Labor market frictions, self-employment and productivity

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Abstract

Unemployment and self-employment/entrepreneurship are two central, closely linked features of urban labor markets in low income economies. They matter both for individual welfare and for aggregate productivity. Yet, the links between the two are poorly understood. Existing models of unemployment are almost exclusively geared towards advanced economies, very rarely address self-employment, and are therefore ill-suited for this context. To address this gap, this paper builds and quantitatively analyzes a model of unemployment and entrepreneurship with labor market frictions and firm heterogeneity.

The quantitative analysis is guided by evidence from urban Ethiopia, a very poor economy with high rates of unemployment and self-employment. It reveals important connections between labor market frictions, self-employment, and productivity. First, making self-employment less attractive not only reduces the self-employment rate, but firm entry overall. It also pushes some otherwise self-employed individuals into the unemployment pool. While firm sizes rise, so does the unemployment rate. Second, reducing labor market frictions not only affects unemployment, as in the standard DMP model. It also motivates the self-employed to switch to job search, thereby reducing the self-employment rate by more than the unemployment rate. The larger pool of job seekers contributes to raising the size of firms and output per person engaged, as there is a shift in the composition of employment and output from self-employment to more productive employer firms. Self-employment and unemployment are thus intimately linked. Labor market interventions affect not only unemployment, but also self-employment and aggregate productivity.

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Ongoing work goes beyond the analysis of a single country and aims to determine how important labor market frictions are in accounting for the observed large variation in self-employment across countries.

Keywords: unemployment, self-employment, occupational choice, entrepreneurship, firm size, productivity

1 Introduction

Labor markets in low income countries differ fundamentally from those in advanced economies. Besides employees and job searchers, they feature a very large number of self-employed or own account workers, many small and few large firms, and a great importance of short term, casual jobs. In Addis Ababa, the capital of Ethiopia, for example, the unemployment rate exceeds 20%, and the entrepreneurship rate (self-employed plus employers) exceeds the unemployment rate. Casual or temporary jobs are almost as prevalent as permanent ones. (See the next Section for details.) The high unemployment rate and the low incomes of the self-employed imply that understanding these outcomes and the potential effects of policies is important. Yet, existing models of labor markets have been tailored to advanced economies, and therefore are not suitable for this purpose. In particular, they do not address self-employment or casual employment. In this paper, I extend the standard Diamond-Mortensen-Pissarides (DMP) search and matching model to be able to capture the high rates of self-employment and casual work and the importance of small firms typical of low income countries.

The first contribution of the paper thus consists in providing the minimum extension of the DMP model required to match key features of low income labor markets. For this, I add three features to the model: 1. occupational choice between wage work or job search on the one hand and entrepreneurship on the other hand, 2. firm heterogeneity, and 3. casual jobs. The first feature clearly is needed to be able to say anything about entrepreneurship and self-employment. The second feature is necessary for a meaningful distinction between the self-employed and employers, and also allows addressing the determinants of the small size of firms in low income economies. The third feature captures the importance of casual jobs in a very simple way.

I then calibrate the model using Ethiopian data, and use it for a quantitative analysis of the determinants of high unemployment and entrepreneurship rates, and the potential impact of policy. The quantitative analysis also highlights how labor market frictions and the possibility of self-employment affect aggregate output and productivity.

Quantitative results are preliminary. At this stage, there are two key takeaways. First,
self-employment not only is a crucial option for many individuals, but also has important aggregate implications. For example, taxing the self-employed and redistributing the revenue lump sum results not only in less self-employment, but also in less firm entry overall, and thus less hiring, lower wages, and higher unemployment. While the greater number of job seekers does make employer firms larger and more numerous, the newly appearing firms are not particularly productive. This combined with higher unemployment leads to a large decline in output. Self-employment in poor countries thus matters not only for individuals, but also for aggregate outcomes, and contributes in a quantitatively significant way to alleviating the effect of frictions in the labor market.

Second, labor market frictions have a strong effect on self-employment here – more so than on the unemployment rate. Lower hiring costs or more efficient matching encourage firm entry and vacancy posting. More firms become employers, and average firm size increases. As a consequence, there are fewer, larger firms in equilibrium. Particularly noticeable is the decline in the self-employment rate. It is larger than that in unemployment. For example, for a decline of expected hiring costs by 23%, the unemployment rate declines by 4.5 percentage points, while the self-employment rate falls by 6.2 percentage points. Unemployment declines less because the effect of more hiring by firms is partially compensated by the larger number of job seekers.

There thus is a strong two-way relationship between self-employment and unemployment. Potential job seekers or entrants compare the two options, so that their relative attractiveness has a strong effect on the number of people engaging in each activity. This implies that improving labor market functioning in low income economies can have multiple benefits: not only reduced unemployment, but also a lower incidence of low-profit own-account work. Making self-employment less attractive, in contrast, can strongly reduce the number of own-account workers, but will lead to a run-up of unemployment unless attractive employment options can easily be found.

The paper is organized as follows. The next Section provides context by presenting key features of low income labor markets, and briefly discussing related literature. Section 3 presents the model. Quantitative results are shown in Section 4. Appendices contain additional details on theory and numerical methods.
2 Context

2.1 The labor market in a low income economy

Table 1 shows the distribution of labor force status and private sector employment across categories in Addis Ababa, the capital and major city of Ethiopia, in 2015. First, it is evident that a large share of individuals reports to be unemployed. Among the employed, less than a quarter of those working in the private sector hold permanent employment of the type typical in advanced economies. A group that is just as large is engaged in casual or temporary employment, while an even larger group consists of “entrepreneurs,” a term that I will use to refer to the union of employers and the self-employed or own-account workers. Evidently, permanent private sector jobs, while considered desirable, are only one of several common forms of employment. Entrepreneurship is just as important, and many workers are engaged in casual work. Anecdotal evidence suggests that this work often serves to finance job search, another very common activity.

Table 1: Distribution of labor force status and private sector employment status in 2015 in Addis Ababa, Ethiopia

<table>
<thead>
<tr>
<th>Labor force status</th>
<th>share (%)</th>
<th>Employment status</th>
<th>share (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employed</td>
<td>47.4</td>
<td>Private permanent</td>
<td>23.7</td>
</tr>
<tr>
<td>Unemployed</td>
<td>14.2</td>
<td>“Entrepreneurs”</td>
<td>32.8</td>
</tr>
<tr>
<td>Not in the labour force</td>
<td>38.3</td>
<td>Self-employed</td>
<td>17.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Employers</td>
<td>16.0</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>23.1</td>
<td>Private casual or temporary</td>
<td>22.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Domestic</td>
<td>12.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Other</td>
<td>8.9</td>
</tr>
</tbody>
</table>


1 Data is from the Urban Employment and Unemployment Survey for 2015 collected by the Ethiopian Central Statistical Agency.

2 This figure is in line with that directly reported by the Ethiopian CSA for Addis Ababa, for example in Central Statistical Agency of Ethiopia (2015). Note that both figures use a “relaxed” definition of unemployment, as advocated by the Ethiopian CSA as appropriate for the local context. The relaxed concept includes persons without work who are available for work, but not actively searching. According to ILO figures, which use a narrower concept, the median unemployment rate in Ethiopia over the period 2004 to 2014 is 16%. Another difference is that this includes rural areas, where unemployment is systematically lower for low income economies.
Due to scarcity of panel data, less is known about flows. For the case of Ethiopia, work by Bigsten, Mengistae and Shimeles (2007) shows significant flows across employment categories over the four-year interval from 2000 to 2004.

Figure shows that firms are generally small. This is certainly true for firm sizes reported in household-level data (left panel, showing the distribution from 2012 UEUS data). But it is even the case in manufacturing, where firms are typically larger (right panel, using data from CSA Large and Medium Manufacturing Industries database).

![Figure 1: The firm size distribution in the Ethiopian economy](image)


Figures are similar in other years and similar countries. For illustration, Table shows data on average firm employment in a few countries, combining local data sources and data from the GEM used by Poschke (2014). Self-employment rates are also higher in poorer economies. (This is documented more systematically by Gollin (2007).)

What is important is that the data refer to urban labor markets, since both private sector employment and unemployment are much less common in rural areas. (Numbers from LFS.) Low income economies are increasingly urbanized (Gollin, Jedwab and Vollrath 2016),

3While the World Bank’s Living Standards Measurement Study has a panel component, it is too small to be informative for the case of urban Ethiopia.

4See for example earlier waves of the UEUS, or the 2013 Labor Force Survey. Over the last decade (with the exception of 2016), economic growth in Ethiopia has been rapid, with increasing urbanization and a slight increase in the size of the private sector. This makes Ethiopia in 2015 more similar to other East African economies.
Table 2: Average firm sizes and self-employment rates in selected countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Average employment</th>
<th>Self-employment rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethiopia (Addis Ababa, UEUS)</td>
<td>2.9</td>
<td>25.9</td>
</tr>
<tr>
<td>Uganda (GEM)</td>
<td>2.0</td>
<td>31.7</td>
</tr>
<tr>
<td>Chile (GEM)</td>
<td>5.8</td>
<td>12.3</td>
</tr>
<tr>
<td>UK (GEM)</td>
<td>6.3</td>
<td>8.5</td>
</tr>
<tr>
<td>USA (SUSB and CPS)</td>
<td>11.9</td>
<td>8.0</td>
</tr>
</tbody>
</table>

Note: GEM data is for firms with < 250 employees.

making the study of their urban labor markets an important topic. However, there is little work that can guide policy in this area. Notably, models of labor markets in advanced economies, which typically inform policy analysis there, like the DMP model and other search models, are of little help for a context where permanent private sector employment is not the model form of employment.

