Barcelona GSE Summer Forum

Balmes Building – Balmes 132, Barcelona

FRACTIONAL BROWNIAN MOTION AND ROUGH MODELS

June 8-9, 2017 Room 504

PROGRAM FOR THURSDAY, JUNE 8

08:45	*Registration and Welcome*
Session 1	
09:20-10:05	MIKKO PAKKANEN (Imperial College London) "Modelling and Forecasting Rough Volatility" (with Mikkel Bennedsen and Asger Lunde)
10:05-10:50	MATHIEU ROSENBAUM (Ecole Polytechnique) "Rough Heston Model: Pricing, Hedging and Microstructural Foundations" (with Omar El Euch)
10:50-11:20	*Coffee-break**
Session 2	
11:20-12:05	DAVID NUALART (Kansas University) "Functional Central Limit Theorem for the Self-Intersection Local Time of the fractional Brownian motion"
12:05-12:50	ARCHIL GULISASHVILLI (Ohio University) "Implied Volatility Skew in Rough Stochastic Volatility Models. Moderate Deviation Regime"
12:50-13:35	WALTER SCHACHERMAYER (University of Vienna) "Can Fractional Brownian Motion Touch a Semi-Martingale? A Phenomenon Arising in Mathematical Finance"
13:35-14:35	*Lunch**
Session 3	
14:35-15:00	RÉMI PEYRE (CNRS & Institut Elie Cartan de Lorraine) "Two-Way Crossing Property for fBm"
15:00-15:45	EHSAN AZMOODEH (University of Helsinki) "Convergence Towards the Second Wiener Chaos"
15:45-16:30	LAURE COUTIN (Université Paul Sabatier, Toulouse): "Invariance for Rough Differential Equations"
16:30-17:00	*Coffee-break**
Session 4	
17:00-17:25	CHRISTOPH CZICHOWSKY (London School of Economics) "Optimal Investment under Rough Volatility"
17:25-18:00	**POSTER SESSION:** GIULIA BINOTTO (Universitat de Barcelona) "Weak Symmetric Integrals with Respect to the Fractional Brownian Motion" (with Ivan Nourdin and David Nualart) BLANKA HORVATH (Imperial College London) "Short Dated Option Pricing with Rough Volatility" CHLOE LACOMBE (Imperial College London) "Asymptotic of Randomised (Fractional) Stochastic Volatility Models" (with Blanka Horvath and Antoine Jacquie) AITOR MUGURUZA (Imperial College London) "On Vix Futures in the Rough Bergomi Model"(with Antoine Jacquier and Claude Martini)

PROGRAM FOR FRIDAY, JUNE 9

Session 5	
09:20-10:05	PETER FRIZ (TU Berlin) "An Application of Regularity Structures to the Analysis of Rough Volatility"
10:05-10:50	IVAN NOURDIN (University of Luxembourg) Asymptotic Behavior of Berry Random Wave Model
10:50-11:20	*Coffee-break**
Session 6	
11:20-12:05	SAMY TINDEL (Purdue University) "Rate of Convergence to Equilibrium for Rough Differential Equations"
12:05-12:50	ANTHONY REVEILLAC (INSA Toulouse) "The Itô-Tanaka trick : a Non-Semimartingale Point of View" (with Laure Coutin and Romain Duboscq)
12:50-13:35	MARTIN GROTHAUS (TU Kaiserslautern) "Mittag-Leffler Analysis: Construction and Applications" (with Florian Jahnert, Felix Riemann and José Luis da Silva)
13:35-14:35	*Lunch**
Session 7	
14:35-15:00	MIREIA BESALÚ (Universitat Pompeu Fabra) "Convergence of Delay Differential Equations driven by a Fractional Brownian Motion for 1/3<H<1/2" (with Giulia Binotto and Carles Rovira)
15:00-15:45	CARLES ROVIRA (Universitat de Barcelona) "Some Results on Stochastic Differential Equations with Non-Negativity Constraint Driven by Fractional Brownian Motion"
15:45-16:30	JORGE LEÓN (CINVESTAV-IPN, Mexico) "Fractional Stochastic Differential Equations with Discontinuous Diffusion"

Workshop Organizers:

- ELISA ALÒS (UPF and Barcelona GSE)
- EULÀLIA NUALART (UPF and Barcelona GSE)

The organizers gratefully acknowledge the financial support of the Universitat Pompeu Fabra through the European Union programme FP7-PEOPLE-2012-CIG under grant agreement 333938 and the Spanish Ministry of Economy and Competitiveness, through the "Severo Ochoa" Programme for Centres of Excellence in R&D"

* Meals are provided by the organization

ABSTRACTS (alphabetical order):

