Joseph's Monetary and Fiscal Policy

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Abstract

This paper considers monetary and fiscal policy in liquidity traps when consumption can be stored. As in the biblical story of Joseph managing seven fat years to prepare for seven lean years, the flexible price allocation uses storage. Even though liquidity traps arise from excess demand for intertemporal substitution, they reduce current consumption and storage. Monetary policy that influences price-level expectations can solve the liquity trap, but it can remain even when inflation expectations are arbitrarily high. In this case, fiscal policy can coordinate households' savings decisions with a bond auction and thereby guide the economy to the flexible-price allocation.

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1 Introduction

On Pharoh's behalf, Joseph accumulated grain during seven fat years to smooth consumption into seven prophetically foreseen lean years. If Joseph had managed a New Keynesian economy without storage and with only monetary policy at his disposal, the seven fat years could have fallen into a liquidity trap. In this paper, I add potentially costly storage to a New Keynesian model with nominal rigidities the monetary and fiscal policies that can be used to avoid a liquidity trap and successfully accumulate assets during fat years for consumption in lean years.

The model features a single household with an endowment of time that has preferences over leisure and a CES aggregate of differentiated consumption goods. A monopolist produces each good using a linear technology with labor as is only input, and it faces a nominal rigidity in choosing its price. As in Woodford (2003), money only serves as a unit of account. The household also has access to a constant-returns-to-scale storage technology for the *aggregate* good. The technology might be costly, as is the case with actual storage, or it could give a positive return, as would be the case if "storage" took place by selling the good abroad, investing the resulting revenue, and repatriating the investment as goods in a future period. Households also trade in a market for one-period risk-free nominal bonds. Since they are in zero net supply, they cannot serve as a real means of intertemporal substitution. A monetary authority sets the interest rate on these bonds subject to a non-negativity constraint, the zero lower bound.

In the liquidity-trap scenario, labor's productivity at producing consumption goods temporarily has a high value for a fixed time span, afterwards it certainly falls to a low "normal" value thereafter. For algebraic simplicity, the time with high productivity is set to one period rather than the biblically-suggested seven, and the time at low productivity lasts forever instead of seven periods. If initial productivity is high enough, then households use the storage technology in the flexible-price equilibrium.

As is familiar from Eggertsson and Woodford (2003), the impact of introducing nominal rigidities heavily depends on whether we hold fixed households' expectations of the future price level or their expectations of future inflation. If either the fixed price-level are high enough, then the equilibrium allocation with nominal rigidities is identical to that with flexible prices. The necessary condition for optimal storage determines the growth rate of real consumption, and the price-level expectations leave room for the monetary authority to clear the bond market at a non-negative interest rate when the initial period's consumption equals its flexible-price level. If the household has fixed expectations for the price level that are too low, then the zero lower bound on nominal interest rates constrains the monetary authority. In this case, the economy undergoes a deflationary recession so that the bond market can clear at a zero nominal interest rate. At its worst, this liquidity trap induces households to engage in no storage at all: Although the liquidity trap is sometimes referred to as a "paradox of thrift," appropriate management of price-level expectations *raises* real aggregate savings.

When expectations of inflation are fixed, then any deflation in the initial period contributes nothing to clearing the bond market. In that case, appropriate expectations management is much less efficacious. If inflation expectations are too low, the economy indeed falls into a liquidity trap in which no storage occurs. However, raising expectations to an arbitrarily high level does not necessarily bring the economy to the flexible-price allocation. Instead, multiple equilibria arise. The best equilibrium does indeed mimic the flexible-price allocation, but the worst equilibrium features no storage at all. A deflationary coordination game underlies this multiplicity: If firms that can change their prices foresee deflation, they choose low prices today, which lowers both real aggregate consumption and marginal cost and thereby confirms their original expectations. A particular fiscal policy can solve this coordination failure: The fiscal authority holds a bond auction with a particular structure, invests the proceeds in the storage technology, and liquidates the bond position in the next period when technology has reverted to its low value. The auction is desgined so that in its unique equilibrium all of the bonds offered for sale are purchased at the market-clearing interest rate. This coordinates expectations on high future consumption, which in turn leads to high current consumption. Under the model's international interpretation, this policy resembles a central bank accumulating foreign reserves to promote exports by devaluing the domestic currency.