2.2 Existing work

There are a few recent papers that study search and matching models with an informal sector or with self-employment. Zenou (2008), Ulyssea (2010), Bosch and Esteban-Pretel (2012), and Meghir, Narita and Robin (2015) analyze firms’ choice of formality versus informality, with a focus on Latin America. They typically assume that both formal and informal firms operate a constant returns to scale technology (so no meaningful firm size distribution arises), and focus on the share of formal jobs and on the aggregate implications flowing from the often greater productivity of formal firms.

Albrecht, Navarro and Vroman (2009), Margolis, Navarro and Robalino (2012) and Narita (2014) are more closely related to this paper, in that they also allow for self-employment. However, they all assume that self-employment has a constant (potentially heterogeneous) payoff, and that self-employment opportunities arrive at an exogenous rate. The exogenous arrival rate implies that the self-employment rate can be endogenous only via a selection effect. This limits variation in the self-employment rates, and it directly ties it to variation in the quality of self-employment projects. Rud and Trapeznikova (2016) also allow for self-employment, but do not model occupational choice. They assume that workers who do not find a job in a constant-returns sector engage in self-employment.

Finally, Kredler, Millan and Visschers (2014) and Delacroix, Fonseca, Poschke and Ševčík (2016) study the joint determination of unemployment and self-employment over the business
cycle in the United States, Canada and Europe.

There also are a few interesting papers studying search behavior and self-employment in developing economies at the micro level. Both Franklin (2016) and Abebe, Caria, Fafchamps, Falco and Franklin (2016) find that reducing search frictions at the individual level improves job search outcomes. At the same time, they document that job search is a distinct activity from casual work or self-employment. Blattman and Dercon (2016) show that unpleasant jobs are often taken temporarily, to finance search for better jobs or future self-employment, and that self-employment is considered desirable.

More broadly related is also Gollin (2007), who quantitatively analyzes variation in the self-employment rate with income across countries, but does not address labor market frictions. Fonseca, Lopez-Garcia and Pissarides (2001) do allow for such frictions, but only study the qualitative effect of business start up costs on entrepreneurship and unemployment.

3 Theory

The main objective of the paper is to develop a benchmark model that can account for key features of labor markets in very poor countries. As shown above, these features are high rates of unemployment, subsistence self-employment, and temporary or casual work, as well as few and relatively small employer firms. This section sets out a model that can generate these features.

I base the model on the Diamond-Mortensen-Pissarides (DMP) model of random search and matching in labor markets, a workhorse model for the analysis of labor markets in developed economies. I set up a version of the model with occupational choice: the unemployed can choose between job search and entering entrepreneurship, and the employed can choose to leave their jobs for entrepreneurship. Entering entrepreneurs do not know the productivity of the firm they are starting in advance. This generates a post-entry split into the self-employed (own-account workers) and employers, who in turn differ in optimal firm size. The assumption of uncertainty about post-entry productivity is in line with the literature on firm dynamics, and is motivated by the large rates of turnover of young firms. The operation of employer firms is modeled in a way similar to Stole and Zwiebel (1996), Cahuc, Marque and Wasmer (2004) and Elsby and Michaels (2013). The unemployed periodically engage in casual work to sustain their job search.

This is the minimum set of features that need to be added to the DMP model to be able to reproduce the above-mentioned facts, and to study the effects of productivity and frictions on unemployment, self-employment, and firm sizes. Clearly, endogenizing the entrepreneur-
ship rate requires giving model agents the ability to choose between entrepreneurship and employment or job search.\footnote{There is a set of work where workers or the unemployed are assumed to enter self-employment or entrepreneurship at an exogenous, fixed probability, e.g. Albrecht et al. (2009), Margolis et al. (2012) and Narita (2014). With post-entry heterogeneity, the self-employment rate in these papers is not necessarily constant. Nonetheless, the fixed entry rate limits how much it can vary.} Allowing for firm heterogeneity allows capturing the difference between own-account workers and employer firms, and it also allows for an effect of frictions not only on the quantity of entrepreneurs, but also on their quality. It also enables the analysis to address the observed small size of firms in low income economies. Finally, casual jobs are introduced in a simple way to reflect how they allow the unemployed to sustain job search for prolonged periods of time.

### 3.1 Model environment

The economy consists of $I$ ($I \in \mathbb{N}_+$) segmented markets indexed by $a_i$ ($i = 1, 2, \ldots, I$). Individuals in $I_a$ are endowed with productive ability (homogeneous across agents within $I_a$) and are born unemployed. They are 4 types of individuals: (i) Firm owner $e_f$, (ii) the Self-Employed $e_s$ (iii) Workers $n$ and (iv) the Unemployed $u$. We also use the same notation $(e_f, e_s, n, u)$ to denote the fractions of individuals in each sector:

$$I_a = e_f(a) + e_s(a) + n(a) + u(a)$$

and

$$\int I_a f(a) \, da = 1$$

where $f$ is the (continuous/discrete) density of $a$.

In this version, we assume $I = 1$ so that

$$e_f(a) + e_s(a) + n(a) + u(a) = 1$$

Hereafter, dependence on $a$ is suppressed unless necessary.

An agent values consumption, has a discount rate of $r$ and dies (or retires) with a fixed rate $\phi$ every period.

**Workers.** Workers start their life unemployed. In any period, the unemployed may either search for a job or start a firm. Similarly, the employed may work in their job, or leave it to start a firm.

Employed workers receive a wage $w$ (more on this below). The unemployed receive...
an exogenous utility flow $b$. To model the presence of casual jobs, we assume that with probability $\delta$, they need to engage in a casual job to sustain themselves. Such a job yields the same utility flow $b$, but does not allow for finding a regular job. If they do not need to take a casual job, they can choose between job search and firm entry.

**Firms.** There are two types of firms, the self-employed and employers. All firms produce a homogeneous good that they sell in a perfectly competitive market. Their production function is

$$y_f(z, n) = z(a \cdot n)^\gamma, \quad \gamma \in (0, 1).$$

Firm-level productivity $z$ differs across firms. It is revealed upon entry, and is constant thereafter. The parameter $\gamma$ captures the degree of decreasing returns to scale in production. In this setting, optimal firm employment is an endogenous, determinate object that depends on the expected wage, labor market tightness, and on a firm’s productivity. The model can thus generate employers of different sizes, which coexist with self-employed/own-account workers.

The self-employed produce with the production function

$$y_s(z) = \zeta z a^\gamma, \quad \zeta \in (0, 1).$$

Note that employers need to hire workers to produce output. Effectively, the maintained assumption is that they spend all their time managing production, and therefore cannot contribute any production labor input of their own. The self-employed in contrast can spend a fraction of their time that is parameterized by $\zeta$ on production, and the rest on administrative tasks related to managing their business.

Active firms of any type may be forced to exit for two reasons: retirement of the owner (with probability $\phi$ in any period), or an exogenous destruction shock, which occurs with probability $\lambda_f$ or $\lambda_s$ in any period.

**The labor market.** Employers hire workers in a frictional labor market by posting vacancies at a per period cost $k_v$. Every period, they choose how many vacancies to post. Given constant firm-level productivity and constant, linear hiring costs (coming from labor market frictions), it is optimal for firms to move to optimal employment directly upon entry. Later on, firms need to replace departing workers. They thus hire for these two reasons.

In the labor market, they meet unemployed workers. The labor market matching tech-
nology is defined by the following Cobb-Douglas function:

\[ M = A \cdot u^\mu v^{1-\mu} \]

where \( A \) is a matching efficiency parameter, \( v \) denotes the number of vacancies, and \( u \) the number of unemployed workers in this labor market.

Let labor market tightness \( \theta \equiv v/u \). Then a firm’s probability of filling a vacancy is \( q(\theta) \equiv A\theta^{-\mu} \), and an unemployed worker’s job finding probability is \( \theta q(\theta) \). Subsequently, any match in a continuing firm may be destroyed with probability \( \xi \).

Upon matching, a firm and a worker bargain over the wage. We adopt the same Nash bargaining protocol for bargaining with “large” firms as Stole and Zwiebel (1996), Cahuc et al. (2004) and Elsby and Michaels (2013).

**Occupational choice.** In any period, workers and the unemployed choose whether to work in their job or search for a job, or whether to start a firm. To start a firm, they need to pay a fixed entry cost \( k_f \). They then receive a draw of productivity \( z \) from a density \( f_U(z) \) or \( f_W(z) \), respectively. After seeing this draw, they decide whether to become an employer firm and hire workers, whether to become self-employed, or whether to go back to the unemployment pool.

