# Sectoral Shocks, Reallocation and Unemployment in Competitive Labor Markets

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**Abstract**

Can sectoral shocks be the cause of business cycles? I investigate this question using a multisectoral business cycle model of labor reallocation and unemployment in the neoclassical tradition of Lucas and Prescott (1974)'s island model and Long and Plosser (1983)'s RBC model. In my economy, sectors are interconnected by input-output relations and by the reallocation of labor. Aggregate and sectoral productivity shocks influence the demand for labor and intermediate inputs in each sector. Workers accumulate sector-specific human capital, which makes reallocation across sectors costly and prevents rapid adjustments to shocks, thus generating unemployment. I find that an economy with both aggregate and sectoral productivity shocks is able to explain a large share of the observed volatility of output and unemployment and can match the patterns of sectoral comovement. An economy with only independent sectoral shocks cannot reproduce these facts. Moreover, impulse-response analysis shows small aggregate effects of even strong sectoral shocks. While input-output links do not propagate and amplify these shocks considerably, they are a very important source of comovement. In addition, because sectoral gross mobility is always high relative to net mobility, frictions to the reallocation of labor play a minor role in the comovement of sectoral variables.

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## 1 Introduction

*A leading question -perhaps the leading question- in macroeconomics since the publication in 1982 of David Lilien's paper, "Sectoral Shifts and Cyclical Unemployment," is whether sectoral, rather than aggregate, shocks are the key factor responsible for fluctuations in the unemployment rate.*

*Jannet Yellen (1989)*

What pulls most sectors of the economy up, and at other times push them down again? Can shocks that occur in a particular industry generate an economy-wide downturn? A long-standing debate among macroeconomists is whether business cycles are the result of a common aggregate component, or originate in shocks to specific sectors which then propagate to the rest of the economy.

The literature has identified several mechanisms that tilt the argument to one side or the other. On the one hand, Lucas (1977) argued that, since aggregate and sectoral macroeconomic variables tend to exhibit a similar behavior across different expansion and contraction episodes, "all business cycles are alike". Therefore, they must be the result of the same underlying force: an aggregate shock. Even if industries do not follow a homogeneous cyclical path, this is not evidence of the effects of sectoral shocks, as sectors can have different loadings on a common component (Abraham and Katz, 1986). Moreover, with many sectors, a law of large numbers holds and the effects of shocks to individual sectors will average out in the aggregate.

On the other hand, a literature built around the multisector RBC model of Long and Plosser (1983) argues that strong sectoral complementarities, in particular complementarities in production due to input-output links, can amplify and propagate the effects of sectoral shocks to the whole economy.[1] The intuition is simple. If a sector is a key provider of inputs for the rest of the economy, a shock to this industry will affect the price of this input and in turn production costs in mosts other sectors as well. In this way, a sectoral shock has aggregate consequences.

A different line of work, starting with Lilien (1982), highlights the role of frictions in the reallocation of labor and unemployment. Shifts in sectoral demands for factors of production, in particular labor, may require a large reallocation of workers across sectors. If this process is subject to frictions, then these limit the possibilities of different sectors to expand and contract production, and thus may shape how the economy reacts to shocks.

In this paper I study the role of sectoral shocks as a source of business cycles using a framework that incorporates all these mechanisms. I develop a multisectoral, business cycle model of labor reallocation and unemployment which combines Lucas and Prescott (1974)'s island model, a classic framework to analyze sectoral shocks and labor reallocation, with the multisectoral RBC model of Long and Plosser (1983). In my model, both aggregate and

[1] See also Scheinkman and Woodford (1994); Jovanovic (1987); Horvath (2000) and Acemoglu et al. (2012).

sectoral TFP shocks influence labor demand and wages in each sector. Input-output links are part of the production process of sectoral goods. Workers develop sector specific human capital, which makes reallocation across sectors costly and prevents rapid adjustments to shocks, generating unemployment.

In my model, unemployment occurs when workers' marginal product of labor is below their reservation wage, which depends endogenously on individual characteristics and on macroeconomic conditions. Workers are unemployed for one of two reasons: either they are in the process of relocating for a better sector, or they are waiting for conditions in their depressed sector to improve. Unemployed workers search for a suitable employment opportunity for the next period, inspecting the different sectors of the economy for their best employment option. In the model, labor markets are competitive but segmented. This means that workers can switch employers at no cost within that labor market and firms can also replace workers at no cost. However, moving to a different labor market is costly for workers in terms of a period of time spent unemployed, with the associated foregone earnings and possible skill depreciation due to the specialization of labor.

Solving for equilibrium in an economy with many segmented labor markets in the presence of aggregate shocks raises an important technical difficulty. When workers decide on whether to work and where to reallocate, they must be able to make rational predictions about the evolution of wages in the different parts of the economy. These wages, in turn, depend on the labor supply and reallocation decisions of all workers in the economy. Put simply, individual decisions depend on the distribution of workers in all sectors of the economy, and with aggregate shocks, this distribution is not time invariant. In quantitative applications involving a non-trivial number of labor markets, using moments to summarize this distribution, as in Krusell and Smith (1998), is not useful, since this would require a large number of moments. In this paper I borrow from a large literature on dynamic discrete choice models with random utility and propose a new procedure which allow me to solve for equilibrium. There are two key steps. First, I introduce idiosyncratic extreme-value shocks to the worker's problem. Using properties of this distribution I can aggregate discrete individual work and mobility choices into the smooth optimization problem of an island-representative agent. Second, I use perturbation methods. These methods can accommodate a very large number of state variables, which in this model is the distribution of workers across markets.

This paper makes three contributions. First, as I just discussed, I propose a solution method for a Lucas and Prescott (1974) type of model with a finite, but large, number of islands and aggregate uncertainty. Second, I analyze the cyclical properties of island models. Although the original paper was developed almost 40 years ago, little is known about its business cycle properties. Third, I investigate how the economy responds to the effects of aggregate and sectoral shocks and use the model to quantify their importance in generating cyclical fluctuations and comovement in the labor market.

I find that a benchmark economy with both sectoral and aggregate shocks explains much of the aggregate and sectoral volatility of output and labor market variables. Moreover, it approximates well the patterns of comovement in the data. Sectoral comovement, the synchronized upward and downward evolution of macroeconomic variables across most industries, is a defining characteristic of the business cycle. However, most macroeconomic models have difficulties matching the cyclical correlation of sectoral employment.[2] Conversely, an economy with *only* independent sectoral shocks can account at most for one half of the volatility of the benchmark economy and generates too little comovement of sectoral variables. I conduct impulse-response exercises and find that sectoral shocks can explain only a small fraction of the ensuing cyclical movement in aggregate GDP and unemployment. On the other hand an aggregate shock explains a substantial part.

In my model, sectors are connected not only by flows of workers across them but also by input-output relations. The recent work by Foerster et al. (2011) argues that it is necessary to take into account these linkages to properly quantify the importance of sectoral shocks. They offer two important results (Table 7). First, an economy with only sectoral shocks can explain a substantial part of the volatility of industrial production but fails to account for the patterns of comovement in sectoral production. Second, an economy with both aggregate and sectoral shocks explains all of the volatility and matches comovement. In their latter economy, aggregate shocks account for 50% of the volatility. My results are in line with their findings, but my application covers a larger part of the US economy and I pay closer attention to comovement of labor market variables.[3]

Sectoral comovement is a central feature of business cycles. However, Christiano and Fitzgerald (1998) argue that most macroeconomic models are unable to generate comovement in sectoral employment, concluding that "the business cycle is still a puzzle". A few mechanisms have been proposed to reconcile the theory with the real world. Boldrin et al. (2001) introduce a key friction into a standard 2-sector RBC model: it takes one period to reallocate labor. On the other hand, Hornstein and Praschnik (1997) introduce input-output links in a 2-sector model and are able to generate comovement in labor across sectors.[4] In this paper I study the role of both input-output linkages and reallocation frictions in generating employment and unemployment comovement. I find that input-output links are more important in generating this comovement than reallocation frictions. The reason for the smaller role of reallocation frictions in the dynamics of macroeconomic variables is that, in the aggregate, reallocation is large and ongoing at all phases of the cycle, with gross mobility always high relative to net changes. Because of this, when a shock hits it propagates quickly across sectors.

[2]See Christiano and Fitzgerald (1998).

[3]In my application I focus on 12 NAICS sectors, which account for 98% of private employment and value added, while Foerster et al. (2011) focus only in the sectors that comprise the Industrial Production Index, which are basically manufacturing, mining and utilities.

[4]Kim and Kim (2006) study the role of input-output linkages in explaining employment comovement across a large set of sectors, but their model does not approximate well the comovement in the data.

The literature on worker mobility and displacement considers several dimensions when defining a labor market: industry, occupation and geography. Most likely, all of them are relevant and a labor market should be defined in terms of the intersection of all three. However, a reasonable level of detail along each of these dimensions would make the analysis computationally very demanding. As the focus of this work is on the effects of sectoral shocks, here I assume that a labor market is an industry or economic sector, with workers having some form of industry specific human capital which may be lost by switching sectors.[5]

In this work, I relate to a large literature started by Lucas and Prescott (1974) on reallocation and unemployment in competitive labor markets, better known as island models. Essentially all this literature analyzes stationary environments where aggregate shocks are ruled out and little is known about cyclical properties of island models. To the best of my knowledge, Veracierto (2008) is the sole exception. By assuming complete markets, using employment lotteries, and introducing random search, he is able to write down the problem as that of a representative household or a social planner, who instructs workers to move out of islands.[6] In addition, in Veracierto (2008), as in most island models, labor market flows and unemployment duration are not defined. The reason is that, in equilibrium, a positive measure of workers are indifferent between being employed or not. This implies that we can interchange the employment status of many agents without altering equilibrium. However, by doing that, key labor market variables like transitions between employment status and unemployment duration are not uniquely pinned down as they depend on individual labor histories. I introduce aggregate shocks into an island model, but I do not assume complete markets or employment lotteries. Moreover, in my model workers are heterogeneous and labor market histories are pinned down from micro-foundations.

On the technical side, I follow a large literature on dynamic discrete choice models with random utility. Recent applications to segmented labor markets include Lee and Wolpin (2006), Kline (2008), Artuc et al. (2010) and Kennan and Walker (2011). Mangum (2010) is the first application, that I am aware of, that specifies a random-utility discrete choice model with unemployment and search-and-matching frictions. He uses the model to study labor reallocation across US states. He solves his model applying the Oblivious Equilibrium concept (Weintraub et al., 2008), greatly simplifying the computation as the distribution of economic conditions is assumed not to be part of the decision problem of the worker.

My model also relates to a recent literature that studies how workers' specific skills affect their employment and reallocation decisions during the business cycle. Carrillo-Tudela

[5] Jacobson et al. (1993); Neal (1995); Lee and Wolpin (2006); Sullivan (2010), among others, find evidence of industry specific human capital. On the contrary, Kambourov and Manovskii (2009b) argue that human capital is mostly occupation specific. Given that I am abstracting from occupational and geographic mobility, the costs I specify here will also capture, to some degree, the effects of occupational and geographic mobility frictions.

[6] This reduces to a stochastic nonlinear control problem which he solves using a linear-quadratic approximation. Judd (1996) discusses the limitations and potential problems that may arise in using linear-quadratic approximations.

and Visschers (2013) develop a model of occupational mobility and reallocation over the business cycle, which shares features of neoclassical island models and search and matching labor models. By assuming constant returns to scale both in the production function and the matching function, together with a free entry condition, their model is very tractable and displays a block recursive structure (Menzio and Shi, 2011). While this specification is certainly convenient, it departs substantially from traditional island models, where congestion and a downward sloping labor demand in each market are central. My model does not have a block-recursive structure, and therefore the distribution of workers over islands is one of the state variables in the decision problem of the workers.

Two recent papers discuss the role of sectoral shocks and frictions to the reallocation of labor on unemployment. Pilossoph (2012) develops a two sector dynamic discrete choice model of unemployment with frictions across and within sectors to study the effects on unemployment of a shock to construction. Wiczer (2013), analyzes whether occupation specific human capital affects the duration of unemployment in a model with search and matching frictions both across and within labor markets. His model is closely related to Carrillo-Tudela and Visschers (2013) with the exception that search over occupations is semi-directed, instead of fully random, and he has occupational shocks affecting (or correlated across) all workers in an occupation.

The paper is organized as follows. Section 2 describes the main assumptions and details of the structural model. Section 3 discusses the computational strategy, providing insights on the technical contribution of the paper. Section 4 summarizes key moments of labor market variables by sector, comovement and mobility measures, which I then link to the structural model in Section 5. In addition, in Section 6 I conduct counter-factual experiments aimed at gauging the importance of sectoral shocks and the different mechanisms at play. Finally, Section 7 concludes.

## 2 An Island Model with Aggregate Shocks

Time is discrete and infinite. There is a *finite* number of islands in the economy which I interpret both as an economic sector and as a labor market, and will use these terms interchangeably. Additionally, each sector produces a different good or service. Production exhibits constant returns to scale and there are no fixed costs nor any entry or exit barriers. The production function in each sector $i = 1, 2, ..., J$ is Cobb-Douglas in labor and intermediate inputs:

$$q(i) = A(i)L(i)^{\alpha_i} \prod_{j=1}^{J} \varphi(j,i)^{(1-\alpha_i)\eta_{ij}}; \qquad \text{where } \eta_{ij} \in [0,1];\ \sum_{j=1}^{J} \eta_{ij} = 1 \tag{1}$$

where $q(i)$ is total output of good $i$, $\varphi(j,i)$ is the amount of good $j$ that is used as intermediate in sector $i$ and $L(i)$ is the amount of labor hired by the firm. $\alpha_i$ and $\eta_{ij}$ are parameters that represent the share of labor (value added) and the share of good $j$ in intermediate expenditures of sector $i$, respectively. Both sectoral and aggregate shocks affect total factor productivity, $A_i$, as follows:

$$A_i = e^{\lambda_i z + \theta_i} \tag{2}$$

where $z$ is the aggregate, economy-wide component, and $\theta_i$ is the sector-specific shock. $\lambda_i$ is a parameter that captures the sensitivity of sector $i$ to the aggregate shock. By modeling shocks in this way, I nest the hypotheses of purely independent sectoral shocks and a common shock, but with sectors having different sensitivities to it as in Abraham and Katz (1986). I assume that $z$ and $\theta_i$ are distributed AR(1) independent of each other.

$$\begin{aligned} z' &= \rho_z z + (1-\rho_z^2)^{0.5}\xi_z & (3)\\ \theta_i' &= \rho_i \theta_i + (1-\rho_i^2)^{0.5}\xi_i & (4) \end{aligned}$$

with $\xi$'s iid, $E[\xi_z^2] = 1,\ E[\xi_i^2] = \sigma_i^2$.[7]

There is a unit measure of infinitely lived workers. Each worker starts the period attached to a sector, which is her labor market, and can work in that sector if she chooses to. Transitioning to a different labor market is costly and partially irreversible in a way that is detailed next.

Workers can be in one of two employment states: (1) working in sector $i$ or (2) unemployed and searching for a labor market where to work in the next period. If working, they obtain labor income which they consume. In the spirit of Lucas and Prescott (1974) I assume that employed workers cannot reallocate to a different labor market. Unemployed workers get consumption in terms of home production or a consumption equivalent value of leisure, and have the option to reallocate to a different sector. Future utility is discounted with factor $\beta$. In addition to aggregate and sector specific shocks, households face idiosyncratic shocks that affect their return to work, their return to home production and their preferences for working on a different sector.

Workers' market income is

$$\begin{cases} w(i)\,\tau\, e^{\sigma_\epsilon\, \epsilon_W} & \text{if work} \\ 0 & \text{if unemployed} \end{cases}$$

and home production is

$$\begin{cases} 0 & \text{if work} \\ b\,\tau\, e^{\sigma_\epsilon\, \epsilon_S} & \text{if unemployed} \end{cases}$$

where $\tau$ is a random and persistent idiosyncratic productivity shock, $\epsilon_W$ is a transitory (iid) shock to work (market) productivity and $\epsilon_S$ is a transitory (iid) shock to home production (non-market productivity). I discuss shortly why I assume two shocks $(\epsilon_W, \epsilon_S)$ to affect a

[7]The variance of the innovation in the common component is normalized to one since it cannot be identified separately from the average level of $\lambda$. See the discussion on identification in Appendix D.

single binary decision and why $\tau$ enters the value of home production. Employed workers use their earnings to buy goods from all sectors. They have logarithmic utility over a CES basket of sector goods:

$$\begin{aligned} u(C) &= \log(C) \\ \text{where} \qquad C &= \kappa \left( \sum_i \psi_i c(i)^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}} \end{aligned}$$

Unemployed workers do not earn a market income, but have logarithmic utility over their home production. In addition, they search for a work opportunity for next period. While searching, they observe and make rational predictions on the economic conditions in the different sectors. The decision on which sector an unemployed worker wishes to join next period is influenced by economic conditions (aggregate and sectoral), the costs of reallocation, and personal preferences for the sectors. These personal preferences are modeled as an idiosyncratic additive preference shock for each sector, denoted by $\varsigma\,\epsilon_j$, where $\epsilon_j$ is an iid shock with zero mean and unit variance and $\varsigma$ is a scale parameter.

The timing for the worker's problem is illustrated in Figure 1. Workers start the period in a particular sector as the result of past location decisions. Aggregate and sector specific shocks are realized. Idiosyncratic productivity shocks (persistent and transitory) are realized. Workers observe these idiosyncratic shocks and the economic conditions in all sectors, and decide whether to work in their current sector or become unemployed and start searching. After this first decision has been made, a second decision stage begins in which unemployed workers search over all sectors, including their own, and decide in which labor market to participate in the next period. Preference shocks for sectors are revealed only at this stage and only for workers that search.

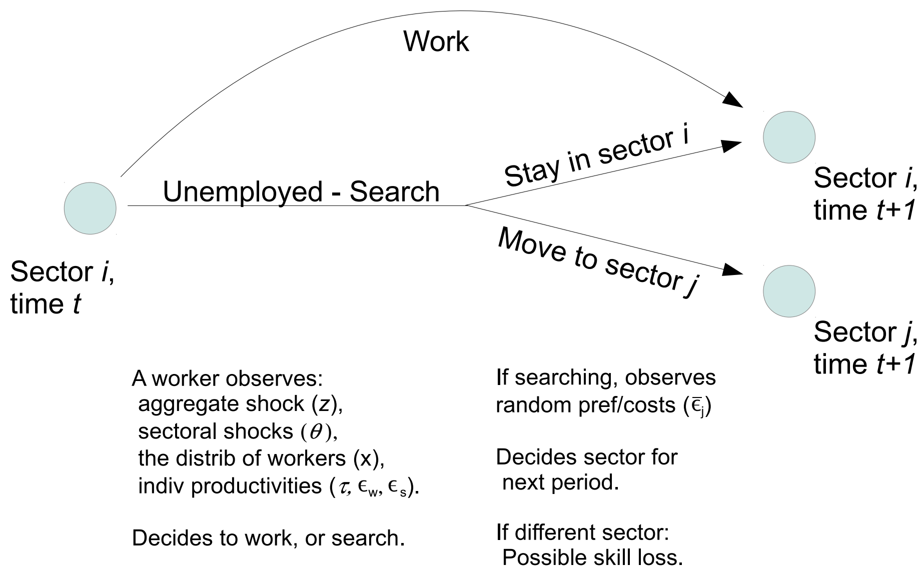
As already mentioned, $\tau$ is a random idiosyncratic productivity component. I assume that $\tau$ has discrete support and its evolution can be described by a Markov chain with transition probability $\pi(\tau'|\tau)$.[8] Workers who switch sectors not only endure a cost in terms of foregone earnings by not working, but also may suffer a cost in terms of their individual productivity. The interpretation is that part of the skills ($\tau$) that a worker has are sector specific (i.e. industry specific human capital) which is destroyed when switching to a different sector.[9] Here, I assume that a worker that switches sector faces, in that period only, a different transition of $\tau$, denoted by $\tilde{\pi}(\tau'|\tau)$, and the assumption is that $\pi(\tau'|\tau)$ first order stochastic dominates $\tilde{\pi}(\tau'|\tau)$. Therefore, on average, workers face costs of switching sectors.[10]

[8] In the context of a single homogeneous labor market, this dimension of worker heterogeneity is in line with Bils et al. (2012) and Krusell et al. (2012).

[9] I make this assumption in an effort reduce the number of state variables in the problem. In this way, $\tau$ captures both general skills and industry-specific skills in reduced form.

[10] While not exactly equal, my assumptions on the evolution on $\tau$ and the costs of switching are similar to Kambourov and Manovskii (2009a) and Alvarez and Shimer (2009). In their models, workers have sector (or occupation) specific experience that is completely lost once a worker changes sector (occupation). The fact that

Figure 1: Timing & Worker's Decisions.




I assume no savings. Markets are incomplete and households cannot insure against any type of shock. Firms and households are price takers in labor and goods markets. The wage per efficiency units of labor in sector $i$ and the price for sectoral good $i$ are set by perfect competition. I normalize all prices and wages by the "ideal" price index stemming from the CES basket.

### 2.1 Firm's problem

As already mentioned, production exhibits constant returns to scale and there are no fixed costs nor any entry or exit barriers for producers. Therefore, the total number of firms in a sector is not pinned-down and, due to symmetry, it suffices to analyze the problem of the "representative firm". In every period the firm solves the following static problem:

$$\max_{L(i),\{\varphi(j,i)\}} p(i)\left(A(i)L(i)^{\alpha_i}\prod_{j=1}^{J}\varphi(j,i)^{(1-\alpha_i)\eta_{ij}}\right) - w(i)L(i) - \sum_{j=1}^{J}p(j)\varphi(j,i)$$

where $w(i)$ is the market wage in sector $i$, and $p(j)$ is the price of sector good $j$.[11] The optimal, minimum-cost demands for labor and intermediate goods conditional on output

here $\tau$ may decrease even if a worker does not change sectors makes the interpretation of $\tau$ as pure experience less straightforward. Instead, $\tau$ is better interpreted as having both general and sector-specific skills.

