

Consumption with Imperfect Perception of Wealth^{*}

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Abstract

Consumers have difficulty tracking their total wealth, or keeping it at the front of their minds when making consumption and saving decisions. In this paper, I show how such imperfect perception of wealth can explain several key deviations of consumption behavior from the permanent income hypothesis, including: excess sensitivity to current income, smaller MPCs out of wealth than out of current income, and excess discounting of future income. Importantly, my approach does not rely on liquidity constraints and can explain the empirical evidence on high-liquidity consumers' deviations from the permanent income hypothesis. I further provide an interpretation of the model in which the consumer has separate mental accounts for her current income and her wealth. Thus, the consumer exhibits behavior similar to a two-asset model, in a one-asset context without borrowing constraints. The friction can be quantitatively important in explaining MPCs, and has substantive macro implications for monetary and redistributive policy. Methodologically, the paper develops a tractable method for incorporating imperfect perception of the endogenous state into an otherwise standard Markov decision problem.

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1 Introduction

Economic models often assume agents perfectly understand their economic circumstances. Yet in practice, people may struggle to recognize their exact situation. In the context of intertemporal consumption and saving problems, the standard model often assumes that the consumer makes her consumption and saving decisions with perfect knowledge of her total wealth. In practice, however, a typical consumer may have difficulty tracking her total wealth, or keeping it at the front of her mind when making consumption and saving decisions.

Empirical evidence also supports such imperfect perception of wealth. Agarwal et al. (2008), Stango and Zinman (2014) and Jiang et al. (2018) find that a typical consumer often neglects her credit card balances, and such neglect leads to suboptimal credit card usage. Moreover, recent literature on Fintech shows that providing information about the consumer's total wealth by aggregating her financial accounts can significantly change her behavior (Levi, 2015; Carlin, Olafsson and Pagel, 2017). In this paper, I study how this empirically grounded, but rarely studied, friction influences the intertemporal consumption and saving decisions, and discuss its macroeconomic implications.

The main result is that imperfect perception of wealth provides a new, behavioral theory for several key deviations of consumption behavior from the permanent income hypothesis (Thaler, 1990; Attanasio and Weber, 2010): (i) excess sensitivity to current income;¹ (ii) smaller MPCs out of wealth than out of current income; (iii) excess discounting of future income. Importantly, my approach does not rely on liquidity constraints, and is particularly suitable for explaining the empirical evidence on high income, *high-liquidity* consumers' deviations from the permanent income hypothesis. For example, in the data, these consumers also exhibit excess sensitivity to current income (Parker, 2017; Kueng, 2018; Fagereng, Holm and Natvik, 2019).

Together, a consumer with imperfect perception of wealth violates the fungibility principle, i.e. different components of income and wealth cannot be collapsed into a single number representing their combined present value. My approach then answers Thaler (1990)'s call for a model that treats the MPCs out of current income, expected future income, and wealth differently. Through a calibration exercise, I further show that imperfect perception of wealth can be quantitatively important in explaining MPCs. By providing a theory about the frictional consumption behavior of the rich, my approach also has important macro implications for the determination of aggregate

¹Papers documenting excess sensitivity to current income include, but are not limited to, Zeldes (1989), Campbell and Mankiw (1989), Souleles (1999), Shapiro and Slemrod (2003), Johnson, Parker and Souleles (2006), Agarwal, Liu and Souleles (2007), Jappelli and Pistaferri (2010), Parker et al. (2013), Agarwal and Qian (2014).

demand.

Framework: imperfect perception of wealth. In a single-agent, one-asset income fluctuation problem without borrowing constraints, I introduce imperfect perception of wealth: the consumer’s perceived current wealth is a weighted average between her actual wealth and a default.² To isolate the friction of interest, I let the consumer have perfect knowledge about her current income state. The decision problem is effectively an intra-personal game among multiple selves with misspecified beliefs, in which each self is in charge of consumption and saving decisions in a period. With perfect perception of wealth, the equilibrium of the game coincides with the standard solution.

