The Inherent Benefit of Monetary Unions*

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Abstract

The desirability of flexible exchange rates is a central tenet in international macroeconomics. We show that, with forward-looking staggered pricing, this result crucially depends on the monetary authority's ability to commit. Under full commitment, flexible exchange rates generally dominate a monetary union (or fixed exchange rate) regime. Under discretion, this result is overturned: a monetary union dominates flexible exchange rates. By fixing the nominal exchange rate, a benevolent monetary authority finds it welfare improving to tradeoff flexibility in the adjustment of the terms of trade in order to improve on its ability to manage the private sector's expectations. Thus, inertia in the terms of trade (induced by a fixed exchange rate) is a *cost* under commitment, whereas it is a *benefit* under discretion, for it acts like a commitment device.

Keywords: monetary union, flexible exchange rates, commitment, discretion, welfare losses, nominal rigidities.

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1 Introduction

A central tenet in international macroeconomics is that flexible exchange rates are desirable because they compensate for the inertia in nominal prices, thereby easing the necessary adjustment in the terms of trade in response to asymmetric disturbances (Friedman 1953, Corsetti, Dedola and Leduc 2011, Farhi and Werning 2012).[1] This argument, which implies that fixed exchange rates are inherently costly, has recently gained renewed interest in light of the observed divergence in macroeconomic performance between the periphery and the core of the European Monetary Union after 2008.

In this paper we revisit the classic dichotomy between flexible exchange rates and monetary unions, within the context of a baseline two-country dynamic New Keynesian model, the workhorse paradigm of the recent optimal monetary policy literature in open economies (Devereux and Engel 2003, Benigno and Benigno 2003, Corsetti and Pesenti 2001).

The key insight of our analysis is that the desirability of flexible exchange rates relative to monetary unions (or, generally, fixed exchange rates) crucially depends on the (in)ability of the monetary authority to commit. If, somewhat unrealistically, the monetary authority can fully commit, flexible exchange rates always implement the constrained efficient allocation. If, however, the monetary authority can only choose its course of action period by period (i.e., it acts under discretion), the previous result is overturned: a monetary union generally dominates flexible exchange rates.

The intuition for the desirability of flexible exchange rates under commitment is well understood. In a baseline New Keynesian model, characterized by forward-looking staggered prices (Woodford 2003, Galí 2015), the monetary authority's inability to commit typically results in a "stabilization bias", i.e., a suboptimal policy response to those disturbances that drive a wedge between the welfare-efficient and the natural level of output. Gains from commitment arise from "policy-induced" inertia in inflation, which, in turn, improves the monetary authority's management of the private sector's expectations. This argument supports the following proposition: under flexible exchange rates, if the monetary authority can commit, there is no tradeoff between the optimal management of (inflation) expecta-

[1] The recent New Keynesian optimal monetary policy literature in open economies has revisited this argument, arguing that, in the presence of local currency price stability of imports, full flexibility of the nominal exchange rate is generally not the welfare maximizing policy (Devereux and Engel 2003, Engel 2011, Corsetti, Dedola and Leduc 2011). Here we purposely abstract from issues related to local currency price stability of imports.

tions and the efficient adjustment of international relative prices in response to asymmetric disturbances.

Our analysis focuses on the case of *lack of commitment*. We show that a fixed exchange rate induces a "commitment-like" inertia in the behavior of the terms of trade and inflation, which mitigates the stabilization bias. As a result, if a credible policy commitment is not feasible, the monetary authority finds it welfare-improving to trade off some flexibility in the adjustment of the terms of trade in order to improve on its ability to manage expectations. This is what we label as "inherent benefit of monetary unions".

To better understand this point, it is instructive to recall that, in a two-country setting with nominal rigidities (and under the assumption of cross-country symmetry), average welfare losses depend not only on the variability of *average* inflation (in a way similar to its closed economy analog), but also on its cross-country *composition*. Thus, inertia in the terms of trade, induced by fixed exchange rates, translates, under discretion, into welfare-enhancing inertia of relative inflation, i.e., precisely of what measures the cross-country composition of inflation. More generally, under discretion, the expectation-management gain stemming from inertia in the terms of trade can outweigh the cost of inefficient adjustment of relative prices, thereby making a monetary union welfare dominant relative to flexible exchange rates. In a nutshell, inertia in the terms of trade, induced by fixed exchange rates, is a *cost* under commitment - because it does not compensate for the underlying stickiness in nominal prices; whereas it is a *benefit* under discretion - because it acts as a commitment device, thereby improving on the policymaker's ability to manage expectations. This result holds for a large range of parameter values, and is especially sharp under two configurations: a sufficiently high degree of nominal price rigidity and a sufficiently high degree of substitutability in internationally traded goods.

Interestingly, while the inertia in the terms of trade has been recognized before as a typical feature of a monetary union (Benigno 2004, Pappa 2004), it was *solely* regarded as a distortion of that regime. Relatedly, Farhi and Werning (2012) emphasize that the inefficiency at the heart of *any* monetary union, and regardless of the underlying degree of completeness in international financial markets, is a structural "lack of insurance", which stems precisely from the suboptimal adjustment in the terms of trade that results from the combination of nominal price rigidity and lack of nominal exchange rate flexibility. Unlike those contributions, we wish instead to emphasize that, in a monetary union, the inertial behavior of the terms of trade can be turned to policymakers' advantage when the latter lack the ability to commit.

Related literature Our paper relates to a large literature analyzing optimal monetary policy in an international setting, and within the context of dynamic optimizing New Keynesian models. Corsetti, Dedola and Leduc (2011) thoroughly survey that literature. A pillar of this research program (a sort of flexible exchange rates *manifesto*) is that under the assumption of (i) cross-country risk sharing, (ii) complete pass-through of exchange rates to import prices, and (iii) full commitment, flexible exchange rates implement the welfare-maximizing policy. The existing literature has typically explored the implications of relaxing (i) and (ii) in order to (re-)assess the desirability of fixed vs. flexible exchange rates (see, e.g., Devereux and Engel 2003, Corsetti, Dedola and Leduc 2011). Our paper differs from the previous ones in that it focuses on the role of relaxing (iii) in determining the desirability of monetary unions vs. flexible exchange rates.

Monacelli (2004), Soffritti and Zanetti (2008), and Groll (2013) study the properties of flexible vs. fixed exchange rates in a New Keynesian open economy model, and show that, with lack of commitment, the classic ranking between flexible and fixed exchange rates can be reversed. The key difference in our paper is that, in order to assess the relative desirability of the two regimes, we frame the analysis within a fully choice-theoretic environment as opposed to relying on ad hoc policy objective functions and/or Taylor-type rules. Benigno (2004) studies optimal monetary policy in a currency area, but under the maintained assumption that the monetary authority can commit, and with no comparison between flexible and fixed exchange rate regimes. Our central focus here is instead on the case of lack of commitment and on the relative desirability of the two regimes. Corsetti, Kuester and Muller (2013) compare the transmission of fiscal disturbances under flexible vs fixed exchange rates (described by simple feedback rules) and highlight the role of nominal anchor played by fixed exchange rates. Cook and Devereux (2014) point out the desirability of fixed exchange rates (or monetary unions) when asymmetric shocks hit a country at the zero lower bound. Our paper shows that, with lack of commitment, the desirability of monetary unions (or fixed exchange rates) holds also in "normal" times, regardless of the occurrence of the zero lower bound.

In focusing on the crucial role played by commitment (or lack thereof), our paper is also related to Chari, Dovis and Kehoe (2015). Our work differs from Chari et al. (2015) in the key factor that, under discretion, drives the welfare dominance of monetary unions relative to flexible exchange rates. In Chari et al. (2015) it derives, in a model with preset prices, from fixed exchange rates allowing to prevent the policymakers' temptation to generate ex-post inflation, as in the classic Barro-Gordon setup. In our case, it derives from the possibility of dampening the stabilization bias in the presence of forward-looking price setting, as emphasized in the more recent New Keynesian literature.

Finally, our paper is also related to a literature, exemplified by Alesina and Barro (2002), which emphasizes that countries, when they lack commitment, may generally benefit from monetary unification. Our paper differs from that strand of the literature in at least two ways. First, the commitment gain in Alesina and Barro (2002) derives from the removal of a typical average inflation bias, whereas the commitment gains from participating to a monetary union arise, in our setup, due to the improved ability of policy to respond to shocks, even in the absence of any source of average inflation bias. Second, and most importantly, the benefit, in Alesina and Barro (2002), of eliminating an average inflation bias is not inherent to a monetary union because it is only obtained if the monetary policy authority after monetary unification is more credible than the one before monetary unification. By contrast, the benefit described in our paper is *inherent* to a monetary union because it is obtained *even if* the monetary policy authority after monetary unification suffers from the same lack of commitment as before unification. So, in our case, two countries that, ex ante, suffer from a lack of commitment gain by establishing a monetary union even if the new common monetary policy authority suffers from the same lack of commitment.

2 A two-country model

We describe a baseline two country model characterized by full financial integration, monopolistic competitive markets and nominal price rigidity (Benigno 2004, Corsetti, Dedola and Leduc 2011). Henceforth we refer to the two countries as Home and Foreign, having measure n and $(1-n)$ respectively. The total mass of households in the world economy is therefore equal to 1.

2.1 Domestic households

Consumption preferences in Home are described by the following composite index of domestic and imported bundles of goods (Faia and Monacelli 2008, De Paoli 2009):

$$C_t \equiv [(1-\gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} \ + \ \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \tag{1}$$

where $\eta > 0$ is the elasticity of substitution between domestic and foreign goods, and $\gamma \equiv (1-n)\alpha$ denotes the weight of imported goods in the Home consumption basket. This weight depends on $(1-n)$, the relative size of Foreign, and on α, the degree of trade openness of Home. In an analogous manner, preferences in Foreign can be described as:

$$C_t^* \equiv [(1-\gamma^*)^{\frac{1}{\eta}} C_{F,t}^{*\frac{\eta-1}{\eta}} \ + \ (\gamma^*)^{\frac{1}{\eta}} \, C_{H,t}^{*\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \tag{2}$$

where $\gamma^* \equiv n \ \alpha^*$.

Each consumption bundle $C_{H,t}$ and $C_{F,t}$ is composed of imperfectly substitutable varieties (with elasticity of substitution $\varepsilon > 1$). Optimal allocation of expenditure within each variety of goods yields:

$$C_{H,t}(i) = \frac{1}{n}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \quad ; \quad C_{F,t}(i) = \frac{1}{1-n}\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t} \tag{3}$$

where $C_{H,t} \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_0^n C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ and $C_{F,t} \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \int_n^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$.

Optimal allocation of expenditure between domestic and foreign bundles yields:

$$C_{H,t} = (1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t; \qquad C_{F,t} = \gamma\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{4}$$

where

$$P_t \equiv [(1-\gamma)P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}} \tag{5}$$

is the CPI index.

A generic household in Home derives utility from consumption and disutility from the production of a continuum of differentiated products indexed by $i \in [0, n)$:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t U\left(C_t\right) - \frac{1}{n}\int_0^n V(Y_t(i), Z_{Y,t}) di \right\} \tag{6}$$

where $Z_{Y,t}$ is a productivity disturbance. To insure their consumption pattern against random shocks at time t households spend $\nu_{t+1,t} \ B_{t+1}$ in nominal state contingent securities, where $\nu_{t,t+1} \equiv \nu(h^{t+1}|h^t)$ is the period-t price of a claim to one unit of domestic currency in state h^{t+1} divided by the probability of occurrence of that state. Each asset in the portfolio B_{t+1} pays one unit of domestic currency at time $t+1$ and in state h^{t+1}.

By considering the optimal expenditure conditions (3) and (4), the sequence of budget constraints assumes the following form:

$$P_t C_t + \sum_{h^{t+1}} \nu_{t+1,t} B_{t+1} \leq B_t + \frac{1-\tau_{H,t}}{n} \int_0^n P_t(i) Y_t(i) di + T_t \tag{7}$$

where $\tau_{H,t}$ is a country-specific tax on firms' profits, and T_t denotes lump-sum transfers (or taxes).

2.2 Risk sharing, the real exchange rate and demand in Foreign

We assume throughout that the law of one price holds, implying that $P_{H,t}(i) = \mathcal{E}_t \ P^*_{H,t}(i)$ and $P_{F,t}(i) = \mathcal{E}_t \ P^*_{F,t}(i)$ for all $i \in [0,1]$, where $\mathcal{E}_t$ is the *nominal* exchange rate, i.e., the price of foreign currency in terms of home currency, and $P^*_{F,t}(i)$ is the price of foreign good i denominated in foreign currency. Importantly, the law of one price does not necessarily imply that purchasing power parity (PPP) holds, unless we make the further restrictive assumption of absence of home bias in consumption.

Under complete markets for state contingent assets, the efficiency condition for bonds' holdings by residents in Foreign reads:

$$\beta \frac{P^*_t \mathcal{E}_t}{P^*_{t+1} \mathcal{E}_{t+1}} \frac{U^*_{c,t+1}}{U^*_{c,t}} = \nu_{t,t+1} \tag{8}$$

Taking conditional expectations of (8) and defining the foreign nominal interest rate $(1+i^*_t) \equiv \left(\mathbb{E}_t \left\{ \nu_{t,t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\}\right)^{-1}$ one can write:

$$(1+i^*_t) = \left[\beta \ \mathbb{E}_t \left\{ \frac{P^*_t}{P^*_{t+1}} \frac{U^*_{c,t+1}}{U^*_{c,t}} \right\} \right]^{-1} \tag{9}$$

Foreign demand for domestic variety i must satisfy:

$$\begin{aligned} C^*_{H,t}(i) &= \frac{1}{n} \left(\frac{P^*_{H,t}(i)}{P^*_{H,t}} \right)^{-\varepsilon} C^*_{H,t} \\ &= \frac{1}{n} \left(\frac{P^*_{H,t}(i)}{P^*_{H,t}} \right)^{-\varepsilon} \gamma^* \left(\frac{P^*_{H,t}}{P^*_t} \right)^{-\eta} C^*_t \end{aligned} \tag{10}$$

Terms of trade and the real exchange rate The terms of trade is the relative price of imported goods:

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} \tag{11}$$

while the real exchange rate is defined as $Q_t = \mathcal{E}_t P_t^* / P_t$. Using equation (5), the terms of trade can be related to the CPI-PPI ratio as follows:

$$\frac{P_t}{P_{H,t}} = [(1-\gamma) + \gamma S_t^{1-\eta}]^{\frac{1}{1-\eta}} \equiv \mathcal{T}(S_t), \tag{12}$$

with $\mathcal{T}_{s,t} \equiv \partial \mathcal{T}(S_t)/\partial S_t > 0$.

The terms of trade and the real exchange rate are linked through the following expression:

$$\begin{aligned} Q_t &= S_t \frac{P_t^*}{P_{F,t}^*} \left(\frac{P_t}{P_{H,t}} \right)^{-1} \qquad (13) \\ &= S_t \, \frac{\mathcal{T}^*(S_t)}{\mathcal{T}(S_t)} \equiv q(S_t), \end{aligned}$$

where

$$\frac{P_t^*}{P_{F,t}^*} = [(1-\gamma^*) + \gamma^* S_t^{\eta-1}]^{\frac{1}{1-\eta}} \equiv \mathcal{T}^*(S_t). \tag{14}$$

Deviations from purchasing power parity (PPP) By using (12), (13) and (14) one can write:

$$Q_t = q(S_t) = \left(\frac{\gamma^* + (1-\gamma^*) \; S_t^{1-\eta}}{(1-\gamma) + \gamma \; S_t^{1-\eta}} \right)^{\frac{1}{1-\eta}}. \tag{15}$$

Notice that if $\gamma = \gamma^* = 1/2$ it follows immediately that $Q_t = 1$ (i.e., PPP holds at all times), regardless of the equilibrium value of S_t.

Risk Sharing Under full international risk sharing, one can combine (8) with the corresponding condition for Home and obtain, after iteration, the following condition linking the real exchange rate to the ratio of the marginal utilities of consumption:

$$\omega_0 \frac{U_{c,t}^*}{U_{c,t}} = \frac{\mathcal{E}_t P_t^*}{P_t} = q(S_t), \tag{16}$$

where ω_0 is a constant that depends on initial conditions, and can be normalized to 1.

