Liquidity Traps, Capital Flows and Currency Wars*

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Abstract

We analyze the role of capital flows and exchange rates in the global economy's adjustment during the Great Recession, a period when many advanced economies, but not emerging markets, were pushed to the zero bound on interest rates. We establish three main results. First, when the North hits the zero bound, capital outflows alleviate the recession by reallocating demand to the South and switching expenditure toward North goods. Second, even a regime of free capital mobility falls short of supporting constrained efficient demand and expenditure reallocations as it induces too little downstream (upstream) flows during (after) the liquidity trap. Third, non-cooperative capital flow management policies are driven by a motive to manipulate terms of trade and are in conflict with the objective of aggregate demand stabilization which would be attained under cooperation. Our results emphasize a novel dimension of policy coordination in a liquidity trap.

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1 Introduction

Following the 2007-2008 financial crisis, the world economy experienced a recession that originated in the United States before spreading to other countries. Central banks responded by engaging in expansionary monetary policy, and interest rates were slashed most dramatically in some major advanced economies, where they hit the zero bound. The policy response was more subdued in emerging economies, which were generally less affected by the financial crisis. The resulting interest rate differential between advanced and emerging economies, however, was associated with a surge in capital inflows and currency appreciation pressure in the latter. Fearing an erosion of external competitiveness, policymakers in some emerging market countries (most notably India and Brazil) adopted measures to slow down capital inflows. Meanwhile, the aggressive response of advanced economies' monetary authorities generated a heated debate about international spillovers and the need for policy coordination in a liquidity trap.[1] The key questions in this debate concerned the desirability of capital flows and associated terms of trade movements. In particular, what role do capital flows play in the global macroeconomic adjustment when the world economy is subject to large asymmetric shocks? Should free capital flows be expected to fulfill this role efficiently? Is the zero bound on interest rates critical in this regard? Should countries actively manage their capital account in these circumstances? Our goal is to make progress on these issues.

To this end, we use a multi-country version of the New Keyensian model of Gali and Monacelli (2005). We assume flexible exchange rates, divide the world economy into two blocks (North and South), and model a liquidity trap as the consequence of a large unanticipated negative demand shock. We use a model with a continuum of countries (rather than a two-country model) in order to highlight policy spillovers both across (North-South) and within (South-South and North-North) country blocks. We adopt a deterministic continuous time formulation, which affords us analytical tractability.

We start by analyzing the optimal monetary policy response (under commitment) of an individual country to a negative demand shock in the presence of a zero lower bound (ZLB) constraint on the interest rate. A large enough shock makes the ZLB bind. By prolonging the period for which the interest rate is kept at zero, optimal policy aims to limiting the size of the initial contraction in output (see Eggertsson and Woodford 2003 and Werning 2012). Monetary policy provides this stabilization by affecting inter-temporal prices, as in a closed economy. Openness provides additional stability by allowing monetary policy to also alter

[1] This debate is commonly associated with statements by Reserve Bank of India's President Raghuram Rajan and Brazil's Finance Minister Guido Mantega. Rajan has repeatedly asked advanced economies to be mindful of international spillovers emanating from their policy decisions and asked for more international policy coordination (see, for instance, Rajan 2010 and Rajan 2014). Mantega is reported to have described the climate as that of an "international currency war" (see Financial Times of September 27, 2010).

intra-temporal prices favorably. Our paper's main insights relate to the way capital mobility interacts with monetary policy in shaping the trajectory of these prices.

During the Great Recession, a large number of advanced economies hit the ZLB, but virtually no emerging economies did. Motivated by this, we model a situation in which only a region of the world economy (the North) experiences a liquidity trap. Under the assumption that all countries conduct monetary policy optimally, we show that the degree of capital mobility critically influences the smoothness of the global macroeconomic adjustment. In particular, the stabilizing effect of openness on the North's output is driven by inter-temporal trade rather than intra-temporal trade between countries. Under free capital flows, the North's temporary desire to save during the liquidity trap is accommodated by an accumulation of claims vis-à-vis the South, who temporarily enjoy cheaper consumption. Capital, thus, flows downstream during the liquidity trap. These flows are accompanied by an exchange rate adjustment on account of the persistent interest rate differentials between the South and the North. South currencies appreciate on impact, and then continuously depreciate during the time spent by the North at the ZLB. In contrast, under closed capital accounts, despite the possibility of intra-temporal trade, the North is unable to save by running a current account surplus and South currencies depreciate rather than appreciate on impact. Thus, by inducing a larger deterioration of the North's terms of trade, greater capital mobility encourages a global switch in expenditures towards the North good in the initial stage of the liquidity trap (when demand is most deficient). This helps reduce the severity of the recession in the North.

Having established the positive result that capital flows promote a smoother adjustment in countries experiencing a liquidity trap, we investigate whether a regime of free capital mobility fulfills this stabilizing role *efficiently*. To this end, we formulate a planning problem in which a global planner chooses a path of taxes or subsidies on downstream capital flows to maximize world welfare, subject to monetary policy being set optimally by individual countries. We find that while away from the zero bound, a regime of free capital mobility is constrained efficient, it is constrained *inefficient* when a region of the world economy faces a binding ZLB. This inefficiency can be traced back to an aggregate demand externality resulting from the combination of output being demand determined and monetary policy being constrained by the zero bound in some countries. Agents do not internalize that their savings decisions have effects on both inter- and intra-temporal prices. In conjunction with nominal rigidities, these decisions affect the level of economic activity. Away from the ZLB, optimal monetary policy is able to address the externality. However, at the ZLB it is unable to do so, and capital flow management can serve as a useful complement.

The efficient capital flow regime entails larger cross-border capital flows than the benchmark with free capital mobility. Capital flow taxes allow exchange rate dynamics to decouple from interest rate dynamics, and thereby relax the ZLB constraint in the North without inflicting much harm on the South. In particular, the taxes allow the implementation of a steeper exchange rate path during the liquidity trap, and a flatter path after the trap. This results in an extra titling of the terms of trade path that shifts expenditure toward North goods precisely when the demand for these goods is low, and away from them when the demand is high. It also shifts expenditure away from and toward South goods, but these effects are offset by monetary policy in the South, which is not constrained by the ZLB.

At first glance, our finding that capital does not flow sufficiently in a liquidity trap seemingly stands in sharp contrast to a recent literature on capital flow management that argues that free capital flows might instead be excessively volatile (see our literature review below). This literature, however, studies capital flow management from the perspective of individual capital flow recipient countries, whereas we take a global efficiency standpoint. To illustrate that this distinction is crucial, we also consider a setting where countries manage capital flows non-cooperatively. In this case, we show that the incentives of individual countries to alter capital flows respond to a desire to manage dynamic terms of trade.[2] In particular, South countries find it optimal to restrict capital inflows during the liquidity trap. Capital flow management policies that are optimal from the perspective of recipients of inflows hence conflict with macroeconomic stabilization in countries experiencing a liquidity trap. A Nash equilibrium where all countries manage their capital account optimally features a form of currency war, with subsidies to outflows by the North and taxes on inflows by the South nearly neutralizing each other. Our analysis thus points to the adverse effects of uncoordinated capital controls in liquidity trap episodes, and highlights the importance of global policy coordination in this area.

The rest of the paper is organized as follows. We conclude the introduction with a review of the related literature. We then describe the model in Section 2. Section 3 highlights the role of capital flows at the zero bound, Section 4 analyzes capital flow efficiency, and Section 5 studies non-cooperative capital flow management. Section 6 concludes.

Related literature The paper first relates to a large literature on optimal policy at the ZLB that developed following the seminal work of Keynes (1936), Krugman (1998) and Eggertsson and Woodford (2003).[3] Our continuous time formulation of the optimal monetary policy problem is most closely related to Werning (2012)'s work in the closed economy context. Especially relevant for our work is the literature on optimal monetary policy at the ZLB open economies. Svensson (2001, 2003, 2004) argues that a *foolproof way* of escaping a liquidity trap

[2]This motive arises in every open economy model where countries have some degree of market power over a good they trade. It applies to capital exporters and importers alike, and prevails independently from zero lower bound considerations.

[3]See, for instance, Eggertsson and Woodford (2004b,a), Eggertsson (2006, 2010), Christiano et al. (2011), Guerrieri and Lorenzoni (2011), Eggertsson and Krugman (2012), Werning (2012), Correia et al. (2013), and Benigno and Fornaro (2015).

in a small open economy involves devaluing the currency, and temporarily adopting a peg and a price-level target. Jeanne (2009), Haberis and Lipinska (2012) and Fujiwara et al. (2013) study two-country models in which a large shock in one country can lead to a worldwide liquidity trap. In a similar context, Cook and Devereux (2013) find that in a global liquidity trap caused by a negative shock at home, the terms of trade may respond perversely and make it optimal for a world planner to raise the foreign interest rate in order to promote expenditure switching to toward home goods. We argue that capital flow taxes might be a complementary instrument to achieve this goal. Perhaps even closer to our work is the paper by Devereux and Yetman (2014), which argues that reducing capital mobility in a liquidity trap is not desirable as long as monetary policy is set optimally. Our paper is consistent with this idea, and goes one step further by showing that laissez-faire is in fact dominated by a policy regime that actively fosters capital flows. Overall, our contribution to the literature on optimal policy at the ZLB is twofold. First, we provide an analytical characterization of optimal monetary policy in an open economy, notably through a comparison of the optimal ZLB exit time across capital flow regimes. Second, we consider capital flow taxes/subsidies as an additional tool to overcome the limitations of monetary policy at the ZLB. In particular, we analytically characterize and compare optimal cooperative and non-cooperative capital flow management regimes, thereby highlighting the importance of international policy coordination.[4]

Second, the paper connects to a large literature on capital flow regulation in emerging markets. Several recent papers have developed arguments in favor of capital account interventions based on imperfections in financial markets (e.g. Caballero and Krishnamurthy 2001, Korinek 2007, 2010, Jeanne and Korinek 2010, Bianchi 2011).[5] Others have shown that imperfections in goods markets may also provide a rationale for the optimal use of capital controls. DePaoli and Lipinska (2012) and Costinot et al. (2014) emphasize the role of market power and dynamic terms of trade management. Farhi and Werning (2012a, 2014) and Schmitt-Grohe and Uribe (forthcoming) stress the role of nominal rigidities. All these papers study optimal capital flow management from the perspective of individual countries. In contrast, we stress the benefits of capital flow taxes/subsidies in promoting efficiency at the level of the world economy.

More generally, our work also speaks to a recent literature on aggregate demand externalities. Farhi and Werning (2013) develop a general theory of aggregate demand externalities in economies with nominal rigidities and constraints on monetary policy, of which Farhi and Werning (2012a,b), Korinek and Simsek (forthcoming) and our paper can be seen as applications, pertaining to, respectively, Mundell's trilemma, fiscal unions, macro-prudential policy

[4]Korinek (2014) (section 5.2) also analyzes the use of capital flow taxes at the ZLB in a small open economy that may fall into a liquidity trap as a result of a drop in the exogenous world interest rate.

[5]Gabaix and Maggiori (forthcoming) also show that in the presence of financial frictions, capital controls can increase the potency of currency market interventions as a tool to combat exchange rate movements generated by financial turmoil.

ahead of a liquidity trap, and stimulative capital flow management during a global liquidity trap.

Finally, the paper also relates to contemporaneous work by Caballero et al. (2015) (CFG) and Eggertson et al. (2015) (EMSS). Like us, these authors study the interplay between international capital flows and liquidity traps. However, their focus is on the steady state analysis of permanent liquidity traps resulting in secular stagnation, while we emphasize transitional dynamics during temporary liquidity trap episodes. As a result, while interest rate policy is permanently impotent in their frameworks, it remains a key determinant of the short-run dynamics in our analysis through forward guidance. With respect to dealing with the multilateral effects of using tools other than monetary policy in a liquidity trap, our papers are complementary: while CFG and EMSS emphasize public debt issuance and fiscal policy, we focus on capital flow management policy and in particular, on the conflict arising between the dictates of global efficiency and the incentives of individual countries in that regard.

2 Model

The world economy consists of a unit mass of countries, separated into two blocks. North economies consist of the countries for which $k \in [0, x]$ and South economies consists of the countries for which $k \in (x, 1]$.[6] Following a large body of literature, we adopt a parameterization featuring unitary inter- and intra-temporal elasticities of substitution. As is well known, this parameterization, popularized by Cole and Obstfeld (1991), results in economies being insular with respect to foreign monetary policy. As a result, it enables us to streamline cross-border spillovers arising from demand shocks and capital flow management policies.[7] We elaborate on these issues at the end of the section.

2.1 Households

In each country k (we will refer to country k as the 'home' country for ease of exposition), there is a representative household with preferences represented by the utility functional

$$\int_0^\infty e^{-\int_0^t (\rho + \zeta_{k,h}) dh} \left[\log \mathbb{C}_{k,t} - \frac{(N_{k,t})^{1+\phi}}{1+\phi} \right] dt, \tag{1}$$

[6] We think of the North block as representing *demand deficient* economies and of the South as standing for the rest of the world.

[7] The Cole-Obstfeld parameterization also has the advantage of decisively improving the tractability of the non-linear model and has been used extensively in the open economy literature. It is known that under this parameterization, the model would have a log-linear structure absent discount rate shocks and capital flow taxes. With these features, the model does not have an exact log-linear structure but it remains analytically tractable.

where $\mathbb{C}_{k,t}$ is consumption, $N_{k,t}$ is labor supply, ϕ is the inverse Frisch elasticity of labor supply, ρ is the (long-run) discount rate and $\zeta_{k,t}$ is a time-varying and country-specific preference shifter. We will refer to a negative realization of ζ_k as a *negative demand shock*, as such a shock lowers the demand for current consumption relative to future consumption (and hence increases the desire to save). The consumption index $\mathbb{C}_{k,t}$ is defined as

$$\mathbb{C}_{k,t} \equiv (1-\alpha)^{1-\alpha}\alpha^{\alpha}\left(C^H_{k,t}\right)^{1-\alpha}\left(C^F_{k,t}\right)^{\alpha} \tag{2}$$

where C^H_k denotes an index of domestically produced varieties, C^F_k is an index of imported goods and α is a home bias parameter representing the degree of openness. Letting $l \in [0,1]$ index varieties, we define $C^H_k \equiv \left[\int_0^1 C^H_k(l)^{\frac{\epsilon-1}{\epsilon}}dl\right]^{\frac{\epsilon}{\epsilon-1}}$, where $C^H_k(l)$ denotes country k's consumption of variety l produced domestically, and $\epsilon > 1$ is the elasticity of substitution between varieties produced within a given country. Similarly, we define $C^F_k \equiv \exp\left(\int_0^1 \log C^j_k dj\right)$ and $C^j_k \equiv \left[\int_0^1 C^j_k(l)^{\frac{\epsilon-1}{\epsilon}}dl\right]^{\frac{\epsilon}{\epsilon-1}}$, where C^j_k (resp. $C^j_k(l)$) denotes country k's consumption of the final good (resp. variety l) produced in country j.

The household's budget constraint is given by

$$\begin{aligned}\dot{a}_{k,t} &= i_{k,t}a_{k,t} + W_{k,t}N_{k,t} + T_{k,t} - \int_0^1 P^k_{k,t}(l)C^H_k(l)dl - \int_0^1\int_0^1 P^j_k(l)C^j_k(l)dldj \\ &\quad + \int_0^1 \left[i_{j,t} - i_{k,t} + \tau_{k,t} - \tau_{j,t} + \frac{\dot{\mathcal{E}}^j_{k,t}}{\mathcal{E}^j_{k,t}}\right]\mathcal{E}^j_{k,t}D^j_{k,t}dj\end{aligned} \tag{3}$$

where $a_{k,t} \equiv \int_0^1 \mathcal{E}^j_{k,t}D^j_{k,t}dj$ are net assets expressed in country k's own currency, $\mathcal{E}^j_{k,t}$ is the nominal exchange rate between country j and k, $D^j_{k,t+1}$ are the bonds issued by country j and held by country k at time t, $W_{k,t}$ is the nominal wage and $T_{k,t}$ denotes lump-sum transfers including the payout of domestic firms. We explicitly allow for taxes and subsidies on capital flows. $\tau_{k,t}$ is a tax on capital inflows (or a subsidy on capital outflows) in country k, and similarly $\tau_{j,t}$ is a tax on capital inflows (or a subsidy on capital outflows) in country j. The proceeds of these taxes are rebated lump sum to the households of country k and j, respectively.