[11]For notational simplicity I omit here any reference to the time period since the problem of the firm is static.

level $q(i)$ are:

$$L(i) = \alpha_i B_i w(i)^{\alpha_i} \prod_{j=1}^{J} p(j)^{(1-\alpha_i)\eta_{ij}} \frac{q(i)}{w(i)} \tag{5}$$

$$\varphi(i,j) = (1-\alpha_i)\eta_{ij} B_i w(i)^{\alpha_i} \prod_{j=1}^{J} p(j)^{(1-\alpha_i)\eta_{ij}} \frac{q(i)}{p(j)} \tag{6}$$

where $B_i = \left[e^{\lambda_i z + \theta_i} \alpha_i^{\alpha_i} (1-\alpha_i)^{(1-\alpha_i)} \prod_{j=1}^{J} \eta_{ij}^{(1-\alpha_i)\eta_{ij}}\right]^{-1}$. The minimum cost to produce an unit of good $i$ is

$$D(i) = B_i w(i)^{\alpha_i} \prod_{j=1}^{J} p(j)^{(1-\alpha_i)\eta_{ij}}$$

### 2.2 Worker's problem

Due to reallocation costs, the worker's problem is dynamic. I deviate from Lucas and Prescott (1974) by introducing additional sources of heterogeneity within sectors. In particular, my assumptions allow me to specify the problem recursively as a dynamic discrete choice model with random utility. The problem of the worker that starts the period in sector $i$ is:

$$U(i,\tau,\epsilon_W,\epsilon_S,\Upsilon) = \max\left\{U_W(i,\tau,\epsilon_W,\Upsilon),\, U_S(i,\tau,\epsilon_S,\Upsilon)\right\} \tag{7}$$

where $U_W(i,\tau,\epsilon_W,\Upsilon)$ and $U_S(i,\tau,\epsilon_S,\Upsilon)$ are the value of working and searching, respectively, for workers in sector $i$ with idiosyncratic productivities $\tau, \epsilon_W, \epsilon_S$. The variable $\Upsilon = \{z, \{\theta_i\}, \{x(i,\tau)\}\}$ summarizes the aggregate state of the economy and contains the aggregate ($z$) and sectoral shocks ($\theta_i$) and the distribution of workers with productivity $\tau$ across sectors ($x(i,\tau)$).

The value of working solves:

$$U_W(i,\tau,\epsilon_W,\Upsilon) = \log(\underbrace{w(i,\Upsilon)\,\tau\, e^{\sigma_\epsilon \epsilon_W}}_{\text{Consumption}}) + \beta\, E_{\epsilon',\tau',\Upsilon'}\left[U(i,\epsilon'_W,\epsilon'_S,\tau',\Upsilon')|\tau,\Upsilon\right] \tag{8}$$

The first term in equation (8) is the period utility from consumption of market goods, the second part is the continuation value. As I ruled out any mobility opportunities while employed, an employed worker will start the next period in the same sector she is in today. As next period's conditions are not known beforehand, expectations are taken with respect to the evolution of the aggregate state $\Upsilon'$ and next period's realization of the idiosyncratic productivities $\tau', \epsilon'_W$ and $\epsilon'_S$.

The value for an unemployed worker that searches is:

$$U_S(i,\tau,\epsilon_S,\Upsilon) = \log(\underbrace{b\,\tau\, e^{\sigma_\epsilon\,\epsilon_S}}_{\text{home prod}}) + E_\epsilon\left[\underbrace{\max_j\left\{E_{\epsilon',\tau',\Upsilon'}\left[\beta\, U(j,\epsilon'_W,\epsilon'_S,\tau',\Upsilon')|\tau,\Upsilon,i,j\right] + \varsigma\epsilon_j\right\}}_{\text{search over sectors}}\right] \tag{9}$$

The first part of equation (9) is the period utility for the unemployed, which is the log of their home production, and the second term is the continuation value. Given that at the time of the first decision (whether to work or search) the preference shocks $\epsilon_j$ are not observed, the expectation is taken with respect of these shocks. The search decision at the second stage can easily be seen in the term inside brackets. Workers choose the best labor market to participate in the next period, and this decision is influenced by economic conditions in the different sectors (reflected by the value $U$, which is common to all workers of similar characteristics) and idiosyncratic preferences (captured by the shock $\epsilon_j$). This shock will create heterogeneity across otherwise-identical workers in their reallocation decisions.[12] To simplify the notation, the terms $i, j$ at the end of the conditional expectation over $U$ capture the assumption that $\tau$ follows a different law of motion if workers switch sectors.

It is worth stressing that unemployed workers search over all sectors, including the worker's current sector. This allows the model to be closer to the data since, as discussed later, not all unemployed workers switch sectors after an unemployment spell.[13] Hamilton (1988); Gouge and King (1997) and Alvarez and Shimer (2011) introduce a similar definition of unemployment in island models which not always involve switching to a different labor market.

From the point of view of a single worker, search over sectors is directed, i.e. once the $\{\epsilon_j\}$ are realized, they choose their most preferred sector. In this way, sectors are an inspection good in the search decision. However, from an *ex-ante* perspective, search can be interpreted as semi-directed, as $\{\epsilon_j\}$ introduce randomness across workers, leading to different mobility patterns.

## 2.3 Aggregation and Equilibrium

I assume that each element in $\{\epsilon_W, \epsilon_S; \{\epsilon_j\}\}$ is distributed standardized Type I Extreme Value. This is a widely used and studied distribution in the literature of dynamic discrete

[12] In real life workers change sectors for many reasons which are not captured by purely economic differences. For example, a worker needs to move to a different city within her firm, which may trigger a sectoral reallocation for the spouse.

[13] In the data, a large fraction of unemployed individuals return to work in the same industry or occupation of last employment. Carrillo-Tudela and Visschers (2013) document that close to 50% of unemployed workers switch *occupation* upon reentering employment. I obtain similar numbers for industry switches. In addition, Fujita and Moscarini (2012) document that a large fraction of the unemployed, both temporarily *and* permanently laid-off, return to work with their previous employer.

choice models (Rust, 1987, 1994).[14] I define $V(i,\tau,\Upsilon) = E_\epsilon\left[U(i,\epsilon_W,\epsilon_S,\tau,\Upsilon\right]$, which is the expected utility of a worker in sector $i$ and persistent productivity $\tau$ which has not yet observed the realization of the shocks $\epsilon_W, \epsilon_S$.

Using the Law of Iterated Expectations, the *ex-ante* problem for the worker can be re-expressed as:

$$V(i,\tau,\Upsilon) = E_\epsilon \max \left\{ \begin{array}{l} \log(w(i,\Upsilon)\,\tau) + \beta\, E_{\tau',\Upsilon'}\left[V(i,\tau',\Upsilon')|\tau,\Upsilon\right] + \sigma_\epsilon\, \epsilon_W, \\ \log(b\,\tau) + \beta\, E_\epsilon\left[\max_j \left\{E_{\tau',\Upsilon'}\left[V(j,\tau',\Upsilon')|\tau,\Upsilon,i,j\right] + \varsigma \epsilon_j\right\}\right] + \sigma_\epsilon\, \epsilon_S \end{array} \right\} \tag{10}$$

It is clear that given $V(i,\tau,\Upsilon)$, and the realization for the $\epsilon$ shocks, we can recover the original problem for the individual worker.[15]

**Proposition 1.** *Assume $\epsilon_W, \epsilon_S, \{\epsilon_j\}$ are distributed iid Type I Extreme Value, then*

$$V(i,\tau,\Upsilon) = \sigma_\epsilon \,\log\left( e^{(\log(w(i,\Upsilon)\,\tau) + \beta\, E[V(i,\tau',\Upsilon')|\tau,\Upsilon])/\sigma_\epsilon} + e^{\left[\log(b\tau) + \beta\,\varsigma\, \log\left(\sum_j e^{E[V(j,\tau',\Upsilon')|\tau,\Upsilon,i,j]/\varsigma}\right)\right]/\sigma_\epsilon} \right) \tag{11}$$

*In addition, let $N(i,\tau,\Upsilon)$ be the proportion of workers in sector $i$ and skills $\tau$ that decide to work, and $M(i,j,\tau,\Upsilon)$ the proportion of workers of skills $\tau$ that choose to search and move from sector $i$ to sector $j$, then*

$$N(i,\tau,\Upsilon) \;=\; \frac{e^{(\log(w(i,\Upsilon)\,\tau) + \beta\, E[V(i,\tau',\Upsilon')|\tau,\Upsilon])/\sigma_\epsilon}}{e^{V(i,\tau,\Upsilon)/\sigma_\epsilon}} \tag{12}$$

$$M(i,j,\tau,\Upsilon) \;=\; \frac{e^{\left[\log(b) + \beta\,\varsigma\, \log\left(\sum_k e^{E[V(k,\tau',\Upsilon')|\tau,\Upsilon,i,k]/\varsigma}\right)\right]/\sigma_\epsilon}}{e^{V(i,\tau,\Upsilon)/\sigma_\epsilon}} \frac{e^{E[V(j,\tau',\Upsilon')|\tau,\Upsilon,i,j]/\varsigma}}{\sum_k e^{E[V(k,\tau',\Upsilon')|\tau,\Upsilon,i,k]/\varsigma}} \tag{13}$$

*Finally, assume that $0 < \sigma_\epsilon < 1$; the total supply of efficiency units of labor in sector $i$ is*

$$L^s(i,\Upsilon) = \sum_\tau \tau\, x(i,\tau)\, N(i,\tau,\Upsilon)^{(1-\sigma_\epsilon)}\, \Gamma(1-\sigma_\epsilon)\, e^{-\sigma_\epsilon\, \gamma} \tag{14}$$

*where $\Gamma(.)$ is the Gamma function and $\gamma$ is Euler's constant.*

*Proof.* In Appendix A □

Proposition 1 is an aggregation result. All the heterogeneity induced by the shocks $\epsilon_W, \epsilon_S, \{\epsilon_j\}$, which affect the work and reallocation decisions, can be integrated out. $V(i,\tau,\Upsilon)$

[14] The main advantage of the Extreme Value distribution, which lead to its popularity, is the closed form solution for the expectation of the maximum of a set of random variables and for the probability of a particular choice being maximal.

[15] Given $V(i,\tau,\Upsilon)$ and the realization for the $\epsilon$ shocks, the problem of the individual worker can be recovered as:

$$U(i,\tau,\epsilon_W,\epsilon_S,\Upsilon) = \max \left\{ \begin{array}{l} \log(w(i,\Upsilon)\,\tau\, e^{\sigma_\epsilon\, \epsilon_W}) + \beta\, E_{\tau',\Upsilon'}\left[V(i,\tau',\Upsilon')|\tau,\Upsilon\right], \\ \log(b\,\tau\, e^{\sigma_\epsilon\, \epsilon_S}) + \beta\, E_\epsilon\left[\max_j \left\{E_{\tau',\Upsilon'}\left[V(j,\tau',\Upsilon')|\tau,\Upsilon,i,j\right] + \varsigma\epsilon_j\right\}\right] \end{array} \right\}.$$

has a structural interpretation as the utility of the "representative agent" of sector $i$ and productivity $\tau$. Contrary to other aggregation results in macroeconomics which lead to a representative agent for the whole economy (for example by using employment lotteries), here there are many representative agents, one per sector ($i$) and skill level ($\tau$). Moreover, as workers move out of some sectors and into others, the proportion of workers that each of these sectoral representative agents represents evolves endogenously in response to the changing economic conditions.

Note that $M(i, i, \tau, \Upsilon)$ is the proportion of unemployed workers in sector $i$ that are searching for a labor market to participate in the next period and find sector $i$ to be their best alternative. There is a close link between $M(i, i, \tau, \Upsilon)$ and the definition of rest unemployment or skill unemployment used by Alvarez and Shimer (2009, 2011).

**Definition.** *A* ***Recursive Competitive Equilibrium*** *in this economy is a set of functions for: prices $p(i, \Upsilon)$ and wages $w(i, \Upsilon)$, labor supply $L^s(i, \Upsilon)$ and demand $L^d(i, \Upsilon)$, production of goods $q(i, \Upsilon)$ , for all markets $i = \{1 \dots J\}$, and demand for intermediate inputs $\varphi_{ij}$ and demand for final consumption goods $c(i, j, \Upsilon)$ for all $i, j = \{1 \dots J\}$; policy functions on employment $N(i, \tau, \Upsilon)$ and mobility $M(i, j, \tau, \Upsilon)$, a value function $V(i, \tau, \Upsilon)$ for all $i$, $j$ and $\tau$ and an operator $G$ over the aggregate state $\Upsilon$ such that:*

- *Given prices and wages, $V(i, \tau, \Upsilon)$ is the maximum lifetime utility of the sector $i$ and skills $\tau$ representative agent (or ex-ante utility of the worker), optimal labor supply decisions in sector $i$ aggregate to $L^s(i, \Upsilon)$, optimal work decisions aggregate to $N(i, \tau, \Upsilon)$, which is the fraction of employed workers in sector $i$ of type $\tau$, and optimal unemployment and mobility decisions aggregate to $M(i, j, \tau, \Upsilon)$, which is the fraction of unemployed movers from sector $i$ to $j$ with type $\tau$. The optimal demands for good $j$ from workers in sector $i$ aggregate to $c(i, j, \Upsilon)$.*

- *Given prices and wages, firms maximize profits. $L^d(i, \Upsilon)$ and $\varphi_{ij}$ are the optimal (cost minimizing) demand for labor and intermediate goods conditional on production level $q(i, \Upsilon)$.*

- *Firms make zero profits in all markets at all times.*

- *All Labor and goods markets clear.*

- *$\Upsilon' = G(\Upsilon)$ and $G$ is consistent with the employment and mobility decisions.*

It is important to note that with a finite number of sectors, both $z$ *and* $\theta$ will have aggregate effects in the economy. The equilibrium is not stationary and the distribution of workers over sectors is not time invariant. These shocks affect the incentives to move as workers trade-off current wages in their sector versus the costs and rewards to moving.

It is convenient to define here the solution to a version of the model with no aggregate uncertainty. A stationary equilibrium in this model requires that $z$ and $\theta$ are equal to their expected value of zero at all times. In this case, the distribution of households over sectors is time-invariant and wages are constant. Nonetheless, workers transit along a stationary distribution by effect of their idiosyncratic productivity and preference shocks, generating gross flows. Gross job creation and destruction will be positive but exactly offset each other at all times in this stationary equilibrium, leading to an invariant level of unemployment. Similarly in a stationary economy, gross mobility across sectors is positive but, on net, will cancel.[16] The characteristics of the stationary version in this model are different from that of Lucas and Prescott (1974) model. This is a direct consequence of the finite number of islands.

### 2.4 Discussion

Given that some assumptions are non-standard, a few comments are in order.

In the model, workers are heterogeneous in four dimensions. The first is the sector they start the period in. This is the standard dimension of heterogeneity in islands models. Second is the persistent component to individual productivity, $\tau$. Third is the idiosyncratic and purely transitory components to market and non-market productivities. And last, for unemployed workers, the idiosyncratic preferences for working in different sectors.

The shocks $\epsilon_W$, $\epsilon_S$ and $\{\epsilon_j\}$ are introduced for computational reasons since, as will become clear in the next section, the model cannot be solved without them. However, their structural interpretation is immediate. $\tau$ allows me to model the costs of switching sectors as affecting skills and earnings in a way that is consistent with empirical evidence.[17] Although these sources of variability provide great flexibility to the model, there is a long tradition in macro and labor economics modeling earnings dynamics with persistent and transitory productivity shocks.[18] I follow this literature and use information on wage dynamics to discipline the parameters that characterize these shocks.

The decision to work or search is binary. In principle we do not need two shocks ($\epsilon_W$, $\epsilon_S$) to influence a single decision as only their difference is relevant for the problem of the worker. However, my empirical strategy uses information on earnings to infer the properties of these shocks. Therefore, in the model these shocks must influence the efficiency units of labor that workers supply. In turn, this will affect the total supply in the market. Keeping both shocks in the way I assumed here is necessary to obtain the closed-form expression for the market labor supply in Proposition 1.

[16] Coen-Pirani (2010) is also able to generate gross flows that are different from net flows in an island model by introducing a binary taste shock.

[17] An alternative specification of moving costs used in the literature is a fixed utility costs. These type of costs would be unobserved and harder to identify separately from $\varsigma$ (the volatility of the preference shocks).

[18] See Meghir and Pistaferri (2011) for a detailed survey on this topic.

I assumed logarithmic flow utility. In this way log-wages and log-individual productivities become additively separable. This is convenient for two reasons. First, it eliminates the incentives for workers to sort over sectors. Katz and Summers (1989) find little evidence of worker's sorting over industries on unobserved characteristics: "Given the absence of a high degree of industrial sorting on the basis of observed labor quality proxies, [education and experience] a high degree of sorting on unobserved characteristics would be surprising" (p. 229). Second, $\epsilon_W$, $\epsilon_S$ enter additively, which is necessary to use the properties of the Extreme Value distribution and obtain the closed-form expressions in Proposition 1.

In the model, productivity $\tau$ affects not only market returns (earnings), but also the returns to home production. The reason is that in this model the difference between the returns to work $U_W$ and not work $U_S$ is the key determinant of the probability of moving in and out of unemployment. When $\tau$ affects home production, as assumed here, this difference does not vary greatly with $\tau$ (will do so if there are non-linear effects of skill losses for switchers), and therefore the probability of becoming unemployed for workers with different values of $\tau$ is not very different. On the contrary, assuming that $\tau$ does not affect home production will lead (under suitable calibration) to a strong selection of workers into employment, with an almost zero probability of unemployment for workers with high $\tau$.[19] Postel-Vinay and Robin (2002) make this same assumption for similar reasons.

In the way I model mobility frictions I am assuming no recall, in the sense that workers cannot recover any skills they lost by switching if they decide to move back to their previous sector. Moreover, switching back to a previously visited sector give no advantage to the worker over those that have never worked in that sector before. In addition, I am assuming ex-post heterogeneity in skills, as workers do not know beforehand their productivity level in other sectors before they work there. In other words, the model is very different from a Roy (1951) model, where workers are *ex-ante* heterogeneous.

## 3 Numerical Solution

The main difficulty in introducing aggregate shocks in island models is technical. There is no difficulty in writing down the model, determining what the state variables are, and defining equilibrium. Yet, because the whole distribution of workers over islands is part of the decision problem of the households, solving the model for a non-trivial number of islands becomes a hard task. In this section I discuss the algorithm and techniques that allow me to solve the model for a large number of islands. As will become evident, the shocks $\{\epsilon_W, \epsilon_S, \{\epsilon_j\}\}$ that affect the two discrete decisions are key elements in the computational solution.

The model does not have a closed form solution and in order to solve it I need to

[19]An alternative specification would be to let $\sigma_\epsilon$ increase with $\tau$, thus increasing the probability of unemployment. This specification would, however, introduce additional parameters that need to be calibrated or estimated.

rely on numerical techniques. The solution to the recursive competitive equilibrium is a set of *functions* $\{x'(i,\tau,\Upsilon), V(i,\tau,\Upsilon), w(i,\Upsilon), p(i,\Upsilon), N(i,\tau,\Upsilon), M(i,j,\tau,\Upsilon), L(i,\Upsilon)\}_{\forall i,j,\tau}$ that solves the following system for any value of $\Upsilon = \{z, \{\theta_i\}, \{x(i,\tau)\}\}$,[20]

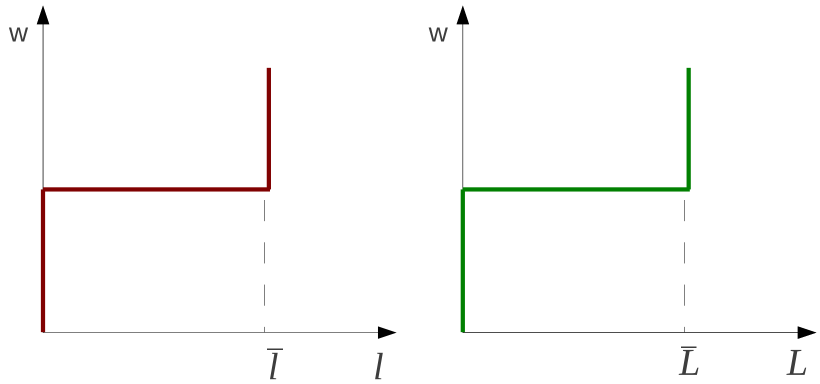
$$
\begin{aligned}
0 &= V(i,\tau,\Upsilon) - \sigma_\epsilon \log\left( e^{(\log(w(i,\Upsilon)\,\tau) + \beta\, E\left[V(i,\tau',\Upsilon')|\tau,\Upsilon\right])/\sigma_\epsilon} + e^{\left[\log(b\tau) + \beta\,\varsigma\,\log\left(\sum_j e^{E\left[V(j,\tau',\Upsilon')|\tau,\Upsilon,i,j\right]/\varsigma}\right)\right]/\sigma_\epsilon}\right) \\
0 &= y(\Upsilon)\,\psi_i^\chi\,\kappa^{\chi-1} p(i,\Upsilon)^{-\chi} - \frac{w(i,\Upsilon)\,L(i,\Upsilon)}{\alpha_i\,p(i,\Upsilon)} + \sum_{j=1}^{J}(1-\alpha_j)\eta_{ji}\,\frac{w(j,\Upsilon)}{p(i,\Upsilon)}\frac{L(i,\Upsilon)}{\alpha_j} \\
0 &= x'(i,\tau,\Upsilon) - \sum_{\hat\tau}\pi(\tau|\hat\tau)\,x(i,\hat\tau)\left(N(i,\hat\tau,\Upsilon) + M(i,i,\hat\tau,\Upsilon)\right) - \sum_{j\neq i}\sum_{\hat\tau}\tilde\pi(\tau|\hat\tau)\,x(j,\hat\tau)\,M(j,i,\hat\tau,\Upsilon) \\
0 &= 1 - \kappa^{-1}\left(\sum_i \psi_i^\chi p(i,\Upsilon)^{1-\chi}\right)^{1/(1-\chi)} \\
0 &= \log(p(i,\Upsilon)) + \log\left(e^{\lambda_i z + \theta_i}\alpha_i^{\alpha_i}(1-\alpha_i)^{(1-\alpha_i)}\prod_{j=1}^{J}\eta_{ij}^{(1-\alpha_i)\eta_{ij}}\right) - \alpha_i\log(w_i) - (1-\alpha_i)\sum_{j=1}^{J}\eta_{ij}\log(p(j,\Upsilon)) \\
0 &= z' - \rho_z\,z - (1-\rho_z^2)^{0.5}\,\xi_z \qquad (15) \\
0 &= \theta_i' - \rho_i\,\theta_i - (1-\rho_i^2)^{0.5}\,\sigma_i\,\xi_i
\end{aligned}
$$

where $\pi(\tau|\hat\tau)$ and $\tilde\pi(\tau|\hat\tau)$ are the conditional transition probability between $\hat\tau$ and $\tau$ for the non-movers and movers respectively. $N(i,\tau,\Upsilon)$, $M(i,j,\tau,\Upsilon)$ and $L(i,\Upsilon) = L^s(i,\Upsilon)$ are defined in (12), (13) and (14) and they should be substituted by their expressions.

