Hierarchies and Promotions in Political Institutions: Accountability and Selection[*]

B. Pablo Montagnes
Emory University

Junyan Jiang
University of Pennsylvania

Stephane Wolton
London School of Economics

Current Draft: April 11, 2017
First Draft: November 11, 2014

[*]A previous version of the paper, which did not include the third co-author, circulated under the title "Tournament Characteristic: Accountability and Selection."

Abstract

Hierarchies are pervasive in political settings. From judges to elected politicians, from activists to bureaucrats, political agents compete to be promoted to higher positions. This paper studies political tournaments and their impact on key aspects of political performance: accountability and selection. While greater tournament size discourages effort, it improves selection. We also discuss the optimal design of tournament as a function of the principal's objectives and features of the environment. We find that tournaments of size two (such as two-candidate elections) are generally suboptimal. Our analysis also highlights that more desirable promotion always increases effort, but reduces the optimal tournament size under some conditions. Our paper provides a host of other comparative statics.

1 Introduction

Hierarchies are pervasive in political settings. Electoral careers often begin locally and then progress towards the provincial or national level (Myerson, 2006). Judges move along the different stratas of the judicial system (Epstein et al., 2003). Authoritarian leaders fill vacancies drawing from administrators at lower levels (Lu and Landy, 2014). Bureaucrats are promoted through various grades of pay and responsibility (Cameron et al., 2013). Political parties have their own set of rules and structures with members competing for positions on party lists or leadership roles (Michels, 1915). Legislators seek ministerial positions (Eggers and Spirling, 2016) or assignments to plum committees (Groseclose and Stewart III, 1998).

Hierarchical structures allow a principal (e.g. a party leader, the president or an authoritarian ruler) to select the highest ability agent (legislator, bureaucrat, local administrator) to fill a vacant position (cabinet post, agency head, provincial position). Hierarchies thus presuppose the possibility for promotions. From the perspective of agents, promotions are prizes to be won by besting fellow contestants. Promotions-hierarchies are thus best understood as political tournaments. This paper examines how this characteristics of political institutions affects two key aspects of political performance: accountability and selection of competence.

Our model features a principal designing a tournament—i.e., deciding the number of agents competing for promotion—and choosing which agent to promote. The principal wants to promote the agent with the highest ability. She does not, however, directly observe agents' skills. Instead, she observes a noisy signal of their types. Agents are unaware of their ability and try to manipulate the principal's signal by exerting costly effort.

Given her objective, the principal conditions her promotion decision on the observable output and is unable to commit to an arbitrarily promotion rule (see Fearon, 1999, for a related argument in electoral settings). We show that in the unique pure strategy equilibrium, all competing agents exert the same effort and the principal always promote the highest performer. As the size of the tournament grows, effort by each agent decreases, but selection improves. Thus, the tournament structure is generated by a the principal's goal of promoting the highest ability agent. This is in contrast to past theoretical treatment of tournaments in the economics literature, in which tournaments are often considered as incentive mechanisms.

By incorporating both effort and selection, our paper identifies the fundamental trade-off faced by a principal when designing a promotions-hierarchies structure. We find that whenever agents' efforts increase the principal's utility, tournaments with 2 competing agents are generally dominated by greater tournaments. This result suggests that two-candidate elections are suboptimal from the electorate's perspective unless the cost of running larger elections is very high. We further establish that the optimal size of a tournament depends critically on whether the principal values total effort or the effort of the promoted agent (equivalently, due to symmetry in equilibrium, average, maximum, or minimum effort). As total effort increases in size, whereas individual effort decreases, the principal always designs a (weakly) larger tournament when she cares about total effort.

Our paper also provides a host of additional comparative statics, some intuitive, some more subtle. Effort is increasing in the value of the prize (as agents find promotion more valuable) and decreasing in the heterogeneity of ability and how informative agents' outputs are about their skills (since in both cases, effort is less likely to determine promotion). In turn, selection always improves with the variance in agents' types as the principal always cares about the highest draw of ability. More surprisingly, even though effort increases with the value of the prize, the optimal tournament size decreases with the desirability of promotion whenever the principal cares about the promoted agent's effort. The key force behind this result is that the principal weights the gain and loss of adding one competitor when designing a tournament. Higher prize exacerbates the negative effect on effort of increasing the pool of competitors. In turn, greater desirability of promotion leads to larger tournament when the principal cares about total effort since bigger prize enhances the gain from increased size.

We plan to use institutional features of the U.S. House and Senate to test our core predictions. Treating Senate seats as promotions for House representatives, we will test whether effort as measured by a legislator's effectiveness score (Volden and Wiseman, 2014) is decreasing in the size of her/his party delegation in the state (i.e., the pool of competitors) and increasing in the probability of winning the Senate race (i.e., the desirability of nomination and thus promotion). In turn, performance in the Senate (either effectiveness, promotion to leadership positions, or electoral winning margins) should be positively correlated with the size of the party-state delegation in the House. To assess how tournament size varies with the value of the prize, we turn to committees.[1] We establish that the most valuable committees (according to Stewart III, 2012) tend to be smaller, especially policy committees where (arguably) party leaders only care about individual legislator's effort on bills.