One simple way to describe the key friction here is that the consumer is effectively inattentive to her endogenous wealth. This is different from the existing applications of inattention and sparsity (Sims, 2003; Gabaix, 2014; Gabaix, 2016*b*) to intertemporal consumption and saving problems: those papers focus on inattention to fundamentals that are exogenous to the consumer, such as income and interest rate; they maintain the consumer’s perfect knowledge of her endogenous wealth. A conceptual difference is: imperfect perception of the endogenous wealth comes from the consumer’s bounded recall, or selective retrieval of memory, about her past consumption and saving decisions; inattention to exogenous fundamentals nevertheless can coexist with perfect recall.

On the methodological front, I develop a tractable approach to incorporate imperfect perception of the endogenous state into an otherwise standard Markov: I summarize the consumer’s friction by her misperception of the current state; I am able to maintain the Markov property of the problem and establish a modified principle of optimality. The methodology can be applied to other Markov decision problems such as investment and price setting.

Main results: excess sensitivity and excess discounting. With some simplifying assumptions, I first establish a key analytical theorem about how consumption with imperfect perception of wealth deviates from the predictions of the permanent income hypothesis. The theorem has three parts: first, the consumer exhibits excess sensitivity to current income; second, she exhibits a smaller MPC out of liquid wealth than out of current income; third, she exhibits excess discounting of current income. Moreover, a larger degree of imperfect perception of wealth leads to a larger deviation from the predictions of the permanent income hypothesis.

Let me briefly explain the intuition. The second part of the theorem, the consumer’s smaller MPC out of liquid wealth than out of current income, comes directly from the consumer’s imperfect

²For the main theorem below to hold, each self’s default wealth can be an arbitrary exogenous function of the income process.

perception of current wealth. The other two, more “interesting” parts of the theorem, instead come from the current self’s concern about future selves’ imperfect perception of wealth.³ Specifically, let me start by focusing on the first, most important part of the theorem: the excess sensitivity to current income. Consider first a positive shock to current income. The current self is worried that, if she increases her savings after the income shock, her future selves cannot perfectly perceive and respond to such changes in wealth. As a result, she instead increases her current consumption more. Intuitively, if the consumer leaves the additional money in her bank account, her future selves will not fully notice the increase. The consumer then consumes more now. By the same token, when a negative income shock hits, the current self is less willing to decrease her savings and instead decreases her current consumption more.

Let me now turn to the third part of the theorem: the excess discounting of future income. Note that the response of current consumption to changes in expected future income leads to changes in savings. With imperfect perception of wealth, however, future selves cannot perfectly perceive and respond to changes in wealth. The current self is then less willing to change her savings and exhibits a smaller consumption response to changes in expected future income.

I further work out additional applications of the main theorem. First, I study the impulse responses of consumption to news about future income. I show imperfect perception of wealth can explain the empirical pattern that the consumption response when the consumer actually receives the income is larger than the consumption response when the news arrives (i.e. excess sensitivity to anticipated current income, such as in Kueng, 2018). Second, I study the case in which the income process is distributed as an AR (1) process. I show the degree of excess sensitivity to the income shock decreases with its persistence. This result is consistent with the empirical evidence that consumption does *not* exhibit excess sensitivity to persistent income shocks (Blundell, Pistaferri and Preston, 2008).

A behavioral complement to liquidity constraints. Traditional explanations of the above deviations from the permanent income hypothesis rely upon liquidity constraints (Carroll, 1997; Gourinchas and Parker, 2002; Kaplan and Violante, 2010, 2014). My approach, on the other hand, does not rely on liquidity constraints, and can be viewed as a behavioral complement to the liquidity-constraint-based explanations. One interpretation of the model is that the consumer has separate mental accounts (Thaler, 1990): one current income account which she perfectly knows, and one wealth account of which she has imperfect perception. The consumer then exhibits be-

³In the language of O’Donoghue and Rabin (1999, 2001), “sophistication,” i.e. the current self’s understanding of her future selves’ mistakes, plays an important role. Moreover, the main theorem can be easily extended to the case with partial sophistication.

havior similar to that in two-asset models (Kaplan and Violante, 2014; Kaplan, Moll and Violante, 2018), in a one-asset context without liquidity constraints. Imperfect perception of wealth then decouples frictional behavior such as excess sensitivity to current income from liquidity constraints, and explains the empirical evidence on high income, *high-liquidity* consumers' deviations from the permanent income hypothesis (Di Maggio, Kermani and Majlesi, 2018; Kueng, 2018; Fagereng, Holm and Natvik, 2019).