Ehsan Azmoodeh (University of Helsinki): Convergence towards the second Wiener chaos

Abstract: We consider elements in the second Wiener chaos, such that their associated Hilbert-Schmidt operator has only finitely many non-zero eigenvalues, as target distributions. The talk deals with generalization of the Nourdin-Peccati analysis [4] for normal approximation. Such generalization immediately runs into several obstacles. We introduce an entirely analytic approach which avoid the classical strategy of solving the associated Stein equation. One of our results provides an upper bound for 2- Wasserstein distance, in terms of finitely many cumulants. Our results can be used to quantify the error in several non-central probabilistic limit statements. In particular, we discuss the behavior of the generalized Rosenblatt process at extreme critical exponent values.
References:
[1] Arras. B., Azmoodeh. E., Poly. G., Swan. Y. (2016) Stein's method on the second Wiener chaos: 2-Wasserstein distance. http://arxiv.org/abs/1601.03301.
[2] Azmoodeh. E., Peccati, G., Poly. G. (2014) Convergence towards linear combinations of chisquared random variables: a Malliavin-based approach. Séminaire de Probabilités (Special volume in memory of Marc Yor), 339-367.
[3] Bai, S., Taqqu, M. (2015) Behavior of the generalized Rosenblatt process at extreme critical exponent values. To appear in Ann. Probab.
[4] Nourdin. I., Peccati, G. (2012). Normal Approximations Using Malliavin Calculus: from Stein's Method to Universality. Cambridge Tracts in Mathematics. Cambridge University.

Mireia Besalú (Universitat Pompeu Fabra): Convergence of delay differential equations driven by a fractional Brownian Motion for 1/3<H<1/2

Abstract: We consider stochastic differential delay equations with hereditary drift driven by a fractional Brownian motion with Hurst parameter 1/3<H<1/2. We show that, when the delay goes to zero, the solutions to these equations converge to the solution for the equation without delay. The stochastic integral with respect to the fraction al Brownian motion can be expressed as a Lebesgue integral using the fractional derivative for the case H>1/2. We use the extension proposed by Hu and Nualart for the case where 1/3<H<1/2. This is a joint work with Giulia Binotto and Carles Rovira

Giulia Binotto (Universitat de Barcelona): Weak symmetric integrals with respect to the fractional Brownian motion.

Abstract: We establish the weak convergence, in the topology of the Skorohod space, of the ν-symmetric Riemann sums for functionals of the fractional Brownian motion when the Hurst parameter takes a critical value H that depends on the symmetric measure ν. As a consequence, we derive a change-of-variable formula in distribution, where the correction term is a stochastic integral with respect to a Brownian motion that is independent of the fractional Brownian motion. This is a joint work with I. Nourdin and D. Nualart.

Laure Coutin (Université Paul Sabatier, Toulouse): Invariance for rough differential equations.

Abstract: In 1942, Nagumo has obtain a necessary and sufficient condition for un convex compact set to be invariant under the solution of an ordinary differential equations. This result was extend by Aubin and Da Prato to the case of stochastic differential equations. The object to this talk is to state and prove the same result for rough differential equations, with some application to fractional Brownian motion. It is based on a join work with N. Marie.

Christoph Czichowsky (London School of Economics): Optimal investment under rough volatility

Abstract: Recently, Gatheral, Jaisson and Rosenbaum argued that "volatility is rough". In this talk, we investigate the impact of this change of paradigm in volatility modelling on optimal investment. This will lead to revisiting classical questions for standard Brownian motion in the case of fractional Brownian motion. The talk is based on work in progress with Denis Schelling.

Peter Friz (TU Berlin): TBA

Martin Grothaus (TU Kaiserslautern): Mittag-Leffler Analysis: Construction and Applications

Abstract: Motivated by the results of infinite dimensional Gaussian analysis, we construct a Mittag-Leffler analysis. This is an infinite dimensional analysis with respect to non-Gaussian measures of Mittag-Leffler type which we call Mittag-Leffler measures. Our results indicate that the Wick ordered polynomials, which play a key role in Gaussian analysis, cannot be generalized to this non-Gaussian case. We provide evidence that a system of biorthogonal polynomials, called generalized Appell system, is applicable to the Mittag-Leffler measures, instead of using Wick ordered polynomials. With the help of an Appell system, we introduce a test function and a distribution space. Furthermore we give characterizations of the distribution space including integrability and convergence criteria. As an application we construct Donsker's delta in a non-Gaussian setting. In the second part, we develop a grey noise analysis. This is a special application of the Mittag-Leffler analysis. In this framework, we introduce generalized grey Brownian motion and prove differentiability in a distributional sense and the existence of generalized grey Brownian motion local times. Grey noise analysis is then applied to the time-fractional heat equation and the time-fractional Schrödinger equation. We prove a generalization of the time-fractional Feynman-Kac formula for distributional initial values. In this way, we find a Green's function for the time-fractional heat equation which coincides with the solutions given in the literature. The results presented in this talk are published in the references below. M. Grothaus, F. Jahnert, F. Riemann and J. L. da Silva. Mittag-Leffler analysis I: Construction and characterization. Journal of Functional Analysis. 268(7), 1876--1903,2015. M. Grothaus and F. Jahnert. Mittag-Leffler analysis II: Application to the fractional heat equation. Journal of Functional Analysis, 270(7), 2732--2768, 2016.