Literature Review. The theoretical analysis of tournaments is not new. Beginning with seminal work by Lazear and Rosen (1981), tournament incentives have been studied extensively (Waldman, 2013). Early results emphasized that tournaments are dominated by (sufficiently rich) contracts when it comes to incentive provision since they introduce noise to compensation schemes (Green and Stokey, 1983). Later studies justify tournaments as an optimal response to particular features of the environment such as common shocks (Maskin et al., 2000) or commitment problems (Malcomson, 1986) or assume a tournament and characterize the optimal prize structure (Krishna and Morgan, 1998 complementing Nalebuff and Stiglitz, 1983). Following Prendergast's (1999) remark on the absence of models that consider selection, some (e.g., Ryvkin, 2010) have focused exclusively on this issue. Our work differs from the previous literature by incorporating both incentives to exert effort and selection effects. Doing so, we can establish optimal tournament size under different assumptions regarding the principal's objective function.[2]

By considering accountability and selection, our work is related to agency models of elections (see Ashworth, 2012, for a review). Canonical political career concerns models (e.g., Ashworth, 2005) are, however, a special case of political tournaments in which only the per-

[1] For evidence that committees can be understood as tournament, see Berry and Fowler (2016).

[2] Other papers have looked at this problem either by considering a contest rather than a tournament (Höchtl et al., n.d.) or restricting both the set of types and actions (Hvide and Kristiansen, 2003). These papers consider optimal tournament exclusively from a selection perspective and so do not study how optimal size varies with the principal's objective.

formance of one agent is observed and compared to a benchmark.[3] Our richer set-up allows us to compare how electoral incentives compare to other type of tournament and generates novel comparative statics relating optimal size of tournament and model parameters.

Further, while electoral accountability models focus on bottom-up accountability, promotions-hierarchies are best understood as top-down accountability problems. Ghosh and Waldman (2010) study how promotions can be used to screen agents. They focus on an one-to-one relationship between firm owners and workers and so have nothing to say about the size effect of promotion tournament. Our approach, instead, is rooted in the idea that top-down accountability is a many-to-one problem and agents are rarely judged in isolation. Montagnes and Wolton (2016) study a related question. However, they suppose that the principal has coarse information about her agents' performances and emphasize the role of punishments (purges in authoritarian countries) rather than rewards.

2 Formal Model of Tournaments with Selection

We consider a one-period game with a principal and a set of available agents with cardinality $\overline{n} > 2$. The principal first decides the number of agents $n \in \{1, \ldots, \overline{n}\}$ to be considered for promotion to a unique higher-up position valued by all agents. The n selected agents then compete by exerting costly effort to manipulate the principal's promotion decision. We thus describe the subgame played by the selected agents as an $n-$agent tournament.

Each agent i is characterized by his ability θ_i. Following career concerns model (e.g., Holmström, 1999; Ashworth, 2005), we assume that θ_i is unknown to all players at the beginning of the game. It is, however, common knowledge that for all $i \in \{1, \ldots, n\}$, θ_i is independently and identically distributed (i.i.d.) over the interval $[-\overline{\theta}, \overline{\theta}]$ (possibly unbounded) according to the cumulative distributive function (CDF) F_θ with variance σ_θ^2

[3] See Ashworth and Shotts (2014); Dewan and Hortala-Vallve (2016) for models in which challengers can also send signals to voters.

and symmetric and single-peaked probability density function (pdf) f_θ.[4] We further assume that F_θ and f_θ are continuously differentiable (i.e., $F_\theta \in \mathcal{C}^2$).

The principal cares about the promoted agent's ability. She, however, only observes a noisy signal of agents' skill before making her promotion decision if this agent is considered for promotion, otherwise she observes nothing. More specifically, an agent considered for promotion produces an output—denoted q_i for all $i \in \{1, \ldots, n\}$—which consists of the sum of the agent's ability θ_i, his effort a_i, and a noise (luck) term η_i:

$$q_i = a_i + \theta_i + \eta_i \tag{1}$$

Effort a_i is unobserved by the principal. Like in career concerns models, agents try to manipulate the signal available to the principal. Unlike career concerns models, n agents simultaneously compete for promotion by exerting effort. The noise term η_i is also unknown to all players, but it is common knowledge that η_i is i.i.d. over the interval $[-\bar{\epsilon}, \bar{\epsilon}]$ (possibly unbounded) according to the CDF $F_\eta \in \mathcal{C}^2$ with variance σ_η^2 and symmetric and single-peaked pdf f_η.[5] Notice that the output function only incorporates independent shocks. This assumption is without loss of generality since, as extensively noted in the literature (e.g., Maskin et al., 2000; Green and Stokey, 1983), any shock that would similarly impact all agents would be differenced out by the principal who evaluates agents on their relative rather than absolute performance.

An agent receives a payoff V if promoted and 0 otherwise. The agent also pays a cost $c(a_i)$ to exert effort and manipulate the signal q_i available to the principal. We assume that $c(\cdot)$ is $\mathcal{C}^2$, increasing, and convex. Agent i's utility function thus assumes the following form (with $\mathbb{I}_{\{promoted\}}$ the indicator function equals to 1 if i is promoted):

$$U_i(a_i) = V\mathbb{I}_{\{promoted\}} - c(a_i) \tag{2}$$

[4] This assumption is consistent with evidence on politicians' skills (albeit in Sweden) collected in Dal Bó et al. (2017).

[5] Our assumption that output is a linear sum of effort, ability and noise is closest to the model of Lazear and Rosen (1981), but see Nalebuff and Stiglitz (1983) for a model where effort and shocks are not linearly separable.

The principal's utility depends on both the ability of the promoted agent and the agents' efforts. We assume that increasing the size of the tournament is costly, with the convex cost function $C_P(n)$. The principal's utility can be represented as

$$U_P(n) = \omega g(a_i, \ldots, a_{\overline{n}}) + (1-\omega)\mathbb{I}_{\{i=promoted\}}\theta_i - C_P(n), \tag{3}$$

where $\omega \in [0,1]$ is the weight on agents' effort, $g(a_i, \ldots, a_{\overline{n}})$ is a continuously function weakly increasing in all its arguments,[6] and $\mathbb{I}_{\{i=promoted\}}$ an indicator function equals to 1 if i is promoted and 0 otherwise.

