Corporate Debt Maturity and Investment over the Business Cycle

Johannes Poeschl*
University of Mannheim.

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Abstract

I document that the share of long-term debt of US corporate non-financial firms is pro-cyclical. Furthermore, the long-term debt share of small firms has a higher standard deviation and correlation with output than the long-term debt share of large firms. I construct a quantitative model in which firms optimally choose investment, leverage, the maturity structure of debt, dividends, and default. Firms face idiosyncratic and aggregate productivity risk. When they choose their maturity structure, firms trade off default incentives and roll-over costs. As a result, financially constrained firms endogenously prefer to issue short-term debt, because they face high default premia on long-term debt. Financially unconstrained firms issue long-term debt, because it has lower roll-over costs. The model, which is calibrated to match cross-sectional moments, can explain about one third of the variation of the aggregate maturity structure, and about sixty percent of the variation of the maturity structure of small firms. Restricting firms to issue only short-term debt or no debt at all can lead to higher average equity values and less default, but does not increase the average firm value.

1 Introduction

The maturity structure of debt of nonfinancial US firms varies widely at the aggregate level: In Figure 1, I plot the aggregate share of long-term debt in total debt for the US economy,

*Center for Doctoral Studies in Economics, University of Mannheim. Email: johannes.poeschl@gess.uni-mannheim.de. Web: https://sites.google.com/site/jpoesc/. I would like to thank my advisor, Klaus Adam, as well as Michèle Tertilt and Johannes Pfeifer for helpful comments. I would further like to thank seminar participants at the Macro seminar, the Corporate Finance seminar and the CDSE seminar at the University of Mannheim as well as at the Workshop in Quantitative Dynamic Economics at the University of Marseille for useful suggestions. I acknowledge support by the state of Baden-Württemberg through bwHPC.

computed using quarterly firm-level data from Compustat. Long-term debt is defined as all debt with a residual maturity of at least one year. It is evident that the maturity of debt decreases during recessions and increases during expansions. For example, the long-term debt share during the financial crisis decreased from 88.75 percent in the first quarter of 2007 to 84.97 percent in the last quarter of 2008. This means that the fraction of due payments on outstanding debt increases exactly when internal funds are most valuable for firms, a seemingly puzzling observation.

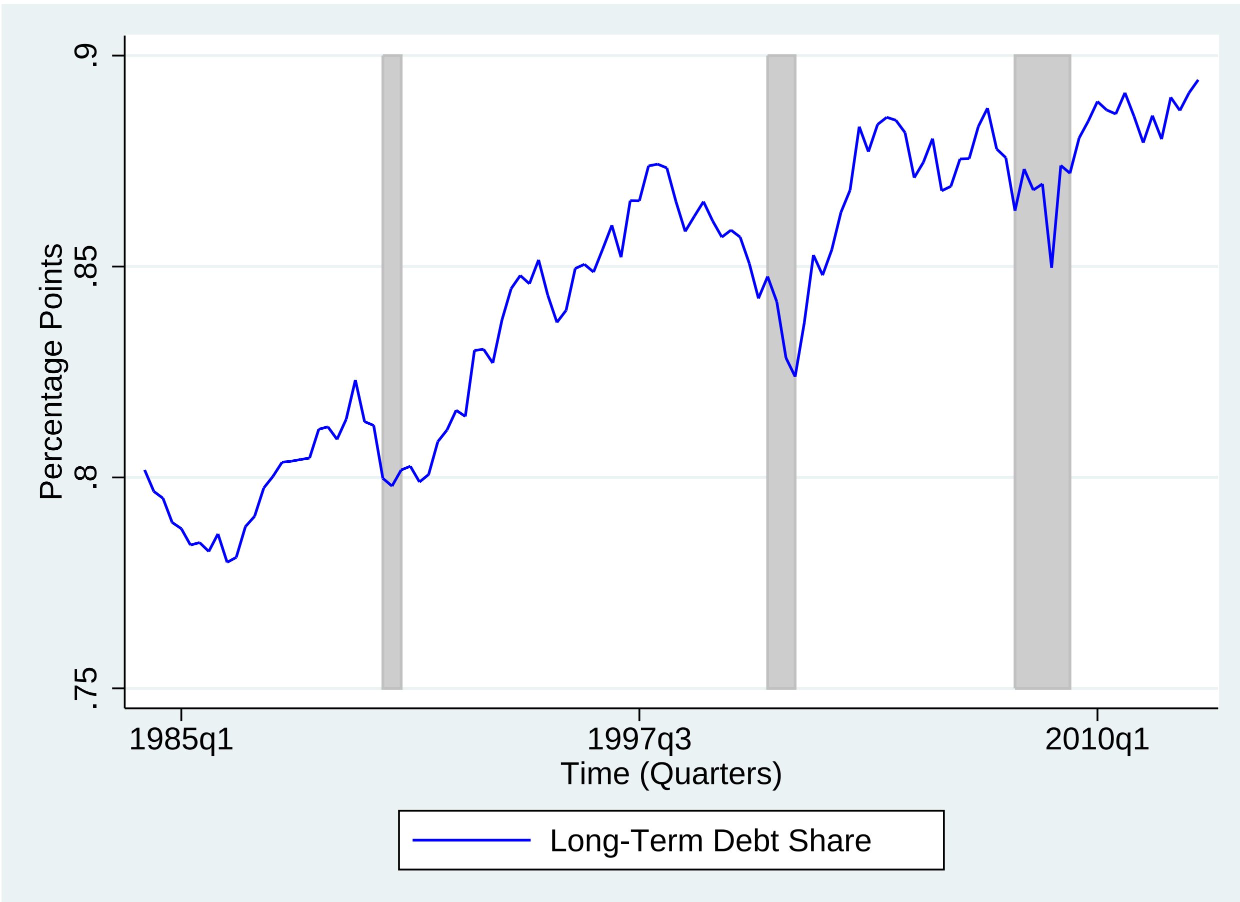

Figure 1: This Figure shows the aggregate share of long-term debt for non-financial firms in the Compustat Quarterly database. Long-term debt is defined as debt with a residual maturity of more than one year. The trend is computed with a Hodrick-Prescott-filter with $\lambda = 1600$.

The maturity structure also varies widely at the firm level: In Figure 2, I plot the time series for the long-term debt share of the smallest 75 percent and the largest 25 percent of firms separately. Figure 2 shows two additional facts: First, it is evident that small firms, denoted by the blue line, have a lower share of long-term debt than large firms. Second, the long-term debt share of small firms is more cyclical than the long-term debt share of large firms, in the sense that the long-term debt share of small firms has a higher standard deviation and the cyclical component of the long-term debt share has a higher correlation with output.[1]

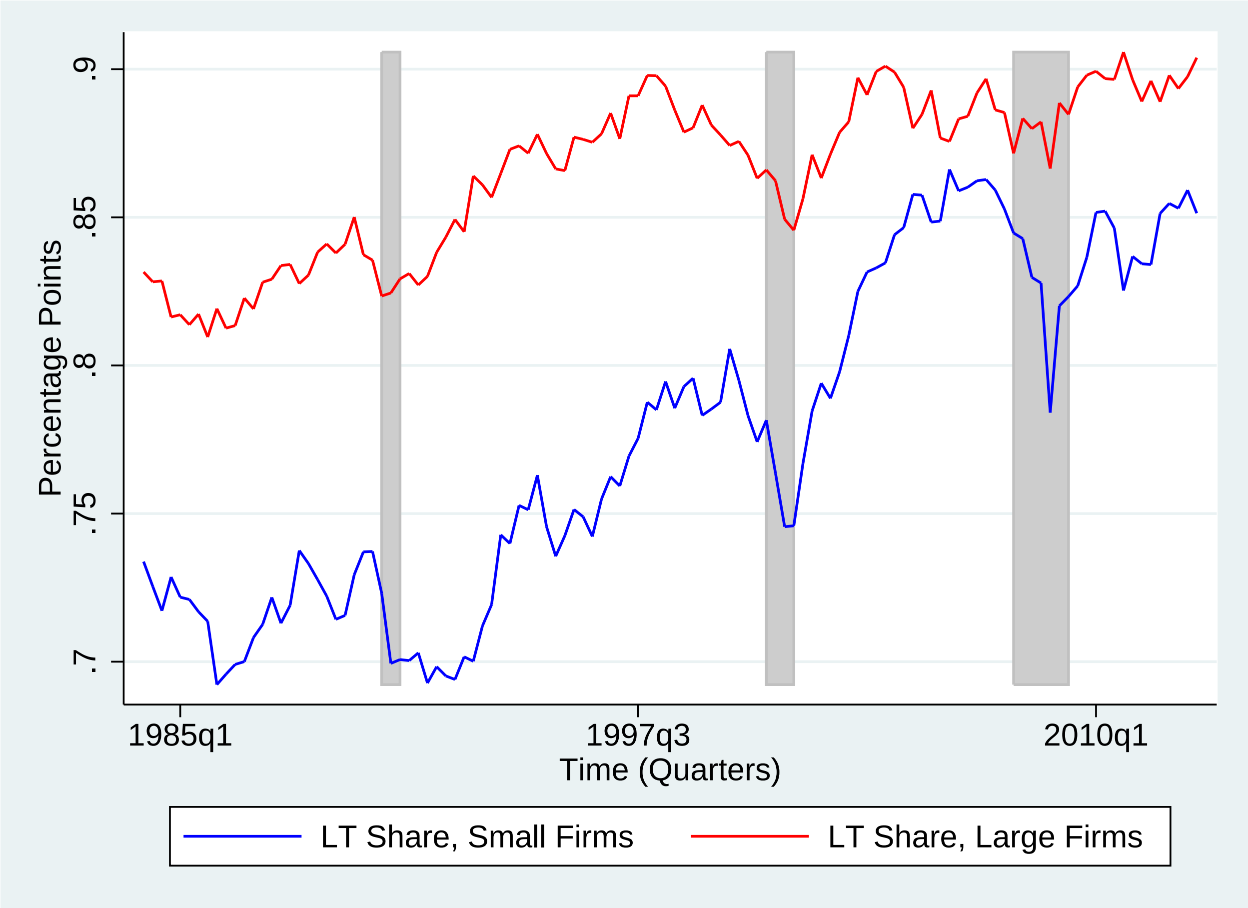

Figure 2: Figure 1 shows the aggregate share of long-term debt for non-financial firms in the Compustat Quarterly database. The upper, red line is the long-term debt share for the largest 25 percent of firms in the Compustat sample, the lower, blue line the long-term debt share for the smallest 75 percent of firms.

Despite these cyclical patterns, the macroeconomic literature has so far not considered endogenous debt maturity choices of firms. Typically, macroeconomic models with financial frictions treat all debt as short-term. Yet a large literature in corporate finance starting with Myers (1977) shows that the maturity structure of outstanding debt, holding the level

[1]Specifically, the standard deviation of the linearly detrended cyclical component of the long-term debt share for small firms is 2.29 percent, for large firms, it is 1.38 percent. The contemporaneous correlation of the long-term debt share of small firms with GDP is 0.42, while the correlation of the long-term debt share for large firms with GDP is 0.0009.

of debt constant, affects investment and financing decisions of firms.[2] The reason for this is that debt overhang, the effect that leveraged firms invest less than otherwise identical unlevered firms, is more severe for long-term debt. One recent paper Gomes, Jermann, and Schmid (2014) has studied the role of long-term debt for business cycles.

In this paper, I go one step further and investigate which factors determine the maturity structure of firms endogenously over the business cycle, at the firm level and in the aggregate. To my knowledge, my paper is the first one which considers dynamic debt maturity choice in a framework with endogenous dynamic investment and leverage choices and aggregate uncertainty. I construct a quantitative dynamic model that allows for a rich capital structure of firms: Firms can issue short-term debt, long-term debt and equity. They accumulate productive capital, which is illiquid due to investment adjustment costs. Due to limited liability, firms can default on outstanding debt.

The main trade-off between short-term debt and long-term is the following: On the one hand, firms wants to keep the expected cost of rolling over debt low. This can be done by issuing long-term debt, since only a small fraction of long-term debt has to be rolled over at face value every period. On the other hand, firms also want to keep the default premium on newly issued debt low. This can be done by issuing short-term debt, since the default premium on short-term debt is endogenously lower for two reasons: First, the probability that the firm defaults before the debt has matured is lower and second, the incentive misalignment between short-term creditors and shareholders is less severe.

As a consequence, firms endogenously choose different types of debt for different debt issuance motives in the model: Firms use short-term debt if they have to issue debt due to shortfalls of internal funds. Firms instead use long-term debt if they issue debt for the purpose of the tax benefit of debt. Since the liquidity constraints in the model are counter-cyclical, while the net tax benefit of debt is pro-cyclical, the aggregate debt maturity structure is pro-cyclical.

The model matches several targeted moments, among them the average share of long-term debt in the cross-section and the default rate. While I calibrate the model to match cross-sectional moments, it can also match the main time series facts outlined in this section, notably the pro-cylicality of the long-term debt share in the aggregate and the higher cyclicality of the long-term debt share for small firms. It can also match the counter-cyclicality of equity issuance and the default rate. Finally, the model can replicate the maturity structure across the firm size distribution well.

Furthermore, I show that the maturity structure of debt has important effects on the

[2]These models treat the maturity structure of debt as an exogenous parameter. For a recent paper that studies the role of debt maturity in a dynamic model, see Titman and Tsyplakov (2007).

investment and financing decisions of firms: As outlined in Myers (1977) and more recently in Diamond and He (2014), there exists an agency problem between creditors and equity owners. In my model, this agency problem arises because a shareholder who maximizes the equity value of firms does not internalize the effect of his choices on the market value of outstanding long-term debt. This agency problem is more severe, the longer is the maturity of debt: For a given level of debt, a larger share of long-term debt leads to lower investment and more debt issuance. However, I show that firms in the model value the ability to issue both short-term debt and long-term debt: Restricting firms to issue only a certain type of debt or no debt at all cannot increase the average firm value across all firms.

2 Review of the Literature

My paper is related to the literature on business cycles in models with financial frictions. Papers in this literature have taken different assumptions on the maturity structure of debt, which then highlight transmission mechanisms of the capital structure on firm decisions.

Khan and Thomas (2013) develop a heterogeneous firm model in which firms issue secured short-term debt if they lack internal funds for investment. They do not consider default decisions. The financial friction in their model takes the form of a collateralized borrowing constraint. Financial shocks take the form of a tighter credit constraint, which lowers credit supply. The main negative effect of leverage on investment operates therefore through credit supply and rollover risk.

Furthermore, the debt issuance motive and hence leverage of firms in their model is negatively related to firm size.[3] By allowing firms to choose between different types of debt for different debt issuance motives, I can endogenously achieve a positive cross-sectional relation between firm size and leverage.

Gilchrist, Sim, and Zakrajsek (2014) study uncertainty shocks in a heterogeneous agent model with financial frictions and defaultable, short-term debt. Different from Khan and Thomas (2013), effective borrowing limits in their model are therefore endogenous. The main channel through which the capital structure affects investment decisions is however again a tightening in the effective supply of credit, here through higher default premia. Consequently, leverage in their model affects investment decisions mainly through rollover risk.

Relative to these papers, my main contribution is to show that firms use more long-term

[3]Specifically, this is true for their baseline firm setup. They improve their cross-sectional fit by adding a second type of firms to the model which instead of optimally choosing how much debt to issue use a simple rule that relates debt issuance to the square of the capital stock of the firm.

debt during expansions and more short-term debt during recessions. Due to debt overhang, firms will not quickly reduce leverage after a negative shock, which is an implication in all of the papers listed above. Instead, they will only reduce long-term leverage slowly and actually issue more short-term debt to cover liquidity shortfalls. Therefore, debt overhang is more important in my model, while credit supply effects are attenuated.

