Optimal capital requirements over the business and financial cycles*

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Abstract

I propose a simple theory of intertwined business and financial cycles, where financial regulation both optimally responds to and influences the business and financial cycles. In this model, government guarantees induce excessive aggregate lending by the financial sector. In response, the regulator sets capital requirements to trade-off expected output against financial stability (lower probability and social cost of a banking sector collapse). This trade-off depends on the state of the economy. Optimal capital requirements are therefore not constant. Because of a general equilibrium effect, optimal capital requirements increase with aggregate banking capital. A regulation that fails to take this effect into account would exacerbate economic fluctuations and result in excessive aggregate lending during a boom. It would also allow for an excessive build up of risk in the financial sector, which implies that, at the peak of a boom, even a small negative productivity shock can trigger a banking sector collapse, then followed by an excessively severe credit crunch.

1 Introduction

Motivation

The recent crisis has exposed how important the interactions between the financial sector (and financial regulation) and the real side of the economy (and macroeconomic policies) can be. Yet, most of the models used by researchers and policy makers to study these two spheres are separate, and there is no consensus on an integrated approach. I develop here a simple theory of intertwined business and financial cycles, where financial regulation both optimally responds to and influences them.

The questions I seek to address are:

- What are the general equilibrium effects of bank capital requirements?

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1 More generally, empirical evidence suggests that risks are built up in the financial system during good times (Borio and Drehmann (2009), and that financial booms do not just precede busts but cause them (Borio 2012). Also, the amplitude of the financial cycle is not constant, and is influenced by financial regulation regimes (Borio and Lowe 2002, Borio 2007). The model I develop here is consistent with those facts.

2 See for instance Goodhart (2010); Woodford (2010); Galati and Moessner (2011).
• Should bank capital requirements be tighter in “good times” and reduced in “bad times”?

• What macroeconomic variables are key for determining the optimal stringency of capital requirements?

To study these questions, I build a model where government guarantees induce excessive aggregate lending by the financial sector. In response, the regulator sets capital requirements to trade-off expected output against financial stability (lower probability and social cost of a banking sector collapse). This trade-off depends on the state of the economy. Optimal capital requirements are therefore not constant. Although other tools could equally be used by the regulator to correct the market failure in the model, the focus on capital requirements is motivated by the current policy debate on their effect on the real side of the economy, and in particular on the “pro-cyclical effects of bank regulation”.

Cyclically adjusted capital requirements have been used in Spain for a while (Jimenez et al. (2013)) and other countries have started to make discretionary adjustments based on the state of the economy. More generally, the introduction of “counter-cyclical capital buffers” is an explicit recommendation of “Basel III”, the latest version of the Basel Committee on Banking Supervision’s international standards for banking regulation. While this is not explicitly stated, the main logic seems the following: If “high” capital requirements are contractionary, such a cost has to be balanced with the benefits in terms of financial stability, or of tax-payer exposure to systemic financial crisis (Kashyap and Stein (2004)). If these costs and benefits are dependent on the state of the economy, optimal capital requirements may vary over the cycle.

Main features of the model

It is an overlapping generation model with savers and bankers. Bankers’ wealth is assumed to be scarce. They are protected by limited liability, they collect deposits and competitively lend to firms, which operate a constant-returns-to-scale risky production function. Bank lending is the only source of firm funding. Firms always make zero profits, and bankers are in effect the residual claimants of the production. Note that the decreasing marginal productivity of physical capital translates into decreasing marginal returns to lending. Or, from a reverse perspective, aggregate bank lending affects the marginal productivity of physical capital. This general equilibrium effect is a key feature of the model.

Deposits are insured. If the banks are not able to repay deposits in full, the regulator compensates depositors for the shortfall and levies lump-sum taxes to break even. As a result, the interest rate paid on deposits is insensitive to the risk taken by the banks. The interaction between this friction and diminishing returns to capital is the heart of a market failure in the model. Correcting this market failure is the rationale for regulation. Deposit insurance is taken in the model as an existing institutional feature. It is a particular and explicit form of government guarantee of bank liabilities, but one could alternatively build a model in which implicit guarantees would endogenously arise due to the inability of the government to fully commit not to bail out bank creditors. It would generate the same distortion and lead to similar results.

Finally, there is a regulator that seeks to maximize expected social welfare, a discounted weighted sum of successive generations of agents’ expected utility. To do so, the regulator can constrain bank lending. In practice, it can choose capital requirements at the beginning of each period to solve the following trade-off: allowing more lending increases expected output (up to a certain point), but it also increases the probability of a banking sector collapse, and the severity thereof.

Main mechanisms

The interest rate paid on deposits is insensitive to the risk taken by the banks. As a result, banks have an incentive to lever up to levels which are socially too high. Although this is not an asset-substitution mechanism, this is essentially reminiscent of the classic result of bank risk-shifting behavior when deposits are insured (Kareken and Wallace (1978)).

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3 For instance, in September 2012, amid credit crunch concerns, the UK Financial Services Authority has decided to soften bank capital requirements for new lending.

4 Which is actually disputed by some prominent economists (see Admati et al. (2010) and Hellwig (2010)).

5 Note that whether deposit insurance is funded by taxpayers ex post or through an ex-ante premium does not matter as long as the premium does not fully reflect the full social cost of the risks taken by the bank.

6 Note that deposit insurance is observed in all the most advanced economies, Demirgüç-Kunt et al. (2008) although coverage may be different. Coverage, in terms of maximum amount per person (or account) has generally been extended during the 2008 crisis. In some cases, it has been fully extended ex-post, including to other types of debt. More recently however, in Cyprus, ex-ante uninsured depositors have been excluded ex-post.
Moreover, the guarantees act as a subsidy and banks do not internalize the full (even risk non-adjusted) expected cost of borrowing; there is a wedge in their expected marginal cost, which makes them willing to issue loans with an excessively low expected return. In fact, this wedge interacts with diminishing returns to capital, which in a competitive equilibrium results in aggregate over-lending, and in the marginal loan having negative NPV. This makes bank balance sheets deteriorate and increases both the probability of a banking sector collapse and the severity thereof. It is important to note that this increase in risk originates in the financial system. A central planner that would like to implement the same level of expected output would do it with an allocation that entails less risk exposure. The excess of output volatility compared to such a benchmark can thus be interpreted as a form of systemic risk.

**Results and insights**

I find that optimal capital requirements are:

- decreasing in expected productivity;
- increasing in aggregate bank capital.

The first result is very intuitive since an increase in expected productivity makes the marginal investment in the economy more profitable. Therefore, it makes the marginal loan more profitable since the probability of default of the borrowers go down. It also positively affects expected consumption, and decreases tax-payer marginal utility. All other things equal, regulation should therefore be less stringent when expected productivity is high. This channel suggests that time-series effect of the Basel II regulation are to some extent desirable.

The second result, which is the main result of the paper, is perhaps less intuitive. On the one hand, more bank capital means that the banking sector can absorb more losses, which suggests that the banking sector could expand. But, on the other hand, there is a general equilibrium effect that dominates the loss absorbing effect. To see the intuition behind the general equilibrium effect, first consider a single (atomistic) bank that doubles its equity base. It should simply be allowed to double the size of its assets. However, if all banks in the economy double their equity base, and if they are allowed to double the size of their assets, this could double aggregate lending in the economy. Given diminishing returns to capital on the real side of the economy and given that banks have incentives to take on too much risk, this really is problematic, as this will decrease marginal returns to an extent that is far from being optimal. In fact, the optimal policy is to let the banking sector expand, but less than proportionally, which corresponds to an increase in capital requirements and resonates with the notion of “counter-cyclical capital buffers” of Basel III.

If this general equilibrium effect is overlooked by the regulator, it exacerbates economic fluctuations and results in systemic risk being created in the financial sector: aggregate bank lending will be excessive during a boom and the banking collapse that may ensue will result in an excessive credit crunch. The prediction that risks are being piled up by the banking sector during good times finds empirical support (see Borio and Drehmann, 2009 for instance).

The dynamics of the model provide a simple theory of intertwined business and financial cycles that goes as follows: productivity shocks affect output and therefore aggregate bank capital accumulation (directly through banks’ retained profits, and indirectly through the wealth that new bankers will be able to inject as fresh capital in the sector). Productivity shocks also affect the socially optimal level of aggregate lending in the next period (through persistence in productivity). This socially optimal level of lending, which will be implemented through optimal capital requirements, also depends on the level of aggregate bank capital. In turn, the optimal capital requirements affect physical capital accumulation and therefore output. They also affect the exposure of the banking sector to productivity shocks and therefore influence bank capital accumulation.

These dynamics deliver periods of “good times”, when productivity, consumption, investment, physical and bank capital are high, and periods of “bad times”, when they are all low. Looking at the comparative statics results tells us that, in good times, high productivity and high consumption advocate for lower capital requirements, but the general equilibrium effect of aggregate bank capital goes in the other direction. It turns out that the latter dominates and optimal capital requirements are tighter in good times.

This result is less general than the comparative static results as it hinges on aggregate bank capital being relatively more pro-cyclical than the optimal level of aggregate lending. However, it conveys an important and
more general policy insight: if aggregate bank capital varies more over the cycle than the “desired” level of aggregate lending, than optimal capital requirements should be higher in good times, and conversely.

**Related literature**

This paper belongs to the strand of literature that builds on Kareken and Wallace (1978) and Dewatripont and Tirole (1994) and sees prudential regulation of banks as a response to excessive risk taking by banks induced by the government guarantees.

The most closely related paper is Repullo and Suarez (2013), which studies optimal bank capital regulation over the cycle and compares it to regulations that resemble Basel I, II, and III. They find that “counter-cyclical buffers” help to mitigate the “pro-cyclical” effects of regulation such as Basel II. In their set-up, optimal capital requirements are always tighter in bad times than in good times. An important feature of their model is that the production function is linear in investment, which does not leave much room for interaction between aggregate bank capital and marginal productivity in the real sector. In other words, this assumption restricts interactions between the financial and the real business cycle, and they do not capture the general equilibrium effects I highlight in this paper, which explains the difference in results.

Starting from a completely different angle (his model has heterogenous banks that take uninsured deposits but face a moral hazard problem) Repullo (2013) also finds that capital requirements should be lowered after an exogenous negative shock bank capital. Here again, the relative contribution of my paper is to endogenize aggregate bank capital and to highlight the general equilibrium effects that result from its interaction with the business cycle.

The paper is also related to the literature on macro-prudential regulation.