A calibration exercise. To illustrate the quantitative potential of imperfect perception of wealth, I conduct a calibration exercise with a more general utility function and a borrowing constraint. I introduce a sufficient-statistic method to calibrate the size of imperfect perception of wealth based on the ratio between the MPC out of wealth and the MPC out of current income. In the benchmark calibration, for a high-income, high-liquidity consumer with imperfect perception of wealth, the MPC out of a dollar of an unanticipated temporary current income shock is 0.146 dollars per year, around 3.89 times its benchmark with perfect perception of wealth. This result then reduces the gap between theoretical predictions and the empirical evidence (e.g. Fagereng, Holm and Natvik, 2019) on the MPC out of current income for consumers with high income and high liquidity.

I further study how imperfect perception of wealth interacts with borrowing constraints. Two new results emerge. First, the consumer with imperfect perception of wealth has a flattened MPC-wealth curve, for two reasons. At a high wealth level, as above, the friction increases the MPC out of current income. At a low wealth level, the consumer's perceived wealth is larger than her actual wealth, and she thinks she is further away from the borrowing constraint than she actually is. She then displays a lower MPC out of current income at a low wealth level than that with perfect perception of wealth. This flattened MPC-wealth curve is also consistent with the empirical evidence in Kueng (2018) and Fagereng, Holm and Natvik (2019). Second, imperfect perception of wealth increases the probability that a consumer hits her borrowing constraint. The intuition is also that, since the consumer's perceived wealth is larger than her actual wealth at a low wealth level, she over-consumes at a low wealth level and is more likely to hit her borrowing constraint. This result may help alleviate one shortcoming of the traditional one-asset model of income fluctuations: it does not generate enough liquidity-constrained households (Kaplan and Violante, 2014).

Macro implications. Imperfect perception of wealth also has important macro implications. First, with imperfect perception of wealth, even rich consumers can behave as "hand-to-mouth" consumers. Such frictional behavior of the rich can be important in determining aggregate demand.

For example, the redistributive policy from the rich towards the poor to stimulate aggregate demand may not be as powerful.

Second, imperfect perception of wealth more generally increases the importance of GE income multiplier because of the excess sensitivity to income. A recent literature shows that the strength of such GE income multiplier might be over-stated because it requires unrealistic coordination among consumers (Angeletos and Lian, 2018, Farhi and Werning, 2017, and Gabaix, 2016*b*). By increasing the importance of GE income multiplier, imperfect perception of wealth then enhances the GE dampening mechanisms studied in the above works. Moreover, such an enhancement of GE dampening increases with the horizon: the friction can then help explain the forward guidance puzzle.

Behavioral literature on intertemporal consumption problems. This paper builds upon the literature on intertemporal consumption problems with inattention (Sims, 2003; Reis, 2006; Luo, 2008; Luo and Young, 2010; Maćkowiak and Wiederholt, 2015; Gabaix, 2016*a*). Those papers study the impact of a consumer’s inattention to her exogenous income, and often find her consumption exhibits a sluggish response to income shocks.⁴ I instead study the impact of a consumer’s imperfect perception of her endogenous wealth. I show how imperfect perception of endogenous wealth leads to unique predictions about the consumer’s MPCs, such as excess sensitivity to current income. Conceptually, imperfect perception of endogenous wealth can come from the consumer’s bounded recall, or selective retrieval of memory, about her past consumption and saving decisions; inattention to exogenous income nevertheless can coexist with perfect recall.⁵

Gabaix and Laibson (2002), Alvarez, Guiso and Lippi (2012) and Abel, Eberly and Panageas (2007, 2013) study intertemporal portfolio choice problems with inattention to stock returns. Their consumers still have perfect recall, and inattention to stock returns alone will not necessarily break the consumer’s Euler equation and will not generate excess sensitivity to current income.

Hyperbolic discounting (Laibson, 1997) can also generate excess sensitivity to current income (Barro, 1999; Ganong and Noel, 2019). However, within the context of one-asset models without borrowing constraints, the fungibility principle is maintained under hyperbolic discounting. That is, all components of income and wealth can still be collapsed into a single number by combining

⁴Sims (2003) also studies the inattention to exogenous initial wealth, which effectively plays the same role as exogenous income.

