

Do Firms Mitigate or Magnify Capital Misallocation? Evidence from Plant-Level Data^{*,**}

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Abstract

Almost two thirds of the cross-plant dispersion in marginal revenue products of capital occurs across plants within the same firm rather than between firms. Even though firms allocate investment very differently across their plants, they do not equalize marginal revenue products across their plants. We reconcile these findings in a model of multi-plant firms, physical adjustment costs and credit constraints. Credit constrained multi-plant firms can utilize internal capital markets by concentrating internal funds on investment projects in only a few of their plants in a given period and rotating funds to another set of plants in the future. The resulting increase in within-firm dispersion of marginal revenue products of capital is hence not a symptom of misallocation within the firm, but rather indicates the mitigating of external credit constraints. Economies with multi-plant firms produce more aggregate output despite *higher* dispersion in marginal revenue products of capital compared to economies with single-plant firms. The efficiency gains from reducing distortions in emerging single-plant-firm economies to the level of developed multi-plant-firm economies are larger than previously thought.

KEYWORDS: Misallocation, Productivity Dispersion, Multi-Plants Firms, Internal Capital Markets.

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1 Introduction

A considerable body of recent research has documented a large, persistent and ubiquitous degree of productivity dispersion across production units, leading in turn to a revival of interest in the causes and consequences of resource misallocation. For instance, [Hsieh and Klenow \(2009\)](#) have found substantial aggregate output increases if one managed to lower this dispersion by improving the allocation of resources across production units. Alternatively interpreting dispersion as a consequence of exogenous increases in uncertainty, [Bloom \(2009\)](#) has shown the recessionary effects of higher productivity dispersion. The implicit assumption in the literatures growing out of these seminal papers is that a high level of productivity dispersion is a sign of resource misallocation in an economy and therefore reduces welfare. But does higher productivity dispersion always depress output and welfare? In this paper, we explore the opposite hypothesis: we show theoretically that removing constraints from the optimization problem of firms may lead to increased productivity dispersion. Novel empirical evidence from data on multi-plant firms supports our hypothesis. We then build a model of investment in multi-plant firms which, calibrated to U.S. manufacturing, attributes up to a quarter of the empirically observed productivity dispersion to optimal firm behavior rather than misallocation. As a consequence, our theoretical and empirical work suggests that the gains from reducing inefficient distortions in developing economies to the level of the U.S. are considerably larger than previously thought.

To illustrate our theoretical argument, consider an economy with many production units which are perfectly identical in terms of technology, productivity and capital. Given their state, each unit would find it optimal to undertake an investment project of a fixed size, but cannot do so for a lack of access to external finance. With this extreme financing constraint, no investment activity takes place and productivity dispersion across units – conceptualized as the dispersion in marginal revenue products of capital – remains zero. Suppose instead that there is a benevolent planner with a limited amount of funds. These funds suffice to finance the investment project in some but not all plants. So the planner optimally finances the investment project in as many plants as the funds permit. Productivity dispersion would then rise, reflecting investment in some plants and no investment in others. A naive interpretation of this higher productivity dispersion would assume that the latter allocation is less efficient than the former where no investment takes place at all. Yet, in reality, the higher dispersion in the latter case is the outcome of a benevolent planner doing his best to overcome a financing constraint by using his ability to freely reallocate funds in order to

generate more investment, capital and output in the aggregate.

This example demonstrates that productivity dispersion – commonly taken as an indicator for insufficient factor reallocation – may indeed rise or fall with efficiency and thus aggregate output, while reallocation intensity and aggregate output are monotonically linked. It is therefore unclear how much of the empirically observed productivity dispersion reflects inefficient distortions and how much reflects (constrained) efficient behavior. This ambiguity warrants an empirical investigation into the origins of productivity dispersion. While research in the above mentioned literatures are concerned with the impact of misallocation and uncertainty on the aggregate economy, there is little work on why and where in the economy such significant dispersion originates in the first place. Yet, identifying the agents and frictions that are mainly responsible for the substantial productivity dispersion is crucial to assessing how much of that dispersion reflects constrained optimal behavior versus true misallocation. This knowledge in turn is relevant for policymakers interested in fostering growth and mitigating recessions. In the present paper, we fill this gap in the literature by jointly studying the dispersion of marginal revenue products of capital between firms *and* between plants within firms. As a starting point, we document two novel empirical findings based on the plant-level data from the U.S. Annual Survey of Manufactures: First, almost two thirds of the overall dispersion in marginal revenue products of capital originates across plants within the same firm rather than between firms. Second, when overall dispersion rises in recessions, we show that it rises primarily within firms rather than between firms. Our findings are quantitatively significant: a typical firm may increase its output by about 25% if it shifted an optimal amount of resources towards high-productivity plants. In strong downturns such as the 1980/82 recession, this potential output gain is even larger.

This focus on both the plant and the firm is relevant for a number of reasons: First, it allows us to highlight the fact that the firm and the market are two fundamentally different allocation mechanisms. In the micro-founded macroeconomic literature, plants are typically treated as independent decision makers. Yet, the allocation of resources across plants could conceivably be affected if they are part of an integrated firm or are required to share scarce firm inputs such as common logistics or external credit. Consequently, understanding how firms work differently than markets may shed new light on the causes of resource (mis)allocation or how the economy copes with aggregate disturbances such as credit or uncertainty shocks.

Second, our findings potentially inform us on whether firms or markets are the most appropriate determinants of allocative efficiency and economic growth: if the dispersion

of marginal revenue products of capital across plants reflects resource misallocation, then our empirical findings may indicate that firms are in fact inferior to markets at allocating inputs. In other words, distortions such as principal-agent problems, transaction costs and asymmetric information may not be mitigated significantly by firms.

Third, the economy-wide effects of aggregate shocks are chiefly shaped by frictions that hamper resource reallocation at the micro level. While a large variety of micro-level frictions has been suggested and studied in the literature, little effort has been made to understand at what granular level these frictions matter quantitatively. Since we are studying the dispersion of marginal revenue products of capital, we give some examples of frictions that interfere with capital reallocation and investment in general. For instance, some studies have highlighted the lumpy nature of investment at the *plant* level (see [Doms and Dunne \(1998\)](#) and [Gourio and Kashyap \(2007\)](#)), while others have found that capital expenditures at the level of the *firm* is fairly smooth ([Eberly et al. \(2012\)](#)). In the literature on uncertainty shocks, [Bloom et al. \(2012\)](#) combine such shocks with non-convex adjustment costs of capital that are most pertinent at the plant level. [Christiano et al. \(2014\)](#), in contrast, focus instead on financial constraints, a class of frictions that are of relevance at the level of the firm. These two separate ways of modeling the effects of uncertainty shocks require empirical productivity dispersion to fluctuate at different aggregation levels. Hence, documenting at which level of aggregation the dispersion of marginal revenue products of capital is most prevalent and volatile could elucidate the type of reallocation frictions that are most relevant.

All of the above reasons are strong arguments to investigate further the extent and dynamics of productivity dispersion. In this paper, our focus is on questions related to the link between within-firm dispersion in marginal revenue products of capital and investment activity. For instance, why do firms tolerate such large dispersion of marginal revenue products of capital across their plants? As an allocation mechanism of scarce resources, are firms less efficient than the market? Or is this dispersion simply optimal given the constraints, both internal and external, faced by the firm? How much do firms discriminate in allocating capital across their plants? At first sight, they appear to do so to a large extent: we find that dispersion in investment rates across plants is also large and more important within than between firms. In line with other studies highlighting the lumpy nature of investment (see [Doms and Dunne \(1998\)](#) and [Gourio and Kashyap \(2007\)](#)), we find that this dispersion is largely driven by investment spikes in a small number of plants. Yet, the correlation between a plant's marginal revenue product of capital and a plant's investment within a firm is not very strong even when the firm invests as a whole. Moreover, we find that this

correlation is even lower in periods of tight credit, exactly when the within-firm dispersion of marginal revenue products of capital is largest. [Eisfeldt and Rampini \(2006\)](#) document a similar mis-timing of marginal revenue products of capital dispersion and capital reallocation between firms. Though we confirm their evidence, we find the larger portion of dispersion and mis-timed reallocation to occur between plants within firms.

This last finding hints at a potential role for *external* financial frictions in shaping the degree of capital misallocation *within* the firm. To investigate this channel, we build a novel dynamic investment model of a multi-plant firm that features technological, organizational and financial interdependencies and constraints. The purpose of the model is threefold. First, we wish to establish how much of the empirically observed dispersion in marginal revenue products of capital and investment rates can be explained by our model. Second, we investigate the relative importance of the various frictions and their interactions as determinants of this apparent misallocation. Third, we study how much of the countercyclical dispersion in marginal revenue products of capital could result from optimal firm behavior.

Because of the focus in the literature on single-plant models, we are required to build a new model of a firm operating multiple distinct plants. Yet, in shaping our framework, we are mindful of the many frictions and imperfections that have been suggested in models of capital accumulation and reallocation (see [Caballero \(1999\)](#) for an overview). The investment literature, for example, has argued that “technological” frictions such as a fixed cost of investing are crucial to replicate the lumpy aspect of investment documented empirically (see among others [Abel and Eberly \(1994\)](#), [Caballero et al. \(1995\)](#), [Cooper and Haltiwanger \(2006\)](#), [Gourio and Kashyap \(2007\)](#)). Others have instead focused on the central role played by financing frictions, arguing that they offer a natural explanation for the documented role of cash flow in investment regressions (see among others [Fazzari et al. \(1988\)](#), [Gilchrist and Himmelberg \(1995\)](#)). Most of these papers have considered frictions in isolation.¹ We are the first to nest real and financial frictions in a model of a multi-plant firm. While some, mostly technological, frictions matter at the level of the plant, others such as financial or organizational imperfections affect the firm as a whole. We argue that modeling a multi-plant structure is crucial to understanding the effects of various frictions and their interactions. For instance, focusing solely on firm-level frictions would counterfactually predict the absence of any dispersion in marginal revenue products of capital across plants within firms. On the other hand, with only plant-level frictions, one would expect to see no difference between

¹[Gomes \(2001\)](#) and [Eisfeldt and Muir \(forthcoming\)](#) who have combined real and financial frictions in a unified model of a firm operating one plant are notable exceptions.

plants in single-unit versus multi-unit firms. Finally, our understanding about which micro-level frictions matter for aggregate outcomes may be affected by the multi-plant structure of firms in the economy.² Though its first aim is to characterize the investment policy of multi-plant firms, our model will also enable us to address questions about the aggregate effects of plant- and firm-level frictions.

In our model, firms operate plants that face both fixed and convex adjustment costs while the firm organizes internal and costly external financing of investment. When firms are credit constrained, they leverage internal capital markets and focus investment on only a few plants *even if most of their plants are equally productive*. This credit constrained investment policy leads to a rise in the dispersion of marginal revenue products of capital within the firm. In other words, had the firm unlimited access to external funds, it would invest in a way to minimize further this dispersion, to the extent allowed by the technological constraints it faces. This is consistent with [Asker et al. \(2014\)](#) and [Bartelsmann et al. \(2013\)](#), who have shown that non-convexities such as fixed investment adjustment costs or overhead labor can lead to endogenous dispersion of capital returns in equilibrium. Our model contains both features, but we show that their impacts can be magnified by the presence of costly external finance. In the face of such financing constraints, we document that the firm’s optimal policy is to allocate internal funds to only a handful of its plants in a given period, postponing investment projects in its other plants. This staggered allocation of scarce resources implies that an externally credit constrained firm will display heightened marginal revenue products of capital and investment dispersion across its plants for several periods. The serial correlation of large investment projects, however, will increase in credit-constrained firms compared to their unconstrained counterparts. Using micro data from the U.S. Annual Survey of Manufactures, we find empirical support for these predictions at the micro and macro levels.

As described above, our model can explain dispersion of marginal revenue products of capital and investment rates within firms qualitatively. But dispersion may result from various other reasons beyond the scope of our model mechanism. We therefore analyze the quantitative predictions of our model. Calibrating it to micro-level moments in our data, we find that introducing financial firm-level constraints increases marginal revenue products of capital dispersion within the firm by 43% compared to dispersion already caused by technological non-convexities at the plant level. This suggests that our mechanism is quite

²We study this question in a companion paper [Kehrig and Vincent \(2015\)](#): “Financial Frictions and Investment Dynamics in Multi-Plant Firms.”

powerful in generating substantial and countercyclical within-firm heterogeneity. This also means that dispersion of marginal revenue products of capital within financially-constrained firms is not necessarily an indicator of misallocation or inefficient investment policies, but rather the by-product of the firm’s effort to mitigate external financial frictions. On the contrary, if the firm did not leverage its internal capital market, dispersion might remain at a lower level, but overall investment, growth and output would be lower as well. Internal capital markets thus help an economy cope more easily with periods of tight financial funding.

In a counterfactual exercise, we find that the existence of internal capital markets in multi-plant firms can increase within-firm dispersion by up to 20%, yet also raise aggregate investment and output by up to 8% and 5%, respectively, relative to an economy in which the pooling of resources within the firm is shut down. We view this finding as a cautionary tale to interpreting higher productivity dispersion as a sign of resource misallocation.

We see our project as a first step into modeling how the organizational structure of a firm may impact the micro-level adjustment of capital, as well as understanding the role of firms for efficiency. Some theoretical research has been done on the efficiency of internal versus external capital markets: [Stein \(1997\)](#), [Malenko \(2012\)](#) study mostly principal-agent problems between a firm’s owner and manager in a single-plant setup. Whether internal capital markets are more or less efficient is theoretically ambiguous: [Gertner et al. \(1994\)](#) show that division managers may exploit imperfect monitoring by firm headquarters to build up “inefficient empires.” On the other hand, [Scharfstein and Stein \(2000\)](#) say that firms are likely to monitor investment spending better than outside financiers, thus improving capital allocation. [Eisfeldt and Papanikolaou \(2013\)](#) stress the importance of organizational or intangible capital at the firm level in order to understand a firm’s productivity, albeit without the multi-plant dimension we are interested in. With the exception of [Lamont \(1997\)](#), [Schoar \(2002\)](#), [Giroud \(2013\)](#), empirical research on within-firm dynamics is scarce. That said, our paper currently falls short of thoroughly studying the general equilibrium effects on multi-plant firms in the spirit of [Thomas \(2002\)](#) and [Khan and Thomas \(2008\)](#). On the empirical front, we believe that taking seriously this firm-plant dichotomy and the joint investment dynamics of plants within a firm offers a new dimension for the researcher to identify various types of frictions.