Martinez-Miera and Suarez (2013) propose a macroeconomic model of endogenous systemic risk-taking in which correlated risk-shifting by some banks gives an incentive to other banks to play it safe, because the more banks that fail at the same time, the larger the scarcity rent after a crisis. Still, the competitive outcome is inefficient, and the optimal (constant) capital requirement trades off output at steady state with the severity of financial crisis (the time it takes to go back to steady state). They do not consider business cycle dynamics but, in their model, reducing capital requirements after a banking crisis reduces the rent earned ex-post by the “last banks standing” and induces thus more systemic risk-taking ex-ante. In contrast, in Dewatripont and Tirole (2012) incentives to gamble for resurrection are stronger after a negative macro-economic shock. Related to this, Morrison and White (2005) study optimal capital requirements in a model with both moral hazard and adverse selection. They find that the appropriate policy response to a crisis of confidence may be to tighten capital requirements. This happens when the regulator’s ability to alleviate adverse selection through banking supervision is relatively low. In my model, there is no asset substitution (or effort) problem, and the primary role of bank capital is to act as a buffer to absorb losses (and therefore protect the tax payer) rather than to correct incentives.

Most of the micro-funded models of macro-prudential regulation that are currently burgeoning are developed around pecuniary externalities mechanisms, such as fire-sale externalities for instance (Bianchi (2011), Brunnermeier and Sannikov (2010), Korinek (2011), Jeanne and Korinek (2010), Jeanne and Korinek (2013), Gersbach and Rochet, 2011, Stein (2012); see Hanson et al. (2011) for an overview). More generally, these models build on debt deflation (Fisher (1933)) and financial accelerator mechanisms (Bernanke and Gertler (1989); Kiyotaki and Moore (1997)). In these papers, borrowing constraints, generally justified by moral hazard concerns, are imposed by lenders. One of the main messages from this approach is that the equilibrium level of lending in the economy may not be efficient because investors fail to internalize the social cost of a binding borrowing constraint in the presence of a pecuniary externality (Geanakoplos and Polemarchakis (1986); Krishnamurthy (2003)). Other papers provide a rationale for a systemic approach to regulation based on financial institutions’ incentives to choose correlated exposures (Farhi and Tirole (2012), Acharya (2009)) on network externalities (Allen et al. (2011)), or aggregate demand externalities (Farhi and Werning (2013)).

My approach differs from these latter approaches in that I set aside incentive problems and maturity mismatch (or liquidity) issues (which can lead to fire-sale externalities) to focus on distortions induced by government guarantees (which lead to solvency problems) and their general equilibrium interactions with the real business cycle.

Other papers that study the distortions caused ex-ante by government guarantees include Merton (1977), Kareken and Wallace (1978), Keeley (1990), Pennacchi (2006). And Gomes et al. (2010) attempts to quantify
the distortions that arise ex-post, when taxes need to be raised to finance the bailouts.

More generally, the paper relates to the literature on bank capital regulation (see Santos (2001) for a survey) and in particular to that on the costs and/or benefits of bank capital requirements. These include Van den Heuvel (2008), Admati et al. (2010), Hellwig (2010), and Morrison and White (2005)). It is however important to stress that the present paper only focusses on the optimal cyclic properties of bank capital requirements, as its highly stylized nature makes it less well suited to study the optimal level of capital requirements.

2 The model

2.1 The environment

There is an infinity of periods, indexed by $t = 0, 1, 2..., $ in which generations of agents born at different dates overlap.

The young generations. There is a measure 1 of agents born at the beginning of each period. These agents are endowed with one unit of labor, which they supply inelastically during the first period of their life for a wage $w_t$. At the end of the period, these agents incur an “ability” shock:

- A share $\eta \ll 1$ of these agents is endowed with “banking ability”. They will be able to set up a bank at date $t + 1$ and invest in its equity.
- The remaining share looses all working ability and retire. I refer to them as savers.

The firms. Each period, there is a continuum of penniless firms that operate a constant-return-to-scale risky production function. Since labor supply is fixed, there are diminishing returns to capital. The production function takes the form:

$$ A_t k_t^\alpha, $$

where $k_t$ is physical capital per worker, $0 < \alpha < 1$, and $A_t$ is a random variable, distributed over a bounded subset of $\mathbb{R}_+^+$ with probability density function $f(A_t)$ and cumulative distribution function $F(A_t)$, that captures aggregate risk. The physical capital fully depreciates in the production process.

Firms compete for workers and for bank funding (to invest in physical capital). They pay a wage $w_t$, and repay $R_t$ per unit of fund. They take these factor prices as given.

Production is risky in the sense that $A_t$ is realized only at the end of the production process. Hence, the realized marginal productivity of physical capital and labor is not known when equilibrium quantities are determined. Assuming that the realized prices correspond to realized marginal productivity, we have at equilibrium that:

$$ \begin{cases} w_t = (1 - \alpha)A_t k_t^\alpha \\ R_t = \alpha A_t k_t^{\alpha-1} \end{cases}, $$

which ensure that firms always make 0 profit.

The old generations of savers. During the second period of their life, savers can choose between depositing their income from the previous period at the bank or using a storage technology. The rate of return to storage is normalized to 0. Deposits are insured by the government. I focus on cases where deposits are in excess supply at equilibrium,\footnote{The economy can be considered as a small open economy with excess savings, facing the worlds interest rate.} so that they are paid the same rate of return as storage. At the end of their second period of life, savers receive their deposit back (either from the bank or the government), pay taxes, if any, and consume and then die. They derive utility from their consumption (see more details below).
**The old generation(s) of bankers.** The measure $\eta$ of agents that receive “banking ability” can use the wage they earned during their first period as equity to set-up a new bank under the protection of limited liability.

Banks raise deposits and compete to lend to firms. Banks are the only source of funds to firms, which are penniless and act competitively. Therefore, the realized marginal return to lending is the realized marginal return to capital $R_t = \alpha A_t k_t^{\eta-1}$. Bankers take the distribution of marginal return to lending as given.

After receiving the proceeds from lending, bankers repay depositors. If they cannot (that is if the book value of equity is negative), they walk away with 0 and leave the economy. After deposit repayment takes place, bankers may lose their banking ability. This happens to them with probability $\delta$, independently and irrespective of their tenure. In that case, they consume all their wealth and also leave the economy. This shock can equally be interpreted as a liquidity shock (Diamond and Dybvig (1983)) that gives bankers an absolute preference for immediate consumption. Bankers that are not hit by the shock (which happens with probability $1-\delta$) allocate their wealth between consumption, risk-less savings (using storage or depositing at another bank), and bank equity. If they are indifferent, I assume the following pecking order: consumption, risk-less saving, and then equity.\(^8\)

Bankers are risk neutral and derive utility from the sum of their lifetime consumption. That is, the utility of a banker $i$ that was born at date $j$ is:

$$E\left[\sum_{t=j}^{\infty} c_{it}\right],$$

where $c_{it}$ denotes his consumption at date $t$.

**The regulator.** The regulator’s objective is to maximize a social welfare function, which is a discounted weighted sum of each generation of two representative agents utility:

$$\sum_{t=0}^{\infty} \beta^t E[\eta u_t + (1-\eta) u_t^s],$$

where $u_t$ and $u_t^s$ respectively denote the utility of the banker and the saver born at date $t$, and $\beta \rightarrow 1$ is a discount factor. The weights correspond to the class of agent’s respective measure.

The regulatory tool I consider is a capital requirement $x_t \in [0,1]$, which constraints bank lending to a multiple of its equity.

I focus on capital requirements several reasons. First of all, capital adequacy ratios are one of the main tools used for prudential purposes. Second, they currently are the core of several policy debates. Third, in the context of this simple model (there is for instance no asymmetry of information), they enable the regulator to implement the social optimum. Last but not least, capital requirements are easy to interpret, they economize on notation, simplify computations, and they make aggregation straightforward. Alternatively, one could consider a (sufficiently progressive) tax on leverage and obtain similar results.\(^9\)

**Bailout tax.** Deposits are insured. That is, if there is a shortfall in a bank’s net worth to repay depositors, the government makes an offsetting transfer to these depositors. I denote by $b_t$ the lump-sum “bailout tax” that the government has to raise to break even (and I impose that the government does break even at each date). For simplicity, I assume here that the full burden of the bailout falls on the savers. An interesting extension would be to study the possibility of splitting the burden of taxation between the current generations. This would improve inter-generational risk-sharing but would impair bank capital accumulation.

**Summary of intra-period time-line.**

- Relevant predetermined variables:
  - $A_{t-1}$, which I assume to be a sufficient statistic for the parameters of $A_t$’s distribution.
  - aggregate bank capital: $e_t$.

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\(^8\)This only simplifies aggregation in the uninteresting, and empirically not relevant, cases where bank equity is plentiful and banks never fail. In those cases, the Modigliani-Miller theorem (Modigliani and Miller (1958)) applies and bank equity is irrelevant.

\(^9\)A deposit insurance premium increasing in the deposit base coupled to a subsidy to bank equity is another example of a set of tools that could replicate the optimal allocation.
• The regulator decides upon $x_t$, the capital requirement imposed on bankers.

• Market activity takes place. Simultaneously:
  
  – Bankers borrow from savers and lend to firms;
  – Firms competitively hire workers, borrow from bankers, and invest in physical capital.

• $A_t$ is realized, production takes place, wages are paid, and the share of capital goes to the bankers.

• If possible, bankers repay deposits. If not, they walk away with nothing, and the regulator compensates depositors for the shortfall, imposing a break-even lump-sum tax on savers.

• Bankers learn whether they keep their are hit by the liquidity shock.

• Bankers that are not hit decide upon consumption and choose their investment portfolio (storage and/or (re)investment in their bank’s equity).

• Consumption takes place. Bankers that are hit by the shock consume all their wealth and leave the economy.

3 Market failure and the role of the regulator

In this section, I highlight the nature of the market failure and I formalize and discuss the problem of the regulator.

3.1 The problem of the banker

Let $v_it$ denote the book value of bank $i$’s equity (the net worth of the bank) at the end of period $t$:

$$v_it \equiv R_t(d_it + e_it) - d_it ,$$

where $d_it$ is the amount of deposit taken by bank $i$ in period $t$ and $e_it$ is its bank capital (or equity) at the beginning of the period. Total lending by the bank is $(d_it + e_it)$. I assume that banks do not hold cash, which is without loss of generality in this context.\(^{10}\) For simplicity, I only consider here inside equity.\(^{11}\)

Let me consider a banker $i$ born at date $j$. During his first period of life, he earns a wage $w_j$. At the end of this period, his problem can be written as follows:

$$\max_{\{c_{it}, e_{it+1}, d_{it+1}\}_{t=j}} E \left[ \sum_{t=j}^{T_i} c_{it} \right]$$

subject to the sequence (for $j \leq t \leq T_i$) of budget constraints, capital requirements, and non-negativity conditions:

$$\begin{align*}
  c_{it} + e_{it+1} + s_{it+1} &= n_{it} \\
  x_{it+1}(d_{it+1} + e_{it+1}) &\leq e_{it+1} \\
  c_{it}, d_{it+1}, e_{it+1} &\geq 0
\end{align*}$$

\(^{10}\) Raising one more unit of deposit and storing it using the safe technology increase by 1 both sides of the balance sheet in all possible states of the world.

