes as χ^i decreases. Therefore, there can be no PSNE with PCP. Finally, suppose that PCP is a unique symmetric NE. Since χ^i and χ_0^i can get only lower than under symmetric DCP, constraint (A46) implies that DCP is dominated by PCP in PSNE. According to Lemma A5, there can also be no PSE with positive measure of LCP.

(3) Suppose there is market i, in which a positive mass of importers are indifferent between PCP and DCP and play mixed strategies. Take an arbitrary small share of firms pricing in producer currency and exogenously switch their invoicing into dollars. The coefficient χ_i does not change, while χ_0 increases. Condition (A46) implies that the firms that were indifferent now strictly prefer DCP, while Condition (A47) implies that the share of LCP can only fall. Since firms (endogenously) switch to dollar in response to the perturbation, the initial equilibrium is not stable. Note there are no indirect effects coming from other markets: as country i is infinitely small, the changes in invoicing of its imports or exports has no impact on other countries. A symmetric argument applies for other types of mixed equilibria. ■

Proof of Proposition 6 It is convenient to use a slightly different notation than in other sections: two currency unions have masses n_1 and n_2 with $n \equiv n_1 + n_2$, the relative exchange rate volatility of pound is $\rho \equiv \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}$, μ_i^k denotes the share of country i imports invoiced in currency k ($\mu_i^1 + \mu_i^2 = 1$). I also define pass-through coefficients as follows: $p_i = \chi_0^i e_i - \chi_1^i e_1 - \chi_2^i e_2$. The equilibrium price index is given by

$$[1-\lambda(\alpha+(1-\gamma)(1-\alpha)\phi)]\,p_i = \gamma[1-\lambda\alpha]\,e_i - \gamma\left[\lambda(1-\alpha)n_1 + (1-\lambda)\mu_i^1\right]e_1 - \gamma\left[\lambda(1-\alpha)n_2 + (1-\lambda)\mu_i^2\right]e_2$$
$$+\lambda\gamma(1-\alpha)\frac{\gamma(1-\lambda)\phi}{1-\lambda(\alpha+(1-\alpha)\phi)}\left[\left(n_1\mu_1^2 - n_2\mu_2^1 - (1-n)\mu_N^1\right)e_1 + \left(n_2\mu_2^1 - n_1\mu_1^2 - (1-n)\mu_N^2\right)e_2\right].$$

Vehicle currency 1 dominates vehicle currency 2 for exporter from j to i iff

$$(1-\alpha)\frac{\text{cov}(\phi p_j + e_1 - e_j, e_1 - e_2)}{\text{var}(e_1 - e_2)} + \alpha\frac{\text{cov}(p_i + e_1 - e_i, e_1 - e_2)}{\text{var}(e_1 - e_2)} < \frac{1}{2}.$$

Using this formula for each bilateral trade flow, we get:

- RoW exports to RoW:

$$(\alpha + (1-\alpha)\,\phi)\,\chi_2^N + \left[1 - \left(\chi_1^N + \chi_2^N\right)(\alpha + (1-\alpha)\,\phi)\right]\rho < \frac{1}{2},$$

- RoW exports to currency unions:

$$(1-\alpha)\,\phi\chi_2^N + \alpha\chi_2^1 + \left[(1-\alpha)\left(1 - \phi\chi_1^N - \phi\chi_2^N\right) + \alpha\left(\chi_0^1 - \chi_1^1 - \chi_2^1\right)\right]\rho < \frac{1}{2},$$
$$(1-\alpha)\,\phi\chi_2^N + \alpha\left(1 + \chi_2^2 - \chi_0^2\right) + \left[(1-\alpha)\left(1 - \phi\chi_1^N - \phi\chi_2^N\right) + \alpha\left(\chi_0^2 - \chi_1^2 - \chi_2^2\right)\right]\rho < \frac{1}{2},$$

- Currency union exporting to RoW:

$$(1-\alpha)\,\phi\chi_2^1 + \alpha\chi_2^N + \left[(1-\alpha)\,\phi\left(\chi_0^1 - \chi_1^1 - \chi_2^1\right) + \alpha\left(1 - \chi_1^N - \chi_2^N\right)\right]\rho < \frac{1}{2},$$
$$(1-\alpha)\left(1 + \phi\chi_2^2 - \phi\chi_0^2\right) + \alpha\chi_2^N + \left[(1-\alpha)\,\phi\left(\chi_0^2 - \chi_1^2 - \chi_2^2\right) + \alpha\left(1 - \chi_1^N - \chi_2^N\right)\right]\rho < \frac{1}{2},$$

- One currency union exporting to the other:

$$(1-\alpha)\,\phi\chi_2^1 + \alpha\left(1 + \chi_2^2 - \chi_0^2\right) + \left[(1-\alpha)\,\phi\left(\chi_0^1 - \chi_1^1 - \chi_2^1\right) + \alpha\left(\chi_0^2 - \chi_1^2 - \chi_2^2\right)\right]\rho < \frac{1}{2},$$
$$(1-\alpha)\left(1 + \phi\chi_2^2 - \phi\chi_0^2\right) + \alpha\chi_2^1 + \left[(1-\alpha)\,\phi\left(\chi_0^2 - \chi_1^2 - \chi_2^2\right) + \alpha\left(\chi_0^1 - \chi_1^1 - \chi_2^1\right)\right]\rho < \frac{1}{2}.$$

(1) Note first that without change in currency choice, ρ has no effect on the global share of pound, while higher n_2 implies a lower one. Next, suppose there is a point, at which the change in currency choice increases the fraction of trade invoiced in currency 1, i.e. there exist trade flow from j to i that switches invoicing from 2 to 1. Parameter ρ is present in only CC block (not in price index). Consider the derivative of the first and the second terms in the CC constraint with respect to ρ

$$(1-\alpha)\left[\phi\chi_2^j + \left(1 - \phi\chi_1^j - \phi\chi_2^j\right)\rho - \left(1 - \phi\chi_0^j\right)\frac{\operatorname{cov}(e_j, e_1 - e_2)}{\operatorname{var}(e_1 - e_2)}\right]$$
$$+\alpha\left[\chi_2^i + \left(1 - \chi_1^i - \chi_2^i\right)\rho - \left(1 - \chi_0^i\right)\frac{\operatorname{cov}(e_i, e_1 - e_2)}{\operatorname{var}(e_1 - e_2)}\right] < \frac{1}{2}.$$

The derivative of each term is clearly positive for all countries except for country 1, for which it is proportional to $\chi_0^1 - \chi_1^1 - \chi_2^1$. This term, however, is non-negative as well:

$$\gamma\lambda\,(1-\alpha)\,(1-n)\left[\frac{\lambda\,(1-\alpha)\,(1-\phi) + (1-\lambda)\,(1-\gamma\phi)}{1 - \lambda\,(\alpha + (1-\alpha)\,\phi)}\right].$$

Thus, as ρ goes up, all constraints become more binding and (everything else equal) can only decrease the use of pound and μ_i^1 (and hence, increase μ_i^2). It follows that χ_0^i is unaffected, χ_1^i falls and χ_2^i rises. According to currency choice inequality, this tightens constraint for currency 1 even further.

Consider next an increase in n_2, assuming that $n_1 + n_2$ remains unchanged. Country sizes n_i are present only in price indices, but not in currency choice inequalities. For given currency choice, χ_1^i and χ_2^i are monotonic in n_1 and n_2 respectively if $\mu_1^1 \geq \mu_2^1$ with derivatives equal to

$$1 - \frac{\gamma(1-\lambda)\phi}{1-\lambda(\alpha+(1-\alpha)\phi)}\left(1-\left(\mu_1^1-\mu_2^1\right)\right)$$

In this case, χ_1^i decreases and χ_2^i increases as n_2 goes up. The currency choice inequalities then tighten with n_2. The argument from above shows that endogenous change in invoicing pattern amplifies fall in global share of pound. It remains to show that inequality $\mu_1^1 \geq \mu_2^1$ indeed holds. The second part of the proposition (proven below) implies that the share of dollar denominated imports from RoW to the first country is not smaller than the one to the second country. Considering only trade between two currency unions we get $\mu_1^1 - \mu_2^1 \geq n_1 - n_1 = 0$.

(2) Consider an increase in n_2, which leaves n unchanged. First, note that price index for any country consists of three terms:

$$p_i \propto \lambda\gamma(1-\alpha)\phi\int_j p_j dj + \lambda\gamma(1-\alpha)\int(e_i-e_j)\,dj + (1-\lambda)\gamma\left[e_i - \mu_i^1 e_1 - \mu_i^2 e_2\right]$$

The first term is the same for all countries, while the second one does not depend on currency of invoicing. The last term, however, implies that starting from the equilibrium where all global trade (except between members of union 2) is denominated in currency 1, μ_i^2 is positive only for $i = 2$. Therefore, χ_2^j is higher and χ_1^j is lower for country 2. The currency choice inequalities imply then $T(b) \leq T(c)$, $T(e) \leq T(f)$ and $T(a) \leq T(c)$, $T(d) \leq T(g)$. But then χ_2^2 remains not lower than any χ_2^j after any changes in a, b, d, e. As long as this is the case, all previous inequalities should hold. Thus, they hold for the whole transition path. The symmetric argument can be made for country 1 with higher χ_1^j and lower χ_2^j implying $T(c) \leq T(f)$, $T(b) \leq T(e)$ and $T(c) \leq T(g)$, $T(a) \leq T(d)$. The comparative statics for ρ can be made in the similar way: the derivative of the LHS of currency choice inequality with respect to ρ is the same for all countries, so that only levels of χ_k^j matter. ■

A.6 Invoicing in terms of currency baskets

This note extends the baseline model by allowing firms to choose from a richer set of invoicing options. First, I provide sharp results for the case when firms are allowed to set prices in an arbitrary basket of currencies. Second, I consider an intermediate case when currency choice is continuous, but as in the basic model, only three currencies can be used for invoicing.

Note that correct interpretation of setting price in terms of currency basket is that e.g. Apple sells iPhone 7 in Germany for 500 dollars plus 300 euros plus 200 swiss francs. The interpretation that it sets price in dollars, euros and francs with probabilities 50%, 30% and 20% respectively is wrong since ex post pass-through of exchange rate shocks conditional on no price adjustment is discrete in this

case. Another wrong interpretation is that firm sells some fraction of products in one currency and some fraction in another currency. If this is the same product, customers will only make purchases using the lowest ex post price. Finally, using different currencies for different products is also a wrong interpretation since profits are separable in products.

A.6.1 Complete basket

Lemma A6 *Suppose that prices are set in terms of basket of arbitrary currencies. Then exporters can achieve the optimal pass-through of exchange rate shocks in every state of the world.*

Proof It sufficient to check that the sum of exchange rate weights in the optimal price is one. Since bilateral exchange rates remain unchanged if all $\{e_i\}$ increase by the same constant, the sum of exchange rate weights in p_i is zero and the sum of weights in $\tilde{p}_{ji}+e_{ki} = (1-\alpha)\left(\phi_j p_j + (1-\phi)\, w_j - a_{Ti} - e_j\right)+ \alpha\left(p_i - e_i\right)$ is one. ■

Thus, even if prices of firms are fully rigid, exporters can construct such invoicing baskets that their prices will move optimally with exchange rates. While this is a strong result, it is important to realize what it does not say:

- While the weights of all exchange rates are positive in the baseline model, in more general environment, they might be negative. From economic perspective, this means that firms are allowed to make transfers to the customers, e.g. a client pays 1200 dollars for the good and gets back 200 euros as a discount.
- While the pass-through of exchange rates into prices is optimal, the pass-through of other shocks is not. In particular, the pass-through is zero for idiosyncratic productivity shocks and even for aggregate shocks as long as they are uncorrelated with movements in exchange rates.
- As long as domestic firms are obliged to set prices in local currency as in the baseline model, the prices of importers and the allocation in tradable sector are different from the flexible-price case.

Proposition A2 *Assume $w_i = a_{Ti} = 0$, domestic firms set prices in local currency, while exporters can use arbitrary baskets of currencies for invoicing. Then*

1. *equilibrium is always unique,*
2. *the share of dollar in the international trade cannot be higher than n,*
3. *relative dollar volatility ρ has no effect on dollar use in international trade,*
4. *high price rigidity $1-\lambda$ decreases the use of dollar and stimulates LCP,*
5. *the share of dollar increases in γ and ϕ and might be not monotone in α.*

Proof Domestic firms set prices in local currency, while importers enjoy the optimal state contingent prices:

$$p_i = \frac{\gamma(1-\alpha)}{1-\gamma\alpha-\lambda(1-\gamma)(\alpha+(1-\alpha)\phi)}(e_i - ne_0).$$

All results except for the second one follow immediately from expressions for p_i and $\tilde{p}_{ji}$. Let $\bar{s}_i$ denote the share of local currency in the optimal basket of exporters. Since $\chi \leq 1$ and $\chi_0 = n\chi \leq n$, the dollar share in trade between third countries is $[\alpha+(1-\alpha)\phi]\chi_0 \leq n$ and the dollar share in the international trade is

$$\frac{1}{1-n^2}\left[(1-n)^2\bar{s}_0 + n(1-n)(\bar{s}_i+\bar{s}_0) + n(1-n)(\bar{s}_j+\bar{s}_0)\right] = \bar{s}_0 + \frac{n}{1+n}(\bar{s}_i+\bar{s}_j)$$

$$= [\alpha+(1-\alpha)\phi]n\chi + \frac{n}{1+n}[1-(\alpha+(1-\alpha)\phi)\chi] = \frac{n}{1+n} + [\alpha+(1-\alpha)\phi]\frac{n^2}{1+n}\chi \leq n.$$

■

The intuition for results 1 and 3 is straightforward: the optimal pass-through of dollar exchange rate depends on the fraction of exporters from the U.S. The share of dollar in the optimal basket is proportional to this pass-through, which is incomplete and therefore cannot be higher than n. Thus, the model with complete basket cannot match empirical fact that the share of DCP is much higher than the share of U.S. in the international trade. In addition, the model predicts that the relative volatility of dollar ρ plays no role because it has zero effect on the optimal pass-through of exchange rate shocks. Also, in contrast to the baseline model, higher price rigidity actually reduces the international use of dollar. This is because lower frequency of price adjustment has direct effect only on domestic producers, while the effect on importers is indirect and decreases the pass-through of exchange rate shocks. Finally, the comparative statics with respect to import share γ and intermediate share ϕ remain unaffected since their main effect comes from the weights of currencies in the optimal basket. Figure A4a provides an illustration of the results, showing the dollar share in trade between third countries.