⁵For other papers on bounded recall, Wilson (2014) studies a sequential problem in which the decision-maker has bounded recall. However, in her world, the decision-maker only makes a final binary decision after receiving a series of signals: the bounded recall is only about the decision maker’s past exogenous signals. Gennaioli and Shleifer (2010) and Bordalo, Gennaioli and Shleifer (2017) instead focus on the psychological foundation of bounded recall and selective retrieval from memory: their models focus on how representativeness heuristics determine “what comes to mind.”

their present value. As a result, hyperbolic discounting alone cannot explain the empirical evidence on different MPCs out of current income versus wealth and excess discounting of future income. Moreover, with hyperbolic discounting, the consumer over-consumes on average compared to her frictionless benchmark. On the other hand, based on my approach, the consumer only exhibits excess sensitivity to income shocks. On average, the consumer's behavior can be frictionless.

In another strand of literature, Laibson (1997) and Angeletos et al. (2001) study the implications of hyperbolic discounting for a two-asset model with an illiquid asset and liquidity constraints. There, the consumer may use the illiquid asset as the commitment device, and endogenously chooses to be at the liquidity constraint to avoid over-spending. In this case, the liquidity-constrained consumer displays excess sensitivity to current income. However, this mechanism cannot explain the evidence on high-liquidity consumers' excess sensitivity to current income. Moreover, the consumer there displays the same MPC out of current income and liquid wealth.

In Mullainathan (2002), Rozsypal and Schlafmann (2017), and Azeredo da Silveira and Woodford (2019), consumption over-reacts to changes in current income because the consumer's expectation of her future income is excessively sensitive to her current income. In this paper, however, the consumer forms a rational expectation about her future income. The excess sensitivity to current income is instead driven by her concern that her future selves cannot perfectly perceive and respond to changes in wealth.

Kőszegi and Rabin (2009) study a two-period consumption and saving model in which the consumer is loss averse over changes in her beliefs about her present and future consumption. Moreover, the fungibility principle is maintained in Kőszegi and Rabin (2009): current income and wealth play the same role. Their consumer exhibits excess sensitivity to increases in current income, but excess smoothness to decreases in current income. Relatedly, Pagel (2017) studies an intertemporal consumption problem in which the consumer obtains news utility over her beliefs about present and future consumption. In her model, consumption initially under-reacts to income shocks and then adjusts with a delay.

Shefrin and Thaler (1988) and Thaler (1990) argue that consumers violate the fungibility principle and exhibit different MPCs out of current income, future income and wealth. They verbally explain this phenomenon based on the hypothesis that the consumer may have several mental accounts, and directly assume that consumers have different degrees of temptation to spend money in different accounts. On the other hand, this paper avoids such an additional assumption. It shows how the fact that the consumer has separate mental accounts itself can explain the empirical deviations from the permanent income hypothesis. Moreover, this paper contributes by

formalizing the idea of mental account in an intertemporal consumption and saving problem.⁶

Finally, a complementary paper (Lian, 2019) studies static multiple-decision problems in which the decision-maker makes each decision with an imperfect understanding of her other decisions. Lian (2019) shows this friction provides a smooth model of narrow bracketing type behavior. In the current paper, I instead study a dynamic Markov decision problem. The current self's imperfect understanding of her past decisions is summarized by her imperfect perception of the endogenous state (wealth). I then commit to the application in the income fluctuations problem, connect to the empirical evidence, and study macro implications.

Layout. Section 2 introduces imperfect perception of wealth in an otherwise standard income fluctuations problem. Section 3 and Section 4 establish and discuss my key analytical results. Section 5 conducts a calibration exercise. Section 6 introduces time-varying interest rates and studies the GE implications. Section 7 concludes. The Appendix contains proofs and additional results.

2 Set up and the Key Friction

I first introduce a single-agent income fluctuations problem. In the tradition of Bewley (1977), the consumer faces idiosyncratic income shocks, and can save and borrow through a risk-free asset. There is a key departure from the standard paradigm: the consumer makes her consumption and saving decisions in each period based on an imperfect perception of her current wealth.