In general, the model equilibrium could feature entry by both some of the unemployed and some workers, only by the unemployed, or only by workers. Which type of equilibrium obtains depends on parameter values. In the quantitative part, we focus on the more realistic equilibrium with entry by both groups. When discussing the theoretical implications of the model, we sometimes discuss the simpler equilibria.

We denote the endogenous fraction of the unemployed (workers) who start a firm in any period by \( \lambda_U \) (\( \lambda_W \)).

**Flows.** There are several endogenous and exogenous flows in the model. The unemployed who decide to search for a job face a probability of \( \delta \) of having to engage in a casual job and returning to unemployed. If they can actually search, they find a job with the endogenous probability \( \theta q \). The fraction of unemployed who choose to start a firm, \( \lambda_U \), is also endogenous, as is the fraction \( \lambda_W \) of workers who start a firm.

Workers who continue in their job can lose it for a variety of reasons. They may retire (and leave the model) with probability \( \phi \). Their match may be destroyed (and the firm continue) with probability \( \xi \), or the entire firm exit (and with it all matches) with probability \( \lambda_f \).

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6 In an extension, we allow workers to go back to their previous job with some probability.
Timing. The following summarizes the timing of events in this economy.

1. If individuals chose to enter, \( z \) is realized (drawn from \( f_W \) for workers and \( f_U \) for the unemployed).

2. Depending on \( z \), they decide whether
   
   (a) to keep the business and post vacancies to reach the optimal employment level,
   
   (b) to be self-employed,
   
   (c) or to exit and go to the unemployment pool.

3. Shocks \((\phi, \lambda_f, \lambda_s, \xi, \delta, \theta \cdot q(\theta), \delta)\) are realized.
   
   - A fraction \( \phi \) of individuals retire. When this hits workers, they leave their firm. When it hits an employer, the firm shuts down.
   
   - A fraction \( \lambda_f \) of firms shut down and the owners and its workers become unemployed.
   
   - A fraction \( \lambda_s \) of the self-employed close the business and they become unemployed.
   
   - Workers leave the existing firms with probability \( \xi \).

4. Value functions are measured and occupational choices take place.

5. Production takes place and payoffs \((w, b, -k_f)\) are realized.

3.2 Agents’ problems and value functions

Every period, agents choose whether to start a firm or not. As a consequence, an equilibrium with entry by both workers and the unemployment requires that for workers, the value of working \( W \) equals the value of starting a firm \( Q_W \), and for the unemployed, the value of job search \( U \) equals that of starting a firm \( Q_U \).

The Unemployed. The values of job search, \( U \), and casual employment, \( U \), are given by

\[
U = b + \frac{1 - \phi}{1 + r} \left[ \theta q(\theta)W + (1 - \theta q(\theta))(\delta U + (1 - \delta)U) \right] \tag{3}
\]

\[
\underline{U} = b + \frac{1 - \phi}{1 + r} [\delta U + (1 - \delta)U] \tag{4}
\]

All unemployed workers, while searching or on casual jobs, have a utility flow \( b \). Job search yields a job next period with probability \( \theta q \). If no job is found, job search is possible
in the period thereafter with probability $1 - \delta$ (equation 3). Casual workers cannot search. In the following period, search will be possible again with probability $1 - \delta$ (equation 4).

Instead of job search, job searchers could also start a firm, which yields value $Q_U$. Since workers are ex ante identical, in an equilibrium with entry by the unemployed it must be true that $U = Q_U$. If this holds, an endogenous fraction $\lambda_U$ of the unemployed start a firm.

**Workers.** The value of working is

$$W = w + \frac{1 - \phi}{1 + r} \left[ (1 - \phi) (1 - \lambda_f) (1 - \xi) W + (1 - (1 - \phi) (1 - \lambda_f) (1 - \xi)) ((1 - \delta)U + \delta U) \right]$$

Workers receives a wage $w$. They keep their job if their firm does not close down and their job is not destroyed. If they lose their job, they become unemployed and either take a casual job or search for a new job in the following period.

As for the unemployed, workers could also start a firm, earning value $Q_W$. In an equilibrium with entry by workers, $W = Q_W$, and an endogenous fraction $\lambda_W$ of workers enter.

**New Entrants into Entrepreneurship.** Entrants pay an entry cost $k_f$ and receive a productivity level $z$. The distribution they draw from depends on their previous employment status. Since $W > U$ in an equilibrium with production, entry by workers only occurs if workers draw from a “better” distribution than the unemployed. Quantitatively, we capture this by assuming that the mean of productivity draws for former workers exceeds that for those coming from unemployment by an exogenous amount $h$. One can think of this as a productivity advantage of entrepreneurs who bring skills or customers from their previous job with them. Once $z$ is realized, entrepreneurs can become an employer, self-employed, or exit.

Employers pay each employee a wage $w$, post vacancies to maintain the optimal labor input $n(z)$, and are subject to an exogenous retirement probability $\lambda_f$ ($\lambda_s$ for the self-employed). The value of running a firm with productivity $z$ at its optimal employment is

$$F_f(z) = \frac{1 + r}{(1 + r) - (1 - \phi)(1 - \lambda_f)} \left( z(an(z))^{\gamma} - n(z)w \right)$$

$$- \frac{k_v}{q(\theta)} n(z) \left( \lambda_W + (1 - \lambda_W)(\xi + (1 - \xi)\phi) \right) + \frac{(1 - \phi)\lambda_f}{(1 + r) - (1 - \phi)(1 - \lambda_f)} U.$$  

Note that this is measured one period ahead because only successful entrants get to produce.
The first term in the second row denotes the cost of hiring workers who depart either for exogenous reasons ($\xi$ or $\phi$) or to start their own firm ($\lambda_W$).

The self-employed do not use outside labor (i.e. $n = 1$), and spend only part of their time on production (reflected in the term $\zeta$). The value of self-employment then is

$$F_s(z) = \frac{1 + r}{(1 + r) - (1 - \phi)(1 - \lambda_s)} \cdot a^\gamma + \frac{(1 - \phi)\lambda_s}{(1 + r) - (1 - \phi)(1 - \lambda_s)} U$$

Entrants can also exit. They then return to the unemployment pool.

Combining these possibilities, the value of entry for each type of individual is given by

$$Q_U = -k_f + \frac{1 - \phi}{1 + r} \left[ \int \max \left( F_f(z) - \frac{k_v}{q(\theta)} n(z), F_s(z), U \right) f_{U,a}(z) dz \right]$$

$$Q_W = -k_f + \frac{1 - \phi}{1 + r} \left[ \int \max \left( F_f(z) - \frac{k_v}{q(\theta)} n(z), F_s(z), U \right) f_{W,a}(z) dz \right]$$

The term subtracted from $F_f(z)$ reflects that firms need to hire up to their optimal employment to obtain $F_f$, which was defined at optimal employment.

![Figure 2: The values of unemployment ($U$), self-employment ($F_s$), and the value of being an employer net of hiring costs ($F_f^{net}(z) = F_f(z) - n(z)k_v/q$), with associated productivity cutoffs](image)

It is clear that $U$ is independent of $z$. With the production structure here, $F_s(z)$ increases linearly in $z$. The net value of entry for employers is increasing and convex in $z$. As a result,
continuation choices as a function of $z$ are as depicted in Figure 2. Entrants with very high levels of productivity become employers. Those with very low levels exit, and those with intermediate levels become self-employed. (This structure is analogous to that in Gollin (2007).) Define two cutoffs: entrants with $z > z_S$ become employers, and those with $z < z_R$ exit. This implies that exit occurs with probability $d_W$ or $d_U$, and entrants become employers with probability $p_W^S$ or $p_U^S$. Formally, these probabilities and cutoffs are defined as follows:

$$z^S = \inf \left\{ z : F_f(z) - \frac{k_v}{q(\theta)} n(z) > F_s(z) \right\}$$

$$p_W^S = \Pr(W) \left( \left\{ z : F_f(z) - \frac{k_v}{q(\theta)} n(z) > F_s(z) \right\} \right)$$

$$p_U^S = \Pr(U) \left( \left\{ z : F_f(z) - \frac{k_v}{q(\theta)} n(z) > F_s(z) \right\} \right)$$

$$d_W = \Pr(W) \left( \{ z : F_s(z) < U \} \right)$$

$$d_U = \Pr(U) \left( \{ z : F_s(z) < U \} \right).$$

### 3.3 Wage Determination

Wage is determined by the Nash bargaining between workers and firms with workers’ bargaining weight $\eta$ (as in Stole and Zwiebel (1996), Cahuc et al. (2004) and Elsby and Michaels (2013)). It can be shown (see Appendix B.1 for detailed derivations) that

$$w = \frac{(r + \phi)}{(1 + r)} U + \frac{\eta}{1 - \eta} \frac{\{(1 + r) - (1 - \phi)(1 - \lambda_f)(1 - \xi)\}}{(1 + r)} R(\lambda_W, d_W) \frac{k_v}{q} \quad (10)$$

where

$$R(\lambda_W) = \frac{(1 + r)(1 - \phi)(1 - \lambda_f)\{1 - (1 - \lambda_W) (\xi + (1 - \xi) \phi) - \lambda_W\}}{(1 + r) - (1 - \phi)(1 - \lambda_f)\{1 - (1 - \phi)(1 - \lambda_f)\{(1 - \lambda_W) (\xi + (1 - \xi) \phi) + \lambda_W\}\}}$$
3.4 Laws of motion