A.6.2 Incomplete basket

Consider next the case when exporter can include only producer currency, local currency and dollar in the invoicing currency:

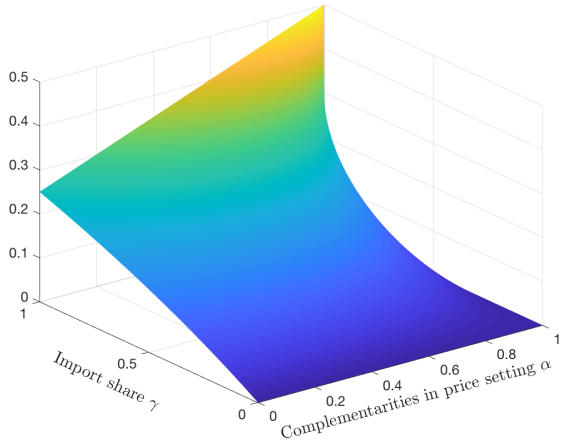
$$\min_s \mathbb{E}\ [\tilde{p}_{ji} + e_{ki}]^2$$

$$\text{s.t. } e_k = s_j e_j + s_i e_i + s_0 e_0, \qquad s_i + s_j + s_0 = 1,$$

$$\tilde{e}_{ji} = (1-\alpha)(1-\phi\chi)e_j + \alpha(1-\chi)e_i + (\alpha+(1-\alpha)\phi)\chi_0 e_0.$$

Notice the sum of weights of e_j, e_i and e_0 in the optimal price $1-(\alpha+(1-\alpha)\phi)(\chi-\chi_0)$ is less than one ($\chi \geq \chi_0$) because of the pass-through of other exchange rate shocks (which drop out by normalization).

(a) All currencies



(b) Three currencies

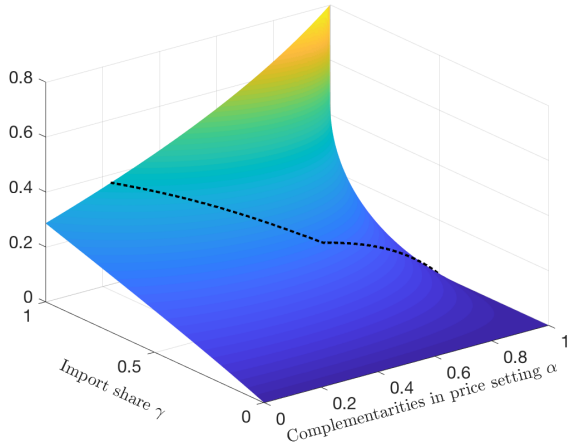


Figure A4: Dollar share in global trade when prices are set in baskets of currencies

Note: plot (a) shows the share of dollars in trade between non-U.S. countries when exporters can use arbitrary baskets of currencies and $n = 0.5$, while plot (b) shows the share of dollar in international trade when exporters can set prices in baskets consisting of only three currencies — producer, local and dollar — and $n = 0$. Parameter values: $\rho = 0.5$, $\phi = 0.5$, $\lambda = 0.5$. The dashed line in plot (b) shows the border of DCP region in the baseline model.

Lemma A7 *The optimal currency weights are*

$$s_j = \bar{s}_j + \frac{\rho}{1+2\rho}\left(1-\bar{s}_j-\bar{s}_i-\bar{s}_0\right) = (1-\alpha)\left(1-\phi\chi\right) + \frac{\rho}{1+2\rho}\left(\alpha+(1-\alpha)\,\phi\right)\left(\chi-\chi_0\right),$$
$$s_i = \bar{s}_i + \frac{\rho}{1+2\rho}\left(1-\bar{s}_j-\bar{s}_i-\bar{s}_0\right) = \alpha\left(1-\chi\right) + \frac{\rho}{1+2\rho}\left(\alpha+(1-\alpha)\,\phi\right)\left(\chi-\chi_0\right),$$
$$s_0 = \bar{s}_0 + \frac{1}{1+2\rho}\left(1-\bar{s}_i-\bar{s}_i-\bar{s}_0\right) = \left(\alpha+(1-\alpha)\,\phi\right)\left[\frac{2\rho}{1+2\rho}\chi_0 + \frac{1}{1+2\rho}\chi\right].$$

Proof Objective function:

$$\mathbb{V}\left[s_j e_j + s_i e_i + \left(1-s_i-s_j\right)e_0 - \bar{s}_j e_j - \bar{s}_i e_i - \bar{s}_0 e_0\right] = \left(s_j-\bar{s}_j\right)^2\sigma^2 + \left(s_i-\bar{s}_i\right)^2\sigma^2 + \left(1-s_i-s_j-\bar{s}_0\right)^2\sigma_0^2.$$

First order conditions:

$$s_j - \bar{s}_j = \left(1-s_i-s_j-\bar{s}_0\right)\rho,$$
$$s_i - \bar{s}_i = \left(1-s_i-s_j-\bar{s}_0\right)\rho.$$

Solving this linear system, one gets the expressions from the lemma. ■

As in the baseline model, other currencies than producer one, local one and dollar account for fraction $1-s_i-s_j-\bar{s}_0$ of $\tilde{e}_{ji}$. When dollar volatility is relatively low, $\rho < 1$, DCP is a better proxy for this fully diversified fraction of optimal price. As result, in both models, dollar share in international trade increases as ρ goes down. In contrast to the benchmark model, however, exporter uses all three

available currencies to proxy for this part of $\tilde{e}_{ji}$ when currency choice is continuous, and the share of dollar is lower. The next proposition characterizes the equilibrium invoicing.

Proposition A3 *Assume that the exporters can set prices in a basket of producer currency, local currency and dollars. Then*

1. *equilibrium is always unique,*
2. *the share of dollar in international trade is positive even if $\rho = 1$, $n = 0$ and $\lambda \to 0$,*
3. *the share of dollar increases as ρ goes down and n goes up.*

Proof The system of equations that defines the pass-through coefficients is

$$[1 - \gamma\alpha - \lambda(1-\gamma)(\alpha + (1-\alpha)\phi)]\chi = \gamma\left[(1-\alpha) - (1-\lambda)(\alpha + (1-\alpha)\phi)\frac{\rho}{1+2\rho}(\chi - \chi_0)\right],$$

$$[1 - \gamma\alpha - \lambda(1-\gamma)(\alpha + (1-\alpha)\phi)]\chi_0 = \gamma\Big[\frac{\gamma(1-\alpha)\phi(1-\lambda)(\alpha + (1-\alpha)\phi)}{1 - (\gamma + (1-\gamma)\lambda)(\alpha + (1-\alpha)\phi)}\frac{1+2\rho n}{1+2\rho}(\chi - \chi_0) +$$
$$+ (1-\lambda)(\alpha + (1-\alpha)\phi)\frac{1+\rho n}{1+2\rho}(\chi - \chi_0) + (1-\alpha)n\Big].$$

Together these expressions imply:

- Aggregate pass-through coefficients χ and χ_0 are unique as shown above, which implies that currency choice is also unique and so is the equilibrium.
- Consider the limit $\rho = 1$, $n = 0$ and $\lambda \to 0$. It follows from the system for pass-through coefficients that $\chi = \frac{\gamma}{1-(1-\gamma)\phi}$ and $\chi_0 = 0$ (LRPT does not depend on currency choice rule). Therefore the share of dollar in international trade is $s_0 = \frac{1}{3}(\alpha + (1-\alpha)\phi)\frac{\gamma}{1-(1-\gamma)\phi} > 0$.
- As expression above shows, it is sufficient to show that χ and χ_0 are decreasing in ρ. Both pass-through coefficients take the form $c_1\frac{1-c_2\frac{\rho}{1+2\rho}}{1-c_3\frac{\rho}{1+2\rho}}$. It is straightforward to check that in both cases $c_3 < c_2$ and therefore, χ and χ_0 increase as ρ goes down.
- Parameter n affects s_0 only through χ and χ_0 and both are increasing in n (use the same trick as in previous part).

■

One surprising result is that the multiplicity of equilibria disappears when currency choice is continuous despite Lemma A7, which shows that the share of dollars in a basket of individual exporter depends positively on the share of dollar invoicing by other firms. The elasticity of exporter's policy function with respect to decisions of other firms (summarized by χ and χ_0) is however lower than in the discrete case and strategic complementarities are not strong enough to generate multiple equilibria.

The second result that dollar is used for invoicing even when it has no advantages over other currencies follows from the simple observation that it is always beneficial to use all available currencies to diversify portfolio. The lower is volatility of dollar ρ, the better it is for diversification and hence, the higher is its share in the international trade. Figure A4b provides an illustration of the results. The dashed line shows the border of DCP region in the baseline model. It follows that the share of dollar is smaller (0%) under discrete choice when economies are relatively closed, but is larger (100%) when openness is high. This is the amplification effect of discrete choice.

A.6.3 Analogy with portfolio problem

There is a clear analogy between portfolio choice and the currency choice:

1. In both cases an agent has to choose a basket of assets/currencies with weights summing to one to minimize volatility of portfolio for given returns. Because of the simplifying assumption that all shocks are mean-zero, expected changes in the basket value are zero and the agent focuses exclusively on variance minimization. In addition, there is no "risk-free" asset, which would make currency choice trivial.

2. When asset markets are complete, the first-best allocation can be achieved. Similarly, as argued above, a full set of invoicing options allows firm to achieve the optimal pass-through.

3. The dominant status of dollar in both international goods markets (as a unit of account) and global financial markets (as the store of value) can be due to similar factors:

   (a) Deviations from complete markets can generate demand for safe asset just like incomplete spanning of currencies (or discreteness) in invoicing choice leads to DCP equilibrium with dollar being used as a proxy for a basket of other currencies.

   (b) The large share of U.S. in two markets mechanically explains high use of dollar and can lead to indirect amplification away from the "complete" benchmark.

   (c) Strategic complementarities between decisions of different agents play crucial role in both cases: the asset cannot be safe if other investors fire-sale it in bad states of the world, while it is more appealing to set prices in dollars when foreign suppliers and competitors use DCP.

4. Finally, there are also similarities in terms of computational techniques used to solve two problems in the context of general equilibrium models. Neither portfolio choice nor invoicing choice are determined under the first order approximation of the equilibrium conditions. Therefore, second order approximation of some optimality conditions is used to solve for steady state invoicing decisions (Engel 2006) and the optimal portfolio allocation (Devereux and Sutherland 2011). Note that the source of convexity of the objective functions is different in two cases: the risk-aversion of investor in portfolio problem and concavity of the profit function in the invoicing problem.

A.7 Alternative models of sticky prices

A.7.1 Calvo pricing

This section shows that the main results about currency choice from Section 3 hold under staggered pricing. As before, I abstract from monetary and productivity shocks. In addition, to simplify the analysis, the exchange rates are assumed to follow random walk, which requires under complete markets that the process for ψ_{it} is random walk.