Gomes and Schmid (2016) develop a heterogeneous agent model that can explain the co-movement between macroeconomic aggregates and credit spreads. In stark contrast to Khan and Thomas (2013), debt in their model takes the form of a perpetual bond with infinite maturity. As a consequence, they emphasize the importance of debt overhang and default risk for aggregate variables in their model. Relative to their paper, I can show that the ability to issue short-term debt reduces debt overhang and therefore leads to less default risk.

The effects of the capital structure on investment in my model are in between those in Gilchrist, Sim, and Zakrajsek (2014) and Gomes and Schmid (2016). By issuing short-term debt and long-term debt, firms can balance roll-over risk and debt overhang. This trade-off is also quantitatively important: It can explain about 30 percent of the variation in the aggregate maturity structure, and about 60 percent of the variation in the maturity structure of the smallest 75 percent of firms. By looking at the maturity structure of debt, I can therefore provide new insights on the transmission mechanisms of leverage on investment dynamics.

There are also several corporate finance papers which study the dynamics of the maturity structure of corporate debt. He and Milbradt (2014a) discuss the dynamics of debt maturity in a continuous time model. They solve their model in closed form and provide a theoretical discussion of the existence of various equilibria. My focus is different: I use a quantitative model to study the dynamics of debt maturity in a setting with rich cross-sectional heterogeneity of firms. Importantly, I discuss the role of aggregate uncertainty and investment for the maturity structure of debt. The focus on investment distinguishes my paper also from Chen, Xu, and Yang (2013), who investigate maturity choice in a He and Milbradt (2014b)-type model with illiquid bond markets and endogenous default. They show that a liquidity-default spiral may lead firms to shorten their maturity structure during recessions, despite the existence of rollover risk. They do however not discuss the dynamics of maturity choice: conditional on the aggregate state variable, debt maturity is static in their model.

There is furthermore a large literature that studies the role of macroeconomic risk for the investment and financing decisions of firms in dynamic models, starting with Gomes (2001). My paper builds on the model of Kuehn and Schmid (2014), who find than in a framework with long-term debt, investment options are crucial to account for the cross section of credit spreads. Hackbarth, Miao, and Morellec (2006) study leverage dynamics with long-term debt and aggregate uncertainty in a continuous-time framework and show that leverage is counter-cyclical. Most other papers consider only short-term debt. Gomes and Schmid (2010) work out the role of real options for the effect of leverage on equity risk and show that the negative correlation between leverage and the presence of growth options helps to explain seemingly contradictory results on the effect of leverage on equity returns. Bolton, Chen, and Wang (2013) study cash holdings in a single-factor model and find that firms issue external funds in times of low funding costs to build up precautionary cash buffers. The financing costs in their model are exogenous. In a similar framework, Eisfeldt and Muir (2014) use an estimated model to provide evidence of a separate financial factor for the build up of precautionary cash buffers through the issuance of external funds. Warusawitharana and Whited (2014) have a similar focus as Eisfeldt and Muir (2014), but provide a behavioral foundation for their financial factor in the form of equity misvaluation.

Moyen (2007) discusses the role of different maturity structures for the quantitative importance of debt overhang. There are some differences between her framework and mine, most notably that in her model, firms hold either only short-term debt or long-term debt. Also, firms that hold long-term debt only issue a perpetual debt claim once at the beginning of their life, whereas firms in my model can dynamically adjust long-term leverage. Diamond and He (2014) also discuss the effect of different maturity structures on debt overhang. Investment in their model is however only a single expansion option.

I proceed as follows: In section 3, I outline the model of the decision problem of an equity-value maximizing firm and the bond pricing equations. Section 4 illustrates the determinants of the maturity structure of debt. Section 5 discusses my calibration strategy. In section 6, I present the numerical results for aggregate debt maturity dynamics. In section 7, I discuss whether restricting what debt firms can issue can lead to normative improvements. Section 8 concludes.

3 Model

Time is discrete: $t = 0, 1, \dots, \infty$. The unit of time is a quarter. There are N firms $i = 1, \dots, N$. The model consists of a common stochastic discount factor, a firm decision problem for each firm and a set of bond pricing equations, which specify the endogenous default premia at the firm level. I derive the stochastic discount factor from an exogenous consumption process and household preferences in section 3.1. In section 3.2, I present the firm problem. In section 3.3, I lay out how the bond prices are determined.

3.1 Aggregate Environment

Aggregate risk in the model takes the form of an aggregate productivity factor, which follows a first-order autoregressive process:

$$\begin{aligned} \ln Z' &= \rho^Z \ln Z + \sigma^Z \varepsilon^Z \\ \varepsilon^Z &\sim \text{i.i.d.} N(0,1) \end{aligned} \tag{3.1}$$

Equity and debt payouts are discounted with the stochastic discount factor

$$\Lambda(Z, Z') = \beta \exp\left(-\sigma \lambda_1 (Z' - Z)\right). \tag{3.2}$$

This discount factor is derived from a household whose consumption process moves with the aggregate productivity process: Household preferences are time-separable with discount factor β. The period felicity function has a constant relative risk aversion σ. The utility function therefore takes the recursive form

$$U(C) = \frac{C^{1-\sigma} - 1}{1 - \sigma} + \beta \mathbb{E}_{Z'}\left[U(C')|Z\right] \tag{3.3}$$

The consumption process is perfectly correlated with the aggregate productivity factor:

$$\ln C = \lambda_0 + \lambda_1 \ln Z \tag{3.4}$$

A security which yields a return of 1 in the next period therefore has the price

$$\frac{1}{1+r} = \beta \mathbb{E}_{Z'}\left[\left(\frac{C'}{C}\right)^{-\sigma} |Z\right] \tag{3.5}$$

where r is the risk free interest rate. Combining equations 3.4 and 3.5 yields the stochastic discount factor in equation 3.2. This discount factor leads to exogenous, time-varying risk premia in the model, which have been established as an important component of bond yields.[4] While the risk premium in the model is exogenous, default premia do reflect the endogenous default decisions of firms in the model, and are therefore endogenously determined as well. I will come back to the bond pricing equation in section 3.3.

Aggregate consumption is exogenous, which by itself is not important for the model results. What matters is that as a consequence, the term structure of risk free interest rates is exogenous. The exogeneity is plausible, since I only model the markets for risky nonfinancial

[4] See for example Chen (2010) or Bhamra, Kuehn, and Strebulaev (2010)

corporate debt and equity. The markets for government debt or household debt, for example, are outside the model. According to the Financial Accounts of the United States, corporate business debt constituted only 17.9 percent of all outstanding debt in 2015. The market value of equity nonfinancial domestic corporations constituted about 27.2 percent of the net wealth of the US for the same year. In addition, the assumption of an exogenous aggregate consumption or an exogenous stochastic discount factor is common in the asset pricing and corporate finance literature. It is for example used in Campbell and Cochrane (1999).

3.2 Firm Problem

Each firm $i = 1, \ldots, N$ is subject to aggregate and idiosyncratic productivity risk. Firms cannot commit and shareholders have limited liability, which means that they can walk away if the value of their stake in the firm is negative. The objective of each firm is to maximize the equity value, which corresponds to the present value of dividends. Firms decide how much to invest in capital, K_i, how much short-term debt $B_{S,i}$ and long-term debt $B_{L,i}$ to issue, how many dividends D_i to pay and when to default.

3.2.1 Technology

The production function of the firm is given by

$$\Pi(K_i, A_i, Z) = A_i Z K_i^{\alpha}, \tag{3.6}$$

where $\alpha < 1$ is a parameter that measures returns to scale at the firm level. The aggregate productivity process Z is common for all firms. The process for idiosyncratic productivity takes the form:

$$\begin{aligned} \ln A_i' &= \rho^A \ln A_i + \sigma^A \varepsilon_i^A \\ \varepsilon_i^A &\sim \text{i.i.d.} N(0,1) \end{aligned} \tag{3.7}$$

The idiosyncratic productivity shocks ε_i^A are uncorrelated over time and across firms. The conditional density function of A_i' is denoted by $f(A_i'|A_i)$.

In addition, firms have to pay a fixed cost ψ. This fixed cost arises only if the firm continues, independent of whether the firm produces or not. This fixed costs can be interpreted as all costs that arise independently of production, for example maintenance costs and administrative overhead. Such a fixed cost is used, for example, in Gomes (2001).

3.2.2 Investment

Capital follows the standard law of motion:

$$K_i' = (1-\delta)K_i + I_i, \tag{3.8}$$

where δ is the depreciation rate and I_i is investment. When installing new capital or selling old capital, the firm has to incur a quadratic capital adjustment cost with functional form

$$AC(K_i, K_i') = \frac{\theta}{2}\left(\frac{K_i'}{K_i} - 1 + \delta\right)^2 K_i \tag{3.9}$$

With these capital adjustment costs, I capture in a simple way that capital is illiquid. This form of capital adjustment costs is familiar from Q-theory, see for example Hayashi (1982). It is still used, for example, in the literature on cash holdings and liquidity management, for example in Bolton, Chen, and Wang (2013) and Eisfeldt and Muir (2014). Furthermore, Bloom (2009) reports that at the firm level as opposed to the plant level, quadratic capital adjustment costs yield a good description of firm level investment behavior.

3.2.3 Debt Financing

The firm can issue short-term debt $B_{S,i}$ and long-term debt, $B_{L,i}$. I find it convenient to use as state variables the total amount of outstanding debt, $B_i = B_{S,i} + B_{L,i}$ and the fraction of long-term debt, $M_i = B_{L,i}/B_i$.

Short-term debt takes the form of a one-period contract. Long-term debt takes the form of a contract with stochastic maturity μ. This formulation is a common trick to economize on state variables. It is for example used in the corporate finance literature in Hackbarth, Miao, and Morellec (2006) and Kuehn and Schmid (2014), but also in the literature on sovereign debt, for example in Hatchondo and Martinez (2009). The stock of long-term debt therefore evolves according to

$$B_{L,i}' = (1-\mu)B_{L,i} + J_i, \tag{3.10}$$

where J_i denotes long-term debt issuance. Long-term debt cannot be repaid early: $J_i \geq 0$. Short-term debt and long-term debt pay respectively coupons c_S and c_L.

Due to limited commitment, debt is risky. Issuance occurs at state-contingent prices $Q_S(\mathcal{S}_i', A_i, Z)$ and $Q_L(\mathcal{S}_i', A_i, Z)$. I will explain how these bond prices are determined in equilibrium in section 3.3.

There are issuance costs for debt. Debt issuance costs are equal for short-term debt issuance and long-term debt issuance. The functional form for debt issuance costs is given by

$$DIC(M_i, B_i, M_i', B_i') = \xi_1 \left(|(1 - M_i')B_i'| + |M_i'B_i' - (1 - \mu)M_iB_i| \right) \tag{3.11}$$

These issuance costs can be interpreted narrowly as flotation fees for new bond issues, or more widely as including bank fees or a constant liquidity premium. Such costs can arise in addition to the endogenous default premium. I focus on a simple linear specification. Typically, the literature considers either a combination of fixed and linear debt issuance costs or one of the two. An example for the former is Kuehn and Schmid (2014). A model which uses a purely linear issuance cost is Titman and Tsyplakov (2007).

3.2.4 Corporate Income Tax

There is a proportional corporate income tax τ. Taxable income is calculated as income less operating costs, depreciation and interest expense. This implies there is a tax benefit for investment as well as debt issuance. As a consequence, from the perspective of the shareholder, debt issuance is cheaper than equity issuance, because a fraction of interest expense is paid by the government. There is therefore an incentive for the firm to increase leverage up to the point where issuance costs and the default premium on newly issued debt cancel out the tax benefit of debt. This mechanism is the primary determinant of leverage in trade-off theory, see for example Kraus and Litzenberger (1973) or Fischer, Heinkel, and Zechner (1989). In this model, it is the reason why large, financially unconstrained firms issue debt.

3.2.5 Dividends and Equity Financing

Dividends are given residually by the budget constraint of the firm,

$$\begin{aligned} D_i = (1 - \tau) & \underbrace{\left[\Pi(K_i, A_i, Z) - \delta K_i - \psi - (c^S(1 - M_i) + c^L M_i)B_i \right]}_{\text{Taxable Income}} \\ & - \underbrace{((1 - M_i) + \mu M_i)B_i}_{\text{Principal Repayment}} + \underbrace{K_i - K_i'}_{\text{Gross Investment}} - AC(K_i, K_i') + \underbrace{Q_S(\mathcal{S}_i', A_i, Z)(1 - M_i')B_i'}_{\text{Revenue from ST Debt Issuance}} \\ & + \underbrace{Q_L(\mathcal{S}_i', A_i, Z)(M_i'B_i' - (1 - \mu)M_iB_i)}_{\text{Revenue from LT Debt Issuance}} - DIC(M_i, B_i, M_i', B_i') \end{aligned} \tag{3.12}$$

where τ is the corporate tax rate, δ is the deprecation rate. ψ is a fixed cost.[5] c^S and c^L are the coupons on short-term debt and long-term debt, respectively. μ is the repayment rate on long-term debt $AC(K_i, K_i')$ is a quadratic capital adjustment cost. $Q_S(\mathcal{S}_i, A_i, Z)$ and $Q_L(\mathcal{S}_i, A_i, Z)$ are the endogenous, state-dependent bond prices for short-term debt and long-term debt, respectively. $DIC(M_i, B_i, M_i', B_i')$ is a debt issuance cost.

Dividends D_i can be negative. In this case, the firm issues equity and has to pay an equity issuance cost. These costs are meant to capture monetary costs, such as underwriting fees, but also non-monetary costs like managerial effort and signaling costs conveyed through the issues. The equity issuance cost consists of a fixed and a linear component, such that there are increasing returns to scale for equity issuance. The functional form is

$$EIC(D_i) = (\phi_0 + \phi_1 |D_i|)\, \mathbb{1}_{(D_i<0)} \tag{3.13}$$

This form of equity issuance costs is for example used in Gomes (2001) and Hennessy and Whited (2007).

3.2.6 Default

Default occurs if the firm does not repay its debt, either for strategic reasons or because the firm cannot raise sufficient funds to repay outstanding liabilities. Since equity owners can simply walk away if the value of owning the firm becomes negative, the value of the firm in default to shareholders is 0.

3.2.7 Recursive Firm Problem

I collect the endogenous state variables of firm i in the tuple $\mathcal{S}_i = (K_i, B_i, M_i)$. K_i is the capital stock of the firm, B_i is the total amount of outstanding debt, and M_i is the fraction of outstanding debt that was issued in the form of long-term debt.

The value of the firm in default is 0, while the value of the firm continues operating is given by $V^C(\mathcal{S}_i, A_i, Z)$. The total firm value is then

$$V(\mathcal{S}_i, A_i, Z) = \max\left\{V^C(\mathcal{S}_i, A_i, Z), 0\right\}. \tag{3.14}$$

[5]It is important that this fixed cost has to be paid by the firm independently of whether the firm produces or not. Otherwise, the production decision of the firm becomes endogenous, which complicates the analysis considerably. Given that the period of the model is a quarter, it is not unrealistic to assume that production overhead has to be paid even if production does no longer occur.In the calibrated model, the case where firms will not want to operate is quantitatively unimportant.