\(^{11}\) See section5 for a discussion and the argument that outside equity issuance would not qualitatively affect the main results of the model.
where \( s_i \) denotes the amount stored by banker \( i \) from date \( t-1 \) to date \( t \), \( T_i \) is the (stochastic) date at which he is hit by the liquidity shock, and \( n_i \) denotes his end-of-period-\( t \) wealth. That is:

\[
n_i = \begin{cases} 
  w_{ij} & : t = j \\
  v^+_i + s_i & : j < t < T_i, 
\end{cases}
\]

where \( v^+_i \) is the realized (or private) value of bank equity, that is the positive part of \( v_i \):

\[
v^+_i \equiv [R_i(d_i + e_i) - d_i]^+.
\]

The difference between the book value and the private value comes from limited liability, which gives bankers the option to walk away with 0 when \( v_i < 0 \).

### 3.2 Incentive distortion and market failure

To stress to source of the market failure, I restrict here the analysis to the lending decision of a bank that is active only one period. I look at the lending decision, given an equity level, and assuming that there is no capital requirement \((x_t = 0)\). Formally, I consider a bank \( i \) that enters date \( t \) with equity \( e_{it} > 0 \) and has to decide upon \( d_{it} \) (recall that banks hold no cash and total lending is therefore \( e_{it} + d_{it} \)).

When risk neutral bankers are active one period only (that is, \( \delta = 1 \)), the optimal level of lending will be the one that maximizes the expected end-of-period value of equity, which can be decomposed as follows:

\[
E[v^+_i] = E[v_i + v^-_i],
\]

where \( v^-_i \) is the negative part of \( v_i \). That is,

\[
v^-_i \equiv [R_i(d_i + e_i) - d_i]^-.
\]

Hence, it is a positive number that captures the shortfall in bank value to repay depositors.

The first order condition (with respect to \( d_{it} \)) for an interior solution is

\[
E[R_t - 1] + \frac{\partial E[v^-_i]}{\partial d_{it}} = 0. \tag{1}
\]

Deriving piecewise gives:

\[
E[R_t] = 1 - \int_0^{R_{it}} (1 - R_t)f(R_t)dR_t, \tag{2}
\]

where \( R_{it} = \left( \frac{d_{it}}{d_{it} + e_{it}} \right) \) is the threshold value for \( R_t \) below which the bank goes bankrupt (it corresponds to \( v_i = 0 \)). The left-hand side is the expected marginal return to lending, and the right-hand side is the (private) expected marginal cost, which exhibits a wedge \((\Delta_{it})\) compared to the return to storage. This wedge reflects the fact the bank does not always repay the deposits in full (it also corresponds to the marginal value of the subsidy from government guarantees). To see this, the right-hand side can be decomposed as follows:

\[
1 - \int_0^{R_{it}} (1 - R_t)f(R_t)dR_t = \int_{R_{it}}^{R_{it}^{\max}} 1f(R_t)dR_t + \int_{R_{it}}^{R_{it}^{\min}} 1f(R_t)dR_t, \tag{3}
\]

where \( R_{it}^{\min} \) and \( R_{it}^{\max} \) are the lower and upper bound for \( R_t \) given \( k_t \) and the distribution of \( A_t \) (recall that \( R_t = \alpha A_t k_t^{\alpha - 1} \)). Equation (3) illustrates well the effect of government guarantees on the private expected marginal cost. When the bank does not default, it repays 1, the nominal value of deposits, but when it defaults, it only...
repays $R_t \leq 1$ per unit of loan, with strict inequality for all $R_t < \hat{R}_t$. So, in expectation, the marginal repayment is strictly smaller than 1 as soon as $\hat{R}_t > R^\text{min}_t$, that is if there is a possible realization for the shock that would lead to default.

**Lemma 1. (Incentive distortion)**

i) $\Delta_t \geq 0$, strictly for $F(\hat{R}_t) > 0$.

ii) $\Delta_t$ is increasing in leverage.

**Proof.** Straightforward.

The first part of Lemma 1 formalizes the result that deposit insurance acts as a subsidy to leveraged lending. The second implies that the marginal subsidy is increasing in leverage (increasing in $d_t$ and decreasing in $e_t$). Figure 1 illustrates these first results.

![Figure 1: The wedge in the banker’s first order condition](image)

This figure illustrates Lemma 1. The downward sloping line is the expected marginal cost for the banker. For a given level of equity ($e_i$), it decreases with total lending (denoted $l_i$) and therefore with deposits $d_i$. The wedge ($\Delta_i$) with respect to the return to storage is then increasing in leverage.

**Lemma 2. (General equilibrium effect) $\Delta_t$ is increasing in $k_t$.**

**Proof.** Straightforward.

The concavity of the production function is crucial to this result. In Kareken and Wallace (1978), the distortion due to deposit insurance is studied in partial equilibrium. This distortion results in an incentive to substitute assets and take on too much risk. Here, bankers also take more risk (through higher leverage), but the key novelty is that, by doing so, they negatively affect the quality of the marginal loans for other banks, and thereby increase the marginal value of the subsidy for all banks in the economy.

Putting these two lemmas together, deposit insurance distorts incentives towards more leverage and, in the context of diminishing returns to capital, towards aggregate over-investment. This is how the interaction between deposit insurance and diminishing returns to capital is at the heart the market failure in this model. An extreme illustration thereof can be formalized as follows:

**Proposition 1. (Negative NPV investment):** Without regulation ($x_t = 0$), the competitive equilibrium exhibits negative NPV investment (that is, $E[R_t] < 1$).

**Proof.** See the proof for the general case (that is, $\delta \leq 1$) in Appendix B.

The intuition for the result is the following. First, Lemma 1 establishes that the first order condition of the bankers cannot be satisfied with $E[R_t] > 1$. Also, it can only be satisfied at $E[R_t] = 1$ if $F(\hat{R}_t) = 0$. Assume it is the case. But $\hat{R}_t$ is increasing in $d_t$ (and decreasing in $e_t$). So, absent regulatory restrictions, there exists a profitable increase in lending, which then decreases the marginal return to capital, and therefore drives down $E[R_t]$ below the return to storage, and it must be that $E[R_t] < 1$ in a competitive equilibrium.
3.3 The problem of the regulator

The role of the regulator is to correct for the distortions exposed above.

In practice, the regulator chooses a sequence of capital requirements \( \{x_t\}_{t=0}^{\infty} \) so as to maximize the social welfare function:

\[
\sum_{t=1}^{\infty} \beta^t E \left[ \eta u_t + (1 - \eta) a_t^u \right],
\]

subject to the market clearing and the break-even conditions:

\[
\begin{cases}
 w_t &= (1 - \alpha)A_t k_t^{\alpha - 1} \\
 R_t &= \alpha A_t k_t^{\alpha - 1} \\
 k_t &= \int (d_i + e_i) \\
 v_t^- &= (1 - \eta) b_t
\end{cases}
\]

where \( v_t^- \equiv \int v_i^- \) denotes the total shortfall in bank value to repay depositors in full, and where gents (bankers and savers respectively) consumption is given by:

\[
\begin{cases}
 c_{it} &= n_{it} - e_{it} + s_{it+1} \\
 c_{st} &= w_t - b_{t+1}
\end{cases}
\]

To complete the specification of the regulator’s problem, one must still specify the stochastic process for \( A_t \) and the utility function of the savers. Let me delay the former and just make here the mild assumption on the latter that \( u_t^s \) is such that it is never socially optimal to fund negative NPV investment (that is \( E[R_t] < 1 \)). In this case:

**Lemma 3.** (Requirements are binding) At the social optimum, capital requirements are binding.

**Proof.** See Appendix B.

When the expected marginal benefit from bank lending equates its expected marginal cost form a social point of view, the former is strictly larger than the later from the bankers’ perspective. In other words, at the social optimum, bankers would like to increase lending and/or decrease equity leaving the other variable unchanged. This result is a direct consequence of the wedge in the bankers first order condition (2), which leads to a pecking order of financing: bankers always prefer to fund investment with insured deposits than with equity.

**Intuition behind the regulator’s problem** Essentially, what the regulator does in this model is to solve for the socially optimal level of aggregate lending, which can then easily be implemented through the capital requirement (since capital requirements are binding at the social optimum, under-lending is not an issue). The socially optimal level of aggregate lending depends on the state of the economy. One way to think about it is that it is the level at which the expected marginal return to lending equates the social expected marginal cost of lending. For instance, anticipating on the analysis, if savers are risk-neutral, the relevant social marginal cost is the return to storage, and the aim of the regulator is to prevent that the marginal return to lending falls below it. This is illustrated in Figure 2. If savers are risk-averse, then the social marginal cost will generally include a risk-premium and there is therefore a trade-off between expected output and the cost (to tax payers) of a banking sector bailout. Again, the aim of the regulator is to prevent negative NPV investment, but the returns have be priced with the correct pricing kernel, which accounts for the social cost of risk. This more general case is illustrated in Figure 3.
When savers are risk neutral, the social marginal cost of lending corresponds to the return to storage. Here the optimal level of total lending is such that the marginal return to physical capital is 1. In this example, this happens when aggregate lending equates $\bar{k}$.

When savers are risk-averse, the relevant social discount factor does not correspond to the return to storage. Therefore, the regulator solves a trade-off. Reducing total lending decreases the risk borne by current savers, but also decreases expected output. Assuming that dynamics effects do not more than offset this, the social optimum is to the left of $\bar{k}$.

**4 Optimal capital requirements over the business and financial cycles**

In this section, I first argue that the model delivers a simple theory of intertwined business and financial cycles where the law of motion for aggregate bank capital plays a crucial role. Then, I analytically derive the optimal capital requirements in the special case where savers are risk neutral. Finally, I analyse the case where they are risk averse.

It is useful to analyze this risk-neutral case because it delivers easy to interpret analytical results. The risk-averse case is more general, but has to be solved numerically. It, by and large, confirms the intuition from the risk-neutral case, but also delivers additional insights on the trade-off between expected output and tax-payer exposure to banking sector collapses.
4.1 Aggregate bank capital

Since optimal capital requirements are binding at the social optimum, \( d_t + e_t = \frac{c_t}{w} \) for all banks, and \( k_t = \frac{c_t}{w} \) in the aggregate, where \( e_t \equiv \int e_t \) is aggregate bank capital. Capital requirements thus can be used to pin down aggregate bank lending, and therefore aggregate physical capital. Since there only is aggregate uncertainty, bank value also aggregates conveniently: \( v_t \equiv \int v_t = \alpha A_t k_t^\alpha - k_t + e_t \), and it follows that the distribution of wealth in the banking sector is irrelevant and that the bailout tax is:

\[
\frac{\alpha A_t k_t^\alpha - k_t + e_t}{1 - \eta}.
\]

Thus, the key state variable is \( e_t \) and it evolves as follows:

\[
e_{t+1} = \frac{v_t^+}{w} + \frac{c_t}{w} - \frac{s_t}{w} + \frac{\eta w}{w},
\]

where \( v_t^+ \), \( s_t \), and \( c_t \) are the aggregate counterparts of \( v_t^+ \), \( s_t \), and \( c_t \). In particular, \( v_t^+ = \frac{\alpha A_t k_t^\alpha - k_t + e_t}{w} \) is the aggregate private value of bank equity.