Assume that prices of all firms are set a la Calvo with the probability of adjustment $1-\lambda$ (note the difference in notation from the baseline model). Start with exporter from country j to country i. Since exchange rates follow random walk, the first order approximation to the adjusted price does not depend on the currency of invoicing (see Gopinath, Itskhoki, and Rigobon 2010) and can be written in destination currency as

$$\hat{p}_{jit} = (1-\beta\lambda)\,\tilde{p}_{jit} + \beta\lambda \mathbb{E}_t \hat{p}_{jit+1},$$

where the optimal static price $\tilde{p}_{jit}$ is the same as in the baseline model. The import price index from j to i aggregates across adjusting and non-adjusting firms

$$p_{jit} = (1-\lambda)\,\hat{p}_{jit} + \lambda\left(p_{jit-1} + \mu^P \Delta e_{ijt} + \mu^D \Delta e_{i0t}\right).$$

The standard manipulations lead to the NKPC:

$$\pi_{it} = \frac{(1-\beta\lambda)(1-\lambda)}{\lambda}\left(\tilde{p}_{it} - p_{it}\right) + \beta\mathbb{E}_t\pi_{it+1} + \gamma\left[\mu^P\left(\Delta e_{it} - n\Delta e_{0t}\right) + \mu^D\left(\Delta e_{it} - \Delta e_{0t}\right)\right],$$

$$\tilde{p}_{it} = (1-\gamma)(1-\alpha)\,\phi p_{it} + \gamma(1-\alpha)\left(\phi p_t + e_{it} - n e_{0t}\right) + \alpha p_{it}.$$

I solve for p_{it} in two steps. First, denote deviations of local variables from global averages with bars:

$$-\beta\mathbb{E}_t\bar{p}_{it+1} + \left[1+\beta+\left(1-(1-\gamma)\,\phi\right)\kappa\right]\bar{p}_{it} - \bar{p}_{it-1} = \kappa\gamma\bar{e}_{it} + \gamma\left(\mu^P+\mu^D\right)\Delta\bar{e}_{it},$$

where $\kappa \equiv \frac{(1-\beta\lambda)(1-\lambda)(1-\alpha)}{\lambda}$ and $\bar{e}_{it} \equiv e_{it} - n e_{0t}$. Rewrite it in terms of lag operator $\mathbb{L}$ and factorize applying Vieta's formula:

$$-\left[\beta\mathbb{L}^{-1} - \left(1+\beta+\left(1-(1-\gamma)\phi\right)\kappa\right) + \mathbb{L}\right] = -\left(1-\beta\varphi\mathbb{L}^{-1}\right)\left(1-\varphi^{-1}\mathbb{L}^{-1}\right)\mathbb{L},$$

$$\varphi = \frac{1+\beta+\varsigma\kappa - \sqrt{\left(1+\beta+\varsigma\kappa\right)^2 - 4\beta}}{2\beta} = \frac{1+\beta+\varsigma\kappa - \sqrt{\left(1-\beta+\varsigma\kappa\right)^2 + 4\beta\kappa\varsigma}}{2\beta} > 0,$$

where $\varsigma \equiv 1-(1-\gamma)\,\phi$ and $\varphi \in (0,1)$. Substitute solution back into the difference equation:

$$\bar{p}_{it} = \varphi\bar{p}_{it-1} + \varphi\mathbb{E}_t\sum_{\tau=0}^{\infty}(\beta\varphi)^\tau\left\{\kappa\gamma\bar{e}_{it+\tau} + \gamma\left(\mu^P+\mu^D\right)\Delta\bar{e}_{it+\tau}\right\}.$$

Since exchange rates follow random walk, we get

$$\bar{p}_{it} = \varphi \bar{p}_{it-1} + \frac{\varphi\kappa}{1-\beta\varphi}\gamma\bar{e}_{it} + \gamma\varphi\left(\mu^P + \mu^D\right)\Delta\bar{e}_{it}.$$

Second, integrate across all countries to get the the second-order difference equation for global price index:

$$-\beta\mathbb{E}_t p_{t+1} + \left[1 + \beta + (1-\phi)\kappa\right] p_t - p_{t-1} = -\gamma\mu^D (1-n)\Delta e_{0t}.$$

Using the same steps as above, obtain solution

$$p_t = \hat{\varphi} p_{t-1} - \gamma\hat{\varphi}\mu^D (1-n)\Delta e_{0t},$$

$$\hat{\varphi} = \frac{1 + \beta + \hat{\varsigma}\kappa - \sqrt{\left(1 + \beta + \hat{\varsigma}\kappa\right)^2 - 4\beta}}{2\beta}, \qquad \hat{\varsigma} \equiv 1 - \phi.$$

Finally, back out dynamics of country i price index from $p_{it} = \bar{p}_{it} + p_t$.

To solve the currency choice problem, consider without loss of generality the case when initial values of all shocks are zero and the optimal preset prices in any currency is zero as well. The ex post price in period t conditional on non-adjustment is therefore e_{ikt} when currency k is used for invoicing. The second-order approximation to the currency choice problem of exporter from j to i is

$$\min_k \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta\lambda)^t \left(\tilde{p}_{jit} + e_{kit}\right)^2.$$

Note that the interpretation that firm chooses currency k to mimic dynamics of the optimal invoicing basket is still valid. It also follows that exporters prefer currency k to currency l iff

$$\sum_{t=0}^{\infty} (\beta\lambda)^t \mathbb{E}_0 \left(\tilde{p}_{jit} - e_{ikt}\right)^2 < \sum_{t=0}^{\infty} (\beta\lambda)^t \mathbb{E}_0 \left(\tilde{p}_{jit} - e_{ilt}\right)^2.$$

Using the fact that exchange rates follow random walk and following the steps from Gopinath, Itskhoki, and Rigobon (2010), the inequality can be rewritten as

$$(1-\beta\lambda)\sum_{t=0}^{\infty} (\beta\lambda)^t \frac{\operatorname{cov}\left(\tilde{p}^k_{jit}, \Delta e_{kl0}\right)}{\operatorname{var}\left(\Delta e_{kl0}\right)} < \frac{1}{2},$$

or after substituting the optimal price as

$$(1-\beta\lambda)\sum_{t=0}^{\infty} (\beta\lambda)^t \frac{\operatorname{cov}\left[(1-\alpha)\left(\phi p_{jt} - e_{jt}\right) + \alpha\left(p_{it} - e_{it}\right) + e_{kt}, \Delta e_{kl0}\right]}{\operatorname{var}\left(\Delta e_{kl0}\right)} < \frac{1}{2}$$

To find covariance terms, I normalize volatilities of non-dollar exchange rates to one and the volatil-

ity of dollar to ρ, and use Yule-Walker equations to compute autocovariance functions:

$$\begin{aligned}
\text{cov}\left(\bar{p}_{it}, \Delta e_{i0}\right) &= \varphi \text{cov}\left(\bar{p}_{it-1}, \Delta e_{i0}\right) + \frac{\varphi\gamma\kappa}{1-\beta\varphi}\text{cov}\left(e_{it}, \Delta e_{i0}\right) + \gamma\varphi\left(\mu^P + \mu^D\right)\text{cov}\left(\Delta e_{it}, \Delta e_{i0}\right), \\
\text{cov}\left(\bar{p}_{it}, \Delta e_{00}\right) &= \varphi \text{cov}\left(\bar{p}_{it-1}, \Delta e_{00}\right) - \frac{\varphi\gamma\kappa n}{1-\beta\varphi}\text{cov}\left(e_{0t}, \Delta e_{00}\right) - \gamma\varphi n\left(\mu^P + \mu^D\right)\text{cov}\left(\Delta e_{0t}, \Delta e_{00}\right), \\
\text{cov}\left(p_t, \Delta e_{00}\right) &= \hat{\varphi} \text{cov}\left(p_{t-1}, \Delta e_{00}\right) - \gamma\hat{\varphi}\mu^D\left(1-n\right)\text{cov}\left(e_{0t}, \Delta e_{00}\right).
\end{aligned}$$

The resulting IRFs are

$$v_{it} \equiv \text{cov}\left(p_{it}, \Delta e_{i0}\right) = \gamma\varphi^{t+1}\left[\frac{\kappa}{1-\beta\varphi} + \left(\mu^P + \mu^D\right)\right] + \frac{1-\varphi^t}{1-\varphi}\frac{\gamma\varphi\kappa}{1-\beta\varphi},$$

$$v_{0t} \equiv \text{cov}\left(p_{it}, \Delta e_{00}\right) = -\gamma\varphi^{t+1}\left[\frac{\kappa n}{1-\beta\varphi} + n\left(\mu^P + \mu^D\right)\right]\rho - \frac{1-\varphi^t}{1-\varphi}\frac{\gamma\varphi\kappa n}{1-\beta\varphi}\rho - \gamma\hat{\varphi}^{t+1}\mu^D\left(1-n\right)\rho,$$

and zero for all other exchange rates. Three inequalities determine invoicing decisions of firms:

$$\begin{aligned}
V^{PCP} < V^{LCP} \quad &\Leftrightarrow \quad \left[\left(1-\alpha\right)\phi - \alpha\right]\frac{\gamma\varphi}{1-\beta\lambda\varphi}\left[\left(1-\beta\lambda\right)\left(\mu^P + \mu^D\right) + \frac{\kappa}{1-\beta\varphi}\right] < 1 - 2\alpha, \\
V^{DCP} < V^{PCP} \quad &\Leftrightarrow \quad \left[\alpha\rho n + \left(1-\alpha\right)\phi\left(1+\rho n\right)\right]\frac{\gamma\varphi}{1-\beta\lambda\varphi}\left[\left(1-\beta\lambda\right)\left(\mu^P + \mu^D\right) + \frac{\kappa}{1-\beta\varphi}\right] \\
&\qquad + \left(\alpha + \left(1-\alpha\right)\phi\right)\frac{\gamma\hat{\varphi}\left(1-\beta\lambda\right)}{1-\beta\lambda\hat{\varphi}}\rho\mu^D\left(1-n\right) > \frac{1}{2}\left(1+\rho\right) - \alpha, \\
V^{DCP} < V^{LCP} \quad &\Leftrightarrow \quad \left[\alpha\left(1+\rho n\right) + \left(1-\alpha\right)\phi\rho n\right]\frac{\gamma\varphi}{1-\beta\lambda\varphi}\left[\left(1-\beta\lambda\right)\left(\mu^P + \mu^D\right) + \frac{\kappa}{1-\beta\varphi}\right] \\
&\qquad + \left(\alpha + \left(1-\alpha\right)\phi\right)\frac{\gamma\hat{\varphi}\left(1-\beta\lambda\right)}{1-\beta\lambda\hat{\varphi}}\rho\mu^D\left(1-n\right) > \alpha - \frac{1}{2}\left(1-\rho\right).
\end{aligned}$$

Proposition A4 *All results about the currency choice from the benchmark model remain true in the mutliperiod model:*

1. *there can be no DCP equilibrium in the closed economy limit* $\gamma \to 0$,
2. *there can be no DCP equilibrium in the flexible price limit* $\lambda \to 0$ *with symmetric countries* $n = 0$, $\rho = 1$,
3. *the DCP region is increasing in* ρ *for* $\lambda \to 0$, $n = 0$,
4. *the DCP region is increasing in* n *for* $\lambda \to 0$,
5. *DCP region is non-empty when prices are sticky* $\lambda > 0$.

Proof

1. In the limit $\gamma \to 0$, the processes for $\bar{p}_{it}$ and p_t have the same AR root, $\varphi \to \hat{\varphi} > 0$. Therefore,

the inequalities reduce to

$$\alpha < \frac{1}{2}, \qquad \alpha > \frac{1}{2}(1+\rho), \qquad \alpha < \frac{1}{2}(1-\rho).$$

The last two expressions imply there are no values of α, for which DCP dominates both PCP and LCP. According to the first inequality, the equilibrium invoicing is PCP if $\alpha < \frac{1}{2}$ and LCP if $\alpha > \frac{1}{2}$.

2. In the limit $\lambda \to 0$, we obtain $\kappa \to \infty$, $\varphi, \hat{\varphi} \to 0$, $\kappa\varphi \to \frac{1}{1-(1-\gamma)\phi}$ and $\kappa\hat{\varphi} \to \frac{1}{1-\phi}$. Add conditions $n = 0$ and $\rho = 1$ and take the limit in the inequalities:

$$\frac{\gamma\left[(1-\alpha)\phi - \alpha\right]}{1-(1-\gamma)\phi} < 1 - 2\alpha \quad \Rightarrow \quad (1 - 2\alpha + \gamma\alpha)(1-\phi) > 0,$$

$$\frac{\gamma(1-\alpha)\phi}{1-(1-\gamma)\phi} > 1 - \alpha \quad \Rightarrow \quad (1-\alpha)(1-\phi) < 0,$$

$$\frac{\gamma\alpha}{1-(1-\gamma)\phi} > \alpha \quad \Rightarrow \quad \alpha(1-\gamma)(1-\phi) < 0.$$

Thus, both PCP and LCP strictly dominate DCP. The only two points, for which firms are indifferent between three options are $\alpha = \gamma = 1$ and $\phi = 1$ as in the baseline model.

3. Note that κ, φ and $\hat{\varphi}$ do not depend on ρ. Therefore, the derivative of the inequalities for DCP vs. PCP/LCP wrt ρ is

$$(\alpha + (1-\alpha)\phi)\frac{\gamma\varphi n}{1-\beta\lambda\varphi}\left[(1-\beta\lambda)\left(\mu^P + \mu^D\right) + \frac{\kappa}{1-\beta\varphi}\right] + (\alpha + (1-\alpha)\phi)\frac{\gamma\hat{\varphi}(1-\beta\lambda)}{1-\beta\lambda\hat{\varphi}}\mu^D(1-n) - \frac{1}{2},$$

which is always negative for $\lambda \to 1$ and $n = 0$.

4. Note that κ, φ and $\hat{\varphi}$ do not depend on n. Therefore, the derivative of the inequalities for DCP vs. PCP/LCP wrt n is

$$(\alpha + (1-\alpha)\phi)\frac{\gamma\varphi\rho}{1-\beta\lambda\varphi}\left[(1-\beta\lambda)\mu^P + \frac{\kappa}{1-\beta\varphi}\right] - (\alpha + (1-\alpha)\phi)\gamma\rho\left[\frac{\hat{\varphi}}{1-\beta\lambda\hat{\varphi}} - \frac{\varphi}{1-\beta\lambda\varphi}\right](1-\beta\lambda)\mu^D,$$

where $\hat{\varphi} > \varphi$. The derivative is positive in the flexible price limit.