The lump-sum rebate $T_{k,t}$ is given in equilibrium by

$$T_{k,t} = -\tau_{k,t}\int_0^1 \mathcal{E}^j_{k,t}D^j_{k,t}dj + \Pi_{k,t}, \tag{4}$$

where $\Pi_{k,t}$ is firm profits.

2.2 Firms

Technology A firm in each economy k produces a differentiated good $l \in [0,1]$ with a linear technology: $Y_{k,t}(l) = AN_{k,t}(l)$. For simplicity, we assume that labor productivity A is constant and identical across countries.

Price setting We assume that the price of each variety is fully rigid, and normalize this price to 1. An implication of this assumption is that the producer price index (PPI) of a country in its own currency is fixed at 1.

The assumption of fully rigid prices can be regarded as an extreme one, but it has the virtue of significantly improving the analytical tractability of the model. This assumption rules out PPI inflation or deflation, but does not eliminate the deflation-recession feedback loop that is a key characteristic of liquidity trap episodes. This is because the relevant measure for that mechanism is CPI inflation rather than PPI inflation, and CPI inflation does respond to fluctuations in the nominal exchange rate.

2.3 Terms of trade and exchange rates

Expenditure minimization leads to the home country's consumer price index (CPI) definition

$$\mathbb{P}_k \equiv (P_k^H)^{1-\alpha}(P_k^F)^{\alpha}, \tag{5}$$

where $P_k^H \equiv \left[\int_0^1 P_k^H(l)^{1-\epsilon} dl\right]^{\frac{1}{1-\epsilon}}$ is the home country's PPI, $P_k^F \equiv \exp\left(\int_0^1 \ln P_k^j dj\right)$ is the price index of imported goods , $P_k^j \equiv \left[\int_0^1 P_k^j(l)^{1-\epsilon} dl\right]^{\frac{1}{1-\epsilon}}$ is country j's PPI, and $P_k^H(l)$ (resp. $P_k^j(l)$) denotes the price of variety l produced in the home country (resp. in country j).[8] A k subscript indicates a price or price index expressed in country k's currency.

$\mathcal{E}_k^j$ is the nominal exchange rate between country k and country j.[9] The law of one price (LOP) implies $P_k^j(l) = \mathcal{E}_k^j P_j^j(l)$. At the level of country j's final good, it implies $P_k^j = \mathcal{E}_k^j P_j^j$. Therefore, the price index of imported goods satisfies $P_k^F = \exp\left[\int_0^1 \ln\left(\mathcal{E}_k^j P_j^j\right) dj\right] = \mathcal{E}_k P^*$, for a world price index $P^* \equiv \exp\left(\int_0^1 \ln P_j^j dj\right)$ and a home country's effective nominal exchange rate $\mathcal{E}_k \equiv \exp\left(\int_0^1 \ln \mathcal{E}_k^j dj\right)$.

[8] Note that $P_k^H \equiv P_k^k$.

[9] An increase in $\mathcal{E}_{k,t}^j$ is a depreciation of country k's currency and an appreciation of country j's currency ($\mathcal{E}_{k,t}^j = 1/\mathcal{E}_{j,t}^k$).

Using the above definitions, the household's budget constraint (3) can be expressed as

$$\dot{a}_{k,t} = i_{k,t}a_{k,t} + W_{k,t}N_{k,t} + T_{k,t} - \mathbb{P}_{k,t}\mathbb{C}_{k,t} + \int_0^1 \left[i_{j,t} - i_{k,t} + \tau_{k,t} - \tau_{j,t} + \frac{\dot{\mathcal{E}}^j_{k,t}}{\mathcal{E}^j_{k,t}} \right] \mathcal{E}^j_{k,t} D^j_{k,t} dj \tag{6}$$

The bilateral terms of trade between country k and country j are defined as the relative price of country j's good in terms of country k's good, $\mathcal{S}^j_k \equiv \frac{P^j_k}{P^k_k}$. The effective terms of trade of country k are defined as $\mathcal{S}_k \equiv \frac{P^F_k}{P^k_k} = \exp\left(\int_0^1 \ln \mathcal{S}^j_k dj\right)$. The bilateral real exchange rate between country k and country j is further defined as the ratio of the two countries' CPIs $\mathcal{Q}^j_k \equiv \frac{\mathcal{E}^j_k \mathbb{P}_j}{\mathbb{P}_k}$, and the effective real exchange rate of country k is defined as $\mathcal{Q}_k \equiv \frac{P^F_k}{\mathbb{P}_k} = \frac{\mathcal{E}_k P^*}{\mathbb{P}_k}$.

2.4 Equilibrium conditions with symmetric North and South blocks

We now present equilibrium conditions from the perspective of a home country k in a case of symmetric North and South blocks. Equilibrium conditions comprise a goods market clearing condition, three equations relating bilateral and effective terms of trade and real exchange rates, a labor market clearing condition, a domestic bond Euler equation, a set of UIP conditions, the country's resource constraint, and a set of bilateral Backus-Smith conditions.

The market clearing condition for for country k's output, defined as $Y_{k,t} \equiv \left[\int_0^1 Y_{k,t}(l)^{\frac{\epsilon-1}{\epsilon}} dl\right]^{\frac{\epsilon}{\epsilon-1}}$, is given by

$$Y_{k,t} = (1-\alpha)\left(\frac{\mathcal{Q}_{k,t}}{\mathcal{S}_{k,t}}\right)^{-1} \mathbb{C}_{k,t} + \alpha x \left(\mathcal{S}_{n,t}\mathcal{S}^n_{k,t}\right) \mathcal{Q}^{-1}_{n,t}\mathbb{C}_{n,t} + \alpha(1-x)\left(\mathcal{S}_{s,t}\mathcal{S}^s_{k,t}\right) \mathcal{Q}^{-1}_{s,t}\mathbb{C}_{s,t}, \tag{7}$$

where the three terms making up demand for country k's good represent domestic demand, foreign demand from North countries, and foreign demand from South countries, respectively.

The effective and bilateral terms of trade are related through $\mathcal{S}_{k,t} = \left(\mathcal{S}^n_{k,t}\right)^x \left(\mathcal{S}^s_{k,t}\right)^{1-x}$, the bilateral real exchange rate is related to the terms of trade through $\mathcal{Q}^j_{k,t} = \left(\mathcal{S}^j_{k,t}\right)^{1-\alpha}$ for $j = \{n, s\}$, and the effective real exchange rate is related to the effective terms of trade through $\mathcal{Q}_k = \mathcal{S}^{1-\alpha}_k$. The labor market clearing condition is given by $N_{k,t} = \frac{Y_{k,t}}{A}$, and the Euler equation for the domestic bond by

$$\frac{\dot{\mathbb{C}}_{k,t}}{\mathbb{C}_{k,t}} = i_{k,t} - \pi_{k,t} - (\rho + \zeta_{k,t}), \tag{8}$$

where $\pi_k \equiv \frac{\dot{\mathbb{P}}_k}{\mathbb{P}_k} = \pi^H_k + \frac{\dot{\mathcal{S}}_k}{\mathcal{S}_k} - \frac{\dot{\mathcal{Q}}_k}{\mathcal{Q}_k}$ is CPI inflation. With fully rigid prices, producer prices are fixed at their $t = 0$ values in own currency terms, and as a result, PPI inflation is always zero: $\pi^H_{k,t} = 0$.[10]

[10]The price (in terms of the home currency) of all varieties produced in each country k is normalized to 1 at

The interest parity condition between the home bond and a North country bond is given by[11]

$$i_{k,t} - \tau_{k,t} = i_{n,t} - \tau_{n,t} + \frac{\dot{\mathcal{E}}_k^n}{\mathcal{E}_k^n}. \tag{9}$$

Finally, country k's budget constraint is

$$\dot{B}_{k,t} = (\rho + \zeta_{n,t} - \tau_{n,t}) B_{k,t} + \mathbb{C}_{n,t}^{-1} \left(\mathcal{Q}_{k,t}^n\right)^{-1} \left[(\mathcal{S}_{k,t})^{-\alpha} Y_{k,t} - \mathbb{C}_{k,t}\right] \tag{10}$$

where $B_{k,t} \equiv \frac{\mathbb{C}_{n,t}^{-1} a_{k,t}}{\mathcal{E}_{k,t}^n \mathbb{P}_{n,t}} = \frac{\mathbb{C}_{n,t}^{-1} \int_0^1 \mathcal{E}_{k,t}^j D_{k,t}^j dj}{\mathcal{E}_{k,t}^n \mathbb{P}_{n,t}}$ is a country's net foreign assets at t measured in terms of a North country's CPI $\mathbb{P}_{n,t}$ and normalized by a North country's marginal utility of consumption $\mathbb{C}_{n,t}^{-1}$. Imposing a no-Ponzi-game condition, this budget constraint can be written in present value form as

$$B_{k,0} = -\int_0^\infty e^{-\int_0^t [\rho + \zeta_{n,h} - \tau_{n,h}] dh} \mathbb{C}_{n,t}^{-1} \left(\mathcal{Q}_{k,t}^n\right)^{-1} \left[(\mathcal{S}_{k,t})^{-\alpha} Y_{k,t} - \mathbb{C}_{k,t}\right] dt \tag{11}$$

For clarity of exposition, we focus on a scenario with symmetric wealth positions, i.e., $B_{k,0} = 0$ for all countries.

The Backus-Smith condition between country k and a representative North country is given by

$$\mathbb{C}_{k,t} = \Theta_{k,t}^n \mathbb{C}_{n,t} \mathcal{Q}_k^n \tag{12}$$

where $\Theta_{k,t}^n$ is a time-varying relative weight, which in the context of the retained Cole-Obstfeld parametrization corresponds to the ratio of expenditure in country k to expenditure in a representative North country.[12,13] Its path satisfies

$$\Theta_{k,t}^n \equiv \Theta_{k,0}^n \exp\left[\int_0^t (\tau_{k,h} - \tau_{n,h} - \zeta_{k,h} + \zeta_{n,h}) \, dh\right]. \tag{13}$$

The path of $\Theta_{k,t}^n$ summarizes the dynamics of the distribution of wealth between countries. $\Theta_{k,t}^n$ is a key variable in our analysis and is closely linked to capital flows. To see this, notice that the (13) implies:

$$\frac{\dot{\Theta}_{k,t}^n}{\Theta_{k,t}^n} = \tau_{k,t} - \zeta_{k,t} - (\tau_{n,t} - \zeta_{n,t}) \tag{14}$$

$t = 0$.

[11] A similar bilateral condition holds between country k and a South country, but this condition can alternatively be derived from (9) and the interest parity condition between a North country and a South country.

[12] This weight is sometimes referred to as a Pareto weight, as it corresponds to the relative weight set on country k in a hypothetical planning problem.

[13] A similar bilateral condition holds between country k and a South country, but this condition can alternatively be derived from (12) and the Backus-Smith condition between a North country and a South country.

Next, consider a scenario in which country k is temporarily less patient than country n for a finite period of time (suppose $\zeta_{n,t} < 0$ and $\zeta_{k,t} = 0$). Absent taxes or subsidies on capital flows ($\tau_{k,t} = \tau_{n,t} = 0$), equation (14) implies that $\dot{\Theta}^n_{k,t}/\Theta^n_{k,t} = \zeta_{n,t} < 0$. Country k temporarily values current consumption more and spends more than country n. It therefore typically borrows from the more patient country n, and consumes relatively less in the future. This coincides with a temporarily higher relative consumption expenditure $\Theta^n_{k,t}$ today, followed by a subsequent decline. During this period, the higher consumption expenditure is made possible by capital flowing into country k. Equation (14) also shows that the magnitude of capital flows can be altered by the use of capital flow taxes. For example, country k can reduce the magnitude of capital inflows by imposing a positive tax ($\tau_{k,t} > 0$), making $\dot{\Theta}^n_{k,t}/\Theta^n_{k,t} = \zeta_{n,t} - \tau_{k,t}$ less negative. Symmetrically, a use of subsidies to outflows by country n increases the magnitude of flows into country k.

The above equilibrium conditions, together with their counterparts for representative North and South countries, can be combined in a way that greatly simplifies the structure of the optimal policy problems we consider in the next sections. This is summarized in the following lemma.

Lemma 1 (Implementability constraints)**.** *Implementable allocations in country k satisfies the consumption-output relationship*

$$\mathbb{C}_{k,t} = \Theta^n_{k,t} \left(\frac{Y_{k,t}}{\Lambda_{k,t}}\right)^{1-\alpha} \left(\frac{Y_{n,t}}{\Lambda_{n,t}}\right)^{\alpha x} \left(\frac{Y_{s,t}}{\Lambda_{s,t}}\right)^{\alpha(1-x)}, \tag{15}$$

and the dynamic IS equation

$$\frac{\dot{Y}_{k,t}}{Y_{k,t}} = i_{k,t} - (\rho + \zeta_{k,t}) - \frac{\alpha x}{\Lambda_{k,t}} \frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \frac{\alpha (1-x) \Theta^n_{s,t}}{\Lambda_{k,t}} \frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}}, \tag{16}$$

for $\Lambda_{k,t} \equiv (1-\alpha) \Theta^n_{k,t} + \alpha x + \alpha (1-x) \Theta^n_{s,t}$, and $\frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}} = \frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}$.

Proof. See Appendix A.1. □

Lemma 1 summarizes the optimal decisions of all private agents in an economy. These equations, along with a description of monetary and capital flow management policy, constitute an equilibrium. (15) implies that home consumption is a geometric average of appropriately normalized home and foreign output levels (adjusted by the expenditure ratio $\Theta^n_{k,t}$), while (16) is a non-linear dynamic New-Keynesian IS curve that relates output growth to the nominal interest rate, the discount rate and the growth of relative expenditure ratios.[14]

[14](15) is obtained by combining the Backus-Smith conditions (12) for countries k and s with the equation

The IS curve (16) is one of the model's key equations and contains important information about the international spillovers at work in the model. Crucially, it reveals that domestic output is independent of foreign monetary policy. A foreign monetary expansion stimulates foreign consumption (through a standard inter-temporal substitution channel) and therefore stimulates demand for the domestic good. At the same time, by generating a domestic currency appreciation, it switches expenditure (by domestic and foreign consumers alike) away from the domestic good. As first noted by Corsetti and Pesenti (2001) in a related model, under the joint assumption of unitary intra- and inter-temporal elasticity of substitution, these two effects exactly cancel out. Hence, the model does not feature spillovers from foreign monetary policy onto domestic output. As is evident from (16), it does, however, feature spillovers from foreign demand shocks and capital flow taxes onto domestic output through their effects on the growth of expenditure ratios. In particular, negative foreign demand shocks are contractionary through their inter-temporal substitution effects on foreign consumption. Similarly, domestic subsidies or foreign taxes on capital inflows are contractionary through their expenditure switching effects on domestic output, as they require a current appreciation of the domestic currency.

3 Capital flows and the zero bound

In a world with integrated financial markets, differences in nominal interest rates across countries are associated with exchange rate dynamics that may reallocate expenditures toward the relatively cheaper goods, both over production locations and over time. The presence of nominal rigidities in turn implies that such reallocations of expenditures impact the level of economic activity. In this section, we describe how monetary policy optimally adjusts to demand shocks originating at home or abroad, and how the induced interest rate differentials lead to global expenditure reallocation.

3.1 Optimal monetary policy in country k

A benevolent monetary authority in country k sets interest rates to maximize the utility of a domestic representative household. Using Lemma 1, the optimal policy problem can be compactly written as:

$$\max \int_0^\infty e^{-\int_0^t \left(\rho+\zeta_{k,h}\right)dh} \left[(1-\alpha)\ln Y_{k,t} - \frac{1}{1+\phi}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right] dt \tag{17}$$

relating the bilateral real exchange rate to the bilateral terms of trade $\mathcal{Q}^j_{k,t} = \left(\mathcal{S}^j_{k,t}\right)^{1-\alpha}$, and the market clearing conditions (A.1) for countries k, s and n. (16) is obtained by differentiating country k's market clearing condition (A.1) and substituting the consumption Euler equations (8) for countries k, s and n. See Appendix A.1 for details.

subject to:

$$\begin{aligned}\frac{\dot{Y}_{k,t}}{Y_{k,t}} &= i_{k,t} - (\rho + \zeta_{k,t}) - \frac{\alpha x}{\Lambda_{k,t}} \frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \frac{\alpha (1-x) \Theta^n_{s,t}}{\Lambda_{k,t}} \frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}} && (18)\\ i_{k,t} &\geq 0. && (19)\end{aligned}$$

with $\Lambda_{k,t} \equiv (1-\alpha)\Theta^n_{k,t} + \alpha x + \alpha(1-x)\Theta^n_{s,t}$, and $\frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}} = \frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}$. (18) is the dynamic IS equation for country k's output, (19) is the ZLB constraint, and $Y_{k,0}$ is free. This is an optimal control problem with control $i_{k,t}$ and state $Y_{k,t}$. The monetary authority's optimal plan is characterized by a two-dimensional system of differential equations in the state variable $Y_{k,t}$ and its co-state $\mu_{k,t}$, consisting of (18) and

$$\dot{\mu}_{k,t} = -\frac{e^{-\int_0^t (\rho+\zeta_{k,t})dh}}{Y_{k,t}} \left\{ (1-\alpha) - \left(\frac{Y_{k,t}}{A}\right)^{1+\phi} \right\} - \mu_{k,t} \frac{\dot{Y}_{k,t}}{Y_{k,t}}, \tag{20}$$

with $\mu_{k,t} i_{k,t} = 0$ and $\mu_{k,0} = 0$. $\mu_{k,t}$ is often referred to as the value of commitment. Proposition 1 characterizes optimal policy in country k in the absence of the ZLB.