2.3 Price setting

Each domestic producer can revise its price at random intervals (Calvo 1983). Let $(1-\theta_H)$ be the probability that a firm can reoptimize its price at any given time t, and $\overline{P}_{H,t}$ the optimally chosen price at time t. Each producer maximizes expected discounted profits:

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_H)^k \, \nu_{t,t+k} \left[\lambda_{t+k}(1-\tau_{H,t+k})\overline{P}_{H,t}(i) Y_{t+k|t}(i) - V(Y_{t+k|t}(i), Z_{Y,t+k}) \right] \tag{17}$$

where (from equilibrium) $\nu_{t,t+k} = \beta^k \frac{U_{c,t+k} P_t}{U_{c,t} P_{t+k}}$ is the stochastic discount factor, $\lambda_{t+k} = \frac{U_{c,t+k}}{P_{t+k}}$ is the marginal utility of nominal revenues, and $Y_{t+k|t}$ is total demand for variety i faced by a firm that last reset its price at time t.

The first order condition yields the optimal price

$$\overline{P}_{H,t}(i) = \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_H)^k \, \nu_{t,t+k} \; \mathcal{M}_{H,t+k} \; V_y(Y_{t+k|t}(i), Z_{Y,t+k}) Y_{t+k|t}(i)}{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_H)^k \, \nu_{t,t+k} \; \lambda_{t+k} Y_{t+k|t}(i)}, \tag{18}$$

where V_y is the marginal disutility from producing output $Y(i)$ and

$$\mathcal{M}_{H,t+k} = \frac{\varepsilon}{(\varepsilon-1)(1-\tau_{H,t+k})}$$

denotes the tax-adjusted optimal markup. We assume that, in order to neutralize the market power distortion in the steady state, $\tau_H = -(\varepsilon-1)^{-1} \equiv \widetilde{\tau}_H$. By construction, then, any deviation of $\tau_{H,t}$ from $\widetilde{\tau}_H$ is an exogenous stochastic source of inefficiency.

We assume that the markup follows the process (in logs):

$$\log \mathcal{M}_{i,t} \equiv \mu_{i,t} = \rho_\mu \mu_{i,t-1} + \iota_{i,t} \; (i = H, F) \tag{19}$$

where $\iota_{i,t}$ is an iid random disturbance, with mean zero and variance υ_{ι_i}.

In any given period, the price from the previous period remains effective for a fraction θ_H of producers. The optimal relative price $\overline{P}_{H,t}/P_{H,t}$ follows:

$$1 = \theta_H \left(\frac{P_{H,t}}{P_{H,t-1}} \right)^{\varepsilon-1} + (1-\theta_H) \left(\frac{\overline{P}_{H,t}}{P_{H,t}} \right)^{1-\varepsilon}. \tag{20}$$

3 Equilibrium

Below we describe the relevant set of equilibrium conditions in log-linearized form (denoted by lower case letters) and for each exchange rate regime - flexible exchange rate and monetary union - respectively. We refer to Appendix A for a derivation of the whole set of primitive conditions. In the expressions below, variables with a superscript (e.g., $\overline{x}_t$) refer to the corresponding values under the first-best or efficient allocation, characterized by flexible prices and the absence of markup shocks (see Appendix B).

3.1 Flexible exchange rates

For a given specification of the two policy instruments $\{i_t,\ i_t^*\}$, an equilibrium under *flexible* exchange rates is a set of endogenous processes $\left\{y_t,\ c_t,\ \pi_{H,t},\ s_t,\ y_t^*,\ c_t^*,\ \pi_{F,t}^*\right\}$ and exogenous processes $\left\{\mu_{j,t},\ j=H,F\right\}$ satisfying the following set of conditions:

- *Aggregate demand*

$$\mathbb{E}_t c_{t+1} = c_t + \sigma^{-1}\left(i_t - \mathbb{E}_t \pi_{H,t+1} - \gamma \Delta s_{t+1}\right) \tag{21}$$

$$\mathbb{E}_t c_{t+1}^* = c_t^* + \sigma^{-1}\left(i_t^* - \mathbb{E}_t \pi_{F,t+1}^* + \gamma^* \Delta s_{t+1}\right) \tag{22}$$

- *Market clearing*

$$y_t = (1-\gamma)c_t + \gamma c_t^* + \gamma(2-\gamma-\gamma^*)\eta s_t \tag{23}$$

$$y_t^* = \gamma^* c_t + (1-\gamma^*)c_t^* - \gamma^*(2-\gamma-\gamma^*)\eta s_t \tag{24}$$

- *Risk sharing*

$$(1-\gamma-\gamma^*)s_t = \sigma\left(c_t - c_t^*\right) \tag{25}$$

- *Aggregate supply*

$$\pi_{H,t} = \beta \mathbb{E}_t \pi_{H,t+1} + (\sigma+\zeta)\kappa\left(y_t - \overline{y}_t\right) - \gamma(2-\gamma-\gamma^*)\left(\sigma\eta-1\right)\kappa\left(s_t - \overline{s}_t\right) + \kappa\mu_{H,t} \tag{26}$$

$$\pi_{F,t}^* = \beta \mathbb{E}_t \pi_{F,t+1}^* + (\sigma+\zeta)\kappa^*\left(y_t^* - \overline{y}_t^*\right) + \gamma^*(2-\gamma-\gamma^*)\left(\sigma\eta-1\right)\kappa^*\left(s_t - \overline{s}_t\right) + \kappa^*\mu_{F,t}, \tag{27}$$

where

$$\pi_{i,t} \equiv p_{i,t} - p_{i,t-1},\ i = H, F$$

$$\kappa \equiv \frac{(1-\theta_H \beta)(1-\theta_H)}{\theta_H(1+\varepsilon\zeta)};\ \kappa^* \equiv \frac{(1-\theta_F \beta)(1-\theta_F)}{\theta_F(1+\varepsilon\zeta)}.$$

$$\sigma \equiv -\frac{U_{cc}C}{U_c};\ \zeta \equiv \frac{V_{yy}Y}{V_y},$$

with σ and ζ assumed equal in both countries.

Notice that the equilibrium characterization (21)-(27) does not feature the nominal depreciation rate, $\Delta e_t \equiv e_t - e_{t-1}$ (with $e_t \equiv \log \mathcal{E}_t$). The equilibrium path of the latter, in fact, can be derived residually from the one of the terms trade. Given $\{\pi_{j,t}\}_{t=0}^{\infty}$ and $\{s_t\}_{t=0}^{\infty}$ from above, one can derive $\{\Delta e_t\}_{t=0}^{\infty}$ using the expression

$$\Delta e_t = \Delta s_t + \pi_{H,t} - \pi^*_{F,t}, \tag{28}$$

holding for all t.[2]

It is useful, in order to eliminate c_t and c_t^*, to combine (23), (24), and (25), to obtain the following equilibrium condition linking the terms of trade to (cross-country) relative output:

$$\Gamma s_t = \sigma\left(y_t - y_t^*\right). \tag{29}$$

where $\Gamma \equiv (\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2 > 0$.

Equation (29) indicates that a rise of domestic output above foreign output requires, in equilibrium, a depreciation of the domestic terms trade. This is the result of two effects: first, holding relative consumption constant, higher output of domestic goods exert a downward pressure on domestic prices; second, since higher domestic output translates, at least in part, into higher relative consumption, this requires a real depreciation to allow for risk sharing, i.e., part of the higher consumption should be shared by foreign households via an increase in their real purchasing power.

3.2 Monetary union

There are two main differences that characterize the equilibrium under a monetary union relative to the case of flexible exchange rates. First, the law of motion (28) can no longer

[2] We assume throughout that the initial price levels, $p_{j,-1}$, are given and all equal to 1. In a steady state with balanced trade, we also have $s = s_{-1} = 1$. These conditions combined allow to pin down e_{-1}, the initial nominal exchange rate level. Combining the latter with the equilibrium path $\{\Delta e_t\}_{t=0}^{\infty}$ allows to derive $\{e_t\}_{t=0}^{\infty}$.

play a mere residual role in pinning down the equilibrium path of the nominal exchange rate. That expression is a necessary cross-equation restriction in the minimal set of equilibrium conditions, so that the fixed exchange rate condition $e_t = 0$ is explicitly accounted for. Second, given that a single monetary authority sets the common policy instrument, henceforth labeled i_t^{MU}, either one of equations (21) and (22) becomes irrelevant for the minimal specification of the equilibrium.

Hence, for a given specification of the policy instrument $\left\{i_t^{MU}\right\}$, an equilibrium under a *monetary union* is a set of endogenous processes $\left\{y_t,\ c_t,\ \pi_{H,t},\ s_t,\ y_t^*,\ c_t^*,\ \pi_{F,t}^*\right\}$ and exogenous processes $\left\{\mu_{j,t},\ j = H, F\right\}$ satisfying the same conditions (23)-(27) along with:

$$\mathbb{E}_t c_{t+1} = c_t + \sigma^{-1}\left(i_t^{MU} - \mathbb{E}_t \pi_{H,t+1} - \gamma \Delta s_{t+1}\right), \tag{30}$$

and the implied law of motion:

$$\Delta s_t = \pi_{F,t}^* - \pi_{H,t}. \tag{31}$$

Finally, notice that equation (29) holds irrespective of the underlying exchange rate regime, and is therefore valid also in the monetary union case.

4 Welfare objective

Under both regimes, we assume that a benevolent monetary authority aims at maximizing world welfare. In the flexible exchange rate regime, in particular, this corresponds to assuming that the monetary authorities of both countries conduct policy under cooperation.

As already well understood in the literature (Galí 2015, Corsetti, Dedola and Leduc 2011), a case of particular interest arises when any (real) inefficiency possibly associated with the flexible price allocation is assumed not to affect the steady state. This is achieved by means of setting the lump sum tax τ_i in order to offset the distortion associated with market power in the goods markets:

$$\tau_i = -\frac{1}{\varepsilon - 1} \equiv \widetilde{\tau}_i < 0 \rightarrow \mathcal{M}_i = 1. \tag{32}$$

In Appendix C we show that, under this assumption, and the additional condition that the degree of trade openness is symmetric across countries ($\alpha = \alpha^*$),[3] the welfare losses

[3] Notice that this further implies

$$n\gamma = (1 - n)\gamma^*.$$

experienced by households in the world economy, appropriately weighted by country size, are, up to second order, given by:

$$\mathbb{W}_t \equiv \sum_{t=0}^{\infty} \beta^t \ \mathbb{V}_t \tag{33}$$

$$\begin{aligned} \mathbb{V}_t \equiv & \frac{(\sigma+\zeta)}{2} \left[n \left(y_t - \overline{y}_t \right)^2 + (1-n) \left(y_t^* - \overline{y}_t^* \right)^2 \right] \\ & - \frac{n\Phi_s}{2} \left(s_t - \overline{s}_t \right)^2 + \frac{\varepsilon}{2} \left[\frac{n}{\kappa} \pi_{H,t}^2 + \frac{(1-n)}{\kappa^*} \pi_{F,t}^{*2} \right], \end{aligned} \tag{34}$$

where $\Phi_s \equiv \frac{\sigma\eta-1}{\sigma}\gamma(2-\gamma-\gamma^*)\Gamma$.

Hence welfare losses depend on the deviation of output from its natural level in each country (which also corresponds to the efficient level given the assumption in (32)), the deviation of the terms of trade from its natural level, and the deviations of domestic inflation (in each country) from its efficient level of zero. Taking unconditional expectations of (33), and letting $\beta \to 1$, we can express unconditional welfare losses (i.e., welfare losses in an average period) as a linear combination of the variances of each argument featured in (34):

$$\begin{aligned} \overline{\mathbb{W}} \equiv & \frac{(\sigma+\zeta)}{2} \left[n \ \text{var} \left(y_t - \overline{y}_t \right) + (1-n) \text{var} \left(y_t^* - \overline{y}_t^* \right) \right] \\ & - \frac{n\Phi_s}{2} \text{var} \left(s_t - \overline{s}_t \right) + \frac{\varepsilon}{2} \left[\frac{n}{\kappa} \text{var} \ \pi_{H,t} + \frac{(1-n)}{\kappa^*} \text{var} \ \pi_{F,t}^* \right]. \end{aligned} \tag{35}$$

Notice that $sign(\Phi_s)$, and therefore the contribution to welfare losses stemming from the variability in the terms of trade gap, depends on the assumption on the inverse elasticity of intertemporal substitution σ and the trade elasticity η, with $\sigma\eta > (<)1$ implying $sign(\Phi_s) > (<)0$.

4.1 A useful special case: symmetric nominal price rigidity

A a useful benchmark arises in the special case of $\kappa = \kappa^*$. Under our maintained assumption that parameters ε and ζ are equal across countries, that special case obtains when the degree of nominal price rigidity is identical across countries, $\theta_H = \theta_F$.

Let

$$\mathcal{Y}_t \equiv n(y_t - \overline{y}_t) + (1-n)(y_t^* - \overline{y}_t^*); \qquad \pi_t \equiv n\pi_{H,t} + (1-n)\pi_{F,t}^* \tag{36}$$

denote respectively the *average* output gap and inflation rate in the world economy (or monetary union), and

$$\widetilde{\mathcal{Y}}_t \equiv y_t - y_t^*; \qquad \widetilde{\pi}_t \equiv \pi_{H,t} - \pi_{F,t}^*, \tag{37}$$

denote the *relative* output gap and inflation rate respectively.

Then, equation (35) can be written as:

$$\begin{aligned}\overline{\mathbb{W}}_{|\kappa=\kappa^*} \equiv & \frac{(\sigma+\zeta)}{2}\left[\underbrace{\operatorname{var}(\mathcal{Y}_t)}_{\substack{\text{average area-wide}\\ \text{output gap}}} + \underbrace{n(1-n)\operatorname{var}\left(\widetilde{\mathcal{Y}}_t\right)}_{\substack{\text{composition}\\ \text{of output gap}}}\right] \\ & - \frac{n\Phi_s}{2}\operatorname{var}(s_t - \overline{s}_t) + \frac{\varepsilon}{2\kappa}\left[\underbrace{\operatorname{var}(\pi_t)}_{\substack{\text{average area-wide}\\ \text{inflation}}} + \underbrace{n(1-n)\operatorname{var}(\widetilde{\pi}_t)}_{\substack{\text{composition}\\ \text{of inflation}}}\right].\end{aligned} \tag{38}$$

Hence we see that, in addition to variations in the terms of trade gap, welfare losses depend on both the *average* level and the *composition* of the area-wide output gap and inflation rate. We will show below that, in this particular case of $\kappa = \kappa^*$, both the average output gap and average inflation are identical across exchange rate regimes. Therefore, the welfare ranking will crucially depend on how monetary policy shapes the composition of the average output gap and inflation under alternative regimes.

Furthermore, in the same special case, the behavior of relative inflation can be conveniently expressed as follows. Combining (26), (27) and (29), one can first write relative inflation as a function of the current terms of trade gap and relative markup shock, as well as of its expected future value:

$$\widetilde{\pi}_t = \Omega\,(s_t - \overline{s}_t) + \kappa\widetilde{\mu}_t + \beta\mathbb{E}_t\widetilde{\pi}_{t+1} \tag{39}$$

where $\Omega \equiv \frac{(\gamma+\gamma^*)(2-\gamma-\gamma^*)(\sigma\eta-1)\zeta+(\sigma+\zeta)}{\sigma}\kappa > 0$, and $\widetilde{\mu}_t \equiv \mu_{H,t} - \mu_{F,t}$.

Next, integrating equation (39) forward, and recalling (19), we obtain:

$$\widetilde{\pi}_t = \Omega\ \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \left(s_{t+j} - \overline{s}_{t+j} \right) \right\} + \frac{\kappa}{1-\beta\rho_\mu} \widetilde{\mu}_t, \tag{40}$$

where we have assumed that $lim_{j\to\infty}\ \mathbb{E}_t \left\{ \beta^j \left(s_{t+j} - \overline{s}_{t+j} \right) \right\} = 0$.

Equation (40) shows that relative inflation depends on the current and expected future values of the terms of trade gap (as well as of the relative markup shocks). Hence, both the volatility and the persistence of the terms of trade gap contribute to the volatility of relative inflation, and therefore to welfare losses.

5 Optimal monetary policy

Next we turn to the central theme of the paper: the characterization of optimal monetary policy under two alternative regimes, flexible exchange rates and monetary union, respectively. For each regime we study two polar cases, depending on the underlying assumption about the ability of the monetary authority to commit. We are particularly interested in studying the case of a Markov perfect equilibrium (henceforth labeled "discretion") in which, under either regime, the monetary authority cannot credibly commit to any future course of actions.