5. Suppose $n = 0$ and $\rho = 1$. Take the limit $\alpha, \gamma \to 1$, which implies $\kappa \to 0$, $\varphi, \hat{\varphi} \to 1$ and show that DCP equilibrium always exists for $\mu^D = 1$.

■

A.7.2 Rotemberg pricing

I argue next that under the second-order approximation, the currency choice problem of individual firms is the same under Rotemberg pricing as in the baseline model, which relies on Calvo pricing. To simplify notation, I suppress indices of origin and destination below.

There are two steps in firm optimization. In the second one, which happens after the shocks are realized, a firm decides how much to adjust its prices. Taking the second-order approximation of the (static) profit function and assuming quadratic costs of price adjustment, the problem of the firm can be written as

$$\min_{p} \left\{ \varphi \left(p - \bar{p} \right)^2 + \left(p - p_0 \right)^2 \right\},$$

where $\varphi < 0$ is a constant determined at the point of approximation, $\bar{p}$ is the optimal price in a given state of the world, p_0 is the value of the preset price, which depends on the value of the exchange rate. The first-order condition implies then that firms chooses a price as a weighted average of the optimal price and the preset price $p = \omega \bar{p} + (1 - \omega) p_0$, where $\omega = \frac{\varphi}{1+\varphi}$. Therefore,

$$p - \bar{p} = (1 - \omega) \left(p_0 - \bar{p} \right), \qquad p - p_0 = -\omega \left(p_0 - \bar{p} \right),$$

and hence, the profit function is proportional to $\left(p_0 - \bar{p} \right)^2$. The first period problem of a firms to choose currency of invoicing is

$$\max \ \mathbb{E} \, \Pi \left(p, s \right) \quad \Leftrightarrow \quad \min \ \left(p_0 - \bar{p} \right)^2.$$

Thus, the currency choice problem is isomorphic to the one in the benchmark case.

A.7.3 Menu cost model

This subsection shows that model predictions remain robust when Calvo pricing is replaced with the endogenous price adjustment. To simplify, I assume as before that prices adjust fully after two periods, while firms optimally choose whether to pay menu costs and to update prices after one period. I use the second-order approximation to firm's profit function and the first-order approximation for price indices.[48] In addition to aggregate exchange rates shocks, firms also experience idiosyncratic productivity shocks, which according to previous studies account for most price adjustments (see e.g. Golosov and Lucas 2007). As in the baseline model, I abstract from monetary and productivity shocks.

I solve the model numerically using the following algorithm. I first guess price function $p_i = p(e_i, e_0)$ for given currency of invoicing. I then estimate deviation of producer's ex-post price from the optimal level $\tilde{p}_{ji}$ in each state of the world and solve for price adjustment decision. Integrating across both idiosyncratic productivity shocks and exchange rates e_j, I then update function $p(\cdot, \cdot)$ and iterate this procedure till convergence. Finally, I compute expected profits of a given exporter under alternative invoicing and check whether conjectured currency choice can be sustained in equilibrium.[49] To implement this algorithm, I use a grid with 31 points for exchange rates and 51 points for idiosyncratic shocks. Following Gopinath and Itskhoki (2010), I calibrate the standard deviation of productivity

[48] For the proof that such approximation is consistent see appendix in Gopinath and Itskhoki (2010).

[49] As is well known (see e.g. Ball and Romer 1991), there are strategic complementarities in price adjustment decisions, which can lead to multiple equilibria. The initial guess for price function is taken from the baseline model and assumes $\lambda = 0$ for the flexible-price limit and $\lambda = 0.5$ for the baseline calibration. The results remain robust for other initial values.

shocks to be 5 times larger than the standard deviation of exchange rates.

Figure A5 reproduces two key results from the baseline model in the extension with menu costs. The left plot shows equilibrium invoicing when menu costs are close to zero and dollar has no fundamental advantages. As in Figures 2b, the equilibrium is unique for most parameter space and no DCP equilibrium exists. The right figure shows instead that the region of DCP is large and close to the one from Figure 5 when prices are sticky and countries are asymmetric.

(a) No-DCP benchmark

(b) Baseline calibration

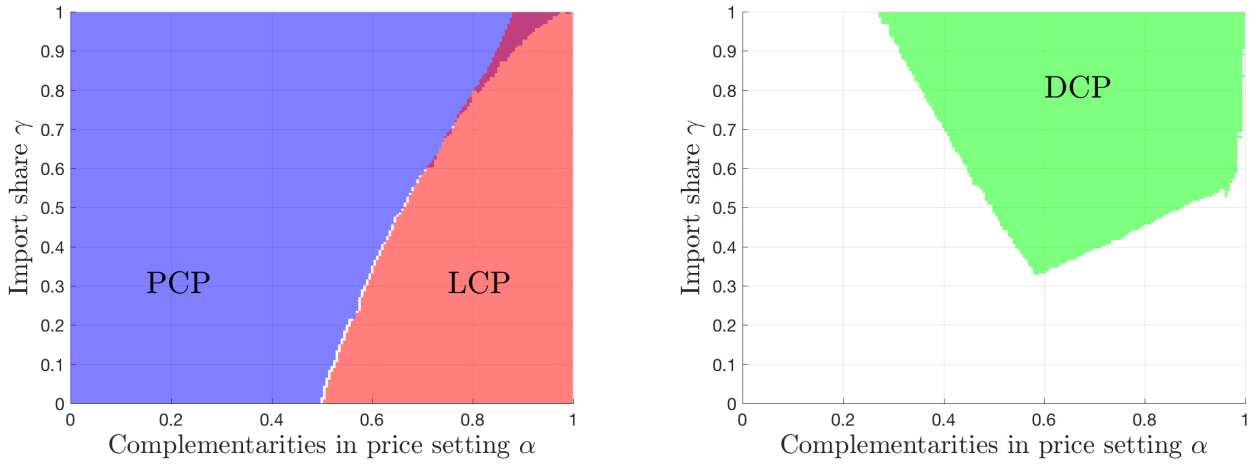


Figure A5: Currency choice in the menu cost model

Note: plot (a) shows DCP region is empty in the limiting case of almost zero menu costs and $n = 0$, $\rho = 1$. Plot (b) shows the region of symmetric DCP equilibrium (other equilibria are suppressed) under the baseline calibration: $n = 0.3$, $\rho = 0.5$ and menu costs are calibrated in such way that the probability of price adjustment is 0.5 for $\alpha = 0.5$, $\gamma = 0.6$, $\phi = 0.5$.

A.7.4 Model with bargaining

This section outlines a model with bargaining between suppliers and buyers and shows that the same equilibrium as in the baseline model can arise even when prices and invoicing currency are chosen jointly by two firms. The extension is based on Hart and Moore (2008) and Gopinath and Itskhoki (2011).

The general equilibrium setup is the same as in the benchmark model. The tradable sector is populated by two types of firms. As before, there is a continuum of manufacturing firms producing intermediate goods in each country. In addition, there are wholesale firms, which combine local and imported products using Kimball aggregator and sell output to final consumers and to firms in tradable sector as intermediate inputs. I assume the most commonly used specification for Kimball demand coming from Klenow and Willis (2007) and use $\Upsilon(\cdot)$ and $h(\cdot)$ below to denote aggregation function and the resulting demand function. Wholesale firms set prices flexibly and charge a constant markup over marginal

costs, i.e. demand for their output is

$$Q_i = P_i^{-\zeta} B_i,$$

where B_i is demand shifter taken as given by individual firms and $\zeta > 1$. Elasticity ζ does not affect optimal price as I show below and therefore, can take arbitrary values. In particular, one can take limit $\zeta \to \infty$ to make wholesale sector perfectly competitive.

Wholesale firms and their suppliers bargain over prices and choose the currency of invoicing before the realization of shocks. After uncertainty is resolved, wholesale firms decide how much inputs suppliers have to deliver. With probability λ, firms experience large enough idiosyncratic shocks to renegotiate prices ex post. The assumption that contract specifies prices, but not quantities is motivated by the result from the optimal contract literature by Hart and Moore (2008): "The parties are more likely to put restrictions on variables over which there is an extreme conflict of interest, such as price, than on variables over which conflict is less extreme, such as the nature or characteristics of the good to be traded."

The marginal costs of production for manufacturers are the same as in the baseline model. The price index for bundle of intermediate goods p_i remains also unchanged because of the combination of two assumptions: (i) prices of all wholesale firms are equal in equilibrium due to symmetry, (ii) wholesale firms charge a constant markup over marginal costs. Denote the marginal costs of wholesalers with R_i. The profits of wholesale firm for given costs are

$$\Pi_i = \frac{B_i}{\zeta^{\zeta} (\zeta - 1)^{\zeta - 1}} R_i^{1-\zeta}.$$

Lemma A8 *The marginal effect of signing a contract with an additional supplier j on marginal costs of wholesaler i is equal*

$$dR_i = D_i P_{ji} h\left(\frac{D_i P_{ji}}{R_i}\right) - R_i \Upsilon\left(h\left(\frac{D_i P_{ji}}{R_i}\right)\right).$$

Proof The equilibrium values of R_i and D_i are characterized by a system of equations:[50]

$$\frac{1}{N}\int_0^n \Upsilon\left(h\left(\frac{D_i P_{ji}}{R_i}\right)\right) dj = 1,$$

$$\frac{1}{N}\int_0^n h\left(\frac{D_i P_{ji}}{R_i}\right)\frac{P_{ji}}{R_i} dj = 1.$$

Take total differential of two equations and use $x_j \equiv \frac{D_i P_{ji}}{R_i}$ to simplify notation

$$\Upsilon\left(h\left(x_n\right)\right) \mathrm{d}n + \int_0^n \Upsilon'\left(h\left(x_j\right)\right) h'\left(x_j\right) x_j \mathrm{d}\log\left(\frac{D_i}{R_i}\right) dj = 0,$$

[50] For simplicity, I assume that demand shifter γ reflects the mass of varieties coming from different countries (extensive margin) rather than the trade flow of a given firm (intensive margin).

$$h\left(x_n\right)x_n\mathrm{d}n+\int_0^n\left[h'\left(x_j\right)x_j^2\mathrm{d}\log\left(\frac{D_i}{R_i}\right)-h\left(x_j\right)x_j\mathrm{d}\log R_i\right]dj=0.$$

Note that $\Upsilon'\left(h\left(x_j\right)\right)=x_j$ in the first condition from definition of $h\left(\cdot\right)$ and that $\frac{1}{N}\int_0^n h\left(x_j\right)x_j dj=1$ in the second condition according to initial equilibrium system. Using these equalities and substituting the first equation into the second one, we obtain

$$\mathrm{d}\log R_i dj=\left[h\left(x_n\right)x_n-\Upsilon\left(h\left(x_n\right)\right)\right]\frac{\mathrm{d}n}{N},$$

which proves the lemma. ■

The benefit of signing a contract for supplier is

$$(P_{ji}-MC_j^i)Q_{ji}=(P_{ji}-MC_j^i)h\left(\frac{D_iP_{ji}}{R_i}\right)R_i^{-\zeta}B_i,$$

where MC_j^i are marginal costs of producer j expressed in currency i. Nash bargaining solution can then be obtained from the following problem:

$$\max_{P_{ji}}\left[\left(P_{ji}-MC_j^i\right)h\left(\frac{D_iP_{ji}}{R_i}\right)R_i^{-\zeta}B_i\right]^{1-\tau}\left[\frac{B_i}{\zeta^\zeta\left(\zeta-1\right)^\zeta}R_i^{-\zeta}\left[D_iP_{ji}h\left(\frac{D_iP_{ji}}{R_i}\right)-R_i\Upsilon\left(h\left(\frac{D_iP_{ji}}{R_i}\right)\right)\right]\right]^{\tau},$$

or equivalently

$$\max_{P_{ji}}\left(1-\tau\right)\log\left[(\frac{R_i}{D_i}x-MC_j^i)h\left(x\right)\right]+\tau\log\left[xh\left(x\right)-\Upsilon\left(h\left(x\right)\right)\right],$$

where τ denotes the bargaining power of wholesaler and $x\equiv\frac{D_iP_{ji}}{R_i}$. The first order condition is

$$\frac{\left(1-\tau\right)\frac{R_i}{D_i}}{\frac{R_i}{D_i}x-MC_j^i}+\frac{\left(1-\tau\right)h'\left(x\right)}{h\left(x\right)}+\frac{\tau\left[h\left(x\right)+xh'\left(x\right)-\Upsilon'\left(h\left(x\right)\right)h'\left(x\right)\right]}{xh\left(x\right)-\Upsilon\left(h\left(x\right)\right)}=0.$$

Multiply all terms by x, use the definition of $h\left(x\right)=\Upsilon'^{-1}\left(x\right)$, which implies $\Upsilon'\left(h\left(x\right)\right)=x$, and definition of $\theta\left(x\right)\equiv-\frac{h'(x)x}{h(x)}$ to rewrite optimality condition as

$$\left(1-\tau\right)\left[\frac{P_{ji}}{P_{ji}-MC_j^i}-\theta\left(x\right)\right]=\tau\frac{h\left(x\right)}{\Upsilon\left(h\left(x\right)\right)-xh\left(x\right)}.$$

Log-linearize equilibrium condition around symmetric deterministic point with all prices being equal $P_{ji}=P=R_i$, $x=D$, $\Upsilon'\left(1\right)=D$, $\Upsilon\left(1\right)=h\left(D\right)=1$:

$$\left(1-\tau\right)\left[\frac{P/MC}{\left(P/MC-1\right)^2}\left(mc_j^i-\tilde{p}_{ji}\right)-\varepsilon\theta\left(\tilde{p}_{ji}-p_i\right)\right]=\tau\frac{\theta D}{\left(1-D\right)^2}\left[\frac{\theta-1}{\theta}-D\right]\left(\tilde{p}_{ji}-p_i\right)$$

where $\varepsilon\equiv\frac{\partial\log\theta(x)}{\partial\log x}$ and $p_i=r_i$. When suppliers have all bargaining power, $\tau=0$, the optimal price is exactly the same as in the benchmark case. More generally, since equation is homogeneous in

$\left(\tilde{p}_{ji}, mc_j^i, p_i\right)$, the optimal price $\tilde{p}_{ji}$ can be written as a weighted sum of marginal costs and local price index as in the baseline model. Moreover, for the aggregator from Klenow and Willis (2007), $D = \frac{\theta-1}{\theta}$ as in the CES case and therefore, optimal price does not depend on distribution of bargaining power τ.