To summarize, the timing of the game is as follows:

0. Nature draws relative abilities (θ_i) and noise (η_i) for each agent $i \in \{1, \ldots, \overline{n}\}$ unobserved by all players;

1. The principal selects the number of agents selected for promotion $n \in \{1, \ldots, \overline{n}\}$. For notational convenience and without loss of generality, we assume that the first n agents are selected;

2. Each selected agent i simultaneously choose his effort a_i;

3. For each selected agent i, the output q_i is observed by all players;

4. The principal promotes one agent among the n selected agents. The game ends and payoffs are realized.

The equilibrium concept is Perfect Bayesian Equilibrium in pure strategies. We suppose that all players consistently anticipate other players' strategies. That is, agent i and the principal have the same expectation about agent j's effort for all $j \neq i$ (this restriction has no effect on the characterization of equilibrium strategies—since all players are correct

[6] Equivalently, the principal may care about agent output rather than their effort. This generally does not affect our results (recall agents not considered for promotion produce no output), but only complicates the exposition (see footnote 12 for more details). We define $g(\cdot)$ over $\overline{n}$ and not n so that the function is well-defined for all tournament sizes.

in equilibrium—but facilitate the proof of existence). Throughout, we also assume that there is sufficient variance in ability and/or noise to guarantee existence and uniqueness of a symmetric equilibrium (this assumption is common in the literature, see Green and Stokey, 1983; Nalebuff and Stiglitz, 1983; Krishna and Morgan, 1998).[7]

As a preliminary, note that given the principal's objective, she always promotes the agent with the highest expected ability. To form a correct assessment of agents' skills, she differences out agents' anticipated effort from the observed outputs. Since her anticipation is correct in equilibrium, we can study separately promotion and effort. We thus forgo the usual (backwards) order of analysis and focus first on agents' effort and how it varies with the model parameters and size of the tournament.[8]

3 Agents' effort

We first consider effort in a tournament of size n. First, agents not competing for promotion (i.e., agents $m \in \{n+1, \overline{n}\}$) do not exert any effort. Second, agent 1 does not exert effort if the tournament is of size 1. Therefore, in what follows, we focus in tournament of size at least 2. We introduce the following notation, let a_j^e be the principal's anticipation of j's effort.

Agent $i \in \{1, \ldots, n\}$ is promoted if and only if the principal expects his ability to be the highest. This occurs whenever he produces the highest output net of anticipated effort, i.e., $q_i - a_i^e \geq \max_{j \neq i} q_j - a_j^e$. Agent i, however, does not observe q_j and must form beliefs about other agents' performances based on (i) the realization of their ability (θ_j) and luck (η_j) and (ii) their effort (a_j). Denote $\hat{a}_j^e$ i's anticipation of j's effort for all $j \neq i$, the probability i is promoted given his output q_i is: $Pr(Promoted|q_i) := Pr\left(q_i - a_i^e \geq \max_{j \neq i}(\theta_j + \eta_j + \hat{a}_j^e - a_j^e)\right)$. Imposing the assumption that anticipated efforts are consistent (i.e., $a_j^e = \hat{a}_j^e$), this reduces

[7] Absent this assumption on variances, selection plays a limited role. The competition between agents resembles am all-pay contest with equilibrium supported by mixed strategies over a support that includes zero effort. We formally characterize the necessary variance condition in **??**.

[8] All proofs are available upon request.

to:

$$Pr(Promoted|q_i) := Pr\left(q_i - a_i^e \geq \max_{j\neq i}(\theta_j + \eta_j)\right) \tag{4}$$

Equation 4 highlights that only the joint effect of θ_j and η_j enter into agent i's assessment. Since the two random variables are independent, it is convenient to merge the terms into a 'combined noise' $\epsilon_j := \theta_j + \eta_j$ for all j. For each j, the combined noise is i.i.d. over $[-\overline{\epsilon}, \overline{\epsilon}]$ ($\overline{\epsilon} = \overline{\theta} + \overline{\eta}$) according to the CDF $F_\epsilon \in \mathcal{C}^2$ with variance $\sigma_\epsilon^2 = \sigma_\theta^2 + \sigma_\eta^2$ and pdf f_ϵ. Observe that due to the independence of θ_i and η_i, f_ϵ is single-peaked and symmetric. Using the independence of ϵ_j's, the probability that i is promoted for a given output q_i is thus $F_\epsilon\big(q_i - a_i^e\big)^{n-1}$.

Agent i, however, only chooses his effort, not his output q_i, which corresponds to the sum of effort a_i and his own combined noise ϵ_i. Since agent i correctly treats ϵ_i as a random variable, he maximizes the following objective function (with $E_{\epsilon_i}(\cdot)$ the conditional operator over ϵ_i):

$$\begin{aligned} K_i(a_i) :=& V E_{\epsilon_i}\big(Pr(Promoted|a_i, \epsilon_i)\big) - c(a_i) \\ =& V \int_{-\overline{\epsilon}}^{\overline{\epsilon}} F_\epsilon\big(\epsilon + a_i - a_i^e\big)^{n-1} f_\epsilon(\epsilon) d\epsilon - c(a_i) \end{aligned} \tag{5}$$

Through usual maximization (our assumption on variances guarantees that the second order condition holds), we can then characterize the agents' effort.

Proposition 1. *There exists a unique equilibrium effort in pure strategies. The equilibrium effort is symmetric—denoted $\overline{a}(n, \sigma_\epsilon, V)$—and characterized by the following equation:*

$$c'\big(\bar{a}\big(n, \sigma_\epsilon, V\big)\big) = V \int_{-\overline{\epsilon}}^{\overline{\epsilon}} f_\epsilon(\epsilon) dF_\epsilon(\epsilon)^{n-1} \tag{6}$$

Our next set of results establishes how the tournament structure—the value of the prize (V), the variance of the combined noise (σ_ϵ), and the size of the tournament (n)—affects the equilibrium effort.