A liquidity trap arises when the demand for risk-free nominal bonds exceeds their supply at the lower bound on nominal interest rates. This paper shows how adding a real technology for intertemporal substitution can substantially alter the analysis of monetary and fiscal policy in a liquidity trap. Although Krugman's (1998) primary analysis was in a closed economy without capital accumultion or international trade, he argued that adding these features to his model would impact his results only little. He dismissed capital accumulation as an outlet for excess savings demand by appealing to large capital adjustment costs. As evidenced by the large movements of inventory investment over the businss cycle, adjustment costs for physical storage are nearly zero. His discussion of using exports to stimulate aggregate demand takes the shortcut "that one can ignore the effect of the current account on the future investment income of the country."[1] This paper shows that accounting for the country's future investment income is crucial for designing appropriate fiscal and monetary policy in a liquidity trap and to understanding the results. The goal of such policy is not to stimulate "aggregate demand." Instead, policy should aim at achieving households' desired goals for intertemporal substitution. With those goals met, current income takes care of itself.

Conventional analyses of the liquidity trap typically follow Krugman's (1998) lead and abstract from storage or any other real investment technology. Christiano, Eichenbaum, and Rebelo (2011) provide an exception by incorporating capital accumulation into an otherwise standard model in which the liquidity trap lasts for a fixed number of periods. They find that

> When the zero bound binds, the real interest rate generally rises. So, other things equal, saving and investment diverge as the real interest rate rises, thus exacerbating the meltdown associated with the zero bound. As a result, the fall in output necessary to bring saving and investment into alignment is larger than in the model without capital.[2]

This paper's results provide two perspectives on this result. First, the "meltdown" in output is larger with storage is larger because *potential* output is larger. If either inflation or price-level expectations are low enough, then output in the liquidity trap is the same with or without capital accumulation. Second, one might infer that appropriate monetary policy is even more effective in a liquidity trap with capital accumulation. The results show that this is true with fixed expectations of future price levels but not if expectations of future inflation are fixed. In the latter case, monetary policy requires an assist from fiscal policy. Importantly, appropriate fiscal policy does not work through the conventional channel of fighting deflation by

[1] See (Krugman, 1998, Page 164).
[2] See (Christiano, Eichenbaum, and Rebelo, 2011, page 81).

raising current marginal cost. Instead, it works by coordinating expectations on the flexible-price equilibrium allocations.

The remainder of this paper proceeds as follows. The next section contains the model's primitive assumptions, and Section 3 presents its flexible-price allocation. Sections 4 and 5 present the results for the model with nominal rigidities and fixed price-level expectations and fixed inflation expectations, respectively. Section 6 develops the interpretation of the model as a small open economy that stores consumption by trading the aggregate good with a large foreign sector. Section 7 contains concluding remarks.

2 Primitive Assumptions

The model features three key features of New Keynesian economies, monopolistic competition so that goods prices are set by specific agents rather than by a Walrasian auctioneer, nominal rigidities which generate a Phillips curve trading off inflation and output, and a competitive nominal bond market with an interest rate set by a monetary authority.

A single representative household populates the model economy. Its preferences over streams of consumption goods and time spent at work are

$$\begin{aligned} U(\{C_t\},\{N_t\}) &= \sum_{t=0}^{\infty} \beta^t \left(\ln C_t + \theta(1-N_t)\right), \text{ with} \\ C_t &= \left(\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}. \end{aligned}$$

Here $\varepsilon > 1$ is the elasticity of substitution between any two of the differentiated goods, each of which has a name on the unit interval. It is well-known that these preferences feature an infinite Frisch elasticity of labor supply. Although this might be an appropriately mimic the results of having sticky nominal wages, I adopt it here mostly for algebraic convenience. There is no uncertainty in this economy, so preferences are defined only over certain streams of consumption and time working. Therefore, risk-aversion plays no role in this analysis. Similarly, I implicitly set the elasticity of intertemporal substitution to one only to avoid unnecessary parameter proliferation.

The technology for producing each of the differentiated goods is the same: one unit of labor yeilds A_t units of the good in question. As specified in the introduction, $A_t = A^L$ for all $t \geq 1$ and $A_0 = A^H > A^L$. The economy's other "production" technology is that for storage. For each unit of the *aggregate* consumption good set aside by the household, $(1 - \delta)$ units of the same good are available in the next period. Written this way, δ is the depreciation rate on storage, although nothing in the analysis below constrains this to be positive. If δ is negative, then storage has a positive return. I interpret this an infinitely-elastic real demand for capital, like that faced by a small open economy. To prevent explosive equilibria, I assume that $1 - \delta \leq \beta^{-1}$.