3.2.8 Continuation Problem

If the firm decides not to default, its problem is then to maximize the present value of dividends by choosing the capital stock K_i', debt B_i', the fraction of long-term debt M_i', and dividends D_i. The value function of a continuing firm i can be summarized as

$$\begin{aligned} V^C(\mathcal{S}_i, A_i, Z) = \max_{K_i', B_i', M_i', D_i} \{ & D_i - EIC(D_i) \\ & + \mathbb{E}\left[\Lambda(Z, Z') \int_{-\infty}^{\infty} V(\mathcal{S}_i', A_i', Z') f(A_i'|A_i) dA_i' | Z \right] \Big\} \end{aligned} \tag{3.15}$$

subject to the constraints $K_i' \geq 0$, $B_i' \geq (1-\mu)M_i B_i$ and $1 \geq M_i' \geq 0$.

3.3 Bond Markets

3.3.1 Setup

The bond markets are competitive. Bonds are discounted with the same discount factor as equity. Both bonds pay a fixed coupon. Coupons are calculated according to

$$c_S = c_L = \frac{1}{\beta} - 1 \tag{3.16}$$

That is, coupons are chosen such that the values of risk-free bond prices in the absence of aggregate risk are both equal to 1.

If the firm decides not to default, firms pay the coupon on the outstanding debt first and the due amount of the principal second.[6] For long-term debt, the outstanding fraction after repayment $(1-\mu)$ is valued by bond-holders at the end-of period market value.

If the firm decides to default on the outstanding debt, the firm is liquidated after production has taken place. There is a cross-default clause: a default on short-term debt triggers a default on long-term debt and vice versa. In addition, there is a pari passu clause: bond holders have equal claims on the liquidation value of the firm, independent of the maturity of their bond. The liquidation value consists of the profits plus the depreciated capital stock. In default, it is not possible to deduct interest expense from taxable income. The recovery value per unit of the bond is therefore

$$R(\mathcal{S}_i, A_i, Z) = \max\left(\chi \frac{(1-\tau)(\Pi(K_i, A_i, Z) - \psi - \delta K_i) + K_i}{B_i}, 0 \right) \tag{3.17}$$

[6] This is important, because the coupon is also paid on the long-term debt that is repaid in the current period.

Default is costly: a fraction $1-\chi$ of the recovery value is wasted. One can think of this fraction as litigation fees, valuations costs and other direct monetary costs of default that have to be incurred by the bondholders. The assumption of a proportional loss in default is common in the literature, see for example Hennessy and Whited (2007).

3.3.2 Bond Pricing Equations

It is useful to define a default threshold set for idiosyncratic productivity. The default threshold set is implicitly defined by

$$a^*(\mathcal{S}_i, Z) = \left\{A \in \mathcal{A} : V^C(\mathcal{S}_i, A, Z) = 0\right\} \tag{3.18}$$

Suppose that the value function is strictly increasing in idiosyncratic productivity. Then, for each $(\mathcal{S}_i, Z)$, the default threshold is unique and equation 3.18 defines a function for the default threshold productivity: For $A \leq a^*(\mathcal{S}_i, Z)$, the firm will default, for $A > a^*(\mathcal{S}_i, Z)$, the firm will continue.

The market prices of the bonds are then

$$\begin{aligned}
Q_S(\mathcal{S}_i', A_i, Z) = \mathbb{E}\Bigg[\Lambda(Z, Z')\Bigg(&\int_{a^*(\mathcal{S}_i', Z')}^{\infty}(1+c_S)f(A_i'|A_i)dA_i' + \\
&\int_{-\infty}^{a^*(\mathcal{S}_i', Z')} R(\mathcal{S}_i', A_i', Z')f(A_i'|A_i)dA_i'\Bigg)|Z\Bigg] \qquad (3.19)\\
Q_L(\mathcal{S}_i', A_i, Z) = \mathbb{E}\Bigg[\Lambda(Z, Z')\Bigg(&\int_{a^*(\mathcal{S}_i', Z')}^{\infty}(\mu + c_L + (1-\mu)Q_L')f(A_i'|A_i)dA_i' \\
&+ \int_{-\infty}^{a^*(\mathcal{S}_i', Z')} R(\mathcal{S}_i', A_i', Z')f(A_i'|A_i)dA_i'\Bigg)\Bigg] \qquad (3.20)\\
Q_L' = Q_L(\mathcal{S}_i'', A_i', Z')&
\end{aligned}$$

That is, bond prices reflect the future default probabilities and the value of the firm in default. Future cash flows are discounted at the stochastic discount factor. Notably, while the short-term bond price only reflects the next period default probability, the long-term bond price captures the entire future path of default probabilities through its dependence on Q_L'.

3.4 Equilibrium

The recursive competitive equilibrium for this economy is given by a set of policy functions $h : (\mathcal{S}_i, A_i, Z) \to \mathbb{R}^3$ for capital, debt, and the share of long-term debt, a default policy characterized by $a^* : (\mathcal{S}_i, Z,) \to \mathbb{R}$, value functions $V_C : (\mathcal{S}_i, A_i, Z) \to \mathbb{R}$ and $V : (\mathcal{S}_i, A_i, Z) \to \mathbb{R}$ and bond price functions $Q_S : (\mathcal{S}'_i, A_i, Z) \to \mathbb{R}$ and $Q_L : (\mathcal{S}'_i, A_i, Z) \to \mathbb{R}$ such that for every firm $i = 1, \ldots, N$

- for any $(\mathcal{S}_i, A_i, Z) \in \mathbb{S} \times \mathbb{A} \times \mathbb{Z}$, given Q_S and Q_L, $h\left(\mathcal{S}_i, A_i, Z\right)$, maximizes the continuation problem in equation 3.15, with the solution to the firm problem given by $V_C\left(\mathcal{S}_i, A_i, Z\right)$.
- for any $(\mathcal{S}_i, A_i, Z) \in \mathbb{S} \times \mathbb{A} \times \mathbb{Z}$, given Q_S and Q_L, the firm chooses a^* such that $V^C(\mathcal{S}_i, A_i, Z) \geq 0 \iff A_i \geq a^*(\mathcal{S}_i, Z)$ and

$$V\left(\mathcal{S}_i, A_i, Z\right) = \begin{cases} V^C\left(\mathcal{S}_i, A_i, Z\right) & \text{if } A_i > a^*\left(\mathcal{S}_i, Z\right) \\ 0 & \text{if } A_i \leq a^*\left(\mathcal{S}_i, Z\right) \end{cases}$$

- for any $(\mathcal{S}'_i, A_i, Z) \in \mathbb{S} \times \mathbb{A} \times \mathbb{Z}$, given h, V_C, V_D and V, $Q_S\left(\mathcal{S}'_i, A_i, Z\right)$ and $Q_L\left(\mathcal{S}'_i, A_i, Z\right)$ are the solutions to the bond pricing equations 3.19 and 3.20.

4 The Determinants of Debt Maturity

In this section, I will outline the determinants of the maturity structure of a single firm. First, I will explain the different channels that determine the maturity structure of the firm. Then, I will discuss how the maturity choice varies over the business cycle.

Throughout this section, I assume that the value function V is once differentiable in K, B, M and A and the bond price functions Q_S and Q_L are differentiable in K', B', M' and A. I further assume that the short-term and long-term bond prices are weakly increasing in A, i.e. $\frac{\partial Q_S}{\partial A} \geq 0$ and $\frac{\partial Q_L}{\partial A} \geq 0$. I do not make these assumptions when I solve the model numerically later on.

I will denote $Q_L = Q_L(\mathcal{S}'_i, A_i, Z)$, $Q_S = Q_S(\mathcal{S}'_i, A_i, Z)$, $V = V(\mathcal{S}_i, A_i, Z)$ and $V^C = V^C(\mathcal{S}_i, A_i, Z)$ to economize on notation.

4.1 The General Case

The optimal maturity choice is given by the first order condition with respect to M' in the continuation problem of the firm presented in equation 3.15. I denote as λ_D the contemporaneous shadow cost of internal funds. The cost of internal funds is positive if the firm has to issue equity to avoid bankruptcy, because equity issuance is associated with issuance costs.[7] Further, I denote as $\lambda_{M,0}$ and $\lambda_{M,1}$ the multipliers for the constraints $M' \geq 0$ and $M' \leq 1$, respectively.

The first order condition for M' is

$$\frac{\partial V^C}{\partial M'} = \Bigg[(Q_L - Q_S)B' + \tag{4.1}$$

$$\frac{\partial Q_S}{\partial M'}(1-M')B' + \frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)\Bigg](1+\lambda_D) +$$

$$\mathbb{E}\left[\Lambda(Z,Z')\frac{\partial V'}{\partial M'}|A,Z\right] = \lambda_{M,1} - \lambda_{M,0} \tag{4.2}$$

The interpretation of this first order condition is that the benefit and the cost of marginally increasing the share of long-term debt must be equal. This decision concerns only the *maturity structure* of debt in the next period, but not the *leverage*. The total amount of debt issuance and hence the leverage choice is given by the first-order condition for total debt, B'.

A useful analogy for the choice of M is a portfolio choice for a given amount of wealth. The cost of issuing marginally more long-term debt is here the opportunity cost of issuing marginally less short-term debt.[8]

4.2 Optimal Maturity Choice without Aggregate Risk

First, I will focus on the main trade-off between short-term debt and long-term debt in a setup without aggregate risk. In this case, the discount factor $\Lambda(1,1) = \beta$ and the risk-free bond prices for short-term debt and long-term debt are, respectively, equal to 1.

The first order condition contains many different terms, so I will consider three different cases: In the first case, I discuss the case of a firm which never defaults. Then, neither short-term debt nor long-term debt is risky. In the second case, the firm may default only after the next period. Then, short-term debt is risk-free while long-term debt is risky. In the third case, the firm may also default in the next period, such that both short-term debt as well as long-term debt are risky.

In this way, I can introduce the channels that affect maturity choice one by one. I will

[7] The non-convex issuance costs for debt and equity require the introduction of this additional multiplier. With respect to equity issuance, the firm problem can be split into three sub-problems: In problem (1) the firm pays dividends. Then, $\lambda_D = 0$. In problem (2), the firm does not pay dividends, but also does not issue equity. Then, $D_i = 0$ becomes one of the equilibrium conditions of the model and λ_D is found residually from the first-order condition for investment. Finally, in problem (3), the firm issues equity. In that case, $\lambda_D = \phi_1$. The value function V^C is the envelope of these three subproblems.

[8] Of course, B' and M' are in the end jointly determined.

first focus on the main trade-off between rollover costs and default risk and then add other channels.

4.2.1 Case I: No Default Risk

In the case of no default risk, and no aggregate risk, the first order condition reduces to

$$\frac{\partial V^C}{\partial M'} = \beta(1-\mu)\xi_1 B' \mathbb{E}\left[(1+\lambda_D')|\mathcal{Y}\right] = \lambda_{M,1} > 0$$

This optimality condition states that the benefit of increasing the long-term debt share is that the firm has to pay less rollover costs, ξ_1, in the next period if it uses relatively more long-term debt. Consequently, a firm that can issue debt without risk wants to set the long-term debt share as high as possible: $M' = 1$, which implies a positive multiplier $\lambda_{M,1} > 0$.

4.2.2 Case II: Risk-Free Short-Term Debt, Risky Long-Term Debt

If default cannot occur in the next period, but may occur after the next period, the first-order condition for the long-term debt share reads[9]

$$\begin{aligned}\frac{\partial V^C}{\partial M'} = & \left[\underbrace{(Q_L - 1)\,B'}_{\text{Change, Marginal Revenue}} + \underbrace{\frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)}_{\text{Change, Intramarginal LT Revenue}}\right](1+\lambda_D) + \\ & \beta\mathbb{E}\left[(1-\mu)(\underbrace{1-Q_L'}_{\text{Change, Endogenous Rollover Cost}} + \underbrace{\xi_1}_{\text{Change, Exogenous Rollover Cost}})B'(1+\lambda_D')|A_i\right] \\ & = 0\end{aligned}$$

In the case with risky long-term debt, there are four important terms: The first two terms describe the change in the revenue from issuing new bonds if the firm decides to issue long-term bonds instead of short-term bonds. These are the costs of issuing a higher share of long-term debt. The last two terms describe the change in future rollover costs if the firm issues marginally more long-term debt. These are the benefits of issuing a higher share of long-term debt. Relative to the last section, the first three terms are new, whereas the exogenous rollover cost also arises in a situation with risk-free short-term *and* long-term debt.

The first term is the *change in the marginal revenue*: If the firm issues marginally more debt as risky long-term debt instead of risk-free short-term debt, it has to incur a default

[9]From this section onward, I focus on the case of an interior solution, so $\lambda_{M,1} = \lambda_{M,0} = 0$.

premium, captured by the term $(Q_L - 1)$. If this default premium is high, the firm prefers to issue short-term debt by setting a low M'. The default premium arises entirely from default risk after the next period. It is instructive to rewrite the bond price for long-term debt as a function of the bond price for short-term debt:

$$\begin{aligned} Q_L &= Q_S + (1-\mu)\beta \underbrace{\mathbb{E}\left[(Q_L' - 1)\,|A_i, Z\right]}_{\leq 0} \\ &= 1 + (1-\mu)\beta \underbrace{\mathbb{E}\left[(Q_L' - 1)\,|A_i\right]}_{\leq 0} \leq 1 \end{aligned}$$

The bond price for long-term debt differs from the bond price for short-term debt, because it incorporates future default risk, which is embedded in the future long-term bond price Q_L'.

The second term captures how a marginally larger long-term debt share affects the *intramarginal revenue from long-term debt issuance*. This effect arises, since a higher share of long-term debt today adversely affects firm policies in the future: The derivative $\frac{\partial Q_L}{\partial M'}$ is given by

$$\frac{\partial Q_L}{\partial M'} = \beta(1-\mu)\mathbb{E}\left[\frac{\partial Q_L'}{\partial K''}\frac{\partial K''}{\partial M'} + \frac{\partial Q_L'}{\partial B''}\frac{\partial B''}{\partial M'} + \frac{\partial Q_L'}{\partial M''}\frac{\partial M''}{\partial M'}|A_i\right]$$

Since the envelope theorem does not apply to the bond price,[10] the effects of current choices on future choices enter the current bond price. It is not possible to find analytic expressions for $\frac{\partial K'}{\partial M}$, $\frac{\partial B'}{\partial M}$ and $\frac{\partial M'}{\partial M}$. In the numerical solution to my model, the policy function for the next period capital stock K'' is decreasing in M', while the policy function for the next period level of debt B'' is increasing in M'. This is because the firm acts only in the interest of the shareholder and does therefore not internalize the effect of its decisions in default states. Since the benefits of investment and the costs of debt issuance arise in the future, the larger the share of firm value that accrues to long-term debt, the lower will be investment and the higher debt issuance. As a consequence, a higher share of long-term debt will in general increase default risk after the next period, which drives down the price of long-term debt today. In this case, $\frac{\partial Q_L}{\partial M'} < 0$.

The third term is the *endogenous rollover cost*. If the firm issues short-term debt, it has to repay the entire amount at the face value in the next period. If the firm instead issues long-term debt, it can roll over a fraction $(1-\mu)$ at the market value. The market value of long-term debt is below the face value because of future default risk. Therefore, being able to roll over long-term debt at the market value leads to lower rollover costs for the firm.

[10]The reason for why the envelope theorem does not apply to the bond prices is that firms do maximize over the market value of equity, but not over the market value of debt.