Proposition 2. *The equilibrium level of effort $a(n, \sigma_\epsilon, V)$ is:*

(i) decreasing in n (i.e., $\overline{a}(\overline{n}, \sigma_\epsilon, V) \leq \overline{a}(\underline{n}, \sigma_\epsilon, V)$ for $\overline{n} > \underline{n} \geq 2$);

(ii) strictly decreasing in σ_ϵ*;*

(iii) strictly increasing in V*.*

Points (ii) and (iii) of Proposition 2 are intuitive. As the prize becomes more attractive (the value of promotion V increases), agents have more incentives to exert effort. As the variance of the combined noise grows, effort is less and less likely to determine the outcome of the tournament. As a consequence, all agents have reduced incentive to exert effort.

The intuition for point (i) is more subtle. As the size of the tournament grows, competition to be promoted becomes more intense. Naive economic reasoning then suggests that effort should increase. This intuition fails to consider the effect of the combined noises on agents' effort choice. When choosing his effort, agent i does not compare his output with every other agent's. Rather, as explained above, he only takes into account his best performing competitor. As the number of competing agents increases, the combined noise associated with agent i's best competitor becomes increasingly likely to take high values (more precisely, $F_\epsilon(\epsilon)^{\overline{n}-1}$ strictly first order stochastically dominates $F_\epsilon(\epsilon)^{\underline{n}-1}$ for all $\overline{n} > \underline{n} \geq 2$).

Better performance by the highest competitor in itself, however, is not sufficient to discourage effort by agent $i \in \{1, \ldots, n\}$. Indeed, agent i's effort equates the marginal cost with the *marginal* gain in promotion probability. His total winning probability has little effect on his effort choice. In turn, the marginal gain in promotion probability depends on the distribution of his own combined noise ϵ_i. Since $f_\epsilon(\cdot)$ is single-peaked and symmetric, agent i's effort is more likely to be complemented by low (in absolute values) rather than large combined noise. Consequently, as the size of the tournament grows, the probability that his output is close to his highest competitor decreases and so does the likelihood that a marginal increase in effort significantly increases his promotion probability.

The reasoning above thus indicates that an agent's performance decreases with the size of the tournament due to the combination of two effects. First, his highest competitor's performance increases. Second, more effort is less and less likely to significantly impact his promotion probability. Absent either of these two effects, the size of tournament has no impact on an agent's effort choice.[9] In particular, when ϵ_i is uniformly distributed so all levels of combined noise are equally likely, the size of the tournament has no effect on agents' effort.[10] As Corollary 1 also shows, 2-agent and 3-agent tournaments also yield the same equilibrium effort as the negative effect of the highest competitor's increased performance is counterbalanced by a positive effect of lower variance which increases the value of marginal effort.

Corollary 1. *The equilibrium effort satisfies* $\overline{a}(n+1, \sigma_\epsilon, V) < \overline{a}(n, \sigma_\epsilon, V)$ *if and only if* $n > 2$ *and there exists* $\epsilon \in [0, \overline{\epsilon}]$ *such that* $f'_\epsilon(\epsilon) < 0$.

Without imposing additional conditions on the shape of agents' cost function $c(\cdot)$, we cannot characterize additional properties of agents' equilibrium effort. Since effort is determined by the marginal cost, any second order prediction depends on the third derivative of the cost function. In the next corollary, we discuss instead some properties of the marginal benefit of effort, which we denote $b(n, \sigma_\epsilon, V) := V \int_{-\overline{\epsilon}}^{\overline{\epsilon}} f_\epsilon(\epsilon) dF_\epsilon(\epsilon)^{n-1}$ (see the right-hand side of Equation 6).

Corollary 2. *The marginal benefit of effort is:*

(i) convex in n *(i.e.,* $b(n'' + k, \sigma_\epsilon, V) - b(n'', \sigma_\epsilon, V) \geq b(n' + k, \sigma_\epsilon, V) - b(n', \sigma_\epsilon, V)$ *for* $n'' > n' \geq 2$ *and* $k \geq 1$*), strictly if* $f'_\epsilon(\epsilon) > 0$ *for some* $\epsilon \in [0, \overline{\epsilon}]$*;*

(ii) strictly convex in σ_ϵ*;*

(iii) neither concave nor convex in V*.*

Point (iii) follows from the risk neutrality of agent. Any increase in the prize linearly increases the benefit from winning the tournament and, in turn, the marginal benefit of effort. Point (ii) follows from the effect of higher variance on $f_\epsilon(\cdot)$. As σ_ϵ increases, the pdf of the combined noise flattens and an increase in effort becomes marginally more likely

[9] Furthermore, as the proof of Proposition 2 shows, if the single peaked assumption is reversed and $f'_\epsilon(\epsilon) > 0$ for all $\epsilon > 0$, the result is reversed. Assuming existence of a symmetric equilibrium, effort is increasing in the size of the tournament.

[10] Hence, the null effect of tournament size on effort found in Orrison et al. (2004) is a knife-edge result which relies on their distributional assumption.

to affect the tournament outcome. This second order effect attenuates the direct effect of variance identified above.

Point (i) indicates that the negative effect on incentives of increased tournament size on effort attenuates as the tournament grows larger. This arises because the effect of adding one more agent on the highest performance (in expectation). The direct effect is that expected highest performance increases. However, this also implies as a second order effect that higher and higher realization of the combined noise are necessary to improve the highest performance. Since $f_\epsilon(\cdot)$ is single-peaked (i.e., low values of ϵ in absolute terms are more likely), such event is less and less likely. Consequently, the expected highest performance increases at a decreasing rate with the number of participants. Since the marginal benefit of effort is inversely related to the highest performance of other agents, it is convex in n.