Product markets conform to the familiar monopolistic competition framework. Each product's monopolist chooses its nominal price taking as given all other products' prices, aggregate income, and the household's demand system for all of the differentiated products. The monopolists' nominal rigidity resembles the sticky information setup of Mankiw and Reis (2002). Each period, half of the economy's producers set their nominal prices for the current and next periods. Unlike in a sticky-price model, the two periods' prices may be different from each other. Mankiw and Reis adopt the Calvo (1983) assumption that the timing of each monopolist's opportunity to set prices is deterministic. To keep the algebra simple, I assume instead that it deterministically arrives every two periods. Mankiw and Reis (2002) assert that sticky information delivers an empirically superior Phillips curve over sticky prices. Its use here has a more modest motivation: By eliminating intertemporal trade-offs in price setting, I focus the analysis on intertemporal substitution and the monetary obsticals to its execution.

The household participates in two other markets, for labor and one-period risk-free nominal bonds. The Walrasian auctioneer sets the wage rate, and the monetary authority sets the nominal interest rate.

3 The Flexible-Price Allocation

The equilibrium allocation when producers face no nominal rigidities serves as a baseline for the subsequent analysis. Denote the price of good $j \in [0, 1]$ with $P_t(j)$. The household's optimal allocation of nominal consumption expenditures across differentiated goods has the familiar form:

$$C_t(j) = C_t \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon}$$

with P_t the aggregate price index

$$P_t \equiv \left(\int_0^1 P_t(j)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}.$$

By construction, $P_t C_t$ is the household's nominal consumption expenditure.

Given the household's demand for differentiated products, we can state the household's optimization problem as choosing sequences of aggregate consumption, hours worked, the face value of bonds in the next period (B_{t+1}), and the quantity of the good available from storage in the next period (S_{t+1}) to maximize utility subject to non-negativity constraints on labor and storage, the initial values of B_t and S_t, and the intertemporal budget constraint,

$$P_t C_t + B_{t+1}/(1 + i_t) \leq W_t N_t + B_t + D_t \tag{1}$$

Here, D_t are dividends returned from the household's ownership of the producers, i_t is the nominal interest rate, and W_t is the nominal wage. Adding an equity market so that households can trade producers' shares would be straightforward but adds nothing to the analysis. Instead, I impose the market-clearing result that the representative household owns all of the producers' shares.

If we denote the Lagrange multipliers on the period t budget constraint, non-negativity constraint on labor, and non-negativity constraint on storage with $\beta^t \lambda_t / P_t$, $\beta^t \lambda_t \nu_t$, and $\beta^t \lambda_t \upsilon_t$, the utility maximization problem yields familiar conditions for optimal labor supply, optimal bond purchases, and optimal storage.

$$\theta C_t = \frac{W_t}{P_t} + \upsilon_t \tag{2}$$

$$1 = \beta(1 + i_t) \frac{P_t C_t}{P_{t+1} C_{t+1}} \tag{3}$$

$$1 = \beta(1 - \delta) \frac{C_t}{C_{t+1}} + \nu_t \tag{4}$$

In the flexible-price baseline, producers always set the optimal monopoly price

$$P_t = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{W_t}{A_t} \tag{5}$$

Given $S_0 \geq 0$ and the productivity sequence A_t, a flexible-price equilibrium is collection of sequences for C_t, N_t, B_{t+1}, S_{t+1}, D_t, W_t, P_t, and i_t such that

1. the choices for C_t, N_t, B_{t+1}, S_{t+1} solve the household's utility maximization problem given D_t, W_t, P_t, and i_t;

2. the price choice satisfies (5);

3. the dividends returned equal producers' profits,

$$D_t = \frac{1}{\varepsilon - 1} \frac{C_t W_t}{A_t};$$

4. the market for labor clears,

$$A_t N_t = C_t + S_{t+1}/(1 - \delta) - S_t;$$

5. the market for bonds clears, so $B_{t+1} = B_0 = 0$; and

6. the interest rate satisfies the zero lower bound constraint $i_t \geq 0$.