In the case discussed in this section, the firm trades off roll-over costs of short-term debt against the long-term default premium and the negative incentive effect of long-term debt issuance. This is the main trade-off I consider and therefore deserves to be discussed in more detail. Consider the case of a firm with low productivity and a low capital stock. This firm issues debt due to a high value of internal funds, that is, since λ_D is high. It will have a low probability of long-term survival, and hence face a high long-term default premium on long-term debt. This is captured by the term $Q_L - 1 < 0$. Furthermore, by issuing long-term debt, such a firm would decrease the incentive for future investment, since a part of that investment would essentially be an intertemporal transfer of current shareholder funds to future bondholder funds. This is captured by the term $\frac{\partial Q_L}{\partial M'} < 0$. If these effects outweigh the rollover costs, such a liquidity-constrained firm will choose a low long-term debt share.

Now consider a firm with a high capital stock and a high productivity. The motive for such a firm to issue debt is not a high value of λ_D, but the tax benefit of debt. Such a firm has a low long-term default probability, and hence $Q_L - 1$ and $\frac{\partial Q_L}{\partial M'}$ will be small. Therefore, such a firm will mostly be concerned about the rollover costs of debt and will issue long-term debt.

So in this model, firms endogenously use different types of debt for different motives: Liquidity constrained firms use short-term debt, while firms which care mostly about the tax benefit of debt use long-term debt. These two motives will later on give rise to the cyclical dynamics of debt maturity: Intuitively, the fraction of firms which issue short-term debt due to liquidity constraints increases in a recession, while the fraction of firms which issue long-term debt due to the tax benefit decreases. The motive to issue debt due to a liquidity shortfall is counter-cyclical, while the tax benefit of debt net of the default premium is pro-cyclical. As a consequence, the aggregate long-term debt share in the model is counter-cyclical.

4.2.3 Case III: Risky Short-Term Debt and Long-Term Debt

If short-term debt is also risky, the first order condition for M' is

$$\frac{\partial V^C}{\partial M'} = \Bigg[\underbrace{(Q_L - Q_S)B'}_{\text{Change, Marginal Revenue}} + \underbrace{\frac{\partial Q_S}{\partial M'}(1 - M')B'}_{\text{Change, Intramarginal ST Revenue}} + \underbrace{\frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)}_{\text{Change, Intramarginal LT Revenue}} \Bigg] (1 + \lambda_D) + \beta \mathbb{E}\left[(1-\mu)(1 - Q'_L + \xi_1)B'(1 + \lambda'_D) || A_i\right] = 0$$

In this case, there are two new terms relative to the case in which only long-term debt is risky: First, the short-term bond price and the long-term bond price also incorporate a premium for the possibility of default in the next period. Similar to the case with only long-term default risk, the long-term bond price can be written as

$$Q_L = Q_S + (1-\mu)\beta\mathbb{E}\left[(Q'_L - 1)\, \mathbb{1}_{(A'>a^*)}|A_i\right]$$

Hence, $Q_L - Q_S = (1-\mu)\beta\mathbb{E}\left[(Q'_L - 1)\, \mathbb{1}_{(A'>a^*)}|A_i\right]$, which is almost exactly the same term as in case II. The difference is that long-term default risk is only relevant when the firm does not default in the next period. The premium for default after the next period is the only change in the marginal revenue that arises when the firm increases the long-term debt share.

Second, if short-term debt is risky, the short-term bond price is also sensitive to the maturity structure of the firm. What matters for the short-term bond price is how a change in the maturity structure of debt affects the probability of default of the firm in the next period: The derivatives of the short-term and long-term bond prices in the case of risky short-term debt and long-term debt are given by:

$$\frac{\partial Q_S}{\partial M'} = \underbrace{[1 + c - R(K,B,a^*,1)]}_{\text{Change in Repayment}} \qquad \underbrace{\frac{\frac{\partial V'}{\partial M'}}{\frac{\partial V'}{\partial A'}} f(a^*|A)}_{\text{Change in Next Period Default Probability}} \qquad > 0$$

$$\frac{\partial Q_L}{\partial M'} = \underbrace{[\mu + c + (1-\mu)Q'_L - R(K,B,a^*,1)]}_{\text{Change in Repayment}} \qquad \underbrace{\frac{\frac{\partial V'}{\partial M'}}{\frac{\partial V'}{\partial A'}} f(a^*|A)}_{\text{Change in Next Period Default Probability}}$$
$$+ \beta(1-\mu) \underbrace{\int_{a^*}^{\infty} \left(\frac{\partial Q'_L}{\partial K''}\frac{\partial K''}{\partial M'} + \frac{\partial Q'_L}{\partial B''}\frac{\partial B''}{\partial M'} + \frac{\partial Q'_L}{\partial M''}\frac{\partial M''}{\partial M'} \right) f(A'|A)dA}_{\text{Change in Long-Term Repayment and Default Probability}}$$

Interestingly, increasing the long-term debt share can have opposite effects on the bond prices: A higher long-term debt share increases the price of short-term debt, because it reduces the probability of default in the next period. However, a higher long-term share also reduces investment in the next period and increases debt issuance in the next period, such that default risk after the next period may actually increase. There are therefore two effects of a higher long-term debt share on the long-term bond price: The next period default probability decreases, but the default probability after the next period increases. In the quantitative version below, the latter effect dominates.

In Figure 3, I depict the bond price as a function of the long-term share for two different levels of debt. All other variables are chosen to be the same. The parameters are from my baseline calibration. In the left panel, the level of debt chosen is high and as a consequence, the next period default probability is high. The long-term bond price and the short-term bond price both increase for the most part in response to an increase in the long-term debt share. This is because such an increase lowers the next period default probability, which is here the dominant effect.

In the right panel, the level of debt is low and therefore the next period default probability is low. The long-term bond price decreases mostly if the long-term debt share increases. This is because a higher long-term debt share in this case leads to a higher long-term probability of default. In contrast to that, the short-term bond price increases, as in the left panel, monotonically with a higher long-term debt share.

In summary, if short-term debt and long-term debt are risky, the trade-off is fundamentally the same as in the case of risk-free short-term debt and risky long-term debt. Relative to the case with risk-free short-term debt, there is an additional benefit of issuing long-term debt, since a higher long-term debt share reduces the next period default probability. However, in the quantitative model below, firms will still prefer to issue short-term debt if they are liquidity constrained.

4.3 Aggregate Risk

In the presence of aggregate risk, the difference in bond prices can be decomposed into two terms:

$$Q_L - Q_S = \underbrace{Q_L^{RF} - Q_S^{RF}}_{\text{Risk-Free Term Spread}} + \underbrace{(Q_L - Q_L^{RF}) - (Q_S - Q_S^{RF})}_{\text{Long-Term Default Premium}}$$

The new first term is the difference in risk-free bond prices. The second term is the long-term default premium. Bonds yield a fixed stream of income which is $1 + c$ for short-term bonds and $\mu + c$ per period for long-term bonds. In the presence of aggregate risk, a marginal unit of consumption more in a recession is more valuable than a marginal unit of consumption in an expansion. Hence, conditional on being in a recession, the fact that shocks are mean reverting imply that risk-free bond prices in a recession are lower than in an expansion. Further, the positive autocorrelation of the shocks implies that the risk-free long-term bond price in the recession is lower than the risk-free short-term bond price.

With this decomposition, the first order condition for the long-term debt share can be rewritten as

$$\begin{aligned}\frac{\partial V^C}{\partial M'} = \Bigg[& \left((Q_L^{RF} - Q_S^{RF}) + (Q_L - Q_L^{RF} - Q_S + Q_S^{RF})\right) B' + \\ & \frac{\partial Q_S}{\partial M'}(1 - M')B' + \frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)\Bigg] (1 + \lambda_D) + \\ & \mathbb{E}\left[\Lambda(Z, Z')\frac{\partial V'}{\partial M'}|A_i\right] = 0\end{aligned}$$

Aggregate risk is important for two reasons. First, the risk-free short-term bond price is higher than the risk-free long-term bond price in a recession and lower in expansions. In other words, the term structure of risk-free bond yields is downward sloping in expansions and upward sloping in recessions. Without the shadow cost of internal funds, λ_D, this would not matter, since firms discount future cash-flows at the same discount factor as creditors. However, a firm that places a high value of internal funds today versus tomorrow, i.e. with $\lambda_D > \lambda_D'$, will prefer short-term debt relative to long-term debt more in a recession.

Second, as outlined in Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010), in the presence of aggregate risk, there is an additional risk-premium on the default premium if the default probability is higher in recessions. Then, cash-flows from the firms to creditors are low exactly when creditors value cash flows highly. This negative covariance between the default probability and the marginal utility of consumption increases the default risk premium on debt.

In summary, aggregate risk introduces a new channel for the determination of the maturity structure through the time variation in the term structure of risk-free rates and emphasizes the importance of the default channel relative to the rollover channel. The fact that the short-term bond price is higher than the long-term bond price in recessions makes short-term debt even more attractive for liquidity constraints in recessions. This channel should amplify the counter-cyclicality of short-term debt issuance. In addition, the higher and more cyclical default premia reduce the net tax benefit of long-term debt issuance, particularly in recessions. This effect should amplify the pro-cyclicality of long-term debt issuance.

5 Calibration

In this section, I will describe the data and the calibration procedure. Due to the high dimensionality of the model, I cannot calibrate all parameters. Therefore, I divide the set of parameters into three subsets: I take the first set from the literature. The second set of parameters is taken from a production function estimation. The third set of parameters is calibrated to match cross-sectional moments. I solve the model using value function iteration. The interested reader will find a detailed description of the solution algorithm in appendix 9.3.

5.1 Data

Firm data are from the CRSP Compustat Quarterly North America database. I focus on US-American firms only. The observation unit is a firm-quarter. I use data from the first quarter of 1984 to the last quarter of 2012. I exclude regulated firms (SIC code 4900-4999), financial firms (SIC code 6000-6999) and non-profit firms (SIC code 9000-9999) from my sample. Furthermore, I exclude those observations which do not report total assets or those which report either negative assets or a negative net capital stock. Finally, I exclude observations where total assets change for more than 100% in absolute value, quarter on quarter. This adjustment is necessary to exclude mergers and acquisitions, which I do not model.

I calculate all flow variables from the cash flow statements of firms. Investment is capital expenditure minus sales of property, plant and equipment. Short-term debt issuance is defined as change in current debt. Long-term debt issuance is long-term debt issuance minus long-term debt reduction. Equity issuance is sale of common and preferred stock minus purchase of common and preferred stock minus dividends. All flows are normalized by lagged total assets. I calculate the share of long-term debt to total debt as long-term debt divided by long-term debt plus current debt. Market leverage is defined as the book value of short-term debt plus the market value of long-term debt divided by the sum of the market value of debt and equity. To calculate the market value of long-term debt, I use the method by Bernanke et al. (1988). The market value of common stock is defined as the share price times the number of shares. The market value of preferred stock is defined by the current dividend for preferred stock divided by the current federal funds rate. This approach is essentially the one pursued in Whited (1992).

The data for the default rate is taken from Ou et al. (2011). I use the default rate for the largest sample, namely all firms from 1920 to 2011, which corresponds to 1.1 percent.

5.2 Simulation Procedure

To compute the firm-level moments from the numerical solution, I simulate a panel of 5000 firms for 5200 quarters. I use a burn-in period of 200 quarters. Defaulted firms are replaced with new firms which draw a new productivity shock from the unconditional productivity distribution. These firms start their life with a short-term debt stock of 1 and a capital stock of 1. Changing these values does not affect the results.

To compute the cumulative default rate in the model, I use the method of Cutler and Ederer (1958) which is used in Ou et al. (2011) to calculate empirical default rates: Essentially, I construct and simulate a panel of $N_0 = 1000$ firms for 225 periods where a firm drops out of the panel if it defaults. I denote the number of firms which default in period t by N_t^d. Hence, the number of firms in the panel evolves according to $N_t = N_{t-1} - N_{t-1}^d$. From this panel, I compute a marginal default rate as

$$d_t^m = \frac{N_t^d}{N_t}$$

The cumulative default probability from 0 to t is then given by 1 minus the survival rate at time t:

$$d_t^c = 1 - \Pi_{s=0}^t (1 - d_s^m)$$

Since the marginal default rates and hence the cumulative default rates are contingent on the aggregate state, I simulate 250 such panels and report the average across panels of the cumulative default rates.

I calculate bond yields as

$$y_{i,t,S} = \frac{1 + c_S}{Q_{i,t,S}} - 1$$
$$y_{i,t,L} = \frac{\mu + c_L}{Q_{i,t,L}} - \mu$$

Credit spreads are given by the difference between risky and risk-free bond yields of identical maturity:

$$cs_{i,t,S} = y_{i,t,S} - y_{i,t,S}^{RF}$$
$$cs_{i,t,L} = y_{i,t,L} - y_{i,t,L}^{RF}$$

When computing average bond yields and credit spreads, I use debt weights $\omega_{i,t} = B_{i,t}/B_t$.

In the data, short-term debt is defined as debt with a maturity of less than 1 year. This definition includes long-term debt with a residual maturity of less than 1 year. In the model, debt with a maturity of less than 1 year is given by

$$(1 - M_{i,t})B_{i,t} + (1 - (1 - \mu)^4)M_{i,t}B_{i,t} \tag{5.1}$$

Therefore, the share of long-term debt in total debt at the firm level is given by

$$\text{ltShare}_{i,t} = \frac{B_{i,t} - (1 - M_{i,t})B_{i,t} - (1 - (1 - \mu)^4)M_{i,t}B_{i,t}}{B_{i,t}} = (1 - \mu)^4 M_{i,t} \tag{5.2}$$

Finally, market leverage at the firm level is calculated as the market value of debt divided by the market value of debt plus the ex dividend value of equity:

$$\text{mktLev}_{i,t} = \frac{Q_{S,i,t}B_{S,i,t+1} + Q_{L,i,t}B_{L,i,t+1}}{V_{i,t} - D_{i,t} - EIC(D_{i,t}) + Q_{S,i,t}B_{S,i,t+1} + Q_{L,i,t}B_{L,i,t+1}} \tag{5.3}$$

The aggregate long-term debt share and market leverage are respectively given by

$$\text{ltShare}_t = \frac{\sum_{i=1}^{N}(1 - \mu)^4 M_{i,t}B_{i,t}}{\sum_{i=1}^{N} B_{i,t}}$$

and

$$\text{mktLev}_t = \frac{\sum_{i=1}^{N}\left(Q_{S,i,t}B_{S,i,t+1} + Q_{L,i,t}B_{L,i,t+1}\right)}{\sum_{i=1}^{N}\left(V_{i,1} - D_{i,t} - EIC(D_{i,t}) + Q_{S,i,t}B_{S,i,t+1} + Q_{L,i,t}B_{L,i,t+1}\right)}$$

5.3 Parameters from the Literature

For the preferences of the representative household, I use a time preference rate, $\frac{1}{\beta} - 1$, of four percent per year and a risk aversion coefficient σ of two. These values are standard in the macroeconomic literature. In the baseline calibration, I set the maturity of long-term debt to five years. This implies a quarterly repayment rate μ of five percent. Following Graham (2000), I set the corporate income tax rate τ to 14 percent. This is substantially lower than the true marginal US corporate income tax rate, but corresponds to about the actual average tax rate of firms in the US. Finally, I set the recovery rate in default to 0.8 as in Kuehn and Schmid (2014).

5.4 Estimated Parameters

I follow Cooper and Haltiwanger (2006) and estimate the production function directly from the data. The details on the estimation procedure are in the appendix.