The first equation in system (15) follows from the definition of the value function of the representative agent of sector $i$ and productivity $\tau$. The second equation uses market clearing in the goods market, where the optimal demands for labor and intermediates have been substituted in. The third equation is the law of motion for the measure of households and highlights the fact that the evolution of $\tau$ for workers that switch sectors is different than for non-switchers. The fourth equation is the definition of the aggregate demand, where the final demand for each good has been substituted. The remaining equations are the equilibrium relation between prices and wages and the law of motion of the aggregate and sectoral shocks.

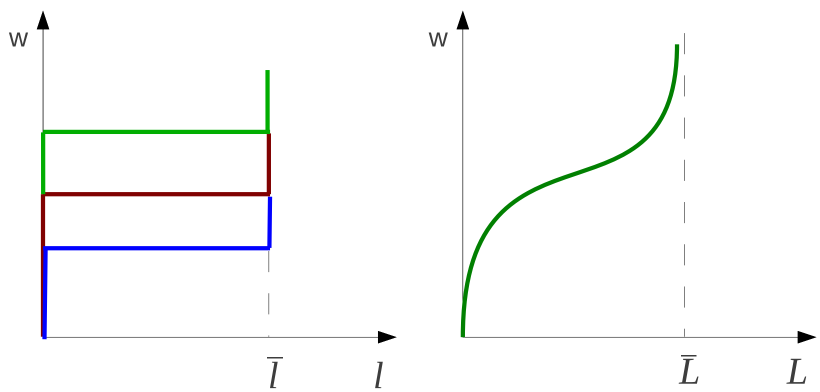
The high dimensionality of the state space and the fact that $\Upsilon$ contains all continuous variables makes problem (15) difficult to solve using standard numerical methods. Even for a small number of sectors the curse of dimensionality has a strong bite. Here I overcome this difficulty using perturbation methods. These methods are used extensively in economics,

[20] Appendix B contains a detailed description of all the equilibrium conditions in the model. The system (15) is simply a re-arrangement that yields a more compact representation of the problem.




(a) individual (b) aggregate

Figure 2: Labor supply with homogeneous workers in the sector




(a) individual (b) aggregate

Figure 3: Labor supply with heterogeneous workers in the sector

mainly in models with a representative agent, and can easily accommodate a very large number of state variables.[21] In the model presented here there are several representative agents, one per sector ($i$) and skill level ($\tau$), and their relative importance (measure) changes in time as the distribution of workers changes. The idea of using perturbation in problems with a distribution of agents and aggregate shocks goes back to Campbell (1998) where he used it in analyzing entry and exit of firms over the business cycle. More recently Reiter (2009) used perturbation in a Bewley-Hugget-Aiyagari model with aggregate shocks. In addition, I will be perturbing the Value Function which is the key element in the extensive margin decision on whether to work and where. Perturbation of the Value function is discussed in Judd (1998) but has seldom been applied in economic problems. A recent exception is Caldara et al. (2011) where they use it to solve a problem with recursive preferences. Kline (2008) uses perturbation methods to solve a model with 3 sectors but no unemployment.

[21] For a complete exposition on perturbation theory in economics see Judd (1998).

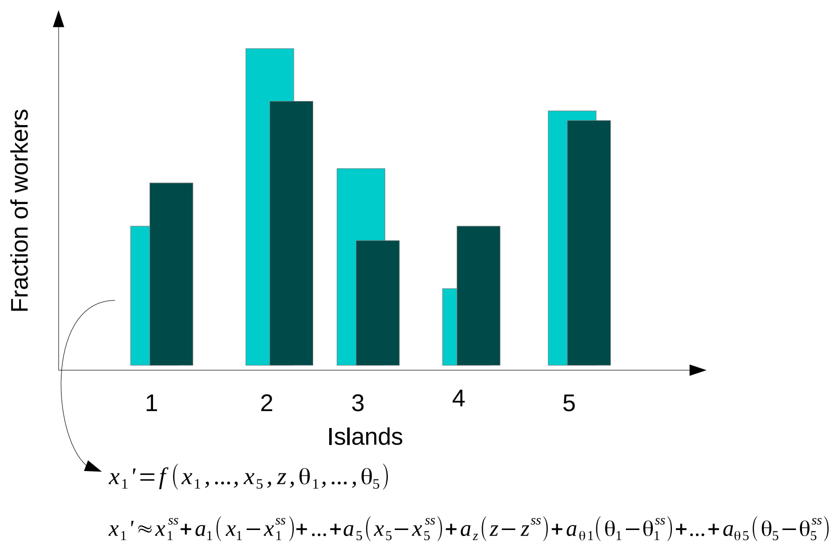



Figure 4: Perturbation of the distribution

In order to use perturbation methods all the functions that characterize the equilibrium must be differentiable, as the technique relies on Taylor's and the Implicit Function theorems. The key role of shocks $\{\epsilon_W, \epsilon_S, \{\epsilon_j\}\}$ is to smooth the discrete problem. Note that in the original Lucas and Prescott (1974) model, all households in the sector have identical preferences with perfect substitution in the work decision. In this case, both at the level of the household and the sector, the solution does not deliver smooth differentiable functions.[22] Figure 2 shows visually the labor supply for the homogeneous case, in which workers have the same reservation wage (panel a), which translates to a non-smooth labor supply in the aggregate (panel b). In the model I develop here, a single worker also has preferences of perfect substitutes on the work decision and over the different sectors, but the introduction of the idiosyncratic shocks $\{\epsilon_W, \epsilon_S, \{\epsilon_j\}\}$ generates smooth functions for the representative agent. This can easily be seen in Figure 3, where in panel (a) I plot the labor supply of three possible individual workers, each with a different reservation wage as a result of different idiosyncratic shocks. For the sector as a whole, the aggregate labor supply is smooth (panel b) as there is a continuum of workers, each with a different reservation wage. While in principle this aggregation can be achieved with any distributional assumption (provided the distribution is continuous and with adequate support) the use of the Extreme Value distribution substantially simplifies the problem by delivering almost closed-form expressions.

Another key element in the solution is to characterize the law of motion for the aggregate state $\Upsilon$. For the shocks $z$ and $\theta$ is trivial. However, for the evolution of the distribution of

[22]The reader can see the shape of the solution in the traditional island model in Alvarez and Veracierto (2000), where it is clear that the Value Function and the labor supply present sharp kinks and areas where the derivative is zero, violating the conditions for the Taylor and Implicit Function Theorems. Note also that the use of employment lotteries do not help overcome the problem. In fact, Alvarez and Veracierto (2000) assume complete markets with employment lotteries.

workers over sectors and skill types, is more difficult. Given that this distribution has a discrete support, the solution will be a function that defines how the mass of workers at each sector-skill combination evolves with changes in the aggregate state.

To build some intuitions on how perturbation techniques work with a distribution, in Figure 4 I plot a possible distribution of workers over sectors. As the number of sectors and types $\tau$ are discrete, the distribution is discrete and can be represented by a histogram where each bar denotes the fraction (measure) of workers on each sector-$\tau$ pair. In this example, the light-colored bars represent the initial distribution and the dark-colored bars represent the distribution after a shock hits. Perturbation methods approximate, by a Taylor expansion, the functions that describes how each of these bars move as the aggregate state moves. In this way, an unknown functional expression is approximated by a polynomial on the variables in $\Upsilon$. The coefficients of this polynomial are functions of the deep parameters of the model and can be found using methods that are now standard in economics.[23] Further technical details on the numerical solution are described in Appendix C.

## 4 Sectoral comovement: empirical evidence

In this section I summarize key moments of the labor market that I will link to the structural model presented in the previous sections.

The choice of how many sectors to study demands a fine balance between data availability, more detailed and disaggregated analysis and computational burden. Labor market data for narrowly defined sectors is harder to come by at the monthly or quarterly frequencies. While it is possible to construct it using microdata, small sectors will contain few observations for each month or quarter, and thus the exercise will be more influenced by measurement error. Therefore, in the present work I restrict the analysis to the 12 main sectors of the US economy. In this work I follow the NAICS classification and analyze the following sectors: construction; manufacture of durables; manufacture of non-durables; wholesale trade; retail trade; transport and warehousing; information and technology; finance, insurance and real state; professional and business services; leisure and hospitality; education and health-care; and other services.[24]

While this level of aggregation may seem too coarse, it is worth noting that, in the data (CPS monthly-matched-records from 1979 to 2011), out of all workers that switch more narrowly defined industries (4 digit NAICS), 80% of the time also switch across these broadly defined groups. In other words, industry switches are almost always across large industry groups.

[23] For example, for a first order approximation one can use Blanchard and Kahn (1980) or Sims (2002) and for a second order approximation Gomme and Klein (2011).

[24] These sectors account for 98% of private employment. Mining and agriculture comprise around 0.5% and 1.5% of employment respectively, and, due to their small size, including them as separate sectors can potentially affect the model's performance.

### 4.1 Multifactor Productivity

I use data on multifactor productivity computed by the Bureau of Labor Statistics (BLS).[25] The concept of multifactor productivity closely matches the definition of sectoral TFP in the model.

Multifactor productivity is computed as the Solow residual from an assumed production function that takes into account sectoral output, factors of production and intermediate inputs. The data on multifactor productivity is annual and the sample covers the years 1987 to 2011. Given that the sample is too short for estimation, I use the proportional Denton method to obtain an interpolated quarterly series for Sectoral TFP. For this I use sectoral wages as an indicator for intra annual movements.[26]

Table 1 shows cyclical properties of sectoral TFP series at the annual and quarterly frequency. While there is a strong comovement across sectors in terms of TFP, there are some industries like manufacturing of non-durables, education and health, and other services, that display a small amount of cyclical volatility and correlation with GDP. On the other hand, TFP in construction and durable manufacturing display large cyclical volatility and correlation. These sectoral differences provide information on the relative importance of a common, economy-wide component for the different sectors. In the next section I use this information on sectoral TFP to estimate a dynamic factor model and quantify the importance of each type of shock.

### 4.2 Labor market moments

Labor market variables also display a high degree of sectoral comovement. I document in Table 2 the cyclical properties of employment and unemployment for the 12 NAICS sectors I study. Sectoral unemployment at time $t$ for sector $i$ is defined as the number of workers that are unemployed at time $t$ whose last job was in sector $i$.

I use microdata from the monthly Current Population Survey (CPS) from 1979 to 2007. The CPS does not have an homogeneous industry classification over time. The classification evolves to better take into account an increasing number of new goods and services. Clear

[25] The data is available at http://www.bls.gov/mfp/. I use the detailed tables for manufacturing and non-manufacturing industries that follow the KLEMS methodology. These tables follow the NAICS classification. Three sectors, Professional and Business Services, Leisure and Hospitality, and Education and Health, are disaggregated into sub-sectors. In these cases I compute the sectoral TFP as the weighted average of these sub-sectors' TFP, using 2002 production as weights.

[26] The objective of the proportional Denton method is to keep the ratio of the estimated quarterly series to an indicator series as constant as possible with the restriction of matching the true annual values. This method has been shown to outperform other alternative interpolation techniques and is used by many statistical agencies. See Bloem et al. (2001) and Chen (2007). I use data on wages from the BLS-CES, which is monthly and I aggregate to the quarterly frequency. Wages are average weekly earnings for production and non-supervisory employees for the NAICS sectors I use in this work and are seasonally adjusted. In addition I HP filter wages with a smoothing parameter of 1600 and filter annual productivity with a smoothing parameter of 6.25, which is the value recommended by Ravn and Uhlig (2002).

Table 1: Moments of Multifactor Productivity

| Sector | Annual Data | | Quarterly Data | |
|---|---|---|---|---|
| | Std Dev | Corr with GDP | Std Dev | Corr with GDP |
| Construction | 0.029 | 0.80 | 0.029 | 0.60 |
| Manuf Non Durables | 0.011 | 0.21 | 0.010 | 0.33 |
| Manuf Durables | 0.014 | 0.80 | 0.013 | 0.73 |
| Wholesale | 0.037 | 0.77 | 0.035 | 0.55 |
| Retail | 0.023 | 0.78 | 0.022 | 0.68 |
| Transport | 0.020 | 0.84 | 0.019 | 0.63 |
| Information | 0.030 | 0.32 | 0.030 | 0.06 |
| Finance & Real Estate | 0.020 | 0.77 | 0.020 | 0.60 |
| Prof & Business Services | 0.020 | 0.86 | 0.020 | 0.67 |
| Leisure & Hospitality | 0.012 | 0.58 | 0.012 | 0.31 |
| Education & Health | 0.011 | 0.39 | 0.010 | 0.33 |
| Other Services | 0.017 | 0.30 | 0.016 | -0.02 |
| GDP | 0.011 | - | 0.012 | - |

Moments are computed for the deviations of the logarithm of the series relative to an HP trend. Quarterly series for Multifactor Productivity is constructed using Denton method, with sectoral average weekly earnings as the indicator variable. For the quarterly series the standard value of 1600 is used for the smoothing parameter in the HP filter, and for the annual series a smoothing parameter of 6.25 is used as recommended by Ravn and Uhlig (2002). The annual sample covers the years 1987-2011 (25 observations) and the quarterly sample covers 1987q4 to 2011q4 (97 observations).

examples are mobile communications and Internet services which became widely available in the late 1990s. In 2003 the classification underwent a major change, moving from the Standard Industrial Classification (SIC) to the North American Industrial Classification System (NAICS). Nonetheless, even within the SIC system there were changes in 1983 and 1992. Here I use the current NAICS definition and adjust years 1979 to 2002 in order to construct an homogeneous industry series. For this I use a cross-walk published by the Bureau of Economic Analysis. While for many industries there is an almost one-to-one mapping between SIC and NAICS, for others the link is not direct, leading to a break in the temporal evolution of the series in 2003. This break is noticeable in the levels of employment and unemployment, but the unemployment rate is almost unaffected.

To minimize the effect of the change in the classification, I proceed in the following way. First, because the sectoral unemployment rates are almost unaffected by the change in the classification, I use it as computed directly from the CPS microdata. Second, I compute a series of total employment aggregating CPS employment for the 12 NAICS industries. This series presents no break. Using data on employment by NAICS sector compiled by the Bureau of Labor Statistics (BLS), I compute time series for the employment shares of each sector.[27] I recover the level of employment using total employment from the CPS (seasonally

[27] The BLS constructs historic series of aggregate employment by NAICS sector using the Current Employ-

adjusted) and these shares. For the series of level of unemployment in each sector, I use the unemployment rate by sector from the CPS microdata (seasonally adjusted) and my measures of employment by sector. I adjust all CPS series using the X13-ARIMA-SEATS program from the Census Bureau.[28]

Table 2: Moments of Labor Market Variables by Sector

| | Employment | | Unemployment | |
|---|---|---|---|---|
| | Std Dev | Corr w/ Tot E | Std Dev | Corr w/ Tot U |
| Construction | 0.027 | 0.88 | 0.112 | 0.86 |
| Manuf Non Durables | 0.008 | 0.67 | 0.104 | 0.84 |
| Manuf Durables | 0.021 | 0.90 | 0.171 | 0.90 |
| Wholesale | 0.010 | 0.79 | 0.128 | 0.82 |
| Retail | 0.008 | 0.85 | 0.087 | 0.90 |
| Transport | 0.009 | 0.85 | 0.114 | 0.86 |
| Information | 0.017 | 0.58 | 0.136 | 0.79 |
| Finance & Real Estate | 0.010 | 0.27 | 0.118 | 0.78 |
| Prof & Business Services | 0.013 | 0.82 | 0.090 | 0.88 |
| Leisure & Hospitality | 0.007 | 0.70 | 0.077 | 0.86 |
| Education & Health | 0.006 | -0.20 | 0.082 | 0.78 |
| Other Services | 0.008 | 0.25 | 0.096 | 0.78 |
| | | | | |
| Total | 0.008 | | 0.092 | |
| GDP | 0.012 | 0.82 | 0.012 | -0.86 |

Moments are computed for the deviations of the logarithm of the quarterly series (employment and unemployment levels) relative to an HP trend with a smoothing parameter of 1600. Unemployment by sector is the number of workers that are unemployed whose last job was in the reported sector. The construction of sectoral employment and unemployment series is described in the text. The sample covers 1979q1 to 2007q4.

Table 2 shows that the cyclical ups and downs in sectoral employment and unemployment tend to be highly synchronized. This phenomenon is known as comovement, and is a defining characteristic of business cycles. The cyclical correlation in employment is higher in construction, manufacturing and trade and lower in services. In addition, Industries with higher correlation with total employment, tend to have a higher volatility of employment. Education and Health displays both a very low cyclical volatility of employment and a negative correlation with total employment. Similar patterns hold for unemployment, although comovement in unemployment is stronger and more homogeneous across industries.

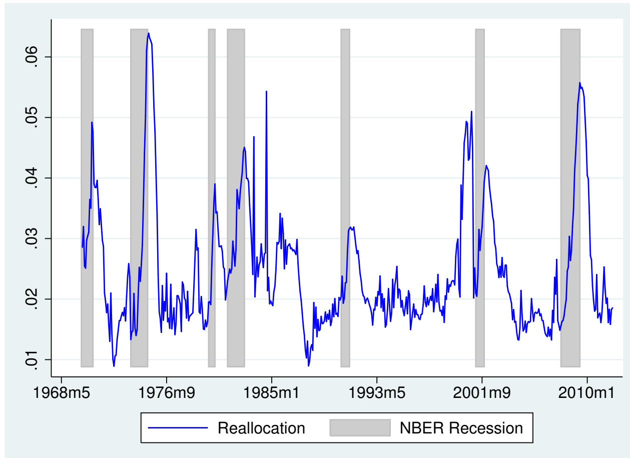
## 4.3 Labor reallocation

Lilien (1982) argued that recessions and unemployment are the result of sectoral shocks, as

ment Survey (CES). These are monthly series starting in the 1970's or earlier (depending on the sector) and are seasonally adjusted. Because of the nature of the CES, these series represent the number of jobs rather than employment.

[28] I use these measures of employment instead of the direct measures on jobs by the BLS-CES, because the series for total employment is less volatile at cyclical frequencies than the series of jobs. Nonetheless, the comovement patterns are similar.

Figure 5: Lilien's Sectoral Reallocation Index.




they increase the need for workers to reallocate from industries with a low labor demand to industries offering better conditions. To support his hypothesis he constructed the following measure of sectoral labor reallocation:

$$R_t = \left[ \sum_i \left( \frac{L_{i,t-1}}{L_{t-1}} \right) (\Delta \log L_{i,t} - \Delta \log L_t)^2 \right]^{1/2}$$

where $R_t$ is the reallocation measure, $L_{i,t}$ is total employment in sector $i$ at time $t$ and $L_t$ is total employment in the economy. What this measure captures are sectoral asymmetries in the process of employment changes. Figure 5 shows the evolution of this index for the US economy.

It is clear from the figure that recessions are times in which sectoral employment changes become more asymmetric.

One measure of reallocation is the net mobility rate. Kambourov and Manovskii (2009b) defined the net mobility rate as the sum of the absolute changes in the level of employment shares by sector. By construction this rate can never be negative. Note that Lilien's measure captures (squared) relative changes in employment shares. In the data these two measures are almost identical, with a correlation of 0.97. Therefore, Lilien's measure captures net mobility.

It is possible to construct alternative reallocation measures focusing only on transitions from unemployment to employment (UE), which are the transitions in the structural model. Using the CPS public use microdata, I compute gross industry mobility measures for all UE transitions. This measure quantifies how many of the workers that transition out of unemployment, do so with a sectoral change. The main advantage of the CPS is that it is the source of official labor market statistics, it has a large sample size at a monthly frequency,

Table 3: Proportion of switchers and non-switchers
UE transitions

| Survey | years | obs | Industry Change | No Industry Change |
|---|---|---|---|---|
| CPS - Monthly | 1979-1994 | 87,434 | 52.6% | 47.4% |
| | 1995-2011 | 82,174 | 51.7% | 48.3% |
| SIPP | 1996-2010 | 24,092 | 45.5% | 54.5% |

Proportion of UE transitions that end with an industry change. Computed using CPS monthly matched records (excludes months where matching is not possible and months where there is a change in the industrial definition) and SIPP.

and microdata is available since the late 1970's. Although not designed specifically as a panel, in the CPS individuals can be matched from month to month for a small number of periods. The limitation of this survey is that attrition and errors in classifying the labor market status can be a source of bias (Elsby et al., 2012).

Another limitation of the CPS, as discussed in Moscarini and Thomsson (2007), is that gross mobility measures are prone to large biases due to coding errors. In 1994 the CPS changed to a "dependent coding" system, substantially reducing the error.[29] Note that a dependent coding system greatly reduces the measurement error for workers that are continuously employed and performing the same task for the same employer. However, the mobility measure I will be focusing on in this work and that is consistent with the model is industry switch after a spell of unemployment, and dependent coding provides no help in reducing spurious transitions.[30] Because of this, as a robustness check I will compare the magnitude of these mobility rates with those from other surveys.

Last, I use the Survey of Income Program Participation (SIPP) which is a series of panels. This survey follows individuals for 3 to 4 years, depending on the panel, and individuals are interviewed every four months. Each interview is called a wave, and workers are asked about their employment status and characteristics of their job (including industry) for the preceding four months, among other things.