Lemma A9 *For Klenow-Willis aggregator, the first-order approximation to the optimal price (19) is the same in the model with bargaining as in the baseline model.*

Finally, because contract is sticky and can be renegotiated only in extreme states of the world, suppliers and wholesalers choose the currency of invoicing to minimize deviations of ex post price from the optimal one. Under the second order approximation, this implies the same invoicing problem as in the benchmark model:

$$\min_k \mathbb{E}\ \left[\tilde{p}_{ji} - e_{ik}\right]^2$$

Thus, the equilibrium conditions for marginal costs, price index, optimal price and currency choice are the same to the first order approximation as the ones in the baseline model, and therefore, two models have the same equilibrium.

A.8 Transmission of shocks

Proof of Proposition 7 Consider a monetary shock w_i. The risk-sharing condition (A38) implies that the depreciation of exchange rate e_i is the same in all countries. Moreover, the pass-through of w_i into prices and quantities (A17)-(A30) is independent from currency regime and is the same for all countries when $n = 0$. The only difference between the U.S. and other countries is therefore coming from the effect of e_i on prices and quantities.

Both export and import elasticity with respect to trade-weighted exchange rate e_i is different for the U.S. than for other countries because of the effect of e_0 on global economy, which is summarized by the partial elasticity

$$\frac{\partial ex_i}{\partial e_0} = \frac{\partial im_i}{\partial e_0} = \left[(1-\gamma)(\theta-1) + \frac{\gamma}{1+\sigma\nu}\frac{(1+\sigma\nu)(\phi-\eta)+\nu\eta}{1-\lambda(\alpha+(1-\alpha)\phi)}\right](1-\lambda)\mu^D,$$

which is positive under DCP. The effect of e_i on CPI inflation is given by $p_i^C = \eta\chi e_i$ for non-U.S. economies and $p_0^C = \eta(\chi-\chi_0)e_0$ for the U.S., which implies that inflation is lower in the U.S. From equation (A31), e_i has no effect on output in non-tradable sector: $y_{Ni} = p_i^C - p_i^N + c_i = w_i - p_i^N$. Equation (A30) implies that the relevant price terms in tradable production are

$$y_i = \frac{\gamma\theta}{1-(1-\gamma)\phi}\left[\left(p_i^I - p_i^E\right) - (p_i - p)\right] - \frac{(1-\gamma)(1-\phi^2)}{1-(1-\gamma)\phi}p_i - \frac{\gamma(1+\phi)}{1-(1-\gamma)\phi}p.$$

Again, the asymmetries across countries come from the partial derivative with respect to e_0:

$$\frac{\partial\left[\left(p_i^I - p_i^E\right) - (p_i - p)\right]}{\partial e_0} = 0, \qquad \frac{\partial p_i}{\partial e_0} = \frac{\partial p}{\partial e_0} = -\chi_0.$$

The stimulating effect on local output is therefore large in the U.S. Finally, Lemma A1 implies that the effect of e_i on net exports of all countries is the same when $n = 0$. ■

Proof of Proposition 8 Consider a monetary shock in the U.S. w_0. The risk-sharing condition (A38) implies that the depreciation of dollar exchange rate e_0 is the same under all invoicing regimes. Moreover, the pass-through of w_0 into prices and quantities (A17)-(A30) is independent from currency regime as well. The only difference in international spillovers under PCP/LCP and DCP come from the effect of e_0 on foreign prices and quantities. Results (1) and (2) then follow immediately from expressions (20)-(22). The price index (A17) implies that higher e_0 decreases p_i and CPI in other economies, and the foreign consumption increases according to (A22). Finally, consider total production of tradable and non-tradable goods. Equation (A31) implies e_0 has no effect on output in non-tradable sector: $y_{Ni} = p_i^C - p_i^N + c_i = w_i - p_i^N$. Equation (A30) implies that the relevant price terms in tradable production are

$$y_i = \frac{\gamma\theta}{1-(1-\gamma)\phi}\left[\left(p_i^I - p_i^E\right) - (p_i - p)\right] - \frac{(1-\gamma)(1-\phi^2)}{1-(1-\gamma)\phi}p_i - \frac{\gamma(1+\phi)}{1-(1-\gamma)\phi}p,$$

where

$$\left(p_i^I - p_i^E\right) - (p_i - p) = \left[\chi - \lambda\left(\alpha\chi + (1-\alpha)(2-\phi\chi)\right) - (1-\lambda)\left(2\mu^P + \mu^D\right)\right] ne_0.$$

Thus, when $\theta n \to 0$, the first term drops out and since both p_i and p fall with e_0 under DCP, the effect on output is positive. ■

A.9 Welfare and policy analysis

A.9.1 Efficient allocation

Proof of Proposition 9 Assume CES aggregator across tradable products, $\alpha = 0$, and no non-tradable sector, $\eta = 1$. The social planner maximizes the global welfare state by state subject to resource and technology constraints:

$$\max \int_0^1 \left(\log C_i - L_i\right)\mathrm{d}i$$

$$\text{s.t.} \qquad C_i + X_i + G_i \leq \left[(1-\gamma)^{\frac{1}{\theta}} e^{-\frac{\gamma}{\theta}\xi_i} Y_{ii}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} e^{\frac{1-\gamma}{\theta}\xi_i} \int_0^1 Y_{ji}^{\frac{\theta-1}{\theta}}\mathrm{d}j\right]^{\frac{\theta}{\theta-1}},$$

$$Y_{ii} + \int_0^1 Y_{ij}\mathrm{d}j \leq A_i \left(\frac{L_i}{1-\phi}\right)^{1-\phi}\left(\frac{X_i}{\phi}\right)^{\phi}.$$

The first-order optimality conditions are

$$C_i = \frac{1-\phi}{\phi}\frac{X_i}{L_i}, \tag{A48}$$

$$\left[(1-\gamma)\, e^{-\gamma\xi_i}\frac{C_i+X_i+G_i}{Y_{ii}}\right]^{\frac{1}{\theta}} = \frac{1}{A_i}\left(\frac{1-\phi}{\phi}\frac{X_i}{L_i}\right)^{1-\phi}, \tag{A49}$$

$$\left(e^{-\xi_i}\frac{1-\gamma}{\gamma}\frac{Y_{ji}}{Y_{ii}}\right)^{-\frac{1}{\theta}} = \frac{A_i}{A_j}\left(\frac{X_i}{L_i}/\frac{X_j}{L_j}\right)^{\phi}. \tag{A50}$$

I show next that equilibrium allocation under PCP and the monetary policy that stabilizes marginal costs in every country satisfies these conditions and therefore, is efficient. First, note that with $\alpha = 0$ and constant marginal costs, both adjusting and non-adjusting firms keep their prices constant in producer currency at $P_{ii} = 1$, so that $P_{ij} = \mathcal{E}_{ji}$. Second, divide labor demand (A3) by demand for intermediate goods (A5) to get expression for real wage

$$\frac{W_i}{P_i} = \frac{1-\phi}{\phi}\frac{X_i}{L_i}.$$

Substitute it into labor supply to show that optimality condition (A48) is satisfied:

$$C_i = \frac{W_i}{P_i} = \frac{1-\phi}{\phi}\frac{X_i}{L_i}.$$

Third, using demand for local goods (A4)

$$Y_{ii} = (1-\gamma)\, e^{-\gamma\xi_i}\left(\frac{P_{ii}}{P_i}\right)^{-\theta}(C_i+X_i+G_i),$$

obtain

$$\left[(1-\gamma)\, e^{-\gamma\xi_i}\frac{C_i+X_i+G_i}{Y_{ii}}\right]^{\frac{1}{\theta}} = \frac{P_{ii}}{P_i}.$$

Combine stable marginal costs condition (A6) together with expression for real wage from above to show

$$P_i = A_i\left(\frac{W_i}{P_i}\right)^{-(1-\phi)} = A_i\left(\frac{1-\phi}{\phi}\frac{X_i}{L_i}\right)^{-(1-\phi)}.$$

Together, the last two equation imply that optimality condition (A49) is satisfied.

Fourth, divide demand for local and foreign goods

$$Y_{ji} = \gamma e^{(1-\gamma)\xi_i}\left(\frac{P_{ji}}{P_i}\right)^{-\theta}(C_i+X_i+G_i).$$

to show

$$\left(e^{-\xi_i}\frac{1-\gamma}{\gamma}\frac{Y_{ji}}{Y_{ii}}\right)^{-\frac{1}{\theta}} = \frac{P_{ji}}{P_{ii}} = \mathcal{E}_{ij}.$$

Substitute expression for P_i from above into the risk-sharing condition (A7) to get equilibrium exchange rate:

$$\mathcal{E}_{ij} = \frac{C_iP_i}{C_jP_j} = \frac{A_i}{A_j}\left(\frac{X_i}{L_i}/\frac{X_j}{L_j}\right)^{\phi}.$$

Combining the last two equations, we get optimality condition (A50).

This completes the proof of efficiency of the allocation given that firms use PCP. I next show using the first-order approximation to the equilibrium system this is indeed the only equilibrium currency choice. Given $\alpha = 0$, the desired price of exporter from j to i in terms of currency k is

$$\tilde{p}_{ji} - e_{ik} = mc_j + e_k - e_j = e_k - e_j,$$

where the last equality follows from marginal costs targeting. It follows that PCP unambiguously dominates any other currency for both exports and domestic firms.

Finally, consider the optimal monetary policy. Complete risk sharing implies $e_{ij} = w_i - w_j$. With marginal costs fully stabilized and $\alpha = 0$, the price index is

$$p_i = \gamma \int_0^1 e_{ij} \mathrm{d}j = \gamma \left(w_i - w \right).$$

Substitute this expression into marginal costs to obtain

$$\left[1 - (1 - \gamma) \phi \right] w_i = a_i + \gamma \phi w.$$

Integrating across countries and substituting result back into the last equation, we get

$$w = \frac{1}{1 - \phi} a, \qquad w_i = \frac{1}{1 - (1 - \gamma) \phi} \left[a_i + \frac{\gamma \phi}{1 - \phi} a \right].$$

It follows that equilibrium exchange rates are $e_i = \frac{1}{1-(1-\gamma)\phi} a_i$. ■

A.9.2 Loss function

Kimball price index To economize on indices, consider a general price index for Kimball demand with demand shifters that is determined by the following system:

$$\int_0^1 \gamma_i e^{z_i} \Upsilon \left(h \left(D e^{x_i} \right) \right) \mathrm{d}i = 1,$$
$$\int_0^1 \gamma_i e^{z_i} h \left(D e^{x_i} \right) e^{x_i} \mathrm{d}i = e^d,$$

where x_i is the log-deviation of $\frac{DP_i}{P}$ from symmetric deterministic point with $P_i = P$ and some constant D, $\int_0^1 \gamma_i \mathrm{d}i = 1$ and z_i are demand shifters such that $\int_0^1 \gamma_i z_i \mathrm{d}i = 0$. Take the SOA to this system. Start

with the first equation:

$$\int_0^1 \gamma_i \Bigg[\Upsilon\left(h\left(D\right)\right) + \Upsilon'\left(h\left(D\right)\right) h'\left(D\right) Dx_i + \Upsilon\left(h\left(D\right)\right) \left(z_i + \frac{1}{2} z_i^2 \right) + \Upsilon'\left(h\left(D\right)\right) h'\left(D\right) Dx_i z_i$$
$$+ \frac{1}{2} \Big(\frac{d\Upsilon'\left(h\left(X\right)\right)}{dX} h'\left(D\right) D^2 + \Upsilon'\left(h\left(D\right)\right) h''\left(D\right) D^2 + \Upsilon'\left(h\left(D\right)\right) h'\left(D\right) D \Big) x_i^2 \Bigg] \mathrm{d}i = 1.$$