Archil Gulisashvilli (Ohio University): Implied volatility skew in rough stochastic volatility models. Moderate deviation regime.

Abstract: The talk presents a joint work with C. Bayer, P. K. Friz, B. Horvath, and B. Stemper. We study correlated rough stochastic volatility models, in which the volatility is described by a function of a Gaussian Volterra process. An important special case of such a volatility process is the exponential of fractional Brownian motion. In our work, we obtain small-time asymptotic formulas in a moderate deviation regime for the call pricing function and the implied volatility in certain rough volatility models. M. Forde and S. Zhang established a large deviation principle for fractional stochastic volatility models, and also found a semi-explicit formula for the rate (energy) function. One of the main results of our work is a sharp asymptotic formula for the Forde-Zhang energy function. This formula generalizes to a non-Markovian setting the known energy expansion due to Y. Osajima. Another main result of our work is a small-time asymptotic formula in the moderate deviation regime for the implied volatility and the implied volatility skew. The skew formula is a generalization of known formulas obtained by E. Alos, J. León, and J. Vives, and M. Fukasawa.

Blanka Horvath (Imperial College London): Short dated option pricing with rough volatility

Abstract: We consider rough stochastic volatility models where volatility has fractional -- worse than diffusion -- scaling. This regime, recently attracted considerable attention both from the statistical and option pricing point of view. With focus on the latter, we sharpen the large deviation results of Forde-Zhang (2016) in a way that allows us to zoom-in around the money while maintaining full analytical tractability. Mathematically speaking, this amounts to prove higher order moderate deviations estimates, recently introduced in the option pricing context by Friz, Gerhold and Pinter (2016).

Chloe Lacombe (Imperial College London): Asymptotic of randomised (fractional) stochastic volatility models

Abstract: In this paper, we study the tails and small-time asymptotics of the randomised forward Stein-Stein model, both in the non-fractional and fractional cases. A path-wise large deviations principle on C([0,T], R) is proved for the volatility and forward log-stock price processes, based on Millet, Nualart and Sanz-Sole's results for stochastic differential equations with a random starting point (1991). In the non-fractional case, using the path-wise large deviations results proved for the forward Stein-Stein model, one can conclude that the tails of the forward distribution and of the forward smile, for large strikes, do not depend on the forward-starting date. Moreover, in the fractional case, under an additional assumption on the right-tail behaviour of the starting point distribution, we prove a path-wise LDP, both for tails and small times. Joint work with Blanka Horvath and Antoine Jacquier.

Jorge León (CINVESTAV-IPN, Mexico): Fractional stochastic differential equations with discontinuous diffusion.

Abstract: In this talk, we study the existence and uniqueness for the solution of a class of stochastic differential equations driven by a fractional Brownian motion with H > 1/2 and a discontinuous coefficient in the diffusion.

Aitor Muguruza (Imperial College London): On vix futures in the rough Bergomi model

Abstract: The rough Bergomi model introduced by Bayer, Friz and Gatheral has been outperforming conventional Markovian stochastic volatility models by reproducing implied volatility smiles in a very realistic manner, in particular for short maturities. We investigate here the dynamics of the VIX and the forward variance curve generated by this model, and develop efficient pricing algorithms for VIX futures and options. We further analyse the validity of the rough Bergomi model to jointly describe the VIX and the SPX, and present a joint calibration algorithm based on the hybrid scheme by Bennedsen, Lunde and Pakkanen. (with Antoine Jacquier and Claude Martini).

Ivan Nourdin (University of Luxembourg): TBA

David Nualart (Kansas University): Functional central limit theorem for the self-intersection local time of the fractional Brownian motion.

Abstract: The purpose of this talk is to discuss some recent results on the asymptotic properties of the self-intersection local time of the multidimensional fractional Brownian motion. First, we will present a functional version of the central limit theorem for the renormalized self-intersection local time of the d-dimensional fractional Brownian motion with Hurst parameter H satisfying 3/4> H>3/(2d). The tightness property is proved using techniques of Malliavin calculus. On the other hand, when the Hurst parameter H is greater than 3/4, we have convergence in mean square to a sum of independent Rosenblatt-type processes.

