Abstract

This paper analyzes the Eurozone financial crisis through the lens of sovereign bond liquidity. Using novel data, I show that repo haircuts on peripheral government bonds sharply increased during the crisis reducing their liquidity and amplifying the rise in their yields. Impulse response functions of SVAR and local projection models exhibit a significant increase of Irish government bond yield following a funding liquidity shock identified with a combination of narrative method and high frequency identification approach. Further, I study the systemic impact of liquidity shock on asset prices and business cycle in a general equilibrium model with financial friction. The model predicts a drop in economic activity and deflation, and confirms the rise in the required returns of illiquid government bonds. Liquidity facilities can alleviate the contractionary effect of the liquidity shock.

JEL codes: E44, E58, G12

Key words: Repo haircuts, liquidity shock, funding constraint
1 Introduction

“Italian bonds are in the perfect storm at the moment. Real money investors are running away and those using Italian bonds to finance will also be clearing the desk now”.

Financial Times 9th November 2011, “LCH Clearnet SA raises margin on Italian bonds”

Why did countries in the periphery of the Eurozone (Greece, Ireland, Italy, Portugal and Spain) pay higher interest rates on public debt than countries in the core during the recent financial crisis? Since the creation of a monetary union has integrated the sovereign debt markets and eliminated the exchange rate risk, two main factors may explain this: credit risk and liquidity.

Credit risk derives from the government’s probability of default. Weak fiscal and macroeconomic fundamentals of a country induce investors to ask higher compensation for holding government debt because of the possibility of suffering losses. In addition, fears of default and self-validating expectations may also drive up yields of government securities issued by those countries that cannot print new currency as predicted by Calvo (1988), Cole and Kehoe (2000) and Corsetti and Dedola (2012).

Liquidity is a broad concept that the traditional theories of Keynes (1936) and Hicks (1962) refer to as the capacity of an asset to store wealth and protect its owner from a shortage of revenue providing a means to smooth consumption. Modern corporate finance distinguishes between market liquidity and funding liquidity. Market liquidity is the facility to obtain cash by selling an asset; when frictions in the secondary market make it difficult to find a buyer the market liquidity is low and the price of the asset deviates from its fundamentals.¹

¹Favero, Pagano, and von Thadden (2010) and Manganelli and Wolswijk (2009) disentangle
This paper instead focuses on the role of funding liquidity, which is the ease with which investors can obtain funding (Brunnermeier and Pedersen (2009)). As they typically borrow against an asset, funding liquidity is considered as the ability of an asset to serve as collateral. I show that government bonds are the prime collateral securities in the European market of repurchase agreements (repos), which is becoming an essential source of funding for the banking system, especially since the onset of the crisis when the increase in counterparty credit risk led to a shift from the unsecured to the secured funding. This forced borrowing banks to post securities in the interbank market, whose value exceeds the loan by a certain amount, the haircut or initial margin, which is the metric that I employ to measure funding liquidity. Given the value of an asset, the lower the haircut the larger the amount of cash that the borrower can obtain by pledging the asset.

Prior to the crisis the perceived safety of government bonds made them good collateral to back banks’ debt, their repo haircuts were low and their function as a medium of exchange compressed their yields. Nevertheless, I show that during the crisis the emergence of sovereign risk led to rises in repo haircuts on peripheral government bonds, reducing their liquidity and capacity to serve as collateral for secured borrowing. The funding of investors shrank along the lines of the mechanism emphasized by Gorton and Metrick (2012) for the US liquidity crisis in 2007 - 2008, leading to a drop in investment. In order to reduce the contraction of their funding, leveraged investors shifted their portfolios towards more liquid bonds of the core with lower haircuts, contributing to the widening of the yield spreads.

\[ \text{the impact of credit risk and market liquidity risk on the evolution of European government bond yields.} \]

\[ ^2 \text{Gennaioli, Martin, and Rossi (2014) find empirical evidence that in a large panel of countries banks hold a sizable amount of government bonds because of their liquidity services.} \]

\[ ^3 \text{Banks could alternatively pledge government bonds for ECB refinancing operations, espe-} \]
In order to assess this conjecture, I analyze empirically the response of 10 year Irish government bond yield to rises in haircuts applied by LCH Clearnet Ltd, the largest European clearing house. I identify a funding liquidity shock via the narrative approach, by reading the the circulars published by LCH Clearnet Ltd concerning variations in haircuts, and the high frequency identification method to address the issue of simultaneity between news in financial markets and the haircut policy of the clearing house. The key identification strategy hinges upon the delay between the announcement and the implementation of changes in haircuts, which implies that haircuts respond to movements in financial variables with one lag, but variations in haircuts affect financial variables instantaneously. I also provide evidence that the funding liquidity shock is not anticipated by market participants.

Impulse responses of SVAR and local projection models exhibit a significant increase of government bond yield following a liquidity shock. These results suggest that the returns on a security incorporate a funding liquidity premium, in line with Bartolini, Hilton, Sundaresan, and Tonetti (2011) who find that differences in the collateral values across asset classes contributes to explain yield spreads in the US, and Ashcraft, Gărleanu, and Pedersen (2011) who show the inverse relation between haircuts in repos with the Federal Reserve and prices of the underlying collateral during the crisis.

Furthermore, I address the following question: How does a funding liquidity shock propagate to the real economy? Building on Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017), I explore this liquidity channel through a DSGE model with...
financial frictions calibrated for Ireland. Similarly to European banks, investors choose to hold sovereign bonds as a way to store liquidity for financing future investment. Since they cannot completely pledge the future returns of investment, the liquidity of their asset portfolio is crucial to determine the amount of investment that can be funded. Thus, even if the returns on public bonds are lower than those on private assets, investors can ease their funding constraint by borrowing against them. However, a liquidity shock calibrated to match the dynamic of repo haircut can suddenly reduce the amount of funding that investors can obtain by pledging government bonds and investment falls. In addition, nominal frictions and the zero lower bound (ZLB) prevent the real interest rate to decline and consumption also falls.

Several papers incorporate the liquidity friction proposed by Kiyotaki and Moore (2012) into general equilibrium models, such as Kurlat (2013), Bigio (2015), Shi (2015), Cui (2016), Ajello (2016), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017). They assume that privately issued assets are subject to a resaleability constraint which limits their liquidity, whereas government bonds are perfectly liquid. I depart from this assumption and introduce two types of government bonds with different degrees of liquidity, in the spirit of Hicks (1939, pag. 146): long-term bonds which are subject to a liquidity constraint and short-term bonds which are not. An exogenous liquidity shock is a tightening of the constraint on long-term

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4The model abstracts from the risk of sovereign default to focus on a pure liquidity channel. Bi (2012); Bi and Traum (2012); Bocola (2016) and Uribe (2006) introduce the probability of default in DSGE models.

5This echoes Hölström and Tirole (1998) who show that firms that cannot pledge any of their future income are willing to pay a premium on assets that are able to store liquidity and help them in state of liquidity shortage. In Aiyagari and McGrattan (1998) government debt enhances the liquidity of households by loosening borrowing constraints.

bonds, which increases the premium that investors are willing to pay for holding short-term bonds. While those papers interpret the liquidity shock as a change in market liquidity and a dry-up of liquidity in the secondary market, in this model it is equivalent to a rise in the repo haircut capturing a change in funding liquidity.

Finally, I analyze the impact of a policy experiment which consists of swapping illiquid government bonds for highly liquid papers (short-term debt or money) through direct purchases or collateralized loans in response to the liquidity shock. This intervention can be thought as the Expanded Asset Purchase Programme (APP) and 3 year Long Term Refinancing Operations (LTROs) implemented by the European Central Bank. The liquidity friction in long-term bonds breaks the irrelevance principle of Wallace (1981) and Eggertsson and Woodford (2003) for open market operations since the government exchanges liquid bonds for illiquid bonds, thus modifying the composition of aggregate portfolio holdings and mitigating the drop in consumption and investment.\(^7\)

The structure of the paper is as follows. Section 2 provides a picture of the European repo market and the funding liquidity of government bonds. Section 3 examines empirically the impact of rises in repo haircuts on government bond yield. Section 4 presents the model. Section 5 shows the calibration and the results of the numerical simulation and Section 6 concludes.

\(^7\)Although this intervention could be interpreted as the Operation Twist implemented by the Federal Reserve in 1961 and 2011, the objective of this policy is to modify the liquidity of the portfolios of the private sector rather than their maturity. Chen, Curdia, and Ferrero (2012) introduce limits to arbitrage and market segmentation between short-term and long-term bonds in the preferred habitat framework. Reis (2015) also evaluates the effect of Quantitative Easing, assuming that short-term bonds are more liquid than long-term bonds since they can be used as collateral in the interbank market together with reserves. In his model the unconventional monetary policy relaxes the constraint of banks by exchanging illiquid long-term bonds for liquid reserves.
2 Funding liquidity of government bonds

This section presents evidence of the importance of government bonds for the funding of European banks, since they are the main collateral securities of the repo market which is becoming essential for banks to meet their liquidity needs. Figure 1 exhibits the extraordinary expansion of the European repo market in the last decade as reported by the European Repo Market Surveys. Repos tripled in the run-up to the crisis and after a short contraction between 2008 and 2009 recovered to their pre-crisis level, reaching around €3 trillion. The size of the European repo market is therefore considerable and close to that of the US market, estimated to be about $4.4 trillion in 2009 based on the average daily amount outstanding of the primary dealers repo financing (See Acharya and Öncü (2012)).

Looking at banks’ balance sheets, Table 2 in the Appendix A.3 shows that repos are a considerable share of European banks’ funding, especially for the largest financial institutions accounting for between 9% and 14% of total liabilities, more than unsecured interbank deposits and long-term debt. Figure 2 compares the dynamics of secured and unsecured borrowing in interbank transactions using data from the European Money Market Survey of the European Central Bank. There is a massive shift of banks’ funding from the unsecured to the secured segment, in particular after the onset of the global financial crisis following the rise in counterparty credit risk. Furthermore, decomposing the repo market by types of arrangements we can observe that bilateral CCP-cleared repos steadily increased, while over-the-counter bilateral repos declined and the tri-party repos account for

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8See the Appendix A.1 and A.2 for the definitions employed for repo contracts and a description of the data sources.
Figure 1: European repo market (billions of euros)

Note: repos reported by banks which participated continuously in all the surveys (borrowing activity). Source: European repo survey (ICMA)

a little share of the market. The enhanced role of clearing houses increases the importance of the quality of collateral since they set repo haircuts as a function of the credit risk of securities. Thus, a change in the credit risk of collateral securities is reflected in variations in haircuts applied by clearing houses affecting funding conditions in the European repo market.