A standard constrained-efficiency approach to optimal policy prescribes that a social planner maximizes world welfare (33) subject to the relevant constraints that characterize the competitive equilibrium, i.e., (21)-(27) under flexible exchange rates, and (23)-(27) together with (30) and (31) in the monetary union case. The optimal policy problem, however, can be characterized in terms of a less constrained problem, under both exchange rate regimes, as we show below.

5.1 Flexible exchange rates

Optimal cooperative policy under flexible exchange rates requires solving the following problem:

$$\max\ \mathbb{W}_t \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathbb{V}_t \tag{41}$$

s.t. (26), (27), (29).

For a given specification of the exogenous processes $\{\mu_{j,t},\ j = H, F\}$, a *flexible-exchange rate* equilibrium under the optimal policy consists in a vector $\{\pi_{H,t},\ \pi^*_{F,t}, y_t,\ s_t,\ y^*_t\}^{OPT,FLEX}$ solving (41). One can then use (21), (22), (23), and (24) to residually obtain $\{c_t,\ c^*_t, i_t,\ i^*_t\}^{OPT,FLEX}$.

We show in Appendix D that, under commitment, the system of equations describing the equilibrium evolution under the optimal policy is given by (26), (27), (29) together with the following targeting rules:

$$(y_t - \overline{y}_t) - (y_{t-1} - \overline{y}_{t-1}) + \varepsilon\pi_{H,t} = 0 \tag{42}$$

$$(y^*_t - \overline{y}^*_t) - (y^*_{t-1} - \overline{y}^*_{t-1}) + \varepsilon\pi^*_{F,t} = 0. \tag{43}$$

On the other hand, under discretion, the complete system of equations that describe the evolution of the welfare-relevant variables is given by (26), (27), (29), together with the following targeting rules:

$$(y_t - \overline{y}_t) + \varepsilon\pi_{H,t} = 0, \tag{44}$$

$$(y^*_t - \overline{y}^*_t) + \varepsilon\pi^*_{F,t} = 0. \tag{45}$$

5.2 Monetary union

Relative to (41) under flexible exchange rates, optimal cooperative policy under a monetary union requires solving the more constrained optimization problem:

$$\begin{aligned} &\max\ \mathbb{W}_t \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathbb{V}_t \\ &\text{s.t. } (26),\ (27),\ (29),\ (31)\ . \end{aligned} \tag{46}$$

For a given specification of the exogenous processes $\{\mu_{j,t},\ j = H, F\}$, a *monetary union* equilibrium under the optimal policy consists in a vector $\{\pi_{H,t},\ \pi^*_{F,t}, y_t,\ s_t,\ y^*_t\}^{OPT,MU}$ solving (46). One can then use (23), (24) and (30) to residually obtain $\{c_t,\ c^*_t, i^{MU}_t\}^{OPT,MU}$.

In the Appendix D we present the general system of first order conditions for the monetary union case, both under commitment and discretion. In the particular case of $\kappa = \kappa^*$, those optimality conditions take a simplified and intuitive form involving only the union-wide average output gap and inflation.

Under commitment, the monetary authority's targeting rule reads:

$$(\mathcal{Y}_t - \mathcal{Y}_{t-1}) + \varepsilon \pi_t = 0, \tag{47}$$

whereas under discretion:

$$\mathcal{Y}_t + \varepsilon \pi_t = 0 \tag{48}$$

Hence, in a monetary union, and under the assumption $\kappa = \kappa^*$, the system of equations that describe the evolution of the welfare-relevant variables consists of (26), (27), (29), (31) together with (47) and (48) respectively under commitment and discretion.

5.3 An irrelevance result

We show next that, under particularly useful conditions, the exchange rate regime (i.e., flexible exchange rates or monetary union) is irrelevant for the equilibrium behavior of average inflation π_t and of the average output gap $\mathcal{Y}_t$. We express this point in the following proposition:

Proposition 1. *Assume $\kappa = \kappa^*$ and $\alpha = \alpha^*$. Then the underlying exchange rate regime is irrelevant for the equilibrium behavior of the area-wide average inflation π_t and output gap $\mathcal{Y}_t$.*

To understand the above proposition, notice that combining (26) and (27), for both exchange rate regimes (flexible exchange rates and monetary union), and irrespective of whether monetary policy is conducted under commitment or discretion, one can write the following area-wide average expression for aggregate supply:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + (\sigma + \zeta)\, \kappa \mathcal{Y}_t + \kappa \mu_t \tag{49}$$

where $\mu_t \equiv n\mu_{H,t} + (1-n)\mu_{F,t}$.

Consider, first, the case of commitment and flexible exchange rates. By taking a weighted average of (42) and (43), and recalling the definitions of average output gap and inflation given in (36), one obtains exactly the targeting rule under commitment (47) that characterizes the monetary union case. Hence, under commitment, and irrespective of the underlying exchange rate regime, an equilibrium in the area-wide average variables is a pair $\{\pi_t, \mathcal{Y}_t\}$ solving (47) and (49) for any given process $\{\mu_t\}$. Similarly, under discretion, one can take a weighted average of (44) and (45) to obtain (48). Hence, under discretion, an equilibrium in the area-wide average variables is a pair $\{\pi_t, \mathcal{Y}_t\}$ solving (48) and (49), for any given process $\{\mu_t\}$.

The fact that, under specific conditions of symmetry, area-wide average inflation and output gap behave identically under both flexible exchange rates and a monetary union is important to highlight that the difference in welfare losses across the two regimes lies precisely in the behavior of the terms of trade and, consequently, of relative inflation. Under those conditions, therefore, it is the *composition* of inflation across countries which lies at the heart of the differences in welfare losses.

6 Inertia in the terms of trade: a commitment device

In this section we show that the key difference between the flexible exchange rate regime and the monetary union regime lies in the different equilibrium behavior of the terms of trade. We can illustrate this case analytically holding constant the symmetry assumption $\kappa = \kappa^*$.

Notice, first, that under all regimes, and irrespective of whether policy is conducted under commitment or discretion, equation (39) holds. We reproduce it here for convenience:

$$\widetilde{\pi}_t = \Omega\left(s_t - \overline{s}_t\right) + \kappa\widetilde{\mu}_t + \beta\mathbb{E}_t\widetilde{\pi}_{t+1} \tag{50}$$

Flexible exchange rates Consider the case of flexible exchange rates. Under commitment, and the assumption $\kappa = \kappa^*$, combining (42) and (43) with (29) yields**:**

$$\varepsilon\widetilde{\pi}_t = -\frac{\Gamma}{\sigma}\left[\left(s_t - \overline{s}_t\right) - \left(s_{t-1} - \overline{s}_{t-1}\right)\right]. \tag{51}$$

Combining (50) with (51) to eliminate $\widetilde{\pi}_t$ yields the following second-order stochastic difference equation for the terms of trade (note that $\overline{s}_t = 0$ if markup shocks are the only shocks):

$$\beta\mathbb{E}_t s_{t+1} - \left(1 + \beta + \frac{\sigma\varepsilon\Omega}{\Gamma}\right)s_t + s_{t-1} = \frac{\kappa\sigma\varepsilon}{\Gamma}\widetilde{\mu}_t. \tag{52}$$

The above equation has a unique stationary representation expressing current s_t as a linear function of lagged s_{t-1} and $\widetilde{\mu}_t$. Hence, under commitment, and even in the presence of purely iid markup disturbances, the terms of trade feature an endogenous degree of persistence.

Under discretion, and once again assuming $\kappa = \kappa^*$, combining (44) and (45) with (29) yields:

$$\varepsilon\widetilde{\pi}_t = -\frac{\Gamma}{\sigma}\left(s_t - \overline{s}_t\right). \tag{53}$$

Combining (53) with (50) to eliminate $\widetilde{\pi}_t$ yields the following first-order stochastic difference equation for the terms of trade :

$$\beta\mathbb{E}_t s_{t+1} - \left(1 + \frac{\varepsilon\sigma\Omega}{\Gamma}\right) s_t = \frac{\kappa\varepsilon\sigma}{\Gamma}\,\widetilde{\mu}_t. \tag{54}$$

The above equation has a unique stationary solution, expressing the terms of trade as a purely forward-looking variable and as a function of the (relative) markup shocks. The main insight stemming from (54), and in stark contrast to the case under commitment, is that under discretion the terms of trade do not feature any degree of endogenous persistence.

To summarize, in a flexible-exchange rate equilibrium under the optimal policy, the dynamic properties of the terms of trade depend on the ability of the monetary authority to commit. Under discretion, the terms of trade are exogenously persistent (i.e., only to the extent that the markup shocks are persistent); under commitment, and regardless of the stochastic properties of the markup shocks, the terms of trade feature endogenous inertia.

Monetary union We now turn to the properties of the terms of trade in a monetary union. Consider first the law of motion for the terms of trade, which holds under a fixed nominal exchange rate:

$$s_t = s_{t-1} + \widetilde{\pi}_t. \tag{55}$$

Notice that the pair of equations (50) and (55) is sufficient to determine an equilibrium in the two endogenous variables $\widetilde{\pi}_t$ and s_t. Importantly, this implies that, under a monetary union, the equilibrium dynamics of the terms of trade are independent of monetary policy[4], and therefore independent of whether monetary policy is conducted under discretion or under commitment.

Combining (50) and (55) to eliminate $\widetilde{\pi}_t$ yields the following second-order stochastic difference equation for the terms of trade:

$$-\beta\mathbb{E}_t s_{t+1} + (1 + \beta - \Omega)\, s_t - s_{t-1} = \kappa\widetilde{\mu}_t. \tag{56}$$

[4]See also Benigno (2004) on this point.

Similar to (52), equation (56) has a unique stationary representation expressing current s_t as a linear function of lagged s_{t-1} and $\widetilde{\mu}_t$. Hence, and regardless of the stochastic properties of the markup shocks, the terms of trade feature endogenous persistence, a key feature shared with a flexible exchange rate regime under commitment. Thus, in a monetary union and *irrespective* of whether or not the policy authority can commit, the terms of trade are intrinsically inertial. The intuition for this result is simple: the lack of nominal exchange rate flexibility combined with nominal price rigidity.

Notice that, while the inertia in the terms of trade in the context of a monetary union has been recognized before (Benigno 2004, Pappa 2004), it was typically regarded as a distortion of that regime. Relatedly, Farhi and Werning (2012) emphasize that the inefficiency at the heart of *any* monetary union, and regardless of the underlying degree of completeness in international financial markets, is a structural "lack of insurance", which stems precisely from the suboptimal adjustment in the terms of trade that results from the combination of nominal price rigidity and lack of nominal exchange rate flexibility.

Unlike those previous contributions, which are centered on the inherent *inefficiency* of a monetary union, we wish to show that the inertia in the terms of trade can be an advantage for policy, rather than a constraint, depending on whether or not the monetary authority can commit. Thus, under commitment, inertia in the terms of trade is *always* a cost. Under discretion, however, inertia in the terms of trade can generally be *beneficial*: it allows the policy authority to trade-off efficiency in the response to asymmetric shocks (the aggregate demand stabilization cost) in order to gain in terms of management of inflation expectations. We turn to clarifying this point below.

6.1 Dynamics

In this section, we study the equilibrium dynamics under flexible exchange rates vis-a-vis a monetary union depending on the ability of the monetary authority to commit. We first describe the numerical calibration employed in our exercises.

Calibration We resort to the following calibration. The baseline parameter values are displayed in Table 1. A value of 0.99 for the discount factor β implies a steady-state real interest rate of around 4.1 percent annually. A value of 7.66 for the elasticity of substitution between differentiated goods ε implies a steady-state markup of prices over marginal costs of 15 percent. The trade elasticity of substitution η is calibrated to 2, which implies nonnegligible degree of substitutability between domestic and foreign goods. Given the well-known uncertainty in the literature about the value of η, we perform robustness exercises below. A value of 0.75 for the probability of not being able to reset the price θ_i implies an average duration of price contracts of four quarters, consistent with much of the empirical evidence based on micro data. Both the degree of trade openness α and the relative size of the Home country n are calibrated to 0.5. These values imply a steady-state share of home-produced goods in the consumption basket, γ and γ^*, of 0.75. Note that the share of home-produced goods is symmetric across countries only under symmetric country size ($n = 0.5$). Asymmetries in country size ($n \neq 0.5$) will lead to asymmetries in the share of home-produced goods ($\gamma \neq \gamma^*$).

Table 1. Baseline calibration

Parameter	Description	Value/Target
β	Discount factor	0.99
σ	Inverse elasticity of intertemporal substitution	1
ζ	Inverse elasticity of producing differentiated good	0.67
n	Relative size of Home country	0.5
α	Degree of trade openness	0.5
ε	Elasticity of substitution btw. differentiated goods	7.66
η	Elasticity of substitution btw. Home and Foreign goods	2
θ_i	Probability of not being able to reset price in country $i = H, F$	0.75
ρ_μ	Persistence of markup shock in country $i = H, F$	0.9
υ_{ι_i}	Variance innovation markup process in country $i = H, F$	0.01

Unless otherwise stated, we assume that all structural parameters indicated above have identical values across countries, including the variance of the innovation to the markup process, which are also assumed to be uncorrelated across countries. This implies that, in our baseline calibration, the symmetry assumptions $\kappa = \kappa^*$and $n\gamma = (1-n)\gamma^*$ both hold (see Proposition 1 above).

Impulse responses Figure 1 shows impulse responses of relative inflation and terms of trade gap to a markup shock in Home under the assumption of commitment (the behavior of the relative output gap is isomorphic to the one of the terms of trade, so it was omitted for the sake of clarity). For each panel, the case of flexible exchange rates (dashed line) is contrasted to the one of a monetary union (solid line). Qualitatively, the behavior of both variables is similar under the two policy regimes: this is an implication of our results derived above, and expressed in particular in equations (52) and (56), which show that, under commitment, the terms of trade exhibit inertia under both flexible exchange rates and a monetary union. In both cases, a rise in the Home markup generates a typical "cost-push driven" tradeoff between higher relative inflation and lower output gap (and/or appreciated terms of trade).[5] In the case of flexible exchange rates, however, the more pronounced terms of trade appreciation, made feasible by the flexibility of the nominal exchange rate, restrains the response of relative inflation when compared to the case of a monetary union: under both regimes, in fact, relative inflation is a function of current and expected future movements in the terms of trade via equation (40). Overall, this effect is welfare increasing, and lies at the heart of the widely accepted optimality of flexible exchange rate regimes relative to monetary unions. Conversely, the gap between the response of the terms of trade under the two regimes is a measure of the inefficient adjustment of international relative prices under a monetary union. That inefficiency has been emphasized as the key one leading to a structural lack of insurance characterizing any monetary union (see e.g., Farhi and Werning 2012).

Figure 2 reports the impulse responses of the terms of trade gap and relative inflation in the case of discretion, which is the case of particular interest for our purposes. Once again solid lines indicate the response under a monetary union, whereas dashed lines indicate the responses in the case of flexible exchange rates.

Two results are worth emphasizing. First, notice that the terms of trade feature an inertial behavior only in the case of a monetary union, as (once again) indicated by equation (56). Under flexible exchange rates, the terms of trade appreciate sharply, and follow a Markov-type path afterwards: in practice they are the mirror image of the autoregressive exogenous markup process. Under a monetary union, and due to the inflexibility of the nominal exchange rate, the appreciation of the terms of trade is muted in the short run, but builds up afterwards. It is as if the monetary authority, in the current period, generated expectations of a future, more pronounced appreciation of the terms of trade (relative to the case of flexible exchange rates). The inertial behavior of the terms of trade in the case of a monetary union is reflected in the behavior of relative inflation. Since relative inflation, via equation (40), is a function of both the current and expected future terms of trade, the expectations of a more prolonged future real appreciation restrain the short-run

[5] See Clarida et al. (1999) for the seminal analysis of so called cost-push shocks in a closed economy environment.