Our paper is organized as follows. In [Section 2](#), we describe the data and show evidence on the importance of the within-firm dimension for the dispersion of marginal revenue products of capital and investment. [Section 3](#) describes our multi-plant model of the firm and analyses its predictions when an external financing constraint is introduced. [Section 4](#) con-

ducts various quantitative exercises geared towards understand the nature of productivity dispersion and carry out counterfactual exercises. In Section 5, we investigate whether the model predictions are borne out in the micro-level data. Section 6 concludes.

2 Dispersion between and within firms

2.1 Variables of interest and data sources

Our main objects of interest are the dispersions of real marginal revenue products of capital and that of real investment rates across plants. They reflect differences in productivity of capital as well as divergence in capital allocation activity, respectively. In the standard economic models generally used in the literature on misallocation, when marginal revenue products of capital ($MRPK$) differ across plants in the economy, reallocating capital from low- $MRPK$ to high- $MRPK$ plants may increase aggregate output. In a standard frictionless economy, agents should choose investment to equate the expected $MRPK$.³ Assuming a Cobb-Douglas production technology, we approximate the marginal revenue product of plant n in time t with its real value added per unit of capital and study the variance of its logarithm: $Var_t(mrpk_{nt}) \equiv Var_t(\log MRPK_{nt}) = Var_t(\log(y_{nt}/k_{nt}))$.

Our data source is the Annual Survey of Manufactures (ASM), an annual Census dataset covering manufacturing plants and described in detail in Appendix A.2. Census collects its manufacturing data at the level of an “establishment” which is defined as a physical business unit at a single location for which the primary activity is production. In this paper, we refer to establishments as “plants.” Each plant also carries information about its parent firm which is defined by Census as the collection of plants under common ownership or control.

2.2 Dispersion of revenue products and investment within firms

The main objective of this empirical section is to document how much of economy-wide dispersion is accounted for by heterogeneity across plants within the same firm in contrast to heterogeneity across firms. By shedding light on the sources of misallocation, the answer has potentially important implications for future research efforts in this literature, as well as the

³For details on the assumptions underlying this see Appendix A.1. Based on Hsieh and Klenow (2009), the misallocation literature usually postulates equalizing revenue total factor productivity ($TFPR$). Like Asker et al. (2014), we focus on the capital allocation problem and hence equalizing $MRPK$. In that context, investment should not necessarily flow towards the units with the highest $TFPR$ if they already operate a large capital stock. In fact, investment should flow to units with the highest expected capital return.

structure and calibration of micro-founded macroeconomic models. Throughout the section, we study not only dispersion in $mrpk$ but also i/k , an object that has received relatively little attention so far in the misallocation literature. This is surprising since investment activity is the direct channel through which firms and plants can overcome the possible misallocation underlying $mrpk$ dispersion. We also present evidence on the cyclical properties of dispersion in $mrpk$ and i/k . In doing so, we expand on [Eisfeldt and Rampini \(2006\)](#), who document the cyclical capital reallocation between firms, by studying the within-firm patterns of $mrpk$ dispersion and capital reallocation over the business cycle.

Our emphasis on contrasting the dispersion across firms to that occurring within the firm is only relevant if a significant portion of aggregate economic activity is accounted for by multi-plant firms. [Table 1](#) provides evidence that it is. While single-plant firms dominate in numbers, it is the multi-plant firms that operate the majority of the capital stock, produce most output and generate most investment. In fact, firms that consist of 20 or more plants operate almost half of all the capital stock in U.S. manufacturing.

Table 1: Economic activity by firm type in U.S. manufacturing

	Share of...			
	plants	value added	capital stock	investment
Single-plant firms	0.719	0.220	0.178	0.215
Multi-plant firms	0.281	0.780	0.822	0.785
Firms with at least...				
... 10 plants	0.131	0.513	0.602	0.548
... 20 plants	0.095	0.398	0.470	0.421
... 40 plants	0.060	0.252	0.296	0.261

Note: The sample underlying this table comprises all establishments in the Census of Manufactures 1972-2007 less administrative records. The share of each variable in multi-plant vs. single-plant firms is computed for each Census year and then averaged across Census years. Non-manufacturing operations of firms are ignored.

We now investigate to what extent marginal revenue products of capital and investment rates differ across plants within multi-plant firms and how it compares to differences across firms. This can be done by decomposing the total variance of marginal revenue products of capital, denoted by V_t , into the variance between firms, denoted by V_t^B , and the average variance between plants within firms, denoted by V_t^W . Since marginal revenue products are expressed in units of industry good per unit of capital, comparing dispersion of $mrpk$ across

industries is not very meaningful. Any arbitrary definition of what constitutes a typical good in an industry will automatically impact the measured $mrpk$ dispersion in the economy. We therefore perform the decomposition within sufficiently homogeneous industries and report results on $mrpk$ dispersion averaged across industries. We define an industry at the 4-digit NAICS level in order to make goods within such industries sufficiently substitutable to compare their marginal revenue products while leaving enough observations in an industry-year cell to reliably study heterogeneity. In a given 4-digit NAICS industry our variance decomposition is then:

$$Var_t(mrpk_{nt}) \equiv V_t = \underbrace{\sum_j \omega_{jt} (mrpk_{jt} - mrpk_t)^2}_{V_t^B \text{ average between-firm}} + \underbrace{\sum_j \omega_{jt} \sum_{n \in j}^{N_j} \omega_{nt}^j (mrpk_{njt} - mrpk_{jt})^2}_{V_t^W \text{ average within-firm}} \quad (1)$$

where n indicates the plant, j the firm and t the year. Variable $mrpk_{njt}$ denotes the logarithm of the marginal revenue product of capital of plant n belonging to firm j in year t , $mrpk_{jt}$ the average $mrpk$ in firm j in an industry, and $mrpk_t$ the average $mrpk$ in a given industry. ω_{njt} is the weight of plant n at time t , ω_{jt} that of firm j and $\omega_{nt}^j = \omega_{njt}/\omega_{jt}$ that of plant n just inside firm j . While unweighted dispersion is our benchmark, we also consider capital weighted dispersion to take into account “economic relevance.” In the former case, we have $\omega_{njt} = 1/N_t$ (where N_t is the number of observations), in the latter $\omega_{njt} = k_{njt}/k_t$ and accordingly for ω_{jt} and ω_{nt}^j . We aggregate the industry-specific between-firm and within-firm decompositions across the 86 industries at the 4-digit NAICS level as detailed in Appendix A.4. The basic results of that decomposition are displayed in Panel A. of Table 2; Panel B. of that same table displays general cross-sectional moments of $mrpk$ and i/k .

Two results from our simple accounting exercise stand out: First, overall dispersion in marginal revenue products is large. In the average industry and year, the standard deviation of its logarithm is 0.915. This means that a plant that is one standard deviation above the mean produces $e^{0.889} \approx 2.4$ times the value added as the average plant with the same capital stock; the difference between the plant at the 90th percentile compared to that at the 10th percentile even implies an $e^{2.134} \approx 8.5$ -fold value-added difference. Reallocating capital to high- $mrpk$ plants in the same industry could hence result in a considerable boost in aggregate output. If one interprets $mrpk$ dispersion as misallocation, this means that the economy potentially foregoes a considerable amount of income. To illustrate this loss,

Table 2: Cross sectional moments of capital and investment

Variable	A. Share of Variance		B. General moments across plants					
	between firms	within firms	Mean	StD	IDR	Skewness	Kelley Skewn.	Excess Kurtosis
<i>mrpk</i>	0.399	0.601	-/-	0.889 (0.002)	2.134 (0.008)	0.424 (0.006)	0.099	1.361 (0.102)
<i>i/k</i>	0.321	0.679	0.096 (0.003)	0.230 (0.002)	0.175 (0.008)	5.234 (0.006)	0.479	38.945 (0.012)

Note: Data underlying Panel A. are our benchmark panel comprising annual plant-level data from the ASM 1972-2009. Moments in Panel B. are based on all Census years 1982-2007 are computed for each industry and years first before being aggregated by industry and the averaged across years. For details see Appendix A.3.

imagine there are two production units in the economy. Calibrating their Cobb-Douglas technology and productivity to the empirically estimated parameters in Section 4, we find that cutting the observed dispersion in marginal revenue products just in half would increase aggregate output by 19.3%.

The second takeaway from our accounting exercise is that the majority of dispersion occurs inside firms. More precisely, over 60% of marginal revenue products of capital dispersion in the typical industry occurs within firms, with the remainder accounted for by between-firm variations. The implication is that most of the benefits from reallocation would take place by shifting of capital across the production plants within a given firm, instead of between firms. Analogously to the above exercise, we document that a plant that is one standard deviation (0.709) above the firm’s average produces twice the value added with the same capital stock as the firm on average.

Interestingly, the cross-sectional distribution of *mrpk* is positively skewed, which is consistent with the presence of credit constraints: some productive plants may have too little capital, which thus drives up their marginal product and skews the *mrpk* distribution to the right. Following Kelley (1947), p. 250, we define the Kelley skewness as $\gamma^{Kelley} = \frac{mrpk^{90} + mrpk^{10} - 2mrpk^{50}}{mrpk^{90} - mrpk^{10}}$. This quantile-based measure of skewness is 0.099 on average which means that the top half of the inter-decile range, $mrpk^{90} - mrpk^{50}$ is about $1.099/0.901 = 22\%$ more spread out as the bottom half of the distribution, $mrpk^{50} - mrpk^{10}$.

Investment rates also differ substantially across production plants, which implies that

allocative activity of capital differs greatly across units within a typical year. Note that since investment rates are measured in percentage points, they are not subject to the same industry-specific measurement problems present with marginal revenue products. The cross-sectional standard deviation of 23% is large given that the average plant in the economy has an investment rate of 9.6%, an indication of the lumpy nature of investment. Like with $mrpk$ dispersion, the majority of investment rate dispersion originates within firms rather than between firms: within the typical industry, the within-firm share of investment rate variance amounts to almost 70%.

2.3 Robustness

In the previous section, we documented that most dispersion in marginal revenue products of capital and investment rates originates within firms rather than between firms. We now study the between-firm/within-firm decomposition for a number of subsamples. The objective is to confirm the robustness to our main empirical result as well as identify possible causes behind the importance of within-firm dispersion. Details about these exercises can be found in Appendix A.5.1. Table 3 provides an overview of our robustness checks.

Table 3: Dispersion of $mrpk$ within and between firms

Sample	$Var(mrpk)$		$Var(i/k)$	
	b/w plants within firms	b/w firms	b/w plants within firms	b/w firms
(1) Full panel	0.601	0.399	0.679	0.321
(2) Census sample	0.566	0.434	0.663	0.337
(3) Mid-age plants	0.595	0.405	0.685	0.315
(4) Balanced panel	0.809	0.191	0.895	0.105
(5) 5-year averages	0.554	0.446	0.656	0.344
(6) 5-plant firms	0.668	0.332	0.786	0.214
(7) Homog. Industries	0.559	0.441	0.689	0.311
(8) y is physical output	0.625	0.376		
(9) K -weighted	0.540	0.460	0.680	0.320
(10) Equipment	0.591	0.409	0.631	0.369
(11) Private firms	0.713	0.287	0.795	0.205

The first row in Table 3 reiterates the baseline result of within-firm versus between-firm dispersion of both marginal revenue products of capital (navy blue on the left) and investment

rates (light blue on the right): about 60% of the variance of $mrpk$ and 68% of the variance in i/k arise within firms.

Ruling out ASM sampling specificities We use the ASM as our benchmark panel. This panel is known to overrepresent large plants, which in turn are more likely to be part of a multi-plant firm. Since we do not know if within-firm dispersion is larger in firms with few or many plants, we repeat our between-firm/within-firm decomposition for the entire Census of Manufactures (every five years). This allows us to study the full sample of manufacturing plants in the economy. Row (2) illustrates our findings: while the within-firm share or dispersion is slightly lower, it still remains dominant at 57% and 66% for $mrpk$ and i/k respectively.

Ruling out life-cycle dynamics Next, we examine if our result is driven by entry, exit or other life-cycle dynamics. Young, presumably more productive plants will be characterized by higher revenue products and will hence attract higher investment rates. The opposite may be true of older plants that the firm keeps operational until the capital stock depreciates away. We therefore redo the decomposition using only “mid-age firms,” which we define as plants that are at least three years old and at least three years away from exit. As Row (3) shows, the within-firm share of dispersion in both variables is almost unchanged and the same is true when we considered a five-year distance to entry and exit. If we turn to a strongly balanced panel (Row (4)), the share of within-firm dispersion is even larger.

Ruling out measurement error In our next robustness check, we want to address the possibility that the high within-firm share primarily reflects measurement error at the plant level. Indeed, transitory plant-level noise would “wash out” at the firm level and thus artificially increase the within-firm share of dispersion. To rule out that this effect drives our result, we construct rolling 5-year windows of averaged $mrpk$ and i/k for each plant. This time aggregation should filter out most of high-frequency noise at the plant level. Performing the between-firm/within-firm decomposition on this averaged data shows that the importance of the within-firm share of dispersion persists. As shown in Row (5) of Table 3, the within-firm share of overall dispersion in an industry are now 55% and 66%, respectively. This suggests that the presence of plant-level measurement error is unlikely to be driving our findings. Further investigations of measurement error can be found in Appendix A.5.1.

Ruling out mechanical aggregation In a world composed of many plants owned and operated by a single firm, the within-firm share of dispersion would be 100% by definition. As such, one could worry that the large share of dispersion occurring within the firm is driven by large entities. This is unlikely since in our benchmark decomposition, each firm receives equal weight, irrespective of its size. Because in our sample small 2-plant and 3-plant firms are much more numerous than large, complex firms, this bias does not probably play a large role. Still, in order to determine whether this mechanical aggregation could be an issue, we recompute the between-/within-firm decomposition for the set of firms that operate exactly five plants.⁴ Row (6) shows that for this set of firms, the within-firm share of dispersion is indeed higher than for the whole manufacturing sector (67% for *mrpk* and 79% for *i/k*), but not dramatically so. This suggests that our main result is not merely driven by the statistical importance of large firms.

