On the Geography of Global Value Chains

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Abstract

This paper studies the optimal location of production for the different stages in a sequential global value chain. We develop a general-equilibrium model featuring a proximity-concentration tradeoff: slicing global value chains across countries allows to better exploit agglomeration economies, but such fragmentation comes at the cost of increased transportation costs. We show that, other things equal, it is optimal to locate relatively downstream stages of production in relatively central or well-connected locations, while upstream stages of production are optimally assigned to more remote locations. We illustrate this result by working out the optimal location of production for a few basic topologies featuring a low number of countries and stages. Exact solutions to the problem for a larger number of countries and stages are computationally complex, but can be obtained using combinatorial optimization tools. We apply the model to study the optimal specialization within chains in eleven countries in *Factory Asia*.

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1 Introduction

In recent decades, technological progress and falling trade barriers have allowed firms to slice up their value chains, retaining within their domestic economies only a subset of the stages in these value chains. The rise of global value chains (GVCs) has dramatically changed the landscape of the international organization of production, placing the specialization of countries *within* global value chains at the center stage. Where in global value chains are different countries specializing? Should countries use policies to place themselves in particularly appealing segments of global value chains? What is the optimal shape of those policies? These are questions being posed in the policy arena for which the academic literature has yet to provide satisfactory answers.

This paper describes ongoing and future research aimed at studying the specialization of countries within global value chains in a world with barriers to international trade. This specific paper will focus on outlining the implications of the existence of exogenously given trade costs for the optimal and equilibrium shape of global value chains. The role of natural trade barriers on the geography of global value chains is interesting in its own right and has been underexplored in the literature. Perhaps more importantly, however, the analysis in this proposal can be viewed as a stepping stone for a future *normative* analysis of the costs and benefits of using certain policies (such as man-made trade barriers) to "move" countries up or down global value chains.

As an illustration of the type of phenomena we can shed light on with our framework, Figure 1 provides motivating evidence of a positive relationship between a country's centrality and their relative downstreamness. It shows that relatively more remote countries (within Asia) specialize relatively upstream in the value chain.[1]

The rest of this (rough) draft is organized as follows. Section 2 describes a general environment in which geography – i.e., the ease with which countries can trade with other countries – shapes the (Pareto) optimal position of countries in global value chains. In section 3, a particular case of the general problem is worked out, which serves to unveil an intuitive relationship between the centrality of countries in space and their downstreamness in global value chains. In section 4 we explore a generalization to a world with multiple supply chains while section 5 presents an alternative probabilisitic framework with a continuum of goods and a continuum of supply chains. Finally, section 6 offers a brief conclusion.

Brief Relationship to the Literature In recent years, a few theoretical frameworks have been developed highlighting the role of the sequentiality of production for the global sourcing decisions of firms. Recent theoretical work in this literature include Harms, Lorz, and Urban (2012), Baldwin and Venables (2013), Costinot *et al.* (2013), Antràs and Chor (2013), Kikuchi *et al.* (2014), and Fally and Hillberry (2014).[2] In those frameworks, trade frictions are however either not modelled

[1] Remoteness is constructed as a GDP weighted average bilateral distance to other countries in the world, where bilateral distance is borrowed from CEPIII. Average upstreamness of overall merchandise exports is also from Antràs et al. (2012) and corresponds to the year 2002.

[2] This literature is in turn inspired by earlier contributions in Dixit and Grossman (1982), Sanyal and Jones (1982), Kremer (1993), and Kohler (2004).

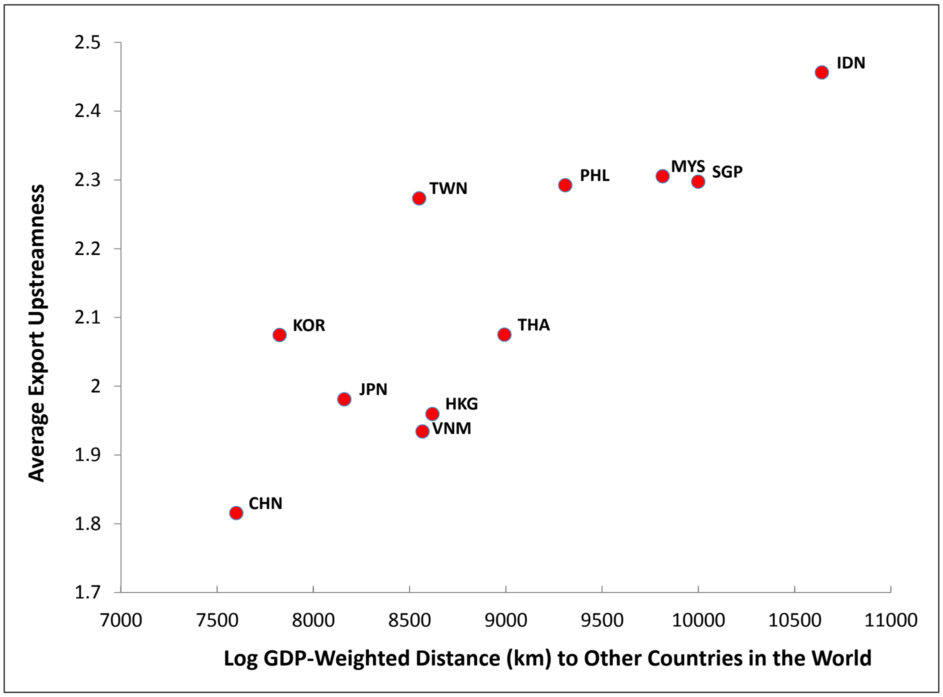

Figure 1: Upstreamness and Remoteness in *Factory Asia*

or modelled in highly stylized ways (i.e., assuming symmetric trade costs across countries). There is also an empirical literature attempting to quantify the role of global value chains in shaping the elasticity of trade flows to trade frictions, as in the work of Yi (2003) or Johnson *et al.* (2014), but that literature also relies on the quantification of relatively simple, low-dimensional models.

2 The Model

2.1 General Environment

There are J countries where consumers derive utility from consuming a final good. The good is produced combining N stages that need to be performed sequentially using a unique composite factor production which We refer to as labor. The last stage of production can be interpreted as assembly and is indexed by N. We will often denote the set of countries $\{1, ..., J\}$ by $\mathcal{J}$ and the set of production stages $\{1, ..., N\}$ by $\mathcal{N}$. Countries differ in their geography, as captured by a $J \times J$ matrix of iceberg trade coefficients τ_{ij}, where τ_{ij} denotes the units of the finished or unfinished good that need to be shipped from i for unit to reach j. We also let countries vary in their size or productivity as captured by the fact that each consumer in country i is endowed with L_i efficiency units of labor. For simplicity, We begin by assuming that a unit measure of consumers populate each country, although population differences can be incorporated into the framework as we will demonstrate later.

Output of stage n in country i is a function of the allocation of country's i labor to that sector, L_i^n, and of the services available in country i of the semi-finished product up to the previous stage

$n-1$. We denote these intermediate input services by c_i^{n-1} and represent the stage-n technology by

$$y_i^n = f_i{}^n\left(L_i^n, c_i^{n-1}\right), \text{ for all } n \in \mathcal{N},\, i \in \mathcal{J}, \tag{1}$$

where we normalize

$$c_i^0 = 1, \text{ for all } i \in \mathcal{J}. \tag{2}$$

Notice that, at this point, we let the function $f_i{}^n$ vary across stages and countries, perhaps reflecting stage asymmetries and Ricardian differences in labor productivity across countries. To make some progress in the characterization of the problem it will be necessary below to place some more structure on this function.

The services of the semi-finished product up to stage n available in country i are in turn determined by country i's absorption of the worldwide production of that stage output. We write this as

$$c_i^n = \sum_{j=1}^{J} \frac{\delta_{ji}^n y_j^n}{\tau_{ji}}, \text{ for all } n \in \mathcal{N},\, i \in \mathcal{J}, \tag{3}$$

where δ_{ji}^n represents the share of country j's stage-n gross output that is shipped to country i. Notice that when evaluating this expression at $n = N$, equation (3) determines consumption of the final good in each country. The welfare of a consumer in country i is then simply given by $u\left(c_i^N\right)$, where $u(\cdot)$ is assumed to be twice continuously differentiable, non-decreasing and weakly concave.

To complete the description of the model, notice that goods-market clearing implies that

$$\sum_{i=1}^{J} \delta_{ji}^n = 1, \text{ for all } n \in \mathcal{N},\, j \in \mathcal{J}, \tag{4}$$

while labor-market clearing imposes

$$\sum_{n=1}^{N} L_i^n = L_i. \tag{5}$$

Characterizing the Pareto optimal allocation of production across stages and countries simply boils down to choosing the $N \times J$ allocations of labor L_i^n and the $N \times J^2$ distribution shares δ_{ji}^n to solve:

$$\begin{array}{ll} \max & W = \sum_{i=1}^{J} \lambda_i u\left(c_i^N\right) \\ \text{subject to} & (1), (2), (3), (4), \text{ and } (5), \end{array} \tag{6}$$

where λ_i is an arbitrary non-negative Pareto weight assigned to country i. The utilitarian optimum corresponds tot the case in which $\lambda_i = 1$ for all i.

2.2 A Proximity-Concentration Tradeoff

As mentioned before, we have placed very little structure on the recursive formulation of technology in equation (1). My main interest in this paper is to isolate the role of geographical features of countries in shaping their average position in sequential global value chains. For this reason, we will for the time being abstract from technological differences across countries and make the function f_i^n common for all countries. This naturally raises the issue of why exactly it might be optimal to fragment value chains across countries under the plausible assumption that trade costs are lowest in domestic transactions, that is, for all i, $\tau_{ii} = \min_j \tau_{ij}$. In fact, if the function f^n features constant returns to scale in L_i^n and c_i^{n-1}, then the trivial solution to program (6) entails zero fragmentation: all countries produce output in all N stages and there is zero trade flows across countries.