Utility and budget. The consumer's utility is given by

$$U_0 \equiv \sum_{t=0}^{T-1} \beta^t u(c_t) + \beta^T v(a_T + y_T), \quad (1)$$

where c_t is her consumption at period $t \in \{0, 1, \dots, T-1\}$, β is her discount factor, the consumption utility function $u(\cdot)$ is strictly increasing and concave, and $v(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$ captures the utility (also concave) from retirement or bequests. The consumer can save and borrow through a risk-free asset, and is subject to the budgets

$$a_{t+1} = R(a_t + y_t - c_t) \quad \forall t \in \{0, \dots, T-1\}, \quad (2)$$

⁶See Farhi and Gabaix (2015), Hastings and Shapiro (2018), and Lian (2019) for models of mental accounting in static contexts. Galperti (2019) provides a theory of good-specific personal budgets driven by self-control concerns in the presence of intratemporal good-specific taste shocks.

where y_t is her (exogenously drawn) income at period t , a_t is her wealth level at the start of period t , and R is the gross interest rate on the risk-free asset.⁷ To isolate the friction of interest, the consumer here is not subject to any borrowing constraints. I study the case with borrowing constraints in the calibration exercise in Section 5.

In each period t , I use the random variable (or vector) \vec{y}_t to summarize the income process. It includes current income y_t and potential news about future incomes $\{y_{t+k}\}_{k \geq 1}$ in period t . If income is drawn independently across periods, $\vec{y}_t = (y_t)$. In more general cases, \vec{y}_t can also include news about future incomes.⁸ I assume $\{\vec{y}_t\}$ is a continuous state Markov chain with transitional density function $\phi_{t+1}(\vec{y}_{t+1} | \vec{y}_t)$ and initial distributional density $\Phi_0^y(\vec{y}_0)$.⁹ The payoff relevant state for the consumer in each period t can then be summarized by

$$(a_t, \vec{y}_t),$$

where \vec{y}_t captures the exogenously drawn income state, and a_t captures the endogenously determined current wealth level based on the consumer's past decisions (except the exogenous initial wealth a_0).

The decision problem and the intrapersonal equilibrium. Following Piccione and Rubinstein (1997) and Harris and Laibson (2001), I study the Markov perfect equilibrium in the intrapersonal game with multiples selves, in which each self $t \in \{0, \dots, T-1\}$ decides on consumption and saving decisions at period t . Here, different selves have the common objective in (1) but each self's subjective belief can be misspecified. This misspecified belief is not arbitrary: the friction is summarized by each self's misperception of the current state; as discussed shortly, she nevertheless understands the law of motion of the exogenous and endogenous state variables. The intrapersonal equilibrium coincides with the standard solution to the income fluctuations problem if all selves have perfect perception of the states.

Definition 1. *An intrapersonal equilibrium consists of 1) For $t \in \{0, \dots, T-1\}$, each self t 's subjective expectation $\tilde{E}_t[\cdot]$, based on her perceived state (a_t^p, \vec{y}_t^p) ; 2) each self t 's consumption rule $\{\mathbf{c}_t(a_t^p, \vec{y}_t^p)\}_{t=0}^{T-1}$. They satisfy the following two conditions:*

I. For $t \in \{0, \dots, T-1\}$, each self t 's subjective expectation $\tilde{E}_t[\cdot]$ differs from the rational

⁷The gross interest rate R is fixed and perfectly known by the consumer. As a result, my approach is different from Gabaix and Laibson (2002), Alvarez, Guiso and Lippi (2012) and Abel, Eberly and Panageas (2007, 2013), which focus on inattention to returns on wealth.

⁸For example, consider the case where a piece of news about future income y_{t+k} arrives at period t . Using s_t to denote the news, $\vec{y}_t = (y_t, s_t)$.

⁹Note that the Markov chain is allowed to be time-inhomogeneous.

expectation because she (mistakenly) thinks the current state is her perceived state (a_t^p, \vec{y}_t^p) with certainty.

On the other hand, each self t correctly understands other selves' (mis-) perception functions and consumption rules, the budget constraint and the transitional density function for the exogenous income process.