The estimated parameters are reported in Table 1. The value for the curvature of the production function is relatively low. Hennessy et al. (2007) uses a value of 0.65, implying less decreasing returns. However, they do not use a fixed production cost in their production function, which leads to increasing returns for low values of capital. The persistence and volatility of the idiosyncratic productivity process are in line with the values used in Kuehn and Schmid (2014) and Katagiri (2014), who report values of 0.85 and 0.98 for the persistence and 0.15 and 0.12 for the volatility of idiosyncratic productivity, respectively. The persistence of the macroeconomic process is low compared to estimates derived from macroeconomic data. However, the aggregate productivity process roughly matches the productivity process in Eisfeldt and Muir (2014), who estimate a persistence of 0.62, using the productivity time series from Fernald (2014). The consumption coefficient $\lambda_C = 0.38$ implies an unconditional consumption volatility of 0.52%.[11] Lustig et al. (2013) report a standard deviation for consumption growth of 0.46%.

5.5 Calibrated Parameters

I calibrate the capital adjustment costs θ, the debt issuance cost parameter ξ_1, the equity issuance cost parameters ϕ_0 and ϕ_1 and the fixed production cost ψ to match a set of cross-sectional moments. I choose to match the average share of long-term debt, the average size and frequency of equity issuance, the cumulative one year default rate and the cross-sectional standard deviation of the investment capital ratio. Table 1 shows the calibrated parameters. Table 2 reports targeted moments from a numerical simulation.

The fixed production cost is crucial to match the default rate in the model. Quantitatively, it corresponds to 10.54% of the steady state capital stock of the model. Kuehn and Schmid (2014) use a linear production cost that corresponds to 4% of the lagged capital stock.

As I show in Table 2, the model matches the average long-term debt share well. The default rate in the model is close to the default rate in the data, with the model default rate at about 1 percent and the data default rate at about 1.1 percent. The standard deviation of investment is a bit too high. This is due to the rapid capital accumulation of firms after a default. The model can also broadly match the size and frequency of equity issuance, with the size being too high and the frequency too low. Overall, while the match between the model and the data is by no means perfect, it delivers plausible numbers for all targeted moments.

[11] $std(\ln(C)) = \lambda_C \sqrt{\frac{\sigma_Z^2}{(1-\rho_Z^2}}$

Name	Value	Role	Reference
Parameters from the Literature			
β	$1.04^{-1/4}$	Discount Factor	4% Annual Risk Free Rate
σ	2	Risk Aversion	
μ	0.05	Lt Debt Repayment Rate	5 Year Maturity
τ	0.14	Corporate Income Tax Rate	Graham (2000)
χ	0.8	Recovery Rate	Kuehn and Schmid (2014)
Calibrated Parameters			
ξ_1	0.0025	Linear Debt Iss Cost	Average Long-Term Debt Share
ϕ_0	0.1	Fixed Eq Iss Cost	Size, Equity Issuance
ϕ_1	0.04	Linear Eq Iss Cost	Frequency, Equity Issuance
ψ	1.8	Fixed Prod Cost	Cumulative 1Yr Default Rate
θ	4	Capital Adj Cost	Standard Deviation, Investment-Capital Ratio
Estimated Parameters			
α	0.35	Prod Function Curvature	
ρ_A	0.95	Persistence, Idio Prod	
σ_A	0.1	Volatility, Idio Prod	
ρ_Z	0.68	Persistence, Agg Prod	
σ_Z	0.02	Volatility, Agg Prod	
λ_C	0.38	Consumption Coefficient	

Table 1: Parameter Choices. This table shows all model parameters, grouped into three categories: The first category shows parameters chosen from the literature, the second category shows parameters from the literature, the third category shows parameters taken from a production function estimation using a dynamic panel data estimator.

6 Results

6.1 Aggregate Results

In this section, I report the implications of the model for the dynamics of the aggregate maturity structure. The aim of this section is to show that the model, which is calibrated to match cross-sectional facts, can also match time series moments. First, I will discuss the time series fit. I also will discuss dynamics for small and large firms separately. Second, I will report aggregate means and correlations and how they are affected by changes in the model assumptions. Third and finally, I will report and analyze the impulse responses to a negative aggregate productivity shock to investigate aggregate dynamics in more detail.

Targeted Moments		
	Data	Model
Default Rate	1.1%	0.9%
Frequency, Equity Issuance > 0	10.1%	19.2%
Average Size, Equity Issuance	12.2%	8.4%
Average Market Leverage	22.9%	15.1%
Average Long-Term Debt Share	63.2%	67.1%
Standard Deviation, Investment	2.5%	4.9%

Table 2: Model Fit. This table collects the targeted cross-sectional moments from a simulation of the model and compares them to their data counterparts. Data moments are from Compustat, with the exception of the default rate, which is taken from Moody's. Model moments are calculated from a Monte Carlo Simulation as explained in the text. Overall, the model provides a reasonable fit for the targeted moments.

6.1.1 Model-implied Time Series

In Figures 4 and 5, I compare the long-term debt share from the data to the model-implied long-term debt share. To compute the model-implied long-term debt share, I feed in a time series of aggregate productivity shocks. I use the time series by Fernald (2014), specifically, I cumulate the productivity deltas for utilization adjusted TFP from his data set. I use a burn-in period of 200 quarters, after which the aggregate productivity process evolves according to the time series from the data. Both the model time series and the data time series I report are HP-filtered with parameter $\lambda = 1600$.

As can be seen in Figure 4, the model can match the timing of the long-term debt share. However, the time series is only one third as volatile as the time series in the data. The contemporaneous correlation between the two time series is 0.28.

Comparing the time series for small firms and large firms in Figure 5 separately, it is clear that the maturity dynamics are driven by small firms: The model can predict about sixty percent of the variation in the long-term debt share of small firms and the correlation between the time series for small firms implied by the model and the one by the data is 0.34. In contrast to that, there is basically no volatility in the long-term debt share of the largest 25 percent of firms in the model. These firms basically issue only long-term debt. From the perspective of the model, this makes sense, since short-term debt becomes only desirable in the presence of default risk: Large firms in the model have a default probability of zero. This is counterfactual and a recognized problem for these class of models. For example, Katagiri (2014) introduces an additional exogenous default shock to match the exit rate of large firms.

6.1.2 Aggregate First Moments

	(1) Baseline Model	(2) Risk-Neutral Investors	(3) No Debt Issuance Costs	(4) No Equity Issuance Costs	(5) No Capital Adjustment Costs	(6) Data
			Means (%)			
LT Share	68.76	67.64	8.34	79.78	79.62	84.46
Market Leverage	14.16	13.94	22.18	30.62	37.63	22.80
Investment/Assets	3.71	3.70	3.7	3.71	4.13	3.20
Default Rate	0.94	0.96	1.01	0.78	3.66	1.1
			Correlations with Output			
LT Share	0.59	0.28	0.70	−0.01	−0.56	0.10
Market Leverage	0.06	−0.46	−0.64	−0.80	0.39	−0.15
Short-Term Debt Issuance/Assets	−0.56	−0.28	−0.40	−0.31	0.38	0.33
Long-Term Debt Issuance/Assets	0.77	−0.03	0.25	0.39	−0.14	0.49
Debt Issuance/Assets	−0.27	−0.28	−0.35	0.17	0.01	0.46
Equity Issuance/Assets	−0.38	−0.76	−0.38	0.05	−0.16	−0.23
Investment/Assets	0.76	0.80	0.71	0.75	0.03	0.45
Default Rate	−0.31	−0.09	−0.17	−0.28	−0.58	−0.33

Table 3: Aggregate Summary Statistics. In this table, I report aggregate summary statistics for various versions of the model and compare them to the data. The baseline model is the model discussed in section 3 and calibrated in 5. In model (2), I set the risk aversion of the representative household to 0. In model (3), I set the debt issuance costs to 0. In model (4), I set the equity issuance costs to 0. In model (5), I set the capital adjustment costs to 0. In the upper panel of the table, I show means for the whole economy, in the lower panel, I show contemporaneous correlations with GDP. The correlations in the data are computed with linearly detrended time series. I detrend GDP using the HP filter, with parameter $\lambda = 1600$.

In Table 3, I report statistics for the aggregate economy. Column (1) shows the statistics for the baseline model, column (6) the statistics for the data. In columns (2) to (5), I shut down different channels of the model to illustrate their importance.

In the first three rows of the first panel, I report the mean of the aggregate long-term debt share, aggregate leverage and aggregate investment. These are the main statistics for the aggregate capital structure and investment in the economy. Finally, in the last row of the first panel, I show the aggregate default rate. Notice that besides the default rate, all of these moments are untargeted: The model is only targeted to match cross-sectional statistics. The baseline model predict a value for the aggregate leverage and long-term debt share that are too low. These problems are related: The tax benefit motive for long-term debt issuance is too weak, which is why large firms choose a too low leverage compared to the data. I will come back to this issue in section 6.2. The model generates reasonable numbers for the aggregate investment-capital ratio and the aggregate default rate.

For the first alternative model version in column (2), I consider the case of a risk-neutral representative household. I set the risk aversion parameter σ in this model to zero. Without risk-aversion, the household's consumption risk is irrelevant. In particular, there is no term structure of risk-free interest rates and default is priced at the risk-neutral default premium. In terms of aggregate first moments, eliminating the risk aversion of the representative household does not have big effects.

In the second alternative model in column (3), I eliminate debt issuance costs. Specifically, I set ξ_1 to zero. In this case, the exogenous component of the debt rollover costs in the first order condition 4.2 is zero. Hence, this model substantially reduces rollover costs. In the absence of debt issuance costs, firms prefer to mostly issue short-term debt to attain the tax benefit of debt. Short-term debt avoids the problem of the incentive misalignment between shareholders and creditors with respect to future investment and debt issuance decisions. As a consequence, the decision short-term debt can be issued at lower default risk premia and hence at lower costs. The model without issuance costs is at odds with the data, because it predicts a very low long-term debt share.

The third alternative model in column (4) has no equity issuance costs. I set ϕ_0 and ϕ_1 equal to zero. Without equity issuance costs, firms can issue equity at all time at the price of one. This shuts down the variation in the shadow cost of internal funds λ^D over time. As a consequence, the motive to issue short-term debt vanishes in this model. This can be seen in the results in row 1: Firms decide to only issue long-term debt. They also issue a substantial amount of long-term debt: Leverage is about twice as high as in the baseline model. The default rate is approximately the same, which is remarkable, since leverage is almost twice as high. This shows that debt overhang problems in the baseline model with long-term debt can substantially increase the default risk of firms.

The model without capital adjustment costs in column (5) also yields results that are very different from the baseline model. In the absence of the quadratic capital adjustment costs, firms will adjust their capital stock flexibly in response to a change in their productivity. As a consequence, shareholders reduce the capital stock after a negative productivity shock and default right away. The default rate in this model is three times higher than in the data. In contrast, firms with capital adjustment costs will only be able to reduce their capital stock gradually and will issue short-term debt after a negative productivity shock to cover shortages of internal funds, i.e. situations in which λ^D is very high. The motive for short-term debt issuance is absent in the model without capital adjustment costs: Firms issue essentially only long-term debt, which is reflected in a long-term debt share of about 80 percent, the highest possible level.

6.1.3 Aggregate Correlations

In the second panel of Table 3, I report the correlations of the aggregate time series of the model with aggregate output. Aggregate output in the model is computed as

$$Y_t = \sum_{i=1}^{N} A_{i,t} Z_t K_{i,t}^{\alpha}.$$

That is, output is the sum of outputs across all N firms. In the data, I compute the correlation between linearly detrended time series aggregated from Compustat Data and HP-filtered real GDP.

The baseline model matches the signs of the correlations of the long-term debt share, long-term debt issuance, equity issuance and the default rate with output.[12] Long-term debt issuance is pro-cyclical, since long-term debt is issued by unconstrained firms due to the tax benefit of debt. In a recession, default premia increase, while the tax benefit is constant. As a consequence, firms will issue less long-term debt.

Short-term debt issuance is strongly counter-cyclical in the baseline model, in contrast to the data. The main motive for short-term debt issuance are financial constraints in the form of a high value of λ_D. As financial constraints are more severe in recessions, short-term debt issuance is higher during recessions. Moreover, since the long-term debt share is inversely proportional to the long-term debt share, the model can by construction either match the cyclicality of the long-term debt share or short-term debt issuance.

Total debt issuance is counter-cyclical in the baseline model, also in contrast with the data. The reason for this is that counter-cyclical short-term debt issuance is too high relative to pro-cyclical long-term debt issuance. This is related to the problem that long-term leverage in the model is too low. A higher long-term leverage would result in more long-term debt issuance and consequently in pro-cyclical debt issuance.

Equity issuance is counter-cyclical in the baseline model as well as in the data. Financially constrained firms who cannot or do not want to issue debt will issue equity instead. This result is in line with the results in Jermann and Quadrini (2012), who also report counter-cyclical equity issuance.

The model with a risk-neutral representative household cannot match the data: It predicts that debt issuance, whether short-term or long-term, are basically acyclical. The

[12] The correlation of the long-term debt share in the data may seem low, especially compared to the apparent cyclical behavior in Figures 1 and 2. The correlation is higher at one lag, at 0.23 and even higher at two lags, with 0.32. In addition, the low contemporaneous correlation is only due to the largest five percent of firms: Excluding the largest five percent, the contemporaneous correlation of the long-term debt share with output increases to 0.29, with a twice lagged correlation of 0.46.

correlation between the long-term debt share and GDP is, however, positive, although more weakly. This shows that for match the long-term debt share, the term structure implied by the stochastic discount factor of the representative investor matters. Furthermore, the default rate moves too little against GDP.

The model without debt issuance cost overall matches the signs of the correlations in the data well. However, in contrast to the data the correlation of short-term debt issuance with GDP and total debt issuance with GDP are negative, since firms in this model essentially only issue short-term debt. This is because in the absence of issuance costs, the main reason for firms to issue short-term debt in this model is not a shortage of internal funds, but the tax benefit of debt. The main trade-off for short-term debt issuance is then between the tax benefit of debt and the default premium. Since the default premium is counter-cyclical, short-term debt issuance is strongly pro-cyclical.

The model without equity issuance costs predicts a positive correlation between total debt issuance and GDP, since firms issue mostly long-term debt. However, the correlation between equity issuance and GDP is also positive in this model. Finally, market leverage is too strongly counter-cyclical and the long-term debt share in this model is essentially acyclical. This shows that variation in the shadow cost of internal funds λ^D is an important driver of aggregate debt maturity dynamics in this model.

The model without capital adjustment costs also fails to match the data: The correlation of the long-term debt share with GDP is strongly negative. Short-term debt issuance is very pro-cyclical. Investment and long-term debt issuance are basically acyclical.

6.1.4 Impulse Responses

I compute the impulse response to a positive and a negative one standard deviation shock to aggregate productivity. For the impulse responses, I simulate an economy with 5000 firms for 250 periods and discard the first 200 periods, for which I keep aggregate productivity constant. The decrease in productivity occurs in period 210.

I show the results to a negative productivity shock first. Figure 6 shows the dynamic response to this decrease in aggregate productivity. Short-term debt issuance decreases on impact, but increases above the stationary level in the period thereafter. On the impact of the productivity shock, firms want to reduce leverage and decrease their capital stock, while very unproductive firms will outright default. All of these effects lower the incentive to issue short-term debt. As the economy recovers, financial constraints become more important as firms start to invest again and will issue short-term debt instead of defaulting after if their idiosyncratic productivity is low.