Having established generally properties of the marginal benefit of effort, we can provide some predictions for the effect of the tournament parameters on effort. The second-order effect of V on effort depends on the sign of $c'''(\cdot)$. In turn, equilibrium effort is convex in both σ_ϵ and the number of agents n whenever $c'''(\cdot)$ is not too large.

4 Tournament and Selection

As highlighted by Equation 3, the principal does not only care about the agents' effort (through $g(a_1, \ldots, a_{\overline{n}})$), but also about the ability of the promoted agent. Because all agents exert the same level of effort $\overline{a}(\cdot)$, the principal simply promotes the best performer. Further, since she correctly anticipates effort, she can recover the combined noise of the best performer, denoted $\epsilon^{\max}(n, \sigma_\theta, \sigma_\eta)$ (i.e., $\epsilon^{\max}(n, \sigma_\theta, \sigma_\eta) = \max_{i \in \{1,\ldots,n\}} q_i - \overline{a}$). From the principal's perspective, the promoted agent's expected ability—denoted $\theta^{max}(n, \sigma_\theta, \sigma_\eta)$—is then for a given output:

$$\theta^{max}(n, \sigma_\theta, \sigma_\eta) := E_\theta(\theta | \epsilon = \epsilon^{max}(n, \sigma_\theta, \sigma_\eta)) \tag{7}$$

When deciding the tournament size, the principal can only condition her decision to select n agents on the expected realizations of agents' ability and luck. From an ex-ante perspective, the important statistic for the principal is:

$$\theta^W(n, \sigma_\theta, \sigma_\eta) := E_{\epsilon^{max}}\Big(E_\theta(\theta|\epsilon = \epsilon^{max}(n, \sigma_\theta, \sigma_\eta))\Big) \tag{8}$$

Our first result is quite intuitive. As the tournament size grows, the (ex-ante) expected ability of the promoted agent $\theta^W(\cdot)$ increases. Given that effort does not affect the principal's promotion decision, adding agents can only increase the probability that a large combined noise is realized. While effort declines in a larger tournament, selection strictly improves.

Proposition 3. *The ex-ante expected ability of the promoted agent increases with the size of the tournament.*

Additional conditions on the distribution of ability θ_i and luck η_i are required to establish further properties of $\theta^W(\cdot)$. As it is common in the literature on career concerns, we impose that θ_i and η_i both are normally distributed (i.e., $\theta_i \sim \mathcal{N}(0, \sigma_\theta^2)$ and $\eta_i \sim \mathcal{N}(0, \sigma_\eta^2)$). Under these assumptions, $\theta^{max}(n, \sigma_\theta, \sigma_\eta)$ takes a very simple form: $\theta^{max}(n, \sigma_\theta, \sigma_\eta) = \frac{\sigma_\theta^2}{\sigma_t^2+\sigma_\eta^2}\epsilon^{max}(n, \sigma_\theta, \sigma_\eta))$. Denoting, $\epsilon^W(n, \sigma_\theta, \sigma_\eta)) = E_{\epsilon^{max}}(\epsilon^{max}(n, \sigma_\theta, \sigma_\eta)))$, we obtain:

$$\theta^W(n, \sigma_\theta, \sigma_\eta) := \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\eta^2}\epsilon^W(n, \sigma_\theta, \sigma_\eta) \tag{9}$$

Under the assumption of normality, we can provide a fuller picture of the impact of tournament characteristics on the ex-ante ability of the promoted agent, $\theta^W(\cdot)$.

Proposition 4. *Suppose θ_i and η_i are normally distributed. The ex-ante expected ability of the promoted agent $\theta^W(n, \sigma_\theta, \sigma_\eta)$ is:*

(i) strictly increasing in σ_θ;

(ii) strictly decreasing in σ_η.

Further, $\theta^W(n, \sigma_\theta, \sigma_\eta)$

(iii) is strictly concave in n.

As the variance in abilities increases, large realization of ability in absolute values become more likely. Since the principal only cares about the highest realisation, not the average of realisations, at the time of promotion, this always benefits the principal. In turn, when the variance of the noise component η increases, high output is more likely to be due to luck than ability. This, therefore, tends to depress the expected ability of the promoted agent. Finally, the last point of the proposition indicates that the gain of adding contestants is decreasing with the size of the tournament. The logic is the same as for the convexity of effort in n. As the expected highest ability rises (i.e., $\theta^W(n, \sigma_\theta, \sigma_\eta)$ increases with n), it becomes less and less likely that the ability of an additional agent will surpass $\theta^W(n, \sigma_\theta, \sigma_\eta)$.

5 Tournament Design

Having discussed both the effects of tournament size on effort (section 3) and selection (section 4), we can now consider how many agents the principal selects to compete in the promotion tournament. In what follow, we refer to 'optimal tournament size' as the number of contestants which maximize the principal's expected utility given agents' unique equilibrium pure strategies.

When choosing the number of competitors, the principal maximizes her ex-ante expected utility taking into account the effect of tournament size on effort and selection. That is, her maximization problem is:

$$\max_{n\in\{1,\ldots,\overline{n}\}} \omega g(\overline{a}(n,\cdot),\ldots,\overline{a}(n,\cdot),0,\ldots,0) + (1-\omega)\theta^W(n,\cdot) - C_P(n) \tag{10}$$

Our first result establishes that tournaments of size 2 are generally suboptimal from the principal's perspective.