It is straightforward to show that there are many flexible-price equilibria, but that they all share a single allocation of consumption, storage, and hours worked. To reduce the number of cases under review, I suppose that $S_0 = 0$ for the remainder of the paper. If the flexible-price allocation also sets $S_t = 0$ for all $t \geq 1$, then consumption and labor supply must satisfy

$$\begin{aligned} C_t &= \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{A_t}{\theta}, \text{ and} \\ N_t &= C_t / A_t. \end{aligned}$$

These outcomes are consistent with bond-market clearing and the zero lower bound for any sequence of price levels that satisfies

$$\beta \frac{P_t A_t}{P_{t+1} A_{t+1}} \leq 1$$

for all $t \geq 0$. In the liquidity trap scenario, this only requires that $P_1 A^L \geq \beta P_0 A^H$. Since $1 - \delta \leq \beta^{-1}$, zero storage yields a non-negative value of ν_t in Equation 4 for $t \geq 1$. To obtain a non-negative value of ν_0, we require that

$$\frac{A^L}{\beta A^H} \geq (1 - \delta). \tag{6}$$

This says that A^H cannot be too far above A^L for storage to remain unused.

Now, suppose that (6) does not hold. In this case, C_0 remains the same, $\nu_0 = 0$, and Equation (4) determines consumption growth between the first two periods giving us

$$C_1 = \beta(1 - \delta) \left(\frac{\varepsilon - 1}{\varepsilon} \right) \frac{A^H}{\theta}$$

By construction, this value for C_1 exceeds that consistent with satisfying Equation (2) with $\upsilon_1 = 0$, so the non-negativity constraint on labor supply binds in period 1 and the C_1 is financed entirely out of storage.

To determine whether or not the household extends is vacation from period 1 into period 2 by setting $S_2 > 0$, we examine again Equation (4) for $t = 1$. Setting $\nu_1 = 0$, this yields

$$C_2 = \beta(1 - \delta) C_1 = \beta^2 (1 - \delta)^2 \left(\frac{\varepsilon - 1}{\varepsilon} \right) \frac{A^H}{\theta}.$$

Period 2 consumption must also satisfy the optimal labor supply condition with a non-negative value of υ_t. If $\frac{A^L}{\beta A^H} \leq \beta(1 - \delta)^2$, then this is indeed the case; so $N_2 = 0$ and $S_2 > 0$. In general, the household's vacation (financed by storage) will last through period T, the largest integer such that

$$\frac{A^L}{\beta A^H} \leq \beta^{T-1} (1 - \delta)^T$$

Given this sequence of consumption, the period 0 intertemporal budget constraint determines N_0. Of course, a complete collapse of work and GDP in period 1 lasting through period T is unrealistic, but the analysis that follows would remain unchanged if we imposed adjustment costs on labor by constraining it with a positive lower bound, $\check{n}$. Similarly, creating the optimal storage requires a surge in labor supply in period 0. For convenience only, I have ignored any upper bound on hours worked.

4 Fixed Price-Level Expectations

The flexible-price baseline shows the Lucas and Rapping (1969) theory of intertemporal substitution an labor supply in action. Temporarily high real wages in period 0 induce the household to expand labor supply, accumulate savings, and raise consumption in future periods. The future consumption boom lowers hours worked and output while it lasts. The rest of this paper shows how producers' nominal rigidities and households' price expectations can interfere with this happy outcome. This requires examining producers' optimal pricing decisions with sticky information and appropriately redefining a competitive equilibrium. Denote the price chosen by a firm in period $t-j$ that will apply in period t with P_t^j, so P_t^0 is the price chosen by producers with a current price choice and P_t^1 is the price for t chosen in period $t-1$ by the the remaining producers.

Since there is no uncertainty, we have that the optimal choices are

$$P_t^0 = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{W_t}{A_t} \;\; \forall t \geq 0 \text{ and} \tag{7}$$

$$P_t^1 = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{W_t}{A_t} \;\; \forall t \geq 1. \tag{8}$$

The expressions in (7) and (8) are identical, but they apply to different periods. The preset price, P_0^1, is part of the economy's initial conditions. We normalize this to equal one. The two price choices determine P_t through its definition.

$$P_t = \left(\frac{1}{2}P_t^{0\;1-\varepsilon} + \frac{1}{2}P^{1\;1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \tag{9}$$