Long-term debt issuance decreases more markedly and more persistently. Since long-term debt issuance is less attractive if default premia are high relative to the tax benefit, firms will reduce long-term debt issuance during the recession. As productivity increases back to the initial level, firms will increase their long-term leverage back to their target level.

Equity issuance initially spikes down by about six percent in the period of the productivity drop, and increases thereafter above the stationary level. Investment also responds very strongly, decreasing by about ten percent in response to the shock.

The long-term debt share moves as a mirror image to short-term debt issuance: it increases on impact in response to the drop in aggregate productivity, decreases thereafter and remains below the stationary level for about 30 quarters. Market leverage increases initially. This effect is almost entirely due to asset prices as the value of equity is more responsive to the drop in aggregate productivity as the value of debt. As firms adjust their leverage downwards over time, market leverage decreases.

The default rate and credit spreads are counter-cyclical. Note that the long-term credit spread responds less than the short-term credit spread to the change in productivity, despite short-term debt having a lower default premium. In this model, this is mainly a composition effect: Since short-term borrowers will on average have lower productivity and a higher probability of default, a sudden recessionary shock will increase their default premia more than the default premia of high-quality, long-term borrowers.

6.2 Untargeted Cross-Sectional Moments

In Table 4, I show the unconditional distributions for the long-term debt share and market leverage of the model. I report the mean, standard deviation and the 25, 50 and 75 percent quantiles of the distribution. Note that I only target the mean for the long-term debt share and leverage. All numbers report percentages. The model can accurately capture that the mean for the long-term share is lower than the median, whereas the mean for market leverage is higher than the median. Furthermore, the model can capture the variability in the long-term debt share well. By construction, the upper quantile of the long-term debt share is too low, since even if the firm only uses long-term debt debt, about 20 percent of the debt will mature within the next year at all times. As for the market leverage, the model capture about half of the variation in the cross-section in the data.

In Table 5, I show the distribution of market leverage and the long-term debt share conditional on firm size. The model can broadly match these untargeted moments as well. I report conditional averages of those two variables for four size quantiles, sorted from smallest to largest. The model correctly predicts that larger firms choose a higher long-term debt share. As for the market leverage ratio, the model can match the market leverage for the

	Mean	Sd	Q25	Q50	Q75
Long-Term Share					
Model	67.1	20.2	57.1	78.2	81.5
Data	63.2	35.9	33.8	76.8	95.1
Market Leverage					
Model	15.1	9.5	7.9	13.4	21.2
Data	22.9	26.3	0.6	12.1	38.1

Table 4: Unconditional Distributions. This table shows moments for the unconditional, marginal distributions for the long-term share of debt, book leverage and market leverage. I report the mean, the standard deviation, and the 25, 50 and 75 percent quantiles for the model and the data. The model captures the fact that the mean long-term share is below the median long-term share and that the mean leverage is above the median leverage. I compute the moments from the model using a panel of 5000 firms over 250 quarters, where I discard the first 50 quarters. The reported numbers are the average over 250 simulations. The data moments are computed from quarterly Compustat data for the time period 1984Q1 to 2012Q4.

smallest firm size quantiles. However, in the model, leverage decreases at first for larger firms, whereas in the data, leverage mostly stays flat until it increases for the largest 25 percent of firms. The reason for this effect is that in this model, the risk-free coupon and hence the tax benefit of debt is relatively low. In the model, large firms issue long-term debt, primarily for the reason of the tax benefit. With a coupon of only about one percent per quarter, issuance costs and endogenous default premia, firms in the model are very cautious in setting their leverage.

6.3 The Cross-Section over the Cycle

To investigate how firms adjust their issuance in response to macroeconomic shocks, I group the firms from the simulated panel into 25 productivity quantiles and compute average short-term debt issuance, long-term debt issuance, equity issuance and the average long-term share for non-defaulting firms within these quantiles. The results are displayed in Figure 8. I plot averages conditional on the aggregate state: The dashed line is the average within a quantile conditional on being in one of the two expansionary states, the solid line the average within a quantile conditional on being in a recession state. Unconditionally, Figure 8 shows that high-productivity firms issue more long-term debt while low-productivity firms issue more short-term debt.

During a recession, the fraction of liquidity-constrained firms increases. This happens

	Firm Size Quartiles			
	1 (Smallest)	2	3	4 (Largest)
Long-Term Share				
Model	43.4	63.4	79.8	81.5
Data	41.9	56.2	70.6	80.9
Market Leverage				
Model	21.9	15.0	8.0	15.7
Data	19.1	20.7	23.6	28.3

Table 5: Conditional Distributions. In this table, I show the cross-sectional distributions of the long-term share of debt and market leverage conditional on the firm size. 1 to 4 denote the size quartiles for the firm distribution in the model and the data, respectively. The quartiles are sorted in increasing size. The numbers report the means of the respective variables, conditional on the firm being in a certain size quantile. The model predicts that large firms use a larger long-term debt share than small firms. The results for market leverage are mixed: The model can match the mean market leverage for small firms and predicts that very large firms (in quartile 4) have a higher leverage than large firms (in quartile 3). However, the data show a monotonic relation between leverage and size, which the model fails to replicate.

because cash flow decreases leading to operating losses since firms still have to cover the fixed cost of production. Additionally, due to the irreversibility of long-term debt, firms cannot immediately adjust their leverage downward toward a sustainable level.

Such liquidity-constrained firms issue short-term debt to cover a part of their liquidity shortfalls: Firms with a productivity of between 20 percent below to 20 percent above the mean productivity level will issue more short-term debt during recessions and more long-term debt during expansions.

It is therefore primarily due to these liquidity-constrained firms that aggregate debt maturity shortens. The fraction of liquidity-constrained firms remains persistently elevated and short-term debt issuance remains above the level before the initial shock.

6.4 Debt Overhang

I now discuss how debt overhang affects investment and debt issuance decisions in the model, and how the maturity structure of debt affects the severity of debt overhang. In Figure 9, I plot the capital and debt policy functions as a function of the maturity structure of outstanding debt for three different values of the level of outstanding debt. The blue line is for a zero debt firm, the red line for a firm with an intermediate level of debt and the yellow line for a firm with a very high level of debt. The other state variables are held constant at the mean level. The idiosyncratic productivity is set to a value below the mean, since the default probability at the mean productivity is very low.

For a zero leverage firm, the maturity structure of debt does not matter for firm decisions. However, for higher levels of outstanding debt, a larger long-term debt share of outstanding debt leads to a lower capital stock and a higher chosen level of debt. The reason for this is that a higher capital stock and a lower leverage are costly to the shareholder today. However, a large part of their future benefit accrues to the creditors instead of the shareholders, either through a lower default risk or through a higher recovery value in default. As a consequence, investment and lower leverage levels if the default rate is high constitute costly transfers from shareholders to creditors. With only short-term debt, this transfer is priced into the bond price of newly issued debt. Since with short-term debt, all future outstanding debt is issued in the current period, the value of this transfer is fully internalized by the shareholder. With long-term debt, however, the effect on the value of outstanding debt carried over from the last period is not internalized by the shareholder today, and as such the firm will use more leverage and a lower capital stock than an unlevered, otherwise identical firm.

7 Policy Experiments

In this section, I discuss whether the ability to choose between short-term debt and long-term debt is beneficial for firms. A second, related question is: Can a policymaker increase the aggregate value of firms by regulating the maturity structure of firms, for example through taxes on specific types of debt issuance? To give a tentative answer to this question, I compare the baseline model to three extreme cases:

In the first alternative model, firms can only issue short-term debt. This model avoids the incentive misalignment problem between shareholders and creditors with respect to future investment and debt issuance decisions. However, there is still an incentive misalignment with respect to the next period default decision.

In the second model, firms can only issue long-term debt. This case avoids the dilution of outstanding long-term debt through newly issued short-term debt. However, it creates an incentive misalignment with respect to future investment, leverage and default decisions.

Finally, in the third model, firms can issue neither short-term debt nor long-term debt. In this case, there is no incentive misalignment. However, firms cannot use the tax benefit of debt or issue cheap short-term debt in response to a liquidity shortfall and must rely on equity, which is expensive because it offers no tax benefit and has high issuance costs.

7.1 Normative Measures

I report three normative measures in the model below: The first one is the unconditional expected value of equity. This statistic is of interest, because shareholders control the firm and therefore directly set the investment policies, capital structure and default decisions of the firm. An increase in the equity value implies that shareholders would be better off if they could restrict themselves to not using a certain type of debt. I compute the unconditional expected value of equity as

$$\mathbb{E}[V] = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} V_{i,t} \tag{7.1}$$

that is, I take the average over the N firms with index i and T quarters with index t in the simulation. I transform this expected utility value of equity into the permanent consumption equivalent for the household according to

$$C^V = \left[(1-\beta)(1-\sigma)\mathbb{E}[V] + 1\right]^{\frac{1}{1-\sigma}}. \tag{7.2}$$

This way of calculating normative measures from the ergodic distribution of the model by averaging over N and T is essentially equal to the approach to calculate welfare in Krusell and Smith, Jr. (1999) in a model with heterogeneous households and aggregate shocks.

The second normative statistic is the total private value of the firm, which I denote by $V_{i,t}^P$. The total firm value is given by the value of current and future payments to shareholders and creditors. In case of default, shareholders get nothing and bondholders receive the recovery value of the firm. The total value is then given by

$$\begin{aligned} V_{i,t}^P = \Bigg[& \underbrace{D_{i,t} - EIC(D_{i,t})}_{\text{Current Payment to Shareholders}} + \underbrace{\left[\mu M_{i,t} + 1 - M_{i,t} + c\right] B_{i,t}}_{\text{Current Payment to Creditors}} \\ & + \underbrace{\mathbb{E}\left[\Lambda(Z_t, Z_{t+1}) V(\mathcal{S}_{i,t+1}, A_{i,t+1}, Z_{t+1}) | A_{i,t}, Z_t\right]}_{\text{Value of Future Payments to Shareholders}} \\ & + \underbrace{\left[Q_L(\mathcal{S}_{i,t+1}, A_{i,t}, Z_t) M_{i,t+1} + Q_S(\mathcal{S}_{i,t+1}, A_{i,t}, Z_t)(1 - M_{i,t+1})\right] B_{i,t+1}}_{\text{Value of Future Payments to Creditors}} \Bigg] \mathbb{1}_{(\text{No Default})} \\ & + \underbrace{\chi\left((1-\tau)(\Pi_{i,t} - \psi_0 - \delta K_{i,t}) + K_{i,t}\right)}_{\text{Value of the Firm in Default}} \mathbb{1}_{(\text{Default})} \end{aligned}$$

The third statistic is the social value of the firm, which includes the total firm value $V_{i,t}^P$ plus the total tax value of the firm, which I denote by $V_{i,t}^{\tau}$. I denote the social value of the firm by $V_{i,t}^S = V_{i,t}^P + V_{i,t}^\tau$. Taking the tax payments of the firm into account is important, because the tax payments are an additional output of the firm that needs to be accounted for. The tax value of the firm is the present value of discounted tax payments, which is given recursively by

$$V_{i,t}^\tau = \left[\underbrace{\tau(\Pi_{i,t} - \psi_0 - \delta K_{i,t} - cB_{i,t})}_{\text{Current Tax Payment, No Default}} + \underbrace{\mathbb{E}\left[\Lambda(Z_t, Z_{t+1}) V_{i,t+1}^\tau | A_{i,t}, Z_t\right]}_{\text{Discounted Value of Future Tax Payments}} \right] \mathbb{1}_{\text{No Default}}$$
$$+ \underbrace{\tau(\Pi_{i,t} - \psi_0 - \delta K_{i,t})}_{\text{Current Tax Payment, Default}} \mathbb{1}_{\text{Default}}$$

As for the equity value, I compute the unconditional expected private and social firm values in consumption equivalent units as in equations 7.1 and 7.2.

7.2 Results

Table 6 shows the results. I show the unconditional expected equity value, the private firm value and the social firm value. In addition, I show the standard deviation of investment, the autocorrelation of investment with GDP and the default rate.

	(1) Baseline	(2) Only ST Debt	(3) Only LT Debt	(4) Only Equity
Equity Value (% Δ from Baseline)		6.45	−1.14	7.49
Private Firm Value (% Δ from Baseline)		−4.22	−3.09	−4.50
Social Firm Value (% Δ from Baseline)		−2.84	−4.09	−2.45
Investment/Assets, Standard Deviation (%)	0.46	0.48	0.48	0.60
Investment/Assets, Correlation with GDP	0.76	0.77	0.76	0.70
Market Leverage (%)	14.18	2.79	14.32	0.00
Default Rate (%)	0.94	0.58	1.11	0.44

Table 6: Normative Results. This table shows how different welfare statistics change relative to the baseline model if the firms cannot use a certain type of debt. In addition, I report aggregate statistics for investment, leverage and the default rate.

While none of the models can improve upon the social firm value comparing the models with only one type of debt to the maturity choice model is still interesting. First, consider the model with only short-term debt. Compared to the baseline model, this model is worse from a firm value perspective. However, the equity value is higher than in the baseline model. The reason for this is that firms choose a much lower leverage level in this model compared to the baseline model. The fact that equity values can be higher if firms restrict themselves to short-term debt is not inconsistent with the observation that firms issue long-term debt in the unconstrained model: A high productivity, unlevered firms will find issuing long-term debt beneficial due to the tax benefit. Ex post, a situation can arise in which a levered firm will not reduce leverage even though an identical, unlevered firm will not issue debt. The reason is that reducing leverage mostly benefits the bondholders and as such is a costly transfer from shareholders to bondholders. In such a situation, the leverage will be lower and hence the equity value will be higher if the firm can restrict itself ex ante not to issue long-term debt.

The model with only long-term debt is worse along all dimensions: Equity values are lower, private and social firm values are lower, market leverage is higher and the default rate is higher than the baseline model. In the model with only long-term debt, firms cannot issue short-term debt if they are financially constrained, so they either have to issue expensive equity or long-term debt. Since long-term debt leads to particularly severe incentive problems in situations in which the default rate is high, long-term debt is also expensive if firms are financially constrained. As a consequence, the default rate is higher. This shows that the ability to issue short-term debt is valuable to firms in this framework.

Finally, I report results for the model with only equity. This model performs slightly better than the model with only short-term debt compared to the baseline model. However, it still seems that firms value the ability to issue debt and endogenously choose their debt maturity structure.

While the models with debt issuance do not differ very much in the cyclicality of investment, the cyclicality of investment in the model with only equity is lower, in the sense that the correlation with GDP is lower. However, the volatility of investment in this model is also higher. Debt overhang seems to affect mostly the default rate in the model, which is approximately halved.

8 Conclusion

I study the determinants of aggregate debt maturity dynamics in a quantitative model with rich cross-sectional heterogeneity. In the model, firms prefer long-term debt, because issuance costs imply that the tax advantage of long-term debt is much bigger than the tax advantage of short-term debt. Maturity dynamics are driven by liquidity constrained firms issuing short-term debt to cover liquidity shortfalls.

The model can explain about 30 percent of the variation in the aggregate maturity structure, and about 60 percent in the variation of the maturity structure of small firms. In addition, it can match the relation between firm size and the maturity structure in the cross-section.

Some interesting extensions of the model are left for future research. For example, I abstract from cash holdings and credit lines, which are additional sources of funds firms can use to reduce the incidence of liquidity shortfalls. There are no labor market frictions in the model, which might be another important reason to issue short-term debt through a working capital requirement as in Jermann and Quadrini (2012).