From the properties of the functions, we have $\Upsilon\left(h\left(D\right)\right) = 1$, $\Upsilon'\left(h\left(D\right)\right) = D$ and $\frac{d\Upsilon'(h(X))}{dX} = \frac{dX}{dX} = 1$. From the definitions of elasticity and superelasticity of demand:

$$\theta\left(X\right) \equiv -h'\left(X\right) \frac{X}{h\left(X\right)} \;\Rightarrow\; h'\left(X\right) = -\theta\left(X\right) \frac{h\left(X\right)}{X},$$

$$\varepsilon\left(X\right) \equiv \frac{d \log \left(-h'\left(X\right) \frac{X}{h(X)} \right)}{d \log X} = h''\left(X\right) \frac{X}{h'\left(X\right)} + 1 + \theta\left(X\right) \;\Rightarrow\; h''\left(X\right) = \left(\theta\left(X\right) + 1 - \varepsilon\left(X\right)\right) \frac{\theta\left(X\right) h\left(X\right)}{X^2}.$$

Substitute these equalities into the SOA:

$$\int_0^1 \gamma_i \left[-\theta D x_i + \frac{1}{2} \left(-\theta D + \left(\theta + 1 - \varepsilon\right) \theta D - \theta D \right) x_i^2 + z_i + \frac{1}{2} z_i^2 - \theta D x_i z_i \right] \mathrm{d}i = 0$$

or equivalently,

$$\int_0^1 \gamma_i \left[x_i + \frac{1}{2} \left(1 - \theta + \varepsilon\right) x_i^2 + x_i z_i - \frac{1}{2\theta D} z_i^2 \right] \mathrm{d}i = 0.$$

Consider next the second equation of the system determining price indices:

$$\int_0^1 \gamma_i \left[h\left(D\right) D + \left(h'\left(D\right) D^2 + h\left(D\right) D \right) x_i + \frac{1}{2} \left(h''\left(D\right) D^3 + 3h'\left(D\right) D^2 + h\left(D\right) D \right) x_i^2 \right] \mathrm{d}i,$$
$$+ \int_0^1 \gamma_i \left[h\left(D\right) D \left(z_i + \frac{1}{2} z_i^2 \right) + \left(h'\left(D\right) D^2 + h\left(D\right) D \right) x_i z_i \right] \mathrm{d}i = D \left[1 + d + \frac{1}{2} d^2 \right].$$

Substitute steady-state values:

$$\int_0^1 \gamma_i \left[\left(1 - \theta\right) x_i + \frac{1}{2} \left(\left(1 - \theta\right)^2 - \varepsilon\theta \right) x_i^2 + \left(1 - \theta\right) x_i z_i + \frac{1}{2} z_i^2 \right] \mathrm{d}i = d + \frac{1}{2} d^2.$$

Multiple the first equation by $1 - \theta$ and subtract from the second one. Assume for simplicity that $D = \frac{\theta - 1}{\theta}$, which is true for CES and Klenow-Willis aggregator. This helps with demand shifters z_i, but does not matter for price terms:

$$-\frac{1}{2} \varepsilon \int_0^1 \gamma_i x_i^2 \mathrm{d}i = d + \frac{1}{2} d^2.$$

Substitute next the definition of x_i into the system of equations:

$$\int_0^1 \gamma_i \left[(d + p_i - p) + \frac{1}{2}(1 - \theta + \varepsilon)(d + p_i - p)^2 + (d + p_i - p) z_i - \frac{1}{2(\theta - 1)} z_i^2 \right] \mathrm{d}i = 0,$$

$$-\frac{1}{2}\varepsilon \int_0^1 \gamma_i (d + p_i - p)^2 \, \mathrm{d}i = d + \frac{1}{2} d^2.$$

Note that to the FOA $d = 0$, which implies by substitution that all second-order terms with d are zero. Under CES assumption, $\varepsilon = 0$, so that $d = 0$ to the SOA as well as to the first one.

Lemma A10 *The SOA to the Kimball price index is*

$$\int_0^1 \gamma_i \left[(p_i - p) + \frac{1}{2}(1 - \theta)(p_i - p)^2 + (p_i - p) z_i + \frac{1}{2(1 - \theta)} z_i^2 \right] \mathrm{d}i = 0,$$

$$-\frac{1}{2}\varepsilon \int_0^1 \gamma_i (p_i - p)^2 \, \mathrm{d}i = d.$$

Consider next the SOA to the relative demand $V_i \equiv e^{z_i} h(e^{x_i})$:

$$\begin{aligned} v_i + \frac{1}{2} v_i^2 &= h'(D) D x_i + \frac{1}{2}\left(h''(D) D^2 + h'(D) D \right) x_i^2 + h'(D) D x_i z_i + h(D) \left(z_i + \frac{1}{2} z_i^2 \right) \\ &= -\theta x_i + \frac{1}{2}(\theta - \varepsilon) \theta x_i^2 - \theta x_i z_i + z_i + \frac{1}{2} z_i^2. \end{aligned}$$

Therefore, using result from Lemma A10

$$\begin{aligned} \int_0^1 \gamma_i \left(v_i + \frac{1}{2} v_i^2 \right) \mathrm{d}i &= -\theta \int_0^1 \gamma_i \left[(d + p_i - p) + \frac{1}{2}(\varepsilon - \theta)(d + p_i - p)^2 + (d + p_i - p) z_i - \frac{1}{2\theta} z_i^2 \right] \mathrm{d}i \\ &= \frac{\theta}{2} \int_0^1 \gamma_i (p_i - p)^2 \, \mathrm{d}i - \frac{1}{2} \frac{1}{\theta - 1} \int_0^1 \gamma_i z_i^2 \mathrm{d}i. \end{aligned}$$

Lemma A11 *The sum of SOA of relative demand is*

$$\int_0^1 \gamma_i \left(v_i + \frac{1}{2} v_i^2 \right) \mathrm{d}i = \frac{\theta}{2} \int_0^1 \gamma_i (p_i - p)^2 \, \mathrm{d}i - \frac{1}{2(\theta - 1)} \int_0^1 \gamma_i z_i^2 \mathrm{d}i.$$

Labor market and intermediates Both labor demand and labor supply equations are exact in logs:

$$c_i = w_i - p_i, \qquad l_i = -\phi(w_i - p_i) - a_i + y_i.$$

Demand for intermediate goods is also exact in logs

$$x_i = y_i - a_i + (1 - \phi)(w_i - p_i).$$

The sum of final and intermediate demand is therefore,

$$(1-\phi)\left(c_i+\frac{1}{2}c_i^2\right)+\phi\left(x_i+\frac{1}{2}x_i^2\right)=\left(1-\phi^2\right)(w_i-p_i)+\frac{1}{2}\left[(1-\phi)+\phi(1-\phi)^2\right](w_i-p_i)^2$$
$$-\phi(1-\phi)(w_i-p_i)a_i+\phi\left[(1-\phi)(w_i-p_i)-a_i\right]y_i+\phi\left(y_i+\frac{1}{2}y_i^2\right)-\phi\left(a_i-\frac{1}{2}a_i^2\right).$$

Goods market The market clearing condition in tradable sector of country i can be written as

$$Y_i=(1-\gamma)\int_0^1 e^{-\gamma\xi_i}h\left(\frac{D_iP_{ii}(\omega)}{P_i}\right)\mathrm{d}\omega\left(C_i+X_i+G_i\right)+\gamma\int_0^1\int_0^1 e^{(1-\gamma)\xi_j}h\left(\frac{D_jP_{ij}(\omega)}{P_j}\right)\mathrm{d}\omega\left(C_j+X_j+G_j\right)\mathrm{d}j$$
$$\equiv(1-\gamma)\int_0^1\left(V_{ii\omega}C_i+V_{ii\omega}X_i+V_{ii\omega}G_i\right)\mathrm{d}\omega+\gamma\int_0^1\int_0^1\left(V_{ij\omega}C_j+V_{ij\omega}X_j+V_{ij\omega}G_j\right)\mathrm{d}\omega\mathrm{d}j.$$

The SOA to this equation is

$$y_i+\frac{1}{2}y_i^2=\left[(1-\gamma)\int_0^1\left(v_{ii\omega}+\frac{1}{2}v_{ii\omega}^2\right)\mathrm{d}\omega+\gamma\int_0^1\int_0^1\left(v_{ij\omega}+\frac{1}{2}v_{ij\omega}^2\right)\mathrm{d}\omega\mathrm{d}j\right]$$
$$+(1-\gamma)\left[(1-\phi)\left(c_i+\frac{1}{2}c_i^2\right)+\phi\left(x_i+\frac{1}{2}x_i^2\right)+\left(g_i+\frac{1}{2}g_i^2\right)\right]$$
$$+\gamma\int_0^1\left[(1-\phi)\left(c_j+\frac{1}{2}c_j^2\right)+\phi\left(x_j+\frac{1}{2}x_j^2\right)+\left(g_j+\frac{1}{2}g_j^2\right)\right]\mathrm{d}j$$
$$+\left[(1-\gamma)\int_0^1 v_{ii\omega}\mathrm{d}\omega\left((1-\phi)c_i+\phi x_i+g_i\right)+\gamma\int_0^1\int_0^1 v_{ij\omega}\mathrm{d}\omega\left((1-\phi)c_j+\phi x_j+g_j\right)\mathrm{d}j\right].$$

Integrate market clearing conditions across countries:

$$\int_0^1\left(y_i+\frac{1}{2}y_i^2\right)\mathrm{d}i=\int_0^1\left[(1-\gamma)\int_0^1\left(v_{ii\omega}+\frac{1}{2}v_{ii\omega}^2\right)\mathrm{d}\omega+\gamma\int_0^1\int_0^1\left(v_{ji\omega}+\frac{1}{2}v_{ji\omega}^2\right)\mathrm{d}\omega\mathrm{d}j\right]\mathrm{d}i$$
$$+\int_0^1\left[(1-\phi)\left(c_i+\frac{1}{2}c_i^2\right)+\phi\left(x_i+\frac{1}{2}x_i^2\right)+\left(g_i+\frac{1}{2}g_i^2\right)\right]\mathrm{d}i$$
$$+\int_0^1\left[(1-\gamma)\int_0^1 v_{ii\omega}d\omega+\gamma\int_0^1\int_0^1 v_{ji\omega}\mathrm{d}\omega\mathrm{d}j\right]\left((1-\phi)c_i+\phi x_i+g_i\right)\mathrm{d}i,$$

where I changed the order of integrations.

According to Lemma A11, $(1-\gamma)\int_0^1 v_{ii\omega}\mathrm{d}\omega\gamma\int_0^1\int_0^1 v_{ji\omega}\mathrm{d}\omega\mathrm{d}j$ is of the second order and therefore, the last term is zero in the SOA. Substitute the result from the proposition into the first term:

$$\int_0^1\left(y_i+\frac{1}{2}y_i^2\right)\mathrm{d}i=\frac{\theta}{2}\int_0^1\sigma_{Pi}^2\mathrm{d}i+\int_0^1\left[(1-\phi)\left(c_i+\frac{1}{2}c_i^2\right)+\phi\left(x_i+\frac{1}{2}x_i^2\right)+\left(g_i+\frac{1}{2}g_i^2\right)\right]\mathrm{d}i-\frac{\gamma(1-\gamma)}{2(\theta-1)}\int_0^1\xi_i^2\mathrm{d}i,$$

where σ_{Pi}^2 denotes dispersion of prices in region i for brevity. Substitute next the expression for consumption and intermediate demand to obtain

$$\int_0^1\left(y_i+\frac{1}{2}y_i^2\right)\mathrm{d}i=\int_0^1\left[(1+\phi)(w_i-p_i)+\frac{1}{2}\left[1+\phi(1-\phi)\right](w_i-p_i)^2-\phi(w_i-p_i)a_i\right]\mathrm{d}i$$

$$+\int_0^1 \left[\frac{\theta}{2}\frac{1}{1-\phi}\sigma_{Pi}^2+\phi\left[(w_i-p_i)-\frac{1}{1-\phi}a_i\right]y_i-\frac{\phi}{1-\phi}(a_i-\frac{1}{2}a_i^2)+\frac{1}{1-\phi}\left(g_i+\frac{1}{2}g_i^2\right)-\frac{\gamma\left(1-\gamma\right)}{2\left(\theta-1\right)\left(1-\phi\right)}\xi_i^2\right]\mathrm{d}i.$$

Loss function The preferences in country i are given by

$$U_i=\log C_i-L_i.$$

The second-order approximation (SOA) to the objective function:

$$U_i=\log C-L+c_i-L\left(l_i+\frac{1}{2}l_i^2\right).$$

Use steady-state values $C=L=1$ and suppress a constant term:

$$u_i=c_i-l_i-\frac{1}{2}l_i^2.$$

Next, substitute in consumption and labor from labor market clearing condition:

$$u_i=\left(1+\phi\right)\left(w_i-p_i\right)+\left(a_i-\frac{1}{2}a_i^2\right)-\frac{1}{2}\phi^2\left(w_i-p_i\right)^2-\phi\left(w_i-p_i\right)a_i-\left(y_i+\frac{1}{2}y_i^2\right)+\left[\phi\left(w_i-p_i\right)+a_i\right]y_i.$$

Integrate across countries and use expression for total output from the goods market clearing to see several terms cancel out:

$$u=\int_0^1\left[\frac{1}{1-\phi}\left(a_i-\frac{1}{2}a_i^2\right)-\frac{1}{2}\left(1+\phi\right)\left(w_i-p_i\right)^2+\frac{1}{1-\phi}a_iy_i\right.$$
$$\left.-\frac{\theta}{2}\frac{1}{1-\phi}\sigma_{Pi}^2-\frac{1}{1-\phi}\left(g_i+\frac{1}{2}g_i^2\right)+\frac{\gamma\left(1-\gamma\right)}{2\left(\theta-1\right)\left(1-\phi\right)}\xi_i^2\right]\mathrm{d}i.$$