The reallocation measure that I compute is the fraction of UE transitions that involve an industry switch. This measure is similar to the one used in Carrillo-Tudela and Visschers (2013), for occupational switches, and in Fujita and Moscarini (2012), for worker recalls. Table 3 shows that nearly one half of unemployed workers go back to work in their previous industry upon reemployment. However, this proportion is somewhat larger in the SIPP.

Two important facts have been recognized in the the literature of occupational and industrial mobility: gross mobility is larger than net mobility, and gross mobility is pro-

[29]In a dependent coding system, the interviewer will ask the household if she still works for the same employer she reported last period. If that is the case, answers for industry and occupation will be taken from the previous month.

[30]In particular, the series of the proportion of workers that switch industries after an UE transition shows almost no different patterns before and after 1994.

Table 4: Cyclical moments of reallocation

| | Std Dev | Corr w/GDP |
|---|---|---|
| Lilien's Reallocation Measure | 0.31 | -0.27 |
| | | |
| UE switch / (UE stay + UE switch) | 0.028 | 0.50 |
| UE stay / labor force | 0.074 | -0.74 |
| UE switch / labor force | 0.058 | -0.45 |

Lilien's Reallocation Measure is described in the text and is constructed from measures of sectoral employment. UE switch is the number of workers that transition from unemployment to employment with an industry switch. Similarly, UE stay is the number of workers that transition from unemployment to employment and find work in the same industry as they were employed before. Computed using CPS monthly matched records. The sample covers 1979 to 2007.

cyclical, while net mobility is counter-cyclical.[31] Table 4 shows the cyclicality of the two mobility measures. Lilien's reallocation measure (net mobility) is counter-cyclical, and gross mobility for UE transitions, is pro-cyclical.

Reallocation is time and resource-consuming for workers as they experience losses in earnings and depreciation of their human capital. Understanding and quantifying these costs is the subject of a very large literature.[32] This literature has mainly focused on displaced workers while I include all the unemployed in my analysis.

I compute earning losses for industry switchers and non-switchers using the SIPP.[33] To do this, I first regress log real wage for all employed workers on observable characteristics (age, age squared, race, sex and education dummies, dummies for months, to capture seasonality, dummy for year to capture cyclical effects, and dummies for broad occupation and industry). For each worker I compute the residual wage from this regression. To compute my measure of wage losses for switchers, I run a regression of the change in log residual wage between the old job and the new on a dummy that indicates if the spell ended with an industry switch and the duration of the non-working spell. I interpret the coefficient on the switching dummy as the wage loss from switching sectors.

Table 5 shows that unemployed workers that become employed earn, on average, 3.5% less if they switch sector. This is after controlling for other factors, like the length of the unemployment spell and differences in average wage payed in different industries. This value is roughly of similar magnitude to those found by Fujita and Moscarini (2012) for per-

[31]See Murphy and Topel (1989); Moscarini and Thomsson (2007); Kambourov and Manovskii (2008) and Carrillo-Tudela and Visschers (2013).

[32]For example, Jacobson et al. (1993); Farber (2011) and Davis and von Wachter (2011) find evidence of substantial earning loses for displaced workers, which, Jacobson et al. (1993); Neal (1995) find to be larger for those workers that switch sectors.

[33]The monthly CPS is not suitable for computing wage changes, as wages are recorded only twice for the individuals in the sample and 12 months apart (outgoing rotations). The CPS Displaced Workers Supplement does not fit my purposes either since this survey includes only displaced workers and not all unemployment.

Table 5: Regression on residual wage changes for UE transitions

| | coeff | std error |
|---|---|---|
| Constant | -.003 | 0.004 |
| Duration of spell (months) | -.007 | 0.002 |
| Industry switch dummy | -.035 | 0.012 |

N. obs = 8568. The dependent variable is the change in log residual wage (current wage vs wage of last employment) for workers that transit from unemployment to employment. Includes a control (dummy) for seam effects. Sample includes only individuals that do not miss any wave in the SIPP. To minimize bias due to right censoring, I only use individuals with EU transitions that start in the first 3 or 6 waves (depending on the SIPP pannel). SIPP panels 1996-2008.

Table 6: Unemployment duration
UE transitions

| Survey | years | obs | Industry Switch | | | | No Industry Switch | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Avg weeks | S.D. | Median weeks | 90th pctile. weeks | Avg weeks | S.D. | Median weeks | 90th pctile. weeks |
| CPS - Monthly | 1979-2011 | 188,703 | 12.8 | 16.7 | 6 | 34 | 10.8 | 14.7 | 5 | 26 |

| Survey | years | obs | Industry Switch | | | | No Industry Switch | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Avg months | S.D. | Median months | 90th pctile. months | Avg months | S.D. | Median months | 90th pctile. months |
| SIPP | 1996-2010 | 25,136 | 3.5 | 3.0 | 3 | 7 | 2.4 | 2.2 | 2 | 5 |

Unemployment duration for workers that transit from unemployment to employment. Industry switch indicates workers that transition from unemployment to employment and find work in an industry different from that of last employment. Top row computed using CPS monthly matched records.

manently separated workers which are not recalled by their firm (see their Table 7). While these loses are lower than those typically found in the literature on displaced workers, it is worth stressing that I am including all types of unemployed workers.

Workers who switch industries spend, on average, nearly one month longer in unemployment as can be seen in Table 6. This magnitude is relatively similar across surveys and is also consistent with the findings of Fujita and Moscarini (2012). A possible interpretation is that because switching industries is costly for workers, they delay switching.

## 5 Empirical Evaluation of the Model

In this section I evaluate how well the structural model is able to match the patterns of comovement and reallocation observed in the data. For this, I first discuss the calibration and estimation of the model parameters.

## 5.1 Parametrization

In the model, the shocks $\epsilon_W$ and $\epsilon_S$ are key forces shifting workers between employment and unemployment. As these shocks are purely transitory, I assume the time period in the model to be half a quarter. This specification allows the model to match better the observed values for the transition probabilities and unemployment duration.

### 5.1.1 Calibration:

I calibrate the discount factor and the CES elasticity of substitution to values usually assumed in the literature. I assume a discount factor of a 5% implied annual interest rate. Also, I set the elasticity of substitution over sector goods ($\chi$) to 2.2, which is the median value estimated by Broda and Weinstein (2006) and is consistent with the estimates by Alvarez and Shimer (2011).[34] In addition, all coefficients $\eta_{ij}$ and $\alpha_i$ in the production function are calibrated to the 2002 Input-Output use table from the Bureau of Economic Analysis. The rest of the parameters will be estimated structurally.

### 5.1.2 Estimation of Sectoral TFP processes:

My assumptions on sectoral TFP and the dynamic process for $z$ and $\theta_i$ follow a large literature on dynamic factor models (Stock and Watson, 2011). In Appendix D I show that the parameters $\lambda_i, \rho_i, \rho_z, \sigma_i$ on the sensitivity of each sector to the common component, the autoregressive parameters and the standard deviation of the sectoral shocks, are identified from observations on sectoral TFP, provided there are three or more sectors.[35]

I use data on multifactor productivity computed by the BLS described in the previous section for the 12 NAICS sectors. I estimate $\lambda_i, \rho_i, \rho_z, \sigma_i$ for $i = 1, ...12$ by maximum likelihood using the Kalman filter. The sample covers 1987q4 to 2011q4. Table 7 contains the results of the estimation.[36]

[34] Broda and Weinstein (2006) estimate this elasticity for a larger number of varieties. It is expected that the fewer options, the harder it is to substitute among them and the elasticity would be lower.

[35] This last point is worth highlighting. Under the assumptions on TFP I make here, in a model with only two sectors (which is widely used in the literature) it is not possible to identify parameters $\lambda$ and gauge the importance of sectoral and aggregate shocks in the economy. Moreover, using a larger number of sectors in the estimation increases the precision on the estimation of the common component, as each sector carries information on the common factor. Two normalizations are needed. First, the variance of $z$ cannot be identified separately from the average level of $\lambda$. For this I normalize the variance of $z$ to one as is usual in the literature. Second, the sign of z cannot be identified separately from the signs of $\lambda$. I do not impose any restrictions for this, and correct ex-post if the estimation for most $\lambda$ is of a counterintuitive sign.

[36] Using the sample 1987q4 to 2007q4 does not affect notably the point estimates, but standard errors are larger.

Table 7: Maximum Likelihood Estimates

| | $\lambda_i$ | $\sigma_i$ | $\rho_i$ |
|---|---|---|---|
| Construction | 0.016 | 0.019 | 0.952 |
| | (0.006) | (0.006) | (0.029) |
| Manuf Non Durables | 0.003 | 0.009 | 0.931 |
| | (0.001) | (0.002) | (0.033) |
| Manuf Durables | 0.013 | 0.008 | 0.896 |
| | (0.004) | (0.002) | (0.043) |
| Wholesale | 0.038 | 0.020 | 0.924 |
| | (0.012) | (0.005) | (0.041) |
| Retail | 0.020 | 0.016 | 0.940 |
| | (0.007) | (0.005) | (0.037) |
| Transport | 0.019 | 0.007 | 0.923 |
| | (0.006) | (0.002) | (0.043) |
| Information | 0.009 | 0.027 | 0.949 |
| | (0.004) | (0.007) | (0.028) |
| Finance & Real Estate | 0.013 | 0.014 | 0.937 |
| | (0.004) | (0.004) | (0.036) |
| Prof & Business Services | 0.014 | 0.011 | 0.963 |
| | (0.005) | (0.004) | (0.024) |
| Leisure & Hospitality | 0.009 | 0.008 | 0.932 |
| | (0.003) | (0.002) | (0.034) |
| Education & Health | 0.006 | 0.013 | 0.956 |
| | (0.002) | (0.005) | (0.037) |
| Other Services | 0.010 | 0.014 | 0.905 |
| | (0.004) | (0.003) | (0.040) |
| | | | $\rho_z$ |
| Aggregate | | | 0.949 |
| | | | (0.032) |

Standard Errors in parenthesis. Estimated using quarterly Multifactor Productivity HP filtered with smoothing parameter 1600. Sample 1987q4-2011q4.

### 5.1.3 Estimation of micro parameters:

Parameters related to the individual preferences, shocks and reallocation costs are estimated by Indirect Inference in order to minimize the distance between a set of targeted moments in the model and their data counterpart.

The parameter $\psi_i$ for each sector is the weight that good from sector $i$ has in the CES preferences of workers. $\psi_i$ is estimated with the goal of matching the average employment shares of each sector. In the data, employment shares for different industries have non-stable long-run trends, and the model cannot capture this. Therefore I match the average employment shares from 2000 to 2007.

In the model workers face transitory and persistent productivity shocks. This brings the model close to a large literature on wage dynamics.[37] I follow the approach of this literature to infer parameters of the processes for $\tau$, $\epsilon_W$ and $\epsilon_S$.[38]

As is usual in the literature of heterogeneous agents, the persistent productivity $\tau$ will be a discrete approximation to an AR(1) process. To compute the autocorrelation and standard deviation for the proposed AR(1) process for $\tau$ ($\rho_\tau, \sigma_\tau$) and the scale parameter on $\epsilon_W$ and $\epsilon_S$ ($\sigma_\epsilon$), I estimate the following auxiliary model both in the observed and simulated data:

$$\begin{aligned} \log(\tilde{w}_t) &= \zeta_t + \vartheta_t \\ \zeta_t &= \rho_\zeta \zeta_{t-1} + \varepsilon_t \\ \vartheta \sim iid;\ E[\vartheta] = 0, Var[\vartheta] = \sigma_\vartheta^2; &\qquad \varepsilon \sim iid;\ E[\varepsilon] = 0, Var[\varepsilon] = \sigma_\varepsilon^2 \end{aligned}$$

where $\tilde{w}_t$ is the residual log real hourly-wage from a standard Mincer regression.[39] The following moments identify the parameters of the earnings dynamics process

$$\begin{aligned} E[\tilde{w}_t^2] &= \frac{\sigma_\zeta^2}{1-\rho_\zeta^2} + \sigma_\vartheta \\ E[\tilde{w}_t\, \tilde{w}_{t-1}] &= \rho_\zeta \frac{\sigma_\varepsilon^2}{1-\rho_\zeta^2} \\ E[\tilde{w}_t\, \tilde{w}_{t-2}] &= \rho_\zeta^2 \frac{\sigma_\varepsilon^2}{1-\rho_\zeta^2} \end{aligned}$$

The expectation is taken over individuals. In sum, parameters $\rho_\tau, \sigma_\tau$ and $\sigma_\epsilon$ are estimated to make these simulated moments (and the corresponding $\rho_\zeta$, $\sigma_\varepsilon$ and $\sigma_\vartheta$) close to their data counterpart. To estimate this earnings dynamics model I need panel data, and for this I use

[37] The recent work by Low et al. (2010) analyzes the effects of these type of earnings shocks on unemployment and the extensive margin of labor supply.

[38] Note that, for technical reasons, there is a single parameter ($\sigma_\epsilon$) affecting the scale of $\epsilon_W$ and $\epsilon_S$, which is convenient as I can use only observed wages to infer that parameter.

[39] I trim the top and bottom 1% of the distribution.

the SIPP. Given that "seam effects" are pervasive phenomenon in longitudinal surveys like the SIPP and may be a source of bias, I use wage differences across SIPP waves in computing the parameters and adjust the parameters accordingly.[40] In addition I further restrict to non-industry movers that are continuously employed, as this is consistent with my model. For estimation, I use the same selection criteria in model simulated data.[41]

I use Rouwenhorst (1995) method to discretize the AR(1) process for $\tau$.[42] This method defines the transition matrix $\pi(\tau'|\tau)$. I use a 9 point Markov chain for the productivity $\tau$. Since there are 12 sectors, this amount to 121 state variables in $\Upsilon$ (108 in the distribution, 12 sector shocks and the aggregate shock). This implies that, for an approximation to the policy functions of the model using a second order perturbation, each function in the solution contains 121 linear terms, 7381 quadratic (cross) terms and a constant.[43] As there are 241 endogenous variables in the model (exclusive of sectoral and aggregate shocks), the total number of coefficients in all the functions that characterize the solution is over 1.8 million.[44]

In the model, unemployed workers that switch sectors may suffer a depreciation of their skills. This is captured by the transition matrix $\tilde{\pi}(\tau'|\tau)$. For simplicity, I assume that a worker that switches sectors moves one step down in the (discrete) distribution of skills $\tau$ with probability $\delta$ and keep their skill level with probability $(1-\delta)$. The timing is important. Here I assume that a worker faces this $\delta$ shock once she reaches the new sector and after observing her new $\tau$. That is, with probability $(1-\delta)$ she will keep this new $\tau$, and with probability $\delta$, she will move one step down in the skill ladder. By modeling costs in this way, only one parameter, $\delta$, needs to be estimated.

Under these assumptions, the transition probability matrix for $\tau$ for those workers that switch sectors, $\tilde{\pi}$, is:

$$\tilde{\pi}_{i,j} = \begin{cases} \pi_{i,j} + \delta\,\pi_{i,j+1} & \text{if } j == 1 \\ (1-\delta)\,\pi_{i,j} + \delta\,\pi_{i,j+1} & \text{if } j > 1 \text{ and } j < 8 \\ (1-\delta)\,\pi_{i,j} & \text{if } j == 9 \end{cases}$$

Parameters $b$, $\varsigma$ and $\delta$ affect moments of unemployment, job exit and finding probabili-

[40] A seam effect occurs when within-wave changes are less frequent than between-wave changes (Young, 1989). In the case of wages, using within wave information biases wage persistence as wages do not change much within waves.

[41] Using the SIPP, I estimate the variance of log residual wages to be 0.17. The estimated variance of the persistent and transitory components are 0.15 and 0.02, respectively. The variance of the persistent component is computed as $Var(\varepsilon)/(1-\rho_\zeta^2) = 0.003/(1-0.99^2)$. These numbers are somewhat lower than the estimates of Floden and Lindé (2001), who decompose residual wage in the same way as here using the Panel Study of Income Dynamics. In their study, the variance of log residual wages is around 0.31 (they do not include industry dummies in their Mincer regression), and the estimated variances for the permanent and transitory component are 0.26 and 0.04, respectively.

[42] See the discussion in Kopecky and Suen (2010) on the advantages of this procedure.

[43] Using Young's Theorem, the number of quadratic terms in each equation is $121 \times 122/2 = 7381$ instead of $121^2 = 14641$.

[44] The endogenous variables are (in parenthesis I indicate the total number used in this application): $x(i,\tau)$ (108), $w(i)$ (12), $p(i)$ (12), $V(i,\tau)$ (108), $y$ (1).

ties, the mobility rate for UE transitions, the wage losses of switchers, and differential duration for switchers. These parameters will be estimated to minimize the distance between the averages of these moments generated by the model and those observed in the data. While the level of total the unemployment rate can easily be pinned down using analytic expressions in Proposition 1, I do not have closed form formulas for industry switches from UE transitions, their wage losses and employment transitions. Therefore I simulate individual's employment histories and wages to quantify these variables.

I do not have a formal proof for identification of the parameters that characterize the dynamics of income and workers' preferences. However, let me describe which moments in the data are informative of the different parameters, in particular $\sigma$ and $\varsigma$. Note that the same level of steady state unemployment can be achieved with: (1) a large value of $\sigma$ and a low value of $b$, or (2) a low value of $\sigma$ and a large value of $b$. So, it is not possible to identify these two parameters out of this moment alone. However, in the model, $\sigma$ affects earnings through the transitory component of productivity. So, data on wages is informative of $\sigma$. In a similar way, the same level of steady sate mobility can be achieved by: (1) a large value of $\varsigma$ and a large value of $\delta$, or (2) a low value of $\varsigma$ and a low value of $\delta$. However, wage losses for those that switch sectors are informative of the value of $\delta$.

Finally, the scale parameter $\kappa$ is targeted to normalize the economy-wide average wage to be approximately one. A summary of the previous description is displayed in Table 8.

### 5.2 Business cycle dynamics

Now I evaluate the model fit and its cyclical properties. Table 9 shows the model's performance in terms of approximating the key aggregate moments in the labor market. The first part of the table shows the fit in terms of the targeted moments. As there are more targets than parameters (over-identification), the fit is not perfect, but still very good with the exception of the job finding rate, which is larger in the model. The wage losses for switchers are smaller in the model than in the data, but still within the confidence interval. As both unemployment and mobility are purely voluntary in the model, imposing higher costs reduce both the proportion of workers that decide to search and switch.

In terms of non-targeted moments, the model is able to reproduce most of the volatility observed in GDP. However, the volatility of employment and unemployment (normalized by the volatility of GDP), are lower than in the data. Part of this is expected given that the model only has two states of employment. To see this, note that in the model at all times $E_t + U_t = 1$. Taking a first order approximation on the logs of the variables, we have that $\hat{E}_t = -\frac{U_{ss}}{E_{ss}}\hat{U}_t$, where the "hat" stands for log-deviations from the steady state. Using that equation and the average value for employment and unemployment, it is clear that the standard deviation of employment should be roughly 0.06 times that of unemployment.

Table 8: Parameters and targeted moments

| Description | parameter | Moments |
|---|---|---|
| Calibrated | | |
| Unit of time | | 1.5 months |
| Number of sectors | | 12 (NAICS sectors) |
| Discount factor | $\beta = 0.994$ | 5% annual interest |
| Elasticity Substitution CES | $\chi = 2.2$ | Median estimated by Broda and Weinstein (2006). |
| Scale param. in prod function | $\kappa$ | $E(w) \approx 1$ (across industry and time) |
| Share of labor in production | $\alpha_i$ | From Input-Output tables (2002). |
| Share of input $j$ in production of $i$ | $\eta_{ij}$ | From Input-Output tables (2002). |
| Estimated | | |
| Share of sector $i$ in final demand | $\psi_i$ | To match sectoral labor shares (2000-2007) |
| Sector sensitivity to aggregate shock | $\lambda_i$ | Estimated by maximum |
| Sector shock autocorrelation | $\rho_{\theta_i}$ | likelihood using BLS |
| Sector shock variance | $\sigma^2_{\theta_i}$ | multifactor productivity |
| Aggregate shock autocorrelation | $\rho_z$ | measures (see Table 7). |
| Autocorrelation for $\tau$ if employed | $\rho_\tau = 0.985$ | 9 skill levels for $\tau$. Estimated to match |
| Standard deviation for $\tau$ if employed | $\sigma_\tau = 0.067$ | moments on wage dynamics using SIPP data. |
| Std. transitory shock | $\sigma_\epsilon = 0.13$ | Match unemployment level, transition |
| Home production / leisure | $b = 0.52$ | probabilities, gross sectoral mobility (UE), |
| Std. Sector Preference | $\varsigma = 0.42$ | wages losses and duration differences |
| Skill depreciation probability | $\delta = 0.09$ | for switchers. |

Table 9: Model Fit: Aggregate Moments

| Targeted Moments | Data | Model |
|---|---|---|
| unemployment rate (level) | 5.6 % | 5.4% |
| mobility rate for UE transitions (level) | 50% | 47% |
| autocorr persistent income shock | 0.985 | 0.984 |
| std persistent income shock | 0.38 | 0.37 |
| std transitory income shock | 0.17 | 0.17 |
| avg wage change if switch | -3.5% | -1.9% |
| Job finding (quarterly) | 77.4% | 90.0% |
| Employment exit (quarterly) | 5.0% | 5.4% |
| Difference in unemp. duration for switchers (months) | 1.1 | 0.9 |
| Non Targeted Moments | | |
| std(log GDP) | 0.013 | 0.011 |
| autocorr(log GDP) | 0.87 | 0.75 |
| std(log unemployment)/std(log GDP) | 7.0 | 5.2 |
| std(log employment)/std(log GDP) | 0.66 | 0.27 |
| std(log job finding)/std(log GDP) | 2.8 | 0.2 |
| std(log employment exit)/std(log GDP) | 5.4 | 6.1 |
| avg unemp duration (months) | 4.1 | 4.4 |
| avg duration if switch (months, UE transitions) | 3.5 | 2.5 |
| avg duration if no-switch (months, UE transitions) | 2.4 | 1.6 |
| correlation(log gdp, log Lilien's Reallocation) | -0.27 | -0.06 |
| correlation(log gdp, log UE switch) | -0.45 | -0.87 |
| correlation(log gdp, log UE stay) | -0.74 | -0.89 |
| correlation(log gdp, log gross mob) | 0.50 | 0.04 |

Data sample 1979-2007. Variables are seasonally adjusted. For standard deviations and correlations, variables are log transformed and HP filtered. Quarterly job finding and employment exit probabilities computed as in Robin (2011).