Remark 1. *The optimal tournament size is strictly less than* 3 *only if* $C_P(3) - C_P(2) > (1-\omega)\big(\theta^W(3, \sigma_\theta, \sigma_\eta) - \theta^W(2, \sigma_\theta, \sigma_\eta)\big)$.

Note that this result does not require any restriction on $g(a_1, \ldots, a_{\overline{n}})$ (besides the assumption that it is weakly increasing in its arguments) or on the distributions of θ_i and η_i. It is a direct consequence of Corollary 1 and Proposition 3. Increasing the size of the tournament from two to three agents has no effect on an agent's effort. Since the principal always gains in term of selection, she prefers not to restrict the set of contestants to two unless the cost of adding a third competitor is very large.

This result has important implications when one interprets elections as a tournament. Effort in this case can be understood as campaign spending meant to manipulate the voter's evaluation of candidates. In this case, $g(a_1, \ldots, a_{\overline{n}})$ is constant in agents' output. Alternatively, effort may correspond to commitment to targeted spending, with $g(a_1, \ldots, a_{\overline{n}}) = \mathbb{I}_{\{i=promoted\}} a_i$ and $c(\cdot)$ the cost of advertising campaign promises (see Prato and Wolton, 2016). Remark 1 indicates that, independently of the interpretation of agents' output, two-candidate elections are generally inefficient from the electorate's point of view. It suggests that there may be some welfare gain for voters from encouraging at least one more candidate to participate in elections (as long as the marginal cost of increasing the number of candidates is relatively small).

To provide more details on the optimal tournament size, we return in all that follows to the case of normally distributed ability (θ) and luck (η). We further impose that the agents' cost functions is quadratic: $c(a) = \frac{a^2}{2\beta}$, with β sufficiently small so the second order conditions hold. This second assumption allows us to identify the direct effect of size on effort, not the effect mediated through the shape of the cost function.[11]

We focus on two natural situations. First, we suppose that the principal preference for effort depends only on the the output of a single agent; that is, $g(\overline{a}(n, \cdot), \ldots, \overline{a}(n, \cdot), 0, \ldots, 0) = \overline{a}(n, \cdot)$. Second, we assume that the principal cares about total effort (equivalently, total output): $g(\overline{a}(n, \cdot), \ldots, \overline{a}(n, \cdot), 0, \ldots, 0) = n\overline{a}(n, \cdot)$.

Optimal tournament size when the principal cares about single effort

This first situation subsumes distinct principal's objectives. It corresponds to a principal valuing average output (since agents play a symmetric strategy). This naturally occurs when the principal delegates policies to subordinate units and is concerned about all entities receiving similar treatment either to avoid inconsistencies, as in the case of judicial decisions, or to ensure some common standards, as in the case of education policies (see Howell, 2014). Such an objective is also consistent with the principal seeking to maximize minimum output.

[11] See the discussion around Corollary 2. All our results hold as long as $|c'''(a)| < \overline{c}$ for all $a \geq 0$, with $\overline{c}$ strictly positive, but not too large.

In domains where negative spill-overs are present (e.g. environmental policy), a principal may want to minimize the damage caused by the worst performer. In the case of antiterrorism policies, a principal wants to strengthen the weakest link (De Mesquita, 2007; Powell, 2007). Finally, the function $g(\overline{a}(n,\cdot),\ldots,\overline{a}(n,\cdot),0,\ldots,0) = \overline{a}(n,\cdot)$ is also consistent with a principal only taking into account the performance of the promoted agent (or equivalently, the best performance). Examples of such objective include elections as discussed above, maximizing the quality of implemented policies (Hirsch and Shotts, 2015), or innovation.[12]

Under our conditions on the cost function, there exists a unique size maximizing the principal's expected welfare, which we denote n_A^*. The next proposition describes how n_A^* varies with the model parameters.

Proposition 5. *When $g(\overline{a}(n,\cdot),\ldots,\overline{a}(n,\cdot),0,\ldots,0) = \overline{a}(n,\cdot)$, Equation 10 has a unique maximum n_A^* with the following properties:*

(i) n_A^* is decreasing with V;

(ii) n_A^* is increasing with σ_θ.

Recall that the expected ability of the promoted agent does not depend on the tournament prize V, whereas an agent's effort is strictly *increasing* in V. Nonetheless, Proposition 5 indicates that the optimal size is decreasing in the value of promotion. To understand this result, observe than when choosing n_A^*, the principal compare the *marginal gain* from increasing the size of the tournament by one agent with the marginal cost. As highlighted in Proposition 2, effort is decreasing with size. Further since effort is linear in V (under our assumption of quadratic cost of effort), as the value of the prize increases so does the difference between $\overline{a}(n,\sigma_\epsilon,V)$ and $\overline{a}(n+1,\sigma_\epsilon,V)$. Therefore, as V increases, it becomes more

[12] If the principal cares about agents' outputs rather than effort, her objective function becomes: $\omega E_{\vec{\epsilon}} g(\overline{a}(n,\cdot)+\epsilon_1,\ldots,\overline{a}(n,\cdot)+\epsilon_n,0\ldots,0) + (1-\omega)\theta^W(n,\cdot) - C_P(n)$, with $\vec{\epsilon} = (\epsilon_1,\ldots,\epsilon_n)$. In this case, when the principal seeks to maximize the lowest (highest) performance, the objective function reduces to using the symmetry: $\omega\overline{a}(n,\cdot) + (1-\omega)\theta^W(n,\cdot) - \omega\epsilon^W(n,\cdot)$ $(\omega\overline{a}(n,\cdot) + (1-\omega)\theta^W(n,\cdot) + \omega\epsilon^W(n,\cdot))$. All our results remain unchanged for the case of maximizing highest performer and require ω sufficiently small for the case of maximizing lowest performer. Focusing on agents' effort permits to keep the objective function the same in all three cases described in the text.

costly, in term of effort reduction, from increasing the size of the tournament by one agent. Hence n_A^* is decreasing in V.