Ruling out multi-product-firm bias Even though our benchmark definition of an industry is fairly fine, products within 4-digit NAICS industries are still heterogeneous. This could potentially lead to spurious differences of marginal revenue products of capital arising from differences in product composition within 4-digit NAICS industries. Whether such within-industry product differences are most likely to occur between or within firms is ambiguous. For robustness purposes, we repeat the decomposition but this time focusing on plants that produce only one homogeneous standardized good. We follow Foster et al. (2008) and consider industries that produce almost perfectly homogeneous goods such as cement, sugar, coffee beans etc.⁵ Naturally, we expect the within-firm share of dispersion to be smaller than in our benchmark since many firms within a given 4-digit NAICS industry are spread out across several of these narrowly-defined product codes. But even in these homogeneous industries, the within-firm share of dispersion in marginal revenue products of capital and investment rates displayed in Row (7) amounts to 56% and 69% respectively.

Ruling out markups Limiting our attention to homogeneous goods has another advantage: it allows us to derive real value added in two ways. In addition to the standard

⁴According to our more restrictive definition of a firm as all plants operated by the same firm within a 4-digit NAICS industry, half the capital stock is operated by firms with five plants or more.

⁵More specifically, these “industries” are defined as the following SIC product codes: Sugar (2061011), Block and Processed Ice (2097011 and 2097051), Gasoline (2911131), Hardwood flooring (2426111), Concrete (3273000), Whole Bean and Ground Coffee (2095111 and 2095117 & 2095118 – later merged into 2095115 – and 2095121), Carbon black (2895011 and 2895000), Bread (2051111, later split into 2051121 and 2051122) and Plywood (2435100, later split into 2435101, 2435105, 2435107 and 2435147).

approach of deflating sales, we can also use the measured physical quantity of production, a meaningful object for these homogeneous product groups. This makes it possible to study how much of dispersion in capital revenue products reflects price differences – due to differential markups or transfer prices – rather than physical productivity differences. Row (8) shows that the within-firm share of marginal revenue products of capital is slightly higher at 63% when using physical output to compute y rather than deflated sales. This suggests that, if anything, prices impact the within-firm and between-firm variances in a way that stake the odds against our main empirical finding. It is also consistent with the fact that plant-level prices and physical productivity are negatively correlated, as documented by [Foster et al. \(2008\)](#).

Demonstrating economic relevance Next, we wish to confirm that our findings are of economic relevance. Instead of decomposing the unweighted variance, we now consider capital weights for the ω 's in equation (1) and redo the decomposition of the dispersion in $mrpk$ and i/k . Row (9) shows that while the within-firm share of capital-weighted dispersion is slightly lower, it remains still dominant at 54% and 68%, respectively.

Examining different capital types In Row (10), we display our decomposition results when focusing only on equipment capital when computing both revenue products and investment rates. Arguably, equipment can be more easily reallocated across production units than structure capital which would lower dispersion. Again, the results of the unweighted between-firm/within-firm decomposition are almost unchanged at 59% and 63%, respectively.

These exercises have confirmed that the importance of the within-firm share of dispersion in revenue products of capital and investment rates is robust to changing the sampling frame in order to account for measurement and aggregation problems, life-cycle dynamics, multi-product firms, markups and transfer prices, the predominance of multi-plant firms with little capital, the type of capital or the sampling of the ASM. In many cases, the within-firm share of overall dispersion was even higher, suggesting that our baseline results may in fact represent lower bounds on the actual within-firm share of dispersion.

2.4 Cyclicity of dispersion between and within firms

So far, we have focused on time-series averages of between-firm and within-firm dispersion in marginal revenue products and investment rates. At the aggregate level, cyclical movements in either dispersion measure are well known: the countercyclical nature of productivity

dispersion has been documented empirically using marginal revenue products of capital in Compustat data by [Eisfeldt and Rampini \(2006\)](#), TFP levels by [Kehrig \(2015\)](#), and TFP innovations by [Bloom et al. \(2012\)](#), while [Bachmann and Bayer \(2014\)](#) have shown that the dispersion in investment rates is procyclical. These findings have important implications for the literatures on Schumpeterian creative destruction, misallocation, development or uncertainty-driven business cycles. Yet, to our knowledge, no one has investigated separately the cyclicalty of dispersion within firms. We close that gap by studying the time-series properties of the various components of dispersion. We first compute detrended measures of the between-firm and within-firm variance⁶ which we then use for time-series analysis. [Table 4](#) displays properties of both the long-run averages, autocorrelations and time series standard deviations for each measure.

Table 4: Dynamic properties of within-firm and between-firm dispersion

	$V^B(mrp_k)$	$V^W(mrp_k)$	$V^B(i/k)$	$V^W(i/k)$
<i>A. Time-series moments</i>				
Average	0.341	0.516	0.009	0.021
Autocorrelation	0.732	0.657	0.667	0.679
Volatility	0.042	0.084	0.006	0.016
<i>B. Cyclicalty</i>				
$Corr(\Delta Y_{t+1}^{mfg}, \dots)$	-0.252	-0.147	0.306	0.330
$Corr(\Delta Y_t^{mfg}, \dots)$	-0.582	-0.304	0.389	0.394
$Corr(\Delta Y_{t-1}^{mfg}, \dots)$	-0.413	-0.229	0.222	0.241

Note: The table reports time-series moments of the between-firm and the within-firm variance for both marginal revenue products of capital and capital reallocation. “Average” denotes the long-run average of each variance term, “Autocorrelation” the annual persistence, $Corr(V_t, V_{t-1})$, “Volatility” the time series standard deviation, $StD(V_t)$, and the the cyclicalty is with respect to the growth rate of aggregate manufacturing value added, denoted ΔY_t^{mfg} .

Consistent with the evidence from [Section 2.2](#), the within-firm portion of the variance in $mrpk$ is larger than that between firms. When studying fluctuations of the two variances over time, we find that the volatility of the within-firm portion is also twice as strong as that of the between-firm portion. This is true even if one compares coefficients of variations instead of variances, and similar patterns are observed in the between-firm and within-firm dispersion of investment rates.

⁶As [Kehrig \(2015\)](#) and [Gopinath et al. \(2015\)](#) we find an upward trend in cross-sectional dispersion.

As can be seen from Panel B. of Table 4, both $V^B(mrp_k)$ and $V^W(mrp_k)$ are countercyclical. This result could have important implications for the uncertainty literature. If one interprets dispersion as a cause of cycles as Bloom et al. (2012) and Christiano et al. (2014), then our within-firm result suggests that cycles manifest themselves within as well as between firms. This means that looking at the granular level of the firm, as is done in Gabaix (2011) or Eisefeldt and Rampini (2006), may underestimate the role of dispersion for aggregate fluctuations. Since fluctuations in heterogeneity at the sub-granular level of plants within firms are larger and as countercyclical, this suggests one should not discard the dynamics inside multi-unit firms when studying business cycles.

2.5 Discussion

More generally, our findings in this section imply that welfare gains from a more efficient allocation of resources would not only stem from reallocation across firms, but also within. This highlights the importance of developing a better understanding of the factors that impede capital from flowing to its most productive use inside the firm. As such, our findings have implications for micro-founded macroeconomic models and their calibration. In much of the literature, the concepts of plants and firms are used interchangeably, with little discussion of their respective roles and constraints. For example, in the empirical uncertainty literature, plants are almost always interpreted as independent decision makers facing various frictions impeding the reallocation of productive capital. Arguably, some frictions such as technological ones as in Bloom (2009), Bloom et al. (2012) are indeed most relevant at the level of the plant. Yet, others are more likely to impact the decisions of firms. This is for example the case of external financing constraints, which affect interactions between firms and their lenders as in Christiano et al. (2014).

What is the link between firm-level financial frictions and mrp_k dispersion? To investigate we issue, we repeat our between-firm/within-firm decomposition on the sample of privately-held firms only. We find that the share of within-firm dispersion for private firms is *higher* than for the whole sample, at 71% and 80% for mrp_k and i/k , respectively. This is despite the fact that privately-held firms tend to operate less plants than their publicly-traded counterparts, which, as we discussed early, leaves less room for within-firm dispersion in the first place.

This result may appear surprising. After all, it could be expected that by impeding the efficient allocation of capital, financial frictions would increase the dispersion of mrp_k across

firms. Yet, our results indicate that firm-level borrowing constraints may in fact shape the allocation of capital across plants *within* the firm. This suggests that the internal capital market of a multi-plant firm could play an important role in overcoming external financial frictions. In the next section, we aim to gain insight into this channel by building a model of a multi-plant firm facing various types of frictions, including financial ones.

3 A model of the multi-plant firm

In this section, we describe, solve, simulate and analyze a simple model of a firm comprised of several plants. We study how various plant- and firm-level frictions interact with the optimal allocation of capital by the firm across its plants. The interdependency of a plant's capital allocation varies with constraints at the level of the firm. At one extreme, the firm is a collection of disconnected plants: decisions are made on a plant-by-plant basis, without any interactions between them. In that case, it does not matter for a plant's investment and production decisions to what firm it belongs to, how many plants that firm operates and what other common factors may characterize the firm. We show that in the presence of frictions at the firm level, however, the firm alters the size and timing of plant-level investment plans which may result in dispersed capital returns across the firm's plants. These constraints at the firm level may prove so strong that, at the other extreme, the firm will treat several perfectly identical plants differently. After describing and solving the problem of the multi-plant firm, we will examine how much dispersion in marginal revenue products of capital a calibrated version of our model can generate and how much marginal revenue products of capital dispersion remains unexplained.

3.1 The problem of the firm

We focus on the basic problem of a firm that operates two plants n , $n = A, B$. We limit our model to only two plants in an effort to keep the numerical analysis of our model, which features non-differentiable investment policies, computationally feasible. A larger number of plants would exponentially increase the size of the state vector of the firm, which must include the capital stock and the technology level of each one of its plants, without adding insight into the underlying fundamental economic mechanisms. We start by describing the technology and constraints at the level of the individual plant before analyzing the problem of the firm. In what follows, lower case letters refer to plant variables, upper case letters to

firm variables and bold upper case letters refer to vectors of a firm’s plant variables.

3.1.1 Technology and frictions at the plant level

Each plant n is characterized by total factor productivity which may include a firm-specific component that is common to both plants. The plant operates a Cobb-Douglas production function which combines the beginning-of-period capital stock k_{nt} and other variable inputs in order to produce output y_{nt} . While capital is fixed throughout the period, we assume that plants can freely choose any other variable inputs in perfectly competitive markets.⁷ This means we can substitute out any static first-order condition for variable inputs and write plant revenues net of variable factor costs as

$$y_{nt} = e^{z_{nt}} k_{nt}^{\alpha}. \quad (2)$$

z_{nt} contains plant (log) total factor productivity and prices of other statically chosen production factors and α is the scaled production elasticity of capital. The productivity level of plant n which belongs to firm j consists of a firm component common to both plants in the firm and an idiosyncratic plant component; both evolve as follows:

$$\begin{aligned} z_{njt} &= \rho^p z_{njt-1} + \eta_{njt} \\ z_{jt} &= \rho^f z_{jt-1} + \eta_{jt}. \end{aligned}$$

with $\mathbb{E}\eta_{njt} = \mathbb{E}\eta_{jt} = 0$, $V(\eta_{njt}) = (V^f)^2$ and $V(\eta_{jt}) = (V^f)^2$.

The capital stock of plant n depreciates every period at rate δ and grows with investment i_{nt} , so it evolves over time according to the conventional expression

$$k_{nt+1} = (1 - \delta)k_{nt} + i_{nt}.$$

As documented in a number of studies (see Cooper and Haltiwanger (1993), Cooper et al. (1999), Doms and Dunne (1998), Caballero and Engel (1999) among others), investment dynamics at the plant level are characterized by lumpiness: multiple periods of inactivity (no or only small amounts of maintenance investment) are followed by “investment spikes.”⁸

⁷Given our Cobb-Douglas production function, flexible factor markets will result in revenue products of production factors that are completely equalized across plants and firms in the economy. This will not be the case, however, for marginal revenue products of capital because of decreasing returns to scale and because capital is chosen one period in advance.

⁸Investment spikes are usually defined as investment rates exceeding 15% or 20%.

The traditional modeling feature used to reproduce this stylized fact is to introduce a fixed cost of investing: the firm must pay a certain cost, ψk_{nt} , if investment is greater than zero. Such costs can arise because investment activity – no matter how small or large – has a disruptive effect on production activities in the short run, for example. The parameter ψ regulates how much revenue is foregone when the plants needs to shut down production in order to install new capital. As a result of aggregation, firm-level investment activity will be less lumpy, as documented in [Eberly et al. \(2012\)](#).

In addition to this non-convex adjustment cost, we include a traditional quadratic adjustment cost. This convex adjustment cost captures the notion that larger investment projects become increasingly disruptive with size.⁹ The parameter γ below captures the importance of this margin.

To summarize, frictions at the plant level will be expressed as:

$$\theta(i_{nt}, k_{nt}) = \left[\psi \mathbb{I} \left\{ \frac{i_{nt}}{k_{nt}} > \vartheta \right\} + \gamma \left(\frac{i_{nt}}{k_{nt}} \right)^2 \right] k_{nt} \quad (3)$$

where \mathbb{I} is an indicator function equal to 1 if the plant investment rate is above ϑ ; ψ is a parameter regulating the foregone sales if the plant undergoes an investment, and γ regulates the impact of the quadratic adjustment cost. Everything is scaled by the plant's capital stock k_{nt} in order to eliminate size differences.

Combining equations (2) and (3) above, plant cash flow is

$$\pi_{nt} = z_{nt} k_{nt}^\alpha - \theta(i_{nt}, k_{nt}). \quad (4)$$

3.1.2 Technology and frictions at the firm level

What sets plants in a multi-unit firm apart from their identical counterparts in single-unit firms? What are the economic benefits the firm provides to its own plants? This section discusses the allocation of funds within the firm.