In my estimation, unemployment is 5.2 times more volatile than GDP, which implies that employment should be roughly 0.3 times as volatile as GDP, which is approximately the value I get. In the data, unemployment is 7.0 times more volatile than GDP, implying that employment should be 0.42 times as volatile as GDP. However, employment is 0.66 times as volatile as GDP. In sum, it would be hard for the model to reconcile both volatilities at the same time.

Although unemployment is somewhat less volatile than the data, there is substantial amplification of shocks into unemployment. This amplification mechanism is similar to that in Robin (2011), which follows the intuition in Hagedorn and Manovskii (2008). In the model agents are heterogeneous and have different gains from working. There is a mass of agents with small gains from working and they are most affected by the aggregate and sectoral shocks.[45]

The biggest departure from the data is in terms of the level and volatility of the job finding rate. In the model all unemployment is voluntary. Moreover, firms are able to hire all the workers they want at the prevailing wages. These features, which are a characteristic of competitive labor markets, imply that the most active margin of employment transitions is the employment exit.

One important pattern in the data that the model is able to capture, albeit not perfectly, is the pro-cyclicality of the gross mobility rate. Industrial gross mobility is pro-cyclical. The reasons for this in the model are not obvious. In the model, expansions are times in which wages are high, and therefore it becomes more costly to reallocate given that reallocation requires an spell of unemployment. This effect alone would deliver a counter-cyclical mobility rate. A second force at play is the change in the composition of the employment pool. In expansions, workers with a high labor elasticity and high reservation wage become employed. These are workers at the lower end of the distribution of skills ($\tau$). As these workers face very low switching loses, they have high moving rates. In expansions their unemployment spell tends to be short and end with an industry switch. Therefore, more of these workers account for the number of UE transitions, increasing the overall mobility rate. In contractions they work less often (fewer UE transitions) and their effect on the mobility rate is lower.

In calibrating and estimating the model I use sectoral information on the production side of the economy only. First, there is sectoral information on value added, the input output matrix and employment shares. Second, the estimated processes for sectoral TFP will capture industry differences. Note, however, that there are no sector-specific parameters in preferences or earnings that affect how workers move from one sector to the other nor their willingness to work. Nonetheless, the model is able to match relatively well the patterns of sectoral comovement in employment and unemployment observed in the data. Figure 6 shows the relative standard deviation and correlation with aggregates for model simulated

[45]Note, however, that I do not need the returns to leisure to be very close to the returns to work.

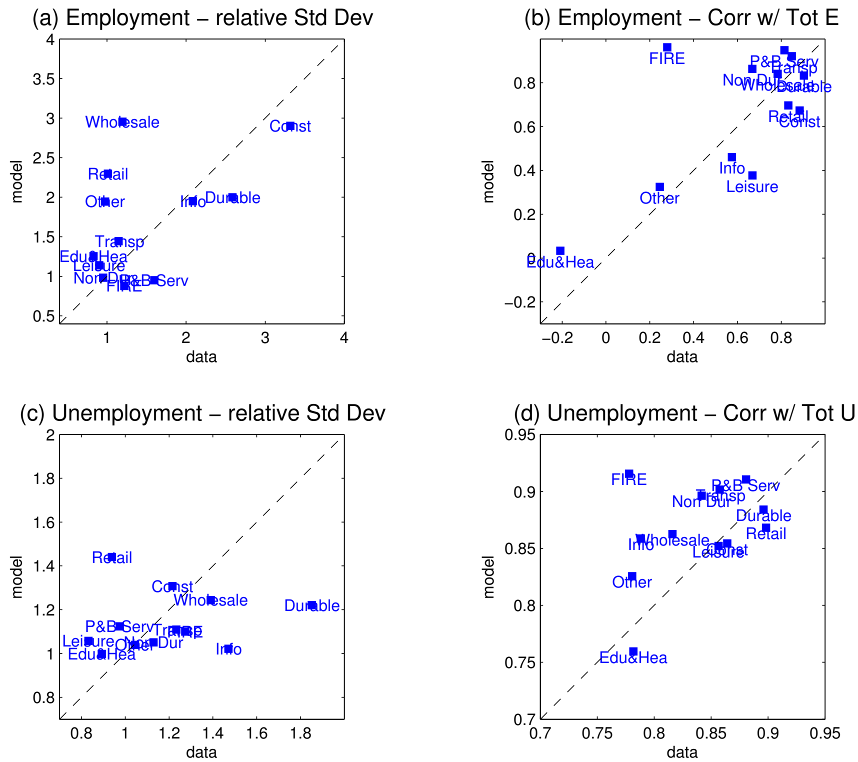



Figure 6: Labor market moments

The graphs show selected moments for model simulated data (y-axis) and real world data (x-axis). Standard deviation and correlations are for the logarithm of the variables in deviations of an HP trend. Simulated and real standard deviations are relative to standard deviations for the aggregate variables. Quarterly frequency and HP filtered with parameter 1600.

data and real world data. If the model was able to match perfectly the real world patterns, all points in the graphs would lie on the 45 degree line.

## 6 Sources of Business Cycles: Aggregate vs. Sectoral Shocks

How important are sectoral shocks in explaining business cycles? Can these shocks account for the strong patterns of comovement in employment and unemployment? The literature has recognized that the effects of sectoral shocks in the aggregate economy can be large due to input-output links. Moreover, these links may induce strong comovement not only in sectoral production, but also in employment. Finally, an additional source of employment comovement discussed in the literature is frictions to the sectoral reallocation of labor.[46]

[46] See for example Horvath (2000); Acemoglu et al. (2012) and Foerster et al. (2011)) on the amplification of sectoral shocks by input-output links, Hornstein and Praschnik (1997) on input-output as a source of comove-

To tackle these questions I investigate the role of the three main mechanisms in the model: the existence of an aggregate shock alongside sectoral shocks, input-output links and frictions to the reallocation of labor. I begin by re-estimating the dynamic factor model for sectoral TFP under two alternative hypotheses. In the first, sectoral TFPs are the sole result of uncorrelated sectoral shocks. In other words, I assume that $\lambda_i = 0$ for all $i$. In the second, sectoral TFPs are the sole result of a single aggregate shock, for which I assume $\theta_i = 0$ for all $i$. In this case, to avoid stochastic singularity I introduce iid classical measurement errors in the measurement of TFPs.

Table 10 shows that when sectoral shocks are uncorrelated (third column), the volatility of aggregate variables at cyclical frequencies is half of that in the benchmark economy. Moreover, net and gross mobility are almost acyclical. On the other hand, the moments of labor market variables in an economy with only a common aggregate shock are similar to the benchmark. In the literature, the pattern of correlation across sectoral variables has typically been used as way to evaluate the fit of models with only sectoral or only aggregate shocks (see footnote 46). Figure 7 compares the patterns of comovement across these economies. When TFP is driven only by uncorrelated sectoral shocks (panels b and e), comovement in employment and unemployment is weaker than observed in the data and in the benchmark economy (panels a and d). Nonetheless, comovement is still positive which highlights the importance of input-output links and labor reallocation.[47] On the contrary, an economy with only a common aggregate shock (panels c and e) displays too much comovement relative the benchmark economy and the data, except for the Education and Health sector, which has a strong negative correlation.[48] The reason is that this sector is only weakly connected to the rest of the economy in terms of input-output links and is the least affected by the aggregate shock (low loading to the common factor). Therefore, in expansions, this sector's productivity is lower relative to the rest of the economy and so are wages. This generates incentives for (some) workers to move out and seek employment in more productive sectors. While this effect is also present in the benchmark economy, it is stronger in an economy with only aggregate shocks.

Overall, the previous exercises show the importance of having both type of shocks, aggregate and sectoral, to jointly explain the volatility of aggregate variables and the comovement of sectoral variables. But, can a strong sectoral shock push the whole economy into a downturn? To answer this question I analyze the response of the economy to different shock impulses. To give sectoral shocks the best chance, in performing these impulse-response ex-

ment, and Boldrin et al. (2001) on comovement due to frictions to the reallocation of labor.

[47] These results are in line with Carvalho (2010) and Foerster et al. (2011), who also find that an economy with independent sectoral shocks can account for only a small fraction of the comovement of sectoral production observed in the data. My results extend theirs to labor market variables.

[48] Although there is only one shock (the aggregate) in this version of the model, the correlations are not perfect since the model is non-linear.

Table 10: The role of aggregate and sectoral shocks

| Targeted Moments | Data | Benchmark | Indep Shocks | Agg Shock |
|---|---|---|---|---|
| unemployment rate (level) | 5.6 % | 5.4% | 5.5% | 5.5% |
| mobility rate (level) | 50% | 47% | 48% | 47% |
| autocorr persistent income shock | 0.985 | 0.984 | 0.984 | 0.985 |
| std persistent income shock | 0.38 | 0.37 | 0.37 | 0.38 |
| std transitory income shock | 0.17 | 0.17 | 0.17 | 0.17 |
| avg wage change if switch | -3.5% | -1.9% | -1.8% | -1.9% |
| Job finding (quarterly) | 77.4% | 90.0% | 89.4% | 89.7% |
| Employment exit (quarterly) | 5.0% | 5.4 % | 5.5% | 5.5% |
| | | | | |
| Non Targeted Moments | | | | |
| std(log GDP) | 0.013 | 0.011 | 0.006 | 0.012 |
| autocorr(log GDP) | 0.87 | 0.75 | 0.76 | 0.75 |
| std(log unemployment)/std(log GDP) | 7.0 | 5.2 | 4.7 | 5.2 |
| std(log job finding)/std(log GDP) | 2.8 | 0.2 | 0.3 | 0.2 |
| std(log employment exit)/std(log GDP) | 5.4 | 6.1 | 5.0 | 5.5 |
| avg unemp duration (months) | 4.1 | 4.4 | 5.0 | 4.3 |
| avg duration if switch (months, UE transitions) | 3.5 | 2.5 | 2.6 | 2.5 |
| avg duration if no-switch (months, UE transitions) | 2.4 | 1.6 | 1.6 | 1.6 |
| correlation(log gdp, log Lilien's Reallocation) | -0.27 | -0.06 | -0.01 | -0.05 |
| correlation(log gdp, log UE switch) | -0.45 | -0.87 | -0.85 | -0.87 |
| correlation(log gdp, log UE stay) | -0.74 | -0.89 | -0.82 | -0.89 |
| correlation(log gdp, log gross mob) | 0.50 | 0.04 | 0.00 | 0.09 |

Column labeled Benchmark shows moments for the model estimated in the previous section where both aggregate and sectoral shocks are active. Column Indep Shocks shows moments for the model with only uncorrelated sectoral TFP shocks, where the independent TFP process was re-estimated in the way described in the text. Column Agg Shock shows moments for the model where sectoral TFPs are driven only by a common aggregate component.

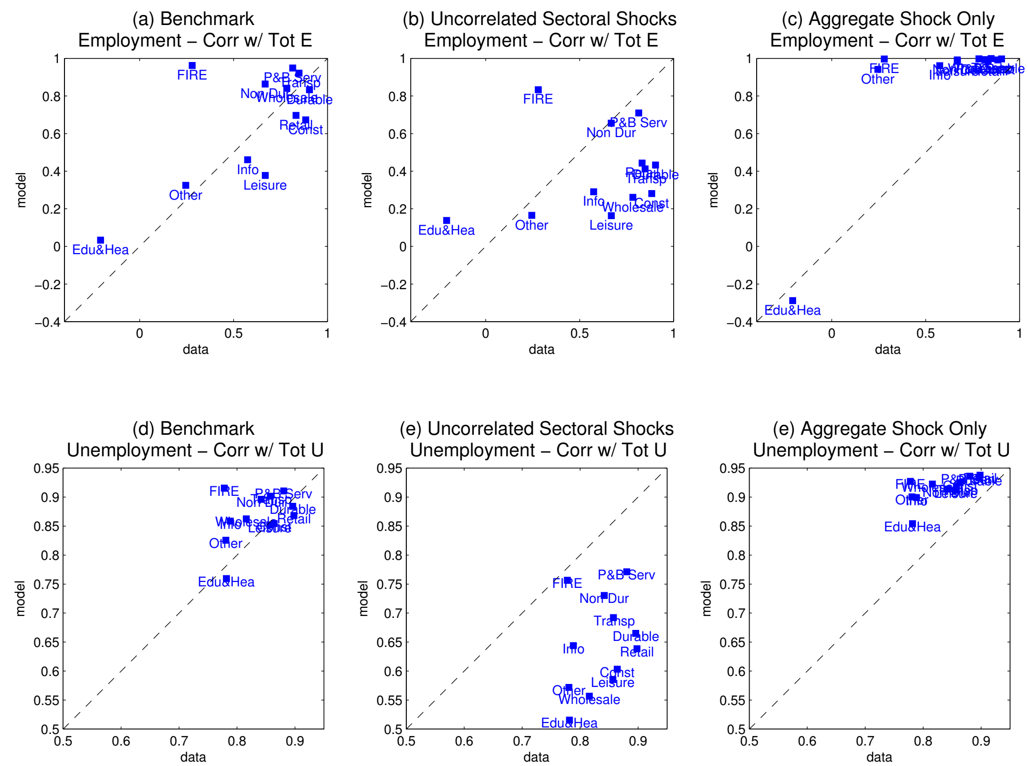



Figure 7: Comovement: Aggregate vs Sectoral Shocks

The graphs show sectoral employment and unemployment correlations with total for model simulated data (y-axis) and real world data (x-axis) under three different economies: the benchmark which includes both aggregate and sectoral shocks, an economy with only uncorrelated sectoral shocks, and an economy with only a common aggregate shock. Quarterly frequency and HP filtered with parameter 1600.

ercises I will use the model with independent sectoral shocks only. The reason is that, in the benchmark economy, the variance of TFP in one sector is the result of both the aggregate shock and its own shock. Therefore, in the benchmark model, the estimated variance of the sectoral shock is smaller than that of the independent shock economy. By using the latter economy, the size of the impulse is larger.

Figure 8 shows the impulse-responses of sectoral TFP shocks. For each sector I plot the evolution of that sector's TFP and Aggregate GDP. While the elasticity of aggregate GDP to the different sectoral shocks is not uniform, in all cases is small. On impact, this elasticity ranges approximately from to 0.1 to 0.30 and depends both on the size of the sector in the total economy and on the strength of the input-output links this sector has with the rest. In all these figures, aggregate GDP is mildly affected by single sectoral shocks.[49] To get more traction, shocks need to hit many sectors and move in the same direction. But if this is the case, these shocks will be correlated, and would resemble an aggregate force.

I now examine the role of frictions to the reallocation of labor. For this I analyze an economy in which workers face no depreciation of their skills when reallocating (i.e. $\delta = 0$). In this economy, it is still the case that workers need to be unemployed for one period in order to switch sectors. Therefore, in this economy frictions are lower than in the benchmark, yet they are not completely absent. Table 11 shows aggregate moments for this economy compared to the data and the benchmark. In this case, the mobility rate is much larger. This is expected given that unemployed workers do not lose skill by switching. At the same time, unemployment duration is much lower and shows no difference whether workers switch industries or not. In the benchmark economy, skill differences and skill depreciation play an important role in shaping transitions into unemployment and duration. In particular, workers at the bottom of the skill distribution face lower expected costs of unemployment and reallocation than workers with high skills. Because of this, in the benchmark economy, there is a larger share of workers with low skills in the unemployment pool and they have higher duration. In the case of $\delta = 0$, these effects are absent. Workers of all skill types face the same costs of unemployment and reallocation and they have short unemployment spells. This same reasoning is also behind the very high job finding rates.

However, the pattern of comovement for this alternative economy is not very different from the benchmark. Figure 9 shows relative standard deviations and correlations of employment and unemployment for the economy with no skill depreciation. Relative to the benchmark, this economy displays a lower volatility of sectoral unemployment and a stronger comovement in unemployment. This is expected given that workers are no longer attached to a particular industry. But, overall, Figure 9 shows that skill depreciation does

[49] The recent work by Caliendo et al. (2013), which account for inter-sectoral and inter-regional trade, also find small effects of large individual sectoral shocks on the aggregate economy. Their model is static and compares differences in steady states.

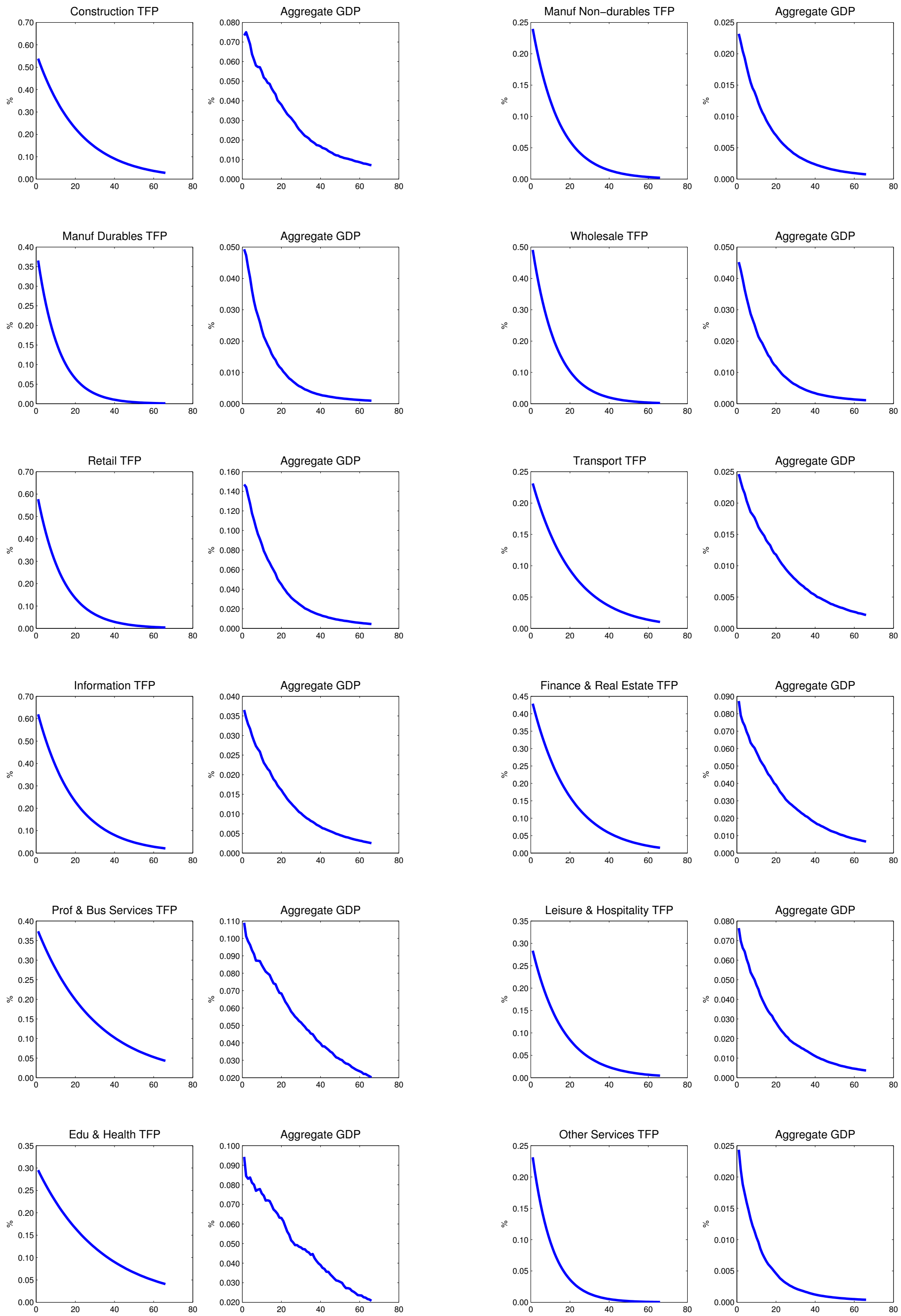



Figure 8: Impulse-Responses for sectoral shocks

Responses to a one-standard-deviation innovation on sectoral TFP in the independent sectoral shocks economy. TFP and GDP are log deviations from trend, in percent. Since the model is non-linear, I compute IRFs for GDP via simulation as in Fernandez-Villaverde et al. (2011).

Table 11: The role of reallocation frictions

| Targeted Moments | Data | Benchmark | No skill depreciation |
|---|---|---|---|
| unemployment rate (level) | 5.6 % | 5.4% | 5.4% |
| mobility rate (level)* | 50% | 47% | 92% |
| autocorr persistent income shock | 0.985 | 0.984 | 0.984 |
| std persistent income shock | 0.38 | 0.37 | 0.37 |
| std transitory income shock | 0.17 | 0.17 | 0.17 |
| avg wage change if switch* | -3.5% | -1.9% | 0.0% |
| Job finding (quarterly) | 77.4% | 90.0% | 94.3% |
| Employment exit (quarterly) | 5.0% | 5.4 % | 5.7% |
| | | | |
| Non Targeted Moments | | | |
| std(log GDP) | 0.013 | 0.011 | 0.012 |
| autocorr(log GDP) | 0.86 | 0.75 | 0.75 |
| std(log unemployment)/std(log GDP) | 7.4 | 5.2 | 5.5 |
| std(log job finding)/std(log GDP) | 2.8 | 0.2 | 0.3 |
| std(log employment exit)/std(log GDP) | 5.4 | 6.1 | 5.5 |
| avg unemp duration (months) | 4.1 | 4.4 | 1.6 |
| avg duration if switch (months, UE transitions) | 3.5 | 2.5 | 1.6 |
| avg duration if no-switch (months, UE transitions) | 2.4 | 1.6 | 1.6 |
| correlation(log gdp, log Lilien's Reallocation) | -0.27 | -0.06 | -0.03 |
| correlation(log gdp, log UE switch) | -0.45 | -0.87 | -0.84 |
| correlation(log gdp, log UE stay) | -0.74 | -0.89 | -0.89 |
| correlation(log gdp, log gross mob) | 0.50 | 0.04 | 0.08 |

*These moments are not targeted for the economy with no skill depreciation as $\delta = 0$ and $\varsigma$ is fixed at the value of the benchmark. Column labeled Benchmark shows moments for the model estimated in the previous section. Column labeled No skill depreciation shows moments for the model where parameter $\delta$ is equal to zero. Quarterly frequency and HP filtered with parameter 1600.