In order to have a well-defined trade off between domestic and foreign sourcing, we assume that the production technology in (1) subsumes agglomeration forces. In particular, we let the productivity of labor in country i at stage n be an increasing function $g\left(L_i^n\right)$ of the allocation of labor to that stage in that country, so that we can write

$$f^n\left(L_i^n, c_i^{n-1}\right) = f^n\left(g\left(L_i^n\right)L_i^n, c_i^{n-1}\right).$$

Even when f^n is homogeneous of degree one in $g\left(L_i^n\right)L_i^n$ and c_i^{n-1}, the presence of external economies now has the potential to bring about gains from international specialization. To illustrate this, consider the case in which

$$f^n\left(L_i^n, c_i^{n-1}\right) = \left(\left(L_i^n\right)^{\phi} L_i^n\right)^{1/n} \left(c_i^{n-1}\right)^{1-1/n}. \tag{7}$$

This formulation of technology is particularly convenient for several reasons. First, it reduces the force of external economies to a single parameter ϕ. Second, when one considers a situation of autarky with no international fragmentation, we can iterate equation (7) together with (3) (while setting, for all n, $\delta_{ji}^n = 1$ if $i = j$, and $\delta_{ji}^n = 0$ otherwise) to obtain:

$$c_i^N = \left(\tau_{ii}\right)^{-(N+1)/2} \left(\prod_{n=1}^{N} \left(L_i^n\right)^{1/N}\right)^{\phi+1}. \tag{8}$$

It thus follows that final-output is simply a *symmetric* Cobb-Douglas aggregator of the labor services of each of the N stages. This constitutes a useful benchmark because it eliminates all stage asymmetries other than their position or 'downstreamness' in the value chain. For this reason, we will stick to the formulation of technology in (7) for the remainder of the paper. Notice that, regardless of the size of Pareto weights and of the particular utility function $u\left(\cdot\right)$, autarky reduces the general problem in (6) to simply maximizing c_i^N in (8) subject to the labor-market constraint (5). It is trivial to verify that the unique autarky solution involves an equal allocation of labor to each stage ($L_i^n = L_i/N$ for all n), implying

$$\left(c_i^N\right)_{autarky} = \left(\tau_{ii}\right)^{-(N+1)/2} \left(\frac{L_i}{N}\right)^{\phi+1}, \tag{9}$$

and a worldwide social welfare equal to

$$W = \sum_{i=1}^{J} \lambda_i u \left((\tau_{ii})^{-(N+1)/2} \left(\frac{L_i}{N} \right)^{\phi+1} \right). \tag{10}$$

To provide a first illustration of the role of the parameter ϕ in generating benefits from international specialization within chains, consider the specific example in which the number of countries coincides with the number of stages and there is complete specialization (denoted by a subscript sp, hereafter). In other words, assume that each stage of production is produced in only one country, and each country produces output of only one stage. Denote by $\ell(n)$ the location (or country) where stage n is produced. It should be clear that this situation is associated with distribution shares, for $n < N$, equal to $\delta^n_{ji} = 1$ for $j = \ell(n)$ and $i = \ell(n+1)$, and equal to $\delta^n_{ji} = 0$ otherwise. In such a case, we can again iterate equation (7) together with (3) to obtain

$$\left(c^N_i\right)_{sp} = \frac{\delta^N_{\ell(N)i}}{\tau_{\ell(N)i}} \prod_{n=1}^{N-1} \left(\tau_{\ell(n)\ell(n+1)}\right)^{-n/N} \left(\prod_{n=1}^{N} \left(L_{\ell(n)}\right)^{1/N} \right)^{\phi+1}. \tag{11}$$

The general problem in (6) then boils down to the following two-step program. First, for a given final-good producing country i, choose the distributional shares of final output $\delta^N_{\ell(N)i}$ to maximizes $\sum_{i=1}^{J} \lambda_i u\left(c^N_i\right)$ subject to (11) and (4). Second, solve for the optimal assignment of stages to countries conditional on the vector of final consumptions being determined by the first step of the program for each potential sequencing of countries along the value chain. We shall devote the next section to analyzing this second stage in some detail.

For now, and to focus attention on the role agglomeration economies play in the model, let us assume that the solution to the first stage delivers an equal division of 'free-on-board' final-good output, so that $\delta^N_{\ell(N)i} = 1/N$. It is trivial to verify that this equal division is optimal under the utiliarian criterion ($\lambda_i = 1$ for all i). The question is then, under which conditions is welfare higher under this specific form of fragmentation than under autarky? Comparing equations (9) and (11) with $\delta^N_{\ell(N)i} = 1/N$, it is easy to verify that global value chains will dominate autarky whenever international trade costs are not much larger than domestic trade costs. To illustrate this, consider the case in which domestic trade frictions are symmetric across countries and captured by $\tau_{ii} = \tau_d$ for all i, while international trade frictions are also symmetric and given by $\tau_{ij} = \tau_f > \tau_d$ for all $i \neq j$. In such a case, we have that for all $i \neq \ell(n)$, $\left(c^N_i\right)_{autarky} < \left(c^N_i\right)_{sp}$ if and only if

$$\left(\frac{\tau_f}{\tau_d}\right)^{(N+1)/2} < N^{\phi} \left(\prod_{n=1}^{N} \left(\frac{L_{\ell(n)}}{L_i} \right)^{1/N} \right)^{\phi+1},$$

while for $i = \ell(n)$, the condition for an improvement is even weaker since this country benefits from assembly being done in its home country). Notice then that as differences in efficiency units of labor across countries are driven down to zero, the right-hand-side of this inequality goes to N^{ϕ}

and autarky is dominated whenever $\phi > 0$ and τ_f is not much higher than τ_d.[3]

3 Complete Specialization in the *Even* Case

Beyond the above discussion, and without putting more structure on the problem, it is virtually impossible to provide a sharp characterization of its solution. To make some progress, we will next focus on the special *even* case in which the number of countries and stages coincide ($J = N$) and in which the equilibrium is one with complete specialization.[4]

3.1 Geography and Specialization

From our derivations above, in this particular case, we can recast the general problem (6) as solving for the assignment of countries to stages, that is $\{\ell(1), \ell(2), ..., \ell(N))$, that maximizes

$$\begin{array}{ll} \max & W = \sum_{i=1}^{N} \lambda_i u \left(\delta^N_{\ell(N)i} \left(\tau_{\ell(N)i}\right)^{-1} \prod_{n=1}^{N-1} \left(\tau_{\ell(n)\ell(n+1)}\right)^{-n/N} \left(\prod_{n=1}^{N} \left(L_{\ell(n)}\right)^{1/N} \right)^{\phi+1} \right) \\ \text{subject to} & \sum_{i=1}^{J} \delta^N_{\ell(N)i} = 1 \end{array} .$$

To simplify the derivations, we will further restrict attention to the case of logarithmic utility, which has the nice property of allowing me to represent social welfare as an additively separable function of the trade frictions incurred at each stage in the value chain. More specifically, in such a case, it is easy to verify that $\delta^N_{\ell(N)i} = \lambda_i/\Lambda$ where $\Lambda = \sum_j \lambda_j$, and setting $\Lambda = N$ without loss of generality, we have

$$W = -\sum_{i=1}^{N} \lambda_i \ln \tau_{\ell(N)i} - \sum_{n=1}^{N-1} n \ln \tau_{\ell(n)\ell(n+1)} + (\phi+1) \sum_{n=1}^{N} \ln L_{\ell(n)} + \sum_{i=1}^{N} \lambda_i \ln(\lambda_i/N). \quad (12)$$

As a result, we can state:

Lemma 1 *In the even case $N = J$, the optimal assignment of stages to countries with complete specialization and logarithmic utility seeks to solve*

$$\min_{\{\ell(n)\}_{n=1}^{N}} H(\ell(1), ..., \ell(N)) = \sum_{i=1}^{N} \lambda_i \ln \tau_{\ell(N)i} + \sum_{n=1}^{N-1} n \ln \tau_{\ell(n)\ell(n+1)}. \quad (13)$$

Notice that the optimal position of a country in the value chain is crucially shaped by the $J \times J$ matrix of trade costs as captured by the coefficients τ_{ij}. Intuitively, the optimal sequencing of production will seek to minimize the trade costs associated with the production process traveling

[3] As our results below will illustrate, autarky will tend to be dominated by some form of global value chains even when countries are dissimilar in size, but an equilibrium with *complete* specialization is not necessarily optimal in that case.

[4] wecall this the *even* case in analogy to the terminology used in the Heckscher-Ohlin framework when considering variants of the model with the same number of sectors and factors (see, for instance, Ethier, 1984).

through each of the J countries, 'visiting' each country exactly one time, and then returning to all countries in the form of a finished product. When put in these terms, the savvy reader will recognize the tight connection between the optimization problem in (13) and the minimal distance Hamiltonian path problem in graph theory, or the associated travelling salesman problem (TSP) in combinatorial optimization. Both of these problems are NP-hard as they entail picking an optimal sequencing out of the $N!$ possible permutations of countries in the value chain.

There are two key differences between the above problem in (13) and these two classical problems. First, the implied transports costs of shipping goods between any two countries i and j is not only a function of the distance and other geographical (or cultural) characteristics of these two countries, but depends also on the stage at which this exchange takes place. As is clear from the second terms in the maximand of (13), the optimal assignment will put a larger weight on reducing trade costs at relatively downstream stages than at stages further upstream. The reason for this is that the costs of transporting costs have been modelled (realistically, we believe) to be proportional to the gross value of the good being transacted, rather than being assumed proportional to the value added at that stage. Because gross output naturally rises along the value chain, so do the distortions arising from shipping goods across borders.[5] This is the first key insight of the model and it suggests that one might expect a positive correlation between centrality and downstreamness (or between remoteness and upstreamness), as illustrated in Figure 1 in the Introduction.

Matters are however a bit more complicated because, as the first term in (13) illustrates, the value chain visits each country not only once, but twice since the finished product is shipped from the assembly location to all countries. This again suggests picking a relatively well connected location to carry out assembly, but that choice is now also shaped by the Pareto weights of each country. Other things equal, a planner would shift the location of assembly towards locations to which it assigns a relatively large weights (provided of course that $\tau_{\ell(N)N} < \tau_{\ell(N)i}$ for all $i \neq \ell(N)$). We do not want to take a particular stance on how large these Pareto weights are, so we will mostly focus on the utilitarian case with $\lambda_i = 1$ for all i. Nevertheless, it is worth emphasizing that variation in Pareto weights will only affect the optimal sequencing of production along countries to the extent to which it shapes the location of production. Or in other words, a small perturbation in the Pareto weights that does not affect the optimal location of assembly will necessarily leave the location of the remaining stages unaffected as well.

A similar result applies when reinterpreting L_i as measuring population (rather than efficiency units of labor per inhabitant) in country i. In such a case, the social welfare function becomes

[5] In terms of the traditional TSP, it is as if the travelling salesman was picking up packages along his route and finding it increasingly costly to travel carrying those packages.

$W=\sum_{i=1}^{J}\lambda_i L_i \ln\left(c_i^N/L_i\right)$, and the minimand in program (13) becomes:

$$H\left(\ell\left(1\right),...,\ell\left(N\right)\right)=\sum_{i=1}^{J}\left(\frac{\lambda_i L_i}{\sum_{i=1}^{J}\lambda_i L_i/J}\right)\ln\tau_{\ell(N)i}+\sum_{n=1}^{N-1}n\ln\tau_{\ell(n)\ell(n+1)}. \tag{14}$$

Clearly, holding geography constant, more populous locations are more attractive locations for assembly. Yet, small changes in population that leave the optimal location of assembly unchanged, will not affect upstream locations either. In fact, variation in population size is irrelevant for the optimal assignment of stages to countries once the location of assembly is pinned down. This result is not entirely obvious given the presence of external economies of scale in the model. Still, as indicated by equation (11), the strength of external economies of scale is kept constant along the value chain, so there is no immediate gain in relocating specific stages (other than assembly) to particularly highly populated countries.