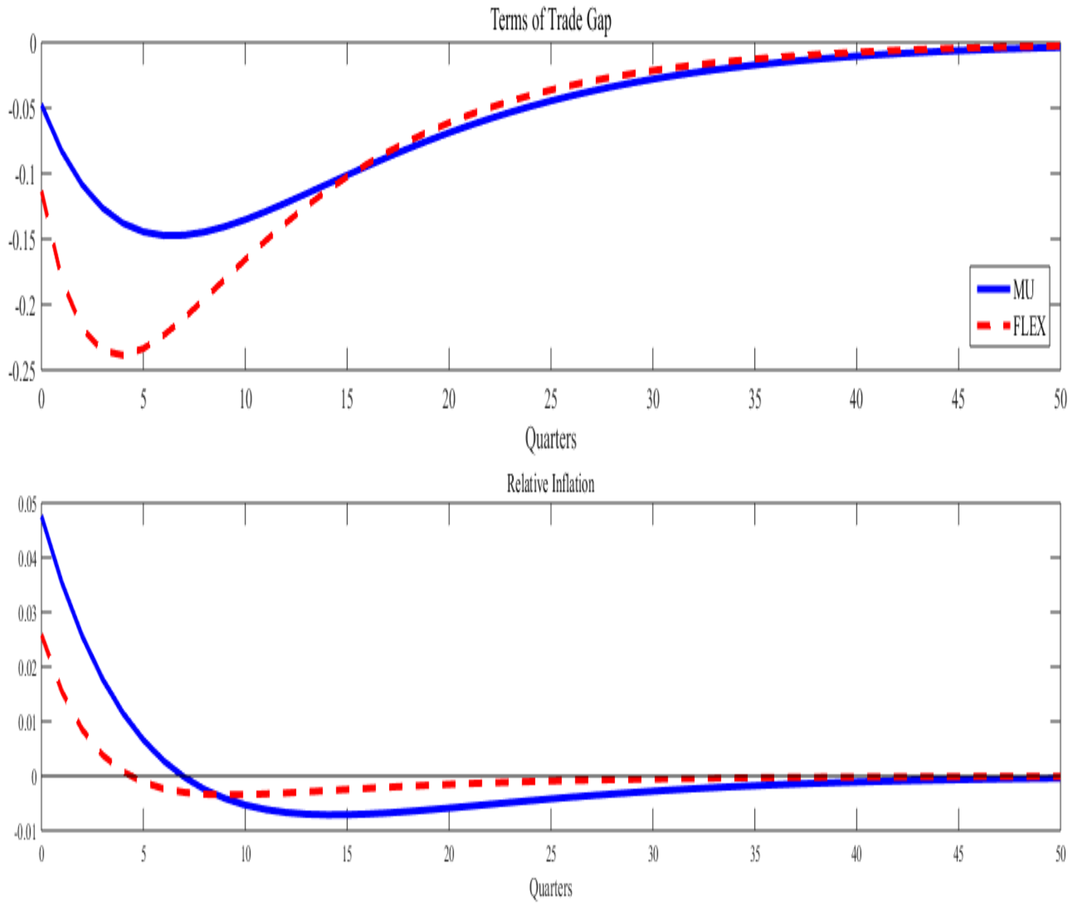

Figure 1: Impulse responses to a positive markup shock in Home under *commitment*: monetary union (solid) vs. flexible exchange rates (dashed). Note: % deviations from steady state.

increase in relative inflation under a monetary union. Overall, inflation is more stable in a monetary union. We will show below that this inherent benefit is critical in generating a welfare improvement in the monetary union case with respect to flexible exchange rates.

7 The (un)desirability of a monetary union

In this section we conduct a thorough comparison of the welfare properties of each exchange rate regime, depending on the ability of the monetary authority to commit.

Figure 3 depicts our key result. It reports the *difference* in (area-wide) welfare losses, $\overline{\mathbb{W}}_{MU} - \overline{\mathbb{W}}_{FLEX}$, between a monetary union and a flexible exchange rate regime, as a function of the underlying degree of nominal rigidity assumed equal across countries ($\theta_H = \theta_F$), and separately for discretion and commitment. Positive values of the welfare loss difference, therefore, indicate that a monetary union entails higher welfare costs than flexible exchange rates.

The dashed line is illustrative of the standard consensus: under commitment, and regardless of the underlying degree of nominal rigidities, a monetary union always entails higher welfare losses relative to flexible exchange rates. This is a plain application of the classic Friedman dictum, whereby, under nominal rigidities, flexible exchange rates compensate for the inertial behavior in goods prices, thereby allowing the economy to replicate the constrained-efficient response of the terms of trade.[6]

However, under discretion, the consensus result is overturned (solid line): a monetary union now entails lower welfare losses than flexible exchange rates. Only for very low degrees of price stickiness, well outside the range of plausible empirical estimates, a monetary union entails higher welfare losses.

Before looking at the factors driving these key results, two further observations are worth noticing. First, and at least under our baseline calibration, the welfare gain of the monetary union regime over the flexible exchange rate regime under discretion is greater than the welfare gain of the flexible exchange rate regime over the monetary union regime under commitment. Second, the welfare loss difference features a U-shaped, non-monotonic, relationship with the degree of price stickiness. The intuition for the non-monotonicity can be easily grasped by looking at the two extreme cases of full price flexibility and full rigidity.

[6] It is worth recalling, however, that the constrained efficient allocation under flexible exchange rates still differs from the first-best flexible price allocation, due to the presence of markup (i.e., inefficient) shocks.

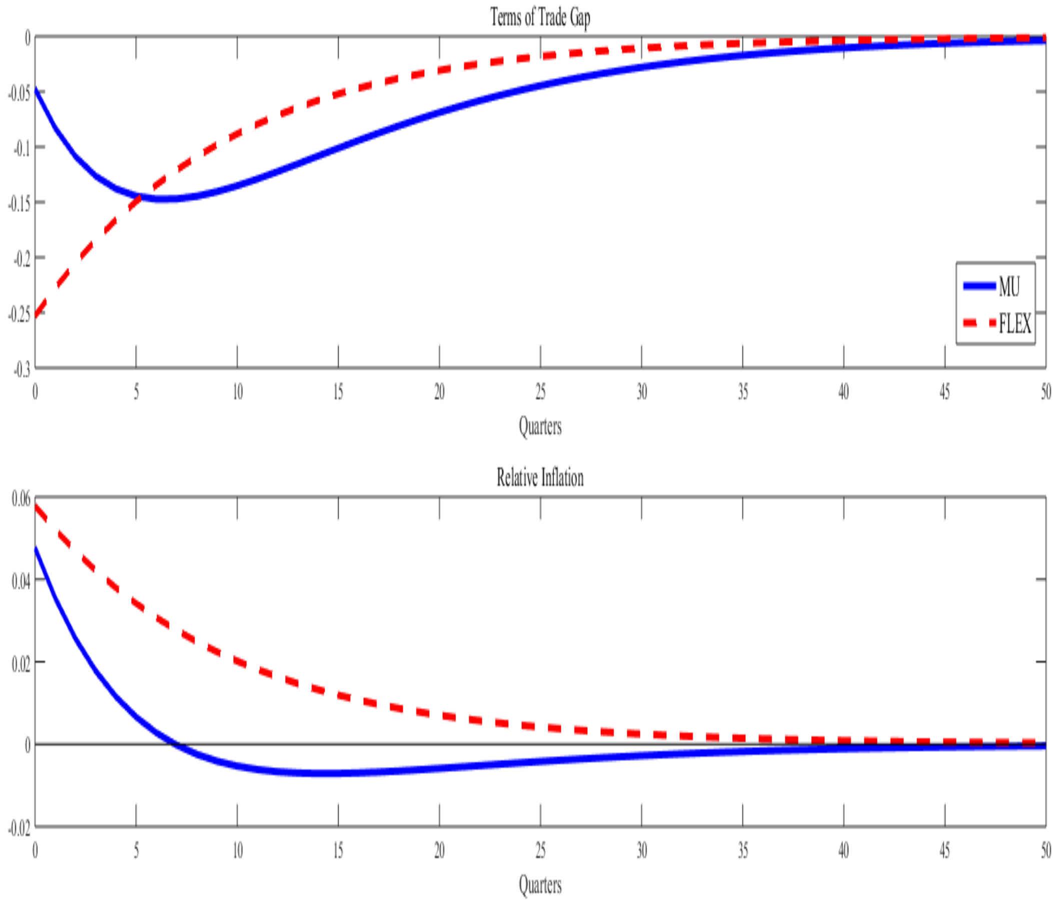

Figure 2: Impulse responses to a positive markup shock in Home under discretion: monetary union (solid) vs. flexible exchange rates (dashed). Note: % deviations from steady state.

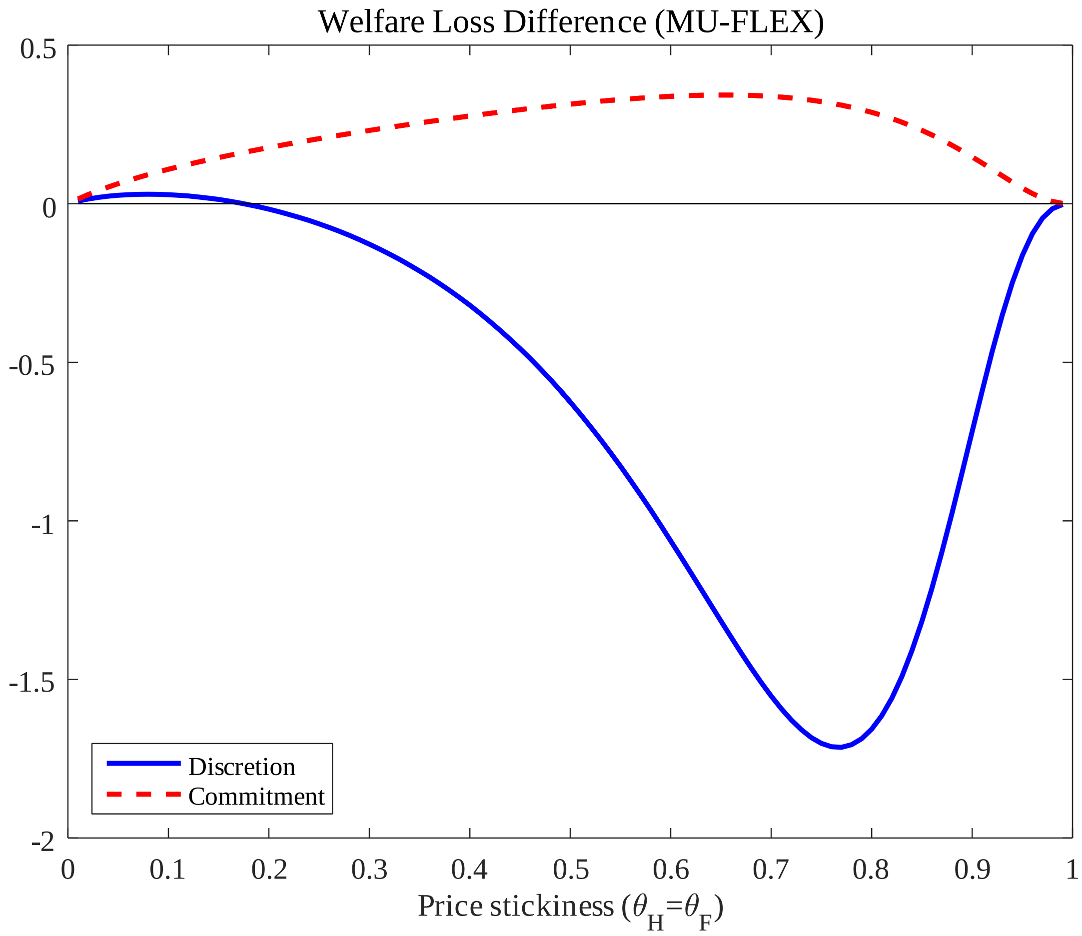

Figure 3: Welfare loss difference $\overline{\mathbb{W}}_{MU} - \overline{\mathbb{W}}_{FLEX}$ as a function of the degree of price stickiness under discretion and commitment. Note: a positive value indicates that flexible exchange rates dominate a monetary union.

Under full price flexibility, the underlying exchange rate regime is irrelevant, because the price flexibility eventually compensates for the inertia in the nominal exchange rate; convresely, full price rigidity implies that inflation is always perfectly constant in equilibrium (and starting from a steady state in which the inflation rate is in line with the assumed target of zero).

To inspect the mechanism more closely, Figure 4 depicts, under commitment, the contribution of the relevant components to the absolute welfare loss: relative inflation, the relative output gap, and the terms of trade gap.[7] In each panel, the case of a monetary union (solid line) is contrasted to the one of flexible exchange rates (dashed line). The figure features a breakdown of the main factors that contribute to the relatively higher welfare cost of monetary unions: first, and foremost, a higher volatility in relative inflation (which increases the overall welfare losses) ; second, a lower volatility of the terms of trade gap (which, under the assumption $\sigma\eta > 1$, and therefore $\Phi_s > 0$, contributes relatively less in decreasing welfare losses). Both terms, in Figure 4, are appropriately weighted by the structural coefficients featured in (38). Clearly, moving from flexible exchange rates to a monetary union entails costs (a higher volatility of relative inflation and a lower volatility of the terms of trade gap, the latter being a cost under the assumption $\sigma\eta > 1$), but also benefits (a lower output gap volatility). As the top left panel shows, under our baseline parameterization, the costs uniformly outweigh the benefits, making a monetary union invariably more welfare costly, regardless of the degree of nominal price rigidity.

By contrast, Figure 5 shows the case of discretion. As shown in the top left panel, a monetary union entails now a lower welfare loss relative to a regime of flexible exchange rates, except for very low degrees of price rigidity. The main driver of this result, as clearly illustrated in the top right panel, is a lower volatility in the relative inflation term. This stems precisely from the inertia in the terms of trade induced by the fixed exchange rate. Noticeably, the welfare advantage of being in a monetary union relative to a regime of flexible exchange rates is non-monotonic in the degree of price rigidity, and reaches a peak around the value for price stickiness (0.75) assumed in our baseline calibration (and allegedly in line with most empirical evidence, especially for the Eurozone[8]). The reason for the non-monotonicity is as follows. The costs, associated with discretionary policy, of taking inflation expectations as given are increasing in the degree of price rigidity because, as the

[7] Recall that under the assumption of symmetry, both the area-wide average inflation rate and output gap are independent of the underlying exchange rate regime (see Proposition 1). Therefore, we do not report them here.

[8] Dhyne et al. (2005).

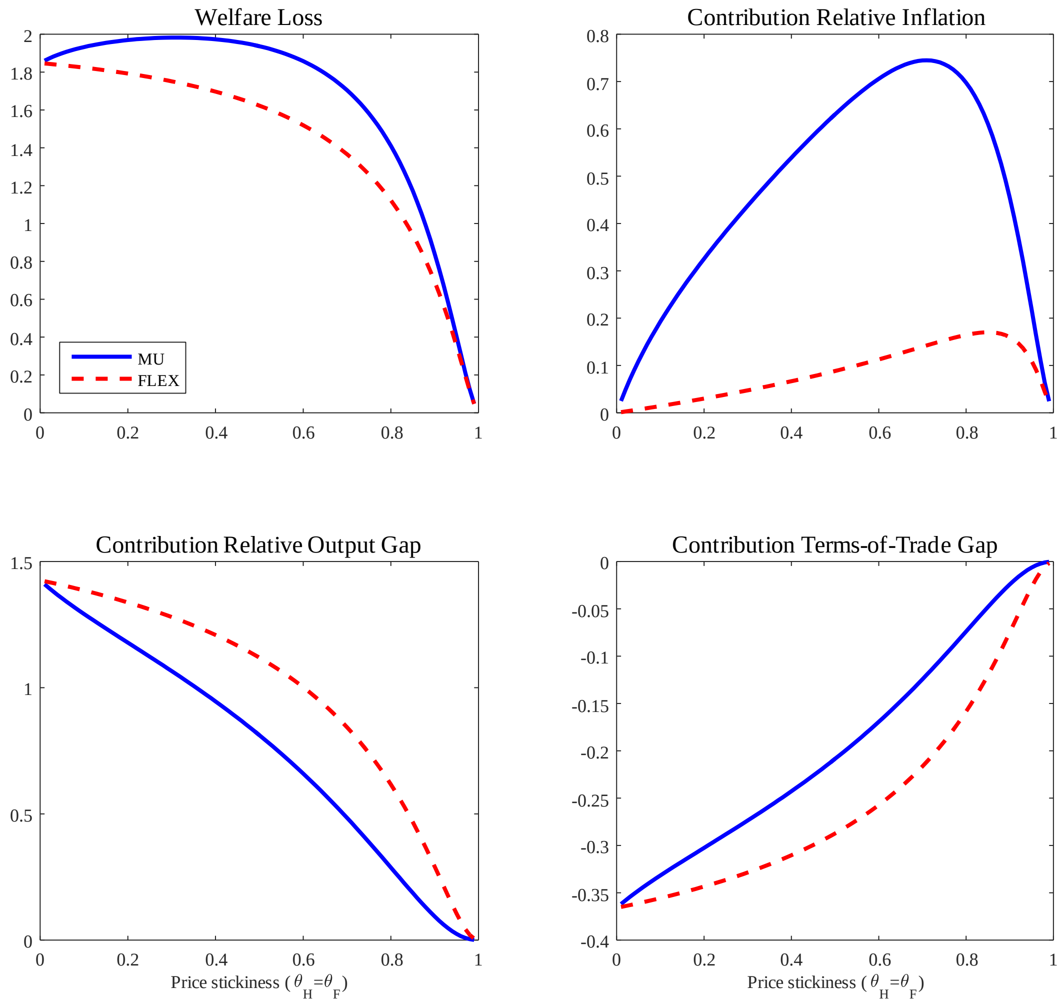

Figure 4: Average welfare loss and its components under *commitment*: monetary union (solid) vs. flexible exchange rates (dashed).

latter increases, price setters become more and more forward-looking. In the limit, however, when prices are perfectly rigid, the gains associated to a better management of inflation expectations disappear, because a cost-push shock becomes ineffective.