Given the flows described above, the distribution of individuals evolves according to the following law of motion:

\[
\begin{bmatrix}
egef_f & 
egef_s & 
n_f & 
u
\end{bmatrix}
= \begin{bmatrix}
(1 - \phi) (1 - \lambda_f), & 0, & (1 - \phi) \lambda_W, & (1 - t) (1 - \phi) \lambda_U \\
0, & (1 - \phi) (1 - \lambda_s), & (1 - \phi) \lambda_W (1 - p^s_W), & (1 - t) (1 - \phi) \lambda_U (1 - p^s_U) \\
0, & 0, & (1 - \phi) \lambda_W, & (1 - t) (1 - \phi) \\
(1 - \phi) \lambda_f + \phi & (1 - \phi) \lambda_s + \phi & \{1 - (1 - \phi) \lambda_W\} & \{\phi + (1 - \phi) (1 - \lambda_U)\} \\
& & (1 - \phi) (1 - \lambda_f) (1 - \xi) & \{t + (1 - t) (1 - \theta \cdot q(\theta))\}
\end{bmatrix} \\
+ \begin{bmatrix}
0, & 0, & 0, & 0 \\
0, & 0, & -(1 - \phi) \lambda_W d_W, & -(1 - t) (1 - \phi) \lambda_U \\
0, & 0, & 0, & 0 \\
0, & 0, & (1 - \phi) \lambda_W d_W & (1 - t) (1 - \phi) \lambda_U 
\end{bmatrix} \\
\equiv T_{-1} \begin{bmatrix}
\negef_f \negef_s \nn_f \\ 
u
\end{bmatrix} + T_{-2} \begin{bmatrix}
\negef_f \negef_s \nn_f \\ 
u
\end{bmatrix}
\]

This process incorporates that a measure $\phi$ of young individuals newly enter unemployment every period. The process is a Markov chain of order 2, not 1, to allow for the case where transitions for failed entrepreneurs, who spend one period in $e_s$, depend on their previous employment status. (Recall though that this is not the main scenario here.)

Productivity distributions are as follows. The productivity distribution of employers who entered from employment and unemployment, respectively, are

\[
\begin{align*}
e^{(W)}_f (z) &= f_{W,a} (z) \mathbf{1}\{z > z^S\} \\
e^{(U)}_f (z) &= f_{U,a} (z) \mathbf{1}\{z > z^S\}
\end{align*}
\]

where $\mathbf{1}$ is the indicator function. Integrating over $z$ gives

\[
\Delta_{f,W} = \lambda_W \int_{z > z^S} f_{W,a} (z) \, dz \cdot n \\
\Delta_{f,U} = (1 - \delta) \lambda_U \int_{z > z^S} f_{U,a} (z) \, dz \cdot u
\]
Then, the productivity distribution of entrepreneurs is

\[ e_f(z) = \left( \frac{\Delta f, W}{\Delta f, W + \Delta f, U} e_f^{(W)}(z) + \frac{\Delta f, U}{\Delta f, W + \Delta f, U} e_f^{(U)}(z) \right) e_f \]

Similarly, for the self-employed,

\[ e_s^{(W)}(z) = \lambda_W f_{W,a}(z) 1 \{ z > z_R, z < z^S \} \]
\[ e_s^{(U)}(z) = \lambda_U f_{U,a}(z) 1 \{ z > z_R, z < z^S \} \]

and

\[ \Delta s, W = \lambda_W \int_{z_R}^{z^S} f_{W,a}(z) \, dz \cdot n \]
\[ \Delta s, U = (1 - \delta) \lambda_U \int_{z_R}^{z^S} f_{U,a}(z) \, dz \cdot u \]

Then,

\[ e_s(z) = \left( \frac{\Delta s, W}{\Delta s, W + \Delta s, U} e_s^{(W)}(z) + \frac{\Delta s, U}{\Delta s, W + \Delta s, U} e_s^{(U)}(z) \right) e_s \]

### 3.5 Equilibrium

In this section, I define an equilibrium for the empirically relevant case with entry by both workers and the unemployed. Definitions for the other two scenarios, with entry by only one of the two groups, are analogous.

A stationary equilibrium with entry by both workers and the unemployed consists in values \( W, U, U', F_f(z), F_s(z), Q_W, Q_U \) and \( (\theta, w, \lambda_W, \lambda_U, (p^S_W, p^S_U), (d_W, d_U)) \) such that

1. values \( W, U, U', F_f(z), F_s(z), Q_W, Q_U \) are given by equations 3 to 9
2. wages fulfill equation 10
3. households are indifferent between occupational choices: \( Q_W = W \) and \( Q_U = U \), and \( \lambda_W > 0 \) and \( \lambda_U > 0 \),
4. firms post vacancies optimally:

\[ v(z) = [(1 - \lambda_W)(\phi + (1 - \phi) \xi) + \lambda_W] \frac{n(z)}{q} \]
for continuing firms and $v(z) = n(z)/q(\theta)$ for new entrants. Optimal employment $n(z)$ is

$$n(z) = (z\gamma a)^{\frac{1}{1-\gamma}} \left( (c_1(\gamma - 1) + 1) \left( \frac{k_v}{q} + w \right) \right)^{-\frac{1}{1-\gamma}},$$

with

$$c_1 = \frac{(1 + r) - (1 - \phi)^2(1 - \lambda_f)(1 - \xi)}{(1 - \eta)(1 + r) + \eta R((1 + r) - (1 - \phi)^2(1 - \lambda_f)(1 - \xi))}$$

gives optimal employment of a firm with productivity $z$.

5. the equilibrium distribution $[e_f, e_s, n, u]'$ is generated by household choices and is stationary:

$$\begin{bmatrix} e_f \\ e_s \\ n \\ u \end{bmatrix} = T_{-1} \begin{bmatrix} e_f \\ e_s \\ n \\ u \end{bmatrix} + T_{-2} \begin{bmatrix} e_f \\ e_s \\ n \\ u \end{bmatrix},$$

where $T_{-1}$ and $T_{-2}$ are given in the previous section, and $e_f(z)$ and $e_s(z)$ are as above.

6. Labor market tightness is generated by unemployment in- and outflows and by firms’ vacancy posting decisions:

$$\theta = \frac{\bar{\lambda}}{q(\theta)} \int n(z)e_f(z)dz \frac{1}{(1 - \delta)u}.$$

where

$$\bar{\lambda} = \phi + (1 - \phi)\lambda_f + (1 - \phi)(1 - \lambda_f)[(1 - \lambda_W)(\phi + (1 - \phi)\xi) + \lambda_W]$$

captures hiring by new and continuing firms.

The key equilibrium variables are $\theta, w, \lambda_U$ and $\lambda_W$. The value functions $W, U$ and $F_s$ depend only on $w$ and $\theta$. Hence, same holds for $z_R$ and $d_W, d_U$. Firm value $F_f$ in addition depends on $\lambda_W$. This carries over to $z_S$ and $p^{S}_W, p^{S}_U$. The value of entry then also depends on $(\theta, w, \lambda_W)$. Finally, $\lambda_U$ matters for the equilibrium unemployment rate and number of employers, and thus for $\theta$.

The equilibrium for the case with entry only by the unemployed can be analyzed graphically, since $\lambda_W$ is fixed at zero here. The other two cases are more complicated, because $\lambda_W$ is also endogenous. While the type of equilibrium depends on parameters, the intuition
gained from the case with entry by the unemployed only is still helpful for the other cases, too.

Figure 3: A sketch of equilibrium determination

The equilibrium can be depicted as three conditions that need to hold simultaneously, and that determine the three key equilibrium variables. (See Figure 3.) The first two conditions are depicted in the top panel. This panel plots the wage curve, which is familiar from the standard DMP model, and the occupational choice (OC) condition. The former shows that workers can bargain higher wages when the labor market is more tight (see equation 10). The latter depicts the combinations of $\theta$ and $w$ at which the equilibrium condition $Q_U = U$
holds. Since $U$ increases in both $\theta$ and $w$, while the value of entry declines in both $\theta$ and $w$, it is clear that this locus is negatively sloped. Since neither the wage curve and nor the OC condition directly depend on $\lambda_U$, these two conditions determine $\theta$ and $w$.

The third key condition, the job creation (JC) condition, then determines the entry rate $\lambda_U$. In this model, where instead of free entry and constant returns there is occupational choice and decreasing returns, this condition is given by equation (11), defining equilibrium $\theta$. The bottom panel depicts equilibrium $\theta$ from the top panel and the JC condition. As usual, job creation declines in the wage. Higher tightness raises the left hand side of the JC condition more than the right hand side. Finally, given $\theta$ and $w$, a larger entry rate $\lambda_U$ implies a larger number of active employer firms, and thus more vacancies. Since $\theta$ raises the LHS of (11) and $\lambda_U$ raises the RHS, the JC curve implies a positive relationship of the two variables. Higher wages shift the curve up. Given $\theta$ from the upper panel, equilibrium $\lambda_U$ can be read off the JC curve in the lower panel.