Proposition 1 (Optimal monetary policy without the ZLB)**.** *In the absence of a zero bound on interest rates, the monetary authority stabilizes domestic output perfectly, achieving* $Y_{k,t} = A(1-\alpha)^{\frac{1}{1+\phi}} \equiv \overline{Y}$, *by setting an initial exchange rate of* $\mathcal{E}^n_{k,0} = \left(\overline{Y}/\Lambda_{k,0}\right)\left(Y_{n,0}/\Lambda_{n,0}\right)^{-1}$ *and an interest rate path given by*[15]

$$\mathcal{I}_k = (\rho + \zeta_{k,t}) + \frac{\alpha x}{\Lambda_{k,t}} \frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} + \frac{\alpha(1-x)\Theta^n_{s,t}}{\Lambda_{k,t}} \frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}} \tag{21}$$

Proof. See text below. □

In the absence of the ZLB (or for small enough shocks), equation (20) indicates that the monetary authority aims to perfectly stabilize output at $Y_{k,t} = \overline{Y}$. By lowering the interest rate in response to a negative (domestic or foreign) demand shock, it stimulates demand for the domestic good through a standard inter-temporal substitution channel and an intra-temporal expenditure switching channel. The inter-temporal substitution channel concerns domestic agents only, while the expenditure switching channel, characteristic of open economy settings, applies to home and foreign agents alike.

We refer to the optimal monetary policy outlined in Proposition 1 as the *unconstrained policy*. A large enough negative demand shock (either at home or abroad) can push the interest

[15] A complete description of the monetary authority's actions requires the specification of an entire exchange rate path or, alternatively, of an initial exchange rate level and a path for the domestic interest rate. The optimal exchange rate path is given by $\mathcal{E}^n_{k,t} = \left(\overline{Y}/\Lambda_{k,t}\right)\left(Y_{n,t}/\Lambda_{n,t}\right)^{-1}$.

rate associated with this policy below 0, leading to a violation of the ZLB constraint. We refer to such a situation as a *liquidity trap* in country k. Proposition 2 describes optimal policy in such a situation.

Proposition 2 (Optimal monetary policy at the ZLB). *Suppose that the interest rate policy prescribed in Proposition 1 violates the ZLB constraint for $t \in [0, T)$ but not for $t \geq T$. Then the ZLB binds, with $i_{k,t} = 0$ for $t \in [0, \widehat{T}_k)$ and $i_{k,t} = \mathcal{I}_k$ for $t \geq \widehat{T}_k$. The ZLB exit time $\widehat{T}_k > T$ and the output path satisfy*

$$0 = \int_0^{\widehat{T}_k} e^{-\int_0^t \left(\rho + \zeta_{k,h}\right) dh} \left[1 - \left(\frac{Y_{k,t}}{\overline{Y}}\right)^{1+\phi}\right] dt, \tag{22}$$

and for $Y_{k,0}$ implicitly defined by (18), (22) and $Y_{k,\widehat{T}_k} = \overline{Y}$, the initial exchange rate is $\mathcal{E}^n_{k,0} = \left(Y_{k,0}/\Lambda_{k,0}\right)\left(Y_{n,0}/\Lambda_{n,0}\right)^{-1}$.

Proof. See Appendix A.2. □

Hence, if the unconstrained policy violates the ZLB for some period of time, it is optimal to keep the interest rate at zero for longer. The commitment to do so, often referred to as "forward guidance," generates a demand boom after the liquidity trap, whose purpose is to alleviate the initial contraction in output. Under optimal policy, an economy with a binding ZLB thus goes through a recession-boom cycle in output. Output growth is positive during the liquidity trap – from 0 to T – and negative between the end of the trap and the ZLB exit time – from T to $\widehat{T}_k$. Furthermore, the ZLB exit time is optimally chosen so as to minimize average deviations from the unconstrained output level $\overline{Y}$.

Our characterization of optimal policy at the ZLB is reminiscent of earlier results in the closed economy literature.[16] In the open economy, monetary policy also operates through an expenditure switching channel, whose precise workings constitute the main focus of our paper. By lowering interest rates, a monetary authority can create an interest rate differential between itself and other economies. This differential induces a depreciation of the home currency, increasing the competitiveness of its exports. The resulting expenditure switching compensates for lower domestic demand and potentially alleviates the demand-driven recession at home. This adjustment of the terms of trade or exchange rates is strongly linked to capital flows.

Is openness an unambiguous blessing? Openness reduces a country's exposure to home demand shocks but increases its exposure to foreign demand shocks. We formally define the exposure of country k to a home demand shock as $\chi_{kk} \equiv \frac{\partial \mathcal{I}_k}{\partial \zeta_{k,t}}$, and its exposure to a foreign demand shock as $\chi_{kn} \equiv \frac{\partial \mathcal{I}_k}{\partial \zeta_{n,t}}$ (or $\chi_{ks} \equiv \frac{\partial \mathcal{I}_k}{\partial \zeta_{s,t}}$). These are natural measures of exposure, as they

[16] See for example, Eggertsson and Woodford (2003) and Werning (2012) for a closed economy analysis.

represent the aggressiveness with which the monetary authority needs to adjust the interest rate to stabilize demand in response to these shocks.

Proposition 3 (Openness and the incidence of demand shocks)**.** *The exposure to home shocks decreases with openness (i.e.* $\frac{\partial \chi_{kk}}{\partial \alpha} < 0$*), but the exposure to foreign shocks increases with openness (i.e.,* $\frac{\partial \chi_{kn}}{\partial \alpha}, \frac{\partial \chi_{ks}}{\partial \alpha} > 0$*).*

Proof. See text below. □

The result follows directly from differentiating the exposure measures with respect to α. On the one hand, an economy hit by a domestic demand shock of a given size is less likely to experience a liquidity trap if it is open than if it is closed. On the other hand, openness creates possibilities that the economy may experience liquidity traps as a result of foreign shocks.[17] Thus, the reduced vulnerability to domestic shocks comes at the expense of an increased vulnerability to foreign shocks. The transmission of shocks is closely tied to capital flows: an economy hit by a negative demand shock exports savings into foreign economies, thereby contributing to the appreciation of their currencies and diverting demand away from locally produced goods. The more integrated economies are, the stronger this channel, as our next result illustrates.

Proposition 4 (Globally spreading liquidity traps)**.** *Under free capital flows, in the limit of no home bias ($\alpha \to 1$), liquidity traps are synchronized across all countries globally.*

Proof. See text below. □

The result follows directly from the fact that under free capital flows and $\alpha \to 1$, the unconstrained interest rate in (21) is equalized across countries and equal to $\mathcal{I}_k = \rho + \frac{x\zeta_{n,t} + (1-x)\Theta^n_{s,t}\zeta_{s,t}}{x + (1-x)\Theta^n_{s,t}}$. The more integrated the world economy, the easier it is for demand shocks to get transmitted accross countries. A direct implication of this result is that under free capital flows and no home bias, any demand shock that pushes a region to the ZLB necessarily also drags the entire world to the ZLB.[18]

To gain insights into the role played by expenditure switching in the global macroeconomic adjustment taking place in a liquidity trap, we put additional structure on the exogenous variables and take a world equilibrium perspective. More precisely, we consider a demand shock that pushes only the North into a liquidity trap and study the unique Nash equilibrium of a

[17] For instance, under free capital flows, a home demand shock in a North economy k leads to an unconstrained interest rate of $\mathcal{I}_k = \rho + [(1-\alpha)\,\Theta^n_{k,t}/\Lambda_{k,t}]\zeta_{k,t}$ (with $0 < (1-\alpha)\,\Theta^n_{k,t}/\Lambda_{k,t} < 1$), which compares with an unconstrained interest rate of $\mathcal{I}_k = \rho + \zeta_{k,t}$ in the closed economy. On the other hand, a foreign (North) demand shock leads to an unconstrained interest rate of $\mathcal{I}_k = \rho + (\alpha x/\Lambda_{k,t})\zeta_{n,t}$ (with $0 < \alpha x/\Lambda_{k,t} < 1$) in the open economy, which compares with an unconstrained interest rate of $\mathcal{I}_k = \rho$ in the closed economy.

[18] This is consistent with the findings of Cook and Devereux (2011), who point out that under the assumption of no home bias and unit elasticities, natural interest rate are equalized across countries.

game in which each country's monetary authority solves (17) subject to (18) and (19). Our focus on such a scenario is motivated by the global economic environment of the Great Recession, during which several key advanced economies, but not emerging markets, were pushed to the ZLB. The analysis of the Nash equilibrium in the next section shows that global adjustment crucially depends on the prevailing capital flow regime.

3.2 Nash equilibrium of the monetary policy game

Following standard practice in the literature, we generate a liquidity trap via a large unanticipated temporary *negative demand shock*.

Assumption 1. *At $t = 0$, agents learn about the path of demand shocks for $t \geq 0$. This path is given by $\zeta_{s,t} = 0$ $\forall t \geq 0$ and*

$$\zeta_{n,t} = \begin{cases} -\bar{\zeta} & \text{for } t \in [0, T), \qquad \text{with } \bar{\zeta} > 0 \\ 0 & \text{for } t \geq T \end{cases}$$

The negative demand shock originates in the North, and prevails from 0 to T. We bound the size of this demand shock to ensure that it is large enough to make the North experience a liquidity trap, yet small enough not to make the South experience one (see Appendix A.3 for details). This structure enables us to characterize the unique Nash equilibrium of the monetary policy game and construct a narrative of the global adjustment following a demand shock that drives the North, but not the South, to the ZLB.

Proposition 5. *Suppose that capital flow taxes are small (in absolute value). Then in the Nash equilibrium of the monetary policy game, the ZLB binds in the North but not in the South.*

Proof. See Appendix A.4. □

Under the maintained assumptions on the size of the demand shock, the ZLB prevents monetary policy from fully stabilizing aggregate demand in the North, but not in the South. For the North, this results in a real interest rate that is "too high" (as in a closed economy) and in an exchange rate that is "too appreciated."

Integrating the dynamic IS equation from $t \geq 0$ to $\widehat{T}_n$ yields an expression for the ratio of the North's output to its unconstrained level, $Y_{n,t}/Y_{n,\widehat{T}_n} = (\Lambda^n_{s,t}/\Lambda^n_{s,\widehat{T}_n})e^{\int_t^{\widehat{T}_n}(\rho+\zeta_{n,h})dh}$. Using this relation to substitute into (22) (specialized for a North economy) yields a single non-linear equation in the North's optimal exit time from the ZLB $\widehat{T}_n$:[19]

$$0 = \int_0^{\widehat{T}_n} e^{-\int_0^t (\rho+\zeta_{n,t})dh} \left[1 - \left(\frac{\Lambda^n_{s,t}}{\Lambda^n_{s,\widehat{T}_n}} \right)^{1+\phi} e^{(1+\phi)\int_t^{\widehat{T}_n} \left(\rho+\zeta_{n,h}\right)dh} \right] dt. \tag{23}$$

The ratio $\Lambda^n_{s,t}/\Lambda^n_{s,\widehat{T}_n}$ in (23) depends on the ease with which capital can flow between countries and thereby promote (or hamper) global adjustment when the North gets pushed to the ZLB. To illustrate this point, we characterize in detail the global adjustment associated with two stylized capital flow regimes: one with free capital flows and another one with closed capital accounts.

Free capital flows The free capital flow regime corresponds to a case where capital flow taxes are zero at all times, i.e. $\tau_{s,t} = \tau_{n,t} = 0\ \forall t$. Capital flows downstream in the early stage of the liquidity trap, and flows upstream in the late stage as well as for a short period after the trap.[20] There is both intra-temporal and inter-temporal trade. North economies reduce their nominal rate to zero and commit to keeping it there until after the trap has ended. South economies also lower interest rates, but not all the way down to zero, and do so only for the duration of the trap. A positive interest rate differential between the South and the North thus prevails during the liquidity trap and persists for a short period after it. During this time, interest parity requires a continuous depreciation of South currencies, i.e., $\dot{\mathcal{E}}^n_{s,t}/\mathcal{E}^n_{s,t} = i_{s,t} - i_{n,t} > 0$, which is typically associated with an appreciation of these currencies on impact. These exchange rate movements induce a terms of trade path that promotes expenditure switching. This expenditure switching has an intra-temporal dimension (North vs. South goods) and an inter-temporal dimension (current vs. future goods). Both work to reallocate demand toward North goods in the initial part of the liquidity trap, precisely when demand for these goods is most depressed. Meanwhile, downstream capital flows allow for a global reallocation of demand: during the trap, North consumption is initially depressed (i.e., is tilting up), but South consumption booms (i.e., is tilting down).[21] Finally, following the trap, the South runs a permanent trade surplus to cover interest payments on the accumulated foreign debt. The solid lines in Figure 1 graphically display global adjustment to the demand shock in the free capital flow regime, confirming the

[19]A sufficient condition for (23) to have a solution larger than T is that North output growth is positive from 0 to T, which itself rules out situations where taxes on upstream flows or subsidies on downstream flows are very large. If a solution to (23) larger than T exists, then it is unique.

[20]In order to determine the direction of capital flows, observe that (1) the current account of a North country is given by $\dot{a}_{n,t} = i_{n,t}a_{n,t} + \alpha(1-x)\left(\Theta^n_{s,t} - 1\right)\left(\mathcal{E}^s_{n,t}\right)^{\alpha(1-x)}\mathbb{C}_{n,t}$, (2) a North country's lifetime budget constraint can be written as $0 = \int_0^\infty e^{-\int_0^t(\rho+\zeta_{n,s})ds}\left(1-\Theta^n_{s,t}\right)dt$, and (3) that under free capital mobility, we have $\dot{\Theta}^n_{s,t}/\Theta^n_{s,t} < 0$ for $0 \le t < T$ and $\dot{\Theta}^n_{s,t}/\Theta^n_{s,t} = 0$ for $t \ge T$.

[21]To see this, observe that during the liquidity trap, from the Euler equations we have $\dot{\mathbb{C}}_{n,t}/\mathbb{C}_{n,t} = \alpha(1-x)i_{s,t} - (\rho - \bar{\zeta}) > 0$ and $\dot{\mathbb{C}}_{s,t}/\mathbb{C}_{s,t} = (1-\alpha x)i_{s,t} - \rho < 0$.

above-described narrative.[22] The stabilizing role of capital flows is best illustrated by comparing the free capital flow outcome to a closed capital account scenario, to which we now turn.