We have so far informally linked the optimal position of a country in global value chains to its geographical centrality. In the next section, we will illustrate this result via a series of examples based on simple network topologies. Before turning to these examples, however, we briefly formalize a version of this downstreamness-centrality nexus using the tools of monotone comparative statics. With that in mind, assume that the easiness of trade between any two countries i and j can be decomposed as follows:

$$\tau_{ij}=\begin{cases}\left(\rho_i\rho_j\right)^{-1} & \text{if } i\neq j\\ \xi\left(\rho_i\rho_j\right)^{-1} & \text{if } i=j\text{, with } \xi<1\end{cases} \tag{15}$$

where we take ρ_i to be an index of the *centrality* of country i. Notice that if country i is more central than country j, then it is cheaper to ship from i to any other country in the world than it is to ship from country j (other than when the destination country is j itself, since $\xi<1$). This is a very strong notion of centrality but it has the virtue of providing the following stark result:

Proposition 1 *Let countries be ordered according to their centrality so that $\rho_1<\rho_2<....<\rho_N$. Then, as long as cross-country differences in the Pareto weights λ_i and population L_i are sufficiently small, the optimal assignment with complete specialization and logarithmic utility is necessarily such that $\ell\left(n\right)=n$, and thus the n-th most central country is assigned the n-th most downstream position in the value chain.*

This result simply follows from the fact that the specification of trade costs in (15) simplifies expression (14) to

$$H\left(\ell\left(1\right),...,\ell\left(N\right)\right)=-\sum_{n=1}^{N}\left(2n-1\right)\ln\rho_{\ell(n)}-\sum_{i=1}^{J}\left(\frac{\lambda_i L_i}{\sum_{i=1}^{J}\lambda_i L_i/N}\right)\ln\rho_i-2\left(1-\xi\right)\frac{\lambda_{\ell(N)}L_{\ell(N)}}{\sum_{i=1}^{J}\lambda_i L_i/N}. \tag{16}$$

In the absence of cross-country differences in the Pareto weights λ_i and population L_i, the last term in (16) reduces to the constant $-2(1-\xi)$ and the function $H(\ell(1), ..., \ell(N))$ is submodular in n and $\ln \rho_{\ell(n)}$. This in turn ensures that the solution $\ell(n)$ is increasing in n and, in fact, $\ell(n) = n$.[6]

How do differences in population or Pareto weights affect the validity of Proposition 1? To answer this question, consider the last term in (16).[7] If a country i is associated with a particularly high λ_i or L_i, and domestic trade costs are sufficiently lower than international ones ($\xi << 1$), then this term indicates that it may be optimal to assign the last stage N to this country regardless of its centrality ρ_i. Intuitively, even if a country is remote relative to the rest of the world, it may be optimal to produce the final good if it hosts a disproportionate large share of the world population, and it is costly to ship that final good from other countries.

An important point to emphasize is that the last term in (16) is a function of the Pareto weight and population of only the assembly country $\ell(N)$. This implies that, beyond possibly affecting the location of assembly, cross-country differences in λ_i and L_i have no bearing on the pattern of specialization derived in Proposition 1. In particular, under the strong notion of centrality in (15), there will continue to be a monotonic relationship between the centrality of the remaining locations and their position in the value chain, with more central locations being closer to assembly. We can formalize this as follows:

Proposition 2 *Consider the subset of locations $J_{\setminus N} = \{i \in 1, .., J : i \neq \ell(N)\}$, where the excluded country $\ell(N)$ is the optimal location of assembly. Let the elements of $J_{\setminus N}$ be ordered according to their centrality so that $\rho_1 < \rho_2 < .... < \rho_{N-1}$. Regardless of the Pareto weights and populations of the countries in $J_{\setminus N}$, the optimal assignment with complete specialization and logarithmic utility is necessarily such that $\ell(n) = n$ for $n \leq N-1$, and thus the n-th most central country is assigned the n-th most downstream position in the value chain leading to (but excluding) assembly.*

The notable feature of Proposition 1 is that, regardless of the population sizes and Pareto weights associated with different countries, the index of centrality ρ_i is a sufficient statistic for the determining the optimal allocation of all stages of production other than assembly across countries. Obviously, this stark result relies in part on the special form that bilateral trade costs take in equation (15). But, as argued above, the positive association between centrality and downstreamness is a more general result of the theoretical framework. We will next attempt to convey this by working out the optimal location of production for a few basic topologies featuring a low number of countries and stages. This will permit a graphical illustration of the link between centrality and downstreamness implied by the model. We will also conclude this section by working out computationally the solution to the problem using proxies for *actual* bilateral trade costs for a sample of twelve countries.

[6] To be more precise, the proof also requires noting that $\rho_{\ell(n)}$ is itself is increasing in $\ell(n)$ given our ordering of countries and that the allocation $\ell(n)$ is one-to-one by assumption.

[7] The second term in (16) also depends on the vectors of Pareto weights and population, but its value is independent of the allocation of countries to stages.

3.2 Centrality and Downstreamness: Some Examples

In this section, we will consider a series of simple low-dimensional examples of the model in which $J = N$ is small. To avoid a large taxonomy of cases, we will restrict attention to the cases with two, three, four, and five countries. We will also restrict attention to situations in which countries are of equal size L and share a common Pareto weight $\lambda_i = 1$. In sum, we will invoke Lemma 1, and solve the program

$$\min_{\{\ell(n)\}_{n=1}^{N}} H(\ell(1), ..., \ell(N)) = \sum_{n=1}^{N-1} n \ln \tau_{\ell(n)\ell(n+1)} + \sum_{i=1}^{N} \ln \tau_{\ell(N)i}, \tag{17}$$

for different (low-dimensional) $N \times N$ symmetric matrices of iceberg trade costs τ_{ij}.

Two countries (N=2) This is the simplest case to consider and is illustrated in Figure 2. The ease of trade between countries 1 and 2 is given by some general value $\tau_{12} = \tau_{21}$, while τ_{11} and τ_{22} capture the domestic ease of trade in each of these countries. It is hard to envision one country being more central relative than the other in this two-country example with $\tau_{12} = \tau_{21}$, but it seems sensible to regard the country with the lowest domestic trade cost as the most central one. Consistent with this notion, using (17) we find that $H(1,2) = \ln(\tau_{12}\tau_{21}\tau_{22})$ and $H(2,1) = \ln(\tau_{21}\tau_{11}\tau_{12})$. As a result, it is optimal to assign assembly to the (more central) country featuring the lowest domestic trade costs (e.g., $H(1,2) > H(2,1)$ whenever $\tau_{22} < \tau_{11}$).

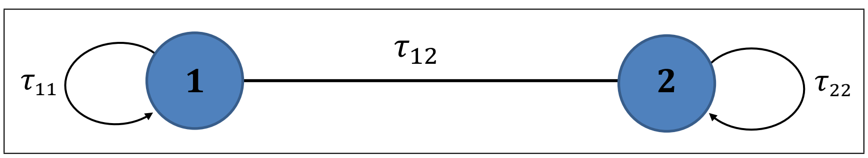

Figure 2: Two-Country Case

Three countries (N=3) Three countries will generically form a triangle in the plane, as illustrated in Figure 3.[8] Domestic trade costs are not essential to determine the equilibrium in this (or any higher dimensional) case, so we will set $\tau_{ii} = 1$ for all i in the remaining examples. With $N = 3$, this leaves only three relevant values in the 3×3 *symmetric* matrix of ease-of-trade parameters, namely τ_{12}, τ_{13}, and τ_{23}. Computing (17) for the six possible permutations of the three countries, it is easy the verify that the relative ranking of τ_{12}, τ_{13}, and τ_{23} uniquely determines which country produces the most upstream stage. More specifically, the pair of countries (i,j) associated with the lowest value of τ_{ij} will be in the two most downstream positions of the value chain, while the remaining country will necessarily specialize in the most upstream stage. Thus, the most remote country will produce the most upstream stage. For example, when $\tau_{12} > \tau_{13} > \tau_{23}$, as depicted in Figure 3, country 1 will necessarily specialize in stage 1.

[8] The analysis remains unchanged when the three countries lie in a straight line, except that this situation imposes a linear constraint on the ease-of-trade coefficients (e.g., $\rho_{12} = \rho_{13} + \rho_{23}$ when country 3 lies between 1 and 2).

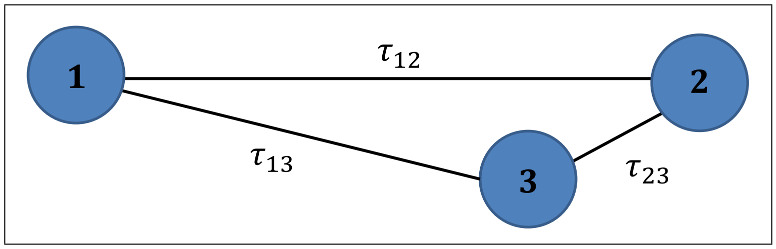

Figure 3: Three-Country Case

Which of the two countries (i, j) associated with the lowest value of τ_{ij} will specialize in assembly? In the example in Figure 3, it seems clear that country 3 is more central than country 2. Indeed, this country features higher *closeness centrality* in the jargon of network theory.[9] Nevertheless, simple computations demonstrate that $H(1,2,3) = \ln\left(\tau_{12}\tau_{13}(\tau_{23})^3\right) = H(1,3,2)$ and thus $1 \to 2 \to 3$ and $1 \to 3 \to 2$ are both optimal sequences of the global value chain. In sum, although it is optimal to assign the n-th most central country to the n-th most downstream stage in the value chain, there exists an alternative optimal solution that reverses the location of the last two stages. Fortunately, this type indeterminacy disappears in higher-dimensional situations, as we will next illustrate.

Four countries (N=4) Moving from three to four countries greatly complicates the characterization of the optimal sequencing of production. This is for at least two reasons. First, because four points in the plane can form a wide range of topologies in which visually determining the relative centrality of each point becomes less straightforward. Second, because the number of possible permutations of countries to consider increases from 6 to 24 when adding a fourth country. To make some progress, we shall focus on the case depicted in Figure 4, in which (i) one of the countries (country 4) is located at the circumcenter of the triangle formed by the other three countries; and (ii) the radius of the circumcircle is smaller than the distance between any two of the countries in the triangle. Denoting the identical trade costs between country 4 and the other three countries by α, the second assumption implies that $\alpha < \min\{\tau_{12}, \tau_{13}, \tau_{23}\}$. The advantage of these assumptions is that they allow one to more easily to rank countries in terms of their centrality, while also reducing the set of parameters relevant for determining the optimal location of production along the value chain. In Figure 4, it seems uncontroversial that country 4 is the most central country and country 1 is the most remote one. Computing (17) for each of the 24 possible permutations we then find that the optimal path is $1 \to 2 \to 3 \to 4$ and thus it begins in the most remote country and ends in the most central one. Note, however, that downstreamness is not strictly monotonic in standard measures of centrality, since country 2 actually features a higher closeness centrality than country 3. This serves to illustrate that the relevant notion of centrality implicit in (17) is more complex than existing ones.