With entry by both workers and the unemployed, the lower panel still can give an approximate indication of the entry rate, but not by source state.

The key comparative statics are familiar. Increases in all firms’ productivity raise all values. For a high enough probability that an entrant becomes an employer, the value of entry increases more than that of unemployment, and the upward shift in the OC curve dominates. Job creation increases for any $\theta, w$, shifting the JC curve right. Higher productivity also raises $F_f$ relative to $F_s$. As a result, $z_S$ declines, and a larger fraction of entrants become employers, implying a further rightward shift of JC. The rightward shift in OC indicates that the net effect is an increase in $\lambda_U$.

The richer model allows for analyzing some effects that are not present in the basic DMP model. For instance, higher entry costs imply lower $Q_U$ relative to $U$. Then lower $\theta$ is required for entry to occur (OC shifts left). This implies lower $\theta$ and $w$. Lower wages shift the JC curve down, implying a reduction in $\lambda_U$, and thus fewer firms. Higher entry costs thus have aggregate implications.

4 Quantitative exercise: labor market frictions, entrepreneurship and productivity

RESULTS IN THIS SECTION ARE PRELIMINARY.

In this section, I analyze the quantitative properties of the model. To do so, I first

Footnote: This is not true anymore when there is entry by workers, too, and $\lambda_W$ is endogenous. In that case, the OC curve also depends on $\lambda_W$. In practice, the effect is weak, so the graphical apparatus is still useful.
Table 3: Calibration: model and data moments

<table>
<thead>
<tr>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate</td>
<td>0.3</td>
</tr>
<tr>
<td>Job finding rate</td>
<td>0.046</td>
</tr>
<tr>
<td>Average employment duration</td>
<td>7.8</td>
</tr>
<tr>
<td>Casual employment</td>
<td>0.15</td>
</tr>
<tr>
<td>Labor income share</td>
<td>0.72</td>
</tr>
<tr>
<td>$b/w$</td>
<td>0.33</td>
</tr>
<tr>
<td>Entrepreneurship rate</td>
<td>0.42</td>
</tr>
<tr>
<td>Self-employment rate</td>
<td>0.34</td>
</tr>
<tr>
<td>Employers</td>
<td>0.08</td>
</tr>
<tr>
<td>Average firm employment</td>
<td>5.3</td>
</tr>
<tr>
<td>Share of small firms</td>
<td>0.97</td>
</tr>
<tr>
<td>4-year entrepreneurship persistence</td>
<td>0.56</td>
</tr>
</tbody>
</table>

calibrate the model using Ethiopian micro data. I then analyze the quantitative role of productivity and frictions in the model. In doing so, I consider counterfactual scenarios and also compare model predictions to cross-country data.

4.1 Calibration

Calibrating the model requires using statistics on rates of unemployment and self-employment, firm sizes, job finding rates, and other statistics on the Ethiopian economy. Given recent growth in the Ethiopian economy, it is important to use recent data. Therefore, I use the Urban Employment and Unemployment Survey (UEUS) for 2015 when possible, and the 2013 Labor Force Survey (LFS) otherwise. Both surveys are conducted by the Ethiopian Central Statistical Agency. Data processing is described in Appendix C. The model time period is set to one month.

As usual in such models, some parameters need to be calibrated outside the model. I set the interest rate such that the annual interest rate is 4%. I set the retirement probability $\phi$ such that the expected duration of working life is 30 years. I set $\mu$, the exponent on unemployment in the matching function, to 0.5. I set $\gamma$, the exponent on labor in the production function, to 0.69. Finally, I set the exogenous firm exit rates $\lambda_f$ and $\lambda_s$ such that the annual rate is 12%. This results in a four-year entrepreneurship to entrepreneurship transition rate of 56%, close to the level of 54% observed over the period 2000 to 2004 (Bigsten et al. 2007, Table 3).

The remaining parameters are calibrated internally to match a set of ten targets. Heuris-
Table 4: Calibration: parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>externally calibrated:</strong></td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>0.04/year</td>
</tr>
<tr>
<td>$\phi$</td>
<td>retirement probability 1/30 p.a.</td>
</tr>
<tr>
<td>$\mu$</td>
<td>matching function 0.5</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>decreasing returns to scale 0.69</td>
</tr>
<tr>
<td><strong>internally calibrated:</strong></td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>matching function 0.08</td>
</tr>
<tr>
<td>$k_v$</td>
<td>vacancy posting cost 6</td>
</tr>
<tr>
<td>$\xi$</td>
<td>separation rate 0.02</td>
</tr>
<tr>
<td>$\delta$</td>
<td>casual jobs 0.26</td>
</tr>
<tr>
<td>$\eta$</td>
<td>worker bargaining power 0.36</td>
</tr>
<tr>
<td>$b$</td>
<td>utility flow of unemployment 60</td>
</tr>
<tr>
<td>$k_f$</td>
<td>entry cost 4000</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>relative self-employment productivity 0.65</td>
</tr>
<tr>
<td>$\mu_z$</td>
<td>productivity distribution 260</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>productivity distribution 114</td>
</tr>
<tr>
<td>$\lambda_f, \lambda_s$</td>
<td>firm exit rates 0.12 p.a.</td>
</tr>
</tbody>
</table>

Typically, one can think of a mapping of targets to parameters as follows: First, the three key labor market parameters $A$, $k_v$ and $\xi$ are set to match the unemployment rate, the job finding rate, and average employment duration. (The first two are computed from the UEUS, the last one from the LFS. Note that the reported number is not average employment spell length, but average total length of employment in a respondent’s history.) The probability of taking a casual job is set to match the share of casual jobs in the data. Note that our model measuring convention is that individuals on casual jobs are not included in the unemployment rate, casual jobs are included in measured duration of unemployment spells.

Second, workers’ bargaining power $\eta$ and the utility flow of unemployment $b$ are set to generate a labor income share of around 0.7, in line with figures reported in Gollin (2002), and a ratio of $b/w$ of around 0.4.

The third set of moments relates to self-employment and entrepreneurship. Here, we set the parameters $k_f$, $\zeta$, $\mu_z$ and $\sigma_z$ to match the entrepreneurship rate, the self-employment rate, average employment, and the share of small firms ($n<10$).

Table 3 shows the model fit to these moments. In the current, preliminary version, the fit is not perfect, but acceptable. Table 4 shows the parameters generated by the calibration exercise.

The model also fits well in some dimensions that have not been targeted. Notably, the
Table 5: Four-year transition matrix between the states of entrepreneurship, employment and unemployment. Data values in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>(e')</th>
<th>(n')</th>
<th>(u')</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e)</td>
<td>0.556</td>
<td>0.112</td>
<td>0.207</td>
</tr>
<tr>
<td></td>
<td>(0.538)</td>
<td>(0.107)</td>
<td>(0.221)</td>
</tr>
<tr>
<td>(n)</td>
<td>0.184</td>
<td>0.337</td>
<td>0.353</td>
</tr>
<tr>
<td></td>
<td>(0.065)</td>
<td>(0.597)</td>
<td>(0.219)</td>
</tr>
<tr>
<td>(u)</td>
<td>0.142</td>
<td>0.337</td>
<td>0.396</td>
</tr>
<tr>
<td></td>
<td>(0.068)</td>
<td>(0.261)</td>
<td>(0.528)</td>
</tr>
</tbody>
</table>

Source: Bigsten et al. (2007). Remaining probability is retirement/transition out of the labor force.

transition matrix for the states of entrepreneurship, employment and unemployment fits the one from the data fairly closely (Table 5). The data matrix is adapted from Bigsten et al. (2007); see Appendix C for details. The first row of the matrix, showing persistence in entrepreneurship and exit rates to both employment (only indirect in the model) and unemployment, are replicated very closely by the model. In contrast, the model overstates entry rates into entrepreneurship, from both employment and unemployment, overstates employment to unemployment transitions, and understates unemployment persistence. This is due to the fact that the transition matrix is for the years 2000 to 2004, a period when the Ethiopian economy was very different from its current state.

The first column of Table 6 shows some equilibrium outcomes from the model, notably the entrepreneurship entry rates \(\lambda_W\) and \(\lambda_U\), the entry success rates \(d_W\) and \(d_U\), and labor market tightness \(\theta\). Monthly entry rates consistent with indifference between entrepreneurship entry and wage work/job search are positive by low. Still, they imply higher four-year transition rates into entrepreneurship than observed over the 2000 to 2004 period, and are the rates required to match observed entrepreneurship rates in 2015. The model somewhat overstates the rates of unemployment and entrepreneurship (the calibration is still very preliminary). The implied job finding rate for workers is 4.6%, essentially as targeted. While the hiring rate for firms is only 14%, the low vacancy posting cost implies that the expected cost of a hire corresponds to 24% of a monthly wage; much lower than evidence on this moment for advanced economies. The driver of high unemployment thus is not so much a high cost of

\[9\] More specifically, it reflects the fact that the ergodic distribution over entrepreneurship, employment and unemployment implied by the data transition matrix is [0.130,0.550,0.32], i.e. it implies much less entrepreneurship and higher unemployment than what is observed in more recent data. As a result, if the model is calibrated to match recently observed entrepreneurship and unemployment rates, it will generate more entrepreneurship entry and a lower persistence of unemployment than found in the data a decade earlier.
hiring for firms as a low efficiency of the matching process.