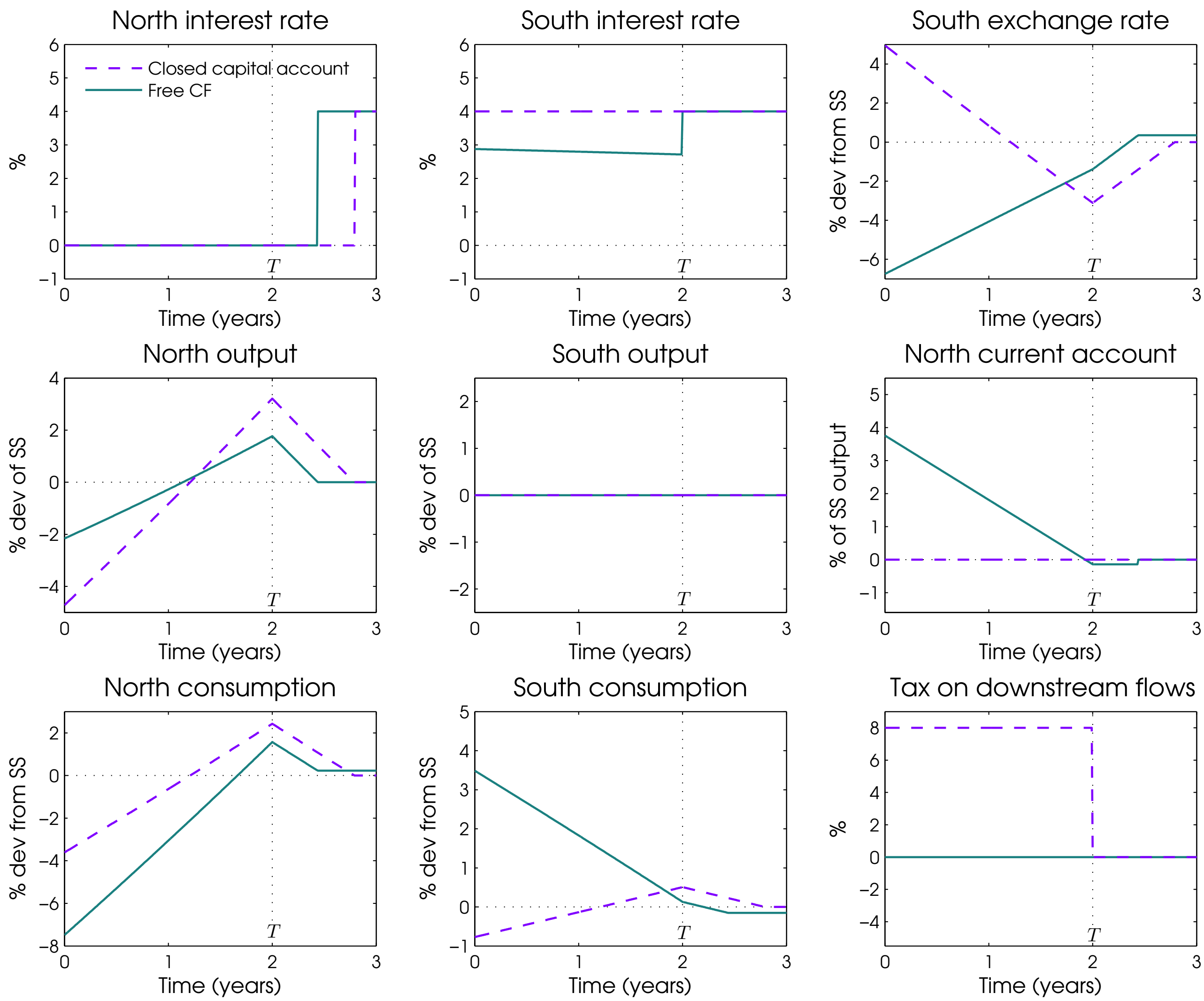

Figure 1: Variable paths under optimal monetary policy in all countries: free capital flows (solid) vs. closed capital accounts (dashed).

[22] The parameterization used to generate the figure relies on standard values from the literature. Following Gali and Monacelli (2005), we set the discount rate to $\rho = 0.04$, the openness parameter to $\alpha = 0.4$, and the inverse Frisch elasticity of labor supply to $\phi = 3$. For parameters pertaining to our liquidity trap scenario, we follow Werning (2012). The duration of the trap is set to $T = 2$ years, and the size of the demand shock is set to $\bar{\zeta} = 0.08$. In a closed economy benchmark, such a shock size would result in a natural real interest rate of -4% for the duration of the liquidity trap. Finally, we set the relative size of the North block, for which there is no natural counterparts in standard models, to $x = 0.4$, aiming to generate a share of advanced economies in world GDP in line with recent figures (alternative plausible values for that parameter deliver similar qualitative results). These parameters satisfy the technical condition (A.9) in Appendix A.3 that guarantees that under free capital flows, the ZLB binds in the North but not in the South. Unless noted otherwise, they are used for all of the following figures.

Closed capital accounts The closed capital account scenario corresponds to a capital flow tax wedge $\tau_{s,t} - \tau_{n,t} = -\zeta_{n,t}$ that exactly shuts down capital flows, i.e. results in $\dot{\Theta}^n_{s,t}/\Theta^n_{s,t} = 0$ for all $t \geq 0$. Unlike the free capital flow case which features both inter- and intra-temporal trade, the closed capital account scenario only features intra-temporal trade. The IS equation (18) for an economy k reduces to $\dot{Y}_{k,t}/Y_{k,t} = i_{k,t} - (\rho + \zeta_{k,t})$, which coincides with the closed economy limit. Consequently, the optimal monetary policy takes the same form as in the closed economy. South interest rate do not move ($i_{s,t} = \rho\ \forall t$), while in the North, the ZLB binds and exit is delayed to $\widehat{T}_n^{\text{closed}} > T$. From 0 to T, North output grows at rate $\dot{Y}_{n,t}/Y_{n,t} = \bar{\zeta} - \rho > 0$, while from T to $\widehat{T}_n^{\text{closed}}$, it grows at rate $\dot{Y}_{n,t}/Y_{n,t} = -\rho < 0$, exactly as in the closed economy. In stark contrast with the free capital flow case, interest parity requires a continuous appreciation of South currencies during the liquidity trap: $\dot{\mathcal{E}}^n_{s,t}/\mathcal{E}^n_{s,t} = \rho - \bar{\zeta} < 0$. The terms of trade thus move "the wrong way" from the perspective of stabilizing aggregate demand stabilization in the North. Furthermore, the lack of capital flows prevents global demand reallocation: during the trap, consumption in both the North and the South is initially depressed (i.e., tilting up).[23] Finally, since the relative expenditure ratio $\Theta^n_{s,t}$ is constant, $\Lambda^n_{s,\widehat{T}_n}/\Lambda^n_{s,t} = 1$ in equation (23), and the characterization of the exit time coincides with that of Werning (2012) under rigid prices. Our next result contrasts this exit time with the one prevailing under free capital flows.

Proposition 6. *The North exits the ZLB earlier under free capital flows than under closed capital accounts (or equivalently, than in a closed economy benchmark):* $\widehat{T}_n^{\textit{free}} < \widehat{T}_n^{\textit{closed}}$.

Proof. See Appendix A.5. □

This result is yet another manifestation of the stabilizing effects of capital flows at the ZLB: not only do free capital flows yield a smoother output path for North economies, they also allow for a faster adjustment process. The dashed line in Figure 1 represents the variables' responses to the demand shock in the closed capital account regime, confirming the above narrative.

To sum up, the comparison of the dynamics under the two regimes sheds light on the stabilizing effects of capital flows when only one region experiences a liquidity trap. Downstream flows reallocate demand globally by inducing a consumption boom in the South at the precise time when demand is deficient in the North. Meanwhile, exchange rate movements associated with these flows foster a reallocation of expenditure toward North goods at the moment when the provision of those goods is the most depressed.

[23] To see this, observe that during the liquidity trap, from the Euler equations we have $\dot{\mathbb{C}}_{n,t}/\mathbb{C}_{n,t} = -[1 - \alpha(1-x)](\rho - \bar{\zeta}) > 0$ and $\dot{\mathbb{C}}_{s,t}/\mathbb{C}_{s,t} = -\alpha x(\rho - \bar{\zeta}) > 0$.

4 Efficient capital flows

The above analysis emphasized the stabilizing role played by capital flows in a scenario where only a region of the world economy faces a binding ZLB. It did not address, however, the question of the efficiency properties of a free capital flow regime. In this section, we tackle this issue by asking whether a regime of managed capital flows can raise welfare in some countries without reducing it in others.

We frame this efficiency question by considering a Ramsey planning problem. The objective of our utilitarian global planner is to maximize global welfare while ensuring that each country gets at least the same level of welfare it enjoyed in the free capital flows case. We endow the planner with two instruments: (i) a transfer $\mathcal{T}$ from North to South countries at date 0, and (ii) a path for taxes (or subsidies) on downstream capital flows $\tau_{s,t}$.[24,25] The planner's choices are restricted by two sets of implementability constraints. First, she must respect all equilibrium conditions characterizing private agents' optimal decisions. Second, she must observe constraints representing optimal monetary policy making by individual countries. The problem is formally described in Appendix A.6.

The advantage of setting up the problem in this way is that we can evaluate the efficiency of a regime of free capital flows by asking a very simple question: *Is the planner's optimal choice characterized by* $\tau_{s,t} = 0$, $\forall t$? If so, then the planner cannot achieve Pareto improvements by distorting international borrowing decisions, and we conclude that a regime of free capital mobility is constrained Pareto efficient. However, if the planner does choose a non-zero tax path, it means that free capital mobility is constrained Pareto inefficient. We now show that the Pareto efficiency of a free capital flow regime depends crucially on whether the North is experiencing a liquidity trap.

Proposition 7 (Constrained inefficiency of free capital flows in a liquidity trap)**.** *During the liquidity trap and up till renormalization of monetary policy in the North, the free capital flow regime is constrained inefficient.*

Proof. See Appendix A.6. □

This result indicates that a regime of active capital flow management Pareto dominates the free capital flows benchmark when one region of the world economy is at the ZLB. The proof is by contradiction and relies on the fact that a zero tax path does not satisfy the planner's optimality conditions. This negative result raises the question of what the efficient capital flow regime looks like. Does it entail more or less flows than the free capital flows benchmark?

[24] We focus on symmetric equilibria where the planner treats countries symmetrically within the North and South blocks.

[25] The relevant effective tax rate for cross-border savings decisions is the tax differential $\tau_{s,t} - \tau_{n,t}$. As a result, in the presence of transfers, the assumption that $\tau_{n,t} = 0$ is without loss of generality.

How does this regime reduce the distortions caused by the ZLB? Our next result provides an analytical characterization of the efficient capital flows regime.

Proposition 8 (Characterization of efficient capital flows). *For a small enough degree of openness α, the optimal capital flow tax path satisfies*

$$\begin{aligned} \tau_{s,t} &< 0 \quad \textit{for } 0 \leq t < T \\ \tau_{s,t} &> 0 \quad \textit{for } T \leq t < \widehat{T}_n \\ \tau_{s,t} &= 0 \quad \textit{for } t \geq \widehat{T}_n \end{aligned}$$

Proof. See Appendix A.7. □

The efficient capital flow regime thus features a subsidy to downstream flows during the liquidity trap, followed by a tax on downstream flows between the end of the trap and the ZLB exit time in the North.[26,27] Thus, from the planner's point of view, in the free capital flow regime, capital does not flow enough from the North to the South during the liquidity trap, and then does not flow enough in the opposite direction between the end of the trap and the North's ZLB exit time.[28] Loosely speaking, the policy intervention, without destabilizing output in the South, achieves a superior output stabilization in the North relative to the free capital mobility benchmark.

Figure 2 illustrates the properties of the efficient capital flow regime by plotting the paths of key macro variables alongside their free capital flow counterparts. The solid line represents the free capital flow regime, while the dashed line represents the efficient regime. The subsidy to downstream flows during the liquidity trap induce a larger expected depreciation of the South currencies, while the subsidy to upstream flows after the trap induces an expected appreciation (rather than continued depreciation) of these currencies. The steeper exchange rate path during the trap translates into a steeper terms of trade path and shifts global expenditure (i.e., originating from both the North and the South) toward North goods in the initial stage of the trap and away from them in the late stage. Consequently, both the initial recession and the ensuing boom in the North are less pronounced than under free capital flows.

It is worth noting that the optimal capital flow tax path also affects demand for South goods: it is contractionary during the liquidity trap and expansionary after the trap. But since the monetary authority in the South is not constrained by the ZLB, it has the potential to adjust and

[26] Equivalently, the efficient regime features a tax on upstream flows during the trap, followed by a subsidy to upstream between the end of the trap and the ZLB exit time in the North.

[27] Note that this analytical characterization of the efficient regime is valid for a small enough degree of openness as measured by α. For larger values of α, we cannot unambiguously sign the path of taxes. However, we can confirm numerically that this characterization also holds for larger values of α in line with the ones commonly used in the literature (see Figure 2).

[28] Recall that in the free capital flow regime, capital only flows (downstream) during the liquidity trap. In particular, capital flows halt at the end of the trap.

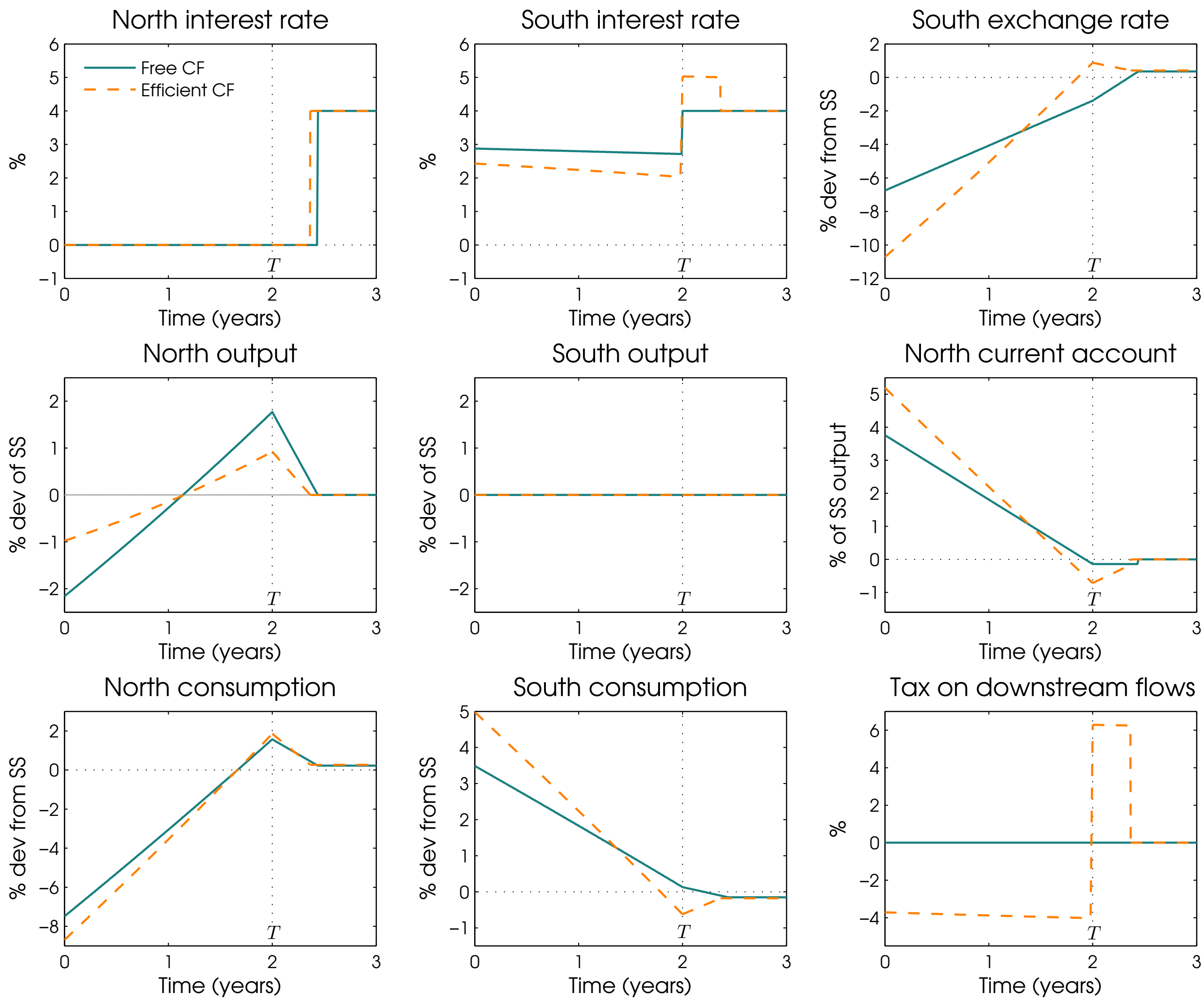

Figure 2: Variable paths under free capital flows (solid) vs. efficient capital flows (dashed).

offset these effects, by being more expansionary during the trap and more contractionary after the trap than in the free capital flow case. In terms of inter-temporal consumption allocations, the subsidy to downstream flows during the trap generates a larger consumption boom there in the South and the larger consumption bust in the North. Current account fluctuations are accordingly magnified under the efficient regime.