7.1 Robustness

In this section, we assess the robustness of our main result (i.e., the desirability of a monetary union under discretion) to variations in some key model parameters.

Relative degree of price stickiness So far we have worked under the assumption of cross-country symmetry in the degree of price stickiness. Figure 6 depicts the effects on the welfare loss difference, $\overline{\mathbb{W}}_{MU} - \overline{\mathbb{W}}_{FLEX}$, of varying (only) the degree of *domestic* price stickiness, while holding constant θ_F, the degree of price stickiness in Foreign. This is shown for several cases, corresponding to alternative values of θ_F. Clearly, with the exception of limiting cases in which the degree of price stickiness is extremely low in both countries (although not necessarily equal), the welfare loss difference takes invariably a negative value, implying a lower welfare loss under a monetary union. We therefore conclude that asymmetries in the degree of price stickiness are not, to any important degree, relevant for the welfare ranking between the two monetary regimes. They only affect the size of the welfare gain of the monetary union regime. Notice also that the maximum welfare gain from a monetary union (i.e., the lowest value in the welfare loss difference) is not necessarily achieved under a symmetric degree of price stickiness.

Persistence of markup shocks Next, we look at how price stickiness affects the welfare loss difference under alternative assumptions on the persistence of the markup shock ρ_μ. In what follows, we return to the baseline assumption of a symmetric degree of price stickiness across countries. Figure 7 shows that a monetary union continues to be desirable for plausible estimates of the degree of price stickiness, though the size of the welfare gain decreases rapidly as the shock persistence decreases. Notice, however, that the lower the degree of persistence of the markup shock, the higher the minimal amount of price stickiness needed to make a monetary union more desirable relative to flexible exchange rates. The role played by the shock persistence is intuitive, given that the inherent benefit of a monetary union is due to the inertia in the terms of trade and its stabilizing effect on inflation expectations. As the shock becomes more and more short-lived, the importance of expectations

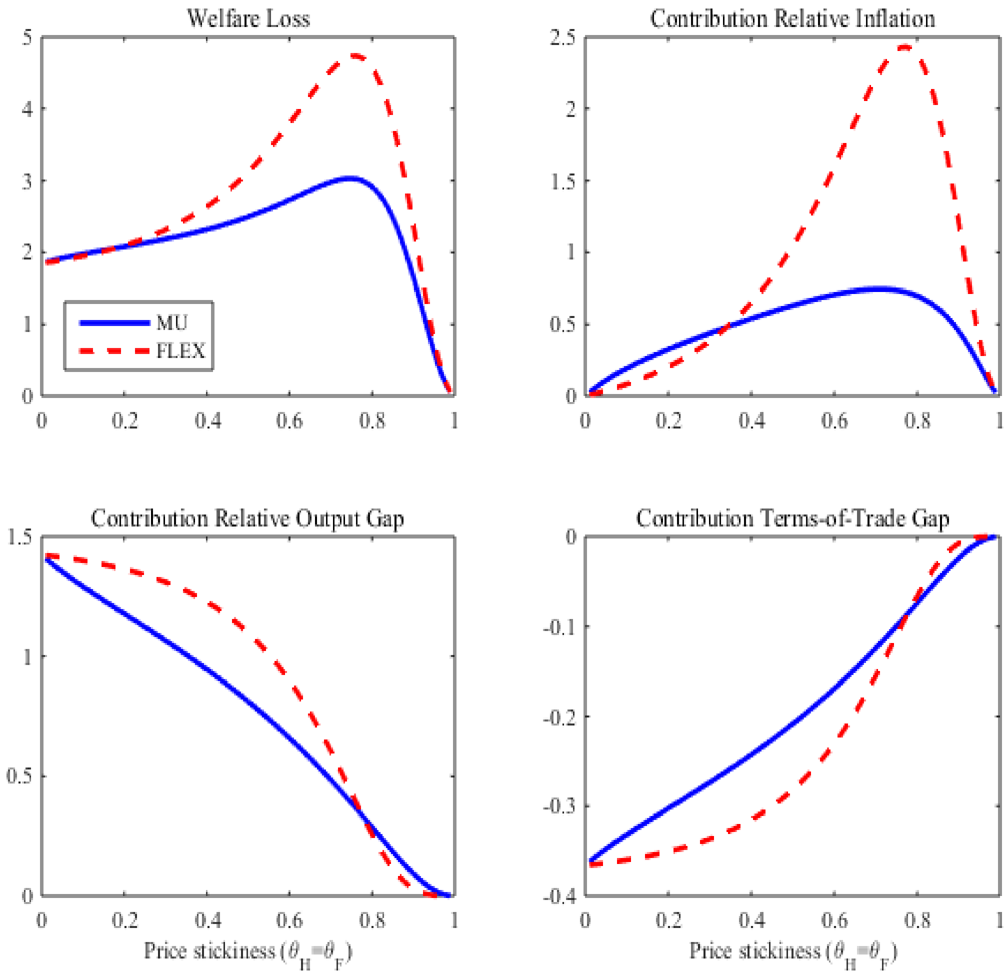

Figure 5: Average welfare loss and its components under *discretion*: monetary union (solid) vs. flexible exchange rates (dashed).

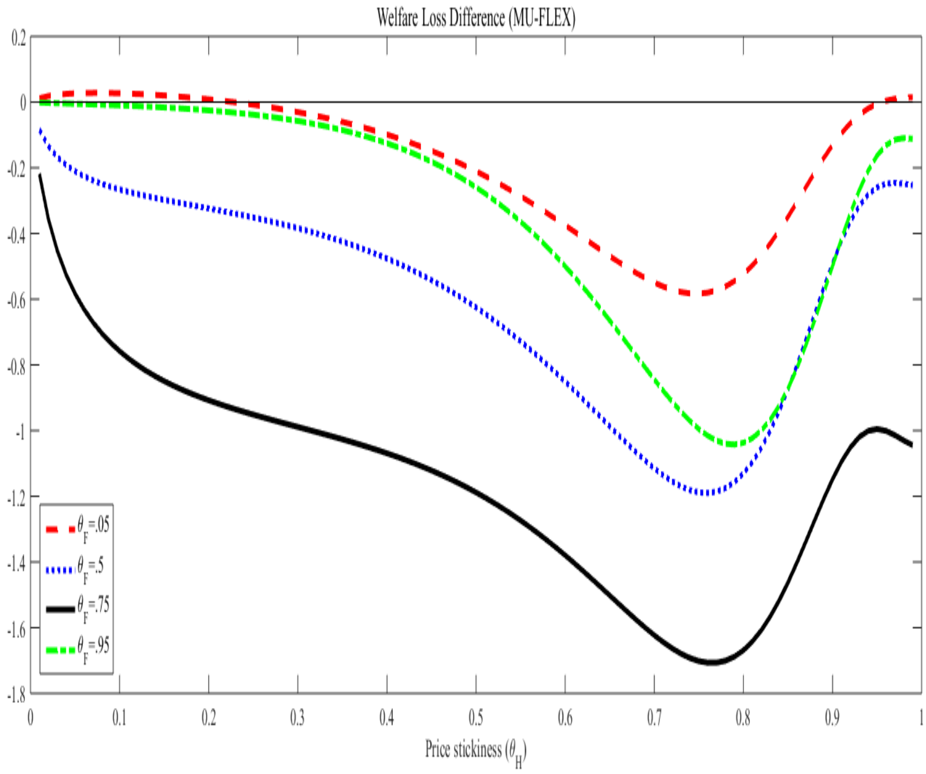

Figure 6: Effect of varying domestic price stickiness on the welfare loss difference $\overline{\mathbb{W}}_{MU} - \overline{\mathbb{W}}_{FLEX}$ under *discretion* and for alternative foreign price stickiness.

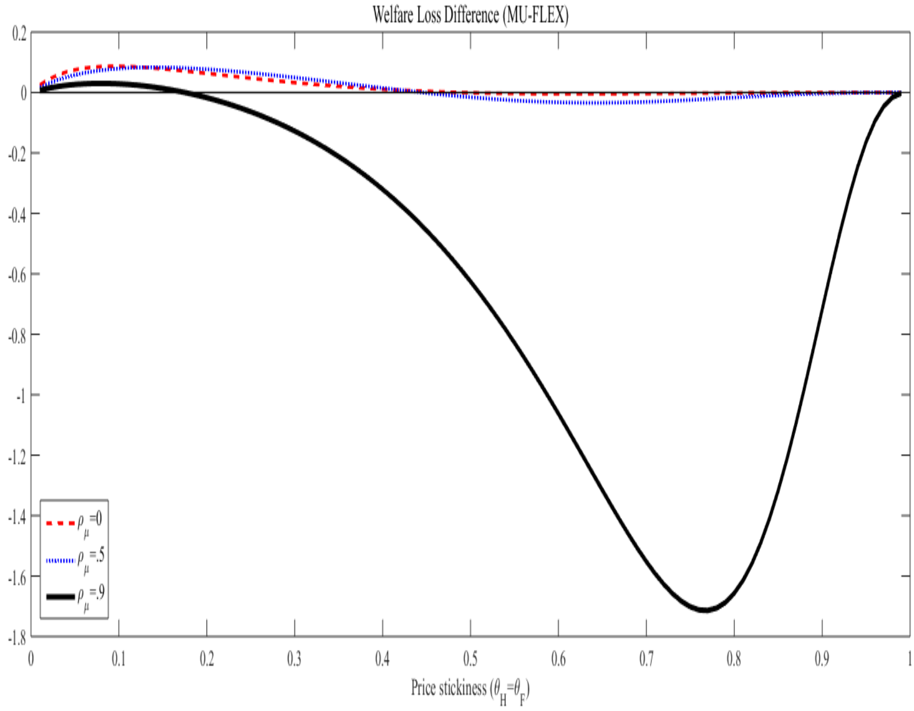

Figure 7: Effect of varying domestic price stickiness on the welfare loss difference $\overline{\mathbb{W}}_{MU} - \overline{\mathbb{W}}_{FLEX}$ under *discretion* and for alternative degrees of persistence of the markup shock.

in general decreases. As a result, the benefit of stabilizing inflation expectations tends to vanish.

Trade elasticity of substitution Figure (8) displays the effect on the welfare loss difference of varying the trade elasticity of substitution η. The relationship is clear-cut: The lower the degree of substitutability of internationally traded goods, the higher the minimal degree of price stickiness needed to make a monetary union more desirable relative to flexible exchange rates, and the smaller is the corresponding welfare gain. As the internationally traded goods become less substitutable for consumers in both countries, the terms of trade become less and less important for price setters and their expectations, due to a decreasing expenditure switching effect. In the extreme case of zero substitutability ($\eta = 0$), households consume the goods in fixed proportions, irrespective of changes in their relative price. In this case, the benefit of policy-induced inertia in the terms of trade ceases to exist. By contrast, if

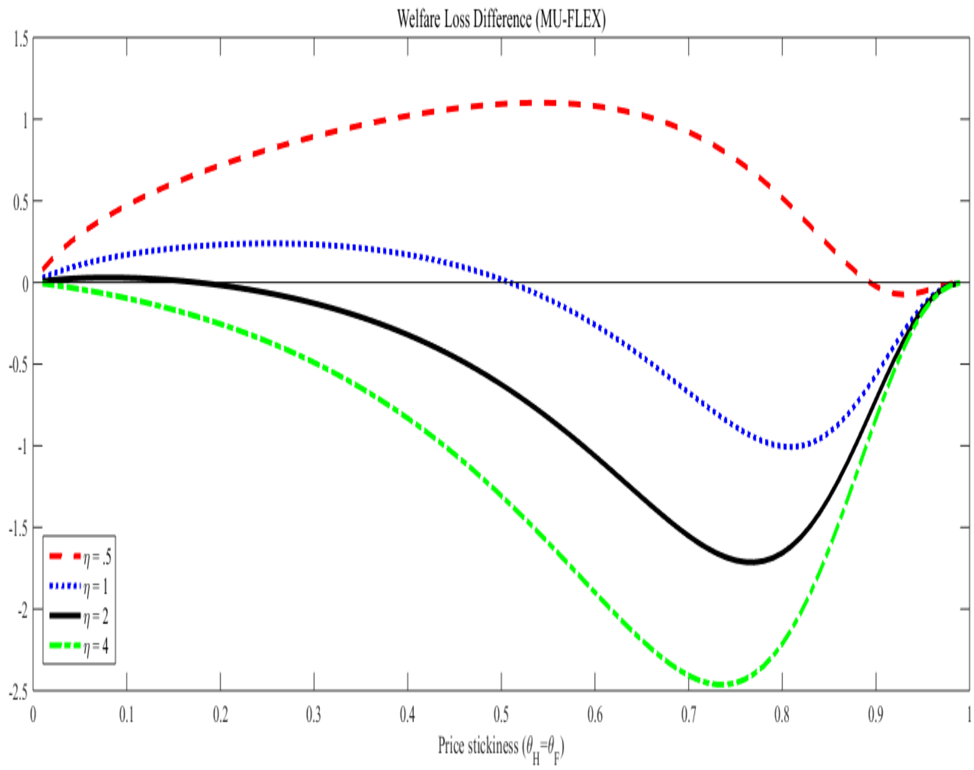

Figure 8: Effect of varying domestic price stickiness on the welfare loss difference $\overline{\mathbb{W}}_{MU} - \overline{\mathbb{W}}_{FLEX}$ under *discretion* and for alternative values of the trade elasticity of substitution η.

the elasticity of substitution increases, the terms of trade become more and more important for price setters due to the increasing expenditure switching effect. As a result, the beneficial effect of terms-of-trade inertia on inflation expectations, and ultimately on inflation, tends to increase. Lastly, notice that, given $\sigma = 1$, Figure 8 displays cases where, respectively, $\sigma\eta < 0$, $\sigma\eta = 0$, and $\sigma\eta > 0$. It therefore becomes clear that this condition, which plays a crucial role in shaping cross-border spillovers of shocks, is not important for the welfare implications of monetary unification.

8 Conclusions

We have studied a classic issue in international monetary economics, namely whether, as originally argued by Friedman (1953), the presence of nominal price rigidity makes an unequivocal (welfare) case in favor of flexible exchange rates. We have shown that the answer to this question hinges crucially on the monetary authority's ability to commit. When the monetary authority lacks commitment, a regime of fixed exchange rates (or monetary union) generally welfare dominates one of flexible exchange rates. This result is in stark contrast with the general consensus whereby the participation to a monetary union entails a genuine inefficiency, in that it precludes the efficient adjustment of international relative prices in response to asymmetric shocks. Our analysis shows that such an inefficiency typically makes a monetary union welfare dominated only under the admittedly extreme assumption of full commitment by the monetary policy authority. Although both polar cases of full commitment and discretion are somewhat extreme, this result highlights the importance of focusing on the policy credibility dimension when employing dynamic microfounded models to assess the desirability of alternative exchange rate regimes.