A firm collects the cash flow from all plants and decides on how to allocate funds across its plants for investment projects. This means firm cash flow is

$$\Pi_t = \pi_{At} + \pi_{Bt} - \Phi. \quad (5)$$

⁹This formulation is similar to assuming lower profitability during large capital adjustments which has been documented by [Power \(1998\)](#), [Sakellaris \(2004\)](#).

Managing firm-wide operations requires overhead costs such as firm management which every firm needs to pay in order to be functional. This overhead fixed cost, denoted by Φ , reflects items such as management costs, overall firm infrastructure, expenditures for R&D, marketing etc. Hence, compared to single-unit firms, multi-unit firms benefit from operating two plants with the same corporate overhead.

Second, while all production and investment activities take place at the level of the individual plant, only the firm is capable of organizing external finance. This assumption is realistic and sensible: while large and complex firms like General Electric operate hundreds of plants, only the firm issues bonds, borrows from banks or raises equity. Typically, the firms then allocate these funds in an internal capital market to individual plants. Consistent with this empirical pattern, we assume that it is the firm that co-ordinates investment plans across all its plants, organizes financing of investment through either internal cash flow or external borrowing and allocates funds to plants where investment is put in place. Only if desired firm-wide investment exceeds firm cash flow, the firm attempts to borrow amount B_{t+1} at net interest rate R_t so that all investment gets financed:¹⁰

$$i_{At} + i_{Bt} = I_t \leq \Pi_t + B_{t+1}. \quad (6)$$

Organizing external financing, however, is an imperfect process. Following the literature on financial frictions, we assume there are two types of frictions: First, there is a financial participation cost, denoted by ζK_t , if the firm wants to borrow at all. This cost reflects the effort to establish a relationship with a lender, expenditures for various information and disclosure requirements and other administrative expenses independent of the loan amount. Like the real investment adjustment cost above, this cost is scaled by the capital stock. Second, we assume that the firm can divert a fraction $1/\eta$ of the loan amount B_{t+1} to its private benefit. This diversion of resources cannot be observed or prevented by the lender. As a consequence, the lender will require collateral which it can seize in case it discovers ex post that the firm did divert funds. We assume that firms pledge a fraction ξ of the value of their capital stock, $K_t = (k_{At} + k_{Bt})$, as collateral. Given the posted collateral, lenders will limit the loan such that the diverted loan amount never exceeds the collateral in order

¹⁰For computational reasons, we do not allow for savings by the firm, except through the accumulation of capital.

to avoid ex post moral hazard unprofitable for the borrower:

$$\frac{B_{t+1}}{\eta} \leq \xi K_t. \quad (7)$$

Note that one can think of $\lambda \equiv \eta\xi$ as the maximum leverage the lender is willing to tolerate. In our quantitative exercise, we will calibrate λ directly, since it is neither possible nor necessary to separately identify η and ξ . We summarize the cost of external finance in the following function

$$\Theta(B_{t+1}, K_t) = \begin{cases} 0 & \text{if } B_{t+1} = 0 \\ \zeta K_t & \text{if } 0 < B_{t+1} \leq \lambda K_t \\ \infty & \text{if } B_{t+1} > \lambda K_t. \end{cases} \quad (8)$$

Note that $\Theta(\cdot)$ captures the net present value of the borrowing cost associated with B_{t+1} . The interest rate payments next periods have a net present value of $\beta R_t B_{t+1}$. Since there is no risk in our model, the firm's borrowing rate R_t is equal to the risk-free interest rate which in turn equals the inverse of the discount rate, so the net present value of the linear part of borrowing costs exactly equals B_{t+1} . Of course, this would change once we introduce any risk of bankruptcy along the lines of [Townsend \(1979\)](#) which would drive up R_t above the risk-free rate and make the middle portion of the borrowing cost in equation (8) monotonically increasing in the borrowing amount B_{t+1} .

Total cost of investment in a given plant depends on the investment amount in that plant, the combined investment in the rest of the firm, whether or not the firm needs to borrow and if it possibly runs into the collateral constraint. The total cost of investment in plant A then consists of fixed and quadratic adjustment costs (real costs $\theta(i_{At}/k_{At})$ in equation (3)) as well as fixed borrowing costs (financial costs $\Theta(B_{t+1}, K_t)$ in equation (8)). The latter part depends on how much the other plant in the firm, plant B , invests as it dictates how fast and how much the firm needs to borrow. Thus, investment in one plant imposes an externality on investment in the rest of the firm because it depletes internal funds and imposes a borrowing cost that is shared by the entire firm.

We plot the total cost of investment in [Figure 1](#) to illustrate the multiple non-convexities and how the interaction of investment across plants shape the cost of investment for the firm. Note that in these plots we assume that ψ , the parameter that regulates the fixed costs of investing in a plant, is “small” in the sense that the minimum investment in one plant can be financed using internal funds of the firm. If they were excessive, even the minimum

investment to justify the fixed investment adjustment costs would require borrowing. In that case, the effective fixed cost of investing in any plant would be $(\psi + \zeta)k_{nt}$.

3.1.3 Firm value and firm policy

We define the vectors of technology levels and capital stocks within the firm as $\mathbf{Z}_t = \{z_{At}, z_{Bt}\}$ and $\mathbf{K}_t = \{k_{At}, k_{Bt}\}$, respectively. Given the plant-level fixed adjustment cost, the firm's state will consist of the distribution of capital stocks \mathbf{K}_t and technology levels \mathbf{Z}_t across plants within the firm. The firm chooses investment in either plant A and B in order to maximize firm value which consists of the net present value of discounted future gross profits net of investment and borrowing costs. When choosing the investment levels in either plant, the firm takes into account the various adjustment costs and whether or not borrowing is required to finance the desired level of investment. The firm's problem can be written in recursive form as:

$$\begin{aligned}
 V(\mathbf{Z}_t, \mathbf{K}_t) &= \max_{i_{At}, i_{Bt}, B_{t+1}} \{ \Pi_t - I_t - \Phi - \Theta(B_{t+1}, K_t) + \beta \mathbb{E}V(\mathbf{Z}_{t+1}, \mathbf{K}_{t+1}) \} \\
 \text{s.t. } k'_{nt} &= (1 - \delta) k_{nt} + i_{nt} \quad \forall n = A, B \\
 B_{t+1} &\leq \lambda K_t. \\
 I_t &\leq \Pi_t + B_{t+1}.
 \end{aligned}$$

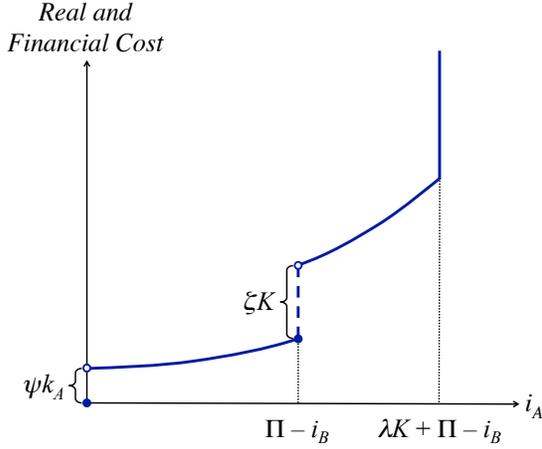
This value function embeds three non-convexities: the fixed investment adjustment cost, ψ , the fixed borrowing cost, ζ , and the collateral constraint λ .¹¹ All three non-convexities give rise to “inaction regions” in the firm's state space where the firm may choose to not change investment and/or borrowing even though the underlying productivity shocks z do. In those regions, the realized marginal revenue product of capital will also differ from that of a frictionless investment model. The three non-convex investment/borrowing costs make the solution of the firm problem fairly complex and there are consequentially at most seven different¹² cases of investment-borrowing decisions each of which reflect investment and borrowing

¹¹We refer to these frictions as non-convexities even though the last one is strictly speaking only a non-differentiability; but it has similar results for the dispersion of marginal revenue products of capital within the firm as the non-convexities.

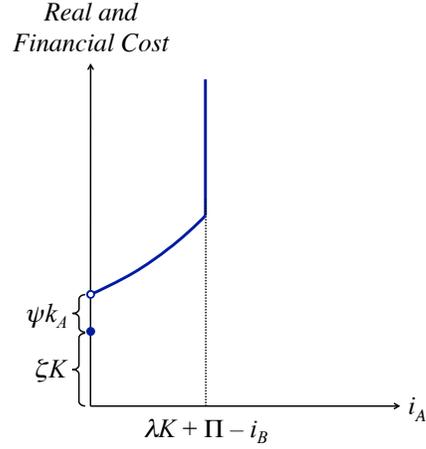
¹²With three binary decisions, there are eight cases, but when neither plant invests the firm would never borrow thus making one case redundant. We restrict the investment fixed adjustment cost ψ to be small. If it were extremely high, even the minimum investment which justifies paying the fixed cost would require firm borrowing and there would only be four cases. Then, there is no role for an internal capital market to finance investment because it never suffices even for the minimum investment in one of the two plants.

Figure 1: Total cost of investment

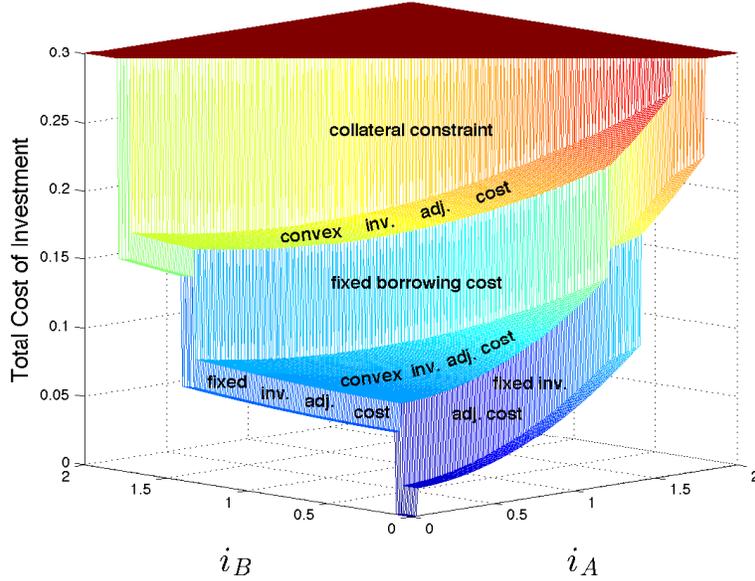
(a) Total cost of investing in plant A when i_B can be fully financed internally



(b) Total cost of investing in plant A when i_B cannot be financed internally



(c) Total investment costs for the firm



Note: Panel (a) on the left displays total cost of investing in plant A when investment in the rest of the firm does not exceed internal funds $0 \leq i_{Bt} \leq \Pi_t$. Then, small amounts of i_{At} can be financed with left over internal funds $(\Pi_t - i_{Bt})$ without incurring borrowing costs. Any investment exceeding that amount makes the cost level jump due to the fixed borrowing cost ζK_t . Panel (b) on the right displays the case when the firm already needs to borrow to finance investment in the rest of the firm. Even zero investment in plant A means fixed and linear borrowing costs $\zeta K_t + R(i_{Bt} - \Pi_t)$. Investment in either case is always limited by the collateral constraint: $B_{t+1} \leq \lambda K \Leftrightarrow i_{At} \leq \lambda K + \Pi_t - i_{Bt}$. Panel (c) shows the the total cost jointly for $k_{At} = 1$, $k_{Bt} = 3$, $\zeta = \psi = 0.02$, $\gamma = 0.04$, $\vartheta = 0.03$, $\lambda = 0.1$ and $\Pi_t = 1.4$.

(in)activity in the various parts of the firm.

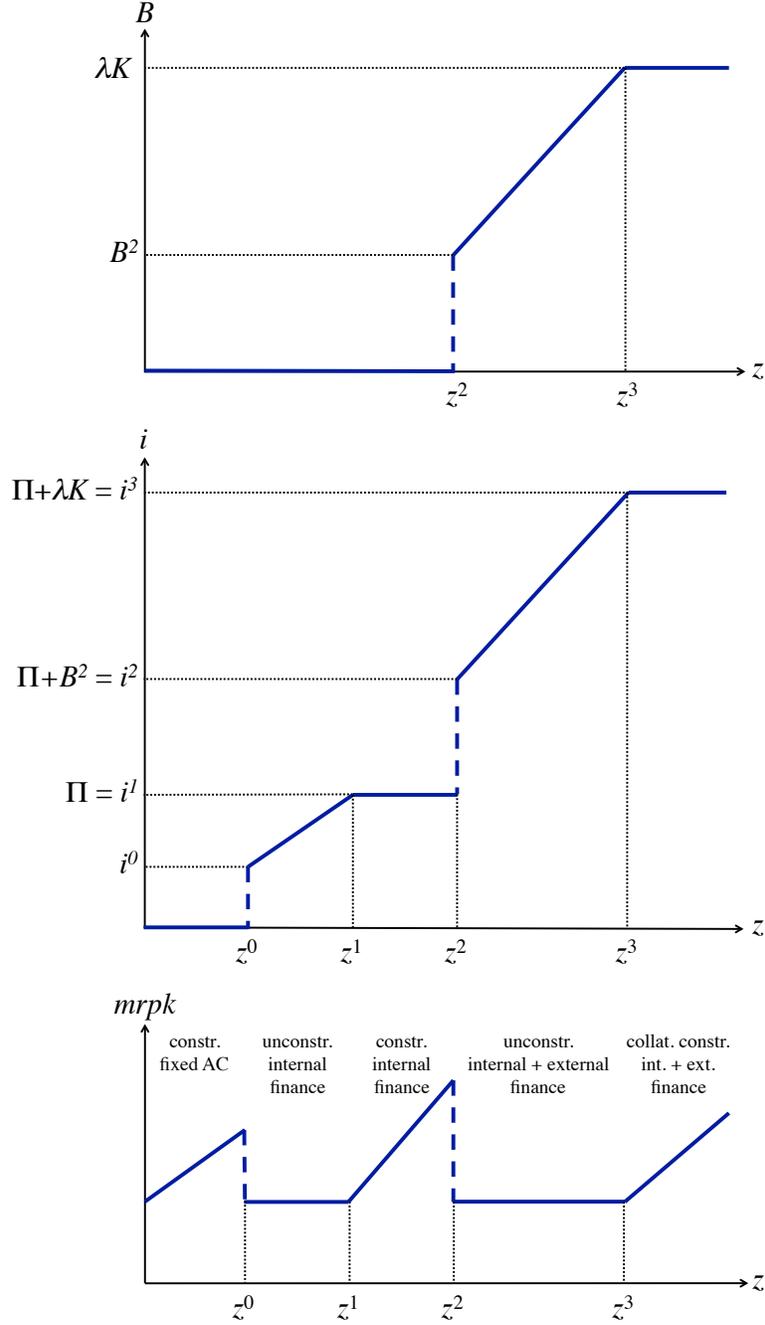
We illustrate the investment policy for plant A and the firm's borrowing policy in Figure 2 for a given amount of investment in plant B . In principle, investment in either plant is monotone in its TFP. The fixed investment adjustment costs prevent small amounts of investment that would not justify paying the fixed cost ψk_{At} . So investment will not change unless z exceeds a threshold $z^0(k_{At})$. At $z^0(k_{At})$ the expected benefit from investing, $\mathbb{E} \frac{\partial V'}{\partial k_{At}} i_{At}$, equals the fixed costs, ψk_{At} . Because the latter are modeled proportional to k_{At} and because of decreasing returns to scale, the first threshold will depend on k_{At} . Note that z^0 does not depend on firm variables. Investment jumps and then increases monotonically in productivity. The quadratic investment adjustment cost regulated by γ increase the marginal cost of investing but do not break the monotonicity. Plant investment increases until the firm's internal funds, Π_t , are exhausted at $z_{At}^1(k_{At}; z_{Bt}, k_{Bt})$. Notice that this second threshold of productivity for plant A depends on investment in the rest of the firm, i.e. i_{Bt} , and vice versa. This is because investment expenditures in the other plants entail a negative externality for plant A as it depletes internal funds of the firm.