Concerning the collateral posted in repo transactions, Figure 3 shows that government bonds are the predominant securities, accounting for around 80% of the total collateral pool. This share was stable during the crisis and represents a structural characteristic of the European repo market, different to the US market where securities issued by the private sector account for a larger share (Krishnamurthy, Nagel, and Orlov (2014)). Looking at the composition of sovereign securities, German bonds are the largest share, although their supply is lower than French and Italian bonds (Eurostat (2013)). In addition, the share of Italian bonds dropped
substantially during the Eurozone crisis, from 10% in December 2010 to 7% in December 2011.

The collateral composition of the European repo market reflects not only the safety of securities but also their liquidity. Figure 4 shows the evolution of 10 year government bond yields of peripheral countries (Ireland, Portugal, Italy and Spain) and the haircuts applied on these securities by LCH Clearnet, the largest European clearing house. Following the rise in bond yields and sovereign risk, haircuts on Irish and Portuguese sovereign securities surged up to 80%, making those bonds almost completely illiquid. Italian and Spanish bonds also experienced rises in haircuts but these were more mitigated. However, it is worth noting that the increase in haircuts on Italian bonds is associated with their decrease in the share of collateral in the repo market (Figure 3).

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9See Figure 11 in Appendix A.1 for a comparison of the volume of repos cleared by the largest European clearing house

10In Armakola, Douady, Laurent, and Molteni (2016) we show that a similar pattern is observed
To summarize, European government bonds have become an essential liquid instrument for banks, especially after the onset of the financial crisis, since they are needed to pledge collateral securities as guarantee of repayment in order to borrow on the interbank market. Therefore, their value incorporates a premium reflecting their capacity to serve as collateral. The increase in repo haircuts reduced their liquidity premium and increased their required returns. Delatte, Fouquau, and Portes (2017) report the sell-off in Irish bonds during the crisis driven by higher collateral requirements. In the next section I investigate empirically this liquidity for haircuts applied by other major European clearing houses.

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11 From the *Financial Times*, November 10, 2010: “The dramatic sell-off in Irish bonds was driven by a fire sale of positions by market participants who were unable to meet collateral requirements enforced by LCH.Clearnet - one of the biggest clearing house - on Wednesday morning. Ireland’s bank were faced with an estimated $1 bn cash-call from LCH Clearnet as a result of its decision to require a deposit of 15 percent against all Irish bond position as indemnity against default. […] In order to avoid the call, many other banks and traders are dumping their bond positions, however.” “Irish Bond Yields Leap after Selling Wave”.
spiral in the Irish economy, by studying the impact of rises in haircuts on Irish government bond yields.

Figure 4: Yields (LHS) and haircuts (RHS) on 10-year government bonds

Source: Bloomberg and LCH Clearnet website

3 Assessing the impact of a funding liquidity shock

The rich dynamics of haircuts on Irish bonds provides a perfect laboratory to assess the interaction between funding liquidity of government bonds and sovereign bond yields. To do so, I study the impulse response function of a funding liquidity
shock with a Structural Vector Autoregression (SVAR) model, including haircuts applied by LCH Clearnet Ltd on 10 year government bonds ($h_t$), the logarithm of 5 year sovereign CDS spread ($CDS_t$) and the logarithm of yield of 10 year government bonds ($yd_t$), in order to deal with endogeneity problems and reverse causality issues since CDS spreads and government bond yield spreads over the German bonds are part of the information set used by this clearing house to settle the level of haircuts.\textsuperscript{12} The inclusion of CDS spread as a measure of credit risk allows to disentangle the funding liquidity channel from the credit risk channel and helps identify a funding liquidity shock.

This exercise is close to the study of Pelizzon, Subrahmanyam, Tomio, and Uno (2016), who analyze the dynamic relation between credit risk and market liquidity on Italian sovereign debt market. They find that this link is reinforced above 500 basis points in the sovereign CDS spread, because of changes in haircuts applied by LCH Clearnet. They also show that the threshold effect disappears after December 2011 when the ECB started the 3 year LTROs providing abundant liquidity to the banking system and conducted moral suasion to the European clearing houses to limit the procyclicality of haircuts, loosening the link between credit risk and market liquidity.

I use data at daily frequency spanning the period from 11/01/2010 to 12/01/2011. Data on CDS spread and government bond yield come from Datastream and Bloomberg, respectively. The sample size is due to the data availability on changes in haircuts and is right censored to exclude the launch of the first 3 year LTRO on 8th December 2011. Other unconventional monetary policies, such as the Secu-

\textsuperscript{12}See LCH.Clearnet Margining Approach (2011): http://www.ecb.europa.eu/paym/groups/pdf/mmcg/Item_1_LCH_Margining.pdf?0fe79f1cef93461dc22566a4e165db44.
rity Market Programme (SMP) and the refinancing operations with full allotment, were already active during this period, but their inception, which is likely to have an impact on market sentiments, precedes the beginning of the sample.¹³

Let \( y_t = [h_t, CDS_t, yd_t]' \) the vector of endogenous variables and consider the reduced form VAR

\[
y_t = A_c + A_1y_{t-1} + \ldots + A_py_{t-p} + u_t
\]

(1)

where \( A_j \) are 3 x 3 matrices of coefficients with \( p \) denoting the number of lags, end \( u_t \) is the 3 x 1 vector of reduced form residuals with \( u_t \sim (0, \Sigma_u) \). I set \( p = 4 \) but results are robust to a longer lag length. Matrices of coefficients and covariance are estimated with Bayesian techniques using non-informative priors (Canova (2007)). I impose limited prior information given the ignorance about the properties of a funding liquidity shock. The Appendix B.2 reports the details of the estimation procedure. Since the components of \( u_t \) may be instantaneously correlated (i.e. \( \Sigma_u \) could be not diagonal), I consider the following structural VAR model with orthogonalized residuals

\[
A_0y_t = A_0^*c + A_1^*y_{t-1} + \ldots + A_p^*y_{t-p} + \epsilon_t
\]

(2)

where \( A_0 \) is the impact matrix, \( A_j^* = A_0A_j \), and \( \epsilon_t \) are the structural shocks with diagonal covariance matrix \( \Sigma_\epsilon \). Reduced form residuals can be expressed as linear combination of structural shocks \( u_t = A_0\epsilon_t \) and \( \Sigma_u = A_0\Sigma_\epsilon A_0^\prime \). Normalizing the

¹³Ghysels, Idier, Manganelli, and Vergote (forthcoming) find that the SMP was effective in reducing yields of peripheral government bonds. Because of the scarcity of public available information regarding the implementation of the SMP, I cannot control for it but we can expect that this policy has dampened the impact of variations in haircuts on government bond yields.
variances of the structural shocks to one (i.e. \( \mathbb{E}(\epsilon_t \epsilon'_t) = I \)) gives

\[
\Sigma_u = A_0 A'_0
\]

In order to impose the necessary restrictions on \( A_0 \) to achieve the identification of structural shocks, I apply a Cholesky decomposition of \( \Sigma_u \) choosing \( \tilde{A}_0 \) as a lower triangular matrix with positive elements on the main diagonal. The recursive structure and zero restrictions on the contemporaneous coefficients find justification by the procedure through which LCH Clearnet Ltd decides and communicates to its members variations in haircuts, which is key to identify a funding liquidity shock, remaining agnostic on the identification of other financial shocks.

LCH Clearnet Ltd notifies all modifications in haircuts at the close of business through the Repo Clear Margin Rate Circulars and their revision is applied one day after the publication of the Circulars. These documents provide information on the date of the announcement, date of implementation and variations in additional margins required. To give a sense of how I proceed, Figure 12 in the Appendix A.2 reports an example of the Circular.

The ordering structure of the variables implies the following assumptions. First, \( h_t \) is predetermined relative to \( CDS_t \) and \( yd_t \), so financial shocks relative to \( CDS_t \) and \( yd_t \) do not affect \( h_t \) within a period. Second, a shock to \( h_t \) instantaneously impacts \( CDS_t \) and \( yd_t \). In other words, haircuts do not respond to movements in other financial variables during the same day, but their variations affect contemporaneously all financial variables included in the model. This identification strategy is based on a similar assumption commonly employed in the empirical literature on fiscal policy that fiscal instruments do not react instantaneously to
variations in other variables, mainly economic activity, because of the outside lag, which is the delay between the decision and the implementation of a certain policy. Nevertheless, with low frequency data the implementations of fiscal policy can be anticipated by private agents, leading to a non-fundamental moving average representation (see Leeper, Walker, and Yang (2013), Ramey (2011), Mertens and Ravn (2012), Favero and Giavazzi (2012)). In this application high frequency data rule out the possibility that market participants may react to announcements of changes in haircuts before their implementation. In this regard, it shares the High Frequency Identification (HFI) approach for monetary policy shocks.

However, a possible issue for the identification of a funding liquidity shock is that market participants can anticipate the decision of the clearing house to change the haircuts. In particular, LCH Clearnert Ltd published indicative thresholds at 450 basis point spread at the 10 year maturity to AAA benchmark, or at 500 basis point at 5 year CDS spread as risks indicators. However, the clearing house also states that these are key indicators to judge the credit risk of a security but do not trigger automatic actions for increases in haircuts and margin calls. Furthermore,

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14 Using different approaches Blanchard and Perotti (2002), Perotti (2005), Fatás and Mihov (2001) assume that government spending is predetermined within the quarter imposing time restrictions in a SVAR. Romer and Romer (2010) and Ramey (2011) also place their narrative fiscal measures as first endogenous variable with a Choleski decomposition.

15 The HFI approach addresses the problem of simultaneity which arises with low frequency data since within a period monetary policy both affects financial variables and responds to their movements (See Bagliano and Favero (1999), Gürkaynak, Sack, and Swanson (2005) Faust, Rogers, Swanson, and Wright (2003), Gertler and Karadi (2015)).


17 See LCH Clearnert (2014) “Frequently asked questions on the sovereign risk framework”. Furthermore, ICMA (2015, pag. 25) argues that “Although CCPs apply more rigorous risk management practices than many market users, their methodologies are often proprietary and therefore opaque, and it is not possible for members to scrutinize these methodologies, despite their critical dependence on them”
Bank of Italy (2013) documents that changes in haircuts applied by LCH Clearnet partly followed discretionary criteria and were unexpected.