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A Appendix A. Full model

Market clearing for domestic variety i must satisfy:

$$\begin{aligned} Y_t(i) &= n\ C_{H,t}(i) + (1-n)\ C^*_{H,t}(i) && \text{(A.1)} \\ &= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left[(1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \frac{(1-n)}{n}\gamma^* \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} C^*_t\right] \\ &= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\gamma)C_t + \frac{(1-n)}{n}\gamma^* Q_t^\eta C^*_t\right] && \text{(A.2)} \end{aligned}$$

Market clearing for foreign variety i must satisfy:

$$\begin{aligned} Y^*_t(i) &= n\ C_{F,t}(i) + (1-n)\ C^*_{F,t}(i) && \text{(A.3)} \\ &= \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} \left[\frac{n}{1-n}\gamma C_t + (1-\gamma^*)Q_t^\eta C^*_t\right] \end{aligned}$$

Inserting (A.1) and (A.3) into the following two equations, respectively

$$\begin{aligned} Y_t &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_0^n Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} && \text{(A.4)} \\ Y^*_t &= \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \int_n^1 Y^*_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}, \end{aligned}$$

yields aggregate demand in each country:

$$\begin{aligned} Y_t &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\gamma)C_t + \frac{1-n}{n}\gamma^* Q_t^\eta C^*_t\right] && \text{(A.5)} \\ Y^*_t &= \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} \left[\frac{n}{1-n}\gamma C_t + (1-\gamma^*)Q_t^\eta C^*_t\right]. \end{aligned}$$

In the particular case of a *symmetric degree of trade openness* across countries ($\alpha = \alpha^*$), we can write aggregate demand in each country as:

$$\begin{aligned} Y_t &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\gamma)C_t + \gamma Q_t^\eta C^*_t\right] && \text{(A.6)} \\ Y^*_t &= \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} \left[\gamma^* C_t + (1-\gamma^*)Q_t^\eta C^*_t\right]. \end{aligned}$$

Assuming $C = C^*$ in steady state, log-linearization of the previous two equations yields

$$y_t = \gamma\eta s_t + (1-\gamma)c_t + \gamma c_t^* + \gamma\eta q_t \tag{A.7}$$
$$y_t^* = -(1-\gamma)\eta s_t + \gamma^* c_t + (1-\gamma^*)c_t^* + (1-\gamma^*)\eta q_t.$$

Inserting the log-linearized version of (15), which is given by

$$q_t = (1-\gamma-\gamma^*)s_t, \tag{A.8}$$

to eliminate q_t yields:

$$y_t = (1-\gamma)c_t + \gamma c_t^* + \gamma(2-\gamma-\gamma^*)\eta s_t \tag{A.9}$$
$$y_t^* = \gamma^* c_t + (1-\gamma^*)c_t^* - \gamma^*(2-\gamma-\gamma^*)\eta s_t. \tag{A.10}$$

B Appendix B. Efficient allocation

The first-best or efficient allocation describes the equilibrium in which prices are fully flexible and in which markups are neutralized at all times with an appropriate subsidy ($\mu_{j,t} = 0$). This efficient allocation provides a useful benchmark in order to assess the welfare implications of the two exchange rate regimes. Again, we assume a symmetric degree of trade openness, i.e., $\alpha = \alpha^*$.

Log-linearizing the risk sharing condition (16) yields[9]

$$\overline{q}_t = \sigma(\overline{c}_t - \overline{c}_t^*) + (z_{C,t}^* - z_{C,t}). \tag{B.1}$$

Combining (A.9), (B.1) and (A.8) yields

$$\sigma\overline{y}_t = \sigma\overline{c}_t + \gamma[(2-\gamma-\gamma^*)(\sigma\eta-1)+1]\overline{s}_t + \gamma(z_{C,t}^* - z_{C,t}). \tag{B.2}$$

Log-linearizing the optimal pricing equation (under flexible prices) in Home yields:

$$\zeta\overline{y}_t = -\gamma\overline{s}_t - \sigma\overline{c}_t + z_{C,t} + \zeta z_{Y,t}. \tag{B.3}$$

Combining the previous two equations to eliminate $\overline{c}_t$ yields:

$$(\sigma+\zeta)\overline{y}_t = \gamma(2-\gamma-\gamma^*)(\sigma\eta-1)\overline{s}_t - \gamma(z_{C,t} - z_{C,t}^*) + z_{C,t} + \zeta z_{Y,t}. \tag{B.4}$$

[9] The following equations contain consumption preference shocks $z_{C,t}$ and $z_{C,t}^*$ that we have abstracted from in the main body of the paper; see also the derivation of the welfare loss function.

The corresponding equation for country F is derived in a completely analogous way. Thus,

$$(\sigma+\zeta)\overline{y}_t^* = -\gamma^*(2-\gamma-\gamma^*)(\sigma\eta-1)\overline{s}_t + \gamma^*(z_{C,t}-z_{C,t}^*) + z_{C,t}^* + \zeta z_{Y,t}^*. \tag{B.5}$$

The efficient terms of trade are given by

$$\left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right]\overline{s}_t = \sigma\left(\overline{y}_t - \overline{y}_t^*\right) - (1-\gamma-\gamma^*)\left(z_{C,t}-z_{C,t}^*\right), \tag{B.6}$$

which is obtained by subtracting the country-specific aggregate demand equations (A.9) and (A.10) from each other and by using the risk sharing condition (B.1) and equation (A.8) to eliminate country-specific consumption and the real exchange rate.

C Appendix C. Welfare loss function

The derivation of the quadratic welfare loss function follows Corsetti, Dedola, and Leduc (2011). The period utility of agents living in country H is given by

$$\mathcal{W}_t = U(C_t, Z_{C,t}) - \frac{1}{n}\int_0^n V(Y_t(i), Z_{Y,t})di, \tag{C.1}$$

where

$$U(C_t, Z_{C,t}) = Z_{C,t}\frac{C_t^{1-\sigma}-1}{1-\sigma} \tag{C.2}$$

$$V(Y_t(i), Z_{Y,t}) = Z_{Y,t}^{-\zeta}\frac{Y_t(i)^{1+\zeta}}{1+\zeta}, \tag{C.3}$$

where $Z_{C,t}$ and $Z_{Y,t}$ are shocks to consumption preferences and to productivity, respectively. A second-order approximation of $U(C_t, Z_{C,t})$ yields

$$U(C_t, Z_{C,t}) = U_C C\left[c_t + \frac{1-\sigma}{2}c_t^2 + c_t z_{C,t}\right] + t.i.p. + O(\|Z\|^3), \tag{C.4}$$

where the term $t.i.p.$ collects all the terms that are independent of monetary policy as well as independent of the exchange rate regime and the term $O(\|Z\|^3)$ groups all the terms that are of third or higher order in the deviations of the various variables from their steady state.

A second-order approximation of $V(Y_t(i), Z_{Y,t})$ yields

$$V(Y_t(i), Z_{Y,t}) = V_Y Y\left[y_t(i) + \frac{1+\zeta}{2}y_t(i)^2 - \zeta y_t(i) z_{Y,t}\right] + t.i.p. + O(\|Z\|^3). \tag{C.5}$$

Under the assumption that $\alpha = \alpha^*$ and that in the steady state $C = C^*$, it follows that $S = Q = 1$ and $Y = C$ in the steady state. Under the additional assumption that the steady state is efficient, i.e., an appropriate subsidy τ^H eliminates the distortion due monopolistic competition, the efficiency of the flexible price allocation implies

$$V_Y Y = U_C C. \tag{C.6}$$

Integrating (C.5) over the differentiated goods i yields

$$\begin{aligned}\frac{1}{n}\int_0^n V(Y_t(i), Z_{Y,t})di =& U_C C\left[y_t + \frac{1+\zeta}{2}y_t^2 - \zeta y_t z_{Y,t} + \frac{1}{2}(\varepsilon^{-1}+\zeta)\operatorname{var}_i y_t(i)\right] \\ &+ t.i.p. + O(\|Z\|^3),\end{aligned} \tag{C.7}$$

where we used

$$\operatorname{var}_i y_t(i) = \mathbb{E}_i y_t(i)^2 - [\mathbb{E}_i y_t(i)]^2 \tag{C.8}$$

to eliminate $\mathbb{E}_i y_t(i)^2$ and the second-order approximation of y_t

$$y_t = \mathbb{E}_i y_t(i) + \frac{1}{2}\frac{\varepsilon-1}{\varepsilon}\operatorname{var}_i y_t(i) + O(\|Z\|^3) \tag{C.9}$$

to eliminate $\mathbb{E}_i y_t(i)$.

Inserting (C.4) and (C.7) into (C.1) yields

$$\begin{aligned}\mathcal{W}_t =& U_C C\left[c_t + \frac{1-\sigma}{2}c_t^2 + c_t z_{C,t} - y_t - \frac{1+\zeta}{2}y_t^2 + \zeta y_t z_{Y,t} - \frac{1}{2}(\varepsilon^{-1}+\zeta)\operatorname{var}_i y_t(i)\right] \\ &+ t.i.p. + O(\|Z\|^3).\end{aligned} \tag{C.10}$$

The log-linear expression of (A.1) implies that

$$\operatorname{var}_i y_t(i) = \varepsilon^2 \operatorname{var}_i p_{H,t}(i). \tag{C.11}$$

And completely analogous to Woodford (2003)

$$\sum_{k=0}^{\infty}\beta^k \operatorname{var}_i p_{H,t+k}(i) = \frac{\theta^H}{(1-\theta^H\beta)(1-\theta^H)}\sum_{k=0}^{\infty}\beta^k \pi_{H,t+k}^2. \tag{C.12}$$

Using the previous two relationships to eliminate $\operatorname{var}_i y_t(i)$ yields

$$\mathbb{E}_t\sum_{k=0}^{\infty}\beta^k\mathcal{W}_{t+k} = U_C C\mathbb{E}_t\sum_{k=0}^{\infty}\beta^k\mathcal{V}_{t+k} \tag{C.13}$$

where

$$\mathcal{V}_t = c_t - y_t + \left(\frac{1-\sigma}{2}c_t + z_{C,t}\right)c_t - \left(\frac{1+\zeta}{2}y_t - \zeta z_{Y,t}\right)y_t$$
$$- \frac{1}{2}\varepsilon(1+\varepsilon\zeta)\frac{\theta^H}{(1-\theta^H\beta)(1-\theta^H)}\pi^2_{H,t} + t.i.p. + O(\|Z\|^3). \tag{C.14}$$

The previous steps can be repeated completely analogously to obtain the corresponding expression for country F:

$$\mathbb{E}_t\sum_{k=0}^{\infty}\beta^k\mathcal{W}^*_{t+k} = U_C C\mathbb{E}_t\sum_{k=0}^{\infty}\beta^k\mathcal{V}^*_{t+k} \tag{C.15}$$

where

$$\mathcal{V}^*_t = c^*_t - y^*_t + \left(\frac{1-\sigma}{2}c^*_t + z^*_{C,t}\right)c^*_t - \left(\frac{1+\zeta}{2}y^*_t - \zeta z^*_{Y,t}\right)y^*_t$$
$$- \frac{1}{2}\varepsilon(1+\varepsilon\zeta)\frac{\theta^F}{(1-\theta^F\beta)(1-\theta^F)}\pi^{*2}_{F,t} + t.i.p. + O(\|Z\|^3). \tag{C.16}$$

The world welfare loss function is given by the weighted average of the country-specific welfare loss functions:

$$\begin{aligned}
\mathbb{V}_t =& n\mathcal{V}_t + (1-n)\mathcal{V}^*_t \\
=& nc_t + (1-n)c^*_t - ny_t - (1-n)y^*_t \\
&+ n\left(\frac{1-\sigma}{2}c_t + z_{C,t}\right)c_t + (1-n)\left(\frac{1-\sigma}{2}c^*_t + z^*_{C,t}\right)c^*_t \\
&- n\left(\frac{1+\zeta}{2}y_t - \zeta z_{Y,t}\right)y_t - (1-n)\left(\frac{1+\zeta}{2}y^*_t - \zeta z^*_{Y,t}\right)y^*_t \\
&- \frac{1}{2}n\varepsilon(1+\varepsilon\zeta)\frac{\theta^H}{(1-\theta^H\beta)(1-\theta^H)}\pi^2_{H,t} \\
&- \frac{1}{2}(1-n)\varepsilon(1+\varepsilon\zeta)\frac{\theta^F}{(1-\theta^F\beta)(1-\theta^F)}\pi^{*2}_{F,t} \\
&+ t.i.p. + O(\|Z\|^3).
\end{aligned} \tag{C.17}$$

C.1 Eliminating the linear terms

The world resource constraint expressed in the currency of country H is given by

$$nP_tC_t + (1-n)\mathcal{E}_tP^*_tC^*_t = nP_{H,t}Y_t + (1-n)\underbrace{\mathcal{E}_tP^*_{F,t}}_{=P_{F,t}}Y^*_t \tag{C.18}$$

Dividing both sides by P_t yields

$$nC_t + (1-n)\underbrace{\frac{\mathcal{E}_t P_t^*}{P_t}}_{=Q_t} C_t^* = n\frac{P_{H,t}}{P_t}Y_t + (1-n)\frac{P_{F,t}}{P_t}Y_t^* \tag{C.19}$$

Note that

$$\frac{P_{H,t}}{P_t} = \left[1-\gamma+\gamma S_t^{1-\eta}\right]^{-\frac{1}{1-\eta}} \tag{C.20}$$

and

$$\frac{P_{F,t}}{P_t} = \frac{P_{F,t}}{\mathcal{E}_t P_t^*}\frac{\mathcal{E}_t P_t^*}{P_t} = \left[\gamma^* S_t^{\eta-1} + 1 - \gamma^*\right]^{-\frac{1}{1-\eta}} Q_t. \tag{C.21}$$

Thus, the world resource constraint can be written as follows:

$$\begin{aligned} nC_t + (1-n)Q_t C_t^* =& n\left[1-\gamma+\gamma S_t^{1-\eta}\right]^{-\frac{1}{1-\eta}} Y_t \\ &+ (1-n)\left[\gamma^* S_t^{\eta-1} + 1 - \gamma^*\right]^{-\frac{1}{1-\eta}} Q_t Y_t^*. \end{aligned} \tag{C.22}$$

The second-order approximation of this expression yields

$$\begin{aligned} nc_t + (1-n)c_t^* - &ny_t - (1-n)y_t^* \\ &= -\frac{1}{2}\left(nc_t^2 + (1-n)c_t^{*^2}\right) + \frac{1}{2}\left(ny_t^2 + (1-n)y_t^{*^2}\right) \\ &\quad + (1-n)\left(y_t^* - c_t^*\right) q_t - n\gamma\left(y_t - y_t^*\right) s_t \\ &\quad + \frac{1}{2}n\gamma(2-\gamma-\gamma^*)\eta s_t^2 \\ &\quad + t.i.p. + O(\|Z\|^3), \end{aligned} \tag{C.23}$$

where we have used the fact that, under $\alpha = \alpha^*$:

$$n\gamma = (1-n)\gamma^*. \tag{C.24}$$

Note that this equality is frequently used in the subsequent manipulations as well. Inserting (C.23) into (C.17) to eliminate the linear terms yields

$$
\begin{aligned}
\mathbb{V}_t = & - n \left(\frac{\sigma}{2} c_t - z_{C,t} \right) c_t - (1-n) \left(\frac{\sigma}{2} c_t^* - z_{C,t}^* \right) c_t^* \\
& + (1-n) \left(y_t^* - c_t^* \right) q_t + \frac{1}{2} \eta n \gamma (2 - \gamma - \gamma^*) s_t^2 - n\gamma \left(y_t - y_t^* \right) s_t \\
& - n \left(\frac{\zeta}{2} y_t - \zeta z_{Y,t} \right) y_t - (1-n) \left(\frac{\zeta}{2} y_t^* - \zeta z_{Y,t}^* \right) y_t^* \\
& - \frac{1}{2} n \varepsilon (1 + \varepsilon\zeta) \frac{\theta^H}{(1-\theta^H \beta)(1-\theta^H)} \pi_{H,t}^2 \\
& - \frac{1}{2} (1-n) \varepsilon (1 + \varepsilon\zeta) \frac{\theta^F}{(1-\theta^F \beta)(1-\theta^F)} \pi_{F,t}^{*2} \\
& + t.i.p. + O(\|Z\|^3). \qquad \text{(C.25)}
\end{aligned}
$$