With this notation in place, we define a sticky-information equilibrium with sticky information as a collection of sequences for C_t, N_t, B_{t+1}, S_{t+1}, D_t, W_t, P_t^0, P_{t+1}^1, P_t, and i_t such that given $P_0^1 = 1$, S_0, and the productivity sequence A_t,

1. the choices for C_t, N_t, B_{t+1}, S_{t+1} solve the household's utility maximization problem given D_t, W_t, P_t, and i_t;

2. P_t^0 satisfies (7);

3. P_t^1 satisfies (8);

4. the dividends returned equal producers' profits,

5. the market for labor clears,

6. the market for bonds clears, so $B_{t+1} = B_0 = 0$, and

7. the interest rate satisfies the Zero Lower Bound constraint $i_t \geq 0$.

This definition has omitted the expressions for producers' profits and labor demand because they are tedious an unimportant for the analysis.

As with flexible prices, we can demonstrate constructively that there are many sticky-information equilibria. In this section, we consider only those with fixed expectations of the *price level*: Households and producers rationally expect $P_t = P^\star$ for all $t \geq 1$. Suppose that (6) holds good, so that the flexible-price allocation sets $S_t = 0$ always. To construct the corresponding equilibrium in the sticky-information economy, suppose first that

$$i_t = \max\{0, \beta^{-1}\frac{P_{t+1}A_{t+1}}{P_t A_t} - 1\}$$

That is, the monetary authority sets the interest rate equal to its value in the flexible-price equilibrium if this is above the zero lower bound. Otherwise, it sets the interest rate to zero. For $t \geq 1$, this sets $i_t = \beta^{-1} - 1$, and the equilibrium consumption equals its flexible-price value. If the zero lower bound also does not constrain this choice in period 0, then the sticky-information equilibrium implements the entire flexible-price allocation. Producers set P_0^0 to one (so that there are no distortions in goods' relative prices) and the economy's output equals its potential.

If instead $P^\star < \beta A^H / A^L$, then the interest rate hits the zero lower bound and the first order condition for optimal bond supply determines

$$P_0 C_0 = \beta^{-1} P^\star \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{A^L}{\theta}.$$

This requires nominal income to fall below its flexible-price value. For this to occur, producers with current price choices set $P_0^0 < 1$, so the economy undergoes deflation. To determine how the price level and real consumption contribute to this nominal contraction, use the given value for P_0C_0 and condition for optimal labor supply in (2) to express the nominal wage as

$$W_0 = A^L \beta^{-1} P^\star \left(\frac{\varepsilon - 1}{\varepsilon}\right).$$

By construction, this is less than its flexible-price value. Using this to determine the nominal marginal cost in (7) yields

$$P_0^0 = P^\star \frac{A^L}{\beta A^H},$$

which is less than $P_0^1 = 1$ by assumption. Plugging this into (9) gives us

$$P_0 = \left(\frac{1}{2} + \frac{1}{2}\left(\frac{P^\star A^L}{\beta A^H}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

which is also less than one. The corresponding value of initial consumption is

$$C_0 = \beta^{-1} \frac{P^\star}{P_0} \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{A^L}{\theta},$$

which is less than its value in the flexible-price allocation.

The traditional interpretation of an equilibrium like this in which the economy falls into a liquidity trap labels C_0 *aggregate demand.* In this story, monetary policy that is made too tight by the zero lower bound lowers aggreate demand through (3), and this reduction in aggregate demand brings about an accompanying deflation. However, the construction of the equilibrium above suggests a very different chain of causation that also begins with (3). Given the price-level expectations $P^\star$, clearing the bond market requires lower nominal income, which in turn requires a lower nominal wage. The lower nominal wage induces those producers currently choosing prices to reduce them. If no prices were preset (as in the flexible-price equilibrium), then the resulting deflation would be enough to reestabish equilibrium in the bond market. However, the deflation raises the markup on those products with preset prices, and this markup increase reduces real income and consumption as well.[3] Phrased more succinctly, the economy undergoes a contraction to clear

[3] This intuition suggests that increasing price flexibility would amplify the deflation but reduce

the bond market. Although this interpretation is non-standard, I find it helpful for understanding liquidity traps with storage.

Since initial consumption never exceeds its flexible-price value, the above equilibrium is consistent with the supposition that $S_t = 0$ always. To generate storage in a sticky-information equilibrium, we must assume that it is used in a flexible-price equilibrium. That is, $A^L/(\beta A^H) < (1-\delta)$. To parallel the analysis without storage, I assume that from period 1 onwards the monetary authority sets i_t to implement the flexible-price allocation given the chosen values of S_1 and P_1^1.