4.2 The intertwined business and financial cycles

In line with the real business cycle tradition (Kydland and Prescott (1982); Long Jr and Plosser (1983)), the exogenous shocks that generate aggregate fluctuations in this model are productivity shocks. However, the frictions relevant to physical capital accumulation are completely different. First, firms have to borrow from banks (they cannot borrow directly from the savers). Second, banks incentives are distorted by deposit insurance. Third, banks are subject to capital requirements. Finally, there are no technological or information frictions to physical capital accumulation other than the fact that input quantities have to be decided before the realization of the shock.

The model delivers a simple theory of intertwined business and financial cycles, with propagation mechanisms that go as follows: Productivity shocks affect output and therefore aggregate bank capital accumulation (directly through banks retained profits, and indirectly through the wealth that new bankers will be able to inject as fresh capital in the sector). Productivity shocks also affect the socially optimal level of aggregate lending at the next period (through persistence in productivity). This socially optimal level of lending, which will be implemented through the optimal capital requirements, also depends on the level of aggregate bank capital. In turn, the optimal capital requirements affect physical capital accumulation and therefore output. They also affect the exposure of the banking sector to productivity shocks and therefore influence bank capital accumulation.

Figure 4 summarizes the main interactions in the model.

Costs of capital. In this model, there are two different measures for the cost of capital, both being endogenously determined at equilibrium.

The cost of capital for firms: this is the cost of borrowing. Firms take it as given and equate it to the marginal return to investment. So, the equilibrium cost of capital for firm corresponds to the marginal return to aggregate physical capital. It therefore depends on expected productivity, on the capital requirement, and on aggregate bank capital.

The private cost of bank capital corresponds to the expected leveraged return to lending (which in turn reflects the marginal productivity of physical capital), including the expected subsidy from deposit insurance (due to the option value of equity). It depends on the same variables, and is typically strictly greater than the cost of capital. This is the case whenever the expected subsidy from deposit insurance is strictly positive, that is, whenever banks may fail with positive probability.

---

13If one would like to study shocks originating in the financial sector, making stochastic the share of agents that become bankers (\( \eta_\) or introducing correlation in the realization of the liquidity shock are obvious options. They would affect bank capital accumulation through fresh capital and retained profits respectively.
Productivity. The model is very stylized and the focus of the analysis is qualitative in essence. To keep things simple, I assume here that \( A_t \) follows a 2-state Markov process, with \( A_t \in \{A_L, A_H\} \), with \( A_L < A_H \), and a transition matrix:

\[
\Omega = \begin{pmatrix}
\omega_L & 1 - \omega_L \\
1 - \omega_H & \omega_H
\end{pmatrix},
\]

where \( 0 < \omega_L < 1 \) and \( 0 < \omega_H < 1 \), \( E[A_t] = 1 \). I denote the conditional expectation: \( \bar{A}_L \equiv E[A_{t+1} | A_t = A_L] \), and I impose \( \bar{A}_L \leq 1 \leq \bar{A}_H \). This modeling strategy captures the idea of persistence in productivity: after a good draw (\( A_H \)) economic prospects are better, that is, expected productivity is relatively high (\( \bar{A}_L \leq \bar{A}_H \)).

Note that we have \( \frac{A_H}{A_L} > \frac{\bar{A}_H}{\bar{A}_L} \), which captures the idea that while prospects are better than average after a good draw, the process always tend to “revert to the mean”.

4.3 The case of risk-neutral savers

4.3.1 Preferences

In this subsection, I assume that savers are risk neutral: they derive linear utility from their consumption, which corresponds to their wage net of taxes:

\[ u_t^s = w_t - b_{t+1}. \]

I also set \( \beta = 1 \), which simplifies expressions.\(^{14}\)

4.3.2 Solving for the optimal capital requirements.

**Lemma 4.** When savers are risk-neutral and \( \beta = 1 \), maximizing the welfare function (4) boils down to maximizing expected output at each period.

**Proof.** See Appendix B.

The intuition is quite simple. First, when savers are risk-neutral, as defined above, their marginal utility is always the same as that of bankers. Therefore, intra-period consumption allocation is irrelevant to social welfare. Second, if \( \beta = 1 \), the social inter-temporal marginal rate of substitution always equate the gross return to storage (that is, 1). Therefore, maximizing expected output at each date must maximize social welfare.

\(^{14}\)This does not affect the qualitative results. In fact, \( \beta < 1 \) is only required in the general case so that the numerical solution converges.
Assumption 1. (Full reinvestment) As long as they are not hit by the liquidity shock, it is optimal for bankers to fully (re)invest their wealth in banking equity.

This assumption, which is verified at equilibrium for all the examples presented below, is very useful because it allows me to derive closed form solutions, which deliver important insights to understand the more general cases. There however exist parameter regions for which it is not verified, and where, at equilibrium, scarcity rents are low and bankers find it optimal to store a positive fraction of their wealth until the next period. The analysis is slightly more complicated in these cases, but does not bring much additional insights.

From Lemma 3, it is optimal for bankers to delay consumption until they are hit by the liquidity shock. Accordingly, aggregate banker consumption at date \( t \) is given by \( \delta v_t^+ \). Also, Assumption 1 implies that \( s_{t,t+1} = 0, \forall t < T_i \). Hence, the law of motion for bank capital simplifies to:

\[
e_{t+1} = (1 - \delta)v_t^+ + \eta w_t
\]

The relevant program (at each date) is then:

\[
\max_{x_t} E[A_t k_t^\alpha - k_t]
\]

s.t.:

\[
\begin{align*}
  k_t &= \frac{\alpha}{\delta} \\
  e_t &= (1 - \delta)v_t^+ + \eta (1 - \alpha)A_{t-1} k_{t-1}^\alpha
\end{align*}
\]

The level of physical capital that maximizes expected output is:

\[
k^*_t = \left(\frac{\alpha}{\delta}A_{t-1}\right)^{\frac{1}{1-\alpha}}, \tag{5}
\]

Note that it does not depend on \( e_t \) (see discussion below).

The optimal capital requirement therefore only depends on expected productivity and aggregate bank capital at the beginning of the period. That is:

\[
x^*_t = x^*(\alpha, A_{t-1}) = \frac{e_t}{(\alpha A_{t-1})^{\frac{1}{1-\alpha}}}, \tag{6}
\]

4.3.3 Characterization of the optimal capital requirements

Proposition 2. In the risk-neutral case, the optimal capital requirement \( x^*_t \) is

i) decreasing in expected productivity \( \bar{A}_{t-1} \);

ii) increasing in aggregate bank capital \( e_t \).

Proof. Straightforward.

The intuition goes as follows: First, \( x^*_t \) decreases with expected productivity simply because higher productivity makes marginal lending more productive. The second part may be a bit counter intuitive as, after all, more bank capital sounds like good news. However, in this risk-neutral set-up, bank capital plays no role (from a welfare point of view) as a buffer against losses. This is why it does not affect the optimal level of aggregate lending. In a sense, it is simply the number to which regulation will apply, its only has a role here as “regulatory capital”. This is why I will call this channel the “regulatory channel”.

In the risk-averse case (or in a version of the model that would incorporate deadweight losses form bank failure), bank capital does play a role as buffer against losses. Therefore, it does affect the optimal level of lending (see Section 4.4 below).

15If current rents are low, to potential larger future rents become relatively more attractive, even if they realize with a small probability (note that bankers pay bailout taxes if they have savings, which does not make savings very attractive). Intuitively, aggregate bank capital is wiped out in a banking crisis, and the “last banker standing” makes a killing during the recovery. Such a mechanism has been studied elsewhere (see Perotti and Suarez (2002) and Martinez-Miera and Suarez (2013)) and is not the focus of the present paper.
Terminology. There is now a consensus that the Basel II regulation has a magnifying effect on real economic fluctuations. This has been dubbed the “procyclical effect of bank regulation” (Kashyap and Stein (2004); Repullo and Suarez (2013)). Basel III, the latest standard in banking regulation, introduced time-varying capital buffers in order to mitigate this “procyclical effect”. This is why, they have been called “counter-cyclical buffers”. This terminology is common in the context of fiscal policy for instance. However, these buffers are supposed to be larger in good times than in bad times, which make them “procyclical” in the business cycle terminology (according to which a variable is “procyclical” if it is positively correlated with the state of the economy).

In this paper, to avoid confusion, I do not use the words “pro-cyclical” or “counter-cyclical” to qualify capital requirements, and I always use the business cycle terminology for other variables.

Pseudo steady states (PSS). Let me define a Pseudo Steady State (PSS) as the state the economy would converge to (under the optimal capital requirements derived above) if realized productivity were always the same (either \( A_H \) or \( A_L \)). That is, for any variable \( a \) of the model, I define its PSS value \( a_{pss} \) (with the index \( pss \in \{L, H\} \)) as follows:

\[
a_{pss} = \lim_{t \to \infty} a_t \bigg|_{\{A_t = A_{pss}\}}
\]

The fixed point of their respective law of motion (implied by an infinite sequence of \( A_H \) or \( A_L \)) pins down the pseudo-steady-state value of the two key state variables. I denote these values \( k_{pss} \) and \( e_{pss} \). They are defined (implicitly in the case of \( e_{pss} \)) by:

\[
\begin{align*}
\left\{ \begin{array}{l}
k_{pss} = (\alpha \bar{A}_{pss})^{1-\alpha} \\
e_{pss} = (1 - \delta) \left[ \alpha A_{pss} k_{pss}^a - k_{pss}^a + e_{pss} \right]^+ + \eta (1 - \alpha) A_{pss} k_{pss}^a
\end{array} \right.
\end{align*}
\]

(7)

Good and bad times. It is easy to show that PSS \( H \) (that is, the PSS that corresponds to an infinite sequence of \( A_H \)) is associated with higher levels of output, consumption, and investment (and of course physical and bank capital) than PSS \( L \). I therefore refer to PSS \( H \) as “good times” and PSS \( L \) as “bad times”. They can respectively be interpreted as the peak of a boom and the trough of a recession.

It is interesting to note that the cost of capital for firms is constant over the cycle. It is indeed equal to the return to storage. However, the “required return” on bank capital, varies over the cycle. This is because the expected subsidy from deposit insurance depends on the state of the economy.

To analyze the properties of optimal capital requirements at the “business/financial cycle frequency”, I first compare them at pseudo steady state, and then conduct a comparative statics exercise at pseudo steady state.

**Proposition 3.** Optimal capital requirements are tighter in good times than in bad times. Formally: \( x^*_{H} > x^*_{L} \).