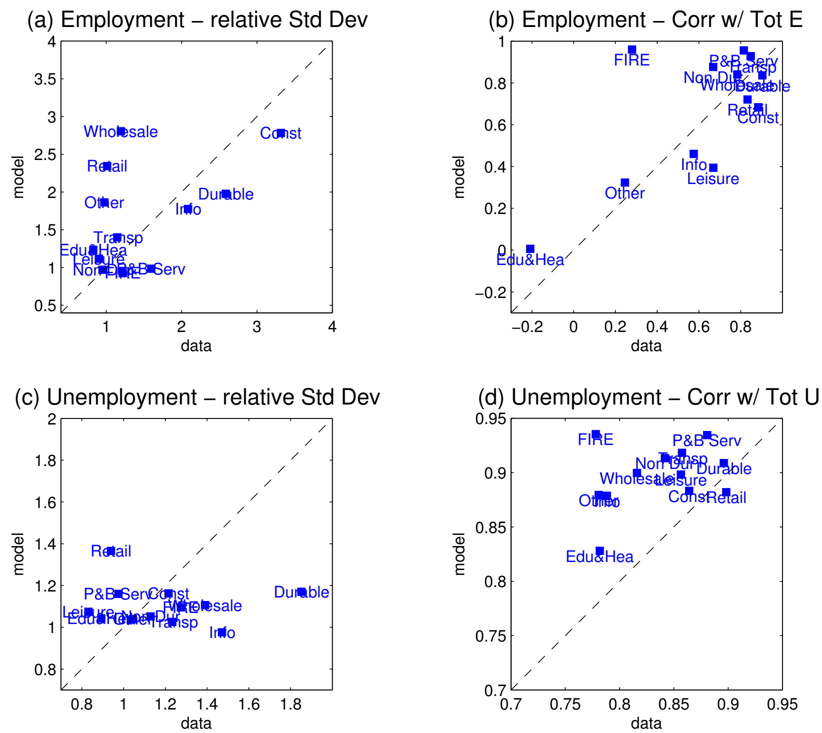



Figure 9: Labor market moments with no skill depreciation
The graphs show selected moments for model simulated data (y-axis) and real world data (x-axis) for the model with no skill depreciation ($\delta = 0$). Standard deviation and correlations are for the logarithm of the variables in deviations of an HP trend. Simulated and real standard deviations are relative to standard deviations for the aggregate variables. Quarterly frequency and HP filtered with parameter 1600.

not seem to be very important in generating comovement.[50] Both in the benchmark economy and in this case, preference shocks are large and ongoing, and gross mobility is always high relative to net changes. Because of this, when a shock hits it propagates quickly across sectors.

I now investigate the role of input-output links. The first step is to compute measures of sectoral TFP that are consistent with a production function that does not use intermediates in production. For this I use the data from the BLS and follow the same methodology em-

[50] I also investigate the patterns of comovement in an economy like Acemoglu et al. (2012) but with elastic labor supply. In this economy there is no unemployment nor any friction to the reallocation of labor and there is a single labor market (wages are equalized across sectors at all times). I find that, for a labor elasticity calibrated to deliver a similar volatility of employment as my benchmark economy, the patterns of comovement are similar to those of figures 6 and 9. Therefore, other forms moving frictions, like unemployment, do not influence much the patterns of comovement either.

Table 12: The role of input output links

| Targeted Moments | Data | Benchmark | No I-O |
|---|---|---|---|
| unemployment rate (level) | 5.6 % | 5.4% | 5.5% |
| mobility rate (level) | 50% | 47% | 47% |
| autocorr persistent income shock | 0.985 | 0.984 | 0.984 |
| std persistent income shock | 0.38 | 0.37 | 0.37 |
| std transitory income shock | 0.17 | 0.17 | 0.17 |
| avg wage change if switch | -3.5% | -1.9% | -1.9% |
| Job finding (quarterly) | 77.4% | 90.0% | 89.6% |
| Employment exit (quarterly) | 5.0% | 5.4 % | 5.5% |
| | | | |
| Non Targeted Moments | | | |
| std(log GDP) | 0.013 | 0.011 | 0.0046 |
| autocorr(log GDP) | 0.87 | 0.75 | 0.75 |
| std(log unemployment)/std(log GDP) | 7.0 | 5.2 | 5.0 |
| std(log job finding)/std(log GDP) | 2.8 | 0.2 | 0.3 |
| std(log employment exit)/std(log GDP) | 5.4 | 6.1 | 5.4 |
| avg unemp duration (months) | 4.1 | 4.4 | 4.6 |
| avg duration if switch (months, UE transitions) | 3.5 | 2.5 | 1.6 |
| avg duration if no-switch (months, UE transitions) | 2.4 | 1.6 | 2.5 |
| correlation(log gdp, log Lilien's Reallocation) | -0.27 | -0.06 | -0.02 |
| correlation(log gdp, log UE switch) | -0.45 | -0.87 | -0.83 |
| correlation(log gdp, log UE stay) | -0.74 | -0.89 | -0.81 |
| correlation(log gdp, log gross mob) | 0.50 | 0.04 | 0.02 |

Column labeled Benchmark shows moments for the model estimated in the previous section. Column labeled No I-O shows moments for the model where there are no input-output links. Quarterly frequency and HP filtered with parameter 1600.

ployed in the construction of sectoral TFP measures, but adapt it to this exercise.[51] Once I have these measures of TFP, I re-estimate the dynamic factor model and recover the parameters $\rho_z, \rho_\theta, \sigma_\theta$ and $\lambda$ and I simulate my model with no input-output links using these new parameters estimates. Table 12 shows aggregate moments for this economy in comparison with the benchmark and the data. The most important deviation is with respect to total volatilities. It is well know that input-output links are an important source of amplification of shocks (Jones, 2013; Bigio and La'O, 2013).

Input-output links also affect sectoral moments. Figure 10 shows that comovement is much lower in this economy relative to the benchmark. In fact, for many sectors, the correlation with aggregate employment is negative. The reason, similar to previous cases, is that those sectors with a higher sensitivity to the aggregate component will offer higher wages in booms than other sectors, generating incentives for workers to move in. With input-output links, sectors hit with an above average productivity shock will demand both more labor

[51] See Footnote 25 for a reference to the data source. The methodology for the construction of these measures which I follow in this counter-factual exercise can be found at http://www.bls.gov/mfp/mprtech.pdf.

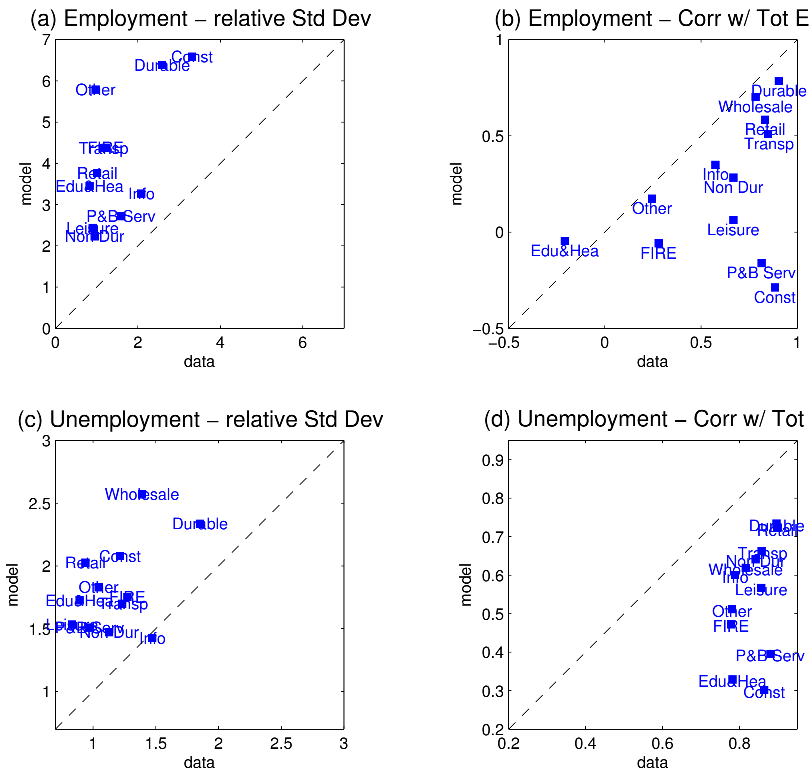



Figure 10: Labor market moments with no input-output links

The graphs show selected moments for model simulated data (y-axis) and real world data (x-axis) for the model with no input-output links. Standard deviation and correlations are for the logarithm of the variables in deviations of an HP trend. Simulated and real standard deviations are relative to standard deviations for the aggregate variables. Quarterly frequency and HP filtered with parameter 1600.

and intermediate goods from other sectors. As labor becomes expensive, they will substitute some of it with intermediates. This increases demand in other sectors as well, pushing wages up and thus increasing the incentives for workers to work and lowering the incentives to reallocate.

Finally I assess the importance of stronger complementarities for the results. For this I change the value of the elasticity of substitution in the utility function ($\chi$) to $0.5$ (it is $2.2$ in the benchmark calibration).[52] Table 13 shows aggregate moments for the data, the benchmark model and two alternative economies, one with $\chi = 0.5$ and both aggregate and sectoral shocks, and the other with $\chi = 0.5$ and only independent sectoral shocks as discussed be-

[52] I am working on relaxing the Cobb-Douglas assumption on the production function. With a CES production function I will also be able to assess the importance of stronger complementarities in production for the results and the patterns of comovement for such economy.

fore. We can see that this type of complementarities, at least for the values used here, does not seem to impact much aggregate moments nor induce a much higher volatility. However, the degree of complementarities has an important impact in sectoral comovement. Figure 11 shows the sectoral correlations for the two counterfactual economies. While the average amount of sectoral correlation in the model approximates well that in the data, the patterns are wrong: sectors with low correlation in the data have a large correlation in the model and vice-versa. The intuition for this is the following. Take a shock that increases TFP in construction (could be either sectoral or aggregate). The price of construction goods will go down and create incentives for a higher demand and consumption. At the same time, because of the stronger complementarities, workers will increase their demand of health and education goods compared to the benchmark and will not consume as much construction goods as in the benchmark economy. This translates into a much higher correlation of health and education variables and lower correlation of construction variables, which is the pattern that emerges in the figure. The main takeaway of this exercise is that the data, and in particular the patterns of comovement, put discipline in the amount of complementarities in the economy.

# 7 Conclusions

In this paper I develop and estimate a multisectoral business cycle model of labor reallocation and unemployment in the neoclassical tradition of Lucas and Prescott (1974)'s island model and Long and Plosser (1983)'s RBC model to study the transmission and amplification of sectoral shocks to the whole economy.

In the model, sectoral and aggregate shocks influence labor demand in each sector. Workers react to the changing economic conditions and decide whether they want to work and in which type of sector they want to work. Switching to another sector is costly for workers for two reasons. First, to reallocate workers endure some unemployment. Second, part of workers' skills are sector-specific and are lost if they change sectors.

Solving for equilibrium in an economy with many segmented labor markets in the presence of aggregate shocks raises an important technical difficulty: when workers decide on whether to work and to which sector to reallocate, they must be able to make rational predictions on what the wage will be in the different parts of the economy. These wages in turn depend on the labor supply and reallocation decisions of all workers in the economy. In other words, individual decisions depend on the distribution of workers in all sectors of the economy, and with aggregate shocks this distribution is not time invariant. For applications involving a non-trivial number of sectors, using moments to summarize this distribution, as in Krusell and Smith (1998), is not useful as this would require a large number of moments. In this paper I borrow from a large literature on dynamic discrete choice models

Table 13: The role of stronger consumption complementarities

| Targeted Moments | Data | Benchmark | $\chi = 0.5$ | $\chi = 0.5$ & only sect. shocks |
|---|---|---|---|---|
| unemployment rate (level) | 5.6 % | 5.4% | 5.4% | 5.5% |
| mobility rate (level) | 50% | 47% | 47% | 48% |
| autocorr persistent income shock | 0.985 | 0.984 | 0.984 | 0.984 |
| std persistent income shock | 0.38 | 0.37 | 0.38 | 0.37 |
| std transitory income shock | 0.17 | 0.17 | 0.17 | 0.17 |
| avg wage change if switch | -3.5% | -1.9% | -1.9% | -1.8% |
| Job finding (quarterly) | 77.4% | 90.0% | 89.9% | 89.6% |
| Employment exit (quarterly) | 5.0% | 5.4 % | 5.4% | 5.4% |
| | | | | |
| Non Targeted Moments | | | | |
| std(log GDP) | 0.013 | 0.011 | 0.011 | 0.006 |
| autocorr(log GDP) | 0.87 | 0.75 | 0.75 | 0.76 |
| std(log unemployment)/std(log GDP) | 7.0 | 5.2 | 5.1 | 4.8 |
| std(log job finding)/std(log GDP) | 2.8 | 0.2 | 0.2 | 0.3 |
| std(log employment exit)/std(log GDP) | 5.4 | 6.1 | 5.3 | 5.0 |
| avg unemp duration (months) | 4.1 | 4.4 | 4.4 | 5.0 |
| avg duration if switch (months, UE transitions) | 3.5 | 2.5 | 1.6 | 1.6 |
| avg duration if no-switch (months, UE transitions) | 2.4 | 1.6 | 2.5 | 2.6 |
| correlation(log gdp, log Lilien's Reallocation) | -0.27 | -0.06 | -0.04 | 0.00 |
| correlation(log gdp, log UE switch) | -0.45 | -0.87 | -0.87 | -0.83 |
| correlation(log gdp, log UE stay) | -0.74 | -0.89 | -0.89 | -0.85 |
| correlation(log gdp, log gross mob) | 0.50 | 0.04 | 0.04 | 0.02 |

Column labeled Benchmark shows moments for the model estimated in the previous section. Column labeled $\chi = 0.5$ shows moments for the model where the elasticity of substitution in CES preferences equals 0.5. Column labeled $\chi = 0.5$ & only sect shocks shows shows moments for the model where the elasticity of substitution in CES preferences equals 0.5 and TFP is driven only by independent sectoral shocks. Quarterly frequency and HP filtered with parameter 1600.

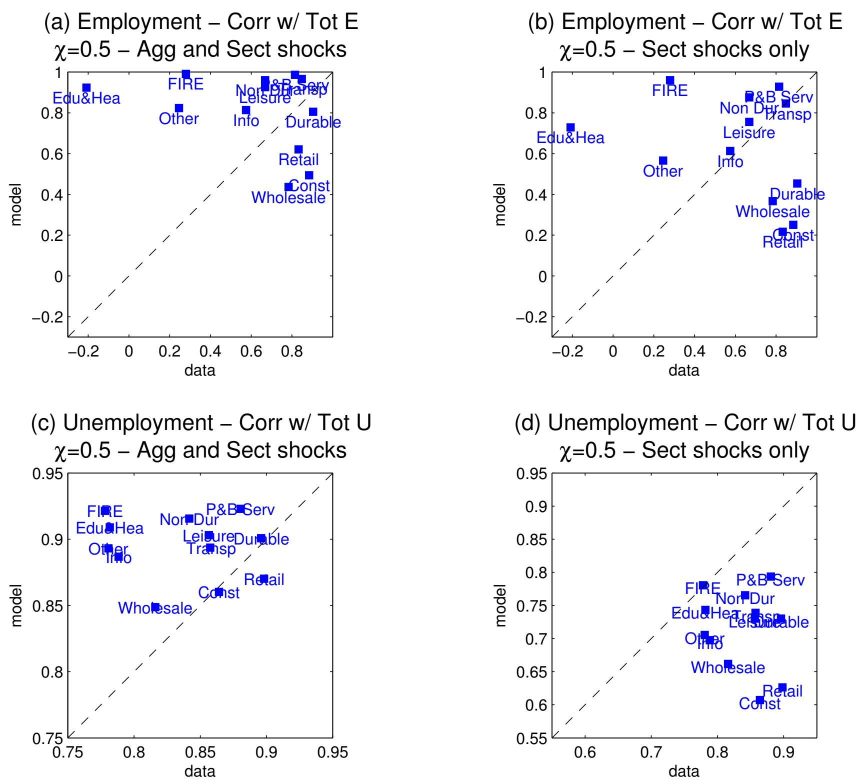



Figure 11: Labor market moments with stronger consumption complementarities

The graphs show correlations for model simulated data (y-axis) and real world data (x-axis). Panels (a) and (c) show, respectively, correlations of employment and unemployment for the model with $chi = 0.5$ and both aggregate and sectoral shocks. Panels (b) and (d) show the same moments but for the model with $chi = 0.5$ and independent sectoral shocks only. Correlations are for the logarithm of the variables in deviations of an HP trend. Quarterly frequency and HP filtered with parameter 1600.

with random utility, and adapt and extend numerical techniques which allow me to solve for equilibrium. There are two key steps. First, I introduce idiosyncratic extreme-value shocks to the worker's problem. Using properties of this distribution I can aggregate discrete individual work and mobility choices into the smooth optimization problem of an sector-representative agent. Second, I use perturbation methods, which can easily accommodate a very large number of state variables, which in this paper is the distribution of workers across markets.

In my analysis I focus on 12 NAICS sectors for the US economy, which account for 98% of private employment and value added. I estimate the model by indirect inference using both aggregate and micro data. I find that the benchmark economy, which includes both aggregate and sector-specific shocks, input-output links and frictions to the reallocation of labor, is able to approximate well the observed volatilities of aggregate variables like GDP and unemployment, the patterns of sectoral co-movement, gross and net reallocation and other moments of labor market variables at business cycle frequencies. On the other hand, an economy with only uncorrelated sectoral productivity shocks cannot match these facts. Input-output links and the presence of an aggregate shock alongside sectoral shocks are important sources of comovement of sectoral labor market variables.

The labor market has two main characteristics: employment, unemployment and many other variables display strong cyclical movements, and workers and jobs are heterogeneous. The solution method I propose here are not exclusive of island economies and can be used to solve for equilibrium in search and matching models with heterogeneous workers or firms. In particular, this approach can be specially useful in models that do not display a block recursive structure.

## Appendix A: Value Function, Labor Supply and Mobility

Here I derive equations (11), (12) and (13) describing the lifetime expected utility the representative agent of the sector, the proportion of agents that decide to work and the proportion of workers that are unemployed and potentially switch to a different sector.

As I discussed in the text, the assumption for each of the realizations of the preference shocks is that they are drawn independently from a Type I Extreme Value distribution. The density and distribution functions for a Type I Extreme Value random variable are:

$$\begin{aligned} f(x) &= e^{-(x+\gamma)-e^{-(x+\gamma)}} \\ F(x) &= e^{-e^{-(x+\gamma)}} \end{aligned}$$

The domain for $x$ is the real line. The constant $\gamma \approx 0.5772$ (Euler's constant) normalizes to get a mean of zero.[53] I assume a total of $J$ islands, with $2 \leq J < \infty$.

[53] The effective variance of the preference shocks will be adjusted by the parameter $\sigma_\epsilon$ in the agent's problem.

The following is the problem for an agent which has observed the realization of the the aggregate shock, the sector productivity shock and his realization of $\tau$, but has not yet observed the realization of his $\epsilon_W$ and $\epsilon_S$ shocks. I call this the problem of the "representative agent" for the sector.

$$V(i,\tau,\Upsilon) \;=\; E_\epsilon \max \left\{ \begin{array}{l} \log(w(i,\Upsilon)\,\tau) + \beta\, E_{\tau',\Upsilon'}\left[V(i,\tau',\Upsilon')|\tau,\Upsilon\right] + \sigma_\epsilon\,\epsilon_W; \\ \log(b\,\tau) + \beta\, E_\epsilon\left[\max_j \left\{E_{\tau',\Upsilon'}\left[V(j,\tau',\Upsilon')|\tau,\Upsilon,i,j\right] + \varsigma\epsilon_j\right\}\right] + \sigma_\epsilon\,\epsilon_S \end{array} \right\}$$

To simplify the notation let me write

$$V_k = E_{\tau',\Upsilon'}\left[V(k,\tau',\Upsilon')|\tau,\Upsilon,i,k\right]$$

Now, the expected utility for an unemployed worker that is searching over sectors (the second stage decision) and has not yet observed his sector preference shocks is:

$$E_\epsilon\left[\max_k \{V_k + \varsigma\,\epsilon_k\}\right] \;=\; \sum_{k=1}^{J}\int_{-\infty}^{\infty}(V_k+\varsigma\,\epsilon_k) f(\epsilon_k)\left[\prod_{j\neq k} F\left(\frac{V_k - V_j}{\varsigma} + \epsilon_k\right)\right] d\epsilon_k$$

$$= \sum_{k=1}^{J}\int_{-\infty}^{\infty}(V_k+\varsigma\,\epsilon_k) e^{-(\epsilon_k+\gamma)-e^{-(\epsilon_k+\gamma)}}\left[\prod_{j\neq k} e^{-e^{-\left(\frac{V_k-V_j}{\varsigma}+\epsilon_k+\gamma\right)}}\right] d\epsilon_k$$

$$= \sum_{k=1}^{J}\int_{-\infty}^{\infty}(V_k+\varsigma\,\epsilon_k)\, e^{-(\epsilon_k+\gamma)}\, e^{-e^{-(\epsilon_k+\gamma)}\,\sum_{j=1}^{J} e^{-\left(V_k-V_j\right)/\varsigma}} d\epsilon_k$$

let's call $\sum_{j=1}^{J} e^{-(V_k-V_j)/\varsigma} = e^{\lambda}$, then

$$= \sum_{k=1}^{J} e^{-\lambda}\left[V_k + \varsigma\,(\lambda-\gamma) + \varsigma\int_{-\infty}^{\infty}(\epsilon_k+\gamma-\lambda)\, e^{-(\epsilon_k+\gamma-\lambda)-e^{-(\epsilon_k+\gamma-\lambda)}} d\epsilon_k\right]$$

$$= \sum_{k=1}^{J} e^{-\lambda}\left[V_k + \varsigma\,\lambda\right]$$

$$= \varsigma\left[\log\left(\sum_{j=1}^{J} e^{V_j/\varsigma}\right)\right]\left[\frac{\sum_{k=1}^{J} e^{V_k/\varsigma}}{\sum_{j=1}^{J} e^{V_j/\varsigma}}\right]$$

$$= \varsigma\left[\log\left(\sum_{j=1}^{J} e^{V_j/\varsigma}\right)\right]$$

Which is inside the last part of equation (11) once we substitute back for what $V_j$ stands for. Proceeding in a similar way, we can recover all of equation (11).