Overall, the model clearly replicates key features of the Ethiopian economy: high rates of unemployment and self-employment, and a preponderance of small or tiny firms.

4.2 The aggregate effects of self-employment opportunities

In the benchmark model, self-employment is a valuable choice for a substantial fraction of agents. What would the economy look like if this choice was not available, or much less attractive? For example, it is sometimes argued that the self-employed are not subject to the same rules and regulations as employers, because of differences in enforcement. In the model, this is reflected in the value of $\zeta$, which governs the relative productivity of employers and the self-employed. Table 6 shows two sets of results: One where $\zeta$ is reduced by 10%, as an example of an economy that is less favorable to the self-employed. And another one where $\zeta$ is set close to zero, to illustrate what an economy without self-employment would look like.

Making self-employment less profitable clearly reduces the value of entry. This results in lower tightness and lower wages. Lower tightness is achieved via a smaller number of firms, which now are larger. The unemployment rate rises, as more individuals opt for job search instead of self-employment, but firms post fewer vacancies. The size of the increase in the unemployment is remarkable. Entirely eliminating self-employment (23% of the labor force in the benchmark) leads to an increase in the unemployment rate by 8 percentage points. That is, one third of those giving up self-employment remain in the unemployment

<table>
<thead>
<tr>
<th></th>
<th>benchmark</th>
<th>reduce $\zeta$ by 10%</th>
<th>set $\zeta$ to zero</th>
</tr>
</thead>
<tbody>
<tr>
<td>tightness $\theta$</td>
<td>0.328</td>
<td>0.303</td>
<td>0.287</td>
</tr>
<tr>
<td>wage/wage$_{bm}$</td>
<td>1.000</td>
<td>0.960</td>
<td>0.934</td>
</tr>
<tr>
<td>aggregate output rel. to bm</td>
<td>1.000</td>
<td>0.898</td>
<td>0.856</td>
</tr>
<tr>
<td>$\lambda_W$</td>
<td>0.011</td>
<td>0.011</td>
<td>0.009</td>
</tr>
<tr>
<td>$\lambda_U$</td>
<td>0.005</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>unemployment rate</td>
<td>0.310</td>
<td>0.348</td>
<td>0.389</td>
</tr>
<tr>
<td>job finding rate</td>
<td>0.046</td>
<td>0.044</td>
<td>0.043</td>
</tr>
<tr>
<td>entrepreneurship rate</td>
<td>0.402</td>
<td>0.309</td>
<td>0.160</td>
</tr>
<tr>
<td>self-employment rate</td>
<td>0.324</td>
<td>0.215</td>
<td>0.000</td>
</tr>
<tr>
<td>employers</td>
<td>0.078</td>
<td>0.094</td>
<td>0.160</td>
</tr>
<tr>
<td>average employment</td>
<td>5.5</td>
<td>7.3</td>
<td>14.1</td>
</tr>
</tbody>
</table>
pool. Self-employment hence not only is an important option for individuals, but also has an important moderating influence on aggregate unemployment.

### 4.3 The effect of labor market frictions

I next come to the main exercise of the paper: how do labor market frictions affect occupational choices? To get a quantitative idea of this, I reduce the expected hiring cost for a worker in the model. This can be done in two ways: via lower vacancy posting costs $k_v$, or higher matching function productivity $A$. The difference is that for given $\theta$, the former approach only makes posting vacancies cheaper for firms, while the latter also raises the job finding probability for workers.

Results are shown in Table 7. The table shows the effects of both small and large changes in $k_v$ and $A$, where each time the change in $A$ is chosen to deliver the same reduction in expected hiring cost as the change in $k_v$.

Lower hiring costs make running a business more attractive. (OC shifts up in the top panel of Figure 3.) Lower costs of creating a match also reduce match surplus, shifting the wage curve down. The net effect is higher tightness, and an ambiguous change in the wage. (Quantitatively, the wage declines in all exercises with lower expected hiring cost that I undertook.) Lower hiring costs also prompt firms to post more vacancies, shifting JC right in the lower panel of Figure 3. The threshold $z_S$ declines, shifting JC further right. The low-hiring cost equilibrium features more, larger employer firms, and less self-employment.

<table>
<thead>
<tr>
<th></th>
<th>benchmark</th>
<th>reduce $k_v$ by 10%</th>
<th>reduce $k_v$ by 50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>tightness $\theta$</td>
<td>0.328</td>
<td>0.371</td>
<td>0.762</td>
</tr>
<tr>
<td>wage/wage$_{bm}$</td>
<td>1.000</td>
<td>0.994</td>
<td>0.985</td>
</tr>
<tr>
<td>aggregate output rel. to bm</td>
<td>1.000</td>
<td>1.007</td>
<td>1.125</td>
</tr>
<tr>
<td>$\lambda_W$</td>
<td>0.011</td>
<td>0.012</td>
<td>0.012</td>
</tr>
<tr>
<td>$\lambda_U$</td>
<td>0.005</td>
<td>0.003</td>
<td>0.000</td>
</tr>
<tr>
<td>unemployment rate</td>
<td>0.310</td>
<td>0.309</td>
<td>0.265</td>
</tr>
<tr>
<td>job finding rate</td>
<td>0.046</td>
<td>0.049</td>
<td>0.070</td>
</tr>
<tr>
<td>entrepreneurship rate</td>
<td>0.402</td>
<td>0.385</td>
<td>0.342</td>
</tr>
<tr>
<td>self-employment rate</td>
<td>0.324</td>
<td>0.306</td>
<td>0.262</td>
</tr>
<tr>
<td>employers</td>
<td>0.078</td>
<td>0.079</td>
<td>0.080</td>
</tr>
<tr>
<td>average employment</td>
<td>5.5</td>
<td>5.8</td>
<td>6.3</td>
</tr>
<tr>
<td>expected hiring cost</td>
<td>0.237</td>
<td>0.228</td>
<td>0.183</td>
</tr>
</tbody>
</table>
The increase in tightness is particularly favorable to job searchers, with the result that their entry rate declines, and falls to zero for large enough increases in the job finding rate. All results are similar when cheaper hiring is achieved via higher $A$ (not shown).

The effect of hiring cost on unemployment found here is of course standard. Aggregate output increases, by much more than the saved hiring costs. What is most striking is that lower hiring costs have a stronger effect on the self-employment rate than on the unemployment rate. These two outcomes are thus intimately linked via the labor market.

4.4 The effect of entry costs and aggregate productivity

4.5 A comparison to cross-country data

5 Conclusion

References


Poschke, M. (2014), ‘The firm size distribution across countries and skill-biased change in entrepreneurial technology’, *manuscript*.


Appendix

A Additional Tables and Figures

B Proofs and derivations

B.1 Detailed Derivation of Wage

Workers’ Surplus

\[ W = w + \frac{1 - \phi}{1 + r} \left[ \{1 - (1 - \phi) (1 - \lambda_f) (1 - \xi)\} U \right. \]

\[ + (1 - \phi) (1 - \lambda_f) (1 - \xi) W \]}

Rewrite this as

\[ \left( 1 - \left( \frac{(1 - \phi)^2 (1 - \lambda_f) (1 - \xi)}{1 + r} \right) \right) W = w + \frac{1 - \phi}{1 + r} \left\{ \{1 - (1 - \phi) (1 - \lambda_f) (1 - \xi)\} U \right. \]

\[ \Leftrightarrow \frac{(1 + r) - (1 - \phi)^2 (1 - \lambda_f) (1 - \xi)}{1 + r} W = w + \frac{1 - \phi}{1 + r} \left\{ \{1 - (1 - \phi) (1 - \lambda_f) (1 - \xi)\} U \right. \]

\[ \Leftrightarrow W = \frac{1 + r}{(1 + r) - (1 - \phi)^2 (1 - \lambda_f) (1 - \xi)} w + \frac{1 - \phi}{(1 + r) - (1 - \phi)^2 (1 - \lambda_f) (1 - \xi)} \left\{ \{1 - (1 - \phi) (1 - \lambda_f) (1 - \xi)\} U \right. \]

\[ \Leftrightarrow W - U = \frac{1 + r}{(1 + r) - (1 - \phi)^2 (1 - \lambda_f) (1 - \xi)} w - \frac{r + \phi}{(1 + r) - (1 - \phi)^2 (1 - \lambda_f) (1 - \xi)} U \]