Aggregate demand externalities and the ZLB Farhi and Werning (2013) argue that *aggregate demand externalities* generated by nominal rigidities and constraints on monetary policy generically render inter-temporal decisions constrained inefficient. In our model, constraints on monetary policy take two forms: (i) the non-cooperative nature of monetary policy setting, and (ii) the ZLB. One might therefore wonder whether it is the ZLB as such, or the non-cooperativeness assumption that renders borrowing and saving decisions constrained inefficient. The following result establishes that the ZLB, rather than the non-cooperativeness, is key in that regard.

Proposition 9 (Constrained efficiency of free capital flows away from the ZLB)**.** *In the absence of the ZLB (or for small enough demand shocks), the free capital flow regime is constrained efficient.*

Proof. See Appendix A.6. □

The result emphasizes that when non-cooperativeness is the only constraint on monetary policy, there is no role for capital flow management from an efficiency perspective. This sharp result is due to the knife-edge nature of the Cole-Obstfeld parameterization we adopt. Absent the ZLB, monetary authorities target and achieve a given constant level of output of $A(1-\alpha)^{\frac{1}{1+\phi}}$. This output targeting prevents capital flow management policies from affecting economic activity and eliminates the global planner's incentive to reallocate consumption intertemporally across countries.

Thus, in our setup, since monetary policy decisions do not entail cross-border externalities, the efficient regime of capital flow management is *not* motivated by inward-looking monetary policy decisions.[29] Instead, it is motivated by externalities associated with private agents' financial decisions interacting with constraints on monetary policy. Private agents take prices as given and do not internalize how their increased desire to save affects the economy as a whole. With nominal rigidities, the fall in demand associated with this increased desire to save pushes the economy into a recession. The monetary authority, which is not a price taker, instead internalizes and attempts to nullify these effects by affecting prices. In the absence of the ZLB, it can lower rates enough to induce sufficient inter-temporal substitution and expenditure switching to eliminate any contraction in output and correct the aggregate demand externality. At the zero bound, however, the monetary authority is unable to lower rates sufficiently to do so. In this scenario, capital flow management can provide an additional stimulus to demand and curtail the severity of the boom-bust cycle. Nevertheless, capital flow taxes are an imperfect substitute and cannot fully eliminate the fall in output without creating other distortions.

Are gift transfers alone desirable? Recall that the global planner also has recourse to a time 0 transfer in addition to capital flow taxes. One might wonder whether she can compensate for the constraints on monetary policy in the North by using only this transfer. Since a transfer

[29] Although the Nash equilibrium of the monetary policy game does not achieve the first best allocation that an unconstrained global social planner would choose, international savings decisions are nonetheless constrained efficient. To further stress the point that the non-cooperative nature of monetary policy is not the reason behind the inefficiency of free capital flows, we show in Appendix B of Acharya and Bengui (2015) that the inefficiency follows a similar pattern under cooperative monetary policy.

of resources from the North to the South renders the North poorer, it could potentially reduce the excess desire to save and improve outcomes. Our next result addresses this issue.

Lemma 2 (Undesirability of gift transfers)**.** *Suppose the global planner has no access to capital flow taxes. Then the optimal date 0 transfer is $\mathcal{T} = 0$.*

Proof. See Appendix A.6. □

The lemma indicates that the transfer is purely compensating and does not address aggregate demand externalities. In order to reduce the excessive desire of the North to save, the planner needs to make the North poorer relative to the future. However, a one-time transfer does not facilitate an intertemporal transfer of wealth. Instead, capital flow taxes allow a planner to transfer purchasing power intertemporally across regions, thus indirectly facilitating an intertemporal transfer of wealth.

5 Capital flow management and currency wars

The previous section established that in a liquidity trap, larger capital flows implemented through capital account interventions could make all countries better off. Would this favorable outcome be achieved in a decentralized setting? We address this question by studying the Nash equilibrium of a game in which domestic monetary authorities choose both monetary and capital flow management policy non-cooperatively. We consider two scenarios: one where only South countries manage their capital account, and one where all countries do so. Section 5.1 characterizes the optimal choices of an individual country, while Sections 5.2 and 5.3 discusses Nash equilibrium outcomes.

5.1 Optimal policy in country k

The optimal policy problem is similar to the problem of the monetary authority in Section 3, but with recourse to an additional instrument: it is able to influence the path of $\Theta^n_{k,t}$ by taxing or subsidizing capital inflows. The problem is formally laid out in Appendix A.8. We characterize the optimal policy choices below.

Proposition 10 (Individually optimal capital flow taxes)**.** *Country k's optimal tax on capital inflows balances a dynamic terms of trade manipulation motive with an aggregate demand stimulation motive, and satisfies:*

$$\tau_{k,t} = \underbrace{\Omega^1_{k,t}\left[(1-x)\,\Theta^n_{s,t}\left(\zeta_{k,t}+\tau_{s,t}\right)+x\left(\zeta_{k,t}-\zeta_{n,t}+\tau_{n,t}\right)\right]}_{\text{dynamic terms of trade management}} + \underbrace{\Omega^2_{k,t}\frac{\dot{Y}_{k,t}}{Y_{k,t}}}_{\text{aggregate demand management}} \tag{24}$$

for $\Omega^1_{k,t}, \Omega^2_{k,t} > 0$.

Proof. See Appendix A.8. □

The first term in (24) reflects incentives to manage dynamic terms of trade and is present regardless of the ZLB. The second term reflects an aggregate demand management motive and is present only when the ZLB binds country k. We now in turn discuss these individual aspects.

Corollary 1 (Optimal taming of capital flow cycles)**.** *The optimal tax on capital inflows is increasing in the home demand shock and decreasing in foreign demand shocks.*

Regardless of the zero bound, faced with a positive home demand shock or negative foreign demand shocks, a country experiences capital inflows and a temporary appreciation of its currency (see ζ_k and ζ_n terms in (24)). As a result, it suffers from a temporarily lower foreign demand for its good. It is then optimal to tax inflows in order to curtail the temporary appreciation and inter-temporally smooth foreign demand for the country's domestic good. This is the dynamic terms of trade manipulation motive of capital flow management identified by Costinot et al. (2014) and also present in Farhi and Werning (2014). Thus, regardless of the zero bound, countries tend to optimally tame capital flow cycles caused by demand shocks.[30] Our next result concerns how countries react to foreign capital flow taxes.

Corollary 2 (Strategic complementarities in capital flow taxes)**.** *The optimal capital flow tax is increasing in foreign capital inflow taxes.*

This result follows naturally from the observation that foreign capital flow taxes have the same effects on a country's capital flows and exchange rate/terms of trade profile as foreign demand shocks. In particular, a positive foreign tax (by North and South countries alike) acts as a negative foreign demand shock, increasing capital inflows and appreciating the home currency. It is thus optimal for the home country to respond to a positive foreign capital flow tax with a positive tax on inflows. Capital flow taxes are thus strategic complements among countries.

The motives discussed so far for managing the capital account are present regardless of the ZLB. We now turn to a motive stressed by Proposition 10 that is specific to ZLB episodes.

Corollary 3 (Aggregate demand stabilizing capital flow taxes at the ZLB)**.** *A country for which the ZLB binds sets a higher tax on inflows when its domestic output is rising, and a lower tax on inflows when its domestic output is falling.*

[30] Note that this mechanism does not hinge on nominal rigidities but on the monopolistically competitive structure of international goods markets.

This result refers to the second term in (24). It reflects the fact that a country where the ZLB binds finds it optimal to adjust its tax on capital inflows to smooth domestic aggregate demand. Given the output profile outlined in Section 3 of a country with a binding ZLB constraint, the tax on capital inflows should be higher during the liquidity trap (i.e., when $\dot{Y}_{k,t}/Y_{k,t} > 0$), and lower following the trap during the time when the monetary policy authority delays exit from the ZLB (i.e., when $\dot{Y}_{k,t}/Y_{k,t} < 0$).

In conclusion, absent (or away from) the ZLB, all countries use capital account management to tame the capital flow cycle caused by asymmetric demand shocks.[31] With the ZLB, however, North countries, in an effort to compensate for the impotency of monetary policy, additionally use capital flow taxes to smooth aggregate demand for their home good. This extra incentive creates a potential conflict between the objectives of North and South economies. The North may try to subsidize capital outflows to combat its liquidity trap, while the South fights back by taxing capital inflows to avoid an excessive appreciation of their currency. This conflict has a flavor of currency wars.

5.2 Capital controls by the South

In the latest global capital flow cycle that constitutes the motivation for our paper, only emerging market countries engaged in capital flow management policies. To analyze such a scenario, we now consider the Nash equilibrium outcome of a game where only South countries are able to optimally set capital flow taxes.[32] We have the following result.

Proposition 11 (South taxes capital inflows)**.** *In the symmetric Nash equilibrium of the capital flow management game between South countries only, the capital flow tax imposed by each South country satisfies*

$$\tau_{s,t} = -\Upsilon_{s,t}\zeta_{n,t}, \tag{25}$$

for some $0 < \Upsilon_{s,t} < 1$. In other words, capital flow management by South countries slows down, but neither shuts down nor reverses capital flows during the liquidity trap.

Proof. See Appendix A.9. □

Given that for each individual South country, (24) indicates that it is optimal to set a positive tax on inflows in repsonse to (i) a negative demand shock in the North, *and* (ii) in response to foreign capital flow taxes, in equilibrium South countries act to slow down capital flows. Each individual South country tries to curtail its own terms of trade improvement by limiting capital inflows. In doing so, it deflects capital flows toward other South countries, which

[31]This is consistent with the common perception that countries may find it optimal to limit capital mobility.

[32]This corresponds to a case where all countries maximize (A.42) subject to (A.43)-(A.46), but North countries in addition face the constraint that $\tau_{k,t} = 0\ \forall t$.

are inclined to act in the same way. Following from this iterative process, the unique symmetric Nash equilibrium of the game features a positive tax on capital inflows by all South economies. This highlights the existence of a conflict not just between North and South countries, but also within the group of South countries. This insight would be absent in a two-country model.

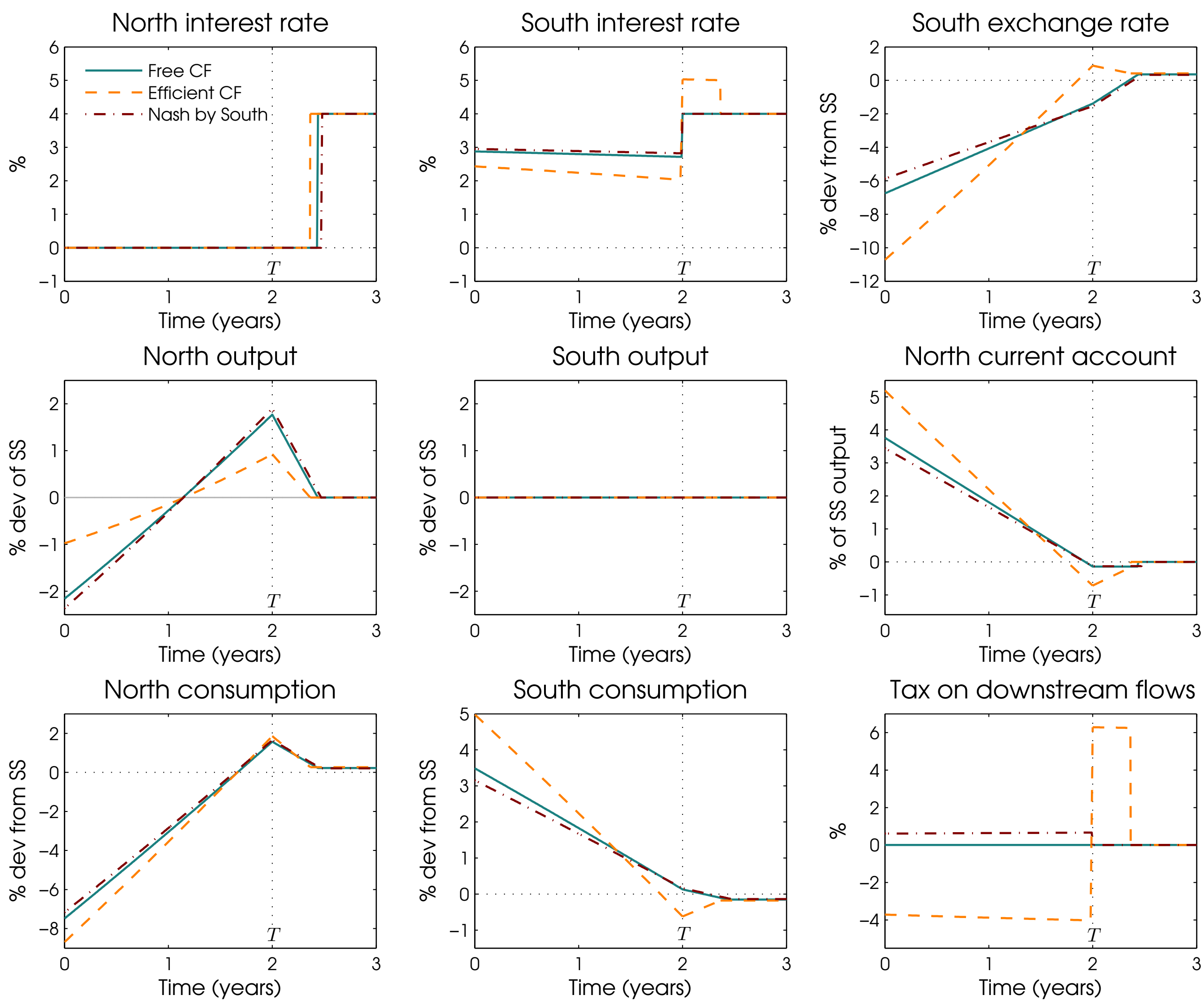

Figure 3: Variable paths under alternative regimes: free capital flows (solid) vs. efficient capital flows (dashed) vs. uncoordinated CF management by South countries (dot-dashed).

Figure 3 shows the paths of key macro variables in this Nash equilibrium and contrasts them with their counterparts in the free capital flow and efficient capital flow regimes. Downstream flows are smaller and the exchange rate path is flatter in the Nash case than under free capital flows. As a result, the North recession is deeper and the ZLB exit is more delayed. Thus, uncoordinated capital account interventions by the South limit the stabilizing forces of capital flows during the liquidity trap and hamper the macroeconomic adjustment in the North.

5.3 An all-out currency war!

When all countries manage their capital account optimally, all the forces laid out in Section 5.1 are simultaneously at play. This is a situation in which a currency war breaks out. In this case, despite being able to analytically characterize the optimal taxes set by each country, we are unable to sign them. In particular, the sign of the tax wedge on downstream flows $\tau_{s,t} - \tau_{n,t}$ depends on the inverse Frisch elasticity parameter ϕ.[33] North countries balance the terms of trade benefits of taxing capital outflows with the aggregate demand stabilization benefits of subsidizing them. The benefits from stabilizing output at the ZLB are increasing in the inverse Frisch elasticity of labor supply ϕ. For larger values of ϕ, the North puts a higher weight on aggregate demand stabilization and thus, subsidizes capital outflows more aggressively. South countries, on the other hand, fight against capital inflows brought about by the demand shock in the North as well as by other countries' taxes on inflows (or subsidies to outflows) by taxing these inflows.

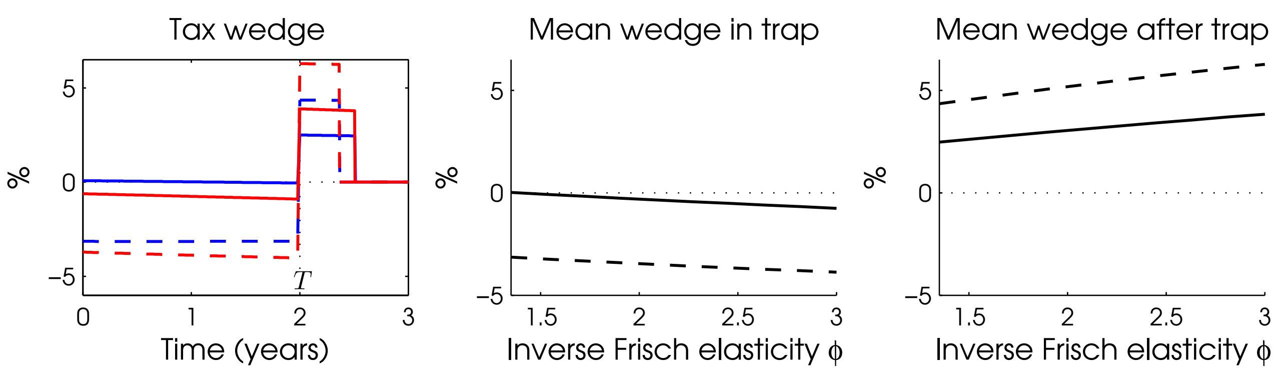

Figure 4: The left panel shows the entire tax wedge path for $\phi = 1.35$ (blue) and $\phi = 3$ (red). The middle panel shows the average tax wedge during the liquidity trap (i.e., between 0 and T) for an inverse Frisch elasticity ranging from $\phi = 1.35$ to $\phi = 3$. The right panel shows the average tax wedge after the liquidity trap while the North is still at the ZLB (i.e., between T and $\widehat{T}_n$) for the same range of ϕ. In all three panels, solid lines represent the tax wedges prevailing in a Nash equilibrium, while dashed lines represent the tax wedge path obtained in the efficient capital flow regime of Section 4.