Her perceived state (a_t^p, \vec{y}_t^p) is given by the perception functions $\mathbf{a}_t^p(\cdot)$ and $\vec{\mathbf{y}}_t^p(\cdot)$, which map the actual state to the perceived state:

$$(a_t^p = \mathbf{a}_t^p(a_t), \vec{y}_t^p = \vec{\mathbf{y}}_t^p(\vec{y}_t)) \quad \forall t \in \{0, \dots, T-1\}. \quad (3)$$

II. For $t \in \{0, \dots, T-1\}$, each self t chooses period t consumption based on her subjective expectation $\tilde{E}_t[\cdot]$, taking other selves' perception functions and consumption rules as given:

$$\mathbf{c}_t(a_t^p, \vec{y}_t^p) = \arg \max_{c_t} u(c_t) + \tilde{E}_t \left[\sum_{k=1}^{T-t-1} \beta^k u(\mathbf{c}_{t+k}(\mathbf{a}_{t+k}^p(a_{t+k}), \vec{\mathbf{y}}_{t+k}^p(\vec{y}_{t+k}))) + \beta^{T-t} v(a_T + y_T) \right], \quad (4)$$

subjective to the budgets in (2).¹⁰

It is worth noting that the final wealth a_T is allowed to be negative, as the utility from retirement or bequests $v(\cdot)$ is defined on the entirety of \mathbb{R} . This guarantees that, even with imperfect perception, the budget in (2) is always satisfied and the intrapersonal problem is always well defined. One can then solve for the intrapersonal equilibrium uniquely via backward induction. The final period also does not play a special role: in Section 3, I show that the consumer's consumption rule converges to a limit when $T \rightarrow +\infty$. Finally, I study the case with non-negative wealth in Section 5.

Here, I allow misperception of both the endogenous wealth ($a_t^p \neq a_t$) and the exogenous income state ($\vec{y}_t^p \neq \vec{y}_t$). I allow for misperception of both states because I want to derive a general modified principle of optimality in Proposition 1. The result is of independent interest. In the main analysis in later sections, I focus on imperfect perception of the endogenous wealth ($a_t^p \neq a_t$). To isolate the friction of interest, I often impose perfect perception of the income state: $\vec{y}_t^p = \vec{y}_t$.

The key friction: misperception of the current state. Inspired by the observation that people often have difficulty perfectly understanding their exact economic circumstances (Gabaix, 2019; Woodford, 2019), I study each self t 's misperception of the current state: her perceived state

¹⁰Note that each self treats her past selves' decisions as given, maximizing the objective in (4) is equivalent to maximizing (1).

(a_t^p, \vec{y}_t^p) may differ from the actual state (a_t, \vec{y}_t) . To minimize the departure from the standard paradigm, such misperception of the current state is the only friction: each self t can correctly predict other selves' (mis-) perception functions and consumption rules, the budget constraint, and the exogenous income process. This means that each self t can correctly predict 1) the law of motion of the exogenous and endogenous state variables, given her perceived current state; 2) the mapping from future states to future consumption. In other words, each self t 's subjective expectation is given by the rational expectation, *as if* the true state is (a_t^p, \vec{y}_t^p) .

Let me use a few examples to further illustrate how each self $t \in \{0, \dots, T-1\}$ forms her subjective expectation. First consider her subjective expectation of the next period wealth, $\tilde{E}_t [a_{t+1}]$. Self t misperceives current income and wealth, but she knows the current consumption, which she decides. She also understands the budget constraint in (2). As a result, self t believes with certainty that

$$\tilde{E}_t [a_{t+1}] = R(a_t^p + y_t^p - c_t). \quad (5)$$

Now let us turn to her subjective expectation of the next period consumption, $\tilde{E}_t [c_{t+1}]$. As self $t \in \{0, \dots, T-2\}$ understands self $t+1$'s (mis-)perception functions $\mathbf{a}_{t+1}^p(\cdot)$ and $\vec{\mathbf{y}}_{t+1}^p(\cdot)$ and consumption rule $\mathbf{c}_{t+1}(a_{t+1}^p, \vec{y}_{t+1}^p)$, she understands the mapping from the period $t+1$ state to c_{t+1} . Her subjective expectation of c_{t+1} is then given by

$$\tilde{E}_t [c_{t+1}] = \int \mathbf{c}_{t+1}(\mathbf{a}_{t+1}^p(R(a_t^p + y_t^p - c_t)), \vec{\mathbf{y}}_{t+1}^p(\vec{y}_{t+1})) \phi(\vec{y}_{t+1} | \vec{y}_t^p) d\vec{y}_{t+1},$$

where I use (5) and the fact that her belief about the future exogenous \vec{y}_{t+1} is given by p.d.f. $\phi(\vec{y}_{t+1} | \vec{y}_t^p)$ based on her current perceived income \vec{y}_t^p .