Firm’s Surplus

\[ F' (n) = \frac{1 + r}{(1 + r) - (1 - \phi) (1 - \lambda_f)} R (\lambda_W, d_W) \left( a \cdot y' (n) - w - n \cdot w' (n) \right) \]

\[ F' (n) = \sum_{j=0}^{\infty} \left( \frac{(1 - \phi) (1 - \lambda_f)}{1 + r} \right)^j \left[ 1 - \{ (1 - \lambda_W) (\xi + (1 - \xi) \phi) + \lambda_W (\phi + (1 - \phi) \{1 - d_W + d_W (1 - \chi) \} (a \cdot y' (n) - w - n \cdot w' (n)) \right] \]
Observe

\[
\sum_{j=0}^{\infty} \left( \frac{(1 - \phi)(1 - \lambda_f)}{1 + r} \right)^j = \frac{1 + r}{(1 + r) - (1 - \phi)(1 - \lambda_f)}
\]

and

\[
\sum_{j=0}^{\infty} \left( \frac{(1 - \phi)(1 - \lambda_f) \{(1 - \lambda_W)(\xi + (1 - \xi)\phi) + \lambda_W(\phi + (1 - \phi)\{1 - d_W + d_W(1 - \chi + \chi\xi)\})\}}{1 + r} \right)^j = \frac{1 + r}{(1 + r) - (1 - \phi)(1 - \lambda_f) \{(1 - \lambda_W)(\xi + (1 - \xi)\phi) + \lambda_W(\phi + (1 - \phi)\{1 - d_W + d_W(1 - \chi + \chi\xi)\})\}}
\]

so that

\[
F'(n) = R(\lambda_W, \lambda_W) \left( a \cdot y'(n) - w - n \cdot w'(n) \right)
\]

where

\[
R(\lambda_W, \lambda_W) = \frac{1 + r}{(1 + r) - (1 - \phi)(1 - \lambda_f)}
\]

\[
= \frac{1 + r}{1 + r \left[ (1 + r) - (1 - \phi)(1 - \lambda_f) \{1 - (1 - \lambda_W)(\xi + (1 - \xi)\phi) + \lambda_W(\phi + (1 - \phi)\{1 - d_W + d_W(1 - \chi + \chi\xi)\})\} \right]}
\]

\[
= \frac{1 + r \{(1 + r) - (1 - \phi)(1 - \lambda_f)\} \{1 - (1 - \phi)(1 - \lambda_f)\} \{(1 - \lambda_W)(\xi + (1 - \xi)\phi) + \lambda_W(\phi + (1 - \phi)\{1 - d_W + d_W(1 - \chi + \chi\xi)\})\}}{(1 + r) - (1 - \phi)(1 - \lambda_f) \{(1 - \lambda_W)(\xi + (1 - \xi)\phi) + \lambda_W(\phi + (1 - \phi)\{1 - d_W + d_W(1 - \chi + \chi\xi)\})\}}
\]

**Nash Bargaining**

\[
(1 - \eta)(W - U) = \eta R(\lambda_W, \lambda_W) \left( a \cdot y'(n) - w - n \cdot w'(n) \right)
\]

\[
\Leftrightarrow (1 - \eta) \left( \frac{1 + r}{1 + r - (1 - \phi)^2 (1 - \lambda_f) (1 - \xi)} w - \frac{r + \phi}{(1 + r) - (1 - \phi)^2 (1 - \lambda_f) (1 - \xi)} U \right) = \eta R(\lambda_W, \lambda_W)
\]

\[
\Leftrightarrow \left[ \frac{(1 - \eta)(1 + r)}{(1 + r) - (1 - \phi)^2 (1 - \lambda_f) (1 - \xi)} + \eta R(\lambda_W, \lambda_W) \right] w = \frac{(1 - \eta)(r + \phi)}{(1 + r) - (1 - \phi)^2 (1 - \lambda_f) (1 - \xi)} U
\]

\[
+ \eta R(\lambda_W, \lambda_W) \left( a \cdot y'(n) - w - n \cdot w'(n) \right)
\]

\[
\frac{(1 - \eta)(1 + r) + \eta R(\lambda_W, \lambda_W) \{(1 + r) - (1 - \phi)^2 (1 - \lambda_f) (1 - \xi)\}}{(1 + r) - (1 - \phi)^2 (1 - \lambda_f) (1 - \xi)} w = \frac{(1 - \eta)(r + \phi)}{(1 + r) - (1 - \phi)^2 (1 - \lambda_f) (1 - \xi)}
\]

\[
+ \eta R(\lambda_W, \lambda_W) \left( a \cdot y'(n) - w - n \cdot w'(n) \right)
\]

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or

\[ w = \frac{(1-\eta)(r + \phi)}{(1-\eta)(1+r) + \eta R(\lambda_W, \lambda_W) \{(1+r) - (1-\phi)^2 (1-\lambda_f) (1-\xi)\}} U \]

\[ + \frac{\eta R(\lambda_W, \lambda_W) \{(1+r) - (1-\phi)^2 (1-\lambda_f) (1-\xi)\}}{(1-\eta)(1+r) + \eta R(\lambda_W, \lambda_W) \{(1+r) - (1-\phi)^2 (1-\lambda_f) (1-\xi)\}} \left( a \cdot y' (n) - n \cdot w' (n) \right) \]

Solve this differential equation to obtain

\[ w = \frac{\bar{C}}{1 - c_1} + \frac{c_1}{1 - c_1} \frac{k_v}{q(\theta)} \]

where

\[ \bar{C} = \frac{(1-\eta)(r + \phi)}{(1-\eta)(1-r) + \eta R(\lambda_W, \lambda_W) \{(1+r) - (1-\phi)^2 (1-\lambda_f) (1-\xi)\}} U \]

\[ c_1 = \frac{\eta R(\lambda_W, \lambda_W) \{(1+r) - (1-\phi)^2 (1-\lambda_f) (1-\xi)\}}{(1-\eta)(1+r) + \eta R(\lambda_W, \lambda_W) \{(1+r) - (1-\phi)^2 (1-\lambda_f) (1-\xi)\}} \]

so that

\[ w = \frac{r + \phi}{1 + r} U + \frac{\eta}{1 - \eta} \frac{(1+r) - (1-\phi)^2 (1-\lambda_f) (1-\xi)}{1+r} R(\lambda_W, \lambda_W) \frac{k_v}{q (\theta)} \]

and

\[ n (z) = \left\{ \left( \frac{c_1 (\gamma - 1) + 1}{z^{\gamma} a^\gamma} \right) \left( \frac{k_v}{q(\theta)} + q \right) \right\}^{-1/(1-\gamma)} \]
Monotonicity of $R(\lambda_W, d_W)$ w.r.t $\lambda_W$

\[
\frac{\partial R(\lambda_W, d_W)}{\partial \lambda_W} = \frac{(1+r)(-(\xi+(1-\xi)\phi)+\phi+(1-\phi)(1-d_W+d_W(1-\chi+\chi\xi)))}{(1+r)-(1-\phi)(1-\lambda_f)((1-\lambda_W)(\xi+(1-\xi)\phi)+\lambda_W\{\phi+(1-\phi)(1-d_W+d_W(1-\chi+\chi\xi)))}}
\]

\[
= \frac{1}{\{(1+r)-(1-\phi)(1-\lambda_f)((1-\lambda_W)(\xi+(1-\xi)\phi)+\lambda_W\{\phi+(1-\phi)(1-d_W+d_W(1-\chi+\chi\xi)))\}}
\]

\[
\left(\text{Numerator of } \frac{\partial R(\lambda_W, d_W)}{\partial \lambda_W}\right) = -(\xi+(1-\xi)\phi)+\phi+(1-\phi)(1-d_W+d_W(1-\chi+\chi\xi)))
\]

\[
= (1+r)\{((1+r)-(1-\phi)(1-\lambda_f)((1-\lambda_W)(\xi+(1-\xi)\phi)+\lambda_W\{\phi+(1-\phi)(1-d_W+d_W(1-\chi+\chi\xi)))\})
\]

\[
+ (1-\phi)(1-\lambda_f)\{(1+r)((1-\lambda_W)(\xi+(1-\xi)\phi)+\lambda_W\{\phi+(1-\phi)(1-d_W+d_W(1-\chi+\chi\xi)))\})
\]

\[
= -(\xi+(1-\xi)\phi)+\phi+(1-\phi)(1-d_W+d_W(1-\chi+\chi\xi)))
\]

\[
= (1+r)\left\{(1+r)-(1-\phi)(1-\lambda_f)\tilde{C}(\lambda_W)\right\}+(1-\phi)(1-\lambda_f)(1+r)\tilde{C}(\lambda_W)
\]

\[
= -(\xi+(1-\xi)\phi)+\phi+(1-\phi)(1-d_W+d_W(1-\chi+\chi\xi)))\right\}(1+r)^2
\]

where

\[
\tilde{C}(\lambda_W) = ((1-\lambda_W)(\xi+(1-\xi)\phi)+\lambda_W\{\phi+(1-\phi)(1-d_W+d_W(1-\chi+\chi\xi)))
\]