Conjecture 1 *Suppose that the distance between country* 4 *and each of the other three countries*

[9] The *closeness centrality* of a node is the reciprocal of the sum of its distances from all other nodes in the graph.

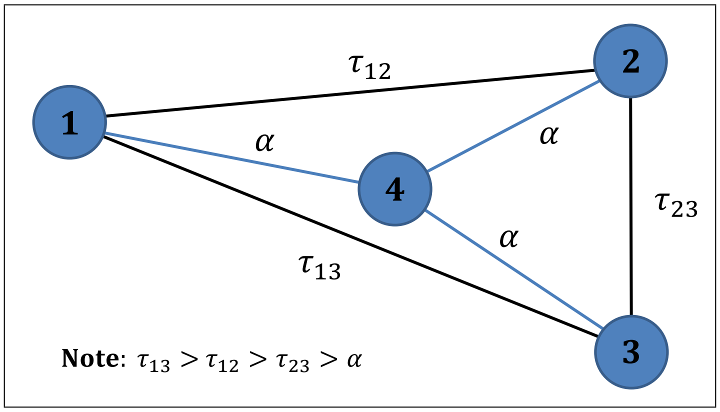

Figure 4: A Simple Four-Country Case

is smaller than the bilateral distance between any two of the other countries. Then it is optimal to have 4 *at the most downstream stage.*

Conjecture 2 *Suppose that the distance between country* 1 *and each of the other three countries is higher than the bilateral distance between any two of the other countries. Then it is optimal to have* 1 *at the most upstream stage.*

Five countries (N=5) The case with 5 countries is significantly more complex than the one with four countries since it entails $5! = 120$ possible permutations and a wide range of possible topologies. With that in mind, we will simply develop a special example that we think serves to sharpen our understanding of the role of centrality in shaping international specialization. For that purpose, consider Figure 5 in which countries 1 and 2 are symmetric and remote, countries 3 and 4 are also symmetric but less remote, and country 5 appears to be relatively central. Note also that the only way to ship back and forth across the two pairs of symmetric countries is via country 5 (at costs β and α, respectively, with $\beta > \alpha$), and thus the latter is not only relatively central, but is also a hub. Yet shipping within each pair of symmetric countries can be done directly without flowing through 5 at cost γ. It is straightforward to verify that for $\alpha < \gamma$, the optimal organization of production is such that the value chain begins in the remote pair of symmetric countries ($1 \rightarrow 2$ or $2 \rightarrow 1$) moving via 5 (but without producing there) to the less remote pair of countries ($3 \rightarrow 4$ or $4 \rightarrow 3$), and concluding with assembly at 5, from which the good is distributed to the other four countries. Thus again we obtain a positive association between downstreamness and centrality. Nevertheless, it is worth emphasizing that if one assumes instead that $\alpha > \gamma$, so the two less remote symmetric countries are closer to each other than they are to 5, a significantly different pattern emerges. In particular, in such a case the value chain continues to begin in the remote pair of countries, but now the third stage is carried out in the central country 5, while the last two stages are carried out in the less remote pair of countries 3 and 4. Intuitively, when γ is low relative to α, the relative centrality of 3 and 4 increases thereby making it more appealing to concentrate the last stages of production in that region. This example illustrates however that effective centrality of a location is shaped in subtle ways by the relative distance across locations.

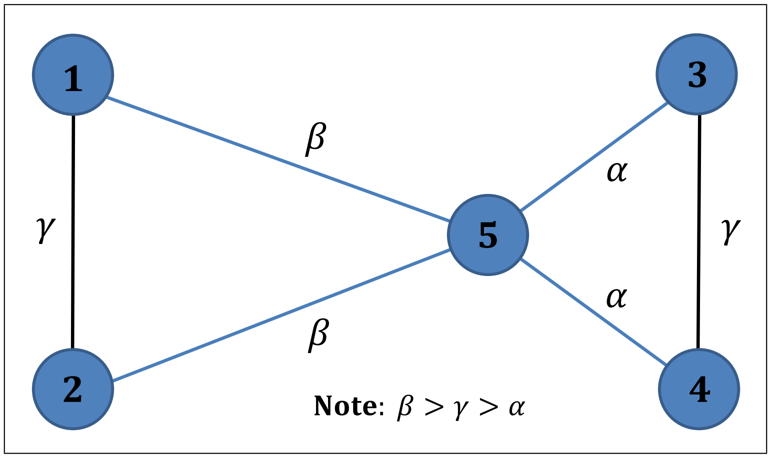

Figure 5: A Five-Country Example with a Hub

Conjecture 3 *Conjectures 1 and 2 above will continue to hold in this case, and actually hold for any N. This may be hard to prove.*

3.3 An Application to *Factory Asia*

As a final illustration of the complete specialization equilibrium in the even case, we solve for the sequential path of production that minimizes equation (14) when using empirical proxies for bilateral trade costs and population sizes (weagain ignore variation in Pareto weights). Because the number of permutations to consider is equal to $J!$ when working with J countries and stages, it is clear that such an exercise requires one to restrict the sample to a relatively small number of countries. Following the lead of the voluminous literature on the Traveller Salesman Problem, one could expand the sample and apply heuristic algorithms to obtain the best solution subject to a given computational time, but we will not do so because focusing on a world equilibrium with complete specialization seems rather far-fetched when the number of countries becomes large.

For the exercise, we will set $J = 12$ and focus on a sample consisting of the 11 largest East and Southeast Asian economies (the same as in Figure 1 in the Introduction) and the United States. We do so because vertical specialization within Asia (also known as *Factory Asia*) has received much attention in recent years. Population shares are drawn from the World Development Indicators for the year 2007 (except for Taiwan, whose 2007 population was obtained from the CIA World Factbook). The iceberg trade friction coefficients τ_{ij} are less straightforward to calibrate as they are shaped by a myriad of factors in the real world.

To make some progress, we invoke the gravity equation literature, which projects these τ_{ij} on a vector of pair-specific variables including distance, contiguity, common language, and so on. More specifically, we let

$$\begin{aligned}\ln \tau_{ij} \quad = \quad & \ln \kappa + \delta_{dist} \ln Distance_{ij} + \delta_{con} Contiguity_{ij} + \delta_{lang} SameLanguage_{ij} + \delta_{col} ColonyLink_{ij} \\ & + \delta_{rta} RegTradeAgreement_{ij} + \delta_{curr} SameCurrency_{ij} + \delta_{dom} DomesticTrade_{ij}, \qquad (18)\end{aligned}$$

where most of these variables should be self-explanatory with the possible exception of $DomesticTrade_{ij}$, which is a dummy variable that takes a value of 1 whenever $i = j$. To recover the δ coefficients in the above specification, one can then appeal to standard gravity equations – emanating, for instance, from the work of Eaton and Kortum (2002) or Anderson and Van Wincoop (2002) – in which bilateral trade costs are shown to be a log-linear function of these iceberg costs τ_{ij}, implying that

$$\delta_v = \frac{1}{\varepsilon} \hat{\delta}_v \text{ for } v = \{dist, con, lang, col, rta, curr\}, \qquad (19)$$

where $\hat{\delta}_v$ is the coefficient on variable v obtained from estimating a gravity equation and $\varepsilon > 0$ is the so-called "trade elasticity". We obtain the values of $\hat{\delta}_v$ from Table 3.4 of Head and Mayer (2014)'s handbook chapter, which reports the median value of each of these parameters from structural estimates of the gravity equation. These coefficients are reproduced in Table 1.

Table 1. Calibration of Trade Friction Parameters $\hat{\delta}_v$

Parameter	$\hat{\delta}_{dist}$	$\hat{\delta}_{con}$	$\hat{\delta}_{lang}$	$\hat{\delta}_{col}$	$\hat{\delta}_{rta}$	$\hat{\delta}_{curr}$	$\hat{\delta}_{dom}$
Value ($\hat{\delta}_v / \varepsilon$)	1.14	-0.52	-0.33	-0.84	-0.28	-0.98	-1.55

We next plug into (18) the values for the key gravity variables recorded in the CEPII gravity dataset for the year 2006 (the most recent one in the dataset). Given the log-linear functional form of the minimand in (14), it should be clear that the particular value of the trade elasticity ε or of the constant κ in (18) are *irrelevant* for determining the optimal sequential location of production. They can thus be set at any arbitrary positive value.[10]

To solve for the path of production that minimizes the function $H(\ell(1), ..., \ell(12))$ in equation (14) we use an exhaustive algorithm that computes $H(\ell(1), ..., \ell(12))$ for each of the 12! (roughly 479 million) possible permutations of economies and picks the one with the lowest value. The optimal sequential value chain is depicted in Figure 6. The value chain sets off in the *remote* United States, then flows to Southeast Asia via the Philippines, stopping in Indonesia, Singapore, Malaysia, and then making its way up to Thailand and Vietnam. From there, the chain flows North via Hong Kong and Taiwan towards Japan and then Korea, before making its final stop in China, where assembly takes place and the finished good is distributed locally and shipped to the remaining 11 countries.

[10] In the calculations below, we set them at $\kappa = 1$ and $\varepsilon = 15$ because this yields empirically reasonable trade costs for the countries in the sample. The 12×12 matrix of trade cost coefficients is reproduced in Table A.1 in the Appendix. The average value of trade costs across country-pairs with $i \neq j$ is equal to 1.77, which is in line with standard values estimated and employed in the literature (see, for instance, Anderson and van Wincoop, 2002, or Melitz and Redding, 2014).