If storage occurs in equilibrium, then the monetary authority must set i_0 so that the return on holding bonds equals that from holding storage. That is

$$i_0 = \frac{P^\star}{P_0}(1-\delta) - 1. \tag{10}$$

If $P^\star \geq 1/(1-\delta)$, then it is possible for the monetary authority to implement the flexible-price outcome while still respecting the zero lower bound. If instead $P^\star < 1/(1-\delta)$, then the monetary authority sets i_0 to zero and $P_0 < 1$. For the reasons noted above, this deflation reduces C_0. If the reduction is small enough, then $\beta(1-\delta)C_0$ still exceeds period 1 consumption when $S_1 = 0$, so storage still occurs. In this case, $P_0 = P^\star(1-\delta)$. This and (9) then give us

$$P_0^0 = \left(2\left(P^\star(1-\delta)\right)^{1-\varepsilon} - 1\right)^{\frac{1}{1-\varepsilon}}$$

Since $P^\star(1-\delta) < 1$ and $\varepsilon > 1$, this solution for P_0^0 is always well defined and less than $P^\star$. The accopanying value for C_0 is

$$C_0 = \frac{A^H}{\theta}\left(\frac{\varepsilon-1}{\varepsilon}\right)\left(2 - \left(P^\star(1-\delta)\right)^{\varepsilon-1}\right)^{\frac{1}{1-\varepsilon}},$$

which is less than its value in the flexible-price allocation.

Reducing $P^\star$ reduces both C_0 and C_1, but the limit of C_0 as P_0 goes to zero is positive. That is, a deflationary recession can only reduce consumption so much . If the associated limiting value of C_1 exceeds the value of C_1 consistent with no storage, then there is positive storage in the sticky-information equilibrium no

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the contraction of real output. It is not hard to verify this by supposing that producers with names in $[0, \phi]$ set prices every period and the remaining producers set price plans every other period. In that extension's liquidity trap, $\partial C_0/\partial\phi$¿0 and $\partial P_0/\partial\phi < 0$.

matter how low $P^\star$ goes. On the other hand, if this limiting value is lower than C_1's value without storage, then $S_1 = 0$ and the sticky-information equilibrium becomes identical to that without storage. For completeness, I summarize these results in the following

Proposition 1 *If $P^\star(1-\delta) < 1$, then the sticky-information equilibrium has $P_0 < 1$ and*

$$C_0 < \frac{A^H}{\theta}\left(\frac{\varepsilon - 1}{\varepsilon}\right)$$

Furthermore, if

$$\frac{A^L}{\beta A^H} < (1-\delta)2^{\frac{1}{1-\varepsilon}}$$

then $S_1 > 0$ for all $P^\star \in [0, 1/(1-\delta)]$. Otherwise, there exists a threshold level $\bar{P}^\star$ defined by

$$A^H\left(2 - \left(\bar{P}^\star(1-\delta)\right)^{\varepsilon-1}\right)^{\varepsilon-1}\Big)^{\frac{1}{1-\varepsilon}}\beta(1-\delta) = A^L$$

such that $S_0 > 0$ if and only if $P^\star > \bar{P}^\star$.

Two aspects of the sticky-information equilibrium are worth noting. First, a liquidity trap that depresses output with disinflation can coexist with positive storage. This contradicts the intuition that liqudity traps arise when the demand for intertemporal substitution opportunities exceeds their supply at the potential output. A more accurate intuition is that the demand for nominal risk-free bonds exceeds their supply at potential output. Second, raising $P^\star$ so that an initial disinflation is not required to equate the returns from bonds to those of storage can have a larger impact on output in the economy with storage, but this reflects an increase in potential output, not a fall in actual output relative to the case without storage.