As with the fixed adjustment cost at the plant level, the fixed borrowing costs, ζK_t , prevent the firm from borrowing small external funds. This means there is another region of productivities $[z_{At}^1(\cdot), z_{At}^2(\cdot)]$ where investment does not respond to productivity. Only at productivities above $z_{At}^2(z_{At}, k_{At}; z_{Bt}, k_{Bt})$ is it worthwhile for the firm to pay the fixed borrowing cost. This third threshold depends on investment needs in the entire firm and hence depends on the levels and the distribution of capital stocks and the productivity shocks. Investment again increases monotonically in productivity above $z_{At}^2(\cdot)$ until productivity threshold $z_{At}^3(z_{At}, k_{At}; z_{Bt}, k_{Bt})$ at which point the firm's borrowing demands are limited by the collateral constraint.

3.2 The effects of external financial constraints in multi-plant firms

In the previous section, we described how investment at the individual plant is affected by firm-level frictions. Understanding these micro-level dynamics is helpful to gain intuition about the *joint dynamics* of marginal revenue products of capital and capital reallocation within the firm, which is central to the model. Next, we study the effect of each firm-level financial constraint on the dispersion of returns and investment rates as well as additional moments that can be empirically checked in the data. We qualitatively explain how interactions between plants shape the firm's investment and borrowing policies, leaving our

Figure 2: Firm borrowing and plant investment



Note: Schematic illustration of firm borrowing, plant investment policy functions and realized marginal revenue products of capital. This illustration assumes that the other plant does not invest at all and that physical adjustment costs ψ are smaller than fixed borrowing costs ζ , so that firms start investing without having to borrow; otherwise z^0 and z^2 would coincide.

quantitative assessment to Section 4.

Our benchmark for the analysis is a version of the model in which the only frictions are the quadratic and fixed investment adjustment costs. This makes our comparisons more meaningful, since in a completely frictionless world the dispersion of the marginal revenue products of capital would be zero. In addition, the non-convexity from the fixed cost is in itself a known source of *mrpk* dispersion (see Asker et al. (2014)).

The framework we described in Section 3 includes two types of financial frictions. The first is a fixed borrowing cost that must be paid every time the firm engages in external borrowing. The impact of this constraint are intuitive. First, our model predicts that fixed borrowing costs lead to investment spikes that are synchronized across plants within the firm. Because accessing capital markets is onerous, the firm minimizes these costs by borrowing less often but larger amounts. It then uses the funds obtained to invest in both plants at once. In those periods of firm-wide investment, the correlation between investment and marginal revenue products of capital is high, while the opposite is true during the long periods of internally-financed small investment projects. The latter tend to dominate, leading to lower overall correlation between investment and marginal revenue products of capital. Finally, because the fixed borrowing costs make investment at the plant level even lumpier, the autocorrelation of investment becomes even more negative, at both the plant and firm level.

While it is enlightening to study its effects, the fixed cost of accessing external capital markets plays little role in our quantitative analysis. Instead, our focus is on a type of financial friction that is more standard in the literature: a collateral constraint that limits the amount of borrowing that can be undertaken by the firm. This constraint is most likely to bind when both plants within the firm are hit by positive technology shocks: the firm would like to invest heavily in both units, yet faces limited internal funds. As a consequence, the firm is forced to focus on one plant at a time, leveraging both internal and external financial markets in the process. In other words, it rotates investment projects across its plants. Seen over time, investment across the individual plants within a firm looks staggered.¹³

What does this staggered investment policy imply for the dispersion of marginal revenue products of capital across plants? By rotating investment across its plants, the firm is in effect creating higher within-dispersion in *mrpk*. More interesting is the fact that this increased dispersion is the optimal response of the firm to the constraints it faces rather than a consequence of inefficient distortions. In fact, were the firm unable to leverage internal

¹³Besley et al. (1993) study the “Rotating Savings and Credit Associations” (Roscas), a financial institution mostly found in developing economies which relies on a similar mechanism.

capital markets by pooling plant revenues, it may not be in a position to invest at all because of its limited access to external finance. The result would be a lower dispersion of $mrpk$ within the firm. In addition, the staggering of investment activity naturally leads to a rise in the within-firm dispersion of investment rates and a drop in their correlation across plants.

The two crucial elements in this beneficial interpretation of $mrpk$ dispersion are the fixed investment adjustment costs ψ and the collateral constraint λ . This leads to investment spikes that the firm can stagger over time thus resulting in higher $mrpk$ dispersion across the plants within the firm. If one of the two crucial components of our model mechanism would be absent, there would not be any role for “beneficial $mrpk$ dispersion.” For example, without a collateral constraint, the firm would have unlimited access to financial resources and invest to equalize $mrpk$ across its plants. In other words, absent distortions à la Hsieh and Klenow (2009), any dispersion in $mrpk$ would be detrimental but merely reflect the presence of the technological fixed investment adjustment costs ψ as in Asker et al. (2014).¹⁴ Conversely, without the presence of fixed adjustment costs, a credit-constrained firm would invest suboptimally low amounts, but would still allocate investment to equalize marginal revenue products across its plants.

Our discussion so far has been purely qualitative. In the next sections, we explore the quantitative relevance of the channels highlighted above and discuss the link between $mrpk$ dispersion and aggregate welfare in a multi-firm economy. Finally, in Section 5.1 we present supporting evidence for the mechanisms at play in our model.

4 Quantitative analysis

4.1 Calibration

Table 5 summarizes the parameter values used to calibrate our model for the quantitative analysis. Most values are based on moments from the ASM dataset and are in line with calibrated parameters generally used in the investment literature.

In order to learn about the production function elasticity α , we extend the structural framework of Cooper and Haltiwanger (2006) to accommodate multi-plant firms. Our GMM estimate puts α at 0.627 which is fairly close to their value. The parameters governing persistence, ρ^p and ρ^f , and volatility, σ^p and σ^f , of the plant and firm shock processes

¹⁴Similar results could be obtained in the case of a constraint that would see the interest rate paid by the firm increase with the size of its borrowing. Under such conditions, the firm would again wish to stagger investment projects in order to minimize the cost of borrowing.

Table 5: Model Calibration

Parameter	Meaning	Value	Target/Source
β	Discount rate	0.95	Long-run real interest rate
α	Production elasticity	0.627	ASM data
ρ^p	TFP persistence plant	0.60	serial correlation of $mrpk_p$: 0.25
ρ^f	TFP persistence firm	0.85	serial correlation of $mrpk_f$: 0.31
σ^p	TFP shock plant	0.25	volatility of $mrpk_p$: 0.33
σ^f	TFP shock firm	0.24	volatility of $mrpk_f$: 0.26
δ	Depreciation rate	0.067	Mean investment rate in ASM data
ψ	Fixed inv. adj. cost	0.039	Cooper and Haltiwanger (2006)
γ	Quadratic inv. adj. cost	0.049	Cooper and Haltiwanger (2006)
ζ	Fixed borrowing cost	10^{-4}	See text
λ	Collateral constraint	0.30	Li et al. (2016)
Φ	Firm fixed cost	0	See text

are chosen to match the persistence and volatility of $mrpk$ at the plant and firm levels in two-plant firms in the ASM.

Of note, the fixed investment adjustment costs are lower than those estimated in Cooper and Haltiwanger (2006), Asker et al. (2014). This is because the fixed borrowing costs ζ reinforce the effects of fixed investment adjustment costs ψ . As a result, estimating a model that does not take into account firm structure will confound the two and bias the inferred value for ψ upwards. In addition, when we use the value from Cooper and Haltiwanger (2006), we find that the minimum investment projects needed to justify the fixed adjustment costs are so large that they usually require borrowing. While this case may be theoretically possible, it seems not very plausible that even minimal investment cannot be financed out of the combined firm's internal funds.

Given the lack of hard evidence to support the calibration of the borrowing fixed cost ζ , we remain conservative and pick a small value equal to only 0.01% of the firm capital stock. Also, we start with no firm fixed production cost Φ , which mostly acts as a scale factor anyway. Regarding the collateral constraint, our benchmark case relies on $\lambda = 0.3$, while we will be experimenting with other values as well to illustrate the key driver of within-firm capital allocation. This value is in line with the evidence from Li et al. (2016) who estimate a dynamic model of the firm with a similar borrowing constraint. They find values of the collateral constraint parameter between 0.22 and 0.32 across manufacturing industries.¹⁵

¹⁵In the case of Li et al. (2016), it is the non-depreciated value of the capital stock that is collateralizable.

4.2 Financial constraints, investment activity and dispersion

Next, we analyze turn our attention to the model simulations. The model is solved and simulated using a value function iteration procedure which is described in detail in Appendix B. We study how the firm’s capital allocation responds to a tightening of their collateral constraint in their capital allocation. As we will see, the firm will leverage its internal capital markets, shaping the dispersion of $mrpk$ of its plants in the process.

In the top panel of Figure 3, we plot the relationship between the tightness of the collateral constraint, determined by the parameter λ , and the degree of dispersion of logged expected marginal revenue products of capital $mrpk_{nt}$ and investment i_{nt}/k_{nt} within the firm. Both moments are normalized to 1 in the scenario where the collateral constraint is not binding ($\lambda = \infty$). As the constraint is tightened (λ gets smaller), dispersion within the typical firm rises sharply, increasing by more than 45% for investment and 20% for marginal revenue products of capital once $\lambda = 0.3$ (the firm cannot borrow more than 30% of its collateralizable capital stock).

Within-firm dispersion reaches a maximum at around $\lambda = 0.2$, before eventually falling as the constraint becomes so binding that the firm rarely invests at all. This can be also be witnessed in the rapid drop of the within-firm dispersion for $\lambda > 0.3$.¹⁶

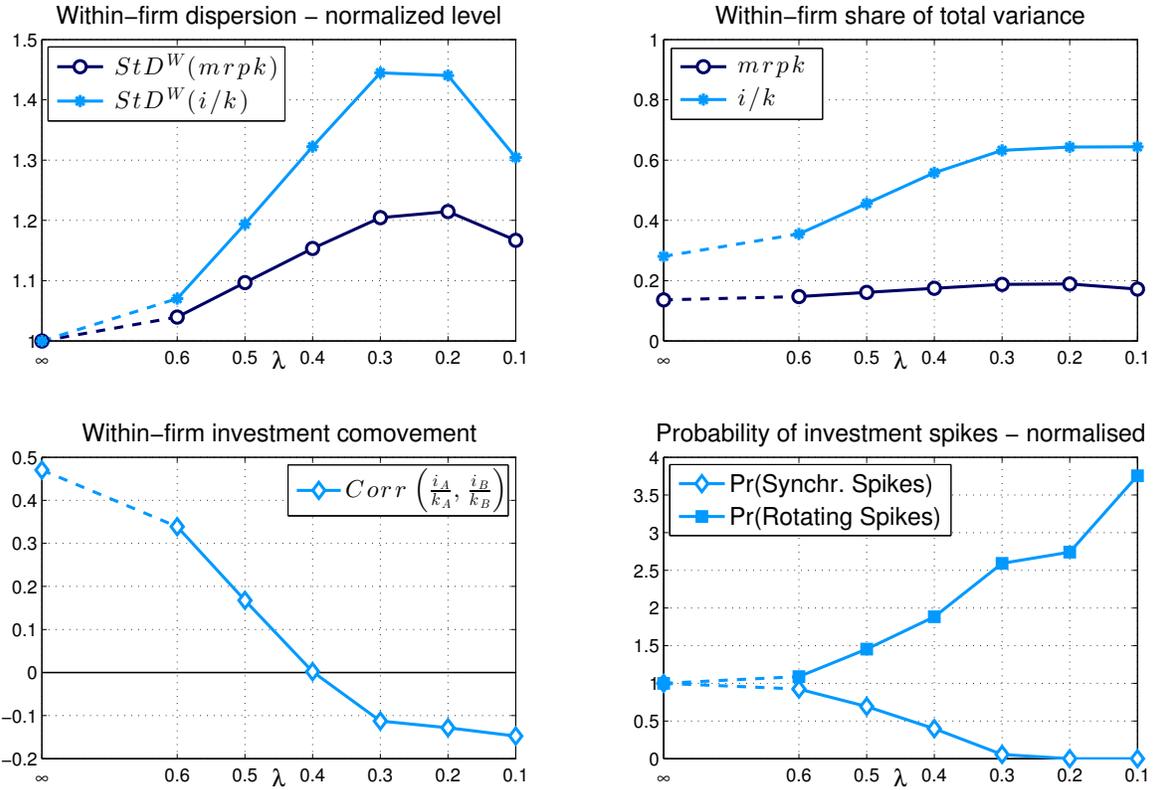
The bottom-left panel of Figure 3 hints at the mechanism behind the increase in dispersion: as the collateral constraint limits the ability of the firm to borrow, the correlation in investment activity across plants within the firm drops dramatically from its benchmark value of +0.48 in the unconstrained case. In fact, for low-enough values of λ , the cross-plant investment correlation turns *negative*, reaching a trough of -0.12 with our parameterization.