Figure 4 displays the tax wedge on downstream flows when all parameters except ϕ are set to the previously specified values.[34] We vary ϕ between 1.35 (from the Real Business Cycle literature) and 3 (from the New Keynesian literature; see, e.g., Gali and Monacelli 2005). Following the argument above, the tax wedge is more negative for larger values of ϕ during the liquidity trap. However, irrespective of the value of ϕ, both during and after the liquidity

[33]This tax wedge is informative because it summarizes the net effects of the capital account interventions by North and South countries.

[34]See footnote 22 for details.

trap, the tax wedge falls short of its value in the efficient regime. Overall, a currency war results in countries attenuating each others' capital account interventions. Thus, uncoordinated capital flow management does not encourage enough capital flows to promote an efficient global adjustment.

6 Conclusion

We argue that when a large region of the world economy experiences a liquidity trap, global capital flows allow for a reallocation of demand and expenditures and are therefore stabilizing. Owing to aggregate demand externalities operating at the zero lower bound, free capital flows are nonetheless constrained inefficient and result in reallocations that are too small. Global efficiency requires larger flows during and after the liquidity trap, to compensate for monetary policy's inability to stimulate aggregate demand in the region where the zero bound on interest rates is binding. Despite pointing to inefficient capital flows in a liquidity trap, our analysis does not support the management of capital flows by individual countries. To the contrary, it suggests that the terms of trade management objectives underlying such policies may interfere with aggregate demand stabilization and thus hamper, rather than promote, a smooth global macroeconomic adjustment. Consequently, the analysis underscores the importance of international policy coordination.

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A Appendix

A.1 Proof of Lemma 1

We start by deriving the consumption expression (15). Substituting the Backus-Smith equations for country k and for a representative South country into the market clearing condition (7), we get

$$Y_{k,t} = \left[(1-\alpha)\,\Theta^n_{k,t} + \alpha x + \alpha\,(1-x)\,\Theta^n_{s,t}\right]\left(\mathcal{S}^s_{n,t}\right)^{\alpha(1-x)}\mathcal{S}^n_{k,t}\mathbb{C}_{n,t}. \tag{A.1}$$

Then, substituting $\mathcal{Q}^n_{k,t} = \left(\mathcal{S}^n_{k,t}\right)^{1-\alpha}$ and (A.1) into (12) to eliminate $\mathcal{Q}^n_{k,t}$ and $\mathcal{S}^n_{k,t}$, we get

$$\mathbb{C}_{k,t} = \Theta^n_{k,t}\,(Y_{k,t})^{1-\alpha}\,(\mathbb{C}_{n,t})^{\alpha}\left[(1-\alpha)\,\Theta^n_{k,t} + \alpha x + \alpha\,(1-x)\,\Theta^n_{s,t}\right]^{-(1-\alpha)}\left(\mathcal{S}^s_{n,t}\right)^{-\alpha(1-x)(1-\alpha)} \tag{A.2}$$

Specializing this expression for representative North and South countries, we have

$$\mathbb{C}_{n,t} = Y_{n,t}\left[1-\alpha\,(1-x) + \alpha\,(1-x)\,\Theta^n_{s,t}\right]^{-1}\left(\mathcal{S}^s_{n,t}\right)^{-\alpha(1-x)} \tag{A.3}$$

$$\mathbb{C}_{s,t} = \Theta^n_{s,t}\,(Y_{s,t})^{1-\alpha}\,(\mathbb{C}_{n,t})^{\alpha}\left[(1-\alpha x)\,\Theta^n_{s,t} + \alpha x\right]^{-(1-\alpha)}\left(\mathcal{S}^s_{n,t}\right)^{-\alpha(1-x)(1-\alpha)} \tag{A.4}$$

Furthermore, specializing the Backus-Smith condition for a representative South country, we have

$$\mathbb{C}_{s,t} = \Theta^n_{s,t}\mathbb{C}_{n,t}\left(\mathcal{S}^n_{s,t}\right)^{1-\alpha} \tag{A.5}$$

(A.2), (A.3), (A.4) and (A.5) are a log-linear system in $\mathbb{C}_{k,t}$, $\mathbb{C}_{n,t}$, $\mathbb{C}_{s,t}$ and $\mathcal{S}^n_{s,t}$ whose solution for $\mathbb{C}_{k,t}$ is given by (15).

Next, we derive the dynamic IS equation (16). Taking logs of the goods market clearing condition (A.1) and differentiating with respect to time, we get

$$\begin{aligned}\frac{\dot{Y}_{k,t}}{Y_{k,t}} = &\left[\frac{(1-\alpha)\,\Theta^n_{k,t}}{(1-\alpha)\,\Theta^n_{k,t} + \alpha x + \alpha\,(1-x)\,\Theta^n_{s,t}} - \frac{1}{1-\alpha}\right]\frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} + \frac{1}{1-\alpha}\frac{\dot{\mathbb{C}}_{k,t}}{\mathbb{C}_{k,t}} \\ &+\alpha\,(1-x)\left[\frac{\Theta^n_{s,t}}{(1-\alpha)\,\Theta^n_{k,t} + \alpha x + \alpha\,(1-x)\,\Theta^n_{s,t}} + \frac{1}{1-\alpha}\right]\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} - \frac{\alpha\,(1-x)}{1-\alpha}\frac{\dot{\mathbb{C}}_{s,t}}{\mathbb{C}_{s,t}} - \frac{\alpha x}{1-\alpha}\frac{\dot{\mathbb{C}}_{n,t}}{\mathbb{C}_{n,t}}\end{aligned} \tag{A.6}$$

Under the maintained rigid prices assumption, CPI inflation is given by

$$\pi_{k,t} = \alpha\frac{\dot{\mathcal{S}}_{k,t}}{\mathcal{S}_{k,t}} = \alpha\left[\frac{\dot{\mathcal{E}}^n_{k,t}}{\mathcal{E}^n_{k,t}} + (1-x)\frac{\dot{\mathcal{E}}^s_{n,t}}{\mathcal{E}^s_{n,t}}\right].$$

Substituting out the exchange rate depreciation terms above using the UIP conditions (9) specialized for country k and for a representative South country, we can write the Euler equation (8) as

$$\frac{\dot{\mathbb{C}}_{k,t}}{\mathbb{C}_{k,t}} = (1-\alpha)i_{k,t} + \alpha\left[x\,(i_{n,t} + \tau_{k,t} - \tau_{n,t}) + (1-x)\,(i_{s,t} + \tau_{k,t} - \tau_{s,t})\right] - (\rho + \zeta_{k,t})$$

Specializing this Euler equation for representative North and South countries, and substituting into the output growth expression (A.6), we obtain the dynamic IS equation (16).

A.2 Proof of Proposition 2

Equation (20) can be rewritten as

$$\frac{d\left(\mu_{k,t}Y_{k,t}\right)}{dt} = -e^{-\rho t+\int_0^t \zeta_{k,h}dh}\left[(1-\alpha)-\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right] \tag{A.7}$$

Defining $\widehat{T}_k$ as the time at which the ZLB stops binding in country k, and integrating both sides of (A.7) from 0 to $\widehat{T}_k$ yields

$$\mu_{k,\widehat{T}_k}Y_{k,\widehat{T}_k}-\mu_{k,0}Y_{k,0} = -\int_0^{\widehat{T}_k} e^{-\rho t+\int_0^t \zeta_{k,h}dh}\left[(1-\alpha)-\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right]dt$$

Since $\mu_{k,0}$ is free and the ZLB (by construction) does not bind anymore at $\widehat{T}_k$, we have $\mu_{k,0}=\mu_{k,\widehat{T}_k}=0$ and therefore

$$0=\int_0^{\widehat{T}_k} e^{-\int_0^t(\rho+\zeta_{k,t})dh}\left[1-\left(\frac{Y_{k,t}}{\overline{Y}}\right)^{1+\phi}\right]dt \tag{A.8}$$

with $\overline{Y}\equiv A\left(1-\alpha\right)^{\frac{1}{1+\phi}}$.

The result that $\widehat{T}_k>T$ is easily established. First, $\widehat{T}_k<T$ can be ruled out, because it would require contradicting (for some $t<T$) the premise that the interest rate policy prescribed in Proposition 1 violates the ZLB for $[0,T)$. Second, $\widehat{T}_k=T$ can be ruled out using the observation that if the interest rate policy prescribed in Proposition 1 violates the ZLB for $[0,T)$, then a binding ZLB requires $\frac{\dot{Y}_{k,t}}{Y_{k,t}}>0$ for $[0,T)$, which implies a strictly positive integral on the right hand side of (A.8).

A.3 Bounds on size of preference shock

We impose that the size of the demand shock satisfies:

$$\rho+\frac{\alpha\left(1-x\right)\rho}{\left[1-\alpha(1-x)\right]\left(\rho-\bar{\zeta}\right)}\left[\rho-\bar{\zeta}e^{-(\rho-\bar{\zeta})T}\right]<\bar{\zeta}<\rho+\frac{\left(1-\alpha x\right)\rho}{\alpha x\left(\rho-\bar{\zeta}\right)}e^{-\bar{\zeta}T}\left[\rho-\bar{\zeta}e^{-(\rho-\bar{\zeta})T}\right]. \tag{A.9}$$

In the limiting case with extreme home bias ($\alpha\to 0$), this condition trivially reduces to $\rho<\bar{\zeta}$, which is analogous to the closed economy condition under which the natural rate becomes negative. In general, the condition depends on the degree of openness α and on the mass x of countries experiencing the demand shock. In particular, a small enough α ensures that the condition (A.9) is satisfied if $\rho<\bar{\zeta}$. Thus, as long as the North block is not too large, a large enough shock in the North need not push the South to the ZLB. Furthermore, the set satisfying condition (A.9) is non-empty iff:

$$T\bar{\zeta} < \ln\left[1 + \frac{1-\alpha}{\alpha(1-x)\alpha x}\right].$$

Loosely speaking, this condition requires that for a given duration of the liquidity trap T the shock $\bar{\zeta}$ is not too large, or equivalently that for a given shock $\bar{\zeta}$, the duration of the trap T is not too long.

A.4 Proof of Proposition 5

We proceed by proving the result for zero capital flow taxes, and then arguing that it must hold as well for small taxes due to the continuity of the interest rate policy function $\mathcal{I}_k$ in $\tau_{k,t}$, $\tau_{n,t}$ and $\tau_{s,t}$. In a symmetric equilibrium with zero capital flow taxes, the interest rate policy function specified in Proposition 1, for $0 \leq t < T$, reduce to

$$\mathcal{I}_n = \rho - \frac{1-\alpha(1-x)}{1-\alpha(1-x)+\alpha(1-x)\Theta^n_{s,t}}\bar{\zeta}, \tag{A.10}$$

$$\mathcal{I}_s = \rho - \frac{\alpha x}{(1-\alpha x)\Theta^n_{s,t}+\alpha x}\bar{\zeta}. \tag{A.11}$$

for a North and for a South country, respectively. Now, noting that under the retained assumption of symmetric initial wealth positions, we have $\Theta^n_{s,0} = \frac{\rho}{\rho-\bar{\zeta}} - \frac{\bar{\zeta}}{\rho-\bar{\zeta}}e^{-(\rho-\bar{\zeta})T}$ and condition (A.9) can be written as

$$\rho + \frac{\alpha(1-x)\rho}{[1-\alpha(1-x)]}\Theta^n_{s,0} < \bar{\zeta} < \rho + \frac{(1-\alpha x)\rho}{\alpha x}\Theta^n_{s,T}.$$

or

$$\frac{1-\alpha(1-x)+\alpha(1-x)\Theta^n_{s,0}}{1-\alpha(1-x)}\rho < \bar{\zeta} < \frac{\alpha x+(1-\alpha x)\Theta^n_{s,T}}{\alpha x}\rho. \tag{A.12}$$

Given that $\Theta^n_{s,T} < \Theta^n_{s,t} < \Theta^n_{s,0}$ for $0 < t < T$, the left inequality in (A.12), together with (A.10), implies that $\mathcal{I}_n < 0$ for $0 \leq t < T$. Similarly, the right inequality in (A.12), together with (A.11), implies that $\mathcal{I}_s > 0$ for $0 \leq t < T$. For $t \geq T$, we have $\mathcal{I}_n = \mathcal{I}_s = \rho > 0$. As a result, the unconstrained optimal policy prescribed in Proposition 1 is feasible for South countries, but not for North countries. These instead follow the constrained policy prescribed in Proposition 2.

A.5 Proof of Proposition 6

Defining the functions

$$f_1(z) \equiv \int_0^T e^{-(\rho-\bar{\zeta})t}\left\{1-\left(\frac{\Lambda^n_{s,t}}{\Lambda^n_{s,z}}\right)^{1+\phi}e^{(1+\phi)\left[\rho(z-t)-\bar{\zeta}(T-t)\right]}\right\}dt,$$

$$f_2(z) \equiv -e^{\bar{\zeta}T}\int_T^z e^{-\rho t}\left\{1-\left(\frac{\Lambda^n_{s,t}}{\Lambda^n_{s,z}}\right)^{1+\phi}e^{(1+\phi)\rho(z-t)}\right\}dt,$$

equation (23) can be written as

$$f_1(\widehat{T}_n) = f_2(\widehat{T}_n) \tag{A.13}$$

The functions satisfy $f_1'(z) < 0$ and $f_2'(z) > 0$, with $f_1(T) > 0$, $f_2(T) = 0$, $\lim_{z\to\infty} f_1(z) = -\infty$, and $\lim_{z\to\infty} f_2(z) = +\infty$.[35] (A.13) therefore has a unique solution $\widehat{T}_n > T$.

Now observe that under free capital flows, $\Lambda^n_{s,t} > \Lambda^n_{s,T}$ for $t < T$ and $\Lambda^n_{s,t} = \Lambda^n_{s,T}$ for $t \geq T$, while under closed capital accounts, $\Lambda^n_{s,t} = \Lambda^n_{s,T} = 1$ for all $t \geq 0$. As a result, $f_1^{\text{free}}(z) < f_1^{\text{closed}}(z)$ and $f_2^{\text{free}}(z) = f_2^{\text{closed}}(z)$ for $z > T$. It must thus be that $\widehat{T}_n^{\text{free}} < \widehat{T}_n^{\text{closed}}$.