C.2 Further manipulations

Add and subtract $\frac{1}{2} n\gamma s_t y_t$ and $\frac{1}{2}(1-n)\gamma^* s_t y_t^*$ to obtain

$$
\begin{aligned}
\mathbb{V}_t = & - n \left(\frac{\sigma}{2} c_t - z_{C,t} \right) c_t - (1-n) \left(\frac{\sigma}{2} c_t^* - z_{C,t}^* \right) c_t^* \\
& + (1-n) \left(y_t^* - c_t^* \right) q_t + \frac{1}{2} \eta n \gamma (2 - \gamma - \gamma^*) s_t^2 - \frac{1}{2} n\gamma \left(y_t - y_t^* \right) s_t \\
& - n \left(\frac{\zeta}{2} y_t - \zeta z_{Y,t} + \frac{1}{2}\gamma s_t \right) y_t - (1-n) \left(\frac{\zeta}{2} y_t^* - \zeta z_{Y,t}^* - \frac{1}{2}\gamma^* s_t \right) y_t^* \\
& - \frac{1}{2} n \varepsilon (1 + \varepsilon\zeta) \frac{\theta^H}{(1-\theta^H \beta)(1-\theta^H)} \pi_{H,t}^2 \\
& - \frac{1}{2} (1-n) \varepsilon (1 + \varepsilon\zeta) \frac{\theta^F}{(1-\theta^F \beta)(1-\theta^F)} \pi_{F,t}^{*2} \\
& + t.i.p. + O(\|Z\|^3). \qquad \text{(C.26)}
\end{aligned}
$$

Add and subtract $n\left(\frac{\sigma}{2}c_t - z_{C,t}\right)y_t$ and $(1-n)\left(\frac{\sigma}{2}c_t^* - z_{C,t}^*\right)y_t^*$ to obtain

$$\begin{aligned}
\mathbb{V}_t = &- n\left(\frac{\sigma}{2}c_t - z_{C,t}\right)(c_t - y_t) - (1-n)\left(\frac{\sigma}{2}c_t^* - z_{C,t}^* + q_t\right)(c_t^* - y_t^*) \\
&+ \frac{1}{2}\eta n\gamma(2-\gamma-\gamma^*)s_t^2 - \frac{1}{2}n\gamma\left(y_t - y_t^*\right)s_t \\
&- n\left(\frac{\sigma}{2}c_t - z_{C,t} + \frac{\zeta}{2}y_t - \zeta z_{Y,t} + \frac{1}{2}\gamma s_t\right)y_t \\
&- (1-n)\left(\frac{\sigma}{2}c_t^* - z_{C,t}^* + \frac{\zeta}{2}y_t^* - \zeta z_{Y,t}^* - \frac{1}{2}\gamma^* s_t\right)y_t^* \\
&- \frac{1}{2}n\varepsilon(1+\varepsilon\zeta)\frac{\theta^H}{(1-\theta^H\beta)(1-\theta^H)}\pi_{H,t}^2 \\
&- \frac{1}{2}(1-n)\varepsilon(1+\varepsilon\zeta)\frac{\theta^F}{(1-\theta^F\beta)(1-\theta^F)}\pi_{F,t}^{*2} \\
&+ t.i.p. + O(\|Z\|^3).
\end{aligned} \tag{C.27}$$

The term in parentheses in front of y_t and y_t^* in the third and fourth line, respectively, can be related to the output gap and the terms of trade gap. Rearranging (B.2), which also holds under sticky prices, yields

$$\gamma\left(z_{C,t} - z_{C,t}^*\right) = \sigma\left(c_t - y_t\right) + \gamma[(2-\gamma-\gamma^*)(\sigma\eta - 1) + 1]s_t. \tag{C.28}$$

Inserting this into (B.4) yields

$$\sigma c_t - z_{C,t} + \zeta\left(y_t - z_{Y,t}\right) + \gamma s_t = (\sigma+\zeta)\left(y_t - \overline{y}_t\right) - \gamma(2-\gamma-\gamma^*)(\sigma\eta-1)\left(s_t - \overline{s}_t\right). \tag{C.29}$$

This equation can be rewritten as follows:

$$\begin{aligned}
&\frac{\sigma}{2}c_t - z_{C,t} + \frac{\zeta}{2}y_t - \zeta z_{Y,t} + \frac{1}{2}\gamma s_t \\
&= (\sigma+\zeta)\left(\frac{1}{2}y_t - \overline{y}_t\right) - \gamma(2-\gamma-\gamma^*)(\sigma\eta-1)\left(\frac{1}{2}s_t - \overline{s}_t\right) - \frac{1}{2}\gamma\left(z_{C,t} - z_{C,t}^*\right).
\end{aligned} \tag{C.30}$$

Repeating the same steps, the foreign analog to this equation is given by

$$\begin{aligned}
&\frac{\sigma}{2}c_t^* - z_{C,t}^* + \frac{\zeta}{2}y_t^* - \zeta z_{Y,t}^* - \frac{1}{2}\gamma^* s_t \\
&= (\sigma+\zeta)\left(\frac{1}{2}y_t^* - \overline{y}_t^*\right) + \gamma^*(2-\gamma-\gamma^*)(\sigma\eta-1)\left(\frac{1}{2}s_t - \overline{s}_t\right) + \frac{1}{2}\gamma^*\left(z_{C,t} - z_{C,t}^*\right).
\end{aligned} \tag{C.31}$$

The left-hand side of the previous two equations corresponds to the terms in parentheses in front of y_t and y_t^* in (C.27). Accordingly, substitution yields

$$\begin{aligned}\mathbb{V}_t = &- n\left(\frac{\sigma}{2}c_t - z_{C,t}\right)(c_t - y_t) - (1-n)\left(\frac{\sigma}{2}c_t^* - z_{C,t}^* + q_t\right)(c_t^* - y_t^*) \\ &- \frac{1}{2}n\gamma\left[s_t - \left(z_{C,t} - z_{C,t}^*\right)\right](y_t - y_t^*) + \frac{1}{2}\eta n\gamma(2-\gamma-\gamma^*)s_t^2 \\ &+ n\gamma(2-\gamma-\gamma^*)(\sigma\eta - 1)\left(\frac{1}{2}s_t - \overline{s}_t\right)(y_t - y_t^*) \\ &- n(\sigma+\zeta)\left(\frac{1}{2}y_t - \overline{y}_t\right)y_t - (1-n)(\sigma+\zeta)\left(\frac{1}{2}y_t^* - \overline{y}_t^*\right)y_t^* \\ &- \frac{1}{2}n\varepsilon(1+\varepsilon\zeta)\frac{\theta^H}{(1-\theta^H\beta)(1-\theta^H)}\pi_{H,t}^2 \\ &- \frac{1}{2}(1-n)\varepsilon(1+\varepsilon\zeta)\frac{\theta^F}{(1-\theta^F\beta)(1-\theta^F)}\pi_{F,t}^{*2} \\ &+ t.i.p. + O(\|Z\|^3). \end{aligned} \tag{C.32}$$

Combining the first-order approximation of the world resource constraint (C.22)

$$nc_t + (1-n)c_t^* = ny_t + (1-n)y_t^* \tag{C.33}$$

with the risk-sharing condition

$$q_t = \sigma\left(c_t - c_t^*\right) + \left(z_{C,t}^* - z_{C,t}\right) \tag{C.34}$$

to eliminate c_t^* yields

$$-\left(c_t - y_t\right) = (1-n)\left[y_t - y_t^* - \frac{1}{\sigma}\left(q_t + z_{C,t} - z_{C,t}^*\right)\right]. \tag{C.35}$$

The foreign analog is given by

$$\left(c_t^* - y_t^*\right) = n\left[y_t - y_t^* - \frac{1}{\sigma}\left(q_t + z_{C,t} - z_{C,t}^*\right)\right]. \tag{C.36}$$

Inserting the previous two equations into (C.32) yields

$$
\begin{aligned}
\mathbb{V}_t =& n(1-n)\left[\frac{\sigma}{2}\left(c_t - c_t^*\right) - q_t - \left(z_{C,t} - z_{C,t}^*\right)\right]\left[y_t - y_t^* - \frac{1}{\sigma}\left(q_t + z_{C,t} - z_{C,t}^*\right)\right] \\
& - \frac{1}{2}n\gamma\left[s_t - \left(z_{C,t} - z_{C,t}^*\right)\right]\left(y_t - y_t^*\right) + \frac{1}{2}\eta n\gamma(2-\gamma-\gamma^*)s_t^2 \\
& + n\gamma(2-\gamma-\gamma^*)(\sigma\eta - 1)\left(\frac{1}{2}s_t - \overline{s}_t\right)\left(y_t - y_t^*\right) \\
& - n(\sigma+\zeta)\left(\frac{1}{2}y_t - \overline{y}_t\right)y_t - (1-n)(\sigma+\zeta)\left(\frac{1}{2}y_t^* - \overline{y}_t^*\right)y_t^* \\
& - \frac{1}{2}n\varepsilon(1+\varepsilon\zeta)\frac{\theta^H}{(1-\theta^H\beta)(1-\theta^H)}\pi_{H,t}^2 \\
& - \frac{1}{2}(1-n)\varepsilon(1+\varepsilon\zeta)\frac{\theta^F}{(1-\theta^F\beta)(1-\theta^F)}\pi_{F,t}^{*2} \\
& + t.i.p. + O(\|Z\|^3). \qquad \text{(C.37)}
\end{aligned}
$$

Using the risk-sharing condition to eliminate the consumption differential $(c_t - c_t^*)$ and rearranging the resulting first line yields

$$
\begin{aligned}
\mathbb{V}_t =& -\frac{1}{2}n(1-n)\left(q_t + z_{C,t} - z_{C,t}^*\right)\left(y_t - y_t^*\right) + \frac{1}{2}n(1-n)\frac{1}{\sigma}\left(q_t + z_{C,t} - z_{C,t}^*\right)^2 \\
& - \frac{1}{2}n\gamma\left[s_t - \left(z_{C,t} - z_{C,t}^*\right)\right]\left(y_t - y_t^*\right) + \frac{1}{2}\eta n\gamma(2-\gamma-\gamma^*)s_t^2 \\
& + n\gamma(2-\gamma-\gamma^*)(\sigma\eta - 1)\left(\frac{1}{2}s_t - \overline{s}_t\right)\left(y_t - y_t^*\right) \\
& - n(\sigma+\zeta)\left(\frac{1}{2}y_t - \overline{y}_t\right)y_t - (1-n)(\sigma+\zeta)\left(\frac{1}{2}y_t^* - \overline{y}_t^*\right)y_t^* \\
& - \frac{1}{2}n\varepsilon(1+\varepsilon\zeta)\frac{\theta^H}{(1-\theta^H\beta)(1-\theta^H)}\pi_{H,t}^2 \\
& - \frac{1}{2}(1-n)\varepsilon(1+\varepsilon\zeta)\frac{\theta^F}{(1-\theta^F\beta)(1-\theta^F)}\pi_{F,t}^{*2} \\
& + t.i.p. + O(\|Z\|^3). \qquad \text{(C.38)}
\end{aligned}
$$

Collecting the terms in the output differential $(y_t - y_t^*)$ in the first two lines yields

$$\begin{aligned}\mathbb{V}_t =& \frac{1}{2}n(1-n)\frac{1}{\sigma}\left(q_t + z_{C,t} - z_{C,t}^*\right)^2 + \frac{1}{2}\eta n\gamma(2-\gamma-\gamma^*)s_t^2 \\ &- \frac{1}{2}n(1-n)\left[\left(1-\frac{\gamma}{1-n}\right)\left(z_{C,t} - z_{C,t}^*\right) + s_t\right](y_t - y_t^*) \\ &+ n\gamma(2-\gamma-\gamma^*)(\sigma\eta - 1)\left(\frac{1}{2}s_t - \overline{s}_t\right)(y_t - y_t^*) \\ &- n(\sigma+\zeta)\left(\frac{1}{2}y_t - \overline{y}_t\right)y_t - (1-n)(\sigma+\zeta)\left(\frac{1}{2}y_t^* - \overline{y}_t^*\right)y_t^* \\ &- \frac{1}{2}n\varepsilon(1+\varepsilon\zeta)\frac{\theta^H}{(1-\theta^H\beta)(1-\theta^H)}\pi_{H,t}^2 \\ &- \frac{1}{2}(1-n)\varepsilon(1+\varepsilon\zeta)\frac{\theta^F}{(1-\theta^F\beta)(1-\theta^F)}\pi_{F,t}^{*2} \\ &+ t.i.p. + O(\|Z\|^3). \end{aligned} \tag{C.39}$$

Using (B.6), which also holds under sticky prices, to eliminate the output differential $(y_t - y_t^*)$ yields

$$\begin{aligned}\mathbb{V}_t =& \frac{1}{2}n(1-n)\frac{1}{\sigma}\left(q_t + z_{C,t} - z_{C,t}^*\right)^2 - \frac{1}{2}n(1-n)\frac{1-\gamma-\gamma^*}{\sigma}\left(z_{C,t} - z_{C,t}^*\right)s_t \\ &- \frac{1}{2}n(1-n)\left(1-\frac{\gamma}{1-n}\right)\frac{(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2}{\sigma}\left(z_{C,t} - z_{C,t}^*\right)s_t \\ &+ n\gamma(2-\gamma-\gamma^*)(\sigma\eta-1)\frac{1-\gamma-\gamma^*}{\sigma}\left(z_{C,t} - z_{C,t}^*\right)\left(\frac{1}{2}s_t - \overline{s}_t\right) \\ &+ \frac{1}{2}n\left[\eta\gamma(2-\gamma-\gamma^*) - (1-n)\frac{(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2}{\sigma}\right]s_t^2 \\ &+ n\gamma(2-\gamma-\gamma^*)\frac{\sigma\eta-1}{\sigma}[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2]\left(\frac{1}{2}s_t - \overline{s}_t\right)s_t \\ &- n(\sigma+\zeta)\left(\frac{1}{2}y_t - \overline{y}_t\right)y_t - (1-n)(\sigma+\zeta)\left(\frac{1}{2}y_t^* - \overline{y}_t^*\right)y_t^* \\ &- \frac{1}{2}n\varepsilon(1+\varepsilon\zeta)\frac{\theta^H}{(1-\theta^H\beta)(1-\theta^H)}\pi_{H,t}^2 \\ &- \frac{1}{2}(1-n)\varepsilon(1+\varepsilon\zeta)\frac{\theta^F}{(1-\theta^F\beta)(1-\theta^F)}\pi_{F,t}^{*2} \\ &+ t.i.p. + O(\|Z\|^3). \end{aligned} \tag{C.40}$$

Using $q_t = (1-\gamma-\gamma^*)s_t$ and collecting the terms in s_t^2 and $\left(z_{C,t} - z_{C,t}^*\right)s_t$, it is straightforward to show that the first four lines of the above expression are equal to $0 + t.i.p.$. As a result, the above expression simplifies to

$$\begin{aligned}\mathbb{V}_t =& - n(\sigma+\zeta)\left(\frac{1}{2}y_t - \overline{y}_t\right)y_t - (1-n)(\sigma+\zeta)\left(\frac{1}{2}y_t^* - \overline{y}_t^*\right)y_t^* \\ &+ n\gamma(2-\gamma-\gamma^*)\frac{\sigma\eta-1}{\sigma}[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2]\left(\frac{1}{2}s_t - \overline{s}_t\right)s_t \\ &- \frac{1}{2}n\varepsilon(1+\varepsilon\zeta)\frac{\theta^H}{(1-\theta^H\beta)(1-\theta^H)}\pi_{H,t}^2 \\ &- \frac{1}{2}(1-n)\varepsilon(1+\varepsilon\zeta)\frac{\theta^F}{(1-\theta^F\beta)(1-\theta^F)}\pi_{F,t}^{*2} \\ &+ t.i.p. + O(\|Z\|^3). \end{aligned} \tag{C.41}$$

Note that for any given variable x_t

$$\begin{aligned}\left(\frac{1}{2}x_t - \overline{x}_t\right)x_t &= \frac{1}{2}\left(x_t^2 - 2x_t\overline{x}_t\right) \\ &= \frac{1}{2}\left(x_t^2 - 2x_t\overline{x}_t + \overline{x}_t^2\right) - \frac{1}{2}\overline{x}_t^2 \\ &= \frac{1}{2}\left(x_t - \overline{x}_t\right)^2 + t.i.p. \end{aligned} \tag{C.42}$$

Thus, the world welfare loss function can be rewritten in its final form:

$$\begin{aligned}\mathbb{V}_t =& -\frac{1}{2}\Bigg(n(\sigma+\zeta)\left(y_t - \overline{y}_t\right)^2 + (1-n)(\sigma+\zeta)\left(y_t^* - \overline{y}_t^*\right)^2 \\ &- n\gamma(2-\gamma-\gamma^*)\frac{\sigma\eta-1}{\sigma}\left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right]\left(s_t - \overline{s}_t\right)^2 \\ &+ n\varepsilon(1+\varepsilon\zeta)\frac{\theta^H}{(1-\theta^H\beta)(1-\theta^H)}\pi_{H,t}^2 \\ &+ (1-n)\varepsilon(1+\varepsilon\zeta)\frac{\theta^F}{(1-\theta^F\beta)(1-\theta^F)}\pi_{F,t}^{*2}\Bigg) \\ &+ t.i.p. + O(\|Z\|^3). \end{aligned} \tag{C.43}$$