In order to confirm that changes in haircut could not be anticipated, I perform two statistical tests. First, I construct a variable $h_{t}^{a}$ for the changes in haircuts on Irish bonds at the date of announcement of their revision, instead of the date of the effective changes as measured by $h_{t}$. I run a Granger causality test of CDS spread and yield spread to 10 year German bonds on the announced variations in haircut $h_{t}^{a}$. If they help predict variations in $h_{t}^{a}$, market participants could anticipate their modifications by looking at these indicators of sovereign risk. Table 3 in the Appendix B.1 shows that CDS spread and yield spread fail to predict announced changes in haircuts, confirming that changes in haircuts did not follow automatically variations in these indicators and were in part discretionary. Second, I run the Hansen (2000) test to assess the presence of a threshold regressing $h_{t}$ on $CDS_{t}$.$^{18}$ Figure 13 in the Appendix B.1 displays the graph on the normalized likelihood ratio sequence as a function of the threshold in CDS spread. The graph provides evidence of a threshold at 562 basis points, substantially higher than the 500 basis point threshold published by LCH Clearnet Ltd as a key indicator of risk. Overall, these tests suggest that variations in haircuts were to a large extent exogenous liquidity surprises.

Figure 5 plots the response of government bond yield and CDS spread to one standard deviation liquidity shock. The reaction of yield is positive and significant up to 0.6 percent after 15 days vanishing after 40 days. The impact on CDS spread is stronger up to 1.7 percent and more persistent, suggesting the presence

$^{18}$I obtain similar results when I regress $h_{t}^{a}$ on $CDS_{t}$. The yield spread to 10 year German bonds was above 450 basis points for the whole period of the sample.
of a negative liquidity spiral during the Eurozone crisis. The surge in sovereign risk induced LCH Clearnet Ltd to set higher haircuts on government bonds, which in turn increased yield and CDS spread, leading to additional rises in haircuts.

Figure 5: Impulse response function of a liquidity shock

As a robustness test, I estimate impulse response by local projections (Jordà (2005)) using the series of haircut as narrative shock. This approach does not require transforming the VAR model into a vector moving average model for the impulse response function using estimated parameters for horizon 0 to iterate forward. Moving average representation of VAR can be non-fundamental in small-scale VAR or because shocks are anticipated, so the advantage of using an observable narrative shock is that allows to compute impulse responses omitting a large amount of information which would be orthogonal to the shock included in the regression. The model is the following

$$y_{dt} = \delta'X_{t-1} + \beta_{shockt} + u_{t}$$  (3)
where $X_{t-1}$ include a constant and the first four lags of the variables $yd_t, CDS_t, h_t$; $\delta$ collects the coefficients; $shock_t$ corresponds to the series of haircut. I use the Newey-West corrections for standard errors because of the serial correlation in the error terms induced by the successive leading of the dependent variable.

Figure 6: Response of government bond yield to a liquidity shock

![Government bond yield](image)

Note: This figure shows the impulse response of 10 year government bond yields to a one percent liquidity shock. The left hand panel shows the impulse response of a linear model. The right hand panel shows the impulse response of a state-dependent model, with blue line representing the response in the regime of low CDS spread and the magenta line the response in the regime of high CDS spread. The dash lines represent 95 percent bands that are based on Newey-West standard error.

Figure 6 shows that the liquidity shock has a substantial effect on government bond yields. A one percent shock in haircuts increases the government bond yield by 4 percent and the impact vanishes after 40 days, analogously to the impulse response estimated with the SVAR. These findings lend support to the presence of a negative liquidity spiral on the Irish sovereign debt market during the crisis which amplified the rise in the yields. Since the high frequency of financial series is essential for the identification of a liquidity shock, I do not consider its impact on

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19When I include a linear and quadratic time trend results remain almost unchanged.
macroeconomic variables in the empirical analysis. However, empirical literature provides evidence that during the crisis the rise in government bond yields negatively affected firms’ capital expenditure through the bank lending channel (see Acharya, Eisert, Eufinger, and Hirsch (2014) and Altavilla, Pagano, and Simonelli (2016)). In the next section I study the transmission of a liquidity shock to the real economy with a model incorporating liquidity frictions.

4 The model

The model is an infinite horizon economy populated by a continuum of households of measure one. The members of each household are either entrepreneurs or workers. The model incorporates nominal rigidities, since prices and wages are set in staggered contracts, and real rigidities with capital adjustment cost along the lines of Smets and Wouters (2007). The Appendix C.1 through C.4 reports the equations for the production process and market-clearing conditions and equilibrium definitions are in Appendix C.5. Households allocate saving across three risk-free financial assets characterized by different degrees of liquidity: equity, long-term and short-term sovereign bonds. The government conducts fiscal policy collecting taxes and conventional monetary policy by setting the nominal interest rate. Long-term bonds are subject to a liquidity shock which is the only shock perturbing the economy. In response to this shock the government may implement an unconventional policy which consists of increasing the supply of one-period bonds that are more liquid than long-term bonds.

20 The mixed frequency VAR approach creates a bridge between high frequency financial variables and low frequency macroeconomic variables but cannot keep the daily frequency of financial variables that I exploit for the identification strategy (see Foroni and Marcellino (2014)).
4.1 Households

Structure. Each household has a unit measure of members indexed $j \in [0, 1]$. At the beginning of each period all members are identical and hold an equal share of the household’s assets. They receive an idiosyncratic shock, iid across members and across time, which determines their profession: entrepreneurs or workers. With probability $\gamma$ they are entrepreneurs. By the law of large number $\gamma$ also represents the fraction of entrepreneurs in the economy. Each entrepreneur $j \in [0, \gamma)$ invests and each worker $j \in [\gamma, 1]$ supplies labor; both types return their earnings to the household and at the end of each period, all members share consumption goods and assets, but resources cannot be reallocated among members within the period.

Preferences. The household’s objective is to maximize the utility function

$$
E_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s, H_s(j)) = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{C_s^{1-\sigma}}{1-\sigma} - \frac{\xi}{1+\eta} \int_{\gamma}^{1} H_s(j) \gamma^{1+\eta} dj \right]
$$

(4)

where $E_t$ denotes the conditional expectation, $\beta$ is the subjective discount factor, $\sigma$ measures the degree of relative risk aversion, $\xi$ is a scaling parameter that can be chosen to match a target value for the steady state level of hours and $\eta$ is the inverse of the Frisch elasticity of the labor supply. Utility depends positively upon the sum of the consumption good bought by household members ($C_t = \int_0^1 C_t(j) dj$) and negatively upon the workers’ labor supply $H_t$.

Portfolio. Households hold physical capital $K_t$ with price $q_t$ that depreciates for a fraction $\lambda$ every period. They lend it to intermediate good producers earning a constant dividend stream $r_t$. They can sell claims on their capital to other households, $S_t^l$, which represents the only liabilities of households, and purchase
claims on other households’ capital, $S^O_t$. Equity issued by the other households ($S^O_t$) and the unmortgaged capital stock ($K_t - S^I_t$) are assumed to yield the same returns, have the same value and liquidity and depreciate at the same rate, so they are perfect substitutes and can be summed together and defined as net equity

\[ S_t = S^O_t + K_t - S^I_t \]  

Households also own government debt with different maturities. Short-term bonds $B^S_t$ are one period securities purchased at time $t$ at price $q^S_t$ that pay an unit return at time $t + 1$. Long-term bonds $B^L_t$ are perpetuities with coupons which decay exponentially as in Woodford (2001) with price $q^L_t$. A bond issued at date $t$ pays $\lambda^{k-1}$ at date $t + k$, where $\lambda \in [0, 1]$ is the coupon decay factor that parametrizes the maturity of long-term bonds, corresponding to $(1 - \lambda \beta)^{-1}$. If $\lambda = 0$ these securities are one period zero coupon bonds and if $\lambda = 1$ they are consols. In the steady state the returns on long-term and short-term bonds are linked by a non-arbitrage condition. The table below summarizes the household’s balance sheet at the beginning of the period.

<table>
<thead>
<tr>
<th>Asset</th>
<th>Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital stock: $q_t K_t$</td>
<td>Equity issued: $q_t N^I_t$</td>
</tr>
<tr>
<td>Other’s equity: $q_t N^O_t$</td>
<td></td>
</tr>
<tr>
<td>Long-term bonds: $q_t B^L_t$</td>
<td></td>
</tr>
<tr>
<td>Short-term bonds: $q_t^S B^S_t$</td>
<td>Net worth: $q_t N_t + q_t^L B^L_t + q_t^S B^S_t$</td>
</tr>
</tbody>
</table>

Finally, at the end of each period households receive real profits $d_t$ and $d^I_t$ from

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21 An alternative interpretation of the long-term debt is that $\lambda$ is the fraction of the outstanding bonds paying a constant coupon of 1 and $(1 - \lambda)$ is the fraction of bonds which mature at each period and for which the government pays back the principal to the bond holder (Chatterjee and Eyigungor (2012)).
intermediate goods producers and capital producers, respectively. For each unit of capital they receive the dividend

$$r_t^K = r_t \frac{d_t + d^l_I}{K_t}$$

Resources allocation. When the asset market and goods market open households allocate their resources and trade assets to finance new investments. The flow of fund constraint of household member j is as follows

$$C_t(j) + q^I_t I_t(j) + T_t + q^L_t \frac{B^L_{t+1}(j)}{p_t} + q^S_t \frac{B^S_{t+1}(j)}{p_t} + q_t[S_{t+1}(j) - I_t(j)] = [1 + \lambda q^S_t] \frac{B^L_t}{p_t} + \frac{B^S_t}{p_t} + [r^K_t + \lambda q_t] S_t + \frac{W_t(j)}{p_t} H_t(j)$$

where $p_t$ denotes the price level, $q^I_t$ is the cost of one unit of new capital in terms of consumption goods, differing from 1 because of capital adjustment cost, $H_t(j)$ and $W_t(j)$ are the working hours and nominal wage for workers j. According to the left side of the budget constraint, the household members allocate resources between purchase of non-storable consumption good, investment in new capital - if they are entrepreneurs -, taxes and net purchase of equity, long-term bonds and short-term bonds. They finance their activities on the right side of the budget constraint with returns on equity, long-term bonds, short-term bonds and wages of differentiated labor - if they are workers.