A.6 Proof of Proposition 7

The problem of the global planner is given by

$$\max_{\{i_{n,t}\geq 0, i_{s,t}\geq 0, \tau_{s,t}, \mathcal{T}\}} \int_0^\infty e^{-\rho t}\left\{x e^{\bar{\zeta}\min[t,T]}\mathbb{W}_{n,t} + (1-x)\,\Xi\mathbb{W}_{n,t}\right\} dt$$

subject to:

$$V_n^{\text{free}} \leq \int_0^\infty e^{-\rho t + \bar{\zeta}\min[t,T]}\mathbb{W}_{n,t}dt \tag{A.14}$$

$$V_s^{\text{free}} \leq \int_0^\infty e^{-\rho t}\mathbb{W}_{s,t}dt \tag{A.15}$$

$$\mathbb{W}_{n,t} \equiv \log\left(\underbrace{\left(\frac{Y_{nt}}{\Lambda_{n,t}}\right)^{1-\alpha(1-x)}\left(\frac{Y_{s,t}}{\Lambda_{s,t}}\right)^{\alpha(1-x)}}_{\mathbb{C}_{n,t}}\right) - \frac{1}{1+\phi}\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}$$

$$\mathbb{W}_{s,t} \equiv \log\left(\underbrace{\Theta^n_{s,t}\left(\frac{Y_{n,t}}{\Lambda_{n,t}}\right)^{\alpha x}\left(\frac{Y_{s,t}}{\Lambda_{s,t}}\right)^{1-\alpha x}}_{\mathbb{C}_{s,t}}\right) - \frac{1}{1+\phi}\left(\frac{Y_{s,t}}{A}\right)^{1+\phi}$$

$$\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} = \tau_{s,t} + \zeta_{n,t} \tag{A.16}$$

$$\frac{\dot{Y}_{n,t}}{Y_{n,t}} = i_{n,t} - (\rho + \zeta_{n,t}) + \frac{\alpha(1-x)\Theta^n_{s,t}}{\Lambda_{n,t}}\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} \tag{A.17}$$

$$\frac{\dot{Y}_{s,t}}{Y_{s,t}} = i_{s,t} - \rho - \frac{\alpha x}{\Lambda_{s,t}}\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} \tag{A.18}$$

[35] A sufficient condition for $f_1(T) > 0$ is that $\frac{\dot{Y}_{n,t}}{Y_{n,t}} > 0$ for $t \in [0,T)$. This follows from the fact that $\mathcal{I}_n(\cdot) < 0$ for $t \in [0,T)$ and is guaranteed to hold under both free capital flows and closed capital accounts.

$$B_{n,0} - \frac{1-x}{x}\mathcal{T} = \alpha(1-x)\int_0^\infty e^{-\rho t+\bar{\zeta}\min[t,T]}\left[1-\Theta_{s,t}^n\right]dt \tag{A.19}$$

$$\dot{\mu}_{n,t} = -\frac{e^{-\rho t+\bar{\zeta}\min[t,T]}}{Y_{n,t}}\left\{1-\alpha-\left[\frac{Y_{n,t}}{A}\right]^{1+\phi}\right\} - \mu_{n,t}\frac{\dot{Y}_{n,t}}{Y_{n,t}} \tag{A.20}$$

$$\dot{\mu}_{s,t} = -\frac{e^{-\rho t}}{Y_{s,t}}\left\{1-\alpha-\left[\frac{Y_{s,t}}{A}\right]^{1+\phi}\right\} - \mu_{s,t}\frac{\dot{Y}_{s,t}}{Y_{s,t}} \tag{A.21}$$

$$\mu_{n,t}i_{n,t} = 0 \tag{A.22}$$

$$\mu_{s,t}i_{s,t} = 0 \tag{A.23}$$

for $\Lambda_{n,t} \equiv 1-\alpha(1-x)+\alpha(1-x)\Theta_{s,t}^n$ and $\Lambda_{s,t} \equiv (1-\alpha x)\Theta_{s,t}^n+\alpha x$. Ξ is an exogenous relative weight assigned by the global planner to South countries. (A.14) and (A.15) are the constraints indicating that the planner must deliver to all countries at least the same level of welfare as under free capital flows (defined as V_k^{free}). (A.16) is the law of motion for the relative expenditure ratio of South to North countries. (A.17) and (A.18) are dynamic IS equations.[36] (A.19) is a North country's inter-temporal budget constraint.[37] (A.20) and (A.21) are the conditions representing optimal monetary policy responses by individual countries.

Let $\Delta_n \geq 0$ and $\Delta_s \geq 0$ be the multipliers on the inequality constraints (A.14) and (A.15), respectively. The Hamiltonian associated with this problem is given by[38]

$$\begin{aligned}
\mathcal{H} &= x(1+\Delta_n)e^{-\rho t+\bar{\zeta}\min[t,T]}\mathbb{W}_{n,t} + (1-x)(\Xi+\Delta_s)e^{-\rho t}\mathbb{W}_{s,t} + \mu_t\Theta_{s,t}^n(\tau_{s,t}+\zeta_{n,t}) \\
&+ \lambda_{n,t}Y_{n,t}\left\{i_{n,t}-(\rho+\zeta_{n,t})+\frac{\alpha(1-x)\Theta_{s,t}^n}{1-\alpha(1-x)+\alpha(1-x)\Theta_{s,t}^n}(\tau_{s,t}+\zeta_{n,t})\right\} \\
&+ \lambda_{s,t}Y_{s,t}\left\{i_{s,t}-\rho-\frac{\alpha x}{(1-\alpha x)\Theta_{s,t}^n+\alpha x}(\tau_{s,t}+\zeta_{n,t})\right\} \\
&+ \varphi_{n,t}\left[-\frac{e^{-\rho t+\bar{\zeta}\min[t,T]}}{Y_{n,t}}\left\{(1-\alpha)-\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}\right\} - \mu_{n,t}\left\{i_{n,t}-(\rho+\zeta_{n,t})+\frac{\alpha(1-x)\Theta_{s,t}^n}{1-\alpha(1-x)+\alpha(1-x)\Theta_{s,t}^n}(\tau_{s,t}+\zeta_{n,t})\right\}\right] \\
&+ \varphi_{s,t}\left[-\frac{e^{-\rho t}}{Y_{s,t}}\left\{(1-\alpha)-\left(\frac{Y_{s,t}}{A}\right)^{1+\phi}\right\} - \mu_{s,t}\left\{i_{s,t}-\rho-\frac{\alpha x}{(1-\alpha x)\Theta_{s,t}^n+\alpha x}(\tau_{s,t}+\zeta_{n,t})\right\}\right] \\
&+ \kappa_{n,t}\mu_{n,t}i_{n,t}+\kappa_{s,t}\mu_{s,t}i_{s,t}
\end{aligned}$$

The state variables are Y_n, Y_s, μ_k, μ_s and Θ_s^n, and we define $\lambda_n, \lambda_s, \varphi_n, \varphi_s$ and μ as the respective co-states. κ_n, κ_s are the multiplier on the equality constraints (A.22) and (A.23). The optimality

[36] See Lemma 1 for details.

[37] $\mathcal{T}$ is the date 0 amount taxed away from a North country. As a result, $\frac{x}{1-x}\mathcal{T}$ is the amount transfered to a South country. Given the consistency condition for initial net foreign assets, $B_{s,0} = -\frac{xB_{n,0}}{1-x}$, a South country's inter-temporal budget constraint can be derived from (A.19).

[38] The budget constraint (A.19) is omitted for convenience, since the first order condition for $\mathcal{T}$ will require that the multiplier on that constraint be zero.

conditions are given by:

$$\frac{\partial \mathcal{H}}{\partial i_{n,t}} = \lambda_{n,t} Y_{n,t} + \kappa_{n,t}\mu_{n,t} \leq 0, \qquad \lambda_{n,t} i_{n,t} = 0 \tag{A.24}$$

$$\frac{\partial \mathcal{H}}{\partial i_{n,t}} = \lambda_{s,t} Y_{s,t} + \kappa_{s,t}\mu_{s,t} \leq, 0 \qquad \lambda_{s,t} i_{s,t} = 0 \tag{A.25}$$

$$\frac{\partial \mathcal{H}}{\partial \tau_{s,t}} = \mu_t \Theta^n_{s,t} + (\lambda_{n,t} Y_{n,t} - \varphi_{n,t}\mu_{n,t}) \frac{\alpha (1-x) \Theta^n_{s,t}}{\Lambda_{n,t}} - (\lambda_{s,t} Y_{s,t} - \varphi_{s,t}\mu_{s,t}) \frac{\alpha x}{\Lambda_{s,t}} = 0 \tag{A.26}$$

$$\begin{aligned} -\dot{\mu}_t &= e^{-\rho t + \bar{\zeta}\min[t,T]} (1+\Delta_n) \left(x \frac{\partial \mathbb{W}_{n,t}}{\partial \Theta^n_{s,t}} + (1-x)\, \Xi^n_{s,t} \frac{\partial \mathbb{W}_{s,t}}{\partial \Theta^n_{s,t}} \right) + \mu_t \frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} \\ &+ (\lambda_{n,t} Y_{n,t} - \varphi_{n,t}\mu_{n,t}) \frac{[1-\alpha(1-x)]\,\alpha(1-x)}{(\Lambda_{n,t})^2} \frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} \\ &+ (\lambda_{s,t} Y_{s,t} - \varphi_{s,t}\mu_{s,t}) \frac{(1-\alpha x)\,\alpha x}{(\Lambda_{s,t})^2} \frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} \end{aligned} \tag{A.27}$$

$$\begin{aligned} -\dot{\lambda}_{n,t} &= e^{-\rho t + \bar{\zeta}\min[t,T]} (1+\Delta_n) \left(x \frac{\partial \mathbb{W}_{n,t}}{\partial Y_{n,t}} + (1-x)\, \Xi^n_{s,t} \frac{\partial \mathbb{W}_{s,t}}{\partial Y_{n,t}} \right) + \lambda_{n,t} \frac{\dot{Y}_{n,t}}{Y_{n,t},} \\ &+ \frac{\varphi_{n,t}}{Y_{n,t}} \frac{e^{-\rho t + \bar{\zeta}\min[t,T]}}{Y_{n,t}} \left[(1-\alpha) + \phi \left(\frac{Y_{n,t}}{A} \right)^{1+\phi} \right] \end{aligned} \tag{A.28}$$

$$\begin{aligned} -\dot{\lambda}_{s,t} &= e^{-\rho t + \bar{\zeta}\min[t,T]} (1+\Delta_n) \left(x \frac{\partial \mathbb{W}_{n,t}}{\partial Y_{s,t}} + (1-x)\, \Xi^n_{s,t} \frac{\partial \mathbb{W}_{s,t}}{\partial Y_{s,t}} \right) + \lambda_{s,t} \frac{\dot{Y}_{s,t}}{Y_{s,t}} \\ &+ \frac{\varphi_{s,t}}{Y_{s,t}} \frac{e^{-\rho t}}{Y_{s,t}} \left[(1-\alpha) + \phi \left(\frac{Y_{s,t}}{A} \right)^{1+\phi} \right] \end{aligned} \tag{A.29}$$

$$-\dot{\varphi}_{n,t} = -\varphi_{n,t} \frac{\dot{Y}_{n,t}}{Y_{n,t}} + \kappa_{n,t} i_{n,t} \tag{A.30}$$

$$-\dot{\varphi}_{s,t} = -\varphi_{s,t} \frac{\dot{Y}_{s,t}}{Y_{s,t}} + \kappa_{s,t} i_{s,t} \tag{A.31}$$

where $\Xi^n_{s,t} \equiv e^{-\bar{\zeta}\min[t,T]} \frac{\Xi+\Delta_s}{1+\Delta_n}$, and

$$\frac{\partial \mathbb{W}_{n,t}}{\partial Y_{n,t}} = \left[1 - \alpha(1-x) - \left(\frac{Y_{n,t}}{A} \right)^{1+\phi} \right] \frac{1}{Y_{n,t}}, \quad \frac{\partial \mathbb{W}_{n,t}}{\partial Y_{s,t}} = \alpha(1-x) \frac{1}{Y_{s,t}}, \quad \frac{\partial \mathbb{W}_{n,t}}{\partial \Theta^n_{s,t}} = -\frac{\alpha(1-x)\,\Phi_t}{\Lambda_{n,t}\Lambda_{s,t}}, \tag{A.32}$$

$$\frac{\partial \mathbb{W}_{s,t}}{\partial Y_{n,t}} = \alpha x \frac{1}{Y_{n,t}}, \quad \frac{\partial \mathbb{W}_{s,t}}{\partial Y_{s,t}} = \left[1 - \alpha x - \left(\frac{Y_{s,t}}{A} \right)^{1+\phi} \right] \frac{1}{Y_{s,t}}, \quad \frac{\partial \mathbb{W}_{s,t}}{\partial \Theta^n_{s,t}} = \frac{\alpha x \Phi_t}{\Theta^n_{s,t}\Lambda_{n,t}\Lambda_{s,t}} \tag{A.33}$$

with $\Phi_t \equiv 1 - \alpha(1-x) + (1-\alpha x)\,\Theta^n_{s,t}$.

A.6.1 Constrained efficiency of free capital flows away from the ZLB

Absent the ZLB, we have $\mu_{n,t} = \mu_{s,t} = 0$ for all t. Then (A.20)-(A.21) require that that $Y_{n,t} = Y_{s,t} = A(1-\alpha)^{\frac{1}{1+\phi}}$ for all t and (A.24)-(A.25) imply that $\lambda_{n,t} = \lambda_{s,t} = 0$ for all t. (A.26) then requires that $\mu_t = 0$ for all t. Using this information in (A.27), we get $\Theta^n_{s,t} = \Xi^n_{s,t}$. Differentiating this equation with respect to time yields $\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} = \zeta_{n,t}$, which implies an optimal choice of $\tau_{s,t} = 0$ for all t. Thus, in the absence of the ZLB, free capital flows are constrained efficient.

A.6.2 Constrained inefficiency of free capital flows at the ZLB

In the sequel, we focus on a case where the ZLB binds only in the North, and later on verify that this is indeed the relevant scenario. We define as $\widehat{T}_n > T$ the time at which the North exits the ZLB. Then, for $t < \widehat{T}_n$, (A.30) implies that $\frac{\varphi_{n,t}}{Y_{n,t}} = \frac{\varphi_{n,\widehat{T}_n}}{Y_{n,\widehat{T}_n}} = \delta_n$. Further, for $t \geq \widehat{T}_n$, we must have $\mu_{n,t} = \lambda_{n,t} = 0$ and $Y_{n,t} = A(1-\alpha)^{\frac{1}{1+\phi}}$. (A.28) then implies

$$\delta_n = -\frac{\alpha x}{(1-\alpha)(1+\phi)}(1+\Delta_n)\left[x + (1-x)\Xi_T\right].$$

Differentiating (A.26) with respect to time yields

$$\begin{aligned}\dot{\mu}_t \Theta^n_{s,t} + \mu_t \dot{\Theta}^n_{s,t} &= -\left(\dot{\lambda}_{n,t} Y_{n,t} + \lambda_{n,t}\dot{Y}_{n,t} - \dot{\varphi}_{n,t}\mu_{n,t} - \varphi_{n,t}\dot{\mu}_{n,t}\right)\frac{\alpha(1-x)\Theta^n_{s,t}}{\Lambda_{n,t}} \\ &\quad - \left(\lambda_{n,t} Y_{n,t} - \varphi_{n,t}\mu_{n,t}\right)\alpha(1-x)\left[1-\alpha(1-x)\right]\frac{\dot{\Theta}^n_{s,t}}{(\Lambda_{n,t})^2}\end{aligned} \tag{A.34}$$

Using (A.27), (A.28), (A.30) and (A.20) to substitute into (A.34) leads to

$$-\Psi\alpha x\left\{\left[1-\alpha(1-x)\right] + \alpha(1-x)\Xi_t\right\}\frac{\Theta^n_{s,t}}{\Lambda_{n,t}} + \frac{\Theta^n_{s,t}}{\Lambda_{n,t}}\left(\frac{Y_{n,t}}{A}\right)^{1+\phi} - \Psi\alpha x\frac{\Phi_t}{\Lambda_{n,t}\Lambda_{s,t}}\left[\Xi_t - \Theta^n_{s,t}\right] = 0 \tag{A.35}$$

for

$$\Psi \equiv \frac{1-\alpha}{\left[1-\alpha(1-x) + \alpha(1-x)\Xi^n_{s,T}\right]\alpha x}.$$

We now prove that under condition (A.9), which guarantees that the ZLB binds in the North under free capital flows, the free capital flow regime is constrained inefficient. We prove the result by showing that a zero capital flow tax path is not a solution to the planning problem. We start by establishing the following intermediate result.

Lemma 3. *Conditional on a zero capital flow tax path $\tau_{s,t} = 0\ \forall t \geq 0$, the optimal transfer is $\mathcal{T} = 0$.*

Proof. We proceed in two steps. First, we show that the optimal transfer is zero for one particular welfare weight. Then, we argue that the optimal transfer must also be zero for arbitrary welfare weights. The relevant planning problem is the one described above, with the differences that $\tau_{s,t}$ is not a control variable, and as a result, (A.26) drops out of the set of optimality conditions, and $\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} = \zeta_{n,t}$.

For $t \geq T$, since $\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} = \zeta_{n,t} = 0$ (A.27) implies

$$\frac{d\left(\mu_t \Theta^n_{s,t}\right)}{dt} = -e^{-\rho t + \bar{\zeta}T}(1+\Delta_n)\frac{\Phi_T}{\Lambda_{n,T}\Lambda_{s,T}}\alpha x(1-x)\left\{\Xi^n_{s,T} - \Theta^n_{s,T}\right\}$$

Integrating from T to ∞, and imposing the terminal condition $\lim_{t\to\infty}\mu_t\Theta^n_{s,t}=0$, we get

$$0=\mu_T\Theta^n_{s,T}-\frac{\Phi_T}{\Lambda_{n,T}\Lambda_{s,T}}\alpha x\left(1-x\right)e^{\left(\bar{\zeta}-\rho\right)T}\left(1+\Delta_n\right)\left\{\Xi^n_{s,T}-\Theta^n_{s,T}\right\}\frac{1}{\rho}$$

Since the state variable $\Theta^n_{s,t}$ is free, we have $\mu_T=0$ and therefore $\Theta^n_{s,T}=\Xi^n_{s,T}$. It follows that $\Theta^n_{s,0}=\Xi^n_{s,0}$.