**Proof.** See Appendix B.

Let me convey the intuition in the most simple case. The optimal capital requirement is \( x_{pss}^* = e_{pss}/k_{pss}^* \), when \( \delta = 1 \), we have:

\[
x_{pss}^* = \eta \frac{(1 - \alpha)}{\alpha} \left( \frac{A_{pss}}{\bar{A}_{pss}} \right)
\]

But we have \( A_H/\bar{A}_H > 1 \) and \( A_L/\bar{A}_L < 1 \) (because productivity is mean reverting). Therefore it must be the case that: \( x_{H}^* > x_{L}^* \). So, what happens is that at the peak of a boom, even though productivity is likely to stay high, it can also go down. And this is the opposite at the trough of a recession. Therefore, optimal capital requirements (which are forward looking) have to be more stringent at the peak, which is in line with the direction taken in Basel III.
Comparative statics at pseudo steady state and elasticity interpretation. Another way to derive useful intuition is to look at the relative point elasticities of $e_{pss}$ and $k_{pss}^*$ with respect to $A_{pss}$. Both $e_{pss}$ and $k_{pss}^*$ depend positively on $A_{pss}$. So, whether $x_{pss}^*$ increases with $A_{pss}$ depends on whether $e_{pss}$ increases faster than $k_{pss}^*$, following an increase in $A_{pss}$. In term of elasticities, we have that:

$$
\epsilon_k = \epsilon_k - \epsilon_{kA},
$$

where the three terms respectively denote the point elasticity of $x_{pss}^*$, $e_{pss}$, and $k_{pss}^*$ with respect to $A_{pss}$ (and where I have dropped the pss subscripts for readability). Let me again provide the intuition for the simple case where $\delta = 1$. In that case, we have:

$$
\begin{align*}
\epsilon_{kA} &= \frac{\epsilon_{kA}}{1-\alpha} \\
\epsilon_{kA} &= 1 + \alpha \epsilon_{kA},
\end{align*}
$$

where $\epsilon_{kA}$ denote the point elasticity of $\bar{A}$, with respect to $A_{t}$.

And again, since $\epsilon_{kA} < 1$ (productivity reverts to its mean), we have that $\epsilon_{kA} > \epsilon_{kA}$, that is aggregate bank capital at pseudo steady state is more sensitive to productivity than the optimal stock physical capital. Therefore $\epsilon_{kA}$ is positive and an increase in $pss$ productivity should thus be associated with tighter capital requirements.

When $\delta < 1$, retained profits are a second component of aggregate bank capital, but profits from leveraged investment are also very sensitive to productivity and we get the same result.\(^{16}\)

Short-run capital requirement adjustments. While in this model optimal capital requirements are tighter in “good times” than in “bad times” (that is, they are tighter after a sufficient long series of good shocks than after a sufficiently long series of bad shocks), it does not necessarily imply that a positive (negative) productivity shock should trigger an immediate capital requirement tightening (loosening). In fact, the direction of the short-run adjustment depends on parameter values. I formally state this result later in the context of the risk-averse case.

4.4 Risk-averse savers

In the risk-neutral case, the loss-absorbing power of bank capital is irrelevant to social welfare. Therefore, bank capital does not affect $k^*$, the socially optimal level of physical capital. However, when savers are risk-averse, the level of aggregate bank capital affects $k^*$ because it affects the extent to which tax-payers are at risk and, therefore, the social risk premium. In this case, the optimal capital requirement solves a trade-off between expected output and the risk borne by the tax-payers.

The first purpose of this subsection is to study this trade-off. Its second purpose is to test the robustness of the results delivered by the risk-neutral case. That is, whether optimal capital requirements are still increasing in aggregate bank capital, and tighter in good times than in bad times.

4.4.1 Preferences

I consider the following utility function for savers:

$$
\bar{u}_t = w_t + \frac{(1 + c_{t+1}^*-w_t)^{1-\rho} - 1}{1-\rho},
$$

where $\rho > 0$ ($\rho = 0$ corresponds to the risk-neutral case).\(^{17}\) This formulation implies that, given their wage, consumer utility is concave in consumption. It corresponds to a normalization, so that marginal utility and its curvature is the same for all generations when no “bailout” occurs. The main reason for doing this is to rule out situations in which the regulator would like the banks to fund negative NPV investment for redistributive purposes.\(^{18}\)

\(^{16}\)When $\delta < 1$, we have $\epsilon_{kA} = \lambda (1 + \alpha \epsilon_{kA}) - (1 - \lambda) \epsilon_{kA}$, where $\lambda \in [0, 1]$ is the share of retained profits in the PSS level of aggregate bank capital. And once again, $\epsilon_{kA} < 1$ ensures that $\epsilon_{kA} > \epsilon_{kA}$.

\(^{17}\)If $\rho = 1$, then $\bar{u}_t = w_1 + \ln[1 + c_{t+1}^*-w_t]$.\(^{18}\) Over-investment in capital generates higher wages, which is a way to transfer resources from bankers to workers.
Remark. Substituting $e_{t+1} = w_t - b_{t+1}$, one gets:

$$u_t^e = w_t + \frac{(1-b_{t+1})^{1-\rho} - 1}{1-\rho}.$$ 

Savers’ utility is linear in their wage, but they incur a convex utility cost of taxation. Hence, this formulation has the second advantage that it also corresponds to a model where savers are risk-neutral, but there are convex distortion costs of taxation. Obviously, the qualitative results would also be the same in a model where taxation is not distortive, but bank failures have convex deadweight costs.\(^\text{19}\)

4.4.2 Methodology

I have solved the problem numerically through value function iterations (see Appendix C for a description of the solution algorithm) for a wide range of parameter values. I report here a series of examples of simulations in order to illustrate the results.

4.4.3 Are optimal capital requirements still increasing in aggregate bank capital?

A simple way to answer this question, is to numerically solve the model holding expected productivity constant. This can be done by assuming that productivity shocks are iid. Then, $k^*_t$ depends on $A_{t-1}$ through its effect on $e_t$ only, and the optimal capital requirement can be expressed as follows:

$$x^*(e_t) = \frac{e_t}{k^*(e_t)},$$

or, in terms of elasticities (assuming that $k^*(e_t)$ is a differentiable function), $\varepsilon_{xe} = 1 - \varepsilon_{ke}$. (Note that in the risk-neutral case we have $\varepsilon_{ke} = 0$.)

On the one hand, $e$ is the numerator of the fraction above, and positively affects $x^*$. This is the “regulatory channel” identified by Proposition 2. On the other hand, an increase in $e$ increases the amount of losses that can be absorbed by bankers and decreases tax-payers risk exposure. This decreases the social risk-premium and allows for a higher $k^*$, which negatively affects $x^*$. I call this channel the “loss-absorbing channel”.

Main result. It turns out that the regulatory channel largely dominates, and capital requirements are therefore increasing in aggregate bank capital in the risk-averse case also. The intuition is best conveyed graphically.

Consider Figure 5. Assume that aggregate bank capital is $e_1$ and the corresponding optimal level of physical capital is $k^*_1$. The optimal requirement is $x^*_1$, and we have here $k^*_1 = e_1/x^*_1$. Now, consider an increase in bank capital from $e_1$ to $e_2$. Clearly, banks can absorb more losses and the tax payer is less at risk. The social marginal cost decreases with the social risk-premium and the new optimum $k^*_2$ is to the right of $k^*_1$. So, increasing $e$ helps to close the gap with $\tilde{k}$, the level of $k$ that maximizes expected output (that is, the level at which expected marginal return equates the return to storage). However, $\tilde{k}$ does not move. Now, imagine that $x^*$ decreases with $e$. This would imply that if you double $e$, than $k^*$ would more than double. But then, it would decrease dramatically the marginal productivity of capital to an extent that would clearly not be optimal (just imagine what happens in the graph at a level of capital twice as large as $k^*_1$). Intuitively, while a doubling of $e$ decreases the risk-premium (because it reduces the marginal utility of wealth, holding investment constant), it increases investment by a factor $1/x$, which is problematic given the diminishing returns. On top of that, it also double the size of the bailout needed holding marginal return constant. So, these two last effects reinforce one another and largely dominate the favorable effect on the risk-premium. To sum up, $k^*$ increases with $e$, but optimality requires that it be much less than proportionally (that is $\varepsilon_{ke} \ll 1$), which translate in an increase in $x^*$. I discuss the robustness of this result and its policy implications in the next section.

\(^{19}\)For instance if $z$ dollar of tax would be needed per dollar of shortfall of bank value, with:

$$z = \frac{(1-v^\gamma)^{1-\rho} - 1}{1-\rho}.$$
The graphic illustrates the effect of an increase of $e$ when savers are risk-averse. An increase in bank capital from $e_1$ to $e_2$ implies that, all other things equal, the tax payer is less at risk, which shifts down the social risk premium. Accordingly, the socially optimal level of physical capital increases to $k_2^*$. However, this increase is less than proportional, which implies that $x$ should increase.

4.4.4 Are optimal capital requirements still tighter in good times?

Let me again consider the problem through the lens of the optimal requirement equation, this time allowing for persistence in productivity. Consider

$$x^*(\bar{A}_t, e_t) = \frac{e_t}{k_t^*},$$

where $e_t = e(A_{t-1}, e_{t-1}, x_{t-1}^*)$ and $k_t^* = k^*(\bar{A}_t, e_t)$ are given by the equilibrium stationary laws of motion. An increase in $A_{t-1}$ positively impacts $e_t$ through higher wages and profits, and affects $k_t^*$ through the persistence in productivity (see Proposition 3 in the risk-neutral case). So, changes in productivity affect the optimal requirements through 3 main channels. First, mechanically through the regulatory channel ($x$ should increase proportionally to $e$ to leave total lending unchanged). Second, through the loss-absorbing channel (more bank capital decreases the social risk-premium, which allows for a loosening of the requirement). And third, through the “expected productivity channel” (better productivity improves the marginal return to capital).20 Whether optimal requirements are tighter in good times depends whether the first channel dominates the two others. In terms of elasticities, we have now (under some differentiability assumptions):

$$\varepsilon_{xA} = \varepsilon_{eA} - (\varepsilon_{kA} + \varepsilon_{ke} \varepsilon_{eA}),$$

where $\varepsilon_{eA}$, $\varepsilon_{kA}$, and $\varepsilon_{ke} \varepsilon_{eA}$ capture the respective intensity of the regulatory, the expected productivity, and the loss absorbing channel.

Figure 6 depicts a first simulation based on the numerical solution of the model. Parameter values are: $\alpha = .33$, $\delta = 0.94$, $\rho = 2$, $\omega_{HH} = \omega_{LL} = 0.95$, $\beta = 0.98$, $\eta = 0.05$, $A_H = 1.09$, $A_L = 0.91$. Persistence is high (0.95) so that it is easy to visually identify Pseudo Steady States. The small circles represent the realizations of $A_t$ (right-hand scale) and the solid line reports the optimal capital requirements (left-hand scale). One can clearly see that in this example optimal capital requirements are tighter in good times than in bad times, and that the optimal short term adjustments are gradual.