Finally, the question of regulation arises naturally, given the agency problem outlined in the model. Can and should regulatory authorities develop rules such that the preferences of bondholders are better reflected in the decisions of firms? The results in this paper suggest that such rules can lead to higher equity values and lower default rates.

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9 Appendix

As in section 4, I assume that the bond prices are differentiable once in K', B', M' and A and value function is differentiable once in K, B, M and A. Further, I assume that the short-term and the long-term bond price are non-decreasing in A.

9.1 Derivation of the Derivatives in the Text

9.1.1 Value Function

The first order condition for the long-term debt share is given by

$$\frac{\partial V}{\partial M'} = \Bigg[(Q_L - Q_S)B' + \\ \frac{\partial Q_S}{\partial M'}(1-M')B' + \frac{\partial Q_L}{\partial M'}\left(M'B' - (1-\mu)MB\right)\Bigg](1+\lambda_D) + \\ \mathbb{E}\left[\Lambda(Z,Z')\frac{\partial V'}{\partial M'}|\mathcal{Y}\right] = \lambda_{M,1} - \lambda_{M,0}$$

The envelope condition yields

$$\frac{\partial V}{\partial M} = (1-\mu)\xi_1 B(1+\lambda_D)$$

if the firm is in a no default state and

$$\frac{\partial V}{\partial M} = 0$$

if the firm is in a default state.

The derivative of the value function with respect to A is given by

$$\frac{\partial V}{\partial A_i} = (1-\tau)K^{\alpha} + \frac{\partial Q_S}{\partial A}(1-M')B' + \frac{\partial Q_L}{\partial A}(M'B' - (1-\mu)MB) + \\ \mathbb{E}\left[\Lambda(Z,Z')\int_{-\infty}^{\infty}\frac{\partial V'}{\partial A_i'}\frac{\partial A_i'}{\partial A_i}f(A_i'|A_i)dA_i'|Z\right]$$

Using $\frac{\partial A_i'}{\partial A_i} = \rho\frac{A_i'}{A_i}$ yields

$$\frac{\partial V}{\partial A_i} = (1-\tau)K^{\alpha} + \frac{\partial Q_S}{\partial A}(1-M')B' + \frac{\partial Q_L}{\partial A}(M'B' - (1-\mu)MB) + \\ \mathbb{E}\left[\Lambda(Z,Z')\int_{-\infty}^{\infty}\frac{\partial V'}{\partial A_i'}\rho\frac{A_i'}{A_i}f(A_i'|A_i)dA_i'|Z\right]$$

in non-default states and

$$\frac{\partial V}{\partial A_i} = 0$$

in default states.

Expanding the recursion, one can see that this value function derivative depends on the entire future path of the derivatives of the production function and the bond prices with respect to how the sequence of productivity changes with a current change in the productivity. The production function derivative is positive. As shown below, the short-term bond price derivative is nonnegative. It not possible to analytically determine the sign of these bond prices. If these derivatives are nonnegative, which I assume and which is the case in simulations, the derivative of the value function with respect to idiosyncratic productivity is positive, i.e. $\frac{\partial V}{\partial A_i} > 0$, if the firm is in a non-default state.

9.1.2 Short-Term Bond Price

The sign of the short-term bond price derivative depends on how the default cutoff varies with the share of long-term debt. The default cutoff function $a^*(K_i, B_i, M_i, Z)$, which exists if the value function derivative with respect to idiosyncratic is positive, i.e. $\frac{\partial V}{\partial A_i} > 0$, is implicitly defined by the equation

$$V(K_i, B_i, M_i, a^*(K_i, B_i, M_i, Z), Z) = 0$$

Using the implicit function theorem, the derivative for a^* is given by:

$$\frac{\partial a^*}{\partial M_i} = -\frac{\frac{\partial V}{\partial M_i}}{\frac{\partial V}{\partial A_i}}$$

Since $\frac{\partial V}{\partial M_i} > 0$ and $\frac{\partial V}{\partial A_i} > 0$, $\frac{\partial a^*}{\partial M_i} < 0$, i.e. the default threshold in the next period is decreasing in M.

With this information and using Leibniz rule, the short-term bond price derivative with respect to the long-term debt share can be computed as

$$\frac{\partial Q_S}{\partial M_i'} = \mathbb{E}\left[\Lambda(Z, Z')\left(R(K_i', B_i', a^*, Z') - (1+c)\right) f(a^*|A)\frac{\partial a^*}{\partial M_i'}|Z\right]$$

Since $1 + c \geq R(K, B, a^*, Z)$, i.e. the creditor cannot recover more than his claim per unit of the bond in default, and $\frac{\partial a^*}{\partial M'} \leq 0$, i.e. the default cutoff for the next period is lower if the long-term share in the next period is higher, this derivative is positive.

9.1.3 Long-Term Bond Price

For the long-term bond price, the derivative with respect to the long-term debt share is given by

$$\frac{\partial Q_L}{\partial M'} = \mathbb{E}\left[\Lambda(Z,Z')\left[[R(K_i', B_i', a^*, Z') - (\mu + c + (1-\mu)Q_L')]\, f(a^*|A)\frac{\partial a^*}{\partial M_i'}\right.\right.$$
$$\left.\left. +(1-\mu)\int_{a^*}^{\infty}\left(\frac{\partial Q_L'}{\partial K''}\frac{\partial K''}{\partial M'} + \frac{\partial Q_L'}{\partial B''}\frac{\partial B''}{\partial M'} + \frac{\partial Q_L'}{\partial M''}\frac{\partial M''}{\partial M'}\right) f(A'|A)dA\right]|Z\right]$$

It is not possible to determine the sign of this derivative, for two reasons: First, it is not necessarily the case that $(\mu+c+(1-\mu)Q_L') > R(K_i', B_i', a^*, Z')$, since the continuation bond price $Q_L' = Q_L((K_i', B_i', M_i', a^*, Z'))$, which represents a part of the claim of the creditor to the firm, might be low.

Second, the future bond price also changes with future firm decisions, which depend on the policy for the long-term debt share today. Since the policy functions are unknown, it is not possible to determine these derivatives.

In general, the long-term bond price can therefore decrease in the long-term debt share. This can be the case in two situations: First, if defaulting would actually lead to a higher payoff for creditors. Second, if a higher long-term share increases default risk after the next period through adversely affecting the firm policies in the next period.

9.2 Estimation of the Production Function

I follow Cooper and Haltiwanger (2006) and rewrite the production function in logs:

$$\log \Pi_{i,t} = \log A_{i,t} + \log Z_t + \alpha \log K_{i,t}$$
$$\log A_{i,t} = \rho_A \log A_{i,t-1} + \sigma_A \varepsilon_{i,t}^A$$

Iterating once and substituting for $\log A_{i,t}$ yields a semi-differenced version of the production function that can be estimated:

$$\log \Pi_{i,t} = \rho_A \log \Pi_{i,t-1} + \alpha \log K_{i,t} - \rho_A \alpha \log K_{i,t-1} + \eta_t - \rho_A \eta_{t-1} + \nu_{i,t} \tag{9.1}$$

where $\nu_{i,t} = \varepsilon_{i,t}^A - \rho_A \varepsilon_{i,t-1}^A$ I use this equation to estimate α. Since this estimation does not permit fixed costs and hence does not allow for negative profits, I use sales instead of profits as a proxy variable.

I recover a residual productivity process from $\log \hat{S}_{i,t} + = \log \Pi_{i,t} - \hat{\alpha} \log K_{i,t}$. Then, I decompose this process into orthogonal aggregate and idiosyncratic component by estimating

$$\log \hat{S}_{i,t} = \log \hat{A}_{i,t} + \beta_t d_t \tag{9.2}$$

where d_t is a time dummy. Then, I estimate ρ_A and σ_A by fitting an AR(1) process to $\log \hat{A}_{i,t}$. Similarly, I fit an AR(1) process to the aggregate series given by the time dummy coefficients $\hat{\beta}_t$ to recover ρ_Z and σ_Z. Finally, I estimate γ_C using the equation

$$\log C_t = \gamma_0 + \gamma_C \log Z_t \tag{9.3}$$

Depending on whether I use the book value or the replacement value for capital, I estimate very different parameters for the returns to scale parameter α. In the baseline, I use the value I estimate using the book value of capital. In this version, the returns to scale are more strongly decreasing.

9.3 Numerical Algorithm

I solve the model using value function iteration. For capital and debt, I use grids with 10 points. For the share of long-term debt, I use 10 grid points. For the idiosyncratic productivity shock, I use 9 and for the aggregate productivity shock 3 grid points. The state grid consists therefore of 27000 grid points. The model is subject to all three curses of dimensionality: First, the state grid is large. Second, the choice grid is large. Third, the presence of two stochastic processes requires many nodes for the calculation of the expectations. In addition, each maximization step requires interpolation on 5 functions.

For the calculation of expectations, I approximate the functions $EV(\mathcal{S}_i', A_i, Z)$, $Q_S(\mathcal{S}_i', A_i, Z)$, and $Q_L(\mathcal{S}_i', A_i, Z)$ using linear interpolation on $(\mathcal{S}_i', A_i)$. I treat A_i as continuous, using Gauss-Legendre quadrature to calculate the expectation over A_i'. I compute these expectations on a Grid with 15 points for K_i', B_i' and M_i' and 9 points for A_i. I use 50 quadrature points to compute the expectations approximating the integrals piecewise in the default and no default regions, using the exact default cutoffs for A. I treat Z as discrete, using the Rouwenhorst algorithm as explained in Kopecky and Suen (2010). That way, I only have to calculate expectations only once outside the maximization step instead of many times within the maximization step. This mitigates the curse of dimensionality for the expectation.

For the maximization step, I use the recursive nature of the solution method to let the firm choose decision variables sequentially. I use 250 grid points on the choice grid for capital, 250 points for debt and 100 for the long-term debt share. The algorithm is as follows:

1. Set up a grid for K, B, M, A and Z.

2. Make an initial guess for Q_S, Q_L, $\tilde{Q}_S$, $\tilde{Q}_L$, EV, B' and M'

3. At iteration n,

   - Given $Q_S^{(n-1)}$, $Q_L^{(n-1)}$, $\tilde{Q}_S^{(n-1)}$, $\tilde{Q}_L^{(n-1)}$, $EV^{(n-1)}$, $B'^{(n-1)}$ and $M'^{(n-1)}$ find $\hat{K}'^{(n)}$.
   - Given $Q_S^{(n-1)}$, $Q_L^{(n-1)}$, $\tilde{Q}_S^{(n-1)}$, $\tilde{Q}_L^{(n-1)}$, $EV^{(n-1)}$, $\hat{K}'^{(n)}$ and $M'^{(n-1)}$ find $\hat{B}'^{(n)}$.
   - Given $Q_S^{(n-1)}$, $Q_L^{(n-1)}$, $\tilde{Q}_S^{(n-1)}$, $\tilde{Q}_L^{(n-1)}$, $EV^{(n-1)}$, $\hat{K}'^{(n)}$ and $\hat{B}'^{(n)}$, find $\hat{M}'^{(n)}$.

4. Check that $(K'^{(n)}, B'^{(n)}, M'^{(n)})$ is superior to $(K'^{(n-1)}, B'^{(n-1)}, M'^{(n-1)})$.

   - If yes, keep the new policy functions:
     $(K'^{(n)}, B'^{(n)}, M'^{(n)}) = (\hat{K}'^{(n)}, \hat{B}'^{(n)}, \hat{M}'^{(n)})$.
   - If no, use the old policy functions:
     $(K'^{(n)}, B'^{(n)}, M'^{(n)}) = (K'^{(n-1)}, B'^{(n-1)}, M'^{(n-1)})$.

5. Given $(K'^{(n)}, B'^{(n)}, M'^{(n)})$, update to find $Q_S^{(n)}$, $Q_L^{(n)}$, $\tilde{Q}_S^{(n)}$, $\tilde{Q}_L^{(n)}$ and $EV^{(n)}$.

6. Iterate until $(Q_L^{(n)} - Q_L^{(n-1)})$ and $(EV^{(n)} - EV^{(n-1)})$ are sufficiently close to zero. I require that the maximum absolute errors should be smaller than 0.01 for both the expected value function and the bond price.

The equilibrium for the infinite horizon model might not be unique. I therefore follow Hatchondo and Martinez (2009) and approximate the infinite horizon value functions by finite horizon value functions for the first period.

The bond price function requires some smoothing to converge. I use a smoothing weight of 0.1 on the bond price and no smoothing on the value function.

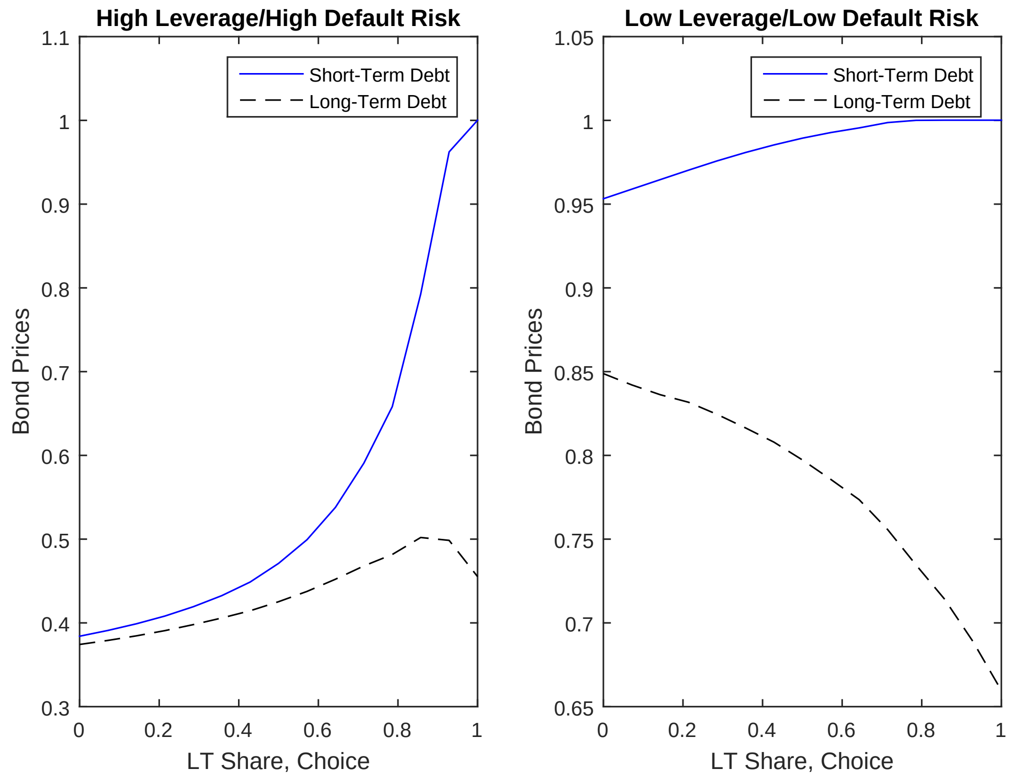

Figure 3: Bond prices as a function of the long-term debt share choice M'. The left panel depicts a situation with high debt and high next period default risk, the right panel depicts a situation with low debt and low next period default risk. Capital, productivity and the aggregate state are the same in the left and the right panel. The short-term bond price is monotonically increasing in the long-term debt share, because a higher long-term debt share reduces future default risk. For the long-term bond price, there are two opposing effects: On the one hand, a higher long-term debt share reduces next period default risk. On the other hand, a higher long-term debt share increases default risk after the next period. The latter channel is particularly important, if next period default risk is low.