The perception functions. Each self t 's perceived state (a_t^p, \vec{y}_t^p) is characterized by the perception functions $\mathbf{a}_t^p(\cdot)$ and $\vec{\mathbf{y}}_t^p(\cdot)$, which map the actual state to the perceived state. These functions are deterministic and increasing in their respective states. In the rest of the paper, I often study the case where the perceived state is a weighted average between the actual state and a default. For example,

$$\mathbf{a}_t^p(a_t) = (1 - \lambda)a_t + \lambda a_t^D, \quad (6)$$

where $\lambda \in [0, 1]$ and a_t^D is the default wealth level at period t . By summarizing the friction based on misperception of the current state, I am able to maintain the Markov nature of the problem: as each self t thinks the current state is her perceived state (a_t^p, \vec{y}_t^p) with certainty, knowledge about past states and decisions are irrelevant for her current decision. As shown shortly, such an

approach allows me to derive a form of modified principle of optimality, facilitating the analysis.

It is also worth noting that I treat the above perception functions as exogenous in the main analysis. In Section 4, I study additional economic implications when I endogenize the perception functions. There, the consumer must pay a cost to perfectly perceive her state.

Relationship with the noisy-signal approach. An alternative way to model misperception of the current state is through noisy signals (Sims, 2003). For example, I can use a noisy signal about wealth to summarize each self t 's knowledge of her current wealth: $s_t = a_t + \epsilon_t$. In this case, each self understands that her signal is noisy and tries to infer the actual state.¹¹ For the main analytical Theorem 1 based on linear consumption rules, as long as incomes and noises are normally distributed, these two ways of modeling misperception of wealth lead to the same MPCs (see Proposition 3 below). To see this equivalence, note that as long as a_t and ϵ_t are normally distributed, self t 's belief about her wealth is given by:

$$E[a_t | s_t] = (1 - \lambda_t)(a_t + \epsilon_t) + \lambda_t a_t^*,$$

where $\lambda_t = \frac{Var(\epsilon_t)}{Var(a_t) + Var(\epsilon_t)} \in (0, 1)$ and a_t^* is her prior about the wealth. Averaging over the noise, each self t 's belief about her wealth essentially takes the same form as the perception function in (6). However, such a noisy-signal-based approach is much harder to extend to non-Gaussian income processes and the more general cases with borrowing constraints in the calibration exercise in Section 5. Therefore, in the main analysis, I use the deterministic perception functions ($\mathbf{a}_t^p(\cdot), \vec{\mathbf{y}}_t^p(\cdot)$) to capture the misperception of states. Gabaix (2014) also uses such deterministic perception functions to capture misperception, with a focus on misperception of the exogenous state.

Relationship with rational inattention. One simple way to describe the key friction here is that the consumer is effectively inattentive to her endogenous wealth. However, this is different from the existing applications of inattention and sparsity (Sims, 2003; Luo, 2008; Luo and Young, 2010; Maćkowiak and Wiederholt, 2015; Gabaix, 2016b) to intertemporal consumption and saving problems. They instead focus on inattention to the exogenous income process, but let the consumption and saving decisions in each period t depend on perfect knowledge of the current,

¹¹This is different from the main case, in which each self t (mistakenly) thinks the current state is her perceived state (a_t^p, \vec{y}_t^p) with certainty.

endogenous, wealth.¹² Using my notation, they study the case where

$$\vec{y}_t^p(\vec{y}_t) \neq \vec{y}_t \quad \text{and} \quad \mathbf{a}_t^p(a_t) = a_t.$$

By contrast, my key friction of interest is the imperfect perception of the current endogenous wealth:

$$\mathbf{a}_t^p(a_t) \neq a_t.$$

In fact, to clarify that my main results are driven by such a friction, in the main analysis in later sections, I impose that $\vec{y}_t^p(\vec{y}_t) = \vec{y}_t$. That is, I study a consumer who perfectly perceives her exogenous income state \vec{y}_t , but has an imperfect perception of her current endogenous wealth.