The result that optimal requirements are tighter in good times is due to aggregate bank capital being “more procyclical” than the socially desirable level of physical capital in the economy. This feature of the model is very robust to changes in parameter values. In particular, it also holds for very high levels of risk aversion

20Note that these two last channels also incorporate the forward looking effect that $x_t$ has on $e_{t+1}$.
(say $\rho = 1000$ for instance). The reason is that while it increases the intensity of the loss-absorbing channel, it also magnifies the regulatory channel (remember that holding $x$ constant, an increase in $e$ increase the size of a bailout proportionally, which can be very costly if risk-aversion is high). Overall, increasing risk aversion substantially affects the level of capital requirements but has little impact on their cyclical properties.\footnote{It is, however, possible to overturn the result by assuming that the curvature of the utility function itself dramatically depends on the state of the economy. For instance, in a case otherwise similar to the ones exposed above, if $\rho$ is exogenously set to 0.5 in good times and 2000 in bad times, than the optimal requirements are slightly tighter in bad times. My conclusion from this exercise is that the intensity of risk-aversion is not a key driver of the cyclical properties of optimal capital requirements.}

The result that optimal short-term adjustments are gradual is however a particular case.

**Figure 6:** Higher requirements in good times

This figure depicts a typical simulation result (100 periods) for the considered set of parameters. The solid line corresponds to $x^*(A_{t-1}, e_t)$, whose values are on the left scale. Small circles give the realizations of $A_{t-1}$, which determines $\hat{A}_t$, and whose values are on the right scale. Clearly, PSS requirements are higher in good times, and transition in $x^*$ is gradual.

4.4.5 Optimal short-term adjustments

The optimal direction of short-term adjustments depends on parameter values. Formally:

**Proposition 4.** $A_t < A_{t-1}$ does not necessarily imply $x^*_{t+1} < x^*_t$, and $A_t > A_{t-1}$ does not necessarily imply $x^*_{t+1} > x^*_t$.

**Proof.** See examples below.\[19\]

Before presenting numerical examples, let me provide the intuition by making the point analytically in the risk-neutral case where $\delta = 1$.

Let’s consider an economy that starts at PSS $H$, and is hit by a negative productivity shock ($A_L$) in $t$. We have:

$$k_t^* = \left(\alpha\hat{A}_H\right)^{-\frac{1}{\alpha}}, \quad k_{t+1}^* = \left(\alpha\hat{A}_L\right)^{-\frac{1}{\alpha}}.$$
Note that $k_t^*$ directly jumps down to its PSS $L$ value.

Now, consider the evolution of $e_t$:

$$e_t = \eta(1 - \alpha)A_H k_H^\alpha, \quad e_{t+1} = \eta(1 - \alpha)A_L k_H^\alpha$$

After the bad shock, bank capital decreases. However, wages still benefit from the high level of physical capital. Hence, they are still above their PSS $L$ level (which in fact they reach after two periods: $e_{t+2} = \eta(1 - \alpha)A_L k_H^\alpha$). Whether the optimal capital requirement should go down or (temporarily) up after a bad shock depends on the relative adjustments. In particular, we have that $x_{t+1}^* > x_t^*$ if

$$\frac{A_H}{A_L} < \left(\frac{\bar{A}_H}{\bar{A}_L}\right)^{-\frac{1}{\alpha}}$$

and conversely. As the numerical examples below show, it is easy to find parameter values for which either case is satisfied.

Remarks.

1. Parameter $\alpha$ and the persistence of the shock ($\omega_H$ and $\omega_L$) both tend to decrease $k_t^*$, which implies a larger decrease in physical capital and make more likely that optimal optimal capital requirements are tighter following a negative shock.

2. Parameter $\delta$ (the probability of being hit by the liquidity shock) determines retained earnings and their share in bank capital. If $\delta$ is small, this share is high, and bank capital becomes more sensitive to a bad shock (the short-term productivity elasticity of bank capital is high). That is, the drop in bank capital $e_{t+1}^*$ increases and it becomes more likely that optimal optimal capital requirements are immediately looser following a negative shock.

3. When $\delta < 1$, the size of the drop in bank capital $e_{t+1}^*$ is affected by limited liability (the drop in $e$ is less sharp than it would be if negative equity would be transferred to the next period).

I now present two examples, in the general case when bankers are risk averse, that are different than the gradual adjustment illustrated by Figure 6.

**Opposite short-term adjustments.** Figure 7 depicts the case when $\delta = 1$ (other parameters are identical to those presented in Figure 6, where $\delta = 0.94$). A higher $\delta$ means a lower productivity-elasticity of bank capital. In fact, following a good shock starting from PSS $L$ for instance, the increase in bank capital is proportionally smaller than that of short-term optimal physical capital, and the optimal policy response is hence to first decrease capital requirements, before to hike them up to the high PSS level.

In this case, the only source of bank capital is fresh capital that comes from the wages of future bankers. From a low state, wages increase in response to a positive shock. However, the capital stock that determines labor marginal productivity is the one of the low PSS. Wages are therefore still below their new PSS level. Since persistence is high (0.95), optimal physical capital stock elasticity is high, and the optimal response to the shock is to decrease capital requirements for one period. Note that this decrease lasts for one period only, because wage transition takes two periods and there is no other source of bank capital.

**Overshooting short-term adjustments.** Figure 8 depicts a case ($\delta = 0.8$) where optimality requires to “overshoot” the capital requirements. In fact, for this value of $\delta$, the short-run productivity-elasticity of bank capital is larger than its long-run elasticity. Since the optimal physical capital short- and long-run elasticities are identical, this implies that capital requirements should increase more in the short-run.

Wages still take two periods to adjust, but a relatively large fraction of bank capital comes from retained profits, which substantially increases the productivity-elasticity of bank capital.

From a low state, after a positive shock, profits are unusually high. They are higher than at the high PSS because the stock of physical capital has not yet adjusted. The relatively low wages reflect in fact the scarcity rent earned by the owner of capital. After a period, physical capital adjusts, and factors’ marginal productivity
In this case again, PSS requirements are higher in good times. The short-run optimal response to a (positive) shock is however to first decrease (increase) capital requirements. This is due to the relatively low productivity-elasticity of bank capital in the short term.

will be at their PSS levels. However, accumulated past profits still make aggregate bank capital larger than its new PSS level for a while. This explains the gradual downward adjustment after the overshooting.

After a negative shock, wages are higher than at PSS $L$, which suggests gradual adjustment. However, the whole banking sector is wiped out, and retained profits will take some time to converge to PSS $L$ level. If the latter effect dominates, capital requirements should optimally drop below their PSS $L$ level first, and then gradually be increased.

5 Discussion, robustness, and policy insights
5.1 The importance of aggregate bank capital

The main result of the paper is also the main policy insight:

**Policy insight 1.** *Optimal capital requirements are increasing in aggregate bank capital.*

To stress again the intuition behind the result, let us consider a single (atomistic) bank that doubles its equity base. It should simply be allowed to double the size of its assets. However, if all banks in the economy double their equity base and are allowed to double the size of their assets, this may double aggregate lending in the economy. Given diminishing returns to capital on the real side of the economy, and given that banks have incentives to take on too much risk, this really is problematic. In practice, when aggregate bank capital increases, the banking sector should be allowed to expand, but less than proportionally, which corresponds to an increase in capital requirements.

If this general equilibrium effect is overlooked by the regulator, for instance if capital requirements adjust to
In this case, the short-run optimal response to a (positive) shock is however to overshoot the capital requirements. This is due to the high productivity-elasticity of bank capital in the short-term implied by the relatively low value of $\delta$.

expected productivity but not to aggregate bank capital (as is for instance the case of Basel II), regulation will magnify the business and financial cycles. There will be over-investment (negative NPV investment) during booms, and systemic risk will build up (the economy will be more fragile to productivity shocks). Then, even a small negative shock is enough to make the banking sector collapses and the credit crunch that ensues is inefficiently severe. In the model, the indicators of systemic risk are a low expected quality of the marginal loan but high profits in the financial sector (relative to the rest of the economy).

This result is very robust as it does not hinge on specific parameter values and is in fact due to very few ingredients. First, there is diminishing return to physical capital in the economy. This is perhaps the most standard assumption in macroeconomics, but is often abstracted from in the literature on banking and financial regulation (some notable exceptions include Martinez-Miera and Suarez (2013); Van den Heuvel (2008)). Second, intermediated credit is not irrelevant. In the model, I assume that banks are the only source of funding for firms, but this is not necessary to generate the result. What really matters is that aggregate bank lending affects aggregate investment in physical capital at the margin. Even though this seems to be a reasonable starting point, this also used to be abstracted from in the macro literature. Now, the real effect of the financial sector really is at the core of current policy debates and there is a fast growing literature on the subject (many papers build for instance on the contribution of Kiyotaki and Moore (1997), Holmström and Tirole (1997), and Bernanke et al. (1999), to study the impact on the economy of financial frictions). Third, and perhaps most importantly, capital requirements are binding, which need not (always) be the case in reality. There is in fact no consensus on the subject (Reppullo and Suarez (2013)). For instance, there is evidence that banks do hold “buffers” above the regulatory level (Gropp and Heider (2010)), but there is also evidence of the relevance of bank capital constraint on the credit supply in general (Bernanke et al. (1991); Thakor (1996); Ivashina and Scharfstein (2010); Aiyar et al. (2012)), and in particular that changes in capital requirements affect bank lending (Jimenez et al. (2013)). And indeed, what matters for my analysis is that the requirements are “essentially” binding. That is that whatever the reason why banks decide not to operate at the limit, moving the limit would affect their behavior. Also, the huge resistance of banks (through lobbying for instance) to structural increases in capital adequacy ratios and the strong evidence of “risk-weight optimization” and regulatory arbitrage by the banks operating under the
Basel II regulation (buying CDS on ABS from AIG was one typical way to explicitly circumvent the regulation for instance, see Yorulmazer (2013)) all point in the direction that capital requirements do constrain bank decisions. In the model, capital requirements are binding because deposit insurance acts as a subsidy to leveraged lending. This is in my view a natural assumption because deposits are insured in most advanced economies, and there is evidence that it does distort their cost of borrowing (Warburton et al. (2013); Demirgüç-Kunt and Detragiache (2002); Ioannidou and Penas (2010)). However, deposit insurance in itself is not a required ingredient either. What matters is that the banks do not fully internalize the social cost of lending. The distortion could also come from an implicit guarantee, which would be the case if banks are able to issue liabilities at a discount when they are perceived as “too big too fail” (empirical evidence does point in that direction, see for instance Laeven (2000); Kelly et al. (2011); Noss and Sowerbutts (2012)), or from other mechanisms such as fire-sale externalities. In all these cases, banks would tend to lend more than what is socially optimal and the main mechanisms would apply.

5.2 Optimal requirements cyclical properties of and link with Basel II and III

The Basel I regulation had very coarse risk categories, and Basel II was conceived to better deal with the cross-sectional variation in risks. The idea was to use variables such as individual loans probability of default to weight bank assets, and then to apply a flat 8% capital requirements on those weighted assets. However, probabilities of defaults they tend to move in the same direction over time (they tend to go up during bad times). Since risk-weights went up on average during bad times, it made the “effective capital requirements” go up too, which then tended to decrease aggregate lending. In my model, the “expected productivity” channel suggests that it is actually desirable.