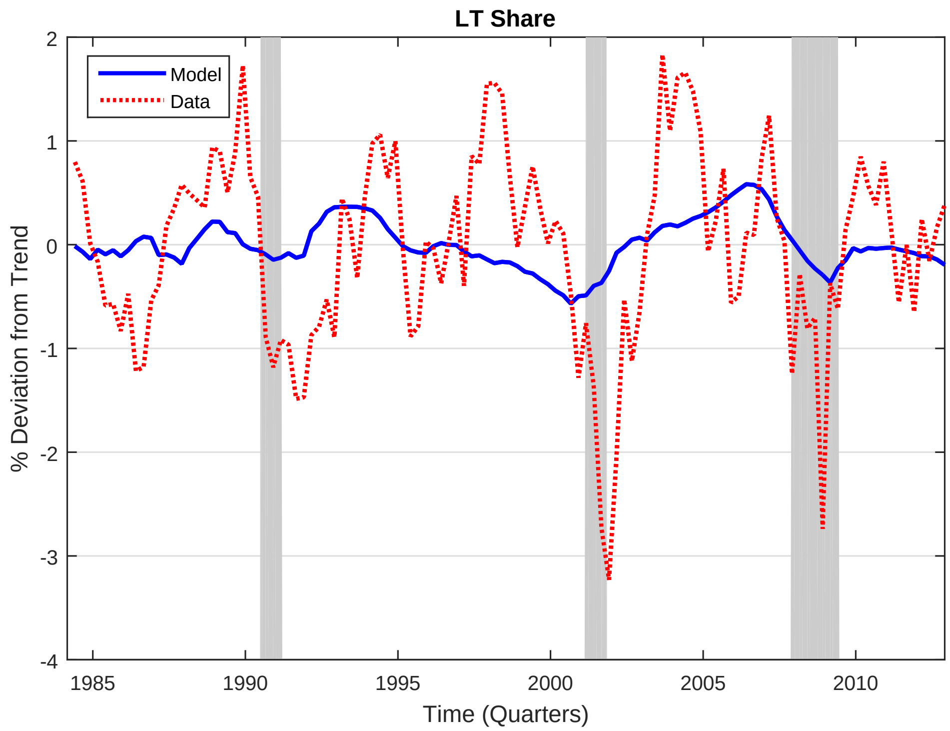

Figure 4: Time series for the aggregate long-term debt share in the model and the data. The red, dashed line is the long-term debt share from the data. The blue, solid line is the model-implied long-term debt share, computed according to the data-consistent expression.

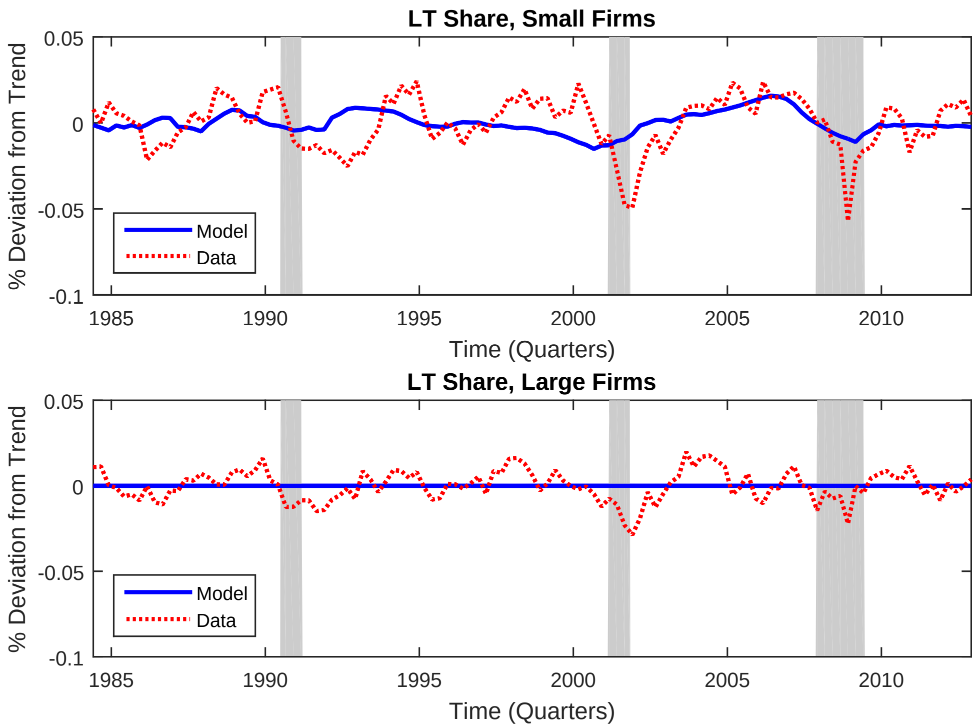

Figure 5: Time series for the long-term debt share in the model and the data for small and large firms. The red, dashed line is the long-term debt share from the data. The blue, solid line is the model-implied long-term debt share, computed according to the data-consistent expression.

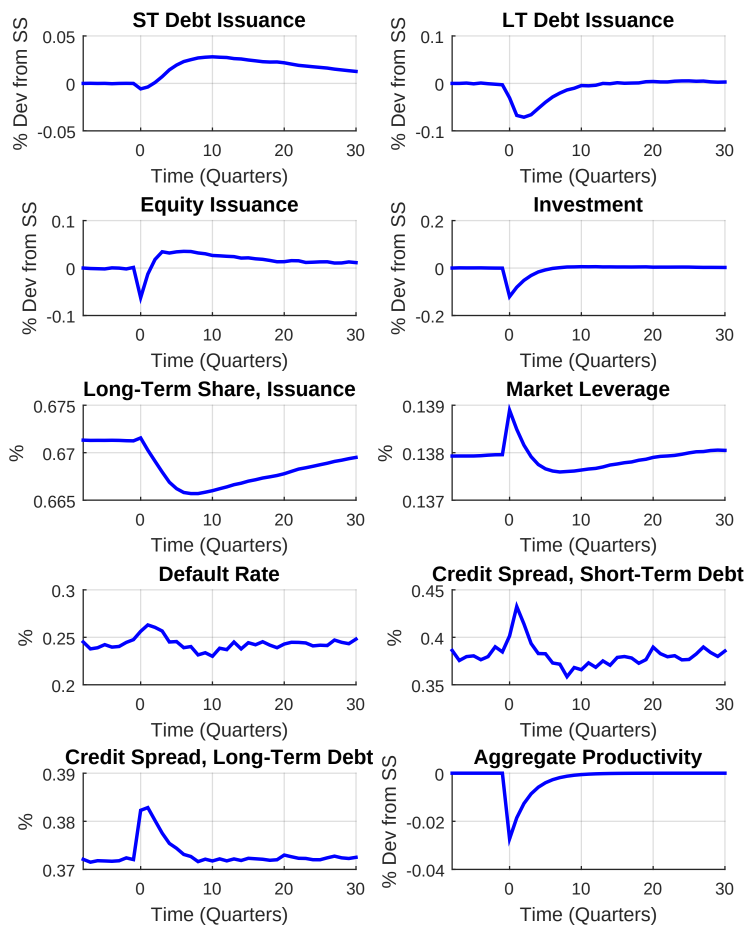

Figure 6: Impulse Response Functions to a 1 standard deviation *negative* productivity shock. The blue solid line reports the average impulse response across 100 simulations. The red dashed lines report the 95 percent confidence interval. Differences arise due to sampling uncertainty caused by different realizations the idiosyncratic shocks. This sampling uncertainty is substantial, even with 10000 simulated firms.

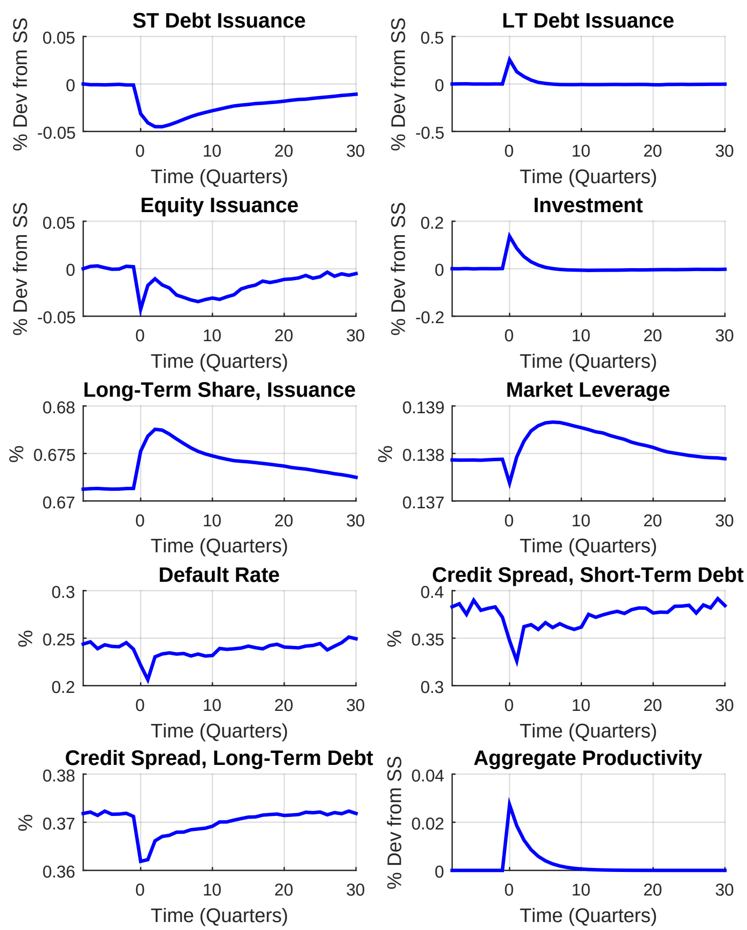

Figure 7: Impulse Response Functions to a 1 standard deviation *positive* productivity shock. The blue solid line reports the average impulse response across 100 simulations. The red dashed lines report the 95 percent confidence interval.

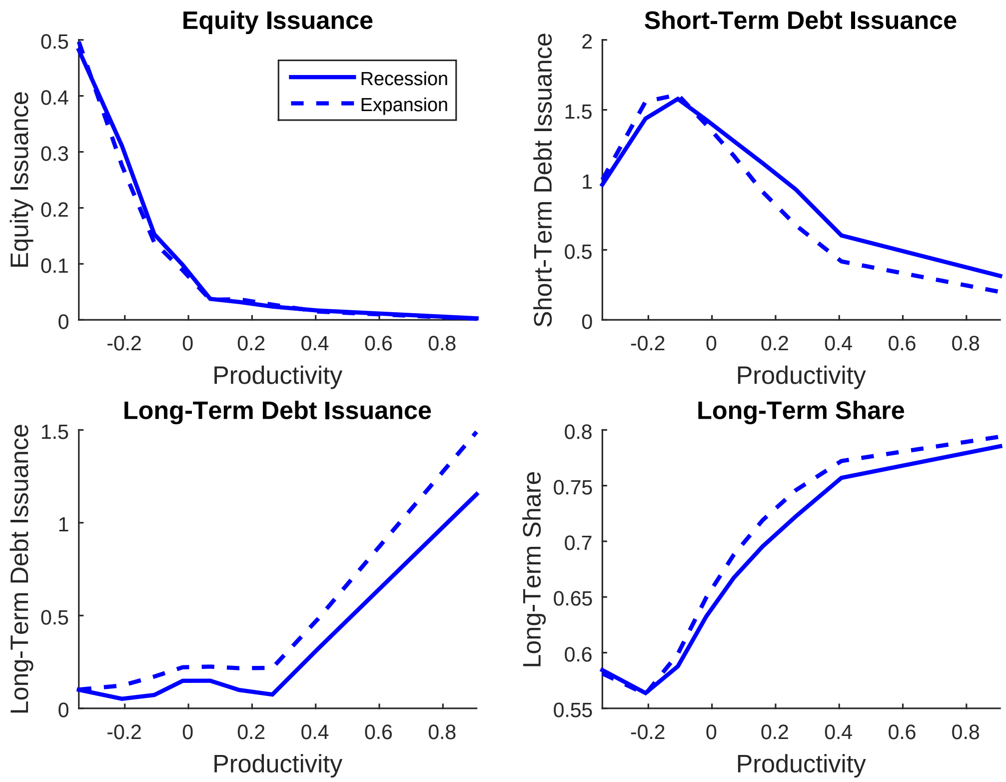

Figure 8: Issuance behavior by productivity quantiles over the business cycle. Figure 8 is computed from simulated data of of a panel of 5000 firms over 1000 periods. The dashed line reports quantiles conditional on aggregate productivity being in the highest state, the solid line the issuance behavior conditional on being in the lowest two state.

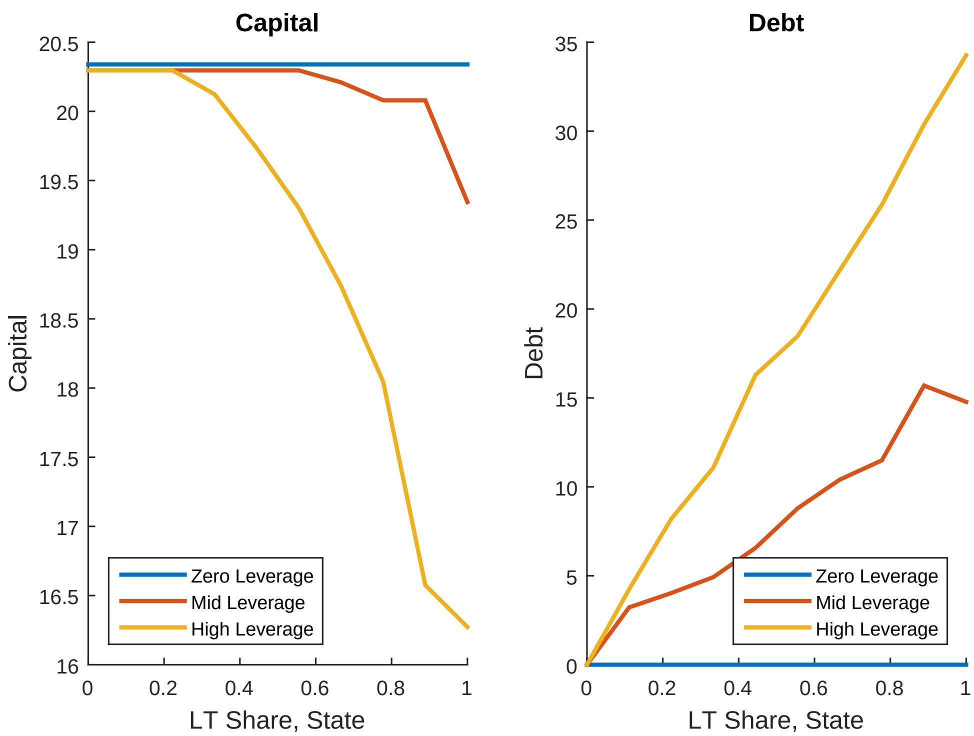

Figure 9: Policy functions for capital and debt as a function of the long-term debt share state variable for three different values of the debt state variable. The capital stock is held constant at the mean level, Idiosyncratic productivity at a low productivity state and aggregate productivity at the mean level. Figure 9 shows that for a given level of debt, firm policies vary with the debt maturity structure of the firm. In particular, for a higher share of long-term debt, firms choose a lower capital stock and a higher level of debt.