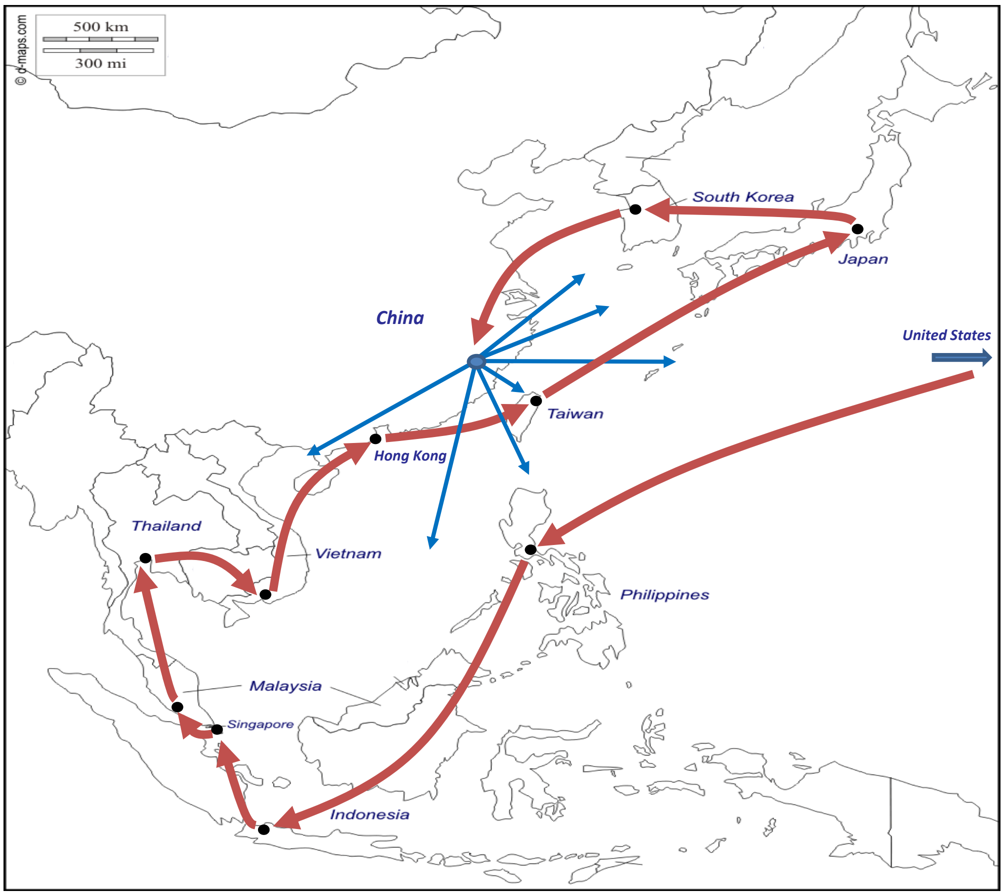

Figure 6: An Optimal Value Chain in *Factory Asia*

Consistent with the qualitative insights of the model, the upstream stages of production are optimal located in relatively remote countries (at least relative to Asia), while the most downstream stages are located close to relatively central and populous countries. It is interesting to compare the ordering of countries in this hypothetical optimal value chain with the *actual* average upstreamness of production of these countries. We do so in Figure 7 by plotting this ordering against the average upstreamness of these countries exports (as in Figure 1). There is a clear positive association between the two and their correlation is very high (0.754).

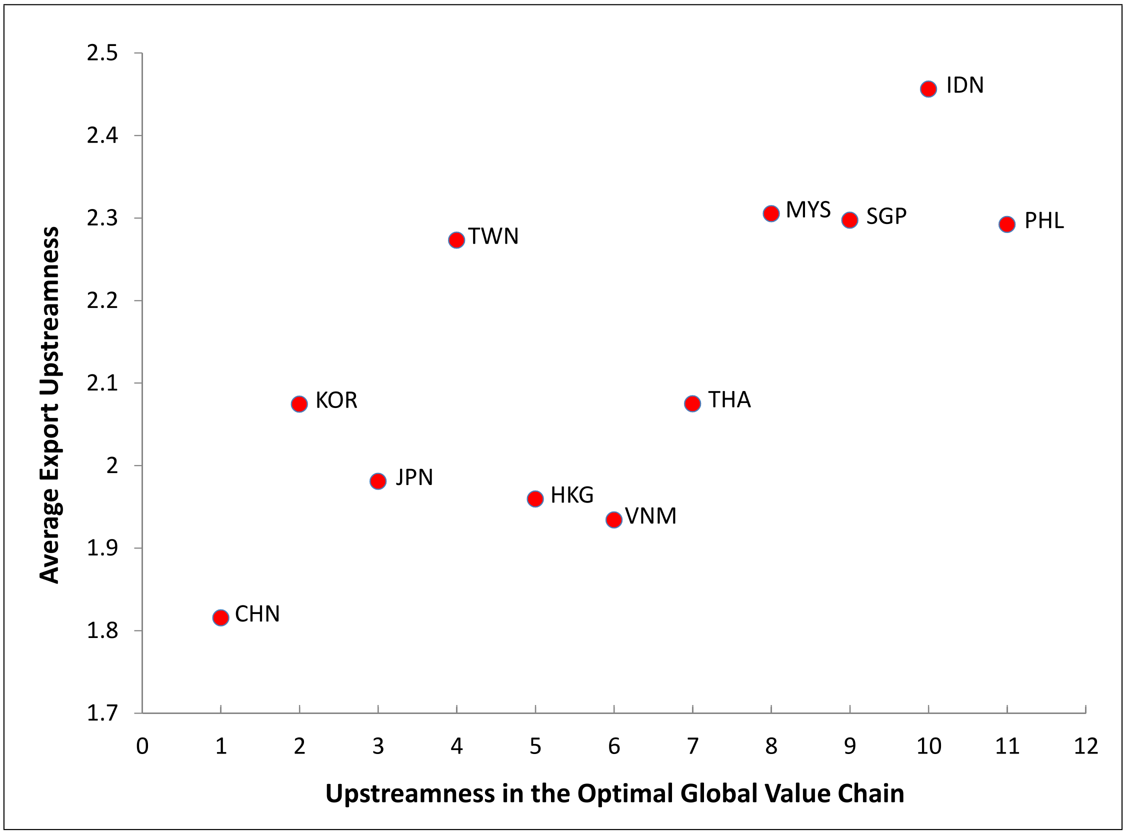

Figure 7: Export Upstreamness and Optimal Position in the Global Value Chain

The results of this quantitative exercise appear to be very robust to alternative measures of the trade costs parameters τ_{ij}. We have, for instance, verified that the results remain unchanged if we set all domestic trade costs τ_{ii} equal to 1 (rather than inferring them from the estimates of the border effect reported in Head and Mayer, 2014). Similarly, the optimal path is unaffected if we simplify the gravity specification in (18) and consider only on the effect of distance (δ_{dist}) and domestic trade (δ_{dom}), or even when only considering the effect of distance and setting $\tau_{ii} = 1$ for all domestic trade costs. As a final illustration of the robustness of the nexus between centrality and upstreamness, it turns out that the value chain begins in the United States and finishes in China not just in the optimal path but also in each of the 25 best performing permutations of countries.[11]

[11] We have also experimented with removing the United States from the sample. In such a cas, China continues to

4 Incomplete Specialization: A Multiple Supply Chain Framework

We have so far focused on characterization Pareto optimal allocations in which there is a single global value chain and each stage is produced in a single country. From a theoretical perspective, this special case is particularly convenient for illustrating some key geographical forces shaping the optimal specialization of countries along value chains. Still, the world economy is one in which several global value chains coexist, countries produce various stages located at different points in the value chain, and in which certain production stages are carried out in multiple countries. Can the framework above accommodate such possibilities? This is not just a matter of checking the robustness of (or enriching the set of) theoretical predictions emanating from the model, but it is also a key factor when judging the usefulness of our model for quantification purposes.

A first approach is a straightforward extension of the *even* case studied above. In particular, imagine that there are now J countries and N stages, but there is potentially a distinct supply chain leading to consumption in each country. We shall however continue to focus on complete specialization equilibria in which each country sources the final good from a *single* supply chain and the supply chain follows a *unique* pathway between its stage $n = 1$ and $n = N$. Notice, however, that countries may now perform various stages for a given value chain, and may even perform different stages for distinct value chains (i.e., value chains that are designed to ultimately service different markets). A key appealing feature of this more general case is that it allows for the coexistence of various global value chains in the world economy, which opens the door for an analysis of circumstances that lead to either domestic value chains, regional value chains or truly global value chains.

Formally, we consider again the general problem in (6) with technology given by (7), and we denote by $\kappa(i)$ the unique value chain leading to consumption of the assembled good in country i. In principle, there can be $K \leq J$ value chains coexisting in the world economy. Denote by $\ell^{\kappa(i)}(n)$ the country that produces stage n for the supply chain from which country i sources the final good. Our assumption that chains follow a unique path implies that two value chains can neither merge (i.e., $c_j^n = \delta^n_{\ell^{\kappa(i)}(n)j} y^n_{\ell^{\kappa(i)}(n)} / \tau_{\ell^{\kappa(i)}(n)j}$) nor split before assembly (i.e., $\delta^n_{ji} \in \{0,1\}$ for all stages $n < N$). Value chains are, however, allowed to split when distributing the final good, just as in our even case, in which remember that a single location was shipping the final good to all other countries in the world. Iterating the production function, we can then conclude that the final consumption available to consumers in country i is given by

$$c_i^N = \frac{\delta^N_{\ell^{\kappa(i)}(N)i}}{\tau_{\ell^{\kappa(i)}(N)i}} \prod_{n=1}^{N-1} \left(\tau_{\ell^{\kappa(i)}(n)\ell^{\kappa(i)}(n+1)} \right)^{-\frac{n}{N}} \left(\prod_{n=1}^{N} \left(L^n_{\ell^{\kappa(i)}(n)} \right)^{\frac{1}{N}} \right)^{1+\phi},$$

be the optimal location of assembly. The main difference in the optimal path is that Korea and Japan now appear in the most upstream positions in the value chain. The correlation between the position of countries in the optimal value chain and their average upstreamness however remains quite high (0.446).

and the social planner simply maximizes $W = \sum_{i=1}^{J} \lambda_i u\left(c_i^N\right)$ subject to $\sum_{i \in \Delta_j^N} \delta_{ji}^N = 1$, where $\Delta_j^N = \left\{i | \ell^{\kappa(i)}(N) = j\right\}$, and subject to the labor market clearing constraint (5). With logarithmic utility, this problem can then be solved in two steps. First, conditional on an allocation of countries and stages to value chains, one can solve for the optimal distribution of final output to countries that share a common source of the assembled good (i.e., $\delta_{\ell^{\kappa(i)}(N)i}^N$) as well as for the amount of labor each country devotes to each value chain's stage (i.e., $L_{\ell^{\kappa(i)}(n)}^n$). Second, one can plug those optimal values and solve for the optimal configuration of value chains across countries and for the set of stages each country performs in each value chain.

We are currently exploring a characterization of this problem to revisit the link between centrality and downstreamness unveiled in the even case in section 3. Perhaps more interestingly, we are also working on computationally implementing this richer framework with data on population sizes and bilateral trade costs across countries. As in the even case described above, the problem under study can be written as a zero-one linear integer programming problem, and standard algorithms can be applied to speedily solve for the optimal allocation of stages to countries even in environments with many countries and stages. Such a quantitative exercise now requires us to take a stance on the parameter ϕ governing the size of agglomeration economies. Our goal is to calibrate the model to data on a large number of economies and conduct counterfactual exercises to explore the extent to which further reductions in trade costs might lead to a further globalization of production processes, with regional value chains giving rise to truly global value chains.

5 Incomplete Specialization: A Probabilistic Approach

We have so far focused on complete specialization allocations in which each stage is produced in a single country. As argued repeatedly above, this is not without loss of generality since the optimal allocation is likely to entail incomplete specialization when trade frictions are sufficiently high or when agglomeration economies are sufficiently weak. Unfortunately, characterizing these allocations is very cumbersome given that small changes in parameter values can lead to discontinuous changes in the optimal allocation of production. The difficulties raised by incomplete specialization allocations are not too distinct from those that appear in multi-country Ricardian models of trade. This suggests that the probabilistic approach of Eaton and Kortum (2002) might be suitable for characterizing these type of equilibria. We will next explore such an approach.