Suppress exogenous terms to simplify the expression:

$$u=\int_0^1\left[-\frac{1}{2}\left(1+\phi\right)\left(w_i-p_i\right)^2-\frac{\theta}{2}\frac{1}{1-\phi}\sigma_{Pi}^2+\frac{1}{1-\phi}a_iy_i\right]\mathrm{d}i.$$

The FOA to the output of an individual country in (A30) implies that the price terms in y_i are

$$y_i=\frac{\gamma\theta}{1-\left(1-\gamma\right)\phi}\left[\left(p_i^I-p_i\right)-\left(p_i^E-p\right)\right]+\frac{\left(1-\gamma\right)\left(1-\phi^2\right)}{1-\left(1-\gamma\right)\phi}\left(w_i-p_i\right)+\frac{\gamma\left(1+\phi\right)}{1-\left(1-\gamma\right)\phi}\left(w-p\right).$$

Substitute this equation and change the signs to obtain the loss function:

$$\mathcal{L} = \int_0^1 \left[\frac{1}{2}(1+\phi)(w_i - p_i)^2 + \frac{1}{1-\phi}\frac{\theta}{2}\sigma_{Pi}^2 - \frac{1}{1-\phi}\frac{\gamma\theta}{1-(1-\gamma)\phi}\left[\left(p_i^I - p_i\right) - \left(p_i^E - p\right)\right] a_i \right. \\ \left. - \frac{(1-\gamma)(1+\phi)}{1-(1-\gamma)\phi}(w_i - p_i)\, a_i \right] \mathrm{d}i + \frac{\gamma(1+\phi)}{1-(1-\gamma)\phi}\frac{1}{1-\phi}(w-p)\, a. \tag{A51}$$

A.9.3 Optimal policy

Proof of Proposition 10 Note that there are no state variables in the model and therefore, the monetary policy affects allocation only in the first period. I therefore focus on one period. The fact that loss function contains only second-order terms implies that the FOA to pricing block and risk-sharing conditions is sufficient. Assuming that invoicing is symmetric across countries and using $\int_0^1 e_i \mathrm{d}i = 0$, the prices are:

$$\begin{aligned} p_{ji} &= \left(\mu^P + \mu^D\right) e_i - \mu^D e_0 - \mu^P e_j, \\ p_i^I &= \left(\mu^P + \mu^D\right) e_i - \mu^D e_0, \\ p_i^E &= -\mu^D e_0 - \mu^P e_i, \\ p_i &= \gamma\left(\mu^P + \mu^D\right) e_i - \gamma\mu^D e_0, \\ p &= -\gamma\mu^D e_0. \end{aligned}$$

In the absence of global shocks, $e_i = w_i$ from the international risk-sharing. It follows that

$$\sigma_{Pi}^2 = \gamma\int_0^1 p_{ji}^2 \mathrm{d}j - p_i^2 = \gamma\int_0^1 \left[\left(\mu^P + \mu^D\right) e_i - \mu^D e_0 - \mu^P e_j\right]^2 \mathrm{d}j - \gamma^2\left[\left(\mu^P + \mu^D\right) e_i - \mu^D e_0\right]^2.$$

Consider first the case of PCP. As long as prices are fully sticky, PCP allows the monetary authoritiesto implement the first-best:

$$w_i = e_i = \frac{1}{1-(1-\gamma)\phi} a_i.$$

The value of the loss function is then

$$\mathcal{L}^{PCP} = -\frac{1}{2}\left[\frac{\gamma(2-\gamma)\theta}{1-\phi} + (1-\gamma)^2(1+\phi)\right]\left(\frac{1}{1-(1-\gamma)\phi}\right)^2 \sigma_a^2.$$

The marginal costs are perfectly stabilized and the currency choice is determined by

$$\tilde{p}_{ji} + e_{ki} = e_k + (1-\alpha)(mc_j - e_j) + \alpha(p_i - e_i) = e_k - (1-\alpha)e_j - \alpha(1-\gamma)e_i \\ \Rightarrow \qquad \alpha \leq \frac{1}{2-\gamma}.$$

Suppose firms choose LCP. Then all prices are fully sticky in currency of destination and the loss

function simplifies to

$$\mathcal{L} = (1+\phi)\int_0^1 \left[\frac{1}{2}w_i^2 - \frac{1-\gamma}{1-(1-\gamma)\phi}w_i a_i\right]\mathrm{d}i,$$

and the FOC is

$$w_i = e_i = \frac{1-\gamma}{1-(1-\gamma)\phi}a_i.$$

The value of the loss function under the optimal policy is

$$\mathcal{L}^{LCP} = -\frac{1}{2}(1-\gamma)^2(1+\phi)\left(\frac{1}{1-(1-\gamma)\phi}\right)^2\sigma_a^2.$$

Thus, firms choose LCP based on

$$\tilde{p}_{ji} + e_{ki} = e_k + (1-\alpha)(mc_j - e_j) + \alpha(p_i - e_i) = \frac{1-\gamma}{1-(1-\gamma)\phi}\left[a_k - \frac{1-\alpha}{1-\gamma}a_j - \alpha a_i\right]$$
$$\Rightarrow \qquad \alpha \geq \frac{1}{2-\gamma}.$$

Assume next that firms choose DCP. Substitute prices into the loss function and exchange rates instead of wages:

$$\mathcal{L} = \int_0^1 \left[\frac{1}{2}(1+\phi)\left((1-\gamma)e_i + \gamma e_0\right)^2 - \frac{1}{1-\phi}\frac{\gamma(1-\gamma)\theta}{1-(1-\gamma)\phi}e_i a_i - \frac{(1-\gamma)(1+\phi)}{1-(1-\gamma)\phi}\left((1-\gamma)e_i + \gamma e_0\right)a_i\right]\mathrm{d}i$$
$$+ \frac{\gamma(1-\gamma)}{1-\phi}\frac{\theta}{2}\int_0^1 (e_i - e_0)^2\,\mathrm{d}i.$$

Integrate and use exchange rate normalization to rewrite it as

$$\mathcal{L} = (1-\gamma)\left[\frac{\gamma\theta}{1-\phi} + (1-\gamma)(1+\phi)\right]\left[\frac{1}{2}\int_0^1 e_i^2\mathrm{d}i - \frac{1}{1-(1-\gamma)\phi}\int_0^1 e_i a_i\mathrm{d}i\right] + \frac{1}{2}(1+\phi)\gamma^2 e_0^2 + \frac{\gamma(1-\gamma)}{1-\phi}\frac{\theta}{2}e_0^2.$$

The FOC with respect to e_i implies

$$e_i = \frac{1}{1-(1-\gamma)\phi}a_i$$

and the optimal value of dollar is $e_0 = 0$. The value of the loss function is then

$$\mathcal{L}^{DCP} = -\frac{1}{2}\left[\frac{\gamma(1-\gamma)\theta}{1-\phi} + (1-\gamma)^2(1+\phi)\right]\left(\frac{1}{1-(1-\gamma)\phi}\right)^2\sigma_a^2.$$

Exporters choose DCP based on

$$\tilde{p}_{ji} + e_{ki} = e_k + (1-\alpha)(mc_j - e_j) + \alpha(p_i - e_i) = e_k - (1-\alpha)e_j - \alpha(1-\gamma)e_i$$
$$\Rightarrow \qquad \frac{1}{2} \leq \alpha \leq \frac{1}{2(1-\gamma)}.$$

Thus, for given parameter values, $\mathcal{L}^{PCP} < \mathcal{L}^{DCP} < \mathcal{L}^{LCP}$. ■

Proof of Proposition 11 Consider the case when the monetary policy of U.S. is exogenous. Since U.S. has zero mass, the objective function of the global planner does not change. Since policy across other countries can be correlated in this case, we obtain $e_i = w_i - w$ (including $e_0 = w_0 - w$) under exchange rate normalization $\int_0^1 e_i \mathrm{d}i = 0$ and $w = \int_0^1 w_i \mathrm{d}i$.

The optimal policy for the U.S. does not depend on currency choice or monetary policy of other countries and implies $w_0 = a_0$ (since there are no intermediate goods in non-tradable sector). Under PCP and LCP, the dollar exchange rate plays no role and the optimal policy and currency choice are the same as under cooperative policy. Assume that firms choose DCP. Substitute prices into the loss function and exchange rates instead of wages (note that e_0 is a function of endogenous w):

$$\begin{aligned}\mathcal{L} = \int_0^1 \Big[& \frac{1}{2}(1+\phi)\left((1-\gamma)(e_i+w)+\gamma w_0\right)^2 - \frac{1}{1-\phi}\frac{\gamma(1-\gamma)\theta}{1-(1-\gamma)\phi} e_i a_i \\ & - \frac{(1-\gamma)(1+\phi)}{1-(1-\gamma)\phi}\left((1-\gamma)(e_i+w)+\gamma w_0\right) a_i + \frac{\gamma(1-\gamma)}{1-\phi}\frac{\theta}{2}\left(e_i+w-w_0\right)^2 \Big] \mathrm{d}i.\end{aligned}$$

The FOC with respect to e_i and w imply

$$\begin{aligned} e_i &= \frac{1}{1-(1-\gamma)\phi} a_i, \\ w &= \frac{\gamma\left[\theta - (1-\phi^2)\right]}{\gamma\theta + (1-\gamma)(1-\phi^2)} w_0, \\ e_0 &= \frac{1-\phi^2}{\gamma\theta + (1-\gamma)(1-\phi^2)} w_0, \\ w_i &= \frac{1}{1-(1-\gamma)\phi} a_i + \frac{\gamma\left[\theta - (1-\phi^2)\right]}{\gamma\theta + (1-\gamma)(1-\phi^2)} w_0. \end{aligned}$$

The monetary policies w_i are therefore positively correlated across countries (including the U.S.). In addition, the volatility of exchange rates against dollar are lower under DCP than PCP:

$$\begin{aligned} e_{i0}^{DCP} &= \frac{1}{1-(1-\gamma)\phi} a_i - \frac{1-\phi^2}{\gamma\theta + (1-\gamma)(1-\phi^2)} a_0, \\ e_{i0}^{PCP} &= \frac{1}{1-(1-\gamma)\phi} a_i - a_0. \end{aligned}$$

The loss function is

$$\mathcal{L} = -\frac{1}{2}\frac{1-\gamma}{1-\phi}\left[\gamma\theta + (1-\gamma)\left(1-\phi^2\right)\right]\left(\frac{1}{1-(1-\gamma)\phi}\right)^2 \sigma_a^2 + \frac{1}{2}\frac{\gamma(1+\phi)\theta}{\gamma\theta + (1-\gamma)(1-\phi^2)}\sigma_{w0}^2,$$

which is higher than losses under PCP by the second term. Firms' currency choice is based on

$$
\begin{aligned}
&p_i = \gamma\left(e_i - e_0\right), \\
&mc_i = \phi p_i + (1-\phi) w_i - a_i = \frac{\gamma(1-\phi)[\theta-(1+\phi)]}{\gamma\theta + (1-\gamma)(1-\phi^2)} w_0 = \gamma\left[\frac{\theta}{1+\phi} - 1\right] e_0, \\
&\tilde{p}_{ji} + e_{ki} = e_k - (1-\alpha) e_j - \alpha(1-\gamma) e_i - \gamma\left[1 - \frac{(1-\alpha)\theta}{1+\phi}\right] e_0,
\end{aligned}
$$

which given uncorrelated exchange rates implies that DCP is always optimal in the limit $\gamma, \alpha \to 1$. ■

A.10 Extensions

A.10.1 Currency choice of domestic firms

Define the global currency pricing (GCP) equilibrium as the one in which all firms in the world (including domestic ones) use dollars for invoicing. In contrast, in DCP equilibrium only exporters price in dollars, while domestic firms use local currency.