Again, staring at the second decision stage, we can compute the proportions of agents that will chose to stay or switch sectors. An agent in sector $i$ that is searching will move to sector $k$ at the end of the period if:

$$V_k + \varsigma\,\epsilon_k > V_j + \varsigma\,\epsilon_j \qquad \forall j$$

By the law of large numbers, the proportion of people that will switch from $i$ to $k$ is

$$\begin{aligned}
Pr\left(V_k + \varsigma\,\epsilon_k > V_j + \varsigma\,\epsilon_j;\, \forall j\right) &= Pr\left(\epsilon_j < V_k/\varsigma\, + \epsilon_k - V_j/\varsigma\,;\, \forall j\right) \\
&= \int_{-\infty}^{\infty} f(\epsilon_k) \left[\prod_{j \neq k} F\left(\frac{V_k - V_j}{\varsigma} + \epsilon_k\right)\right] d\epsilon_k \\
&= e^{-\lambda} \int_{-\infty}^{\infty} e^{-(\epsilon_k + \gamma - \lambda)}\, e^{-e^{-(\epsilon_k+\gamma-\lambda)}} d\epsilon_k \\
&= \frac{1}{e^{\lambda}} \\
&= \frac{e^{V_k/\varsigma}}{\sum_{j=1}^{J} e^{V_j/\varsigma}}
\end{aligned}$$

Following these same steps for an agent facing the initial decision of whether to work or search, we can get equations (12) and (13) for $N(i, \tau, \Upsilon)$ and $M(i, j, \tau, \Upsilon)$, respectively.

### Labor Supply

In much of the literature on discrete choice models, $\epsilon_W$ and $\epsilon_S$ are interpreted as pure preference shocks to the utility of working and not working. However, in this paper I assume that these shocks are idiosyncratic (and transitory) productivity shocks, and $\epsilon_W$ affect the number of efficiency units of labor workers sell in the market (and therefore earnings). Under logarithmic utility and the multiplicative structure I use here, the problem of the worker is identical under both interpretations. However, labor supply and equilibrium wages will depend on which assumption we make since the total number of efficiency units of labor sold in the market will be different. In addition, when mapping data on wages to model wages, the different interpretations will matter for the empirical strategy and estimation.

In what follows I make a slight abuse of notation and call $U_W = \log(w(i,\Upsilon)\,\tau) + \beta\, E\left[V(i, \tau', \Upsilon')|\tau, \Upsilon\right]$ and $U_S = \left[\log(b) + \beta\,\varsigma\, \log\left(\sum_j e^{E[V(j,\tau',\Upsilon')|\tau,\Upsilon,i,j]/\varsigma}\right)\right]$. The labor supply in sector $i$ is

$$L^s(i, \Upsilon) = \sum_{\tau} \tau\, x(i, \tau) E\left[e^{\sigma_\epsilon\, \epsilon_W}\, | \, U_W + \sigma_\epsilon\, \epsilon_W > U_S + \sigma_\epsilon\, \epsilon_S\right] \tag{16}$$

where in the last part I use the law of large numbers to get the supply of efficiency units of labor for the workers of sector $i$ and type $\tau$ that decide to work.[54] Therefore,

$$L^s(i,\Upsilon) = \sum_{\tau} \tau\, x(i,\tau) \int_{-\infty}^{\infty} e^{\sigma_\epsilon \epsilon_W} f(\epsilon_W) F\left(\frac{U_W - U_S}{\sigma_\epsilon} + \epsilon_W\right) d\epsilon_W \tag{17}$$

where $f(.)$ and $F(.)$ are the pdf and cdf of the Type I Extreme Value distribution as discussed previously. Now I will focus on the integral in equation (17).

$$\begin{aligned}
\int_{-\infty}^{\infty} e^{\sigma_\epsilon \epsilon_W} f(\epsilon_W) F\left(\frac{U_W - U_S}{\sigma_\epsilon} + \epsilon_W\right) d\epsilon_W &= \int_{-\infty}^{\infty} e^{\sigma_\epsilon \epsilon_W} e^{-(\epsilon_W+\gamma)-e^{-(\epsilon_W+\gamma)}} e^{-e^{-\left(\frac{U_W-U_S}{\sigma_\epsilon}+\epsilon_W+\gamma\right)}} d\epsilon_W \\
&= \int_{-\infty}^{\infty} e^{\sigma_\epsilon \epsilon_W} e^{-(\epsilon_W+\gamma)-e^{-(\epsilon_W+\gamma)}} e^{-e^{-\left(\frac{U_W-U_S}{\sigma_\epsilon}+\epsilon_W+\gamma\right)}} d\epsilon_W \\
&= \int_{-\infty}^{\infty} e^{\sigma_\epsilon \epsilon_W} e^{-(\epsilon_W+\gamma)} e^{-e^{-(\epsilon_W+\gamma)}} e^{-e^{-\left(\frac{U_W-U_S}{\sigma_\epsilon}\right)} e^{-(\epsilon_W+\gamma)}} d\epsilon_W \\
&= \int_{-\infty}^{\infty} e^{\sigma_\epsilon \epsilon_W} e^{-(\epsilon_W+\gamma)} e^{-\left[1+e^{-\left(\frac{U_W-U_S}{\sigma_\epsilon}\right)}\right] e^{-(\epsilon_W+\gamma)}} d\epsilon_W
\end{aligned}$$

let $e^{\lambda} = 1 + e^{-\left(\frac{U_W-U_S}{\sigma_\epsilon}\right)}$, then

$$\begin{aligned}
&= \int_{-\infty}^{\infty} e^{\sigma_\epsilon \epsilon_W} e^{-(\epsilon_W+\gamma)} e^{-e^{-(\epsilon_W+\gamma-\lambda)}} d\epsilon_W \\
&= \frac{1}{e^{\lambda}} \int_{-\infty}^{\infty} e^{\sigma_\epsilon \epsilon_W} e^{-(\epsilon_W+\gamma-\lambda)} e^{-e^{-(\epsilon_W+\gamma-\lambda)}} d\epsilon_W \\
&= \frac{e^{\lambda-\gamma}}{e^{\lambda}} \int_{-\infty}^{\infty} e^{-(1-\sigma_\epsilon)\epsilon_W} e^{-e^{-\epsilon_W} e^{(\lambda-\gamma)}} d\epsilon_W
\end{aligned}$$

Now with a change of variables $x = e^{-\epsilon_W}$, then

$$= \frac{e^{\lambda-\gamma}}{e^{\lambda}} \int_{0}^{\infty} x^{-\sigma_\epsilon} e^{-x e^{(\lambda-\gamma)}} dx$$

the integral in the previous expression converges provided that $\sigma_\epsilon < 1$.[55]

$$\begin{aligned}
&= \frac{e^{\lambda-\gamma}}{e^{\lambda}} \Gamma(1-\sigma_\epsilon) e^{(\lambda-\gamma)(\sigma_\epsilon-1)} \\
&= \left(\frac{1}{e^{\lambda}}\right)^{(1-\sigma_\epsilon)} \frac{\Gamma(1-\sigma_\epsilon)}{e^{\sigma_\epsilon \gamma}}
\end{aligned}$$

[54] Note that if $\epsilon_W$ is interpreted as preferences which do not affect supply, then this expectation would be $E\left[1 | U_W + \sigma_\epsilon \epsilon_W > U_S + \sigma_\epsilon \epsilon_S\right]$ as all workers of this type will supply a single unit of labor. This expectation then reduces to $N(i,\tau,\Upsilon)$ which is the usual expression.

[55] Given the interpretation of $\epsilon_W$ as a transitory income shock, a vast literature on wage dynamics find that the standard deviation of this shock satisfies this requirement.

where $\Gamma(.)$ is the Gamma function. Given the definition for $\lambda$, we have that $(1/e^{\lambda}) = N(i,\tau,\Upsilon)$. Substituting back into (17), then:

$$L^s(i,\Upsilon) = \sum_{\tau} \tau\, x(i,\tau)\, N(i,\tau,\Upsilon)^{(1-\sigma_\epsilon)} \frac{\Gamma(1-\sigma_\epsilon)}{e^{\sigma_\epsilon}\gamma}$$

# Appendix B: Equilibrium Conditions

Here I specify the full set of equilibrium conditions of the model:

### Firms

$$\begin{aligned}
q_{i,t} &= e^{\lambda_i z_t + \theta_{i,t}} \, (L^d_{i,t})^{\alpha_i} \prod_{j=1}^{J} (\varphi^d_{ji,t})^{(1-\alpha_i)\eta_{ij}} \\
L^d(i,\Upsilon) &= \frac{\alpha_i}{e^{\lambda_i z + \theta_i} \alpha_i^{\alpha_i} (1-\alpha_i)^{(1-\alpha_i)} \prod_{j=1}^{J} \eta_{ji}^{(1-\alpha_i)\eta_{ij}}} w(i,\Upsilon)^{\alpha_i} \prod_{j=1}^{J} p(j,\Upsilon)^{(1-\alpha_i)\eta_{ij}} \frac{q(i,\Upsilon)}{w(i,\Upsilon)} \\
\varphi^d(i,j\Upsilon) &= \frac{(1-\alpha_i)\eta_{ij}}{e^{\lambda_i z + \theta_i} \alpha_i^{\alpha_i} (1-\alpha_i)^{(1-\alpha_i)} \prod_{j=1}^{J} \eta_{ij}^{(1-\alpha_i)\eta_{ij}}} w(i,\Upsilon)^{\alpha_i} \prod_{j=1}^{J} p(j,\Upsilon)^{(1-\alpha_i)\eta_{ij}} \frac{q(i,\Upsilon)}{p(j,\Upsilon)} \\
\log(p(i,\Upsilon)) &= \alpha_i \log(w_i) - \log\left( e^{\lambda_i z + \theta_i} \alpha_i^{\alpha_i} (1-\alpha_i)^{(1-\alpha_i)} \prod_{j=1}^{J} \eta_{ij}^{(1-\alpha_i)\eta_{ij}} \right) + (1-\alpha_i) \sum_{j=1}^{J} \eta_{ij} \log(p(j,\Upsilon))
\end{aligned}$$

where the first equation is the production function, the second and third are the optimal demand for labor and intermediate inputs, respectively, and the last equation comes from a free entry condition (in logs) such that the income per unit of good $i$ (the price) is equal to the minimum cost of producing it.

### Workers

$$
\begin{aligned}
V(i,\tau,\Upsilon) &= \sigma_\epsilon \log\left( e^{(\log(w(i,\Upsilon)\,\tau)+\beta\, E\left[V(i,\tau',\Upsilon')|\tau,\Upsilon\right])/\sigma_\epsilon} + e^{\left[\log(b)+\beta\,\varsigma\,\log\left(\sum_j e^{E\left[V(j,\tau',\Upsilon')|\tau,\Upsilon,i,j\right]/\varsigma}\right)\right]/\sigma_\epsilon}\right) \\
N(i,\tau,\Upsilon) &= \frac{e^{(\log(w(i,\Upsilon)\,\tau)+\beta\, E\left[V(i,\tau',\Upsilon')|\tau,\Upsilon\right])/\sigma_\epsilon}}{e^{V(i,\tau,\Upsilon)/\sigma_\epsilon}} \\
M(i,j,\tau,\Upsilon) &= \frac{e^{\left[\log(b)+\beta\,\varsigma\,\log\left(\sum_k e^{E\left[V(k,\tau',\Upsilon')|\tau,\Upsilon,i,k\right]/\varsigma}\right)\right]/\sigma_\epsilon}}{e^{V(i,\tau,\Upsilon)/\sigma_\epsilon}} \frac{e^{E\left[V(j,\tau',\Upsilon')|\tau,\Upsilon,i,j\right]/\varsigma}}{\sum_k e^{E[V(k,\tau',\Upsilon')|\tau,\Upsilon,i,k]/\varsigma}} \\
L^s(i,\Upsilon) &= \sum_\tau \tau\, x(i,\tau)\, N(i,\tau,\Upsilon)^{(1-\sigma_\epsilon)}\, \Gamma(1-\sigma_\epsilon)\, e^{-\sigma_\epsilon\,\gamma} \\
y_j^d(i,\Upsilon) &= [L^s(i,\Upsilon) w(i,\Upsilon)]\, \psi_j^\chi\, \kappa^{\chi-1} p(j,\Upsilon)^{-\chi} \\
L^s(i,\Upsilon) w(i,\Upsilon) &= \kappa\left(\sum_j \psi_i y_j^d(i,\Upsilon)^{\frac{\chi-1}{\chi}}\right)^{\frac{\chi}{\chi-1}}
\end{aligned}
$$

where $V(i,\tau,\Upsilon)$ is the lifetime utility function of the representative agent for the sector $i$ with type $\tau$, $N(i,\tau,\Upsilon)$ is the proportion of workers of sector $i$ with type $\tau$, $M(i,j,\tau,\Upsilon)$ is the proportion of movers from sector $i$ with to sector $j$ of type $\tau$ and $L^s(i,\Upsilon)$ the labor supply of sector $i$. $y_j^d(i,\Upsilon)$ is the optimal demand of good $j$ made by workers of sector $i$. Since workers in all sectors have the same preferences they will consume the same basket but rescaled by their income. The equation states that all income is spent in the CES good.

### Aggregation and Market Clearing

$$
\begin{aligned}
q(i,\Upsilon) &= y^s(i,\Upsilon) + \sum_j \varphi^s(i,j,\Upsilon) \\
y^s(i,\Upsilon) &= y^d(i,\Upsilon) \\
\varphi^s(i,j,\Upsilon) &= \varphi^d(i,j,\Upsilon) \\
L^s(i,\Upsilon) &= L^d(i,\Upsilon) \\
y(\Upsilon) &= \sum_i L^s(i,\Upsilon) w(i,\Upsilon) \\
P \equiv 1 &= \kappa^{-1}\left(\sum_i p(i,\Upsilon)^{1-\chi}\psi_i^\chi\right)^{1/(1-\chi)}
\end{aligned}
$$

the first equation states that production of good $i$ is allocated to the supply of final demand of good $i$ and intermediates. Equations two to four say that the supply and demand of goods and labor must equalize in all markets. Finally the last two equations are definitions of total output and the ideal price index.

**Laws of motion**

$$\begin{aligned}
x'(i,\tau,\Upsilon) &= \sum_{\hat{\tau}} \pi(\tau|\hat{\tau})\, x(i,\hat{\tau}) \Big(N(i,\hat{\tau},\Upsilon) + M(i,i,\hat{\tau},\Upsilon)\Big) + \sum_{j\neq i} \sum_{\hat{\tau}} \tilde{\pi}(\tau|\hat{\tau})\, x(j,\hat{\tau})\, M(j,i,\hat{\tau},\Upsilon) \\
z' &= \rho_z\, z + (1-\rho_z^2)^{0.5}\, \varepsilon_z \\
\theta_i' &= \rho_i\, \theta_i + (1-\rho_i^2)^{0.5}\, \varepsilon_i
\end{aligned}$$

where the first equation describes the evolution of the measure of workers of type $\tau$ on island $i$, which depends on their work and search decisions. And the last two are the shock processes.

## Appendix C: Further Details of the Numerical Solution

To apply perturbation theory, let me re-write the process for $z$ in the following way:[56]

$$\begin{aligned}
z_t &= \rho_z z_{t-1} + \upsilon\, (1-\rho_z^2)^{0.5} \xi_{z,t} \\
\theta_{i,t} &= \rho_i \theta_{i,t} + \upsilon\, (1-\rho_i^2)^{0.5} \xi_{i,t} \qquad \forall i
\end{aligned}$$

The new variable $\upsilon$ is the perturbation parameter. When $\upsilon = 1$ aggregate shocks $\{z, \theta\}$ are active and when $\upsilon = 0$ aggregate shocks are shut-down and the model is stationary. As a reminder, the solution to the dynamic general equilibrium in this economy is characterized by a set of functions for the endogenous variables. To understand the computational method, I make the dependence of the solution on the perturbation parameter explicit: $\{x'(i,\tau,\Upsilon;\upsilon), V(i,\tau,\Upsilon;\upsilon), w(i,\Upsilon;\upsilon), p(i,\Upsilon;\upsilon), y(\Upsilon;\upsilon)\}_{\forall\, i,\tau}$.

To keep the exposition simple, in what follows I will only perform a second order perturbation. That is, I approximate by a second order Taylor expansion these functions that characterize the equilibrium. This Taylor expansion is done around the point in which $\upsilon = 0$, i.e. the stationary model. Higher orders can easily be achieved but the notation gets cumbersome. Let me define $\Upsilon^{ss} = \Big\{0, 0, \ldots, 0, \{x^{ss}(i,\tau)\}_{\forall i,\tau}\Big\}$. The second order approximation to the Value Function in (15) is:

[56] Here I adapt the notation used in Caldara et al. (2011) to the setup developed here.

$$
\begin{aligned}
V(i,\tau,\Upsilon;\upsilon) \approx\ & V^{ss}(i,\tau) + V^{ss}_z(i,\tau)\, z + \sum_{j,\tilde{\tau}} V^{ss}_{x(j,\tilde{\tau})}(i,\tau)\, \hat{x}(j,\tilde{\tau}) + \sum_j V^{ss}_{\theta_j}(i,\tau)\, \theta_{j,t} + V^{ss}_{\upsilon}(i,\tau)\, \upsilon + \\
& \frac{1}{2}\left[ V^{ss}_{z,z}(i,\tau)\,(z)^2 + \sum_{j,\tilde{\tau}} V^{ss}_{z,x(j,\tilde{\tau})}(i,\tau)\, z\, \hat{x}(j,\tilde{\tau}) + \sum_j V^{ss}_{z,\theta_j}(i,\tau)\, z\, \theta_{j,t} + V^{ss}_{z,\upsilon}(i,\tau)\, z\, \upsilon \right] + \\
& \frac{1}{2}\sum_{k,\bar{\tau}} \Bigg[ V^{ss}_{z,x(k,\bar{\tau})}(i,\tau)\, z\hat{x}(k,\bar{\tau}) + \sum_{j,\tilde{\tau}} V^{ss}_{x(k,\bar{\tau}),x(j,\tilde{\tau})}(i,\tau)\, \hat{x}(k,\bar{\tau})\, \hat{x}(j,\tilde{\tau}) + \\
& \qquad \sum_j V^{ss}_{x(k,\bar{\tau}),\theta_j}(i,\tau)\, \hat{x}(k,\bar{\tau})\, \theta_{j,t} + V^{ss}_{x(k,\bar{\tau}),\upsilon}(i,\tau)\, \hat{x}(k,\bar{\tau})\, \upsilon \Bigg] + \\
& \frac{1}{2}\sum_k \left[ V^{ss}_{z,\theta_k}(i,\tau)\, z\, \theta_{k,t} + \sum_{j,\tilde{\tau}} V^{ss}_{\theta_k,x(j,\tilde{\tau})}(i,\tau)\, \theta_{k,t}\, \hat{x}(j,\tilde{\tau}) + \sum_j V^{ss}_{\theta_k,\theta_j}(i,\tau)\, \theta_{k,t}\, \theta_{j,t} + V^{ss}_{\theta_j,\upsilon}(i,\tau)\, \theta_{j,t}\, \upsilon \right] + \\
& \frac{1}{2}\left[ V^{ss}_{z,\upsilon}(i,\tau)\, z\, \upsilon + \sum_{j,\tilde{\tau}} V^{ss}_{\upsilon,x(j,\tilde{\tau})}(i,\tau)\, \upsilon\, \hat{x}(j,\tilde{\tau}) + \sum_j V^{ss}_{\upsilon,\theta_j}(i,\tau)\, \upsilon\, \theta_{j,t} + V^{ss}_{\upsilon,\upsilon}(i,\tau)\, \upsilon^2 \right]
\end{aligned}
$$

where $V^{ss}(i,\tau) = V(i,\tau,\Upsilon^{ss};0)$, $V^{ss}_q(i,\tau) = \frac{\partial V(i,\tau,\Upsilon^{ss};0)}{\partial q}$, $V^{ss}_{q,r} = \frac{\partial^2 V(i,\tau,\Upsilon^{ss};0)}{\partial q\, \partial r}$ and $q, r \in \{z, \{\theta_j\}, \{x(j,\tilde{\tau})\}\}$. And $\hat{x}(k,\bar{\tau}) = x(k,\bar{\tau}) - x^{ss}(k,\bar{\tau})$. The previous expression can be simplified using Young's theorem and making $\upsilon = 1$. Approximations for the other endogenous variables, $\{x'(i,\tau,\Upsilon;\upsilon), w(i,\Upsilon;\upsilon), p(i,\Upsilon;\upsilon), y(\Upsilon;\upsilon)\}_{\forall i,\tau}$, can be written in a similar way.