5 Fixed Inflation Expectations

Eggertsson and Woodford (2003) find that optimal monetary policy closely resembles price-level targeting, and that deviating from this in favor of strict inflation targeting can be particularly damaging in a liquidity trap. While a full consideration of optimal monetary policy is well beyond this paper's scope, the results with fixed inflation expecations reinforce this message. In particular, *a liqidity trap can arise even when inflation expectations are consistent with the flexible-price allocation.*

Denote households' and producers' common inflation expectations with $\pi^\star = P_{t+1}/P_t$ for all $t \geq 0$. The definition of a sticky-information equilibrium above applies to the present case with fixed inflation expecations. When the flexible-price allocation uses no storage, neither does the sticky-information equilibrium. In this case, (3) in period 0 becomes

$$1 = \beta(1+i_0)\frac{1}{\pi^\star}\frac{C_0}{A^L}\theta\left(\frac{\varepsilon}{\varepsilon-1}\right).$$

If $\pi^\star$ is high enough then the monetary authority can set C_0 to its value in the flexible-price allocation with a non-negative interest choice. Otherwise, the zero lower bound binds and the economy falls into a liquidity trap. Period 0 consumption must fall to clear the bond market, and this occurs through a disinflation that raises markups of producers with currently fixed prices.[4]

When the flexible-price allocation uses storage, the sticky-information equilibrium with fixed inflation expectations might use storage. For this, a *necessary* condition is that inflation expectations are high enough. To see this, note that if storage is used in equilibrium then the condition for bond-market clearing in period 0 becomes

$$\frac{1+i_0}{\pi^\star} = 1-\delta$$

The only endogenous variable appearing here is i_0. If $\pi^\star$ is so high that no non-negative value of i_0 satisfies this, then storage is not used in equilibrium and the sticky-information equilibrium (if it exists) is given by the analysis for the case without storage. For the remainder of this section, I presume that inflation expectations are indeed high enough to facilitate storage.

The next task is to demonstrate that there exists multiple rational-expectations equilibria when storage is feasible. I first do so constructively. With the result in place, it can then be given an economic interpretation. To begin, assume that

$$\frac{A^L}{\beta A^H} > \left(\frac{1}{2}\right)^{\frac{1}{\varepsilon-1}}(1-\delta).$$

[4]If the required fall in C_0 is too large, then no feasible disinflation can bring it to the level required by bond-market clearing. In this case, no sticky-information equilibrium with fixed inflation expectations and output at potential beyond period 0 exists. This reflects a general principle of this model: The worst damage disinflation can do is equivilant to removing firms with preset prices from the market.

This bound on the size of the liquidity trap ensures that some deflation can achieve the required drop in output. Next, select any value of

$$C_1 \in \left(\frac{A^L}{\theta} \frac{\varepsilon - 1}{\varepsilon}, \beta(1-\delta) \frac{A^H}{\theta} \frac{\varepsilon - 1}{\varepsilon} \right)$$

The lowest value in this range is that associated with no storage, and the highest value is that from the flexible-price allocation with storage. This selection automatically determines C_0. The optimal labor supply condition (2) then determines the real wage W_0/P_0. Together, the conditions for optimal flexible prices (**??**) and price aggregation (9) imply that the initial price level solves

$$P_0 = \left(\frac{1}{2} + \frac{1}{2} \left(\frac{\varepsilon - 1}{\varepsilon} \beta^{-1} (1-\delta)^{-1} \frac{C_1}{A^H} P_0 \theta \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

The above condition on the size of the liquidity trap guarantees that P_0 is well-defined. Therefore, we have an equilibrium consistent with the chosen value of C_1.

Mechanically, multiple equilibria arise because the condition for optimal storage only determines consumption's growth rate, and the condition for optimal bond purchases is then redundant so long as inflation expectations are consistent with the zero lower bound on interest rates. Since financial markets do not determine C_0, a deflationary "sunspot" can take over the job. If producers with currently flexible prices expect a deflationary recession to lower their marginal costs, then they set low prices and thereby bring about the deflation they forecast.

Given fixed inflation expectations, monetary policy can do nothing to prevent such a sunspot deflation. Therefore, I turn to fiscal policy. Christiano et al. (2011) focus on "wasteful" goverment spending's ability to fight deflation by raising marginal cost. While this seems sensible in the present context with deflationary sunspots, I focus instead on using fiscal policy to coordinate expectations on the flexible-price allocation without raising marginal cost. For this, I suppose that the fiscal authority also has access to the storage technology, and that it offers a quantity of nominal bonds with nominal face value equal to $\pi^\star S_1$ (where S_1 is storage in the flexible-price allocation) to the households in an auction. In the auction, households can submit multiple bids, each for a single bond, and he S_1 bids with the highest interest rates acquire are fulfilled *at the highest interest rate of a winning bid.* If the number of bids with interest rates at or below the winning bid exceeds the quantity of bonds available, then bids with the highest winning interest rate are randomly fulfilled. It invests the proceeds in the storage technology, and it uses the resulting resources to liquidate the debt in period 1. (If the proceeds are insufficient for this liquidation, then the fiscal authority levies lump-sum taxes in period 1 to recover the shortfall.)