The second panel at the bottom illustrates the mechanics behind the drop in correlation. Here, we plot two additional moments. The first one represents the probability of observing synchronized investment spikes, i.e. investment rates above 15% in both plants at the same time. As the collateral constraint is tightened and access to internal finance is limited, the firm cannot allocate large amounts of capital to both plants at the same time, even following a positive firm-level shock that increases internal funds. Conceivably, an option for the firm would be to invest smaller amounts in both firms to restrain how much it needs to borrow on capital markets. But because of the presence of investment fixed costs, the firm would rather

Also, while Rampini and Viswanathan (2013), D’Acunto et al. (2016) also provide information on leverage, their approach is not model-based which makes it difficult to transpose their findings into our framework.

¹⁶We find a very similar pattern if we instead compute the dispersion of 5-year moving averages of logged expected $MRPK$ thus being consistent with Panel (c) in Table 3.

Figure 3: Quantitative effects of financial constraints on the multi-plant firm economy



Note: This figure plots various dispersion and capital allocation moments within firms as a function of the external collateral constraint λ . Top panels display the resulting dispersion we are interested in explaining, bottom panels highlight the mechanism behind that: more constrained firms use the internal capital market to switch towards “rotating investment spikes” within the firm.

instead opt for desynchronization of investment activity across its plants. That is, in the event of a large positive firm-level shock, the firm allocates capital first to the plant with the highest (expected) return on capital, then does the same for the second plant in the following period. This is why the frequency of “rotating investment spikes,” i.e. an investment spike in one plant followed by a spike in the other plant next period, rises as external financing becomes more limited. This is the key mechanism driving the rise in within-firm dispersion. The likelihood of “rotating investment spikes” in the constrained economy ($\lambda = 0.3$) is 2.5 times as high as it is in the unconstrained economy ($\lambda = \infty$). Conversely, “synchronized investment spikes” virtually disappear in the constrained economy as the firm finds itself unable to gather the internal and external funds to finance investment projects in both plants at the same time.

4.3 Dispersion and aggregates: the social value of multi-plant firms

In the previous section, we studied how a multi-unit firm optimally allocates capital across its plants when it is faced with limited access to external financing in addition to the traditional frictions found in the literature. We found that a credit constrained firm tends to rotate investment activity across its plants to limit borrowing activity. As a result, dispersion of investment but also marginal revenue products of capital within the firm rises.

In what follows, we ask ourselves a different but related question: could the rise in dispersion observed within the firm be socially optimal, i.e. associated with higher aggregate output? After all, the staggering of investment projects helps the multi-plant firm overcome external financial frictions in a way that single-plant firms cannot. This is because when plants are arranged into firms, it becomes possible for them to pool financial resources, to leverage the internal capital market that the firm creates. On their own, single plants can only count on their own revenues. As a result, could it be that multi-plant firms generate both higher *mrpk* dispersion *and* higher output?

To investigate the social value of the firm in the presence of external financing constraints, we simulate two almost identical economies, but for the presence of internal capital markets. The first economy is similar to the exercise we have done so far: we simulate a panel of 1,000 two-plant firms for 1,000 periods, where each firm can pool capital across its plants to post as collateral. The second economy also consists of 1,000 two-plant firms subjected to the same shocks. This time, however, each plant within the firm is considered to be a standalone unit:

it must borrow on its own by posting its plant-specific capital stock as collateral. Besides that, it faces exactly the same constraints and frictions as the plants in multi-unit firms. In other words, we create a “wall” between the plants that bars them from pooling financial resources when access to external financial markets is too expensive. This exercise allows us to specifically isolate the role of the firm in creating internal capital markets which enables new investment projects across its plants.

We then investigate two questions. First, do two-plants firms with internal capital markets produce more dispersion in marginal revenue products than firms without any internal ability to pool resources? Second, how much more aggregate output can the economy with internal capital markets produce? Naturally, two-plant firms with internal capital markets cannot do worse since they convexify the choice set and can always reproduce the allocation of the single-plant economy. Our objective is to determine how large these gains can be.

Table 6: Comparing multi-plant (MUF) and single-plant (SUF) firms

Moment	No frictions; $\lambda = \infty$	$\lambda = 0.3$	
	(MUF = SUF)	MUF	SUF
<i>A. Plant-level moments</i>			
Share of within-firm dispersion of $E[\log(y/k)]$	0.138	0.176	0.111
Share of within-firm dispersion of i/k	0.299	0.632	0.356
$Corr(i/k_{At}, i/k_{Bt})$	0.437	-0.113	0.317
<i>B. Aggregate Moments</i>			
Aggr I	0.557	0.556	0.534
Aggr K	0.832	0.831	0.797
Aggr Y	0.172	0.171	0.167
Aggr V	342.7	342.6	332.8

Table 6 and Figure 4 compare a number of moments across the two economies. To identify the role of credit frictions, we contrast allocations under $\lambda = \infty$ (collateral constraint not binding) to $\lambda = 0.3$ (collateral constraint binding as estimated in U.S. manufacturing). In this latter scenario, the single-plant and multi-plant firm economies are by definition identical. This exercise is similar in spirit to Buera and Moll (2015) who conceptualize a credit crunch as a tightening collateral constraint and examine as what kind of aggregate distortion such a credit crunch manifests itself.

In line with our previous results, we find that the presence of financial frictions raises dispersion in a world of multi-unit firms due to the optimal decision of rotating investment

spikes across plants. With standalone plants, however, dispersion does not rise as much as it does in the economy with internal capital markets. Also, while the correlation of investment across plants turns negative within multi-unit firms, it increases somewhat when the plants cannot pool resources and need to borrow on their own.

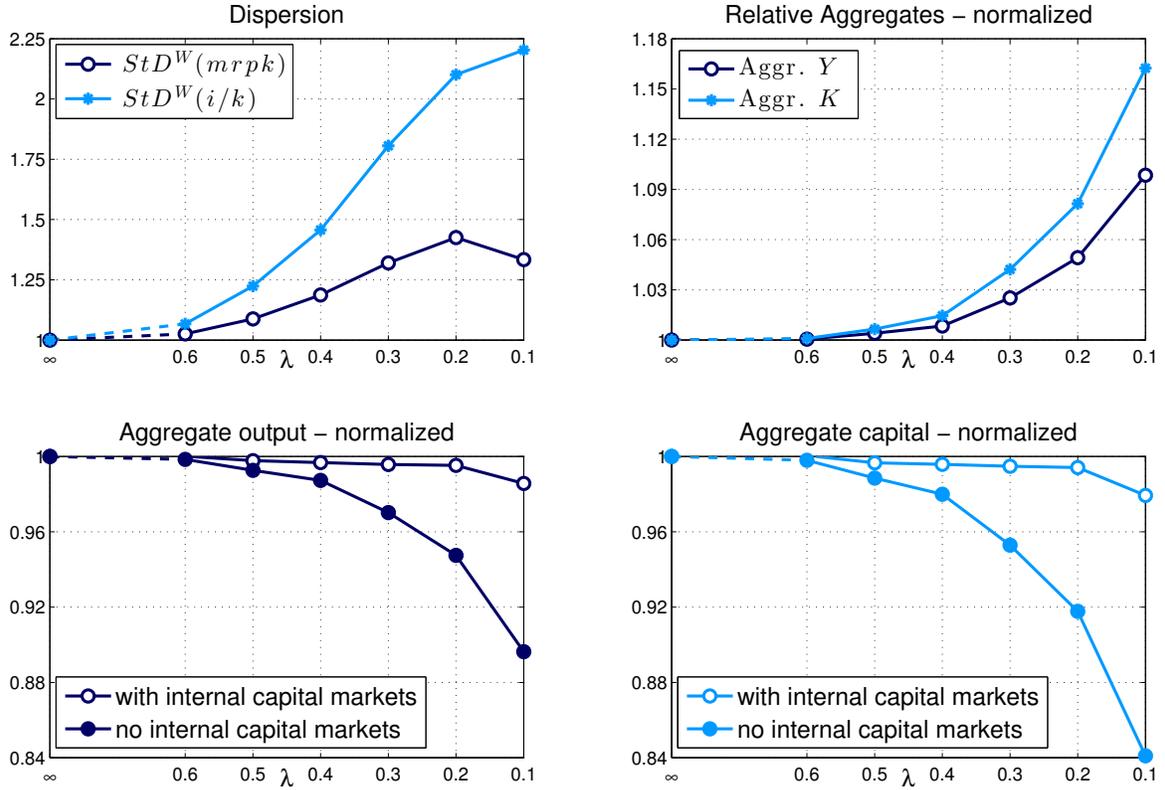
The bottom panel of the table pertains to the aggregates across our economies: we sum up output, investment and capital across all firms for each period, and take the time-series average. One can notice a striking result: the multi-plant firm is capable of undoing most if not all the effects of financial frictions for a wide range of values of λ , with aggregate variables remaining almost unchanged relative to the unconstrained benchmark. This is true even if the presence of collateral constraints *raises* dispersion of marginal revenue products of capital. This indicates that a multi-plant firm's internal capital market is quite a powerful tool to overcome frictions in external markets. On the other hand, in an economy populated by plants that cannot pool resources, tightening the collateral constraint (lowering λ) leads to sharply lower aggregate output *and* lower dispersion. Our results indicate that a higher dispersion of marginal revenue products of capital need not be associated with lower aggregate output.

We investigate this finding further by simulating the impact of a tightening of the collateral constraint in our two economies. The left panel of Figure 4 shows the relative dispersion of *mrpk* as λ decreases, i.e. borrowing becomes more limited. In line with our previous findings, we can see that as the constraint tightens, the multi-plant firm optimally decides to generate 30% *more* dispersion, even if it could replicate the standalone-plants case by shutting down internal capital markets. This optimal strategy at the firm level translates into 4.2% more aggregate capital and 2.8% more aggregate output relative to an identical economy in which plants cannot pool internal resources to fund targeted investment projects and access external financing.

To summarize, our model illustrates that a higher dispersion of marginal revenue products of capital can be an outcome of constrained efficient behavior by multi-plant firms. This higher dispersion is not a sign of resource misallocation in the economy, but instead of an optimal decision by the firm to stagger investment activity in order to circumvent financial constraints.

Our findings therefore cast doubt on the notion that dispersion in marginal revenue products of capital necessarily indicates inefficiencies. In our calibrated model, up to 23% = $0.3/1.3$ of the total within-firm dispersion of *mrpk* in an economy composed of two-plant firms can be traced back to the optimal response to external financial frictions.

Figure 4: Quantitative effects in the multi-plant-firm vs. single-plant-firm economy



Note: Each value corresponds to the ratio of a specific moment in the multi-plant-firms economy (MUF) relative to the same moment for the economy composed of firms that cannot pool capital across plants (SUF). Ratios of average within-firm marginal revenue products of capital dispersion (left panel) and aggregate investment, capital and output (right panel) are plotted for various values of the collateral constraint parameter λ .

4.4 Rethinking the gains from eliminating misallocation

- Previous sections showed that measured dispersion reflects both inefficient distortions and constrained efficient behavior of multi-plant firms.
- Developing countries have almost no multi-plant firms while in the U.S. multi-plant firms account for the majority of economic activity.
- Thus, dispersion in developing economies reflects solely distortions while in the U.S. almost a quarter of dispersion can be attributed to constrained-efficient firm behavior (see quantitative analysis in Section 4.3 above).
- Fostering output in developing countries can be done in two ways:
First, give them multi-plant firms, so these firms can reallocate internally; this would increase dispersion, output and welfare. Difference in returns dispersion now reflects distortions only, not differences in firm complexity; this difference is larger than previously thought.
Second, reduce inefficient distortions. Because they were previously underestimated, bringing down distortions to U.S. levels comes with larger output and welfare gains.
- Conventional welfare gains exercises only consider the second avenue and underestimated the true dimension of distortions.
- So, there are two additional output and welfare gains here:
First, gain from giving developing economies a more complex firm structure which facilitates within-firm reallocation and increases output (previously not considered).
Second, gain which results from bringing down distortions to U.S. levels (previously underestimated).
- We intend to quantify both gains.

4.5 Discussion

4.5.1 Multi-plant firms in general equilibrium

The previous section explained how the staggered investment policy of multi-plant firms depends on fixed investment adjustment costs. While a plethora of research has documented the lumpiness of investment at the level of the individual plant (see e.g. [Doms and Dunne](#)

(1998), Cooper and Haltiwanger (2006), Gourio and Kashyap (2007)), work by Thomas (2002), Khan and Thomas (2008) has shown that general equilibrium forces make the micro-level investment spikes irrelevant at the aggregate level. Investment spikes that rotate across plants in the firm are at the core of the “beneficial dispersion of $mrpk$ ” in our partial equilibrium model. Hence it is a valid question to ask if general equilibrium forces would make our model mechanism irrelevant.

We argue that the organizational structure of multi-plant firms itself makes investment smoothing motives that play out in general equilibrium less pressing. How is that? The firm will allocate investment across its plants in a way to maximize the value of the firm. Doing so will involve allocating investment projects to different parts of the firm which smooths the return of the firm as a whole. The more plants a firm has, the more it can smooth its returns. But the firm’s return would be more volatile – and thus disliked by the firm owner, the household – when the firm operates a single plant and this single plant undergoes an investment spike as in the above-mentioned papers. We hence argue that the staggered investment spike behavior of multi-plant firms in our model world would not change their investment behavior much if factor prices were endogenous in general equilibrium. This also means that a lot of the endogenous dispersion in capital revenue products and investment rates *within* multi-plant firms would not change much.