A key assumption of the model is the presence of the following funding constraints which limit the financing of new investments by entrepreneurs and deter-
mine the different degree of liquidity of the assets

\[ S_{t+1}(j) \geq (1 - \theta)I_t(j) + \lambda S_t \] \hspace{1cm} (7)

\[ B_{t+1}^L(j) \geq (1 - \phi_t)B_t^L \] \hspace{1cm} (8)

\[ B_{t+1}^S(j) \geq 0 \] \hspace{1cm} (9)

Inequality 7 means that the entrepreneur can issue claims on the future output of investment but only for a fraction \( \theta \in [0, 1] \). This borrowing constraint implies that investment is partially funded internally and entrepreneurs have to retain \( 1 - \theta \) as own equity. In addition, equity is assumed to be completely illiquid since entrepreneurs cannot sell it to obtain more resources to invest. Hence, the entrepreneurs’ equity holding at the start of the period \( t+1 \) must be at least the sum of \( (1 - \theta)I_t \) and depreciated equity \( \lambda N_t \), where \( \lambda \) is the inverse depreciation rate.\(^{22}\)

The entrepreneur can acquire additional resources by disposing of a fraction \( \phi_t \in [0, 1] \) of long-term bonds, so a resaleability constraint imposes to keep the residual \( (1 - \phi_t) \) of bonds in his portfolio (inequality 8). \( (1 - \phi_t) \) is equivalent to the haircut in a repo transaction since it determines the amount of liquidity that the entrepreneur can obtain by pledging sovereign securities in secured borrowing. In other words, the entrepreneur cannot borrow against the entire bond holding

\(^{22}\) Nezafat and Slavik (2011) model a financial shock as a tightening in the credit conditions and a drop in \( \theta \) and assume that equity/capital is completely liquid. In our set-up the assumption that equity is illiquid means that entrepreneurs cannot issue equity on the unmortgaged capital stock and cannot sell any of others’ equity remained.
because of the presence of the haircut.

Inequality 9 implies that short-term bonds are not subject to resaleability constraint and are fully liquid, but entrepreneurs cannot borrow from the government.\footnote{Similarly, inequalities 7 and 8 ensure that receipts from trading equity and long-term bonds are strictly positive, which prevents the entrepreneur from going short on these securities.} $\phi_t$ is the key parameter of the model characterizing the liquidity of financial assets. We can think that it takes value 0 for equity, value 1 for short-term bonds and an intermediate value for long-term bonds. The assumption on the diverse resaleability of equity and bonds reflects the different liquidity of privately issued securities, which are scarcely used as collateral, and sovereign bonds, which are largely pledged by European banks for repo transactions, and gives government bonds a special liquidity function. The dynamic of the model follows a reduction in $\phi_t$, which is paramount to a rise in the repo haircut on sovereign bonds.

At the end of the period, the assets of households are given by

\begin{align}
S_{t+1} &= \int S_{t+1}(j) \, dj \\
B_{t+1}^S &= \int B_{t+1}^S(j) \, dj \\
B_{t+1}^L &= \int B_{t+1}^L(j) \, dj \\
K_{t+1} &= \lambda K_t + \int I_t(j) \, dj
\end{align}

Next, the specific functions of entrepreneurs and workers are taken into account.
4.1.1 Entrepreneurs

The entrepreneur \( j \in [0, \gamma) \) does not supply labor, so \( H_t(j) = 0 \) in equation 6 to get his budget constraint. In order to acquire new capital he can either produce it at price \( q_t^I \) or buy it in the market at price \( q_t \). For the rest of the model I assume that \( q_t > q_t^I \) in order to focus on the economy where the liquidity constraints bind, thus limiting the ability of the entrepreneur to finance investments. In this case, the entrepreneur will use all the available liquidity for new investment projects to maximize the households’ utility. Accordingly, he minimizes the equity holding by issuing the maximum amount of claims on the investment return to reduce the size of the downpayment as implied by constraint 7. The entrepreneur also sells the maximum amount of bonds as allowed by constraint 8, because their expected returns are lower than those on new investment. As a result, in equilibrium the liquidity constraints all bind and the entrepreneur does not consume goods within the period:

\[
S_{t+1}(j) = (1 - \theta)I_t(j) + \lambda S_t
\]

(14)

\[
B_{t+1}^L(j) = (1 - \phi)B_t^L
\]

(15)

\[
B_{t+1}^S(j) = 0
\]

(16)

\[
C_t(j) = 0
\]

(17)

Given the solutions for entrepreneurs, \( S_{t+1}(j), B_{t+1}^L(j), B_{t+1}^S(j), C_t \) for \( j \in [0, \gamma) \), equations 14, 15, 16 and 17 can be plugged into equation 6 to derive the function
of investment for entrepreneurs

\[ I_t(j) = \frac{\gamma_t^K N_t + [1 + \lambda \phi_t q_t^L \frac{B_t^L}{p_t} + q_t^S \frac{B_t^S}{p_t} - T_t]}{q_t^I - \theta q_t} \]  

(18)

The nominator represents the maximum liquidity available for the entrepreneurs deriving from the return on papers (equity and long-term bonds), sales of the resaleable fraction of long-term bonds after depreciation, sales of short-term bonds and the dividends net to taxes. The denominator is the difference between the price of one unit of investment goods and the value of equity issued by the entrepreneur, which indicates the amount of own resources necessary to finance one unit of investment. Equation 18 shows that a drop in \( \phi_t \) reduces the leverage of entrepreneurs and impacts directly on their investment. Aggregating by entrepreneurs total investment is

\[ I_t = \int_0^\gamma I_t(j) \, dj = \gamma \frac{\gamma_t^K N_t + [1 + \lambda \phi_t q_t^L \frac{B_t^L}{p_t} + q_t^S \frac{B_t^S}{p_t} - T_t]}{q_t^I - \theta q_t} \]  

(19)

4.1.2 Workers

Workers \( j \in [\gamma, 1] \) do not invest, so \( I_t(j) = 0 \). They supply labor as demanded by firms at a fixed wage; the union who representing each type of worker sets wages on a staggered basis. To determine the asset and consumption choices of workers, I first derive the household’s decision for \( N_{t+1}, B_{t+1}^L, B_{t+1}^S \) and \( C_t \), taking \( W_t \) and \( H_t \) as given. Knowing the solution for entrepreneurs, \( N_{t+1}(j), B_{t+1}^L(j), B_{t+1}^S(j) \) and \( C_t(j) \) can be determined for workers, given the aggregate consumption and asset holding.
The problem of households

To solve the model for the household, I aggregate the workers’ and entrepreneurs’ budget constraint

\[
C_t + q_t^I I_t + T_t + q_t[I_{t+1} - I_t] + q_t^L \left[ \frac{B_t^{L+1}}{p_t} - \lambda \frac{B_t^L}{p_t} \right] + q_t^S B_t^{S+1} = \left[ r_t + \lambda q_t \right] S_t + \frac{B_t^L}{p_t} + \frac{B_t^S}{p_t} + \int_1^{\gamma} \frac{W_t(j)}{p_t} H_t(j)
\]  

Households maximize the utility function (5) by choosing \(C_t, S_t + 1, B_t^{L+1}\) and \(B_t^{S+1}\) subject to the aggregate budget constraint and the investment constraint. The first order conditions for equity, long-term bonds and short-term bonds are respectively

\[
U''_{c,t} = \beta \mathbb{E}_t \left\{ U'_{c,t+1} \left[ \frac{r_{t+1}^{K} + \lambda q_{t+1}}{q_t} + \gamma (q_{t+1} - q_{t+1}^{L}) \frac{r_{t+1}^{K}}{q_{t+1}^{L} - \theta q_{t+1}} \right] \right\}
\]  

\[
U''_{c,t} = \beta \mathbb{E}_t \left\{ \frac{1}{\pi_{t+1}} U'_{c,t+1} \left[ \frac{1 + \lambda q_{t+1}^L}{q_t^L} + \gamma (q_{t+1} - q_{t+1}^L) \frac{1 + \lambda \phi_{t+1} q_{t+1}^S}{q_t^S} \right] \right\}
\]  

\[
U''_{c,t} = \beta \mathbb{E}_t \left\{ \frac{1}{\pi_{t+1}} U'_{c,t+1} \left[ \frac{1}{q_t^S} + \gamma (q_{t+1} - q_{t+1}^L) \frac{1}{q_t^L} \right] \right\}
\]  

where \(\pi_t\) is the inflation rate defined as \(\pi_t = \frac{P_{t+1}}{P_t}\). The choice of sacrificing one unit of consumption today to purchase a paper gives a payoff which is composed of two parts. The first is the returns on the asset: \(r_{t+1}^{K} + \lambda q_{t+1}\) for equity, \(\frac{1 + \lambda q_{t+1}^L}{q_t^L}\) for long-term bonds and \(\frac{1}{q_t^S}\) for short-term bonds. The second part is a “liquidity premium”, deriving from the fact that papers in the entrepreneurs’ portfolio relax their investment constraint. This premium is a function of the leverage \(\frac{\gamma}{q_t^L - \theta q_t}\), the distance between the price of equity and the price of capital goods, and the liquid
returns of each asset. The bond holding eases the financing constraints more than the equity holding, which makes bonds more valuable for entrepreneurs.

4.2 The government

The government conducts monetary and fiscal policy separately, following exogenous policy rules, and faces the following intertemporal budget constraint

\[ q_t^L B_t^{L+1} + q_t^s B_t^{S+1} + T_t = (1 + \lambda q_t^L) B_t^L + B_t^S \quad (24) \]

The debt repayment is financed by the issue of new debt and net taxes \( T_t \). A solvency condition links taxes with the outstanding beginning-of-period long-term debt in term of deviation from the steady state to satisfy the government intertemporal budget constraint. The government adjusts fiscal policy when the debt level goes up as in Chen, Curdia, and Ferrero (2012) and Davig and Leeper (2007)

\[ T_t - T = \psi_T \left( \frac{B_t^L}{p_t} - \frac{B_t^L}{p} \right) \quad (25) \]

where \( \psi_T > 0 \) measures the elasticity of fiscal policy to variations in the size of the debt. Short-term debt does not enter into the solvency condition to avoid that the unconventional policy is counteracted by higher taxes. However, quantitatively results do not change by including the short-term debt since the adjustment of taxes to debt is gradual (\( \psi_T \) is small) and by assumption the steady-state value of \( B_t^s \) is zero. The government sets the nominal interest rate following the feedback

\[ \text{Cui (2016) analyzes the trade-off of issuing more liquid public debt financed via distortionary taxes.} \]
rule constrained by the zero lower bound condition

\[ r_t = \max \left( \pi_t^\psi, 1 \right) \tag{26} \]

where \( \psi > 1 \). Unconventional policy consists of purchasing illiquid long-term bonds by issuing liquid short-term bonds. The supply of one-period bonds is a function of the proportional deviations of liquidity from the steady state

\[ \frac{B_{t+1}^S}{p_t} = \psi_B \left( \frac{\phi_t}{\phi} - 1 \right) \tag{27} \]