B.2 Derivation of Average Employment Duration $T(u)$

At any given period, individuals belong to one of the four sectors $e$ (entrepreneurs), $n$ (workers), and $u$ (the unemployed), and $o$ (the retired) and the assumption that they face a fixed probability of retirement $\phi$, implies that their employment duration in the future only depends on their current state but is independent of their past history on how long the individual has spent in each section. Denote by $T(s)$ ($s \in \{e, n, u, o\}$) the future employment duration of an individual given her/his current state is $s$. Once they are put in $o$, there will be no transition to any other sector; hence

\[
T(o) = 0
\]
Observe that an employment duration is incremented only if they transition to \( n (e \to n, n \to n, \text{or} u \to n) \). Hence, \([T(e), T(n), T(u)]\)' follows the following process:

\[
\begin{bmatrix}
    T(e) \\
    T(n) \\
    T(u)
\end{bmatrix} = \begin{bmatrix}
    \Pr(e \to e) & \Pr(e \to n) & \Pr(e \to u) \\
    \Pr(n \to e) & \Pr(n \to n) & \Pr(n \to u) \\
    \Pr(u \to e) & \Pr(u \to n) & \Pr(u \to u)
\end{bmatrix} \begin{bmatrix}
    T(e) \\
    T(n) + 1 \\
    T(u)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
    \Pr(e \to e) & \Pr(e \to n) & \Pr(e \to u) \\
    \Pr(n \to e) & \Pr(n \to n) & \Pr(n \to u) \\
    \Pr(u \to e) & \Pr(u \to n) & \Pr(u \to u)
\end{bmatrix} \begin{bmatrix}
    T(e) \\
    T(n) \\
    T(u)
\end{bmatrix} + \begin{bmatrix}
    \Pr(e \to n) \\
    \Pr(n \to n) \\
    \Pr(u \to n)
\end{bmatrix}
\]

Note that since \( T(o) = 0 \), \( T(o) \) can be omitted from the equation (the transition matrix already incorporates the retirement probability, i.e. \( \Pr(s \to e) + \Pr(s \to n) + \Pr(s \to u) = 1 - \phi \) for \( s = e, n, u \)).

C Data

D Numerical solution method

Our solution method consists of three stages (1) Given \((\theta, \lambda_W)\), solve the system of linear equations on \( W, U, \text{and} w \) for \((d_W, \{W, U, w\})\). (2) Given the first step, compute \( Q_W \) and \( Q_U \). (3) Find \((\theta, \lambda_W)\) with which the equilibrium conditions are met.

First Stage:

In the first step, given \( \theta \) and \( \lambda_W \), we solve for \((d_W, \{W, U, w\})\). Given \((\theta, \lambda_W, d_W)\), \((W, U, w)\) is the unique solution to the following system of linear equations:

\[
W = w + \frac{1 - \phi}{1 + r} \left\{ \left[1 - (1 - \phi)(1 - \lambda_f)(1 - \xi) \right] U + (1 - \phi)(1 - \lambda_f)(1 - \xi) W \right\}
\]

\[
U = b + \frac{1 - \phi}{1 + r} \left\{ \left[ (1 - \theta \cdot q(\theta)) U + \theta \cdot q(\theta) W \right] \right\}
\]

\[
w = \left( \frac{r + \phi}{1 + r} \right) U + \\
+ \frac{\eta}{1 - \eta} \frac{\left\{ (1 + r) - (1 - \phi)(1 - \lambda_f)(1 - \xi) \right\}}{(1 + r)} R(\lambda_W, d_W) \frac{k_v}{q}
\]

where

\[
R(\lambda_W) = \frac{(1 + r)(1 - \phi)(1 - \lambda_f) \{1 - (1 - \lambda_W)(\xi + (1 - \xi) \phi) - \lambda_W\}}{\{(1 + r) - (1 - \phi)(1 - \lambda_f)\} \{1 - (1 - \phi)(1 - \lambda_f)\} \{(1 - \lambda_W)(\xi + (1 - \xi) \phi) + \lambda_W\}}
\]
\[ R^* (\lambda_W, d_W) = \frac{\eta \{ (1 + r) - (1 - \phi) (1 - \lambda_f) (1 - \xi) \}}{1 - \eta} R(\lambda_W) \]

or

\[
\begin{bmatrix}
1 - \frac{(1 - \phi)^2 (1 - \lambda_f) (1 - \xi)}{1 + r}, & - (1 - \phi) \{ 1 - (1 - \phi) (1 - \lambda_f) (1 - \xi) \}, & -1 \\
- \frac{(1 - \phi) \theta \cdot q(\theta)}{1 + r}, & 1 - \frac{(1 - \phi)(1 - \theta \cdot q(\theta))}{1 + r}, & 0 \\
0, & - \frac{(r + \phi)}{1 + r}, & 1
\end{bmatrix}

\[
\begin{bmatrix}
W \\
U \\
w
\end{bmatrix}
= \begin{bmatrix}
0 \\
b \\
R^* (\lambda_W, d_W) \frac{k_v}{q}
\end{bmatrix}
\]

where

\[
R^* (\lambda_W, d_W) = \frac{\eta \{ (1 + r) - (1 - \phi) (1 - \lambda_f) (1 - \xi) \}}{1 - \eta} \frac{(1 + r) (1 - \phi) (1 - \lambda_f) \{ 1 - (1 - \lambda_W) (\xi + (1 - \xi) \phi) - \lambda_W (\phi + (1 - \phi) \{ 1 - d_W + d_W (1 - \chi) \}) (1 - \phi) (1 - \lambda_f) \{ 1 - (1 - \phi) (1 - \lambda_f) (1 - \xi) \} \} - \lambda_W (\phi + (1 - \phi) \{ 1 - d_W + d_W (1 - \chi) \}) (1 - \phi) (1 - \lambda_f) (1 - \xi) \} + \lambda_W (\phi + (1 - \phi) \{ 1 - d_W + d_W (1 - \chi) \}) (1 - \phi) (1 - \lambda_f) (1 - \xi) \} \}
\]

Let

\[
G(\theta) = \begin{bmatrix}
1 - \frac{(1 - \phi)^2 (1 - \lambda_f) (1 - \xi)}{1 + r}, & - (1 - \phi) \{ 1 - (1 - \phi) (1 - \lambda_f) (1 - \xi) \}, & -1 \\
- \frac{(1 - \phi) \theta \cdot q(\theta)}{1 + r}, & 1 - \frac{(1 - \phi)(1 - \theta \cdot q(\theta))}{1 + r}, & 0 \\
0, & - \frac{(r + \phi)}{1 + r}, & 1
\end{bmatrix}
\]

so that

\[
G(\theta) \begin{bmatrix}
W \\
U \\
w
\end{bmatrix} = \begin{bmatrix}
0 \\
b \\
R^* (\lambda_W, d_W) \frac{k_v}{q}
\end{bmatrix}
\]

Note that \( R^* (\lambda_W, d_W) \) is monotonic in \( \lambda_W \) hence \{\( W, U, w \) \( (\theta_i, \lambda_W ; d_W) \)\} is monotonic in \( \lambda_W \). This means that it suffices to check the boundary of \( \lambda_W \) (Appendix).

Given \( \theta \in \{ \theta_i \} \) and \( \lambda^*_W = \Delta_W, \lambda_W \), solve for \( (d_W, \{ W, U, w \}) \):

\[
d_W = \Pr_W (z < z_S (\theta, \lambda^*_W))
\]

\[
\begin{bmatrix}
W \\
U \\
w
\end{bmatrix} = [G(\theta)]^{-1} \begin{bmatrix}
0 \\
b \\
R^* (\lambda^*_W, d_W) \frac{k_v}{q}
\end{bmatrix}
\]

where

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\[ d_W(\theta_t) = \sum_{i=1}^{I_s} \Pr(W(z = z_i) 1(z_i) \left\{ \left[ 1 - \chi + \chi \left\{ 1 - (1 - \phi)(1 - \lambda_f) (1 - \xi) U \right\} \right] + \chi \left( 1 - \phi \right) (1 - \lambda_f) (1 - \xi) U \right\} \]

\[ F_s(z) = \frac{1 + r}{(1 + r) - (1 - \phi) (1 - \lambda_s)} (a^\gamma) + \frac{(1 - \phi) \lambda_s}{(1 + r) - (1 - \phi) (1 - \lambda_s)} U \]

**Second Stage:**
Given \( \theta \) and \( \lambda_W \) and using \((d_W, \{W, U, w\})\), one can compute \( Q_W \) and \( Q_U \).

**Value of Firms**

\[ F_f(z) = \frac{1 + r}{(1 + r) - (1 - \phi) (1 - \lambda_f)} \left( z \cdot (a \cdot n(z))^\gamma \right) - \left( w + ((1 - \lambda_W) (\xi + (1 - \xi) \phi) + \lambda_W \{ \phi + (1 - \phi) \right) \]

**Value of Entry by Workers**

\[ Q_W = \frac{1 - \phi}{1 + r} \left[ -k_f + \int \max \left( F_f(z) - \frac{k_u}{q(\theta)} n(z), F_s(z), \Lambda(U, W) \right) f_{W,a}(z) \right] \]

**Value of Entry by the Unemployed**

\[ Q_U = \frac{1 - \phi}{1 + r} \left[ -k_f + \int \max \left( F_f(z) - \frac{k_u}{q(\theta)} n(z), F_s(z), U \right) f_{U,a}(z) \right] \]

**Third Stage:**
3. gives a bound for \( \theta: [\theta_{min}, \theta_{max}] \). Find \((\theta_t, \lambda_W, d_W) \in [\theta_{min}, \theta_{max}] \times [0, 0.1] \times [0, 1] \) such that \( Q_W(\theta_t, \lambda_W) = W \) or (/and) \( Q_U(\theta_t, \lambda_W) = U \) holds.