Now, fix the welfare weight to the symmetric value $\Xi=(\bar{\zeta}e^{\left(\bar{\zeta}-\rho\right)T}-\rho)/(\bar{\zeta}-\rho)$, and consider the relaxed problem where the constraints requiring that all countries are at least as well off as in the free capital flow scenario (A.14) and (A.15) are dropped. In this case, $\Delta_n=\Delta_s=0$ and the solution to the planner's problem features $\Theta^n_{s,0}=\Xi^n_{s,0}=\Xi=(\bar{\zeta}e^{\left(\bar{\zeta}-\rho\right)T}-\rho)/(\bar{\zeta}-\rho)$, and therefore entails a zero transfer. Since this optimal plan trivially satisfies the constraints (A.14) and (A.15), it is also the solution to the more constrained version of the problem including these two extra constraints.

To prove that a zero transfer is also optimal for arbitrary welfare weights, we proceed by contradiction. Suppose that a zero transfer is not optimal. This requires that there is a non-zero transfer that makes either the North or the South (or both) better off without making neither of the two worse off. But if this were the case, then for the symmetric welfare weight $\Xi=(\bar{\zeta}e^{\left(\bar{\zeta}-\rho\right)T}-\rho)/(\bar{\zeta}-\rho)$, the optimal transfer cannot be zero either. This is a contradiction with the result above. It follows that the optimal transfer is zero for arbitrary welfare weights. □

We now proceed to show by contradiction that a zero capital flow tax path is not a solution to the planning problem. Suppose it was the case. Then, by the principle of optimality, Lemma 3 implies that an optimal transfer of $\mathcal{T}=0$. It then follows that $\Theta^n_{s,t}=\Xi^n_{s,t}=e^{-\bar{\zeta}\min[t,T]}(\bar{\zeta}e^{\left(\bar{\zeta}-\rho\right)T}-\rho)/(\bar{\zeta}-\rho)$. The allocations thus coincide with those of the free capital flow regime, and Proposition 5 established that in this case the ZLB binds in the North but not in the South. (A.35) then becomes

$$\Lambda_{n,t}=\Lambda_{n,T}\frac{1}{1-\alpha}\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}.$$

Differentiating this expression with respect to time yields

$$\frac{\alpha\left(1-x\right)\Theta^n_{s,t}}{\Lambda_{n,t}}\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}=\left(1+\phi\right)\frac{\dot{Y}_{n,t}}{Y_{n,t}}.$$

For $0\leq t<T$, we have $\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}=-\bar{\zeta}<0$ but $\frac{\dot{Y}_{n,t}}{Y_{n,t}}>0$, while for $T\leq t<\widehat{T}_n$, we have $\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}=0$ but $\frac{\dot{Y}_{n,t}}{Y_{n,t}}<0$. In either case, we have a contradiction.

A.7 Proof of Proposition 8

The labor wedge is defined for the good of any country $k\in\{n,s\}$ as

$$\omega_{k,t}\equiv 1-\left(\mathcal{S}^n_{k,t}\right)^{\alpha}\left(\mathcal{S}^n_{s,t}\right)^{-\alpha(1-x)}\frac{\mathbb{C}_{k,t}N^{\phi}_{k,t}}{A} \tag{A.36}$$

Using this definition for a representative North and South country, we can write $\omega_{n,t} = 1 - \frac{\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}}{\Lambda_{n,t}}$ and $\omega_{s,t} = 1 - \frac{(1-\alpha)\Theta^n_{s,t}}{\Lambda_{s,t}}$. Then, (A.35) can be rewritten as

$$\Theta^n_{s,t} = \Psi \frac{\Xi^n_{s,t} - \left[\alpha x + (1-\alpha x)\,\Xi^n_{s,t}\right] \frac{1-\alpha x}{1-\alpha} (1-\omega_{s,t})}{(1-\omega_{n,t})}. \tag{A.37}$$

Differentiating this equation with respect to time, we obtain

$$\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} = \frac{\zeta_{n,t}\Xi^n_{s,t} - \frac{1-\alpha x}{1-\alpha}\left\{(1-\alpha x)\,\zeta_{n,t}\Xi^n_{s,t}(1-\omega_{s,t}) - \left[\alpha x + (1-\alpha x)\,\Xi^n_{s,t}\right]\dot{\omega}_{s,t}\right\}}{\Xi^n_{s,t} - \left[\alpha x + (1-\alpha x)\,\Xi^n_{s,t}\right]\frac{1-\alpha x}{1-\alpha}(1-\omega_{s,t})} + \frac{\dot{\omega}_{n,t}}{1-\omega_{n,t}}$$

and therefore

$$\tau_{s,t} = \frac{\alpha x\,(1-\omega_{s,t})\,\zeta_{n,t} + \left[\alpha x + (1-\alpha x)\,\Xi^n_{s,t}\right]\dot{\omega}_{s,t}}{\frac{1-\alpha}{1-\alpha x}\Xi^n_{s,t} - \left[\alpha x + (1-\alpha x)\,\Xi^n_{s,t}\right](1-\omega_{s,t})} + \frac{\dot{\omega}_{n,t}}{1-\omega_{n,t}} \tag{A.38}$$

In the limit of extreme home bias, we have $\frac{\dot{\omega}_{s,t}}{\omega_{s,t}} = 0$ and the first term vanishes. Thus

$$\lim_{\alpha\to 0} \tau_{s,t} = \frac{\dot{\omega}_{n,t}}{1-\omega_{n,t}}, \tag{A.39}$$

and therefore

$$\lim_{\alpha\to 0} \tau_{s,t} = \begin{cases} (1+\phi)\left(\rho - \bar{\zeta}\right) < 0 & \text{for } 0 \le t < T \\ (1+\phi)\,\rho > 0 & \text{for } T \le t < \widehat{T}_n \\ 0 & \text{for } t \ge \widehat{T}_n \end{cases} \tag{A.40}$$

Thus, by continuity, for small enough α, we must have

$$\lim_{\alpha\to 0} \tau_{s,t} \begin{cases} < 0 & \text{for } 0 \le t < T \\ > 0 & \text{for } T \le t < \widehat{T}_n \\ = 0 & \text{for } t \ge \widehat{T}_n \end{cases} \tag{A.41}$$

A.8 Proof of Proposition 10

Using Lemma 1, the problem of country k's monetary policy authority can be written as maximizing

$$\int_0^\infty e^{-\int_0^t (\rho+\zeta_{k,s})ds} \left[\ln\left(\Theta^n_{k,t}\right) + (1-\alpha)\ln\left(\frac{Y_{k,t}}{\Lambda_{k,t}}\right) - \frac{1}{1+\phi}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right] dt \tag{A.42}$$

subject to

$$\frac{\dot{Y}_{k,t}}{Y_{k,t}} = i_{k,t} - (\rho + \zeta_{k,t}) - \frac{\alpha x}{\Lambda_{k,t}}\frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \frac{\alpha(1-x)\,\Theta^n_{s,t}}{\Lambda_{k,t}}\frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}} \tag{A.43}$$

$$\frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} = \tau_{k,t} - \zeta_{k,t} - (\tau_{n,t} - \zeta_{n,t}), \quad (A.44)$$

$$i_{k,t} \geq 0, \quad (A.45)$$

$$B_{k,0} = \alpha \int_0^\infty e^{-\int_0^t (\rho+\zeta_{n,s}-\tau_{n,s})ds} \left[\Theta^n_{k,t} - x - (1-x)\Theta^n_{s,t}\right] dt, \quad (A.46)$$

with $\Lambda_{k,t} \equiv (1-\alpha)\Theta^n_{k,t} + \alpha x + \alpha(1-x)\Theta^n_{s,t}$, and $\frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}} = \frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}$. (A.43) is the dynamic IS equation, (A.44) is the law of motion for the expenditure ratio $\Theta^n_{k,t}$, (A.45) is the ZLB constraint and (A.46) is the country's budget constraint.

The policy authority's problem is an optimal control problem with states $Y_{k,t}, \Theta^n_{k,t}$ and controls $i_{k,t}, \tau_{k,t}$. Defining the respective co-state variables as $\mu^\theta_{k,t}, \lambda^y_{k,t}$, and the multiplier on the country budget constraint as Γ_k, the Hamiltonian is given by

$$\begin{aligned}\mathcal{H}_{k,t} &= e^{-\int_0^t(\rho+\zeta_{k,t})dh}\left[\ln\left(\Theta^n_{k,t}\right) + (1-\alpha)\ln\left(\frac{Y_{k,t}}{(1-\alpha)\Theta^n_{k,t} + \alpha x + \alpha(1-x)\Theta^n_{s,t}}\right) - \frac{1}{1+\phi}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right] \\ &+\lambda^y_{k,t}Y_{k,t}\left\{i_{k,t} - (\rho+\zeta_{k,t}) + \frac{\alpha(1-x)\Theta^n_{s,t}\left[\zeta_{k,t} - (\tau_{k,t}-\tau_{s,t})\right] + \alpha x\left[(\zeta_{k,t}-\zeta_{n,t}) - (\tau_{k,t}-\tau_{n,t})\right]}{(1-\alpha)\Theta^n_{k,t} + \alpha x + \alpha(1-x)\Theta^n_{s,t}}\right\} \\ &+\mu^\theta_{k,t}\Theta^n_{k,t}\left[\tau_{k,t} - \zeta_{k,t} - (\tau_{n,t} - \zeta_{n,t})\right] + \nu_{k,t}i_{k,t} \\ &-\alpha\Gamma_k\left\{e^{-\int_0^t(\rho+\zeta_{n,h}-\tau_{n,h})dh}\left[\Theta^n_{k,t} - x - (1-x)\Theta^n_{s,t}\right]\right\}\end{aligned}$$

The optimality conditions are

$$\frac{\partial \mathcal{H}_{k,t}}{\partial i_{k,t}} = \lambda^y_{k,t}Y_{k,t} + \nu_{k,t} = 0 \quad (A.47)$$

$$\frac{\partial \mathcal{H}}{\partial \tau_{k,t}} = \mu^\theta_{k,t}\Theta^n_{k,t} - \lambda^y_{k,t}Y_{k,t}\frac{\alpha(1-x)\Theta^n_{s,t} + \alpha x}{\Lambda_{k,t}} = 0 \quad (A.48)$$

$$-\dot{\lambda}^y_{k,t} = \frac{e^{-\int_0^t(\rho+\zeta_{k,t})dh}}{Y_{k,t}}\left[(1-\alpha) - \left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right] + \lambda^y_{k,t}\frac{\dot{Y}_{k,t}}{Y_{k,t}} \quad (A.49)$$

$$\begin{aligned}-\dot{\mu}^\theta_{k,t} &= e^{-\int_0^t(\rho+\zeta_{k,h})dh}\left(\frac{1}{\Theta^n_{k,t}} - \frac{(1-\alpha)^2}{\Lambda_{k,t}}\right) - (1-\alpha)\lambda^y_{k,t}\left(\frac{Y_{k,t}}{\Lambda_{k,t}}\right)\left[\frac{\dot{Y}_{k,t}}{Y_{k,t}} - i_{k,t} + (\rho+\zeta_{k,t})\right] \\ &+\mu^\theta_{k,t}\frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \alpha\Gamma_k e^{-\int_0^t(\rho+\zeta_{n,h}-\tau_{n,h})dh}\end{aligned} \quad (A.50)$$

$\nu_{k,t}i_{k,t} = 0$ and $\nu_{k,t} \geq 0$.

Differentiating (A.48) with respect to time, we get

$$\dot{\mu}^\theta_{k,t}\Theta^n_{k,t} + \mu^\theta_{k,t}\dot{\Theta}^n_{k,t} - \left(\dot{\lambda}^y_{k,t}Y_{k,t} + \lambda^y_{k,t}\dot{Y}_{k,t}\right)\frac{\alpha(1-x)\Theta^n_{s,t} + \alpha x}{\Lambda_{k,t}} + \frac{\lambda^y_{k,t}Y_{k,t}\Theta^n_{k,t}(1-\alpha)}{\Lambda_{k,t}}\left[-\frac{\dot{Y}_{k,t}}{Y_{k,t}} + i_{k,t} - (\rho+\zeta_{k,t})\right] = 0 \quad (A.51)$$

Substituting (A.49) and (A.50) into (A.51), and simplifying, leads to

$$\alpha\Gamma_k = -e^{\int_0^t(\zeta_{n,s}-\zeta_{k,s}-\tau_{n,s})ds}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\left\{\frac{(1-\alpha)}{\Lambda_{k,t}} - \frac{1}{1-\alpha}\frac{1}{\Theta^n_{k,t}}\right\} + \frac{e^{\int_0^t(\zeta_{n,s}-\zeta_{k,s}-\tau_{n,s})ds}}{\Theta^n_{k,t}}\alpha\left[1-\frac{1}{1-\alpha}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right]. \tag{A.52}$$

Differentiating equation (A.52) with respect to time, substituting the $\frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}}$ and $\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}$ terms, and isolating $\tau_{k,t}$ leads to (24), for

$$\Omega^1_{k,t} \equiv \frac{(1-\alpha)^2\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\Theta^n_{k,t}}{\Omega^3_{k,t}+\Omega^4_{k,t}\left[1-\frac{1}{1-\alpha}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right]}$$

$$\Omega^2_{k,t} \equiv \frac{(1-\alpha)\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}(1+\phi)\,\Lambda_{k,t}\left\{x+(1-x)\,\Theta^n_{s,t}\right\}}{\Omega^3_{k,t}+\Omega^4_{k,t}\left[1-\frac{1}{1-\alpha}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right]}$$

$$\Omega^3_{k,t} \equiv \left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\left\{(1-\alpha)^2\left(\Theta^n_{k,t}\right)^2+\left[x+(1-x)\,\Theta^n_{s,t}\right]\left[(1-\alpha)\,\Theta^n_{k,t}+\Lambda_{k,t}\right]\right\} > 0$$

$$\Omega^4_{k,t} \equiv \left(\Lambda_{k,t}\right)^2(1-\alpha) > 0.$$

It can easily be verified that $\Omega^3_{k,t}+\Omega^4_{k,t}\left[1-\frac{1}{1-\alpha}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right]$ is necessarily strictly positive, so that $\Omega^1_{k,t}$ and $\Omega^2_{k,t}$ too are strictly positive.

A.9 Proof of Proposition 11

We proceed by conjecturing that in the Nash equilibrium of the policy game, the ZLB binds in the North but not in the South, and later verify this conjecture. Imposing $\tau_{n,t}=0$ and symmetry among South countries in (24) for $k=s$ yields

$$\tau_{s,t} = \frac{(1-\alpha)^2\,\Theta^n_{s,t}\left[(1-x)\,\Theta^n_{s,t}\tau_{s,t}-x\zeta_{n,t}\right]}{(1-\alpha)^2\left(\Theta^n_{s,t}\right)^2\left[x+(1-x)\,\Theta^n_{s,t}\right]\left[2(1-\alpha)\,\Theta^n_{s,t}+\alpha x+\alpha(1-x)\,\Theta^n_{s,t}\right]}, \tag{A.53}$$

Solving for $\tau_{s,t}$ yields (25), for

$$\Upsilon_{s,t} \equiv \frac{(1-\alpha)^2 x\Theta^n_{s,t}}{(1-\alpha)^2 x\Theta^n_{s,t}+\left[1-\alpha^2+2\alpha(1-x)\right]x\Theta^n_{s,t}+\alpha x^2+\left[2-\alpha-x\right](1-\alpha x)\left(\Theta^n_{s,t}\right)^2}. \tag{A.54}$$

Since $0<\Upsilon_{s,t}<1$, for $0\le t<T$ we have $-\bar{\zeta}<\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}<0$. As a result $\Theta^n_{s,0}$ is lower than under free capital flows, and $\Theta^n_{T,0}$ is higher than under free capital flows. It is then straightforward to verify that, as under free capital flows, condition (A.9) guarantees that $\mathcal{I}_s>0$ and $\mathcal{I}_n<0$ for $0\le t<T$, which validates the conjecture that the ZLB binds in the North but not in the South.