When the liquidity of long-term bonds dries up the government steps in. The intervention does not affect the liquidity constraint, but modifies the portfolio composition of the private sector. The price of the nominal short-term bond is the inverse of the nominal rate, so the government by setting the nominal interest rate, also sets the price of short-term bonds.

\[ q_t^S = \frac{1}{r_t} \tag{28} \]

5 Numerical simulation

5.1 Parametrization

I calibrate the model at quarterly frequency to match the economy of Ireland because of the rich dynamic of the haircuts on Irish bonds, which is used to calibrate the process of the liquidity parameter \( \phi_t \) (see Figure 4). Table 1 reports the calibrated values.
<table>
<thead>
<tr>
<th>Definition</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences</strong></td>
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<td></td>
</tr>
<tr>
<td>Household discount factor</td>
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</tr>
<tr>
<td>Relative risk aversion</td>
<td>$\sigma$</td>
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<tr>
<td>Inverse Frish elasticity</td>
<td>$\eta$</td>
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<tr>
<td><strong>Production and investment</strong></td>
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<td></td>
</tr>
<tr>
<td>Capital share of output</td>
<td>$\alpha$</td>
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<tr>
<td>Adjustment cost parameter</td>
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<tr>
<td>Probability of investment opportunity</td>
<td>$\gamma$</td>
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</tr>
<tr>
<td>Inverse depreciation rate / Bond maturity parameter</td>
<td>$\lambda$</td>
<td>0.973</td>
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<td><strong>Nominal frictions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price and wage calvo probability</td>
<td>$\zeta_\pi = \zeta_w$</td>
<td>0.75</td>
</tr>
<tr>
<td>Price and wage steady-state markup</td>
<td>$\delta_\pi = \delta_w$</td>
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</tr>
<tr>
<td><strong>Financial frictions</strong></td>
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<td>Borrowing constraint</td>
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<tr>
<td>Liquidity constraint</td>
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<tr>
<td>Autoregressive coefficient of liquidity</td>
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</tr>
<tr>
<td>Size of liquidity shock</td>
<td>$\sigma^\phi$</td>
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</tr>
<tr>
<td>Steady-state of liquidity share</td>
<td>$ls$</td>
<td>0.43</td>
</tr>
<tr>
<td><strong>Policy rule</strong></td>
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<td></td>
</tr>
<tr>
<td>Monetary policy rule coefficient</td>
<td>$\psi_\pi$</td>
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</tr>
<tr>
<td>Transfer rule coefficient</td>
<td>$\psi_T$</td>
<td>0.1</td>
</tr>
<tr>
<td>Government intervention coefficient</td>
<td>$\psi_B$</td>
<td>-0.127</td>
</tr>
</tbody>
</table>

The steady state of liquidity $\phi$ is 0.75, equivalent to one minus the haircut on 10-year Irish bonds before the crisis. I estimate the stochastic process for $\phi_t$ as an AR(1) process from the dynamics of the Irish haircut during the period of crisis. The autoregressive coefficient $\rho^\phi$ is found to be 0.99 and the standard deviation of the residuals $\sigma^\phi$ is 1.3. These values measure respectively the persistence of the
freeze in the repo market and the size on the liquidity shock. The other parameter characterizing the financial frictions $\theta$ describes the fraction of investment financed externally. Since entrepreneurs represent broadly the banking system in channeling resources to the production sector of the economy, I consider $\theta$ as the ratio of banks’ external finance, defined as the sum of deposits, long-term debt and equity, over total assets. I construct the average of this ratio for the 18 largest financial institutions for which Bankscope reports information on repos which is 0.5 (Table 2 in the Appendix A.3). The liquidity share in this economy is defined as:

$$l_{st} = \frac{\phi Q_t^{L} B_{t+1}^L}{\phi Q_t^{L} B_{t+1}^L + q_t P_t K_{t+1}}$$  \hspace{1cm} (29)$$

The nominator is the liquid part of public debt computed as the total of Irish government gross liabilities times the liquidity parameter. The denominator is equal to the value of the total productive capital. Data are taken from the OECD Economic Outlook. The average of this ratio during the period 2000 and 2011 is 0.43, which is taken as the steady state value of the liquidity share. The parameter $\lambda$ pins down the duration of long-term bonds given by $(1 - \lambda \beta)^{-1}$. I set $\lambda = 0.973$ to match the average maturity of the Irish debt, which is 6.9 years (Eurostat (2013)).

Other parameters are standard in the literature: the discount factor $\beta = 0.99$, the inverse Frish elasticity of labor supply $\eta = 1$, the capital share $\alpha = 0.4$, the arrival rate of investment opportunity in each quarter $\gamma = 0.05$. The degree of monopolistic competition in labor and product markets is calibrated symmetrically assuming a steady state markup of 10% ($\delta_p = \delta_w = 0.1$). The average duration of price and wage contracts is 4 quarters ($\zeta_p = \zeta_w = 0.75$). Concerning the policy
rules, the feedback coefficient on inflation in the monetary policy rule $\psi_x$ is 1.5 in order to guarantee a uniquely determined equilibrium. The government finances most of the intervention through emission of new short term debt and transfers slowly adjust to the government net wealth position ($\psi_T = 0.1$). The coefficient for the intensity of the government intervention $\psi_B$ is -0.127, the same value used by Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017).

5.2 Results

5.2.1 The impact of a liquidity shock

I first analyze the economy in which the government does not respond to the negative liquidity shock. Figure 7 shows the response of real and financial variables to the calibrated liquidity shock $\phi_t$.

Output, consumption and investment simultaneously fall by 8.01%, 7.45% and 10.92%, respectively. The contraction of liquidity impacts directly on investments by tightening the entrepreneurs’ funding constraint, shrinking the funds they can obtain from borrowing against the bonds. Moreover, it is amplified by the fall in the value of equity that increases the required downpayment, reducing the leverage of entrepreneurs. The drop in the price of equity (-14.23%) and long-term bonds (-8.42%) has a negative wealth effect on consumption. In particular, the presence of nominal rigidities is a key element for the fall in consumption, because with a flexible price the contraction in the economic activity would generate deflationary expectations leading to negative real interest rate and boosting consumption, as observed in the model of Kiyotaki and Moore (2012). The liquidity shock results in a large and persistent decline in the price of final goods (-6.12%).
Concerning asset prices, the contraction in the liquidity of long-term bonds leads to a "flight to liquidity" towards the more liquid short-term bonds, as indicated by the jump in the liquidity spread (13.78%), defined as the difference between the price of short-term and long-term bonds. In addition, the strong persistence of the shock induces entrepreneurs to anticipate lower future liquidity, thus reinforcing this mechanism. The tightening of funding constraints increases the demand and price of short-term bonds, since they are the most liquid assets, but not of equity, since it is completely illiquid. Nominal frictions are at the root of the fall in the value of long-term bonds. In the Kiyotaki and Moore (2012) setting a negative liquidity shock on equity leads to an increase in equity price. Shi (2015) explains that this puzzling result is due to the rise in the demand for
assets, including the liquid parts of equity, which in their model are partly liquid. Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017) shows that incorporating nominal rigidities allows equity price to drop following the tightening of liquidity constraint, which they define as resaleability constraint.

In this model, the liquidity shock is akin to a tightening of margin requirements in Gârleanu and Pedersen (2011) and Ashcraft, Gârleanu, and Pedersen (2011), which increases the shadow cost of capital and drives up required returns of assets, lowering investment and output.

5.2.2 The effects of the policy intervention

I consider now the scenario in which the government reacts to the drop in $\phi_t$ by issuing more short-term bonds. Figure 8 compares the impulse responses to a negative liquidity shock in the case that the government does not intervene (blue line) and in the case in which it does react (red line).

The model predicts that this unconventional policy alleviates the contractionary effect of the shock substantially. Output decrease by 5.73%, consumption by 6.17% and investment by 5.35%. The fall in consumption drastically lessens, mainly because the reduction in the price of equity is less pronounced, which also reduces the deleveraging of entrepreneurs. The increased supply of liquid short-term bond relaxes the funding constraint of entrepreneurs. Moreover, the implemented policy succeeds in reducing the deflationary effect of the liquidity shock (-2.89%). Paradoxically, the policy intervention does not reduce the drop in the value of long-term bonds following the liquidity shock. Shi (2015) notes that when the government injects liquidity following a liquidity shock on equity, relaxing the entrepreneurs’ financing constraint the value of equity falls. The same
mechanisms works for a liquidity shock on long-term bonds. The rise in the price of short-term bond is smaller because the government increases their supply. Overall, the response of the liquidity spread to the shock does not change with the policy intervention.

### 5.2.3 The zero lower bound

Finally, I solve the model imposing that the zero lower bound is binding. When monetary policy is constrained by the zero lower bound the impact of the liquidity shock and the effect of unconventional policy are both amplified. Figure 9 displays the impulse response to the liquidity shock if the zero lower bound is binding with (red dashed line) and without (blue dashed line) the policy intervention.
Figure 9: The zero lower bound

Note: The figure shows the response to a calibrated liquidity shock in the log-deviation from steady states in percentage points for 12 quarters with and without policy intervention at the zero lower bound.

On impact, the drop in output and consumption is similar to what I obtain ignoring the zero lower bound, as shown by Figure 9 (-8.55% and -7.19%, respectively), but is far more persistent. The nominal interest rate cannot reach the negative region and conventional monetary policy cannot boost consumption expenditure through a reduction in the real interest rate, mitigating the impact of the liquidity shock, as in the case in which the zero lower bound does not bind. This in part explains the strong deflationary pressure following the liquidity shock. The fall in investment is deeper (-15.80%) and also more persistent. Concerning asset prices, the price of equity falls more steeply following the stronger reduction in the demand for new capital, while the magnitude of the liquidity spread is similar to the scenario without zero lower bound. Because of the zero lower bound

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constraint, the effectiveness of unconventional policy is greater than it would be if it were possible for the government to lower the policy rate below zero. In particular, this measure avoids the economy remaining in a region of low consumption and deflation.

6 Conclusions

This paper has explored the liquidity channel of the Eurozone sovereign debt crises. It has shown that government securities play a key role as collateral in the secured interbank market, which is a primary source of funding for banks to meet their liquidity needs. Nevertheless, during the crisis repo haircuts on peripheral government bonds grew substantially following the rise in sovereign risk.