$V^{ss}(i,\tau)$, $V^{ss}_q(i,\tau)$, $V^{ss}_{q,r}(i,\tau)$ are the coefficients in the approximate solution which are not known and must be solved for. To do this we use the equilibrium conditions in (15). The system in (15) can be re-written more compactly as:

$$
F(\Upsilon;\upsilon) = H(\{V(i,\tau,\Upsilon), E_{\Upsilon}[V(i,\tau',\Upsilon')], x(i,\tau), x'(i,\tau,\Upsilon), w(i,\Upsilon), p(i,\Upsilon), \theta_i\}_{\forall i,\tau}, z, y;\upsilon) = \mathbf{0}
$$

where $F$ is a function which links implicitly the solution of the model with the state variables.[57]

Note that

$$
F(\Upsilon^{ss};0) = H(\{V(i,\tau,\Upsilon^{ss}), V(i,\tau,\Upsilon^{ss}), x(i,\tau,\Upsilon^{ss}), x(i,\tau,\Upsilon^{ss}), w(i,\Upsilon^{ss}), p(i,\Upsilon^{ss}), 0\}_{\forall i,\tau}, 0, y^{ss};0) = \mathbf{0}
$$

is just the solution of the stationary version of the model where aggregate shocks $z, \theta$ are shut-down. This defines a non-linear system in the variables $\{x^{ss}(i,\tau), V^{ss}(i,\tau), w^{ss}(i), p^{ss}(i)\}_{\forall i,j,\tau}$, where each of them is a scalar. While it is possible to solve for the stationary values with any standard non-linear solver, sometimes gradient based methods may fail in finding the solution (general equilibrium) in a model like this one that has a large number of markets and high substitution. Therefore, I found useful to use the following algorithm which describes a *tatonnement* process and turns out to be very robust and fast:

[57] I omit variables $N^{ss}(i,\tau), M^{ss}(i,j,\tau)$ since they are a function of the rest.

1. Guess a vector of sector prices $p^{ss}(i)$ (normalized such that the ideal price index for the CES aggregate good is 1). Use the linear system of log prices and log wages to get the vector of wages $w^{ss}(i)$.

2. Solve the problem of the representative worker for the sector $i$ and productivity $\tau$. To do this use Value Function Iteration to solve equation (11), guessing an initial vector of $V$ and iterate until convergence. This yields $V^{ss}(i,\tau)$. Using these variables, get $N^{ss}(i,\tau)$ and $M^{ss}(i,j,\tau)$, which are the proportion of workers in sector $i$ and type $\tau$ that decide to work and that decide to search and move to sector $j$, respectively.

3. Guess a distribution of workers over sectors at the beginning of the period ($x$).

4. Use $N^{ss}(i,\tau)$, $M^{ss}(i,j,\tau)$ and $x$ to update the distribution of agents over sectors for the beginning of the next period $x'$. Compare $x'$ with $x$ and if they are not approximately equal, use $x'$ as a new guess and go back to step 3. If they are equal, then this is $x^{ss}(i,\tau)$.

5. Compute supply and demand for sector goods and check if markets clear. If not, go back to step 1 using Gauss-Seidel or a similar method.

The derivatives of $F$ with respect to each of the elements in the state space $\Upsilon$ and $\upsilon$:

$$F_q(\Upsilon^{ss};0) = \mathbf{0}; \qquad \text{for } q = \{z, \{\theta_j\}, \{x(j,\tilde{\tau})\}, \upsilon\} \tag{18}$$

define a quadratic system of equations in the unknowns $\left\{x_q^{ss}(i,\tau), V_q^{ss}(i,\tau), w_q^{ss}(i), p_q^{ss}(i), y_q^s s\right\}_{\forall i,\tau}$. We need to use the solution that satisfies Blanchard and Kahn (1980) conditions, provided it exists.

For the elements of the second order, we need to take second order derivatives on $F$ and equalize to zero,

$$F_{q,r}(\Upsilon^{ss};0) = \mathbf{0}; \qquad \text{for } q,r = \{z, \{\theta_j\}, \{x(j,\tilde{\tau})\}, \upsilon\} \tag{19}$$

which defines a linear system in $\left\{x_{q,r}^{ss}(i,\tau), V_{q,r}^{ss}(i,\tau), w_{q,r}^{ss}(i), p_{q,r}^{ss}(i)\right\}_{\forall i,j,\tau}$.

In this way, we recover all the coefficients of the second order Taylor expansion.

## Order of approximation

The goodness of the approximation depends on the shape of the unknown functions. While all numerical methods provide only an approximation to the true solution, a finite order perturbation may deliver poor approximations away from the stationary world.[58] Nonetheless, it has been shown that in many economic applications perturbation methods perform reasonably well (Aruoba et al. (2006) and Caldara et al. (2011).)

[58]Note that for differentiable functions, Taylor's theorem states that the approximation converges to the true function globally as the order of the perturbation tends to infinity, making the approximation truly global.

At a computational level, one important question is whether an approximation of order one is sufficient. There are several reasons to use perturbation of a higher order. On the one hand, in the model $x'(i,\tau,\Upsilon), N(i,\tau,\Upsilon), M(i,j,\tau,\Upsilon)$ are guaranteed to be strictly between zero and one. However, in the approximate solution this may not hold and some large shocks may lead to implausible values for these variables.[59] A higher order approximation may capture the curvature of this functions better, and reduce the probability of getting implausible values in the simulations.

Besides a more accurate approximation, there is other important reason to use higher order perturbations. As is well known, a linear approximation to the solution displays Certainty Equivalence and risk plays no role in the agents decisions. In the context of simple representative agent economies, a first order perturbation provides a reasonably good approximation to the dynamics of the endogenous variables.[60] However, in this model a first order perturbation may provide a poor approximation, even with standard shocks processes. The reason is that workers' decision on where to locate (and the value of working versus not) is done at the extensive margin and differences in the risk of the different choices affect this decision. Since flows are proportional to stocks, the dynamics of the key variables in the model will in turn be affected by the characteristics of the shock process, an element that is completely missed with a first-order/certainty equivalent solution. In other words, in this model, in contrast with much of the literature of perturbation in representative agents economies, risk can be of first order importance for accurately describing the dynamics of the labor market.

## Appendix D: Identification of sectoral and aggregate shock properties

Identification of the properties of sectoral and aggregate shocks is central for the question I pursue in this paper. Here I characterize the dynamics of the sectoral and aggregate shocks and the sensitivities of each sector to the aggregate component are identified, up to normalization, provided we observe sectoral TFP for three or more sectors.

I follow the discussion in Stock and Watson (1992), which is part of the first generation of dynamic factor models (Stock and Watson, 2011). Let $A_{i,t}$ represent TFP at time $t$ for sector $i$, which is observed, and let $z_t$ and $\theta_{i,t}$ be the aggregate and sectoral shocks, which

[59] It is well known that perturbation techniques cannot handle inequality constraints. Therefore, a constraint like $M(i,j,\tau,\Upsilon) \geq 0$ cannot be enforced.

[60] In simple representative agent models, with no stochastic volatility or strong non-linearities, a linear approximation to the policy function generally misses the level of the endogenous variables but captures approximately well the changes. Therefore, only welfare calculations can be severely affected (Kim and Kim, 2003).

are unobserved. The assumptions are:

$$\begin{aligned}
\log(A_{i,t}) &= \lambda_i z_t + \theta_{i,t} \\
z_t &= \rho_z z_{t-1} + (1-\rho_z^2)^{0.5}\xi_{z,t} \\
\theta_{i,t} &= \rho_i \theta_{i,t} + (1-\rho_i^2)^{0.5}\xi_{i,t}
\end{aligned}$$

where $\xi$ are the innovations which are distributed iid independent of each other, and $E[\xi_{z,t}^2] = 1$; $E[\xi_{i,t}^2] = \sigma_i^2$. Note that the variance of $\xi_z$ cannot be identified separately from the average level of $\lambda$ and a normalization is required. This is inconsequential for the analysis and therefore I normalize the variance of $\xi_z$ to one, as is usual in the literature. Under these assumptions, we have:

$$\begin{aligned}
E[z_t] &= 0 \\
E[\theta_{i,t}] &= 0 \\
E[\theta_{i,t-h}\, z_{t-k}] &= 0 \qquad \forall h,k \\
E[z_t^2] &= 1 \\
E[\theta_{i,t}^2] &= \sigma_i^2 \\
E[z_t\, z_{t-h}] &= \rho_z^h \qquad h \geq 1 \\
E[\theta_{i,t}\, \theta_{i,t-h}] &= \rho_i^h \sigma_i^2 \qquad h \geq 1
\end{aligned}$$

The parameters $\lambda_i, \rho_i, \rho_z, \sigma_i$ can be identified directly from the following set of moments in $\tilde{A}_{i,t} = \log(A_{i,t})$, which is observed,[61]

$$\begin{aligned}
E[\tilde{A}_{i,t}^2] &= \lambda_i^2 + \sigma_i^2 \\
E[\tilde{A}_{i,t-1}\, \tilde{A}_{i,t}] &= \lambda_i^2 \rho_z + \rho_i \sigma_i^2 \\
E[\tilde{A}_{i,t}\, \tilde{A}_{j,t}] &= \lambda_i \lambda_j \\
E[\tilde{A}_{i,t}\, \tilde{A}_{j,t-1}] &= \lambda_i \lambda_j \rho_z
\end{aligned}$$

In addition, under the assumption that $\xi$ are normally distributed, we can use the Kalman filter to estimate the parameters by maximum likelihood, which is the procedure I use for the estimates in Table 7.

In matrix notation, the state space representation is:

$$\tilde{A}_t = C s_t \tag{20}$$
$$s_t = K\, s_{t-1} + \Omega\, \xi_t \tag{21}$$

[61]This is just an example. Clearly, the parameters can be recovered using other moments.

where

$$
\begin{aligned}
s_t &= \begin{bmatrix} z_t & \theta_{1,t} & \dots & \theta_{J,t} \end{bmatrix} \\
\xi_t &= \begin{bmatrix} \xi_{z,t} & \xi_{1,t} & \dots & \xi_{J,t} \end{bmatrix} \\
C &= \begin{bmatrix} \lambda_1 & 1 & 0 & \dots & 0 \\ \lambda_2 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_J & 0 & 0 & \dots & 1 \end{bmatrix} \\
K &= \begin{bmatrix} \rho_z & 0 & \dots & 0 \\ 0 & \rho_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \rho_J \end{bmatrix} \\
\Omega &= \begin{bmatrix} (1-\rho_z^2)^{0.5} & 0 & \dots & 0 \\ 0 & (1-\rho_1^2)^{0.5} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & (1-\rho_J^2)^{0.5} \end{bmatrix}
\end{aligned}
$$

and

$$
\begin{aligned}
E[\xi_t] &= \begin{bmatrix} 0 & 0 & \dots & 0 \end{bmatrix}' \\
Var[\xi_t] &= \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_1^2 & 0 & \dots & 0 \\ 0 & 0 & \sigma_2^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_J^2 \end{bmatrix}
\end{aligned}
$$

Equation (20) is the observation (measurement) equation and (21) is the transition equation.

# References


Abraham, K. and Katz, L. (1986). Cyclical unemployment: Sectoral shifts or aggregate disturbances? *Journal of Political Economy*, 94(3):507–522.

Acemoglu, D., Carvalho, V., Ozdaglar, A., and Tahbaz-Salehi, A. (2012). The network origins of aggregate fluctuations. *Econometrica*, 80(5):1977–2016.

Alvarez, F. and Shimer, R. (2009). Unemployment and human capital. *University of Chicago, mimeo*.

Alvarez, F. and Shimer, R. (2011). Search and rest unemployment. *Econometrica*, 79(1):75–122.

Alvarez, F. and Veracierto, M. (2000). Labor-market policies in an equilibrium search model. *NBER Macroeconomics Annual 1999*, page 265.

Artuc, E., Chaudhuri, S., and McLaren, J. (2010). Trade shocks and labor adjustment: A structural empirical approach. *American Economic Review*, 100(3):1008–45.

Aruoba, S., Fernandez-Villaverde, J., and Rubio-Ramirez, J. (2006). Comparing solution methods for dynamic equilibrium economies. *Journal of Economic Dynamics and Control*, 30(12):2477–2508.

Bigio, S. and La'O, J. (2013). Financial frictions in production networks. *University of Chicago Booth Working Paper*.

Bils, M., Chang, Y., and Kim, S.-B. (2012). Comparative advantage and unemployment. *Journal of Monetary Economics*, 59(2):150–165.

Blanchard, O. and Kahn, C. (1980). The solution of linear difference models under rational expectations. *Econometrica: Journal of the Econometric Society*, pages 1305–1311.

Bloem, A. M., Dippelsman, R., and Mæhle, N. Ø. (2001). *Quarterly national accounts manual: concepts, data sources, and compilation*. International Monetary Fund.

Boldrin, M., Christiano, L. J., and Fisher, J. D. (2001). Habit persistence, asset returns, and the business cycle. *American Economic Review*, pages 149–166.

Broda, C. and Weinstein, D. (2006). Globalization and the gains from variety. *The Quarterly Journal of Economics*, 121(2):541–585.

Caldara, D., Fernandez-Villaverde, J., Rubio-Ramirez, J., and Yao, W. (2011). Computing DSGE models with recursive preferences and stochastic volatility. *Review of Economic Dynamics*.

Caliendo, L., Rossi-Hansberg, E., Parro, F., and Sarte, P.-D. (2013). The impact of regional and sectoral productivity changes on the us economy. *Working Paper*.

Campbell, J. (1998). Entry, exit, embodied technology, and business cycles. *Review of Economic Dynamics*, 1(2):371–408.

Carrillo-Tudela, C. and Visschers, L. (2013). Unemployment and endogenous reallocation over the business cycle. *Working Paper*.

Carvalho, V. M. (2010). Aggregate fluctuations and the network structure of intersectoral trade. *Working Paper*.

Chen, B. (2007). An empirical comparison of methods for temporal distribution and interpolation at the national accounts. *BEA Papers*, (0077).

Christiano, L. J. and Fitzgerald, T. J. (1998). The business cycle: it's still a puzzle. *Economic Perspectives-Federal Reserve Bank Of Chicago*, 22:56–83.

Coen-Pirani, D. (2010). Understanding gross worker flows across us states. *Journal of Monetary Economics*, 57(7):769–784.

Davis, S. J. and von Wachter, T. M. (2011). Recessions and the cost of job loss. *NBER Working Paper*.

Elsby, M., Hobijn, B., and Sahin, A. (2012). On the importance of the participation margin for labor market fluctuations. *Working Paper*.

Farber, H. S. (2011). Job loss in the great recession: Historical perspective from the displaced workers survey, 1984-2010. *NBER Working Paper*.

Fernandez-Villaverde, J., Guerron-Quintana, P., Rubio-Ramirez, J., and Uribe, M. (2011). Risk matters: The real effects of volatility shocks. *American Economic Review*, 101(6).

Floden, M. and Lindé, J. (2001). Idiosyncratic risk in the United States and Sweden: Is there a role for government insurance? *Review of Economic Dynamics*, 4(2):406–437.

Foerster, A., Sarte, P., and Watson, M. (2011). Sectoral versus aggregate shocks: A structural factor analysis of industrial production. *Journal of Political Economy*, 119(1).

Fujita, S. and Moscarini, G. (2012). Recall and unemployment. *Working Paper*.

Gomme, P. and Klein, P. (2011). Second-order approximation of dynamic models without the use of tensors. *Journal of Economic Dynamics and Control*, 35(4):604–615.

Gouge, R. and King, I. (1997). A competitive theory of employment dynamics. *The Review of Economic Studies*, 64(1):1–122.

Hagedorn, M. and Manovskii, I. (2008). The cyclical behavior of equilibrium unemployment and vacancies revisited. *American Economic Review*, 98(4):1692–1706.

Hamilton, J. D. (1988). A neoclassical model of unemployment and the business cycle. *The Journal of Political Economy*, pages 593–617.

Hornstein, A. and Praschnik, J. (1997). Intermediate inputs and sectoral comovement in the business cycle. *Journal of Monetary Economics*, 40(3):573–595.

Horvath, M. (2000). Sectoral shocks and aggregate fluctuations. *Journal of Monetary Economics*, 45(1):69–106.

Jacobson, L., LaLonde, R., and Sullivan, D. (1993). Earnings losses of displaced workers. *The American Economic Review*, pages 685–709.

Jones, C. I. (2013). Misallocation, economic growth, and input-output economics. In *Advances in Economics and Econometrics, Tenth World Congress*.

Jovanovic, B. (1987). Micro shocks and aggregate risk. *The Quarterly Journal of Economics*, 102(2):395–409.

Judd, K. (1998). *Numerical methods in economics*. MIT press.

Judd, K. L. (1996). Approximation, perturbation, and projection methods in economic analysis. *Handbook of computational economics*, 1:509–585.

Kambourov, G. and Manovskii, I. (2008). Rising occupational and industry mobility in the united states: 1968-97. *International Economic Review*, 49(1):41–79.

Kambourov, G. and Manovskii, I. (2009a). Occupational mobility and wage inequality. *Review of Economic Studies*, 76(2):731–759.

Kambourov, G. and Manovskii, I. (2009b). Occupational specificity of human capital. *International Economic Review*, 50(1):63–115.

Katz, L. and Summers, L. (1989). Industry rents: Evidence and implications. *Brookings Papers on Economic Activity. Microeconomics*, 1989:209–290.

Kennan, J. and Walker, J. (2011). The effect of expected income on individual migration decisions. *Econometrica*, 79(1):211–251.

Kim, J. and Kim, S. (2003). Spurious welfare reversals in international business cycle models. *journal of International Economics*, 60(2):471–500.

Kim, K. and Kim, Y. S. (2006). How important is the intermediate input channel in explaining sectoral employment comovement over the business cycle? *Review of Economic Dynamics*, 9(4):659–682.

Kline, P. (2008). Understanding sectoral labor market dynamics: An equilibrium analysis of the oil and gas field services industry. *Department of Economics, UC Berkeley, mimeo*.

Kopecky, K. and Suen, R. (2010). Finite state Markov-chain approximations to highly persistent processes. *Review of Economic Dynamics*, 13(3):701–714.

Krusell, P., Mukoyama, T., Rogerson, R., and Şahin, A. (2012). Is labor supply important for business cycles? *National Bureau of Economic Research*.

Krusell, P. and Smith, Jr, A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):867–896.

Lee, D. and Wolpin, K. (2006). Intersectoral labor mobility and the growth of the service sector. *Econometrica*, 74(1):1–46.

Lilien, D. (1982). Sectoral shifts and cyclical unemployment. *The Journal of Political Economy*, pages 777–793.

Long, J. B. and Plosser, C. I. (1983). Real business cycles. *The Journal of Political Economy*, pages 39–69.

Low, H., Meghir, C., and Pistaferri, L. (2010). Wage risk and employment risk over the life cycle. *American Economic Review*, 100(4):1432–1467.

Lucas, R. and Prescott, E. (1974). Equilibrium search and unemployment. *Journal of Economic Theory*, 7(2):188–209.

Lucas, R. E. (1977). Understanding business cycles. *Carnegie-Rochester Conference Series on Public Policy*, 5(0):7 – 29.

Mangum, K. (2010). A dynamic model of local labor markets. *unpublished, Department of Economics, Duke University*.

Meghir, C. and Pistaferri, L. (2011). Earnings, consumption and life cycle choices. *Handbook of Labor Economics*, 4:773–854.

Menzio, G. and Shi, S. (2011). Efficient search on the job and the business cycle. *Journal of Political Economy*, 119(3):468–510.

Moscarini, G. and Thomsson, K. (2007). Occupational and job mobility in the US. *The Scandinavian Journal of Economics*, 109(4):807–836.

Murphy, K. and Topel, R. (1989). The evolution of unemployment in the United States: 1968-1985. *NBER Macroeconomics Annual*.

Neal, D. (1995). Industry-specific human capital: Evidence from displaced workers. *Journal of labor Economics*, pages 653–677.

Pilossoph, L. (2012). A multisector equilibrium search model of labor reallocation. *Working Paper*.

Postel-Vinay, F. and Robin, J.-M. (2002). Equilibrium wage dispersion with worker and employer heterogeneity. *Econometrica*, 70(6):2295–2350.

Ravn, M. O. and Uhlig, H. (2002). On adjusting the Hodrick-Prescott filter for the frequency of observations. *The Review of Economics and Statistics*, 84(2):371–375.

Reiter, M. (2009). Solving heterogeneous-agent models by projection and perturbation. *Journal of Economic Dynamics and Control*, 33(3):649–665.

Robin, J. (2011). On the dynamics of unemployment and wage distributions. *Econometrica*, 79(5):1327–1355.

Rouwenhorst, G. K. (1995). Asset pricing implications of equilibrium business cycle models. In Cooley, T., editor, *Frontiers of Business Cycle Research*, pages 294–330. Princeton University Press.

Roy, A. D. (1951). Some thoughts on the distribution of earnings. *Oxford economic papers*, 3(2):135–146.

Rust, J. (1987). Optimal replacement of GMC bus engines: An empirical model of Harold Zurcher. *Econometrica: Journal of the Econometric Society*, pages 999–1033.

Rust, J. (1994). Structural estimation of Markov decision processes. *Handbook of econometrics*, 4:3081–3143.

Scheinkman, J. A. and Woodford, M. (1994). Self-organized criticality and economic fluctuations. *The American Economic Review*, 84(2):417–421.

Sims, C. A. (2002). Solving linear rational expectations models. *Computational Economics*, 20(1):1–20.

Stock, J. H. and Watson, M. W. (1992). A probability model of the coincident economic indicators. *Leading Economic Indicators: New Approaches and Forecasting Records*, page 63.

Stock, J. H. and Watson, M. W. (2011). Dynamic factor models. *Oxford Handbook of Economic Forecasting. Oxford University Press, USA*, pages 35–59.

Sullivan, P. (2010). Empirical evidence on occupation and industry specific human capital. *Labour Economics*, 17(3):567–580.

Veracierto, M. (2008). On the cyclical behavior of employment, unemployment and labor force participation. *Journal of Monetary Economics*, 55(6):1143–1157.

Weintraub, G. Y., Benkard, C. L., and Roy, B. V. (2008). Markov perfect industry dynamics with many firms. *Econometrica*, 76(6):1375–1411.

Wiczer, D. (2013). Long-term unemployment: Attached and mismatched. *Working Paper*.

Yellen, J. (1989). The Beveridge curve: comment. *Brookings papers on economic activity*, 1989(1):1–76.

Young, N. (1989). *Wave seam effects in the SIPP*. US Department of Commerce, Bureau of the Census.