Proposition A5 *Assume that domestic firms optimally choose the currency of invoicing and $n = 0$. Then*

1. *in the flexible price limit $\lambda \to 1$, the region of GCP is the subset of DCP, is non-empty as long as $\rho < 1$ and is increasing in γ, ϕ and α,*
2. *in the limit of fully rigid prices $\lambda \to 0$, the region of DCP is a subset of GCP.*

Proof As before, the import price index is

$$
p_i^I = \lambda\left[(1-\alpha)(\phi p + e_i) + \alpha p_i\right] + (1-\lambda)\left[\left(\mu^P + \mu^D\right) e_i - \mu^D e_0\right].
$$

Denote the currency choice of domestic firms with $\hat{\mu}$. Note that PCP and LCP coincide for domestic firms and therefore it is sufficient to focus on $\hat{\mu}^{DCP}$. The price index for local goods is therefore

$$
p_i^D = \lambda\left[(1-\alpha)\phi + \alpha\right] p_i + (1-\lambda)\hat{\mu}^D\left(e_i - e_0\right).
$$

Solve for the price index of individual country:

$$
p_i = \frac{\gamma\lambda(1-\alpha) + \gamma(1-\lambda)\left(\mu^P + \mu^D\right) + (1-\gamma)(1-\lambda)\hat{\mu}^D}{1 - \lambda\left(\alpha + (1-\alpha)(1-\gamma)\phi\right)} e_i - \frac{(1-\lambda)\left[\gamma\mu^D + (1-\gamma)\hat{\mu}^D\right]}{1 - \lambda\left(\alpha + (1-\alpha)\phi\right)} e_0.
$$

In the flexible price limit, the currency of invoicing of both exporters and domestic firms has no effect on equilibrium prices. Therefore, the aggregate price index and the currency choice of exporters remain the same as in the baseline model:

$$
p_i = \frac{\gamma}{1 - (1-\gamma)\phi} e_i.
$$

The currency choice of domestic producers is determined by

$$\tilde{p}_{ii} + e_{ki} = e_k - \frac{(1-\phi)(1-\gamma\alpha)}{1-(1-\gamma)\phi} e_i.$$

The volatility of the optimal price expressed in domestic currency and dollars is therefore

$$V^{PCP/LCP} = \left[1 - \frac{(1-\phi)(1-\gamma\alpha)}{1-(1-\gamma)\phi}\right]^2, \qquad V^{DCP} = \rho + \left[\frac{(1-\phi)(1-\gamma\alpha)}{1-(1-\gamma)\phi}\right]^2.$$

It follows that local firms choose DCP if $2\frac{(1-\phi)(1-\gamma\alpha)}{1-(1-\gamma)\phi} < 1-\rho$, which is more likely when ρ is low and γ, ϕ and α are high. In particular, if $\rho < 1$, both exporters and domestic suppliers set prices in dollars in two limiting cases: $\phi \to 1$ and $\alpha, \gamma \to 1$.

Consider next the case with $\lambda > 0$. Start with the following observation: in the PCP (LCP) equilibrium in the baseline model domestic firms are also following PCP (LCP). Because of strategic complementarities, this gives these equilibria the highest chances, i.e. if PCP (LCP) equilibrium cannot be sustained when $\hat{\mu}^{DCP} = 0$, there is no way to support it with $\hat{\mu}^{DCP} = 1$. On the other hand, it might be easier to sustain the DCP equilibrium if domestic firms choose DCP. Indeed, $\hat{\mu}^{DCP} = 1$ increases both χ and χ_0 relative to the baseline model, which makes DCP more attractive for importers. A necessary and sufficient condition to sustain such equilibrium is however that domestic firms choose DCP. Since $e_i = e_j$ for local firms,

$$\tilde{p}_{ii} + e_{ki} = e_k - \left[(1-\alpha)(1-\phi\chi) + \alpha(1-\chi)\right] e_i - \left(\alpha + (1-\alpha)\phi\right)\chi_0 e_0.$$

It follows,

$$V^{PCP/LCP} = \left[1 - (1-\alpha)(1-\phi\chi) - \alpha(1-\chi)\right]^2 + \left(\alpha + (1-\alpha)\phi\right)^2 \chi_0^2 \rho,$$

$$V^{DCP} = \left[(1-\alpha)(1-\phi\chi) + \alpha(1-\chi)\right]^2 + \left[1 - \left(\alpha + (1-\alpha)\phi\right)\chi_0\right]^2 \rho.$$

DCP dominates local currency if

$$\left(\alpha + (1-\alpha)\phi\right)(\chi + \rho\chi_0) > \frac{1+\rho}{2},$$

where

$$\chi + \rho\chi_0 = \frac{\gamma\lambda(1-\alpha) + (1-\lambda)}{1 - \lambda\left(\alpha + (1-\alpha)(1-\gamma)\phi\right)} + \frac{(1-\lambda)\rho}{1 - \lambda\left(\alpha + (1-\alpha)\phi\right)}.$$

Consider the limit of fully rigid prices $\lambda = 0$: $\chi + \rho\chi_0 = 1 + \rho$ and therefore, condition simplifies to $\alpha + (1-\alpha)\phi > \frac{1}{2}$, which is always satisfied for $\alpha > 0.5$ or $\phi > 0.5$. At the same time, DCP dominates PCP and LCP for importers if

$$(1-\alpha)\gamma\phi + \alpha(1+\rho\gamma) > \frac{1+\rho}{2},$$

$$(1-\alpha)(1-\gamma\phi)\rho+\alpha(1-\gamma)(1+\rho)<\frac{1+\rho}{2}.$$

I argue next that GCP equilibrium exists for these parameters as well. Prove by contradiction. Condition for GCP does not depend on γ, while conditions for DCP relax as γ becomes larger. Therefore, take $\gamma=1$

$$(1-\alpha)\phi>\left(\frac{1}{2}-\alpha\right)(1+\rho),$$

$$(1-\alpha)\phi>\left(\frac{1}{2}-\alpha\right)-\frac{1}{2\rho}.$$

If $\alpha>0.5$, the GCP equilibrium exists and we arrive to contradiction. If $\alpha<0.5$, then conditions are relaxed for $\rho=1$

$$(1-\alpha)\phi>(1-2\alpha),$$

$$(1-\alpha)\phi>-\alpha.$$

The second condition is always satisfied, while the first one implies $\alpha+(1-\alpha)\phi>1-\alpha>0.5$, so GCP equilibrium exists and we again arrive to contradiction. ■

A.10.2 Monetary and productivity shock

Proposition A6 *Assume that monetary shocks follow random walk and that asset markets are either complete or consist of one bond. Then*

1. *if $\lambda\to1$, $\rho<1$, DCP is the only possible equilibrium in the limit $\sigma_m^2\to\infty$,*
2. *if $n=0$, a proportional increase in volatility of unexpected monetary shocks in all countries expends the DCP region.*

Proof Substitute the aggregate price index (A17) into the desired price (A43) to obtain

$$\tilde{p}_{ji}+e_{ki}=e_k-(1-\alpha)\left[(1-\phi\chi)e_j-(1-\phi+\phi\chi_w)w_j\right]-\alpha\left[(1-\chi)e_i-\chi_w w_i\right]-(\alpha+(1-\alpha)\phi)\left(\chi_0 e_0-\chi_{\bar{w}}nw_0\right).$$

Consider first the limiting case when monetary shocks dominate any other shocks in the economy, and according to (A38) and (A41), equilibrium exchange rate is $e_i\approx w_i$. This implies

$$\tilde{p}_{ji}+e_{ki}=e_k-(1-\alpha)\phi(1-\chi-\chi_w)e_j-\alpha(1-\chi-\chi_w)e_i-(\alpha+(1-\alpha)\phi)(\chi_0-\chi_{\bar{w}}n)e_0,$$

where

$$1-\chi-\chi_w=\frac{(1-\lambda)\left(1-\gamma\left(\mu^P+\mu^D\right)\right)}{1-\lambda(\alpha+(1-\gamma)(1-\alpha)\phi)}\geq0,$$

$$\chi_0-\chi_{\bar{w}}n=\frac{\gamma(1-\lambda)}{1-\lambda(\alpha+(1-\gamma)(1-\alpha)\phi)}\left[\left(n\mu^P+\mu^D\right)+\frac{\lambda(1-\alpha)\phi\left[n+\gamma(1-n)\mu^D\right]}{1-\lambda(\alpha+(1-\alpha)\phi)}\right]\geq0.$$

In the flexible price limit $\lambda\to1$, both coefficients converge to zero and $\tilde{p}_{ji}+e_{ki}=e_k$. While firms are indifferent between all currencies when $\rho=1$, an arbitrary small volatility advantage is sufficient to

guarantee that DCP is used for any values of other parameters. More generally, cov $(w_i, e_i) > 0$ under both complete markets and one internationally traded bond. Therefore, the effective weight of producer and local currency in the optimal price goes down as the volatility of monetary shocks increases, and exporters are more likely to choose DCP. ■

The case of productivity shocks in tradable sector are more nuanced. In the baseline model with complete asset markets, log-linear preferences and exogenous process for w_i, the productivity shocks are uncorrelated with movements in exchange rates and therefore have no effects on exporters' currency choice. When asset markets are incomplete, on the other hand, the correlation between unexpected changes in TFP and exchange rates can have either sign. When productivity shocks are highly persistent, the wealth effect dominates and nominal exchange rate appreciates in response to positive productivity shock (for details see Corsetti, Dedola, and Leduc 2008, Itskhoki and Mukhin 2017). When productivity in country of origin or destination goes up, the desired price of exporter $\tilde{p}_{ji}$ falls because of lower marginal costs and competitor prices. Invoicing in producer or local currency is less attractive in this case, and the chances of DCP go up.

A.10.3 Inflation targeting

Consider the case when monetary authorities stabilize consumer price index rather than nominal wages. Assuming away productivity shocks in both sectors, we get

$$p_i^C = \eta p_i + (1-\eta)\, p_i^N = \eta\left(\chi e_i - \chi_0 e_0 + \chi_w w_i - \chi_{\bar{w}} w\right) + (1-\eta)\,\lambda w_i.$$

Since monetary policy is correlated across countries in this case due to common e_0 and w terms, the equilibrium exchange rates are given by

$$e_i = w_i - w + \psi_i.$$

Sibstitute this expression into the CPI index:

$$p_i^C = \eta\left(\chi\left(w_i - w + \psi_i\right) - \chi_0\left(w_0 - w + \psi_0\right) + \chi_w w_i - \chi_{\bar{w}} w\right) + (1-\eta)\,\lambda w_i.$$

Integrating this equality across i and using the policy rule in the U.S. and other countries, obtain the system

$$\left[\eta\left(\chi_0 + \chi_w - \chi_{\bar{w}}\right) + (1-\eta)\,\lambda\right] w = \eta\chi_0\left(w_0 + \psi_0\right),$$

$$\left[\eta\left(\chi - \chi_0 + \chi_w\right) + (1-\eta)\,\lambda\right] w_0 + \eta\left(\chi - \chi_0\right)\psi_0 = \eta\left(\chi - \chi_0 + \chi_{\bar{w}}\right) w,$$

which can be solved to obtain

$$\left[\eta\left(\chi_0 + \chi_w - \chi_{\bar{w}}\right) + (1-\eta)\,\lambda - \frac{\eta^2\chi_0\left(\chi - \chi_0 + \chi_{\bar{w}}\right)}{\eta\left(\chi - \chi_0 + \chi_w\right) + (1-\eta)\,\lambda}\right] w = \eta\chi_0 \frac{\eta\chi_w + (1-\eta)\,\lambda}{\eta\left(\chi - \chi_0 + \chi_w\right) + (1-\eta)\,\lambda}\psi_0,$$

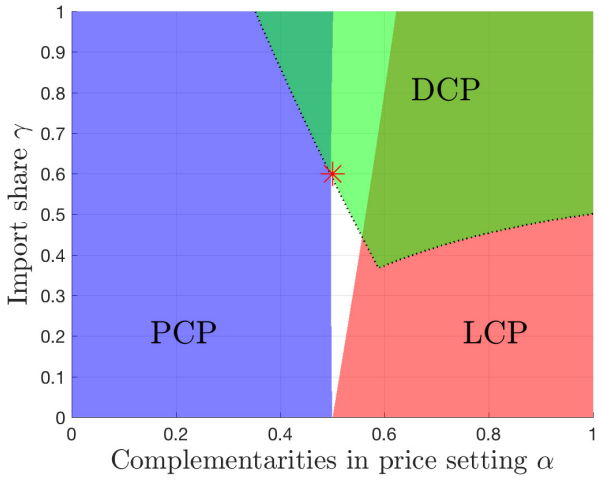


Figure A6: Currency choice under inflation targeting

Note: the figure shows symmetric equilibria when monetary authorities in all countries stabilize CPI and the only shocks are financial ones. The parameter values are taken from the baseline calibration.

$$w_0 = \frac{\eta\left(\chi - \chi_0 + \chi_{\bar{w}}\right)}{\eta\left(\chi - \chi_0 + \chi_w\right) + (1-\eta)\lambda} w - \frac{\eta\left(\chi - \chi_0\right)}{\eta\left(\chi - \chi_0 + \chi_w\right) + (1-\eta)\lambda} \psi_0.$$

Denote solution to this system with $w = k\psi_0$ and $w_0 = k_0\psi_0$ and substitute it into CPI of individual country to solve for w_i

$$w_i = \eta \frac{\chi_0\left(k_0 + 1\right) + \left(\chi + \chi_0 + \chi_{\bar{w}}\right)k}{\eta\left(\chi + \chi_w\right) + (1-\eta)\lambda} \psi_0 - \frac{\eta\chi}{\eta\left(\chi + \chi_w\right) + (1-\eta)\lambda} \psi_i \equiv l_0\psi_0 + l\psi_i.$$

The equilibrium values of exchange rate e_i then follows from the risk-sharing condition. Given these values, one can then solve the currency choice problem of individual firm that minimizes

$$\tilde{p}_{ji} + e_{ki} = e_k + (1-\alpha)\left(\phi p_j + (1-\phi) w_j - e_j\right) + \alpha\left(p_i - e_i\right)$$

$$= e_k - (1-\alpha)\left(\left(1 - \phi\chi\right) e_j - \left(1 - \phi + \phi\chi_w\right) w_j\right) - \alpha\left(\left(1-\chi\right) e_i - \chi_w w_i\right) - \left(\alpha + (1-\alpha)\phi\right)\left(\chi_0 e_0 + \chi_{\bar{w}} w\right).$$

Figure A6 shows the resulting symmetric equilibria under the baseline calibration.