I studied the consequences of a reduction in sovereign bond liquidity with a model incorporating liquidity frictions to simulate the impact of a rise in haircuts. The model exhibits a fall in output, investments, consumptions and a rise in the liquidity spread, suggesting a flight-to-liquidity from the less liquid to the more liquid bonds. The contractionary effects of this shock can be alleviated by a policy response consisting of issuing more short-term bonds to provide investors with more liquid assets in order to relax their funding constraint. This measure is more effective when the monetary policy is constrained by the zero lower bound.

I have assessed empirically the impact of a shock to haircuts on government bond yields by estimating the impulse response function of a SVAR model and the regime-dependent impulse response function by local projections. A liquidity shock increases the yield significantly, especially in the state of high stress in the sovereign debt market, confirming the prediction of the model.
Of course several other macroeconomic and financial shocks played an important role for the Eurozone crisis, contributing to the contraction of economic activity and the drop in the value of peripheral sovereign bonds. However, the liquidity channel magnified the tension in the sovereign debt markets and reinforced the transmission to the real economy.

The theoretical model can be extended in several dimensions. One is to introduce the risk of government default to endogenize the liquidity parameter as a function of the sovereign default, following the literature on fiscal limit (see Bi (2012) and Bi and Traum (2012)). When the level of the debt approaches the fiscal threshold the probability of default increases, reducing the sovereign bond liquidity. This set up would allow to analyze the joint interaction between sovereign risk and liquidity. A second extension would be to consider an open economy with two countries conducting independent fiscal policy and sharing monetary policy in the typical framework of a monetary union in order to study the impact of a liquidity shock on the bond issued by one country and the possible policy responses of the central bank. I leave these extensions for further research.

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A  Data appendix

A.1  Definitions for repurchase agreements

A repo transaction is an agreement between two parties on the sale and subsequent repurchase of securities at an agreed price. It is equivalent to a secured loan, with the main difference that legal title of securities passes from the cash borrower to the cash lender which may re-use them as collateral in other repo transactions. In order to protect the lender from the risk of a reduction in the value of collateral, repos involve overcollateralization and the difference between the value of the loan and the value of collateral is the haircut or initial margin. The haircut takes account of the unexpected loss that the lender may face due to the difficulty of selling the collateral security in response to a default by the borrower. Accordingly, it provides a measure of market liquidity of collateral from the standpoint of the lender and a measure of from the standpoint of the borrower since determines the amount of cash that can be raised given the value of collateral.

Figure 10 shows an example of bilateral repo. At time t, the cash borrower (securities dealer, commercial bank, hedge fund) posts €100 securities as collateral and receives a €90 loan from the cash lender (commercial bank, investment fund, money market fund) with a haircut of 10%. At time t+k, the borrower returns the cash with an interest of 1.1% (the repo rate) and receives back the collateral. If repo is used to finance the purchase of a security, the haircut is equivalent to the inverse of the leverage. In order to hold €100 securities the investor can borrow up to €90 from the repo lender and must come up with €10 of its own capital, so the maximum leverage is 10. A rise in haircut by 10 percentage points reduces the borrower’s funding to €80 and its leverage to 5.

According to the involvement of intermediaries between the lender and the borrower, repos can be distinguished in two types. In bilateral repos, the lender and the borrower transact directly with each other, selecting the collateral, initiating the transfer of cash and securities, and conducting collateral valuation. In tri-party repos, a third party intermediates the transaction providing operational services to the parties, in particular the selection and valuation of collateral securities, but does not participate in the risk of transaction.

The determinants of haircuts vary according to the repo structure. In repos that are not
Figure 10: Bilateral repo contract

cleared by a Central Clearing Counterparty (CCP), the haircut reflects mainly the creditworthiness of the borrower. Instead in repos involving a CCP which bears the counterparty credit risk, haircuts are settled on the basis of the CCP’s internal rules and depend on the market risk of collateral.

A.2 Data

Because of the lack of comprehensive information on the European repo market, we use different sources. First, Bankscope, which provide banks’ balance sheet data at annual frequency showing the amount of repos and reverse repos held by credit intermediaries. It allows to compare different funding instruments, but lacks important breakdowns (such as counterparty, maturity and currency) preventing a more granular analysis and does not distinguish between repos in the interbank market from ECB monetary policy operations. Second, the European Repo Market Survey published semi-annually by the International Capital Market Association (ICMA) since
2001, which asks a sample of 67 banks in Europe for the value of their repo contracts that were still outstanding at close of a certain business date excluding repos transacted with central banks.\textsuperscript{25}

It reports information on the size and composition of the European repo market, including the type of repo traded, the rates, the collateral, the cash currency and the maturity. Third, the Euro Money Market Survey, an yearly survey published by the ECB since 2002 covering 101 banks, which decomposes the repo market in three segments: CCP-based, over-the-counter bilateral, and triparty. We collected data on haircuts to 10-year government bonds applied by the LCH Clearnet, which is the largest European clearing house (see Armakola, Douady, Laurent, and Molteni (2016)), and we identified changes in haircuts with a narrative approach by reading the RepoClear Margin Rate Circulars through which they communicate the variation to its members.\textsuperscript{26} Finally, data on yields, CDS spreads and bid-ask spread come from Bloomberg.

\textsuperscript{25}From the data of the European Repo Market Survey we subtracted reverse repos in order to focus the analysis on the liability side of banks’ balance sheets.

\textsuperscript{26}http://secure-area.lchclearnet.com/risk_management/ltd/margin_rate_circulars/repoclear/default.asp
A.3 Banks’ balance sheet

I use information on banks’ balance sheet from Bankscope in order to measure the share of repos compared to other sources of funding. Since Bankscope does not distinguish repos in the private interbank market from the refinancing operations of central banks I consider 2010 in order to avoid the 3 year LTROs implemented in 2011 and 2012. Repos account for a large share of banks’ liabilities especially for the largest five banks for which Bankscope report data on repos. I use banks’ balance sheet in order to calibrate the parameter $\theta$ for the borrowing constraint.

Table 2: Funding structure of European commercial banks in percentage of total liabilities (2010)

<table>
<thead>
<tr>
<th>Bank</th>
<th>Deposits</th>
<th>Interbank</th>
<th>LT debt</th>
<th>Repos</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNP Paribas</td>
<td>26.62</td>
<td>7.07</td>
<td>6.19</td>
<td>10.48</td>
</tr>
<tr>
<td>Barclays Bank Plc</td>
<td>23.41</td>
<td>5.89</td>
<td>9.89</td>
<td>13.26</td>
</tr>
<tr>
<td>Banco Santander</td>
<td>45.04</td>
<td>4.69</td>
<td>16.92</td>
<td>9.60</td>
</tr>
<tr>
<td>Société Generale</td>
<td>24.47</td>
<td>7.62</td>
<td>8.74</td>
<td>9.58</td>
</tr>
<tr>
<td>UBS AG</td>
<td>24.13</td>
<td>2.13</td>
<td>10.04</td>
<td>12.52</td>
</tr>
<tr>
<td>UniCredit SpA</td>
<td>42.99</td>
<td>10.83</td>
<td>16.96</td>
<td>3.39</td>
</tr>
<tr>
<td>Credit Agricole Corporate</td>
<td>14.43</td>
<td>10.63</td>
<td>0.70</td>
<td>5.61</td>
</tr>
<tr>
<td>Intesa Sanpaolo</td>
<td>30.84</td>
<td>10.32</td>
<td>27.99</td>
<td>1.99</td>
</tr>
<tr>
<td>Banco Bilbao</td>
<td>43.79</td>
<td>6.01</td>
<td>14.16</td>
<td>8.89</td>
</tr>
<tr>
<td>Commerzbank AG</td>
<td>34.32</td>
<td>12.01</td>
<td>13.72</td>
<td>7.13</td>
</tr>
<tr>
<td>Danske Bank</td>
<td>23.22</td>
<td>5.19</td>
<td>27.66</td>
<td>7.87</td>
</tr>
<tr>
<td>Skandinaviska Enskilda Banken</td>
<td>35.5</td>
<td>7.42</td>
<td>14.01</td>
<td>2.14</td>
</tr>
<tr>
<td>Bankia SA</td>
<td>36.78</td>
<td>3.43</td>
<td>29.6</td>
<td>11.56</td>
</tr>
<tr>
<td>Svenska Handelsbanken</td>
<td>29.21</td>
<td>8.06</td>
<td>30.27</td>
<td>0.49</td>
</tr>
<tr>
<td>Fortis Bank</td>
<td>41.18</td>
<td>7.68</td>
<td>5.50</td>
<td>4.30</td>
</tr>
<tr>
<td>Abbey National Treasury Services Plc</td>
<td>52.99</td>
<td>13.42</td>
<td>13.85</td>
<td>2.79</td>
</tr>
<tr>
<td>KBC</td>
<td>50.76</td>
<td>7.35</td>
<td>10.58</td>
<td>8.53</td>
</tr>
<tr>
<td>Banca Monte dei Paschi</td>
<td>32.35</td>
<td>9.41</td>
<td>24.56</td>
<td>9.86</td>
</tr>
</tbody>
</table>

Note: Deposits = costumer deposits; Interbank = interbank deposits; LT debt = long-term debt
Source: Bankscope
Figure 11: Volumes of repos cleared by CCPs in 2011 (millions of euro)

Source: Statistical Data Warehouse (ECB)
**B  Econometric model**

**B.1  Identification of liquidity shock and statistical tests**

Figure 12: Example of Repo Clear Margin Rate Circular
Table 3: Granger causality tests

<table>
<thead>
<tr>
<th>Hypothesis test</th>
<th>Result</th>
<th>F statistics</th>
<th>Critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do CDS spreads Granger-cause variations in haircut ?</td>
<td>No</td>
<td>2.25</td>
<td>3.88</td>
</tr>
<tr>
<td>Do yield spreads Granger-cause variations in haircut ?</td>
<td>No</td>
<td>0.40</td>
<td>3.88</td>
</tr>
</tbody>
</table>

Note: The number of lags is selected using the Bayesian Information Criterion (BIC) considering a maximum of 10 lags. Tests are performed at the significance level of 0.05. If the F-statistics is lower than the critical value, we accept the null hypothesis that variable X does not Granger-causes variable Y.

Figure 13: Confidence interval construction for threshold

Note: The threshold test statistics is plotted for the regression $h_t = \alpha_0 + \alpha_1 CDS_t$. The figure displays a graph of the normalized likelihood sequences as a function of the threshold ($CDS_t$). The dotted line plots the 95% critical value.
Figure 14: Repo haircut and 500 bp threshold of CDS spread

Note: The blue line plots the haircuts on 10-year Irish government bonds applied the LCH Clearnet Ltd. The dash red vertical lines represent the episodes when the CDS spread of Irish government bonds breached the 500 basis point threshold.