To see that this fiscal policy refines the equilibrium set down to that with the flexible-price allocation, first note that the households anticipate being able to sell any bond acquired in the auction in the bond market for $(1 + i_0)^{-1}$, so the bond market's price cannot be below this otherwise it would be oversubscribed and bidders at the highest winning interest rate could increase their payoffs by lowring their bid interest rates marginally and thereby discretely increasing their probabilities of purchasing the bond and making a profitable arbitrage sale in the secondary bond market. On the other hand, the price cannot be above $(1 + i_0)^{-1}$ because the household would do better by not bidding at all and purchasing the bond in the secondary market. Therefore, the price must equal $(1 + i_0)$. Second, note that the entire bond offering must be purchased in any equilibrium. Otherwise, a bidder could bid an interest rate slightly above i_0, win a bond, and sell it for a profit in the secondary market.

The real resources collected by the fiscal authority in its bond auction depend on P_0. The lower the price level is, the greater is the quantity of resources available. Therefore, this fiscal policy is always self-financing and never needs to rely on lump-sum taxes in period 1. Therefore, we can be sure that the real resources available to households in period 1 from their bond purchases are at least S_1, which in turn guarantees that C_1 is no less than its value in the flexible-price allocation. For these resources to be greater than their flexible-price value would require a deflation in period 0, which lowers C_0. This would make the household unwilling to invest in bonds, which contradicts the result that the fiscal authority places all of its bonds with the public. Therefore, the resources available in period 1 from households bonds exactly equal S_1, and consumption in period 0 therefore exactly equals C_0.

There are two points worth making regarding this fiscal policy. First and foremost, it does not require the government to engage in "wasteful" government spending to raise producers' marginal costs. Instead, the fiscal authority uses its position as a "large" player to coordinate households' savings so that consumption in period 1 is raised, which thereby raises period 0 consumption. Phrased differently, the policy ensures that households' intertemporal substitution is efficient given their interest rates and inflation expectations, and this in turn ensures that deflationary equilibria cease to exist. Second, it is important that the fiscal authority be prepared to sell bonds at an interest rate above i_0, even if this does not occur in equilibrium. Otherwise, the bond issue might not be fully subscribed, in which case the coordination benefits of the auction are lost. Put differently, the fiscal authority should not set a reserve price equal to i_0, either implicitly or explicitly.

6 An International Interpretation

Although I have heretofore interpreted "storage" literally, it is possible to interpret this more metaphorically as a stand-in for using international trade to achieve intertemporal substitution. For this, suppose that the economy is small relative to a large foreign sector. The aggregate good can be shipped either to or from the foriegn sector at the iceberg transportation cost τ. The foreign sector has a perfectly-stable currency that can be invested at a rate of return i^f, which is constant. Then, if we define δ with

$$(1-\delta) = (1-\tau)^2(1+i^f)$$

we can interpret storage as shipping aggregate good sabroad, selling them, investing the proceeds in foreign bonds, and repatriating the proceeds in the next period by shipping the aggregate good back home.[5]

This international interpretation of the model brings two points into focus. First, since the aggregate good is traded, any deflation resulting from a liquidity trap is bounded below by the iceberg transportation cost. In this case, if a sticky-inflation equilibrium without trade would require $P_0 < (1-\tau)$, then no sticky-inflation equilibrium with those price expectations will exist. Phrased differently, price-level or inflation expectations can only be so sticky in an open economy, because foreign prices constrain deflation. Second, in the open economy the fiscal policy presented above strongly resembles the exchange-rate policies of many central banks that accumulate foreign reserves in order to "devalue" their currencies and thereby "promote export-driven growth." Therefore, we might better understand those policies as

[5] For this equivalence to be complete, the length of the vacation T should equal 1.

implmenting the intertemporal substitution their citizens desire. The resulting exports are not merely an increment to "aggregate demand" but instead intrinsic to the policy of effecting intertemporal substitution.

7 Conclusion

To be completed.

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