D Appendix D. Derivation of targeting rules

D.1 Flexible exchange rate regime

Under discretion, the Lagrangian is given by

$$\begin{aligned}
\mathcal{L} = &-\frac{1}{2}\Bigg(n(\sigma+\zeta)\left(y_t-\overline{y}_t\right)^2 + (1-n)(\sigma+\zeta)\left(y_t^*-\overline{y}_t^*\right)^2 \\
&- n\gamma(2-\gamma-\gamma^*)\frac{\sigma\eta-1}{\sigma}\left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta+(1-\gamma-\gamma^*)^2\right]\left(s_t-\overline{s}_t\right)^2 \\
&+ n\frac{\varepsilon}{\kappa}\pi_{H,t}^2 + (1-n)\frac{\varepsilon}{\kappa^*}\pi_{F,t}^{*2}\Bigg) \\
&+ n\varphi_{H,t}\Big(\pi_{H,t}-(\sigma+\zeta)\kappa\left(y_t-\overline{y}_t\right)+\gamma(2-\gamma-\gamma^*)(\sigma\eta-1)\kappa\left(s_t-\overline{s}_t\right)-f_t\Big) \\
&+ (1-n)\varphi_{F,t}^*\Big(\pi_{F,t}^*-(\sigma+\zeta)\kappa^*\left(y_t^*-\overline{y}_t^*\right)-\gamma^*(2-\gamma-\gamma^*)(\sigma\eta-1)\kappa^*\left(s_t-\overline{s}_t\right)-f_t^*\Big) \\
&+ \vartheta_t\Big(\left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta+(1-\gamma-\gamma^*)^2\right]\left(s_t-\overline{s}_t\right)-\sigma\left[\left(y_t-\overline{y}_t\right)-\left(y_t^*-\overline{y}_t^*\right)\right]\Big),
\end{aligned} \tag{D.1}$$

where $\varphi_{H,t}$, $\varphi_{F,t}^*$, and ϑ_t are the respective Lagrange multipliers and

$$\kappa = \frac{(1-\theta^H\beta)(1-\theta^H)}{\theta^H(1+\varepsilon\zeta)} \tag{D.2}$$

$$\kappa^* = \frac{(1-\theta^F\beta)(1-\theta^F)}{\theta^F(1+\varepsilon\zeta)} \tag{D.3}$$

$$f_t = \kappa\hat{\mu}_t^H + \beta\mathbb{E}_t\pi_{H,t+1} \tag{D.4}$$

$$f_t^* = \kappa^*\hat{\mu}_t^F + \beta\mathbb{E}_t\pi_{F,t+1}^*. \tag{D.5}$$

The first-order conditions with respect to $\pi_{H,t}$, $\pi^*_{F,t}$, y_t, y^*_t, and s_t are given by

$$
\begin{aligned}
0 =& -\frac{\varepsilon}{\kappa}\pi_{H,t} + \varphi_{H,t} && \text{(D.6)}\\
0 =& -\frac{\varepsilon}{\kappa^*}\pi^*_{F,t} + \varphi^*_{F,t} && \text{(D.7)}\\
0 =& -n(\sigma+\zeta)\left(y_t - \overline{y}_t\right) - n(\sigma+\zeta)\kappa\varphi_{H,t} - \sigma\vartheta_t && \text{(D.8)}\\
0 =& -(1-n)(\sigma+\zeta)\left(y^*_t - \overline{y}^*_t\right) - (1-n)(\sigma+\zeta)\kappa^*\varphi^*_{F,t} + \sigma\vartheta_t && \text{(D.9)}\\
0 =& n\gamma(2-\gamma-\gamma^*)\frac{\sigma\eta-1}{\sigma}\left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right]\left(s_t - \overline{s}_t\right)\\
&+ n\gamma(2-\gamma-\gamma^*)(\sigma\eta-1)(\kappa\varphi_{H,t} - \kappa^*\varphi^*_{F,t})\\
&+ \left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right]\vartheta_t. && \text{(D.10)}
\end{aligned}
$$

Combining the first-order conditions to eliminate the Lagrange multipliers yields the following two targeting rules:

$$
\begin{aligned}
0 &= \left(y_t - \overline{y}_t\right) + \varepsilon\pi_{H,t} && \text{(D.11)}\\
0 &= \left(y^*_t - \overline{y}^*_t\right) + \varepsilon\pi^*_{F,t}. && \text{(D.12)}
\end{aligned}
$$

Under commitment, the Lagrangian is given by

$$
\begin{aligned}
\mathcal{L} =& U_C C\mathbb{E}_t\sum_{t=0}^{\infty}\beta^t\Bigg[-\frac{1}{2}\Bigg(n(\sigma+\zeta)\left(y_t-\overline{y}_t\right)^2 + (1-n)(\sigma+\zeta)\left(y^*_t - \overline{y}^*_t\right)^2\\
&- n\gamma(2-\gamma-\gamma^*)\frac{\sigma\eta-1}{\sigma}\left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right]\left(s_t-\overline{s}_t\right)^2\\
&+ n\frac{\varepsilon}{\kappa}\pi^2_{H,t} + (1-n)\frac{\varepsilon}{\kappa^*}\pi^{*^2}_{F,t}\Bigg)\\
&+ n\varphi_{H,t}\Bigg(\pi_{H,t} - (\sigma+\zeta)\kappa\left(y_t-\overline{y}_t\right) + \gamma(2-\gamma-\gamma^*)(\sigma\eta-1)\kappa\left(s_t-\overline{s}_t\right)\\
&- \kappa\hat{\mu}^H_t - \beta\mathbb{E}_t\pi_{H,t+1}\Bigg)\\
&+ (1-n)\varphi^*_{F,t}\Bigg(\pi^*_{F,t} - (\sigma+\zeta)\kappa^*\left(y^*_t-\overline{y}^*_t\right) - \gamma^*(2-\gamma-\gamma^*)(\sigma\eta-1)\kappa^*\left(s_t-\overline{s}_t\right)\\
&- \kappa^*\hat{\mu}^F_t - \beta\mathbb{E}_t\pi^*_{F,t+1}\Bigg)\\
&+ \vartheta_t\Bigg(\left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right]\left(s_t-\overline{s}_t\right) - \sigma\left[\left(y_t-\overline{y}_t\right) - \left(y^*_t-\overline{y}^*_t\right)\right]\Bigg)\Bigg],
\end{aligned}
\qquad \text{(D.13)}
$$

The first-order conditions with respect to $\pi_{H,t}$, $\pi^*_{F,t}$, y_t, y^*_t, and s_t are given by

$$0 = -\frac{\varepsilon}{\kappa}\pi_{H,t} + \varphi_{H,t} - \varphi_{H,t-1} \tag{D.14}$$

$$0 = -\frac{\varepsilon}{\kappa^*}\pi^*_{F,t} + \varphi^*_{F,t} - \varphi^*_{F,t-1} \tag{D.15}$$

$$0 = -n(\sigma+\zeta)\left(y_t - \overline{y}_t\right) - n(\sigma+\zeta)\kappa\varphi_{H,t} - \sigma\vartheta_t \tag{D.16}$$

$$0 = -(1-n)(\sigma+\zeta)\left(y^*_t - \overline{y}^*_t\right) - (1-n)(\sigma+\zeta)\kappa^*\varphi^*_{F,t} + \sigma\vartheta_t \tag{D.17}$$

$$\begin{aligned} 0 =& n\gamma(2-\gamma-\gamma^*)\frac{\sigma\eta-1}{\sigma}\left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right]\left(s_t - \overline{s}_t\right) \\ &+ n\gamma(2-\gamma-\gamma^*)(\sigma\eta-1)(\kappa\varphi_{H,t} - \kappa^*\varphi^*_{F,t}) \\ &+ \left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right]\vartheta_t. \end{aligned} \tag{D.18}$$

Combining the first-order conditions to eliminate the Lagrange multipliers yields the following two targeting rules:

$$0 = (y_t - \overline{y}_t) - \left(y_{t-1} - \overline{y}_{t-1}\right) + \varepsilon\pi_{H,t} \tag{D.19}$$

$$0 = (y^*_t - \overline{y}^*_t) - \left(y^*_{t-1} - \overline{y}^*_{t-1}\right) + \varepsilon\pi^*_{F,t}. \tag{D.20}$$

D.2 Monetary union regime

Under *discretion*, the Lagrangian is given by

$$\begin{aligned} \mathcal{L} =& -\frac{1}{2}\Bigg(n(\sigma+\zeta)\left(y_t - \overline{y}_t\right)^2 + (1-n)(\sigma+\zeta)\left(y^*_t - \overline{y}^*_t\right)^2 \\ &- n\gamma(2-\gamma-\gamma^*)\frac{\sigma\eta-1}{\sigma}\left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right]\left(s_t - \overline{s}_t\right)^2 \\ &+ n\frac{\varepsilon}{\kappa}\pi^2_{H,t} + (1-n)\frac{\varepsilon}{\kappa^*}\pi^{*2}_{F,t}\Bigg) \\ &+ n\varphi_{H,t}\Big(\pi_{H,t} - (\sigma+\zeta)\kappa\left(y_t - \overline{y}_t\right) + \gamma(2-\gamma-\gamma^*)(\sigma\eta-1)\kappa\left(s_t - \overline{s}_t\right) - f_t\Big) \\ &+ (1-n)\varphi^*_{F,t}\Big(\pi^*_{F,t} - (\sigma+\zeta)\kappa^*\left(y^*_t - \overline{y}^*_t\right) - \gamma^*(2-\gamma-\gamma^*)(\sigma\eta-1)\kappa^*\left(s_t - \overline{s}_t\right) - f^*_t\Big) \\ &+ \vartheta_t\Big(\left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right]\left(s_t - \overline{s}_t\right) - \sigma\left[(y_t - \overline{y}_t) - (y^*_t - \overline{y}^*_t)\right]\Big) \\ &+ \psi_t\Big(s_t - s_{t-1} - \pi^*_{F,t} + \pi_{H,t}\Big), \end{aligned} \tag{D.21}$$

where $\varphi_{H,t}$, $\varphi^*_{F,t}$, ϑ_t, ψ_t are the respective Lagrange multipliers. The first-order conditions with respect to $\pi_{H,t}$, $\pi^*_{F,t}$, y_t, y^*_t, and s_t are given by

$$0 = -n\frac{\varepsilon}{\kappa}\pi_{H,t} + n\varphi_{H,t} + \psi_t \tag{D.22}$$

$$0 = -(1-n)\frac{\varepsilon}{\kappa^*}\pi^*_{F,t} + (1-n)\varphi^*_{F,t} - \psi_t \tag{D.23}$$

$$0 = -n(\sigma+\zeta)\left(y_t - \overline{y}_t\right) - n(\sigma+\zeta)\kappa\varphi_{H,t} - \sigma\vartheta_t \tag{D.24}$$

$$0 = -(1-n)(\sigma+\zeta)\left(y^*_t - \overline{y}^*_t\right) - (1-n)(\sigma+\zeta)\kappa^*\varphi^*_{F,t} + \sigma\vartheta_t \tag{D.25}$$

$$\begin{aligned} 0 =& n\gamma(2-\gamma-\gamma^*)\frac{\sigma\eta-1}{\sigma}\left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right](s_t - \overline{s}_t) \\ &+ n\gamma(2-\gamma-\gamma^*)(\sigma\eta-1)(\kappa\varphi_{H,t} - \kappa^*\varphi^*_{F,t}) \\ &+ \left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right]\vartheta_t \\ &+ \psi_t - \psi_{t+1}. \end{aligned} \tag{D.26}$$

In the special case in which the degree of price stickiness is equal across countries ($\kappa = \kappa^*$), combining the first-order conditions to eliminate the Lagrange multipliers yields the following targeting rule:

$$0 = n\left(y_t - \overline{y}_t\right) + (1-n)\left(y^*_t - \overline{y}^*_t\right) + \varepsilon\Big(n\pi_{H,t} + (1-n)\pi^*_{F,t}\Big). \tag{D.27}$$

Under *commitment*, the Lagrangian is given by

$$
\begin{aligned}
\mathcal{L} =& U_C C \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Bigg[-\frac{1}{2} \Bigg(n(\sigma+\zeta)\left(y_t - \overline{y}_t\right)^2 + (1-n)(\sigma+\zeta)\left(y_t^* - \overline{y}_t^*\right)^2 \\
& - n\gamma(2-\gamma-\gamma^*)\frac{\sigma\eta - 1}{\sigma}\left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right]\left(s_t - \overline{s}_t\right)^2 \\
& + n\frac{\varepsilon}{\kappa}\pi_{H,t}^2 + (1-n)\frac{\varepsilon}{\kappa^*}\pi_{F,t}^{*2} \Bigg) \\
& + n\varphi_{H,t}\Bigg(\pi_{H,t} - (\sigma+\zeta)\kappa\left(y_t - \overline{y}_t\right) + \gamma(2-\gamma-\gamma^*)(\sigma\eta-1)\kappa\left(s_t - \overline{s}_t\right) \\
& - \kappa\hat{\mu}_t^H - \beta\mathbb{E}_t\pi_{H,t+1} \Bigg) \\
& + (1-n)\varphi_{F,t}^*\Bigg(\pi_{F,t}^* - (\sigma+\zeta)\kappa^*\left(y_t^* - \overline{y}_t^*\right) - \gamma^*(2-\gamma-\gamma^*)(\sigma\eta-1)\kappa^*\left(s_t - \overline{s}_t\right) \\
& - \kappa^*\hat{\mu}_t^F - \beta\mathbb{E}_t\pi_{F,t+1}^* \Bigg) \\
& + \vartheta_t\Bigg(\left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right]\left(s_t - \overline{s}_t\right) - \sigma\left[\left(y_t - \overline{y}_t\right) - \left(y_t^* - \overline{y}_t^*\right)\right] \Bigg) \\
& + \psi_t\Bigg(s_t - s_{t-1} - \pi_{F,t}^* + \pi_{H,t} \Bigg) \Bigg],
\end{aligned}
\tag{D.28}
$$

The first-order conditions with respect to $\pi_{H,t}$, $\pi_{F,t}^*$, y_t, y_t^*, and s_t are given by

$$0 = -n\frac{\varepsilon}{\kappa}\pi_{H,t} + n\varphi_{H,t} - n\varphi_{H,t-1} + \psi_t \tag{D.29}$$

$$0 = -(1-n)\frac{\varepsilon}{\kappa^*}\pi_{F,t}^* + (1-n)\varphi_{F,t}^* - (1-n)\varphi_{F,t-1}^* - \psi_t \tag{D.30}$$

$$0 = -n(\sigma+\zeta)\left(y_t - \overline{y}_t\right) - n(\sigma+\zeta)\kappa\varphi_{H,t} - \sigma\vartheta_t \tag{D.31}$$

$$0 = -(1-n)(\sigma+\zeta)\left(y_t^* - \overline{y}_t^*\right) - (1-n)(\sigma+\zeta)\kappa^*\varphi_{F,t}^* + \sigma\vartheta_t \tag{D.32}$$

$$
\begin{aligned}
0 =& n\gamma(2-\gamma-\gamma^*)\frac{\sigma\eta-1}{\sigma}\left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right]\left(s_t - \overline{s}_t\right) \\
& + n\gamma(2-\gamma-\gamma^*)(\sigma\eta-1)(\kappa\varphi_{H,t} - \kappa^*\varphi_{F,t}^*) \\
& + \left[(\gamma+\gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2\right]\vartheta_t \\
& + \psi_t - \beta\psi_{t+1}.
\end{aligned}
\tag{D.33}
$$

In the special case in which the degree of price stickiness is equal across countries ($\kappa = \kappa^*$), combining the first-order conditions to eliminate the Lagrange multipliers yields the following targeting rule:

$$
\begin{aligned}
0 = & n\left[(y_t - \overline{y}_t) - \big(y_{t-1} - \overline{y}_{t-1}\big)\right] + (1-n)\left[(y_t^* - \overline{y}_t^*) - \big(y_{t-1}^* - \overline{y}_{t-1}^*\big)\right] \\
& + \varepsilon\Big(n\pi_{H,t} + (1-n)\pi_{F,t}^*\Big).
\end{aligned}
\tag{D.34}
$$