However, bank capital also tends to be low in bad times. This is not taken into account by Basel II and tighter capital requirements applied to a smaller numerator can therefore dramatically contract credit, seriously magnifying economic fluctuations. This is the rationale for Basel III’s “counter-cyclical buffers”. These buffers are supposed to mitigate the effect of increased risk weights. My results strongly suggest that this goes in the right direction. It is worth to note that while other studies lead to similar conclusions, the underlying mechanisms are completely different (fire-sale and other pecuniary externalities, network externalities, asset substitution problem...). In fact, my results suggest that these buffers should more than offset the effect of increased risk weights (so that effective capital requirements be in fact looser in bad times). However, for reasons that I develop below, I would not take this result at face value. I would rather take it as a hint that the general equilibrium effects discussed above are potentially powerful, and I would limit myself to the following statement:

Policy insight 2. The capital buffers of Basel III go in the right direction.

One of the reasons for not taking at face value the result the capital requirements should overall be tighter in good times is that, while it is robust in the context of the model, it may be more specific to the model itself. Indeed, the laws of motion for \( e_t \) and \( k_t \) are extremely stylized and abstract from many ingredients that are potentially relevant (see below). However, that the joint dynamics of \( e_t \) and \( k_t^* \) are key to the optimal stance of bank capital regulation is a quite general insight and would extend to other models where banks do not internalize the full social cost of borrowing and bank activity has an impact on the quality of the marginal loan in the economy. Therefore:

Policy insight 3. The joint dynamics of aggregate bank capital and the socially optimal level of aggregate lending are key to the optimal stance of capital regulation.

Discussion on robustness and avenues for future research I discuss here a series of modeling choices and assumptions that affect the law motions for \( k_t \) and \( e_t \) and therefore potentially the results:

1. Full-reinvestment policy: this is an assumption that I made in order to derive closed form solutions. It is verified for a wide range of parameters, and for all example presented in the paper. When it is not verified
(which is more likely to happen if $\delta$ is low and $\eta$ is high), then there are states of the economy where bankers find it preferable to store part of their wealth until the next period. This affects bank capital accumulation, and my numerical explorations suggest that this generally attenuates its pro-cyclicality. However, I did not find example where it overturned the results. Note also that:

(a) When $\delta$ is very low and $\eta$ very high, then bank capital is not scarce. And then, the Modigliani-Miller theorem applies and bank capital is irrelevant. I ruled out these cases as empirically irrelevant.

(b) For simplicity, I have assumed that banker savings (if any) are not taxed in case of a bailout. If it were the case, this would strongly reduce their incentives to save (since these savings are actually most valuable when there is a bailout). This would therefore make the full-reinvestment assumption even more likely to be verified.

2. No outside equity: allowing bankers to issue outside equity would of course affect the law of motion for $e_t$. However, the regulatory channel would still apply. Second, savers are risk averse and the return to bank equity is negatively correlated with their marginal utility, which make such assets little attractive to them. Still, given that the expected return is generally larger than that of storage they may still be willing to bear that risk. But, in that case, their willingness to do so is likely to be stronger in good times when there is less risk. Therefore, allowing for outside equity issuance is unlikely to decrease the pro-cyclicality of aggregate bank capital.

3. Risky wages: in this model, while labor is hired at the beginning of the period, the wage is determined later, once productivity has realized. If it were determined at the beginning of the period, it would only affect transition dynamics. It therefore does not matter for the main results.

4. Adjustment costs to physical capital: the same applies, they would only affect transition dynamics.

In general, a more sophisticated law of motion for aggregate bank capital would be in my view a crucial ingredient of a more quantitative study of these issues. However, our current understanding of bank capital structure decisions is at best incomplete, especially in general equilibrium (see the discussion in Allen and Carletti (2013), and Repullo and Suarez (2013) for instance). In fact the recent crisis has challenged previous theories (Acharya et al. (2011); He et al. (2010)), perhaps because changes in remuneration practices, and financial innovations have greatly reshaped incentives, and there are no widely accepted historical stylized fact about the dynamics of aggregate bank capital. Even if it was the case, this is not clear that in such a changing environment using past stylized facts as a definite guide to modeling is the most relevant approach. There is therefore a huge scope for future research.

To sum up, the main purpose of this paper is to highlight the different mechanisms by which changes in economic conditions affect the optimal stringency of capital requirements. Which effect dominates is ultimately an empirical question that is beyond and above it scope.

6 A formal definition of Systemic risk

Since the recent crisis, the notion of systemic risk has been at the center of many discussions and studies. This concept has often been linked to the magnifying effect due to various source of contagion in the financial sector (fire-sale externalities, interconnectedness, etc.). It has also been linked to ways by which the financial sector builds up risk in the run up to the crisis.

The present model can provide of formal definition of systemic risk that falls in the latter category. Denote $\bar{k}$ the level of aggregate lending that maximizes expected output at an arbitrary date. It is given by:

$$\bar{k} = (\alpha E[A])^{1+\gamma}. $$

For simplicity, assume that $E[A] = 1$, $VAR[A] = \sigma^2$. Expected output is:

$$\frac{22}{24}$$

There are in fact huge measurement issues as the relevance book value suffers from the huge lags in loss recognition, and the market value does include the option value of equity, which can be substantial. See Korinek and Kreamer (2013) for recent US data (market value) based on the Federal Reserve Data base.
\[ E[y(\bar{k})] = \bar{k}^\alpha - \bar{k} \]

and its variance is:

\[ \text{VAR}[y(\bar{k})] = \sigma^2 \bar{k}^{2\alpha} \]

Hence, up until \( \bar{k} \), increasing \( k \) increases expected output and its variance. There is therefore a trade off. However, passed \( \bar{k} \) while output variance keeps on increasing, expected output falls. So, passed that point, there no longer is a trade-off: reducing investment both reduces risk and increases expected output!

Now, consider two different level of aggregate lending, one on each side of \( \bar{k} \):

\[ k_1 < \bar{k} < k_2 \]

such that \( E[y(k_1)] = E[y(k_2)] \). Then, we have:

\[ \text{VAR}[y(k_1)] < \text{VAR}[y(k_2)] \]

And one can define the systemic risk associated with \( k_2 \) as the difference between the two:

\[ \sigma^2 (k_2^{2\alpha} - k_1^{2\alpha}) \]

This metric measures the amount of excessive output variance at \( k_2 \). It measures the reduction in variance that is possible, keeping expected output constant. Variance is here excessive in the sense that it is not rewarded by an increase in expected output.

References


7 Appendices

Appendix A: Equilibrium

Equilibrium definition

I am interested in a symmetric rational expectation equilibrium that consists of two blocks.

- The first block is simply a sequence of standard (static) competitive equilibria in the labor and physical capital market. They imply the following conditions:
  - \( w_t = (1 - \alpha)A_t k_t^\alpha \) (labor market)
  - \( R_t = \alpha A_t k_t^{\alpha - 1} \) (the physical capital market)

- The second block is a dynamic equilibrium in the banking sector associated to a stochastic process for \( A_t \) and a capital requirement rule \( \epsilon_t \). Such an equilibrium is recursively defined by:
– bankers’ decision rules for \( e_{t+1}, s_{t+1} \), and \( c_t \) that maximize their expected remaining life-time utility given their rational expectations on the distribution of current and future variables, where these decisions rules, as a fraction of their end-of-period wealth \( n_{it} \), are stationary functions of \((A_t, n_t)\) and of the realization of their idiosyncratic liquidity shock.

– the law of motion for aggregate bank capital: \( e_{t+1} = v_t^+ - c_t - s_t + \eta w_t \)

– the law of motion for savers consumption: \( c_t^* = (1 - \eta)w_{t-1} - v_t^+ \)

– the loan-market clearing conditions: \( k_t = \int (d_{it} + e_{it}) \)

– the sufficiency of savings condition: \( \int d_{it} \leq (1 - \eta)w_{t-1} \) (excess supply of savings)

**Solving the general case**

At the end of the period \( t - 1 \), a banker \( i \) that was not hit by the liquidity shock has to allocate his wealth between consumption \((c_{it-1})\), risk-less savings \((s_t)\), and bank equity \((e_t)\).

He maximizes:

\[
E[m_t n_{it}]
\]

where \( m_t \) is a stochastic discount factor (endogenously determined) that captures the marginal utility of wealth at the end of date \( t + 1 \), and \( n_{it} \) is the end-of-period-\( t \) wealth of the banker. That is, if he is born at date \( j \),

\[
n_{it} = \begin{cases} w_t & : t = j \\ v_{it}^+ + s_t & : j < t \leq T \end{cases}
\]

I assume (and derive later the conditions for it to be true) that \( e_{it} = 0 \), for all \( t < T \), so that \( s_{it} = n_{it-1} - e_{it} \).

Then, the problem of the banker can be written as follows:

\[
\max_{e_{it}, d_{it}} E \left[ m_t \left( (R_t - 1)d_{it} + R_t e_{it} \right)^+ \right] + E \left[ m_t (n_{it-1} - e_{it}) \right] \tag{9}
\]

subject to:

\[
\begin{align*}
 & x_{it}(e_{it} + d_{it}) \leq e_{it} \quad (1) \\
 & e_{it} \leq n_{it-1} \quad (2)
\end{align*}
\]

where \( d_{it} = l_t - e_{it} \) is the amount of deposits raised by the banker. Without loss of generality, I assume that bankers do not hold cash.

First, note that, since banker can always choose to consume, we have \( m_t \geq 1 \), for all \( t \). If the capital requirement constraint is binding with strictly positive probability at some date \( s \geq t \), then \( E[m_t] > 1 \) which implies that \( c_{it} = 0 \) for all \( t < T \). Second, note that bankers also take \( m_t \) as given. In fact, either the banker is hit by the retirement shock (and chooses to consume) and \( m_t = 1 \), or he is not and \( m_t \) only depends on aggregate variables. This is because bankers are risk-neutral and have the same time horizon, and all activities are perfectly scalable. Since \( m_t \) reflects the best possible use of wealth going forward, \( m_t \) must be identical for all bankers at a competitive equilibrium under these conditions.

Omitting from now on the \( t \) subscripts for the sake of readability, let me define \( \hat{R}_t = \frac{d_t}{c_t + d_t} \) the threshold realization for the return to lending below which the bank goes bankrupt.\(^{23}\) Similarly, \( R_{\text{max}} = \alpha A_{\text{max}} k^{\alpha - 1} \) and \( R_{\text{min}} = \alpha A_{\text{min}} k^{\alpha - 1} \) denote that boundary values of \( R \).