B.2 Bayesian estimation

Given N different variables in a vector \( y_t = (y_{1t}, ..., y_{Nt})' \), consider the following VAR:

\[
y_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + u_t
\]

where \( t = 1, \ldots, T \) and \( u_t \sim (0, \Sigma_u) \). Each equation has \( M = Np + 1 \) regressors. By collecting the coefficient matrices in the \( N \times M \) matrix \( A = [A_c, A_1, ..., A_p] \) and defining \( x_t = (1, y_{t-1}', ..., y_{t-p}')' \) as a vector containing an intercept and \( p \) lags of \( y_t \), the VAR can be written as

\[
Y = X\Phi + A
\]

where \( Y = [y_1, ..., y_T]' \), \( X = [x_1, ..., x_T]' \) and \( A = [A_1, ..., A_T]' \) are matrices of dimension \( T \times N \), \( T \times M \) and \( T \times N \) respectively. Defining the following OLS estimations \( \hat{A} = (X'X)^{-1}X'Y \), \( \hat{a} = vec(\hat{A}) \), \( \hat{S} = (Y - X\hat{A})'(Y - X\hat{A}) \), and \( \hat{\Sigma}_u = \), we consider the non-informative version of the
natural conjugate prior:

$$\alpha | \Sigma, y \sim N(\alpha^*, V^*)$$

$$\Sigma^{-1}|y \sim W(S^{-1}, T - K)$$

where $$V^* = \Sigma_u \otimes (X'X)^{-1}$$ and $$a^* = \hat{a} + Q$$. Q is the Choleski factor of $$V^*$$, i.e. $$V^* = QQ'$$.
C Additional Model Details and Derivations

C.1 Firms

Competitive final good producers combine differentiated intermediate goods \( Y_{it} \), for \( i \in [0, 1] \) into a single homogeneous final good \( Y_t \), using a constant return to scale technology

\[
Y_t = \left[ \int_0^1 Y_{it}^{1+\lambda_f} \, di \right]^{1+\lambda_f} \tag{30}
\]

where \( \lambda_f > 0 \). They take input prices \( P_{it} \) and output prices \( P_t \) as given and their demand for the generic \( i^{th} \) intermediate good is

\[
Y_{it} = \left[ \frac{P_{it}}{P_t} \right]^{-\frac{1}{\lambda_f}} Y_t \tag{31}
\]

The zero profit condition for competitive final-goods producers implies that the aggregate price level is

\[
P_t = \left[ \int_0^1 P_{it}^{-\frac{1}{\lambda_f}} \, di \right]^{-\lambda_f} \tag{32}
\]

The monopolist firm \( i \) hires labor services and rent capital from households to produce the intermediate good using the following technology

\[
Y_{it} = K_{it}^\alpha H_{it}^{1-\alpha} \tag{33}
\]

where \( \alpha \in (0, 1) \). \( K_{it} \) denotes the capital services and \( H_{it} \) the quantity of labor hired by the \( i^{th} \) intermediate-good producer, which sets prices \( P_{it} \) subject to Calvo (1983) scheme frictions, taking the rental rate of capital \( r_t \) and the real wage \( \frac{W_t}{P_t} \) as given. With probability \( 1 - \zeta_p \), the firm can reset its price and with probability \( \zeta_p \) cannot. By the law of large numbers, the probability of changing the price corresponds to the fraction of firms that rest the price, so each period a randomly selected fraction of firms \( 1 - \zeta_p \) can reoptimize the price \( \tilde{P}_{it} \) to maximize the present discount value of profits.
C.2 Labor market

The labor market mirrors the structure of the good market. Competitive labor agencies aggregate differentiated labor inputs into a homogeneous single labor service $H_t$ according to the technology

$$H_t = \left[ \left( \frac{1}{1 - \gamma} \right)^{\frac{\lambda_w}{1 + \lambda_w}} \int_{\gamma}^{1} H_t(j) \, \frac{1}{1 + \lambda_w} \right]^{1 + \lambda_w}$$  \hspace{1cm} (34)

where $\lambda_w > 0$. Labor agencies sell labor services $H_t$ to intermediate good producers for the nominal wage rate $W_t$. The first order condition for labor services determines the demand curve for the $j^{th}$ labor type:

$$H_t(j) = \frac{1}{1 - \gamma} \left[ \frac{W_t(j)}{W_t} - \frac{1 + \lambda_w}{\lambda_w} \right] H_t$$  \hspace{1cm} (35)

Labor unions represent all types of workers and set the wage rate $W_t(j)$ for the specific labor input $j$ taking as given the demand for their specific labor input and subject to the Calvo scheme frictions on a staggered basis, taking as given the demand for their specific labor input. Each period, labor agencies are able to reset the wage $W_t(j)$ with probability $1 - \zeta_w$ and with probability $\zeta_w$ they cannot and the wage remains fixed. By the law of large number, the probability of changing the wage corresponds to the fraction of workers whose wages change. Households supply whatever labor is demanded at that wage. If labor agencies can modify the wage, they will choose the wage $\tilde{W}_t$ to maximize their utility function.

C.3 Capital-good producers

The creation of new capital is delegated to competitive capital-good producers who transform consumption goods into investment goods. They choose the amount of investment goods to maximize the profits taking the price of investment goods $p^I_t$ as given

$$D_t^I = \left\{ p^I_t - \left[ 1 + \frac{I_t}{T} \right] I_t \right\} I_t$$  \hspace{1cm} (36)

The price of investment goods differ from the price of consumption goods because of the adjust-
ment costs, which depends on the deviations of actual investment from its steady-state value. \( \Gamma(.) I_t \) reflects the adjustment cost and \( \Gamma(.) \) is a measure of technology illiquidity, capturing the difficulty to undo investment. We assume that \( \Gamma(1) = \Gamma'(1) = 0 \) and \( f''(1) > 0 \). The first order condition for this problem is

\[
p_t^I = 1 + \Gamma \left( \frac{I_t}{I} \right) + \Gamma \left( \frac{I_t}{I} \right) \frac{I_t}{I} \tag{37}
\]

### C.4 Optimality conditions in good markets and labor market

At each period \( 1 - \zeta_p \) intermediate goods firms set the price \( \tilde{P}_{it} \) to maximize the present discounted value of profits

\[
D_{it+k} = P_{it+k} Y_{it+k} - w_{t+k} H_{it+k} - r_{t+k} K_{it+k}
\]

subject to the demand for its own good (31) and conditional on not changing its price. Intermediate good producers, first choose the optimal amount of inputs (capital and labor) and they minimize the costs, \( w_t H_{it} - r_t K_{it} \), subject to the production of intermediate goods (33). The first order condition is

\[
\frac{K_{it}}{H_{it}} = \frac{K_t}{H_t} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t} \tag{39}
\]

Since the marginal capital-labor ratio is independent of firm-specific variables, then the marginal cost, \( mc_{it} \), i.e. the Lagrange multiplier on the constraint, is also independent of firm-specific variables

\[
mc_{it} = mc_t = \left( \frac{r_t}{\alpha} \right)^\alpha \left( \frac{w_t}{1-\alpha} \right)^{1-\alpha} \tag{40}
\]

In a second step, the \( (1 - \zeta_p) \) firms that can change the price, will choose \( \tilde{P}_{it} \) to maximize

\[
\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} C_s^{-\sigma} \left[ \frac{\tilde{P}_{it}}{P_s} - (1 + \lambda_f) mc_s \right] Y_s(i) = 0 \tag{41}
\]
We focus on a symmetric equilibrium in which all firms choose the same price $\tilde{P}_t = \bar{P}_t$. Let $\tilde{p}_t = \tilde{P}_t/P_t$ the optimal price level. The first order condition for optimal price settings becomes

$$E_t \sum_{s=t}^{\infty} \left( \beta \zeta_p \right)^{s-t} C_s^{-\sigma} \left[ \frac{\tilde{p}_{it}}{\pi_s} - (1 + \lambda_f)mc_s \right] \left( \frac{\tilde{p}_{it}}{\pi_{t,s}} \right) Y_s = 0$$

By the law of large numbers, the probability of changing the price coincides with the fraction of firms who change the price in equilibrium. From the zero profit condition, 32, inflation depends on the optimal reset price according to

$$1 = (1 - \zeta_p)\tilde{p}_t^{1+\gamma} + \zeta_p \left( \frac{1}{\pi_t} \right)$$

Finally, since the ratio of capital-output is independent of firm-specific factors, the aggregate production function is

$$K_t^{1-\gamma} H_t^{1-\gamma} = \int_0^1 Y_{it} \, di = \sum_{s=0}^{\infty} \zeta_p (1 - \zeta_p)^{t-s} \left( \frac{\tilde{p}_{t-s}}{\pi_{t-s}} \right)^{-\frac{s-t}{1+\gamma}} Y_t$$

where $K_t = \int_0^1 K_i \, di$ and $H_t = \int_0^1 H_i \, di$. Each period, $1 - \zeta_w$ labor agencies are able to reset the wage $W_t(j)$ to minimize the present discount value of disutility of work

$$E_t \sum_{s=t}^{\infty} \left( \beta \zeta_p \right)^{s-t} \left[ C_s^{1-\sigma} \frac{\xi}{1-\sigma} \int_0^1 H_s(j)^{1+\nu} \, dj \right]$$

subject to (20 and 35). The first order condition for this problem is

$$E_t \sum_{s=t}^{\infty} \left( \beta \zeta_p \right)^{s-t} C_s^{-\sigma} \left[ \frac{W_{t+s}(j)}{P_s} - (1 + \lambda_w) \frac{\omega H_s(j)^{\nu}}{C_s^{1-\sigma}} \right] H_s(j) = 0$$

We focus on a symmetric equilibrium in which all agencies choose the same wage. Let $w_t = \frac{W_t}{P_t}$.