Mikko Pakkanen (Imperial College London): Modelling and forecasting rough volatility

Abstract: In their recent paper ("Volatility is rough", arXiv:1410.3394) Jim Gatheral, Thibault Jaisson, and Mathieu Rosenbaum argued that financial market volatility should be modelled by stochastic processes that are rougher than Brownian motion. In my talk, I will first present some new corroborative empirical evidence of the roughness of volatility and also discuss some of the intricacies of estimating the roughness of spot volatility. I will then introduce a new stochastic volatility model, building on the so-called Brownian semistationary (BSS) process, which is able to conveniently reproduce both the roughness and highly persistent long-term behaviour of volatility. Finally, I will discuss two parsimonious parameterisations of the BSS model and demonstrate their remarkable performance in intraday volatility forecasting. Joint work with Mikkel Bennedsen and Asger Lunde.

Rémi Peyre (CNRS & Institut Elie Cartan de Lorraine): Two-way crossing property for fBm

Abstract: Motivated by the theoretical study of financial models involving fBm (cf. W. Schachermayer's talk), it was until recently an open question to tell whether fBm has the so-called "two-way crossing (TWC) property": a random process adapted to a filtration is said to satisfy (TWC) when it is not possible to find a stopping time after which the process would locally go upwards or downwards (i.e., the stopping time would be a local minimum or maximum at right). In a work of mine to appear in *Bernoulli*, I managed to prove that fBm actually satisfies (TWC): actually, one can even prove that a modified local law of the iterated logarithm holds true after any stopping time. In this short talk I will explain the general ideas behind this result, which may be used to try and prove (TWC) for a broader class of random processes.

Anthony Reveillac (INSA Toulouse): The Itô-Tanaka trick : a non-semimartingale point of view

Abstract: Since the 80's it is well-known that ill-posed deterministic equations like ODEs can become well-posed if a random force is added to it. One of the main ingredient in that context is the so-called "Itô-Tanaka trick". This formula, that we will recall, relies on the relationship between parabolic PDEs and Markov processes. In this talk we will prove that the Itô-Tanaka trick finds a non-martingale counterpart that allows one to derive regularization effects for more general noises like for instance fractional ones. This talk is based on a joint work with Laure Coutin and Romain Duboscq

Mathieu Rosenbaum (Ecole Polytechnique): Rough Heston model: Pricing, hedging and microstructural foundations

Abstract: It has been recently shown that rough volatility models, where the volatility is driven by a fractional Brownian motion with small Hurst parameter, provide very relevant dynamics in order to reproduce the behavior of both historical and implied volatilities. However, due to the non-Markovian nature of the fractional Brownian motion, they raise new issues when it comes to the risk management of derivatives. Using an original link between nearly unstable Hawkes processes and rough volatility models, we explain in this talk how to price and hedge options in the rough version of the Heston model. This is joint work with Omar El Euch.

Carles Rovira (Universitat de Barcelona): Some results on stochastic differential equations with non-negativity constraint driven by fractional Brownian motion.

Abstract: We recall some results about stochastic differential equations with nonnegativity constraints, driven by a fractional Brownian motion with Hurst parameter H. We deal first with stochastic delay equations, considering the cases H>1/2 and 1/3<H<1/2. The case without delay is also considered.

Walter Schachermayer (University of Vienna): Can fractional Brownian motion touch a semi-martingale? A phenomenon arising in Mathematical Finance

Abstract: Recent joint work with C. Czichowsky, R. Peyre and J. Yang has shown the existence of a shadow price process when studying portfolio optimization problems involving stock price processes driven by a fractional Brownian motion. In particular, a remarkable phenomenon arises which seems puzzling, also independently of the finance context. A fractional Brownian motion may touch in a one-sided way a semimartingale which is, in fact, a martingale under an appropriate measure change.

Samy Tindel (Purdue University): Rate of convergence to equilibrium for rough differential equations.

Abstract: In this talk we wish to give an account on the problem of the rate of convergence to equilibrium for ergodic stochastic differential equations driven by a fractional Brownian motion with Hurst parameter $H\in(1/3,1)$ and multiplicative noise component σ. When σ is constant and for every $H\in(0,1)$, it was proved by Hairer that, under some mean-reverting assumptions, such a process

converges to its equilibrium at a rate of order $t^{-\alpha}$ where $\alpha\in(0,1)$ (depending on H). In this talk, we will show how to extend this result to a multiplicative noise in an irregular situation. We will mainly focus on the general mechanism one should adopt in this context with long range dependence, and we will show how to construct the coupling we need for our purposes.