Denoting \( \hat{\lambda}_{it} \) and \( \lambda_{2t} \) the two inequality constraints respective multipliers, the first order conditions with respect to \( d_t \) and \( e_t \) are respectively:

\[
\begin{align*}
\int_{\hat{R}_t}^{R_{\text{max}}} m(R - 1) f(R) dR + \hat{\lambda}_{4t} x &= 0 \\
\hat{\lambda}_{4t} x &= 0
\end{align*}
\]

\(^{23}\) If \( e_t = d_t = 0 \), then 1 set \( \hat{R}_t = 0 \).
\begin{equation}
\int_{\hat{R}_i}^{R_{\text{max}}} m f(R) dR - \int_{R_{\text{min}}}^{R_{\text{max}}} m f(R) dR + \lambda_{1i}(x - 1) + \lambda_{2i} = 0
\end{equation}

Rearranging the integrals, and substituting the first condition in the second one gives the following two optimality conditions:

\begin{equation}
\int_{\hat{R}_i}^{R_{\text{max}}} m(R - 1) f(R) dR + \lambda_{1i} x = 0 \quad (10)
\end{equation}

\begin{equation}
\lambda_{2i} = \lambda_{1i} + \int_{R_{\text{min}}}^{\hat{R}_i} m f(R) dR \quad (11)
\end{equation}

which allow me to state two lemmas that will prove useful to equilibrium characterization and to proof the several results of the paper.

**Lemma 5. (Pecking order):** \( \lambda_2 \geq \lambda_1 \), strictly if \( R_{\text{min}} < \hat{R}_i \).

**Proof.** The integral in condition (11) is always positive (strictly when \( R_{\text{min}} < \hat{R}_i \)).

This results implies that bankers (strictly) prefer to finance the marginal loan with deposits rather than equity. So, if \( e_i \leq n_t - 1 \) is binding, the capital requirement must be binding as well.

**Lemma 6. (Second derivative):** The second derivative of (9) with respect to \( d_i \) is strictly positive when \( e_i > 0 \) and \( R_{\text{min}} \leq \frac{d_i}{e_i + d_i} < R_{\text{max}} \).

**Proof.** Deriving (9) a second time with respect to \( d_i \) gives:

\[
\begin{cases}
0 ; & \frac{d_i}{e_i + d_i} < R_{\text{min}} \\
-\frac{m(\hat{R}_i - 1)}{e_i + d_i} ; & R_{\text{min}} \leq \frac{d_i}{e_i + d_i} < R_{\text{max}} \\
0 ; & R_{\text{max}} \leq \frac{d_i}{e_i + d_i}
\end{cases}
\]

This implies that if \( x < 1 - R_{\text{min}} \), the first and second order conditions cannot be simultaneously satisfied for an interior value of \( d_i \). So, the capital requirement must in fact be binding for any \( e_i \).

**Equilibrium characterization** Based on Lemmas A1 and A2, in a symmetric equilibrium, decision rules can be characterized as follows:

- **(CASE 1)** If \( \lambda_{2i}(A_{t-1}, n_{t-1}) < 0 \) then \( \lambda_{1i}(A_{t-1}, n_{t-1}) < 0 \): bankers reinvest all their wealth in banking equity \((s_{it+1} = 0 \text{ for all } i)\), and leverage to the maximum.

  - In this case , \( e_{it+1} = n_t \) and \( d_{it+1} = n_t \frac{1 - x_{i+1}}{x_{i+1}} \) for \( i \) such that \( t < T_i \) and \( c_{it} = n_t \) for \( i \) such that \( t = T_i \).

- **(CASE 2)** If \( \hat{\lambda}_{2i}(A_{t-1}, n_{t-1}) = 0 \) then bankers are indifferent between bank equity and savings. If \( R_{\text{min}} < \hat{R}_i \), then \( \lambda_{1i}(A_{t-1}, n_{t-1}) < 0 \): the capital requirement constraint is binding as well and the level of aggregate lending is pined down by:

\begin{equation}
\int_{\hat{R}_i}^{R_{\text{max}}} m(R - 1) f(R) dR - x \int_{R_{\text{min}}}^{\hat{R}_i} m f(R) dR = 0
\end{equation}
• (CASE 3) If \( \lambda_{32}(A_{t-1}, n_{t-1}) = 0 \) but \( R_{\text{min}} \geq \hat{R}_t \), then \( \lambda_{31}(A_{t-1}, n_{t-1}) = 0 \). In this case, Modigliani-Miller applies, and there is a big indeterminacy. This is to pin down the equilibrium in this case that I have assumed that when they are indifferent, bankers choose first to consume, then to save, and then to invest in equity. Therefore, they operate at the lowest possible equity ratio, in which case aggregate bank capital is indeed pinned down by:

\[
\int_{R_{\text{min}}}^{R_{\text{max}}} m(R-1) f(R) dR = 0
\]

Note that this sub-case requires \( x_t \in \left( (1 - R_{\text{min}}(k^*_t)), \frac{k^*_t}{c_t} \right) \), that is the capital requirement should be tight enough so that no bank can fail, but their must be enough aggregate banker wealth to finance the desired level of investment given that requirement. Note that if this case were to hold at all \( t \), then Modigliani-Miller applies for all \( t \) and \( k^* \) corresponds to the risk-neutral case. But then, bankers would be all indifferent. Since I assume that they consume in this case, then this reduces \( c_t \) and this cannot be an equilibrium.

In this paper, I mainly focus on a typical equilibrium for which we are in CASE 1 for all \( t \).

Appendix B: Proofs

Proposition 1. Without regulation \((x_t = 0)\), the competitive equilibrium exhibits negative NPV investment (that is: \( E[R_t] < 1 \)).

Proof. From Lemma A1 (pecking order) we know that bankers prefer to fund lending with deposits than equity. Hence, when \( x = 0 \), we have that \( e_t = 0 \) and therefore \( \hat{R}_i = 1 \). But then, since \( m \geq 1 \),

\[
\int_{\hat{R}}^{R_{\text{max}}} m(R-1) f(R) dR > 0
\]

unless \( R_{\text{max}} \leq 1 \), which necessarily implies that \( E[R] < 1 \).

Note that the result easily extend to \( x > 0 \) but small, and that \( R_{\text{max}} < 1 \) cannot be an equilibrium. \(\square\)

Lemma 3. At the social optimum, capital requirements must be binding.

Proof. this directly follows from Lemmas 5 and 6 (and, for the irrelevant cases where Modigliani-Miller always apply, from the assumption that bankers consume if they are indifferent). \(\square\)

Lemma 4. When savers are risk-neutral and \( \beta = 1 \), maximizing the welfare function \((??)\) boils down to maximizing expected output at each period.

Proof. I propose here the proof for the case of full reinvestment policy (Assumption 1). The proof for the general follows exactly the same logic but is more cumbersome.

The welfare function is \( E \left[ \sum_{t=1}^{\infty} \delta v_t \right] \), with \( v_t = w_{t-1} - v_{t-1}^- / (1 - \eta) \) and \( v_t = \sum_{t=1}^{\infty} c_{t, t+s} \), where \( c_{t, t+s} \) corresponds to the consumption at \( t+s \) of bankers a representative of the bankers born at \( t \).

Step 1: Note that \( \sum_{t=1}^{\infty} \sum_{s=1}^{\infty} \delta c_{t, t+s} = \sum_{t=1}^{\infty} C_t \), where \( C_t \) is the aggregate consumption of all bankers in \( t \), which corresponds to \( \delta v_t^- \). Hence, the welfare function becomes: \( E \left[ \sum_{t=1}^{\infty} \delta v_t^- \right] \), or:

\[
E \left[ \sum_{t=1}^{\infty} \delta (\alpha A_t k_t^\alpha - k_t + e_t)^+ \right] = (1 - \eta) w_{t-1} - (\alpha A_t k_t^\alpha - k_t + e_t)^-.
\]

Step 2: Assume that date \( s \) is the first date (after \( t-1 \)) at which \( v_t \) is negative. That is \( v_t^- = 0 \) for all \( t < s \), and \( v_s^- = 0 \) for \( t = s \). Let me write down all the terms in this sum that depend on \( k_t \), directly and through \( e_{t+1} = (1 - \delta) (\alpha A_t k_t^\alpha - k_t + e_t) + \eta (1 - \alpha) A_t k_t^\alpha e_t \).
which is always satisfied since shocks are not fully persistent.

**Proposition 3.** Optimal macro-prudential capital requirements are higher in good times than in bad times. Formally: \( x_{H}^* > x_{L}^* \).

**Proof.** First, note that \( v_{H}^* \) is always positive since \( \bar{A}_H > 1 \) and \( A_H > \bar{A}_H \). If \( v_{L}^* > 0 \), for \( ss = L, H \):

\[
x_{pss}^* = \frac{1}{\delta} \left[ (\alpha(1-\delta) + \eta(1-\alpha))A_{pss}(\alpha\bar{A}_{pss})^{-1} - (1-\delta) \right].
\]

Hence, I have that:

\[
x_{H}^* > x_{L}^* \iff \frac{A_H}{A_L} > \frac{\bar{A}_H}{\bar{A}_L},
\]

which is always satisfied since shocks are not fully persistent (\( \omega_L, \omega_H < 1 \)).

If \( v_{L}^* = 0 \), I have:

\[
x_{L}^* = \eta(1-\alpha)A_Lk_{L}^{\alpha-1},
\]

and I need to show that:

\[
(\alpha(1-\delta) + \eta(1-\alpha))A_{H}k_{H}^{\alpha-1} - (1-\delta) > \delta\eta(1-\alpha)A_{L}k_{L}^{\alpha-1}
\]

which can be rewritten:

\[
(1-\delta) \left( \alpha A_{H}k_{H}^{\alpha-1} - 1 \right) + \eta(1-\alpha)A_{H}k_{H}^{\alpha-1} > \delta\eta(1-\alpha)A_{L}k_{L}^{\alpha-1}
\]

and is satisfied because the definition of \( k^*, k_{pss}^* \), and \( \bar{A}_L < 1 < \bar{A}_H \) imply \( \alpha A_{H}k_{H}^{\alpha-1} > 1 > \alpha A_{L}k_{L}^{\alpha-1} \). \( \square \)
Appendix C: additional material

The solution algorithm

The detailed algorithm is the following:

- **Initialization**
  - Set parameters \( \{ \alpha, \beta, \delta, \eta, \rho, A_H, A_L, \omega_H, \omega_L \} \), Set \( n = 0 \);
  - Set a grid for \( e \) and \( k \);
  - Set an initial value function \( V(e, A) = V_0 \);
  - Assume: \( e_{t+1} = (1 - \delta)v^+_t + \eta(1 - \alpha)A_t k^*_t \) and \( k_t = e_t/x_t \).

- **Repeat until convergence** (That is: \( V_{n+1} \simeq V_n \))
  - \( V_{n+1}(e_t, A_t) = \max_k E_t \left[ \delta v^+_t + (1 - \eta) \left( (1 - \alpha)A_{t-1} k^*_{t-1} + u(1 - b_t) \right) + \beta V_{t+1}(e_{t+1}, A_{t+1}) \right] \);
  - \( n = n + 1 \);

- **Solution validation**
  - \( k^*(e_t, A_{t-1}) \equiv \arg \max_k V_n(e_t, A_{t-1}) \)
  - \( x^*(e_t, A_{t-1}) = e_t/k^*(e_t, A_{t-1}) \)
  - Simulate and check that:
    - bankers do not want to deviate (that is \( c_{it} > 0 \), for some \( i \) and some \( t < T \)).
    - there are excess savings