From equation 35 the law of motion of real wage is

$$w_t^{1+\gamma} = (1 - \zeta_w)\tilde{w}_t^{1+\gamma} + \zeta_w \left( \frac{\tilde{w}_{t-1}}{\pi_t} \right)^{-\frac{1}{1+\gamma}}$$

58
C.5 Market-clearing conditions and equilibrium definitions

The market-clearing conditions for composite labor and capital use are

\[ H_t = \int_0^1 H_{it} di \]

and

\[ K_t = \int_0^1 K_{it} di \]

The law of motion of capital is

\[ K_{t+1} = \lambda K_t + I_t \] (48)

We consider the following identity equation between capital and net equity

\[ K_{t+1} = N_{t+1} \] (49)

The resource constraint can be expressed as

\[ Y_t = C_t + \left[ 1 + \Gamma \left( \frac{I_t}{T} \right) \right] I_t \] (50)

Finally, considering the aggregate expression for \( D_t \) and \( D^l_t \) the investment function can be written as

\[ I_t = \chi \frac{r_t N_t + [1 + \lambda \phi_t Q^{L^t}_t] \frac{B^L_t}{p^L_t} + Q^{S^t}_t \frac{B^S_t}{p^S_t} + Y_t - w_t H_t - r_t K_t + p^l_t I_t - I_t[1 + \Gamma \left( \frac{I_t}{T} \right)] - T_t}{p^l_t - \theta q_t} \] (51)

To solve the model we define \( L_{t+1} = \frac{B^L_t}{p^L_t} \) as real long-term bonds. The liquidity parameter \( \phi_t \) follows an exogenous AR(1) process and there are 4 endogenous state variables: the aggregate capital stock, the nominal short-term bond, the real long-term bond and the real wage rate from the previous period \( (K_t, B^S_t, L_t, w_{t-1}) \). The recursive competitive equilibrium is defined as 9 endogenous quantities \( (I_t, C_t, Y_t, K_{t+1}, N_{t+1}, B^S_{t+1}, L_{t+1}, H_t, T_t) \) and 11 prices \( (q_t, Q^{L^t}_t, Q^{S^t}_t, \ldots) \).
\( p_t^I, r_t, R_t, \bar{w}_t, w_t, \bar{p}_t, \pi_t, mc_t \) as a function of state variables \((K_t, B_t^S, L_t, w_{t-1}, \phi_t)\), which satisfies the 19 equilibrium conditions (19, 21, 21, 21, 21, 21, 21, 22, 23, 37, 48, 49, 50, 24, 25, 26, 27, 38, 39, 41, 42, 43, 45, 46). Once all market clearing conditions and the government budget constraint are satisfied, the household budget constraint is satisfied by Walras’ Law.

### C.6 Steady state

In the steady state economy there is change in the resaleability of bonds, nominal price level, prices and endogenous variables. The steady state version of the Euler conditions are respectively

\[
\beta^{-1} = \frac{r + \lambda q}{q} + \frac{\gamma(q - 1)r}{q(1 - \theta q)} \quad (A.1)
\]

\[
\beta^{-1} = 1 + \frac{\lambda Q_L}{Q^L} + \frac{\gamma(q - 1)}{q(1 - \theta q)} \frac{1 + \lambda \phi Q_L}{Q^L} \quad (A.2)
\]

\[
\beta^{-1} = \frac{1}{Q^S} + \frac{\gamma(q - 1)}{q(1 - \theta q)} \frac{1}{Q^S} \quad (A.3)
\]

where in the steady state \( p_t^I = 1 \) because \( \Gamma(1) = \Gamma'(1) = 0 \). The non-arbitrage condition between short-term and long-term bonds in steady state implies

\[
\frac{1}{Q^S} = \frac{1 + \lambda Q_L}{Q^L} \quad (A.4)
\]

The capital-labor ratio is

\[
\frac{K}{H} = \frac{\alpha}{1 - \alpha} \frac{w}{r} \quad (A.5)
\]

Since in the steady state all firms charge the same price, \( \bar{p} = 1 \) and real marginal cost is equal to the inverse of markup

\[
mc = \left( \frac{r}{\alpha} \right)^{\alpha} \left( \frac{w}{1 - \alpha} \right)^{1 - \alpha} = \frac{1}{1 + \delta_f} \quad (A.6)
\]

Plugging these two equations into the production function at the steady state we obtain the
capital-output ratio which is a function of the rental rate of capital.

\[
\frac{Y}{K} = \frac{(1 + \zeta f)s}{\alpha}
\]

Equation (A.6) can be rewritten as a function of the rental rate

\[
w = (1 - \alpha) \left( \frac{1}{1 + \delta f} \right)^{\frac{1}{\delta f}} \left( \frac{\alpha}{\gamma f} \right)^{\frac{1}{\gamma f}}
\]

In the steady state, the real wage is equal to a markup over the marginal rate of substitution between labor and consumption

\[
w = (1 + \delta w) \left[ \frac{H (1 - \gamma)}{\gamma - \sigma} \right]
\]

Assuming that \(B^S = 0\) and considering \(K = N\), the investment function in steady state is

\[
I = \gamma \frac{rK + (1 - \lambda\phi Q^L)B^L + \delta f}{1 - \theta q} - T
\]

Steady-state investment is the depreciated steady-state capital

\[
\frac{I}{K} = (1 - \lambda)
\]

The resource constraint is

\[
Y = C + I
\]

Finally, from the government budget constraint the steady state tax is

\[
T = B(Q^B - \lambda Q^B - 1)
\]

C.7 Log-linear approximation

Let \(\hat{x}_t = \log \left( \frac{x_t}{\bar{x}} \right)\), where \(x\) is the steady state value of \(x_t\). The log-linearized equilibrium conditions are the following:
Investment

\[(1 - \gamma)\lambda p_i^t + (1 - \theta q)\lambda L_t - \theta \lambda q \hat{q}_t - \gamma \lambda \phi q \hat{Q}_L - \gamma(1 + \lambda Q L) \frac{L}{K} \hat{L}_t + \gamma(1 + \lambda Q L) \frac{L}{K} \hat{\pi}_t - \gamma Q S \hat{B}_S - \gamma r \hat{N}_t + \gamma T \hat{K}_t = 0\]

(B.1)

Euler equation for equity

\[-\sigma \hat{C}_t = -\sigma \mathbb{E}_t[\hat{C}_{t+1}] - \dot{q}_t + \beta \frac{r}{q} \left(1 + \gamma \frac{q - 1}{1 - \theta q}\right) \mathbb{E}_t[\hat{\pi}_{t+1}] + \beta \gamma r \left(1 - \frac{1}{1 - \theta q}\right) \mathbb{E}_t[\hat{r}_{t+1}]
- \beta \gamma r \left(1 - \frac{1}{1 - \theta q}\right) \mathbb{E}_t[\hat{q}_{t+1}]\]

(B.2)

Euler equation for long-term bonds

\[-\sigma \hat{C}_t = -\sigma \mathbb{E}_t[\hat{C}_{t+1}] - \hat{Q}_L - \mathbb{E}_t[\hat{\pi}_{t+1}] + \beta \lambda \gamma \frac{q - 1}{1 - \theta q} \phi \mathbb{E}_t[\hat{q}_{t+1}] + \beta \left[\lambda + \gamma \frac{q - 1}{1 - \theta q}\right] \mathbb{E}_t[\hat{Q}_{L_{t+1}}]
- \beta \left[\gamma \left(1 - \frac{1}{1 - \theta q}\right) \mathbb{E}_t[\hat{q}_{t+1}]\right] - \beta \left[\gamma \left(1 - \frac{1}{1 - \theta q}\right) \mathbb{E}_t[\hat{q}_{t+1}]\right]\]

(B.3)

Euler equation for short-term bonds

\[-\sigma \hat{C}_t = -\sigma \mathbb{E}_t[\hat{C}_{t+1}] - \hat{Q}_S - \mathbb{E}_t[\hat{\pi}_{t+1}] + \beta \gamma \frac{(1 - \theta)q}{1 - \theta q^2} \mathbb{E}_t[\hat{q}_{t+1}] - \beta \gamma \frac{(1 - \theta)q}{1 - \theta q^2} \mathbb{E}_t[\hat{q}_{t+1}]\]

(B.4)

The resource constraint

\[\hat{Y}_t = \frac{L}{Y} \hat{L} + \frac{C}{Y} \hat{C}_t\]

(B.5)

The marginal cost

\[\hat{mC}_t = (1 - \alpha) \hat{w}_t + \alpha \hat{r}_t\]

(B.6)

The Phillips curve

\[\hat{\pi}_t = \frac{(1 - \zeta_f \beta)(1 - \zeta_f)}{\zeta_f} \hat{mC}_t + \beta \mathbb{E}_t[\hat{C}_{t+1}]\]

(B.7)
The capital-labor ratio

\[
K_t = \hat{w}_t - \hat{r}_t + \hat{H}_t \tag{B.8}
\]

The law of motion for aggregate wages

\[
\hat{w}_t = (1 - \zeta_w)\hat{w}_t + \zeta_w(\hat{w}_{t-1}) \tag{B.9}
\]

Wage-setting decisions

\[
\left(1 + \eta \frac{1 + \delta_w}{\delta_w}\right)\hat{w}_t - (1 - \zeta_w \beta)\eta \frac{1 + \delta_w}{\delta_w} \hat{w}_t = (1 - \zeta) \left(1 + \eta \frac{1 + \delta_w}{\delta_w}\right) E_t[\hat{w}_t + \hat{\pi}_{t+1}] \tag{B.10}
\]

Aggregate production function

\[
\hat{Y}_t = \alpha \hat{K}_t + (1 - \alpha)\hat{H}_t \tag{B.11}
\]

The first order condition for capital producers

\[
\hat{p}_t = \Gamma''(1)\hat{I}_t \tag{B.12}
\]

Identity condition of capital

\[
K_{t+1} = (1 - \lambda)\hat{I}_t + \lambda \hat{K}_t \tag{B.13}
\]

Government budget constraint

\[
\frac{T}{K} \hat{l}_t = \frac{L}{K} (1 + \lambda Q^L)\hat{L}_t - \frac{L}{K} (1 + \lambda Q^L)\hat{\pi}_t + \hat{B}_t^S + (1 + \lambda)(Q^L L)\hat{Q}_t^L + Q^L L \hat{L}_{t+1} + Q^S \hat{B}_{t+1} \tag{B.14}
\]

Tax rules

\[
\frac{T}{K} \hat{l}_t = \psi_T \left[ \frac{L}{K} (\hat{L}_t - \hat{\pi}_t) \right] \tag{B.15}
\]
The interest rule rules

\[ \hat{R}_t = \psi \hat{\pi}_t \]  \hspace{1cm} (B.15)

Government rule for issuing short-term bonds

\[ \hat{B}_t^S = \psi \hat{\phi}_t \]  \hspace{1cm} (B.15)

Price of short-term bonds

\[ \hat{R}_t^S = -\log(Q^s) \]  \hspace{1cm} (B.15)