4.5.2 Multi-plant firms and investment adjustment cost

The organizational structure of multi-plant firms also has important consequences for the nature of adjustment costs. The landmark paper of Cooper and Haltiwanger (2006) has structurally estimated a variety of physical investment frictions; Cooper and Ejarque (2003) have expanded this work to include financial frictions in the investment model. Like the overwhelming majority of these papers assume that individual plants are independent decision makers. If plants belong to the same firm, however, our model shows that credit constrained firms smooth out investment over time by rotating investment spikes across their plants. Credit constraints thus filter through the organizational structure of the firm to smooth out investment *at the firm level*. This makes the firm look like as if it was subjected to investment flow adjustment costs of the form $\kappa(I_t/I_{t-1})^2$. Such costs have been found to fit the dynamics of investment at more aggregate levels much better than other types of adjustment costs (see Christiano et al. (2005)). It appears that our model of investment spikes within multi-plant firms are a novel micro foundation for such adjustment costs at the firm or more aggregate levels. With the exception of Lucca (2007), we are unaware of alter-

native micro-foundations. How structurally estimated investment adjustment costs such as flow adjustment costs or partial irreversibilities would change once one takes into account the within-firm interdependencies of individual plants, is an interesting and important question for future research.

5 Empirical model support

The previous sections documented a set of new facts and proposed a new theoretical model whose quantitative analysis challenged the notion that high dispersion in marginal revenue products of capital necessarily reflects misallocation. In fact, we showed in the previous section that economies with multi-plant firms may enable more aggregate investment and produce higher aggregate output while at the same time increasing dispersion in revenue products. With our calibrated model capable of explaining a considerable quantity of the empirically observed data facts, we now want to demonstrate its empirical relevance more directly. To do that, we verify the crucial features of our model mechanism in firms which operate exactly two plants as in our model. The crux of the model mechanism is a collateral constraint which limits external finance. The tighter this financing constraint, the more relevant a firm’s internal capital market becomes which the firm uses to finance investment in at least a subset of its plants. This means that the more financially constrained a firm, the more dispersed are its marginal revenue products of capital as well as investment rates and the more prevalent are “rotating investment spikes” as opposed to “synchronized investment spikes.” We check for these patterns in two-plant firms in the data. As an empirical proxy for the tightness of the collateral constraint, we choose the amount of the pledgeable collateral which we identify as proportional to the plant’s capital stock.

5.1 Empirical evidence I: Staggering investment

We first study how a firm responds to a firm-wide technology shock, that is, all the firm’s plants experience a boost in plant productivity. Controlling for a plant’s capital stock such a firm-wide technology shock should lead to “synchronized investment spike” if the firm has a large debt capacity. But, when the firm is credit constrained, such a firm-wide technology shock should lead to “rotating investment spikes.”

We first estimate the likelihood of synchronized investment spikes after a firm-wide tech-

nology shock using the following probit model:

$$Pr(Y_{jt}^{sync} = 1|X_{jt}) = \Phi(X_{jt}\beta) \quad (9)$$

where X_{jt} is a vector of controls which includes is the firm’s level of the productivity shock, η_{jt} , and the dummy variable Y_{jt}^{sync} is defined as follows

$$Y_{jt}^{sync} = \begin{cases} 1 & \text{if } \frac{i_{At}}{k_{At}} > 0.15 \text{ and } \frac{i_{Bt}}{k_{Bt}} > 0.15 \\ 0 & \text{otherwise.} \end{cases}$$

We focus on the firm’s investment response to a firm shock because this provides the cleanest example of the changing investment patterns as the financial constraint tightens. We estimate equation (9) separately for ten size deciles of two-plant firms where size is defined as the value of the pledgeable collateral. We then evaluate the marginal likelihood of a synchronized investment spike in the wake of a firm-wide technology shock, i.e. a shock experienced by both plants. To ease disclosure requirements we smooth the results of these deciles using a cross-sectional rolling-window average estimate of the five adjacent deciles. Normalizing the probability of a “synchronized investment spike” to unity for the least financially constrained firms, we plot the normalized probabilities in the left panel of Figure 5 (line with hollow diamonds). It shows that the financially most constrained firms are only 80% as likely as the least constrained firms to respond to a firm-wide technology shock with an investment spike in both plants.

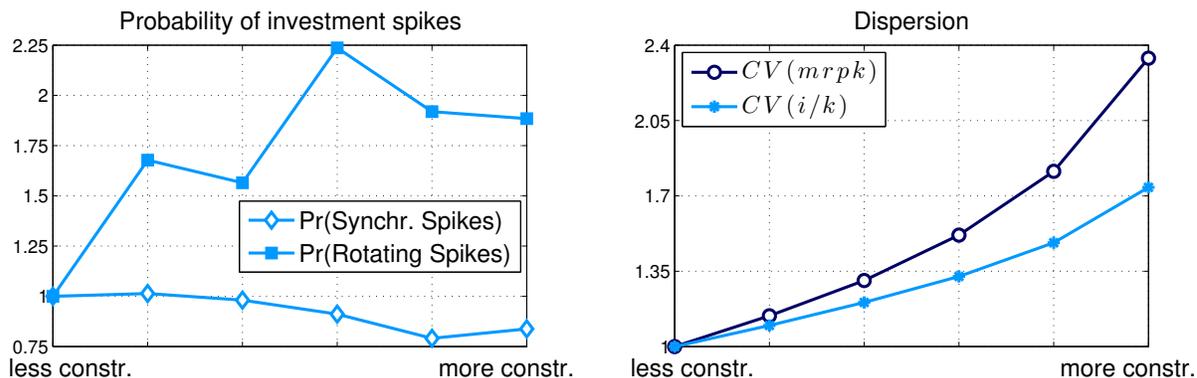
In a similar vein, we estimate probit models where we regress a dummy variable if the firm experiences a “rotating investment spike,” that is, both plants experience an investment spike in the wake of a firm productivity shock, but just in subsequent periods. We estimate

$$Pr(Y_{jt}^{rotate} = 1|X_{jt}) = \Phi(X_{jt}\beta) \quad (10)$$

where X_{jt} is a vector of controls which includes is the firm’s level of the productivity shock, η_{jt} , and the dummy variable Y_{jt}^{rotate} is defined as follows

$$Y_{jt}^{rotate} = \begin{cases} 1 & \text{if } \frac{i_{At}}{k_{At}} > 0.15 \text{ and } \frac{i_{Bt}}{k_{Bt}} < 0.15 \text{ and } \frac{i_{At+1}}{k_{At+1}} < 0.15 \text{ and } \frac{i_{Bt+1}}{k_{Bt+1}} > 0.15 \\ 1 & \text{if } \frac{i_{At}}{k_{At}} < 0.15 \text{ and } \frac{i_{Bt}}{k_{Bt}} > 0.15 \text{ and } \frac{i_{At+1}}{k_{At+1}} > 0.15 \text{ and } \frac{i_{Bt+1}}{k_{Bt+1}} < 0.15 \\ 0 & \text{otherwise.} \end{cases}$$

Figure 5: Dispersion and the nature of investment spikes after firm TFP shocks



Note: Left panel: normalized probabilities of “synchronized investment spikes” and “rotating investment spikes” after firm technology shocks by collateral size. Right panel: Dispersion of marginal revenue products of capital and investment rates by collateral size.

Again, we evaluate the marginal probabilities, smooth them across deciles and normalize the probability in the least constrained group to unity. The results are plotted in the left panel of Figure 5 (line with solid squares). In line with the model, the financially most constrained firms are more likely to respond to firm shocks with a rotating investment spike. That is, the firms contemporaneously invests in only one plant *and* invests in the other plant in the subsequent period. The empirical difference of the likelihood of a rotating investment spike increases a lot with our proxy measure of financial constraint: The most constrained firms are twice as likely to experience a rotating investment spike as financially unconstrained ones. All in all, we view this evidence as strong support for the presence of the key model mechanism.

5.2 Empirical evidence II: Within-firm dispersion

Lastly, we consider the within-firm dispersion in marginal revenue products of capital and capital reallocation. According to our model, firms which are more credit constrained should experience more dispersion while firms with a lot of pledgeable collateral should have an easier time managing to equate returns. We compute the two dispersion measures by decile of pledgeable collateral and divide by the mean level in each decile to account for level differences. Unlike the standard deviation, the resulting coefficient of variation is dimensionless and can be easily compared across deciles. Again, we smooth out the results across deciles

as in the previous subsection and plot the results in the right panel of Figure 5.

Indeed, the within-firm dispersion measures show that investment dispersion monotonically increases in our proxy for credit constraints: the most constrained firms have investment dispersion within firms that is about 1.7 times as large as that for the least constrained firms. The difference in marginal revenue products of capital is even stronger: the most constrained firms are about 2.5 times as dispersed as the least constrained firms.

6 Conclusion

This paper showed that dispersion in marginal revenue products of capital need not indicate distortions. Motivated by the evidence that dispersion mostly occurred within firms rather than across firms, we built a model of a firm operating several plants. Such firms dispose over an internal capital market that helps easing external financial constraints and supports aggregate investment, capital and output. Most importantly, economies with multi-plant firms may well exhibit more dispersion in marginal revenue products of capital than economies with single-plant firms, but still produce more aggregate output with the same technologies.

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Appendix

A Additional empirical evidence

A.1 Investment and equalizing *MRPK*

The assumption that investment should ideally be undertaken to equalize marginal revenue products of capital rests on some assumptions which we work out in this appendix. In a standard frictionless economy with decreasing returns to scale, agents should choose investment to equate the expected capital return. To understand how equating capital returns relates to equating marginal revenue products consider the expression of the expected capital return. This return is defined as the proceeds of one unit of capital at the end of next period – the value of undepreciated capital plus its marginal revenue product – divided by the cost of next period’s capital in the current period. Defining industry output as the numéraire, we denote the price of next period’s capital k' in terms of this period’s numéraire by $P_t^{k'}$, the industry-wide depreciation rate by δ_t , the marginal revenue product of capital in plant n and year t by $MRPK_{nt}$, real value added by y_{nt} and the capital stock by k_{nt} . Then the expected gross return, $\mathbb{E}\mathcal{R}_{nt+1}$, in a given year and industry is

$$\mathbb{E}\mathcal{R}_{nt+1} = \mathbb{E} \frac{P_{t+1}^k (1 - \delta_t) + MRPK_{nt+1}}{P_t^{k'}}.$$

We assume all units in an industry face the same price of capital, $P_t^{k'}$, and the same depreciation rate δ_t . Then the only source of heterogeneity in returns stems from differences in expected marginal revenue products of capital, $MRPK_{nt+1}$. In a large set of models with Cobb-Douglas technology, this object is proportional to the expected average product of capital, $\mathbb{E} \frac{y_{t+1}}{k_{t+1}}$. Since we do not measure the expected marginal revenue product of capital, we approximate it by the realized marginal revenue product of capital. This is a good approximation if capital is chosen one period in advance, all other inputs are chosen statically and total factor productivity is sufficiently persistent. Only unexpected innovations to profitability will then make the realized and the expected marginal revenue product of capital different. All these assumptions are plausible and widely used in the macroeconomic and investment literature. From now on we study the logarithm of marginal revenue products of capital which is denoted by lower case letters: $mrpk_{nt} \equiv \log(MRPK_{nt})$. Given our above assumptions, we measure dispersion in marginal revenue products of capital as: $Var_t(mrpk_{nt}) = Var_t(\log(y_{nt}/k_{nt}))$.

A.2 Data

We mainly use confidential data on manufacturing establishments collected by the U.S. Census Bureau which comprise the 1972-2009 Annual Survey of Manufactures (ASM), the Census of Manufactures (CMF) from 1972-2007 and the Longitudinal Business Database

(LBD) from 1976-2009. These data inform us about age, output, capital stocks, investment expenditures and other inputs at the level of the individual establishment. In the manufacturing sector, Census defines an “establishment” as a business location where the principal activity is production; we hence think of an “establishment” as a production plant. The Census data also contain information about the ownership of each plant (denoted by the variable `FIRMID`) which allows us to construct the hierarchical plant structure of “firms” necessary for our main object of interest, the within-firm and between-firm component of heterogeneity in returns, productivity and reallocation.

From the Census of Manufactures (CMF) and the Annual Survey of Manufactures (ASM) we construct a large dataset of plants in the U.S. manufacturing sector. In order to obtain a consistent longitudinal panel, we limit attention to the ASM and the ASM portion of the CMF data (identified by establishment type `ET=0`). We prefer the ASM over the CMF as our benchmark dataset because we want to test dynamic implications of our model of investment in multi-plant firms the highest possible frequency. Many aspects of our mechanism would disappear at the quinquennial frequency of the CMF. By focusing on the ASM portion in all years, we automatically eliminate all administrative observations (identified by `AR=1`) which are imputed mainly off industry means and would thus corrupt moments of the distribution we are interested in. Our resulting panel spans the years 1972-2009, which allows us to study the long-run features of the dispersion of marginal revenue products of capital and reallocation. Every year, we observe about 55k plants which total up to 2.1 million observations.

We combine the Census data with industry-level data from several publicly available sources: input and output price deflators from the NBER-CES Manufacturing Industry Database (NBER-CES), various asset data from the the Capital Tables published by the Bureau of Labor Statistics (BLS) and the Fixed Asset Tables published by the Bureau of Economic Analysis (BEA). Unless otherwise noted, all datasets are at annual frequency. Most of the information contained in the non-Census datasets (BEA, BLS, NBER-CES) other than the manufacturing data are merely needed to estimate productivity and the replacement value of capital at current market conditions.

For each plant in these data, we construct real value added, the real capital stock and real investment. To obtain real value added, y_{nt} , we first compute nominal value added as sales less intermediate and energy inputs, correct for inventory changes and resales¹⁷, and deflate the resulting measure by the 6-digit NAICS shipment price deflator from the NBER-CES manufacturing database. The real capital stock, k_{nt} , is the sum of structure and equipment capital each of which are expressed as real replacement values at current market conditions. These replacement values are computed individually for structure and equipment capital with the perpetual inventory method using investment expenditures and depreciation rates. When a plant is observed for the first time, we initialize its capital stock at its book value which is transformed as follows. First, we convert nominal book values into nominal market values and then deflate this measure using BLS’s price deflators for capital

¹⁷Resales are goods purchased from another producer and resold in an unchanged condition. Correcting for them means we assess productivity of the plant as a producer rather than its productivity as a trader.

goods at the 3-digit NAICS industry level.¹⁸ Like capital, we compute real investment, i_{nt} , as the sum of real structure and equipment investment by deflating the respective nominal investment expenditures by the 3-digit NAICS industry investment price deflators from the BLS. Our capital measure denotes the beginning-of-year stock values while our investment and value added measures refer to flow values during the year. To avoid outliers driving our results about dispersion and the investment-productivity link, we drop the 1% tails of the productivity and investment rate distributions in a given 4-digit NAICS industry.