5.1 Theoretical Framework: Preliminaries

Readers familiar with Eaton and Kortum's (2002) work might envision the following natural modification of our initial planning program in (6). First, instead of considering a single value chain, consider a world with a very large number (formally a continuum of measure 1) of value chains indexed by $z \in (0, 1)$. Let utility in each country be defined as a function of the consumption of the finished products associated with the continuum of value chains, i.e., $u\left(\left\{c_i^N(z)\right\}_{z=0}^{1}\right)$. As in

Eaton and Kortum (2002), we will focus on the case in which the function $u(\cdot)$ is a CES aggregator over the continuum of varieties, so

$$u\left(\left\{c_i^N(z)\right\}_{z=0}^1\right)=\left(\int_0^1\left(c_i^N(z)\right)^{(\sigma-1)/\sigma}dz\right)^{\sigma/(\sigma-1)},\quad \sigma>1,$$

where σ equals the common elasticity of substitution across varieties. Second, assume that each country is given a value-chain-specific production technology $f_{i,z}^n(\cdot)$ that relates output of stage n in country i in value chain z to country's i labor allocation to that stage and value chain, and to the services available in country i of the semi-finished product up to the previous stage $n-1$ in value chain z, i.e.,

$$y_i^n(z)=f_{i,z}^n\left(L_i^n(z),c_i^{n-1}(z)\right),\text{ for all }n\in\{1,..,N\},\ i\in\{1,..,J\},\ z\in(0,1),$$

with initial condition $c_i^0(z)=1$ for all $i\in\{1,..,J\}$ and $z\in(0,1)$. Equations (3) and (4) can be analogously adapted to a multiple value-chain environment, while the labor market clearing condition (5) simply needs to integrate the allocations $L_i^n(z)$ across both stages n and value chains z.[12]

It should be clear that if the production technology $y_i^n(z)$ were to be common across value chains, we would revert back to the same exact optimal allocation as in the single value chain example. Assume instead that stage-n labor productivity in country i is heterogeneous across value chains and given by the reciprocal of a unit labor requirement $a_i^n(z)$. Adopting a symmetric Cobb-Douglas production technology analogous to the one in (7), we then have

$$f_{i,z}^n\left(L_i^n(z),c_i^{n-1}(z)\right)=\left(\frac{L_i^n(z)}{a_i^n(z)}\right)^{1/n}\left(c_i^{n-1}(z)\right)^{1-1/n}. \tag{20}$$

We have so far shown that extending the general framework to a multi-stage environment is straightforward. The solution to the general program is however not simpler than before, as it depends in rich way on the matrix of trade costs τ_{ij}, the labor force in each country i, and the $N\times J$ infinitely-dimensional vectors of labor productivity $1/a_i^n(z)$.

The key insight from Eaton and Kortum (2002) is that by treating labor productivity parameters as stochastic and drawn independently from type II (or Fréchet) extreme-value probability distribution, the characterization of a standard multi-country Ricardian model is greatly simplified, since one can express the equilibrium allocations as a function of the few parameters that govern

[12] To be more precise, equation (3) would become

$$c_i^n(z)=\sum_{j=1}^J\frac{\delta_{ji}^n(z)\,y_j^n(z)}{\tau_{ji}},\text{ for all }n\in\mathcal{N},\ i\in\mathcal{J},\text{ and }z\in(0,1),$$

equation (4) would be modified to $\sum_{i=1}^J\delta_{ji}^n(z)=1$,for all $n\in\mathcal{N}$, $j\in\mathcal{J}$, and $z\in(0,1)$, and equation (5) would become $\int_0^1\sum_{n=1}^N L_i^n(z)\,dz=L_i$.

this underlying probability distribution in different countries. In the present context, however, such an approach still leaves us with a highly complex problem to solve. To see this, consider the optimal organization of a value chain that leads to consumption of the finished country in a given country i. Which country should produce the assembled good consumed in i? Naturally, it should be whichever country $j^*(i)$ can provide that good to i at lowest cost. In Eaton and Kortum's (2002) model such a cost depends only on trade costs between $j^*(i)$ and i, on labor productivity of country $j^*(i)$, and on this country's wage (which equals the shadow cost of labor in their competitive model). With multiple production stages, however, the cost of servicing consumers in i from $j^*(i)$ will also depend on the cost of providing the good finished up to stage $N-1$ to country $j^*(i)$. Such a cost will depend on trade costs, wages and labor productivity associated with the country $k^*(j^*)$ from which the good finished up to stage $N-1$ is supplied at the least cost to j^*, but also on the cost faced by $k^*(j^*)$ when procuring the previous $N-2$ stage. Clearly, this process could be iterated until reaching the initial stage of production.

In sum, the minimum cost of providing a given finished good z to consumers in country i will depend on trade costs, wages, and of the realization of labor productivity draws along the value chain. More specifically, with the symmetric Cobb-Douglas technology in (20), such a cost will be a function of the product of the productivity draws associated with the optimal locations of production. Even when these productivity levels are drawn independently across countries and stages from a Fréchet distribution, the resulting distribution of the overall minimum cost of servicing consumers in country i will not admit a simple representation.[13] More generally, the distribution of the minimum cost at which product z finished up to an arbitrary stage n can be procured from a given country j cannot be characterized in closed form as a function of the deep parameters of the distributions from which the productivity levels are drawn. It is worth stressing that these complications persist even when countries are assumed to draw a common productivity level $1/a_i(z)$ for all stages n within a given value chain.

5.2 Theoretical Framework: A Feasible Approach

In order to make some progress, we next relax one of the assumptions we made above in adapting the Eaton and Kortum (2002) framework to the current sequential, multi-stage environment. In particular, the above formulation assumed that the the firm organizing a given value chain (or the planner in the planner formulation) had knowledge *before* making any location decision of the precise productivity level with which all stages of production in that value chain could be produced in different countries. We will now explore a situation in which the firm or the planner only learns the particular realization of $1/a_i^n(z)$ in different countries i when the good has been produced up to stage $n-1$. This might reflect, for instance, that stage n labor productivity is a function of the *compatibility* between labor in different countries and certain characteristics of the product finished up to stage $n-1$, with these product characteristics being observed only *after* manufacturing at

[13] The minimum of Fréchet distribution is also distributed Fréchet, but the products of Fréchet distributions is not.

stage $n-1$ takes place.[14]

To illustrate the difference this assumption makes, consider an example with only two stages, input production (stage 1) and assembly (stage 2). Conditional on procuring the input from country k, the unit cost of production of the finished product in country j is given by

$$c_2^j = \left(c_1^k \tau_{kj}\right)^{1/2} \left(w_j a_2^j (z)\right)^{1/2}$$

where c_1^k is the cost of production in country k, and is given by

$$c_1^k = w_k a_1^k (z) .$$

Assume that the each country j draws productivity levels $1/a_1^j (z)$ and $1/a_2^j (z)$ independently from the Fréchet distribution

$$\Pr(a_n^j (z) \geq a) = e^{-T_j a^\theta}, \text{ for } n = \{1, 2\}, \text{ with } T_j > 0. \tag{21}$$

As in Eaton and Kortum (2002), T_j governs the state of technology in country j, while θ determines the variability of productivity draws across countries, with a lower θ being associated with higher dispersion in productivity across countries.

Consider now the cost-minimizing way to service consumers in a given country i. With full ex-ante knowledge of the productivity draws $1/a_1^k (z)$, for all $k \in \mathcal{J}$, and $1/a_2^j (z)$ for all $j \in \mathcal{J}$, firms would choose the pair of locations $k^* (i)$ and $j^* (i)$ that solves

$$(k^* (i), j^* (i)) = \arg \min_{(k,j)} \left(a_1^k (z) w_k \tau_{kj} a_2^j (z) w_j (\tau_{ji})^2\right)^{1/2} .$$

As argued before, even when the productivity levels are drawn independently from Fréchet distributions, the distribution of the product $a_1^k (z) a_2^j (z)$ is not Fréchet and thus characterizing the problem remains complicated.

What we propose is to instead assume that firms only learn the actual realizations of assembly productivity $1/a_2^j (z)$ after input production has been carried out (and the location k is predetermined). The location of assembly is only decided after observing this vector of possible assembly productivity levels. Delaying this decision is of course optimal given the circumstances. In such a case, conditional on the location of input production k, the location of assembly $j^* (i)$ will simply seek to solve

$$j^* (i) = \arg \min_{j \in \mathcal{J}} \left(c_1^k \tau_{kj} a_2^j (z) w_j (\tau_{ji})^2\right)^{1/2} .$$

The problem thus reduces to a simple variant of the one-stage Eaton and Kortum (2002) framework, and thus the probability that a given country j is chosen is as the assembly location for a value

[14] The approach of building some form of ex-ante uncertainty in the Eaton and Kortum (2002) framework is similar in spirit to the one pursued by Tintelnot (2013) and Antràs, Fort and Tintelnot (2014).

chain that sets off in country k and is seeking to service consumers in i is given by

$$\Pr\left(j=\ell\left(2\right)\mid k=\ell\left(1\right);i\right)=\frac{T_{j}\left(\tau_{kj}w_{j}\left(\tau_{ji}\right)^{2}\right)^{-\theta}}{\Omega_{i|k}}$$

where

$$\Omega_{i|k}=\sum_{j'\in\mathcal{J}}T_{j'}\left(\tau_{kj'}w_{j'}\left(\tau_{j'i}\right)^{2}\right)^{-\theta}. \tag{22}$$

Furthermore, the expected price that consumers in i will pay, conditional again on input production in k, reduces to

$$\mathbb{E}\left[\left(c_{1}^{k}w_{j}\tau_{kj}\left(\tau_{ji}\right)^{2}a_{2}^{j}\left(z\right)\right)^{1/2}\right]=\left(c_{1}^{k}\gamma\left(\Omega_{i|k}\right)^{-1/\theta}\right)^{1/2},$$

where $\gamma=\Gamma\left(1+1/2\theta\right)$ and Γ is the gamma function (see footnote 18 in Eaton and Kortum, 2002). Notice thus that the term $\Omega_{i|k}$ serves as a sufficient statistic for the expected cost of assembly associated with a value chain that begins with input production in country k and is aimed at servicing consumers in country i. From equation (22), we see that this expected cost is disproportionately lower the lower are the relative bilateral trade costs between country k and assembly countries j that either (i) have low wages, (ii) feature high technology levels, or (iii) are close to consumers in i. Rolling back to the decision of where to locate input production, firms choose $k^{*}\left(i\right)$ to solve

$$k^{*}\left(i\right)=\arg\min_{k}\left(a_{1}^{k}\left(z\right)w_{k}\gamma\left(\Omega_{i|k}\right)^{-1/\theta}\right)^{1/2}. \tag{23}$$

The key difference between this formulation and the one with full ex-ante information is that the problem (23) no longer depends on the actual realization of $a_{2}^{j}\left(z\right)$, but rather on the exogenous parameters shaping the term $\Omega_{i|k}$ in (22). Invoking again the Fréchet results in Eaton and Kortum (2002), we can then state that the probability that country k is chosen as the location of input production in a value chain aimed at servicing consumers in i is given by

$$\Pr\left(k=\ell\left(1\right);i\right)=\frac{T_{k}\left(w_{k}\right)^{-\theta}\Omega_{i|k}}{\sum_{k'}T_{k'}\left(w_{k'}\right)^{-\theta}\Omega_{i|k'}},$$

where remember that $\Omega_{i|k'}$ is given in (22) for any $i\in\mathcal{J}$ and $k\in\mathcal{J}$. Applying the law of total probability, we can then conclude that the the probability that country j is chosen as the location of assembly in a value chain aimed at servicing consumers in i is given by

$$\begin{aligned}\Pr\left(j=\ell\left(2\right);i\right) &= \sum\nolimits_{k}\Pr\left(j=\ell\left(2\right)\mid k=\ell\left(1\right);i\right)\times\Pr\left(k=\ell\left(1\right);i\right)\\ &= \frac{\sum_{k}T_{k}\left(w_{k}\right)^{-\theta}T_{j}\left(\tau_{kj}w_{j}\left(\tau_{ji}\right)^{2}\right)^{-\theta}}{\sum_{k'}T_{k'}\left(w_{k'}\right)^{-\theta}\Theta_{i|k'}}.\end{aligned}$$

Given a matrix of trade costs τ_{ij}, a vector of wages w_j, a vector of technology levels T_j, and a value for the parameter θ, these probabilities can then be easily computed.