A similar logic explains why the optimal size is increasing in the variance of ability even though σ_θ tends to depress effort. An increase in variance reduces the loss in term of effort from increasing the size of the tournament. Further, greater variance in abilities not only increase the (expected) quality of the promoted agent, it also enhances the selection gain from adding one agent as the chance of getting a high ability agent increases.

Optimal tournament size when the principal cares about total effort

Situations when the principal values total effort are quite common.A profit-maximizing firm seeks to maximize the performance of its semi-independent divisions competing in different markets. Similarly, an editor wants to increase the quality of reporting of all journalists. A totalitarian leader looks to maximize all party members' performance either regional heads (Maskin et al., 2000; Li and Zhou, 2005; Lu and Landy, 2014) or ranks-and-files (Montagnes and Wolton, 2016).

The next Lemma establishes that while individual effort is decreasing and convex in n, total effort is increasing and concave in n.

Lemma 1. *Total equilibrium effort* $n\overline{a}(n, \sigma_\theta, V)$ *is strictly increasing (i.e.,* $(n+1)\overline{a}(n+1, \sigma_\theta, V) > n\overline{a}(n, \sigma_\theta, V)$*) and strictly concave (i.e.,* $(n''+1)\overline{a}(n''+1, \sigma_\theta, V) - n''\overline{a}(n'', \sigma_\theta, V) < (n'+1)\overline{a}(n'+1, \sigma_\theta, V) - n'\overline{a}(n', \sigma_\theta, V)$ *for all* $n'' > n'$*).*

We then can use Lemma 1 to establish the properties of the optimal tournament size—denoted n_T^*—when the principal cares about total effort.

Proposition 6. *When* $g(\overline{a}(n, \cdot), \ldots, \overline{a}(n, \cdot)) = n\overline{a}(n, \cdot)$*, Equation 10 has a unique maximum* n_T^* *with the following properties:*

(i) $n_T^* \geq n_A^*$*;*

(ii) n_T^* *is increasing with* V*;*

(iii) n_T^* *is decreasing with* σ_η*.*

When the principal cares about individual effort, an increase in tournament size only improves selection. In turn, when she values total effort, increasing the size of tournament has a positive effect on the effort ($n\overline{a}(n, \cdot)$) and selection ($\theta^W(n, \cdot)$) components of her objective function. As a direct consequence, the principal has more incentives to increase the size of the tournament in the latter case and this is reflected in point (i) of Proposition 6.

Points (ii) and (iii) follow from a similar reasoning as for Proposition 5. Observe, however, that since total effort is increasing with n, an increase in V tends to magnify the benefit of adding contestants. Consequently, n_T^* is increasing with V. The impact of σ_η, in turn, is due to its negative impact on effort (which attenuate the positive effect of increasing tournament size) and selection (as higher performances are more likely to be due to luck than ability).

Notice that Proposition 6 says nothing about the effect of an increase in the variance of abilities. This results from the conflicting effects of increased σ_θ on the marginal benefit of adding a participant. Greater variance in abilities, by depressing effort (Proposition 2), mitigates the gain in total effort, whereas by improving $\theta^W(n, \sigma_\theta, \sigma_\eta)$ (Proposition 4), it enhances the selection gain.

6 Discussion

Any hierarchical structure presupposes promotion, the selection of the agent(s) most able to perform at the higher level. In turn, promotion generates incentives akin to tournaments. Consequently, tournament should thus primarily be understood as selection mechanism, rather than a scheme to promote effort.

Not surprisingly, a large body of work on tournaments in economics has long established that (sufficiently complete) contracts always generate higher output than tournaments (Nalebuff and Stiglitz, 1983; Green and Stokey, 1983; Prendergast, 1999). But this literature misses much of the story. As in elections (Fearon, 1999), effort in promotions-hierarchies is a by-product of selection incentives as competing agents attempt to manipulate the principal's evaluation of their ability.

Because promotions are intrinsically a selection mechanism, the principal cannot commit to fixed promotion standards, beside promoting the agent with the highest ability. Any setting which presupposes other promotion rules fall short of the Fearon's (1999) critique.

Agents' efforts do not matter at the time of promotion, but they do affect the design of the tournament. Importantly, we cannot evaluate the effectiveness of a tournament without a proper understanding of the principal's objective. When she values total effort, tournaments tend to be large and generate low individual effort. When she values individual effort, tournaments are relatively smaller and guarantee relatively high effort. When the principal's objective is unobservable to researchers, the relationship between size and value of promotion can help to recover it. Indeed, the correlation is positive if the principal values total effort.

7 Conclusion

Promotions-hierarchies are prevalent in political institutions. In this paper, we model promotions as tournaments and examine their effects on key aspects of political performance: accountability and selection. So doing, we highlight that the goals of selecting able agents and encouraging costly effort are often in conflict. Our analysis provides a host of comparative statics. While effort always increases with the value of the prize, the optimal size may decrease with the desirability of promotion. The effect of greater variance in abilities depends on the principal's objective. And tournament of size 2 (such as two-candidate elections) are generally dominated by larger tournaments.

References

Ashworth, Scott, "Reputational dynamics and political careers," *Journal of Law, Economics, and Organization*, 2005, *21* (2), 441–466.

_ , "Electoral accountability: recent theoretical and empirical work," *Annual Review of Political Science*, 2012, *15*, 183–201.

_ **and Kenneth W Shotts**, "Challengers and Electoral Accountability," 2014.