A firm is defined as all manufacturing plants within the same FIRMID¹⁹ in a given year and 4-digit NAICS industry. The FIRMID defines the collection of plants under common ownership or control. All plants of subsidiary firms are included as part of the owning or controlling firm. If the same firm is active in several industries, we define each industry operations as separate firms. Our within-firm dispersion measures are hence an understatement because we ignore the between-industry component if within-firm dispersion.

A.3 Empirics of cross-sectional moments

This appendix details how the cross sectional moments underlying Table 2, Panel B. were computed. First, we compute cross-plant moments \mathcal{M}_{it} and their standard errors in a given industry i and year t . \mathcal{M}_{it} stands for the cross-sectional standard deviation, inter-decile range, skewness, Kelley skewness and the excess kurtosis. We adopt the formulae for the first four moments, the inter-quantile range and their standard errors from Kendall and Stuart (1987). Kelley skewness is a quantile based measure of skewness whose predecessor was proposed by Kelley (1947).

Every cross-sectional moment is computed by industry and by year. To get along-run industry-specific moments, we first aggregate over years in order to take out any industry-specific trends. As Kehrig (2015), Gopinath et al. (2015) we note an upward trend in dispersion and – to a lesser extent – in skewness and a downward trend in kurtosis. Behold that the cross-plant standard deviation increases about 10 log points per decade; both between-firm and within-firm dispersion increase evenly, so that there is no discernible trend in the within-firm share of the overall industry variance. The cross-plant skewness becomes more positive over time: Kelley skewness increases from around zero (unskewed) to 0.25 (right tail about 1.66 times as wide a bottom tail) in 2007. We compute the typical cross-sectional moment in a given NAICS-4 industry in 1990 which corresponds to the middle of our sample.

Then, we aggregate across industries using that industry’s average share in value added: $\mathcal{M}_t = \sum_i \omega_{it} \mathcal{M}_{it}$. Standard errors are computed according to this aggregation: $SE_{\mathcal{M}_t} = \sqrt{\sum_i (\omega_{it} SE_{\mathcal{M}_{it}})^2}$. This yields the moments within the average industry in the middle of our sample.

¹⁸For more details about the primary data and their transformation needed to obtain measures of the real capital stock and to estimate productivity, see the description in the appendix to Kehrig (2015).

¹⁹Song et al. (2015) identify firms off the EIN, the employer identification number, which comes from tax records. Since we are interested in organizational control rather than tax liability and because the same FIRMID may operate hundreds of EINs for tax purposes, we prefer FIRMID to indicate firms.

A.4 Empirics of between-firm and within-firm moments

In this section, we detail how we compute the within-firm and between-firm dispersion in marginal revenue products of capital and capital reallocation which underly Table 2, Panel A. and the robustness exercises in Section 2.3.

First, we decompose the overall variance in marginal revenue products of capital into three components: one between industries (reflecting differences in measurement and the definition of capital and value added), one between firms in a given industry and one across plants within a firm and industry. We define firms that operate plants in separate industries as different firms, thus biasing the true within-firm component of dispersion downward.

$$\begin{aligned}
 V_t &= \sum_n \omega_{njit} (mrpk_{njit} - mrpk_t)^2 \\
 &= \underbrace{\sum_i \omega_{it} (mrpk_{it} - mrpk_t)^2}_{V_t^{Ind} \text{ between-industry}} + \underbrace{\sum_i \omega_{it} \sum_{j \in i} \omega_{jt}^i (mrpk_{jit} - mrpk_{it})^2}_{V_t^B \text{ average between-firm}} + \underbrace{\sum_i \omega_{it} \sum_{j \in i} \omega_{jt}^i \sum_{n \in j, i}^{N_j} \omega_{nt}^{ji} (mrpk_{njit} - mrpk_{jit})^2}_{V_t^W \text{ average within-firm}} \quad (11)
 \end{aligned}$$

where n indicates the plant, j the firm, i the 4-digit NAICS industry and t the year. $mrpk_{njit}$ denotes the marginal revenue product of capital of plant n belonging to firm j and industry i in year t , $mrpk_{jit}$ the average return in firm j in industry i , $mrpk_{it}$ the average return in industry i , and $mrpk_t$ the average level of returns in the economy.

An industry's level of marginal revenue product of capital is determined by the level of P_t^k and the asset bundle it typically reflects in that industry. This and other industry-specificities in measurement will artificially drive V^{Ind} – an object which we ignore for its lack of economic meaning. In our empirical analysis in Section 2, we focus on V_i^B and V_i^W of plants with at least two plants only as it is meaningful to compare them and how much of the dispersion in marginal revenue products of capital within an industry originates within firms as opposed to between firms in that same industry: $\mathcal{W}_i \equiv \frac{V_i^W}{V_i^W + V_i^B}$. When computing an “aggregate” number for \mathcal{W} , we compute the average of industry ratios which is weighted by ω_i , i.e. that industry's share in plants or capital, depending if we are looking at unweighted or capital weighted dispersion.

Although investment rates do not suffer from the industry-specific measurement issues like marginal revenue products of capital, we proceed in a similar way to assess between-firm and within-firm investment rate dispersion.

A.5 Robustness

A.5.1 Accounting for measurement error

In Section 2.3 we already dealt with some measurement error. If plant-level variables are measured with noise, then the firm-level averages will be measured more precisely thus artificially inflating the within-firm variance. Time aggregation should filter out that type of

measurement error. Because time aggregation cannot deal with persistent measurement error, we now consider that type. To do that, we consider marginal revenue products of capital which are computed using separate measures of capital and values added. Our alternative measures come from different datasets or are separately measured variables in our baseline dataset. We have:

- K^{TAB} – we use appropriately deflated values of variable TAB instead of the perpetual inventory method;
- Y^{IRS} – we use administrative data on sales from the IRS instead of TVS from CMF/ASM;
- Y^{PCU} – we use collected data on actual production from the Plant Capacity Utilization Survey (PCU) instead of TVS.

These alternative measures should be correlated with our original measures of K and Y in the ASM (since they measure the same underlying object), but they should still be different due to different coverage or handling by the statistical agency. We recompute marginal revenue products of capital using the three alternative measures and redo the cross-sectional within-firm between-firm decomposition on these alternative measures. If dominance of within-firm share is true, then this should show up in all of these measures. Song et al. (2015) follow a similar procedure.

Table 7: Accounting for measurement error

	Alt. Measure	$Corr \left(\log \left(\frac{y}{k} \right)^{bench}, \log \left(\frac{y}{k} \right)^{alt} \right)$	$\left(\frac{V^W}{V^W+V^B} \right)^{bench}$	$\left(\frac{V^W}{V^W+V^B} \right)^{alt}$
I: CMF 1972-2007	K^{TAB}	0.979	0.563 (0.008)	0.556 (0.005)
II: CMF 2002-2007	Y^{IRS}	0.990	0.538 (0.005)	0.542 (0.005)
III: ASM 1974-2007	Y^{PCU}	0.494	0.581 (0.015)	0.626 (0.017)

Note: Table displays the within-firm share of overall dispersion for alternative measures of value added Y – collected either from tax records or separately measured in the Plant Capacity Utilization Survey (PCU) – and capital K (real replacement value at current market prices directly computed from book values instead from the perpetual inventory method). Correlation of the computed marginal revenue products of capital measures are positive, some are high and the within-firm share of overall marginal revenue products of capital dispersion is not statistically different at the 95% except when using value added from the PCU which yields an even higher within-firm share. Error bands constructed from averaging across 86 NAICS-4 industries.

Since using these alternative measures limits our sample at times, we also recompute the within-firm between-firm decomposition using our original data so we are comparing the moments for the same underlying sample where we have both our benchmark measure as well as the alternative. It turns out, the differences in the within-firm share are marginal and almost always lie in the 95% error bands of the other measure. Only when using value added from the PCU the benchmark differs from the alternative which yields an even higher within-firm share. Error bands constructed from averaging across 86 NAICS-4 industries. We conclude from this exercise that our main result of the within-firm dispersion accounting for the largest portion in overall dispersion does not go away when using alternative measures of output and capital.

A.5.2 Marginal versus average revenue products

Our empirical work of Section 2 aimed at measuring marginal revenue products of capital which are the relevant measure of what should be equalized across production units. But in the data, one can measure only average revenue products of capital; like the literature, we approximate the dispersion of marginal revenue products with average revenue products. This approximation is usually justified with a Cobb-Douglas production function where average revenue products are proportional to marginal ones. But they are not if technology is not multiplicative or if it is Cobb-Douglas with overhead inputs. Bartelsmann et al. (2013) documented that overheads in production are quite powerful in explaining differences between micro production units and they showed significant aggregate consequences. Most overhead inputs are likely at the headquarter level of a firm rather than the plant level. Though this cast some doubt that all our results could be driven by constant inputs, we don't want to dismiss this possibility.

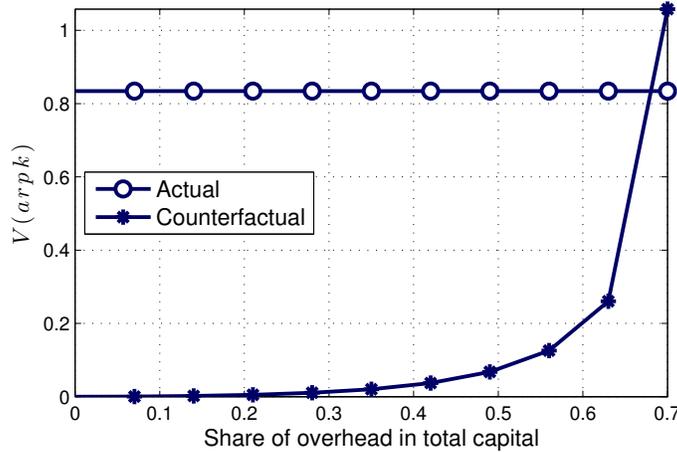
Overheads Any constant input requirements at the plant level are hard to identify empirically. We therefore carry out a quantitative exercise to examine how large overheads would have to be in order to explain all or a portion of the empirically observed dispersion. Suppose the true technology is $y_n = z_n(k_n - \bar{k})^\alpha$. In that case, the average revenue product of capital can be written as $arpk_n = mrpk_n - \log \alpha + \log(1 - \bar{k}/k_n)$. Further suppose that marginal revenue products – which we cannot measure – were completely equalized. Then, the entire variance of average revenue products would reflect the differential share of overheads across firm of different capital size:

$$V(arpk_n) = V\left(\log\left(1 - \frac{\bar{k}}{k_n}\right)\right). \quad (12)$$

We simulate a firm size distribution realistically assuming that capital – unlike employment – is distributed log-normally. We consider how large the right hand side variance in equation (12) is for different levels of \bar{k} .

Figure 6 plots the RHS of equation (12) as a function of $\mathbb{E}[\bar{k}/k_n]$. Naturally, when $\bar{k} = 0$, then this variance will be zero and the observed variance of average revenue products must

Figure 6: How much overheads are necessary to explain the observed $V(arpk)$?



Note: Simulation of the right hand side of equation (12) against the share of overhead ion total inputs.

be caused by the variance of marginal revenue products. This does not change much for low and moderate levels of overheads. Even if half of all inputs are overheads, less than a tenth of the empirically observed variance in average revenue products can be explained by overheads. Clearly, this much overhead inputs at the level of the plant is unreasonable. Only if the average share of overhead in total inputs approaches 70% can the observed variance be explained by overheads. We conclude that overheads may only play a limited role in explaining the long-run dispersion of average revenue products of capital.

Non-unitary elasticity of substitution Another empirically plausible alternative to a simple Cobb-Douglas production function would be a constant elasticity of substitution production function. Suppose $y_n = \left[\alpha k_n^{\frac{\sigma-1}{\sigma}} + (1-\alpha)x_n^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$ where σ is the elasticity of substitution and x_n the variable inputs of plant n . In that case, the average revenue product of capital can be written as $arpk_n = \sigma[mrpk_n - \log \alpha]$ and the variance of average and marginal revenue products as:

$$V(arpk_n) = \sigma^2 V(mrpk_n). \tag{13}$$

The true dispersion of marginal revenue products could then be much lower is the elasticity of substitution is larger than unity. However, Oberfeld and Raval (2014) that estimates that elasticity of substitution at the plant level, puts that number significantly smaller than 1 suggesting that the empirically measured dispersion of average revenue products would be a lower bound on that of marginal revenue products. We conclude that a non-unitary elasticity of substitution would most plausibly measure only a portion of the true dispersion

of marginal revenue products.

B Numerical solution of model

To solve the model, we discretize the plant-level capital stock using a N_k -point grid, where $N_k = 100$ to produce the results in this paper. This implies that the two-plant capital grid is comprised of a total of 10,000 points. In addition, the shock process is approximated by an 8-point grid, from the combination of plant-specific and firm-specific Markov chain processes.

We use a hybrid iterative method to solve the model. First, we iterate and maximize over the (k'_A, k'_B) pairs of plant-specific capital until convergence of the policy function. Then, we continue iterating until changes in the value of the firm between iteration steps is below a given threshold for all states.²⁰ We then ensure that the policy function is indeed stable. This method, while not particularly computationally efficient, allows us to handle the numerous non-convexities of our model. We also tested that lowering or increasing N_k did not have any meaningful impact on our results.

Next, we simulate a single two-plant firm over 100,500 periods, throw out the first 500 observations to allow for burn-in and then create a panel of 500 two-plant firms with the simulations that were kept. This approach is appropriate because there are no aggregate shocks in our setup: with uncorrelated firm-level shocks, we are not required to simulate a panel of firms period-by-period. Simulated moments are computed on this firm panel.

²⁰Because we report the value of firms under various economic environments, we cannot solely rely on the convergence of the policy function.