Although we have focused on a two-stage example, the above derivations naturally extend to a multiple stage environment with $N > 2$. The algebra of those cases is more cumbersome but relatively straightforward. In particular, let us use the simpler notation $A_j = T_j (w_j)^{-\theta}$ and $\rho_{ij} = (\tau_{ij})^{-\theta}$ and define the destination-market specific term

$$\begin{aligned}\Theta_i &= \sum_{\ell(1)\in\mathcal{J}} A_{\ell(1)} \sum_{\ell(2)\in\mathcal{J}} \rho_{\ell(1)\ell(2)} A_{\ell(2)} \sum_{\ell(3)\in\mathcal{J}} \left(\rho_{\ell(2)\ell(3)}\right)^2 A_{\ell(3)} \cdots \\ &\quad \cdots \sum_{\ell(N-1)\in\mathcal{J}} \left(\rho_{\ell(N-2)\ell(N-1)}\right)^{N-2} A_{\ell(N-1)} \sum_{\ell(N)\in\mathcal{J}} \left(\rho_{\ell(N-1)\ell(N)}\right)^{N-1} A_{\ell(N)} \left(\rho_{\ell(N)i}\right)^N . \end{aligned} \tag{24}$$

This turns out to be a sufficient statistic for how the matrix of bilateral trade costs as well as technology and wages worldwide shape the cost at which finished goods produced in value chains of length N are available to consumers in country i. Furthermore, the probability that a given country ℓ^* will be at position 1 of that value chain that ends up servicing consumers in i is given by removing the first summation sign and evaluation Θ_i at $\ell(1) = \ell^*$, i.e.,

$$\Pr(\ell(1) = \ell^*; i) = \frac{A_{\ell^*} \sum_{\ell(2)\in\mathcal{J}} \rho_{\ell^*\ell(2)} A_{\ell(2)} \cdots \sum_{\ell(N)\in\mathcal{J}} \left(\rho_{\ell(N-1)\ell(N)}\right)^{N-1} A_{\ell(N)} \left(\rho_{\ell(N)i}\right)^N}{\Theta_i}.$$

The numerator of the (unconditional) probability of country ℓ^* being at any position $n > 1$ in the value chain can similarly be computed by removing the summation sign over $\ell(n) \in \mathcal{J}$ and by evaluating Θ_i in (24) at $\ell(n) = \ell^*$.[15]

This approach also allows one to easily compute the probability that a specific N-stage value chain aimed at servicing country i will run sequentially through a *given* set of countries $\ell(1)$, $\ell(2)$, and so on until $\ell(N)$. Importantly, and quite differently from the complete specialization allocation studied earlier in this paper, these N countries might not be distinct from each other, and in fact they could all be country i, thus leading to a purely domestic value chain. In any case, the likelihood of a given sequence of countries is given by

$$\Pr(\ell(1), \ell(2), ..., \ell(N); i) = \frac{\prod_{n=1}^{N-1} A_{\ell(n)} \left(\rho_{\ell(n)\ell(n+1)}\right)^n \times A_{\ell(N)} \left(\rho_{\ell(N)i}\right)^N}{\Theta_i}, \tag{25}$$

and thus corresponds to the contribution of that particular permutation of countries to the index

[15] For instance, the (unconditional) probability that country ℓ^* is at position 2 of the value chain leading up to consumers in i is given by

$$\Pr(\ell(2) = \ell^*; i) = \frac{\sum_{\ell(1)\in\mathcal{J}} A_{\ell(1)} \rho_{\ell(1)\ell^*} A_{\ell^*} \sum_{\ell(3)\in\mathcal{J}} \left(\rho_{\ell^*\ell(3)}\right)^2 A_{\ell(3)} \cdots \sum_{\ell(N)\in\mathcal{J}} \left(\rho_{\ell(N-1)\ell(N)}\right)^{N-1} A_{\ell(N)} \left(\rho_{\ell(N)i}\right)^N}{\Theta_i}.$$

Θ_i.

5.3 Downstreamness and Centrality

The nature of the equilibrium with incomplete specialization is quite distinct from the allocations with complete specialization we described above. In particular, in the current variant of the model, countries contribute value added at various (potentially disconnected) stages of the value chain and each stage, including assembly, takes place in multiple locations even within value chains that end up producing an identical final good variety. Despite these differences, the effect of geography on the pattern of specialization is notably similar to the one in the models in sections 3 and **??**. In particular, taking logs of equation (25), we obtain

$$-\ln \Pr\left(\ell\left(1\right), \ell\left(2\right), ..., \ell\left(N\right); i\right) = \theta \sum_{n=1}^{N-1} n \ln \tau_{\ell(n)\ell(n+1)} - \theta N \ln \tau_{\ell(N)i} + \ln \Theta_i - \sum_{n=1}^{N} \ln A_{\ell(n)},$$

which is remarkably similar to the function $H\left(\ell\left(1\right), ..., \ell\left(N\right)\right)$ in (17) we were seeking to minimize in section 3. It is then perhaps not too surprising that the nexus between centrality and downstreamness studied in section 3 will carry over to this variant of the model with incomplete specialization.

In order to formalize this insight, let us define the average upstreamness of production of a given country l in value chains that seek to serve consumers in country i, by

$$U\left(\ell; i\right) = \sum_{n=1}^{N} \left(N - n + 1\right) \times \frac{\Pr\left(\ell = \ell\left(n\right); i\right)}{\sum_{n'=1}^{N} \Pr\left(\ell = \ell\left(n'\right); i\right)}. \tag{26}$$

The index $U\left(\ell; i\right)$ is thus a weighted average distance of country ℓ from final consumers in value chains that service consumers in country i, and it is closely related to measure proposed by Antràs et al. (2012). Although $U\left(\ell; i\right)$ in equation (26) uses probabilities rather than expenditure shares as weights, it can be verified that, as in Eaton and Kortum (2002), these are identical given symmetry in production and the properties of the Fréchet distribution.

Our next goal is to formalize the connection between the measure of upstreamness $U\left(\ell; i\right)$ and the centrality of country j. As in section 3, the structure of equation (25) already hints at a negative association between the two, since high values of τ (i.e., low values of ρ) in relatively downstream (high n) stages have a disproportionately negative effect on the likelihood of a given permutation of countries forming an equilibrium value chain.

In order to develop a more precise formulation of this results, we will again adopt the additively separable specification of trade costs in (15), so that we can easily rank countries according to their centrality ρ_j. In such a case, the probability of a given country ℓ being at any position n in the value chain turns out to reduce to

$$\Pr\left(\ell=\ell\left(n\right)\right)=\frac{A_{\ell}\left(\rho_{\ell}\right)^{2n-1}}{\sum_{j\in\mathcal{J}}A_{j}\left(\rho_{j}\right)^{2n-1}}, \tag{27}$$

independently of the final destination i of the final good. We can thus define a country's average upstreamness $U\left(\ell\right)$ independently of i. It can be shown that equation (27) is supermodular in n and ρ_{ℓ}, and thus we can conclude that:

Proposition 3 *The average upstreamness $U\left(\ell\right)$ of a country in global value chains is decreasing in its centrality $\rho\left(\ell\right)$.*

5.4 An Empirical Illustration: *Factory Asia*

Although a full-fledged quantification of the model is beyond the scope of this paper, we next illustrate how one can empirically implement the simple form of equation (25) determining the likelihood of different configurations of countries in global value chains. To do so, we will return to the *Factory Asia* example in section 3.3, where we solved for the optimal path of a value chain that had to travel through the U.S. and the 11 largest economies in East and Southeast Asia, visiting each country exactly once. In a similar vein, we will next explore the average position $U\left(\ell;i\right)$ of these 12 countries in a world with multiple heterogeneous value chains, and in which particular value chains will involve only a subset of the 12 countries and in which countries may contribute more than one stage to those chains. For simplicity, and consistently with the spirit of the solution in Figure 6, we will focus on the case in which $i=China$.[16]

In order to compute the probabilities that shape the index $U\left(\ell;i\right)$ for the $J=12$ countries, all that is required are empirical proxies for the terms $A_{j}=T_{j}\left(w_{j}\right)^{-\theta}$ and $\rho_{ij}=\left(\tau_{ij}\right)^{-\theta}$. On a first pass, we will ignore technology and wage differences across countries and set $A_{j}=A$ for all j. Although this will lead to grossly counterfactual trade shares, these simpler calculations will turn out to provide indices of upstreamness that are remarkably similar to those that arise when calibrating the vector of A_{j}'s in a more sensible way.

Turning to the bilateral trade costs parameters ρ_{ij}, we calibrate those in a manner analogous to that in section 3.3 by using a gravity-equation approach. In particular, we invoke again equations (18) and (19), so that we can write ρ_{ij} as a function of the coefficients on some key variable in standard gravity equations:

$$\rho_{ij}=\left(\kappa\left(Dist\right)^{\hat{\delta}_{dist}}\left(Con_{ij}\right)^{\hat{\delta}_{con}}\left(Lang_{ij}\right)^{\hat{\delta}_{lang}}\left(Col_{ij}\right)^{\hat{\delta}_{col}}\left(Rta_{ij}\right)^{\hat{\delta}_{rta}}\left(Curr_{ij}\right)^{\hat{\delta}_{curr}}\left(Dom_{ij}\right)^{\hat{\delta}_{dom}}\right)^{-\theta/\varepsilon}.$$

The coefficients $\hat{\delta}_{v}$ are again extracted from Table 3.4 in Head and Mayer (2014). There is however an important added difficulty relative to the approach we followed in section 3.3. More specifically, the solution to the combinatorial optimization problem in that section was unaffected by multi-

[16] In 2007, China accounted for 65% of the population of the 11 Asian countries in the sample.

plicative or power transformations of the trade costs parameters τ_{ij}. For this reason, we was able to simply resort to the coefficient estimates $\hat{\delta}_v$ without having to take a stance on the multiplicative parameter κ or the trade elasticity ε. Unfortunately, in the current probabilistic environment, the same is not true because the probabilities in (25) and (26) are indeed affected by power transformations of τ_{ij}.[17] This implies that one needs to now take a stance on the value of the ratio ε/θ. In a standard Eaton and Kortum (2002) framework, this ratio would be exactly equal to 1, but in our multi-stage environment, it makes sense to impose a higher value for this ratio. To see this, notice for instance that in equation (25), the exponent on τ_{ij} rises from a value of $-\theta$ for $n = 1$ to $-\theta N$ for $n = N$. It is not obvious how one would aggregate the different effects of τ_{ij} on bilateral trade flows into a single 'trade elasticity', so for simplicity we will just set $\varepsilon/\theta = N$ in the calculations below. The implied matrix of bilateral trade cost coefficients τ_{ij} for the case $N = 3$ and $\theta = 5$ is reproduced in Table A.1 of the Appendix.