Berry, Christopher R and Anthony Fowler, "Cardinals or Clerics? Congressional Committees and the Distribution of Pork," *American Journal of Political Science*, 2016, *60* (3), 692–708.

Bó, Ernesto Dal, Frederico Finan, Olle Folke, Torsten Persson, and Johanna Rickne, "Who Becomes a Politician?," 2017.

Cameron, Charles M, John M de Figueiredo, and David E Lewis, "Public Sector Personnel Economics: Wages, Promotions, and the Competence-Control Trade-off," 2013.

Clerk of the House of Representative, "List of Standing Committees: 112th Congress," 2012. `http://history.house.gov/Congressional-Overview/Profiles/112th/`.

Dewan, Torun and Rafael Hortala-Vallve, "Electoral competition, control and learning," *British Journal of Political Science*, 2016.

Eggers, Andrew C and Arthur Spirling, "Party cohesion in Westminster systems: inducements, replacement and discipline in the house of commons, 1836–1910," *British Journal of Political Science*, 2016, *46* (03), 567–589.

Epstein, Lee, Jack Knight, and Andrew D Martin, "The norm of prior judicial experience and its consequences for career diversity on the US Supreme Court," *California Law Review*, 2003, pp. 903–965.

Fearon, James D, "Electoral accountability and the control of politicians: selecting good types versus sanctioning poor performance," *Democracy, accountability, and representation*, 1999, *55*, 61.

Ghosh, Suman and Michael Waldman, "Standard promotion practices versus up-or-out contracts," *The RAND Journal of Economics*, 2010, *41* (2), 301–325.

Green, Jerry R and Nancy L Stokey, "A Comparison of Tournaments and Contracts," *The Journal of Political Economy*, 1983, *91* (3), 349–364.

Groseclose, Tim and Charles Stewart III, "The value of committee seats in the House, 1947-91," *American Journal of Political Science*, 1998, pp. 453–474.

Hirsch, Alexander V and Kenneth W Shotts, "Competitive Policy Development," *The American Economic Review*, 2015, *105* (4), 1646–1664.

Höchtl, Wolfgang, Rudolf Kerschbamer, Rudi Stracke, and Uwe Sunde, "Incentives vs. selection in promotion tournaments: Can a designer kill two birds with one stone?"

Holmström, Bengt, "Managerial incentive problems: A dynamic perspective," *The Review of Economic Studies*, 1999, *66* (1), 169–182.

Howell, William G, "Presidential Prescriptions for State Policy: Obamas Race to the Top Initiative," 2014.

Hvide, Hans K and Eirik G Kristiansen, "Risk taking in selection contests," *Games and Economic Behavior*, 2003, *42* (1), 172–179.

Krishna, Vijay and John Morgan, "The winner-take-all principle in small tournaments," *Advances in applied microeconomics*, 1998, *7*, 61–74.

Lazear, Edward P and Sherwin Rosen, "Rank-Order Tournaments as Optimum Labor Contracts," *The Journal of Political Economy*, 1981, *89* (5), 841–864.

Legislative Clerk, "The Senate of the United States: Committee and Subcommittee Assignments," 2012. `https://www.senate.gov/general/resources/pdf/112_committee_subcommittee_assignments.pdf`.

Li, Hongbin and Li-An Zhou, "Political turnover and economic performance: the incentive role of personnel control in China," *Journal of public economics*, 2005, *89* (9), 1743–1762.

Lu, Xiabobo and Pierre F. Landy, "Show Me the Money: Interjurisdiction Political Competition and Fiscal Extraction in China," *American Political Science Review*, 8 2014, *108*, 706–722.

Malcomson, James M, "Rank-order contracts for a principal with many agents," *The Review of Economic Studies*, 1986, *53* (5), 807–817.

Maskin, Eric, Yingyi Qian, and Chenggang Xu, "Incentives, information, and organizational form," *The Review of Economic Studies*, 2000, *67* (2), 359–378.

Mesquita, Ethan Bueno De, "Politics and the suboptimal provision of counterterror," *International Organization*, 2007, *61* (01), 9–36.

Michels, Robert, *Political parties: A sociological study of the oligarchical tendencies of modern democracy*, Hearst's International Library Company, 1915.

Montagnes, B. Pablo and Stephane Wolton, "Mass Purges: Top-Down Accountability in Autocracy," 2016.

Myerson, Roger B, "Federalism and Incentives for Success of Democracy," *Quarterly Journal of Political Science*, 2006, *1*, 3–23.

Nalebuff, Barry J and Joseph E Stiglitz, "Prizes and incentives: towards a general theory of compensation and competition," *The Bell Journal of Economics*, 1983, pp. 21–43.

Orrison, Alannah, Andrew Schotter, and Keith Weigelt, "Multiperson tournaments: An experimental examination," *Management Science*, 2004, *50* (2), 268–279.

Powell, Robert, "Defending against terrorist attacks with limited resources," *American Political Science Review*, 2007, *101* (03), 527–541.

Prato, Carlo and Stephane Wolton, "The Voters' Curses: Why We Need Goldilocks Voters," *American Journal of Political Science*, 2016, *60* (3), 726–737.

Prendergast, Canice, "The provision of incentives in firms," *Journal of economic literature*, 1999, *37* (1), 7–63.

Ryvkin, Dmitry, "The selection efficiency of tournaments," *European Journal of Operational Research*, 2010, *206* (3), 667–675.

Stewart III, Charles, "The Value of Committee Assignments in Congress Since 1994," 2012. `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2035632`.

Volden, Craig and Alan E Wiseman, *Legislative Effectiveness in the United States Congress: The Lawmakers*, Cambridge University Press, 2014.

Waldman, Michael, "Classic promotion tournaments versus market-based tournaments," *International Journal of Industrial Organization*, 2013, *31* (3), 198–210.