The first row of Table 2 presents the results of computing average upstreamness $U(\ell; i)$ in equation (26) for each of the twelve countries when the destination country i is China. As is clear from the Table, the most downstream country is China itself followed by Hong Kong. On the other hand, the United States and the Philipines are, on average, the most upstream countries in value chains that lead to consumers in China. These patterns very much resonate with the optimal path in the equilibrium with complete specialization described in section 3.3 (see the third row of Table 2). This is made clear in the left panel of Figure 8, where $U(\ell; i)$ is plotted against the position of countries in the value chain in Figure 6. The correlation between the two measures is indeed very high (0.702). It is also interesting to note that $U(\ell; i)$ is also highly positively correlated with the measure of average export upstreamness computed in ACFH (and reproduced in the fourth column of Table 2) despite the fact that our computation of $U(\ell; i)$ focuses on value chains that lead to Chinese consumers, rather than to consumers worldwide. Perhaps for this reason, the one country for which $U(\ell; i)$ differs the most relative to ACFH's measure is the United States.

Table 2. Average Country Upstreamness $U(\ell; i)$ in *Factory Asia* $(N = 3)$

	CHN	HKG	IDN	JPN	KOR	MYS	PHL	SGP	THA	TWN	USA	VNM
With $A_j = A$	1.59	1.86	2.68	2.42	2.13	2.47	2.48	1.96	2.43	2.11	2.88	2.40
With Calibrated A_j	1.77	2.04	2.75	2.55	2.34	2.65	2.67	2.40	2.58	2.18	2.83	2.53
Position in Fig. 6	1	5	10	3	2	8	11	9	7	4	12	6
ACFH Upstreamness	1.82	1.96	2.46	1.98	2.07	2.31	2.29	2.30	2.07	2.27	2.15	1.93

It is definitely encouraging to observe that the optimal position of countries in value chains appears to be similar in equilibria with complete and incomplete specialization. Nevertheless, the equilibria underlying the computation of $U(\ell; i)$ in the first row of Table 2 have some undesirable properties. Most notably, the probabilities with which the equilibrium value chains travel through

[17] Fortunately, the parameter κ continues to be irrelevant and need not be calibrated.

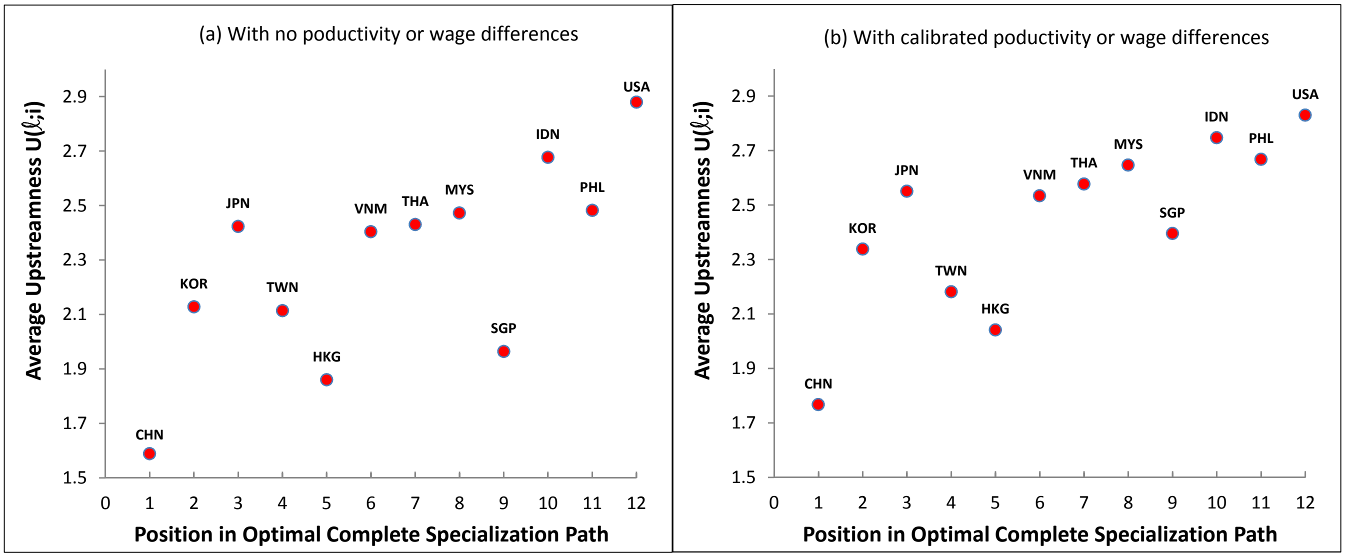

Figure 8: Average Country Upstreamness: Complete versus Incomplete Specialization Cases

China look way to small. For instance, with $N = 3$, the probability that stage N (assembly) is located in China is only 0.241, while the probabilities of stage 1 or 2 being located in China being 0.067 and 0.079, respectively. The reason for this is that both Hong Kong and Singapore feature relatively low trade frictions when exporting to China, while at the same time the domestic trade costs of these two economies are particularly low, thus making them ideal production spots in the presence of multi-stage production.[18] A second concern with the results obtained when $A_j = A$ is that the high correlation visible in Figure 8 gradually fades away as one increases the number of stages N. When $N = 7$, for instance, the correlation is down to 0.062. The reason for this, again, is that absent technology or wage differences across countries, production is more and more concentrated in Hong Kong and Singapore as N becomes larger.

In order to address these limitations, we next explore a simple approach to attempt to build in differences in the term $A_j = T_j (w_j)^{-\theta}$ into the computation of equilibrium value chains. For that purpose, we will rely on the estimates of the sourcing potential of countries in the work of Antràs, Fort and Tintelnot (2014). That paper develops a quantitative multi-country model of the global sourcing decisions of U.S. firms and then estimates the combination of foreign technology, wages and trade costs vis a vis the U.S. that best fits the relative propensity of U.S. firms to source inputs from various countries, including the United States. To be more precise, Antràs, Fort and Tintelnot (2014) recover the following object

$$\xi_j = \frac{T_j (w_j \tau_{US,j})^{-\theta}}{T_{US} (w_{US} \tau_{US,US})^{-\theta}} = \frac{A_j \rho_{US,j}}{A_{US} \rho_{US,US}}$$

based on a firm-level gravity-style regression of normalized foreign country market shares on country

[18] When $N = 1$, China's market share rises to a more plausible value of 0.630.

fixed effects. Using their estimated sourcing potentials ξ_j and our previous estimates of $\rho_{US,j}$ for all countries $j \in J$ (including the U.S.), we can thus recover all the ratios A_j relative to the one for the United States. Because the equilibrium of our probabilistic model of value chains is unaffected by proportional changes in all the A_j's we can safely normalize $A_{US} = 1$. The calibrated values of A_j appear in Table A.2 in the Appendix.

The results of incorporating differences in technology and wages into our calculations of average upstreamness for $N = 3$ appear in the second row of Table 2. As anticipated before, the implied changes are relatively modest with the correlation between the first and second rows of Table 2 being 0.956. The main difference between the two rows is that Singapore now appears much more upstream than when assuming a common A_j across countries. This in turn implies that the positive correlation between average upstreamness in this equilibrium with incomplete specialization and the one in section 3.3 with complete specialization is now even higher and stands at 0.766. This better fit is also easily visualized by comparing the left and right panels of Figure 8. Beyond these small changes on equilibrium average upstreamness, the main improvement associated with the calibration of the A_j's is that the realtive prevalence of China itself in value chains leading to Chinese consumers now appear much more sensible. For instance, when $N = 3$, assembly occurs in China with probability 0.817, with the probability being 0.391 and 0.624 for stages 1 and 2. Furthermore, even when N is increased from 3 to 7, the correlation between upstreamness in the incomplete and complete specialization cases remains quite high at 0.574.

6 Conclusion

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Appendix

Table A.1. Calibrated Bilateral Trade Costs τ_{ij} in the *Factory Asia* Example ($N = 3$; $\theta = 5$)

	CHN	HKG	IDN	JPN	KOR	MYS	PHL	SGP	THA	TWN	USA	VNM
CHN	1.46	1.66	1.90	1.78	1.71	1.83	1.82	1.84	1.84	1.70	2.03	1.76
HKG	1.66	1.07	1.85	1.82	1.79	1.77	1.69	1.78	1.76	1.62	2.01	1.72
IDN	1.90	1.85	1.42	1.92	1.91	1.64	1.78	1.66	1.77	1.86	2.08	1.76
JPN	1.78	1.82	1.92	1.35	1.59	1.91	1.84	1.88	1.89	1.69	2.02	1.87
KOR	1.71	1.79	1.91	1.59	1.26	1.89	1.82	1.86	1.87	1.75	2.02	1.85
MYS	1.83	1.77	1.64	1.91	1.89	1.39	1.77	1.39	1.63	1.80	2.08	1.70
PHL	1.82	1.69	1.78	1.84	1.82	1.77	1.33	1.74	1.77	1.72	1.90	1.73
SGP	1.84	1.78	1.66	1.88	1.86	1.39	1.74	1.02	1.71	1.80	2.00	1.71
THA	1.84	1.76	1.77	1.89	1.87	1.63	1.77	1.71	1.28	1.81	2.07	1.64
TWN	1.70	1.62	1.86	1.69	1.75	1.80	1.72	1.80	1.81	1.25	2.04	1.78
USA	2.03	2.01	2.08	2.02	2.02	2.08	1.90	2.00	2.07	2.04	1.52	2.06
VNM	1.76	1.72	1.76	1.87	1.85	1.70	1.73	1.71	1.64	1.78	2.06	1.39

Table A.1. Calibrated Values of A_j/A_{US} in the *Factory Asia* Example

	CHN	HKG	IDN	JPN	KOR	MYS	PHL	SGP	THA	TWN	USA	VNM
A_j/A_{US}	1.21	0.15	0.28	0.32	0.31	0.21	0.06	0.09	0.30	0.60	1.00	0.34