There is a conceptual difference between inattention to the exogenous income process and imperfect perfect of the endogenous wealth: as further discussed below, imperfect perception of endogenous wealth comes from the consumer's bounded recall, or selective retrieval of memory, about her past consumption and saving decisions; inattention to the exogenous income nevertheless can coexist with perfect recall. One can view my approach as a tractable method to use the analytical tools in the inattention literature, such as the perception function in (6), to study the implications of bounded recall and selective retrieval of memory. As later analysis shows, unlike inattention to the exogenous income process, my approach leads to a violation of the Euler equation at the *individual* level and unique predictions about the consumer's MPCs, such as excess sensitivity to current income.

The modified principle of optimality. As mentioned above, summarizing the friction by misperception of the current state, I will be able to maintain the Markov property of the problem and establish a form of principle of optimality, facilitating the analysis. Specifically, given each self's consumption rules $\{\mathbf{c}_t(a_t^p, \vec{y}_t^p)\}_{t=0}^{T-1}$ and perception functions $\{\mathbf{a}_t^p(a_t), \vec{y}_t^p(\vec{y}_t)\}_{t=0}^{T-1}$ in the intrapersonal equilibrium, I can define the *objective* value at each period $t \in \{0, \dots, T-1\}$ as a function of the current state (a_t, \vec{y}_t) ,

$$V_t(a_t, \vec{y}_t) \equiv u(\mathbf{c}_t(\mathbf{a}_t^p(a_t), \vec{y}_t^p(\vec{y}_t))) + E_t \left[\sum_{k=1}^{T-t-1} \beta^k u(\mathbf{c}_{t+k}(\mathbf{a}_{t+k}^p(a_{t+k}), \vec{y}_{t+k}^p(\vec{y}_{t+k}))) + \beta^{T-t} v(a_T + y_T) \right], \quad (7)$$

where $E_t[\cdot]$ averages over all potential realizations of exogenous income given the current state (a_t, \vec{y}_t) . Note that the objective value here is *not* the value of a frictionless consumer. It is calculated

¹²Sims (2003) also studies the inattention to exogenous initial wealth, which effectively plays the same role as exogenous income.

based on the consumption rules and perception functions of a consumer with imperfect perception of wealth. Finally, for the last period T , we have $V_T(a_T, \vec{y}_T) = v(a_T + y_T)$.

I can also define each self t 's *subjective* value for each period $t \in \{0, \dots, T-1\}$ as a function of her perceived state (a_t^p, \vec{y}_t^p) , based on her subjective expectation $\tilde{E}_t[\cdot]$:

$$W_t(a_t^p, \vec{y}_t^p) \equiv u(\mathbf{c}_t(a_t^p, \vec{y}_t^p)) + \tilde{E}_t \left[\sum_{k=1}^{T-t-1} \beta^k u(\mathbf{c}_{t+k}(\mathbf{a}_{t+k}^p(a_{t+k}), \vec{y}_{t+k}^p(\vec{y}_{t+k}))) + \beta^{T-t} v(a_T + y_T) \right]. \quad (8)$$

Now, I can express the optimal consumption rules and the above objective and subjective value functions recursively, and establish my modified principle of optimality.

Proposition 1 (Modified Principle of Optimality). *In the intrapersonal equilibrium, for $t \in \{0, \dots, T-1\}$, each self t 's consumption $\mathbf{c}_t(a_t^p, \vec{y}_t^p)$ and perceived value $W_t(a_t^p, \vec{y}_t^p)$ are, respectively, the optimal policy and the maximized value of the following problem given her perceived state (a_t^p, \vec{y}_t^p) :*

$$\max_{c_t} u(c_t) + \beta \int V_{t+1}(R(a_t^p + y_t^p - c_t), \vec{y}_{t+1}) \phi(\vec{y}_{t+1} | \vec{y}_t^p) d\vec{y}_{t+1}. \quad (9)$$

Moreover, for $t \in \{0, \dots, T-1\}$, the objective value $V_t(a_t, \vec{y}_t)$ satisfies

$$V_t(a_t, \vec{y}_t) = u(\mathbf{c}_t(\mathbf{a}_t^p(a_t), \vec{y}_t^p(\vec{y}_t))), \\ + \beta \int V_{t+1}(R(a_t + y_t - \mathbf{c}_t(\mathbf{a}_t^p(a_t), \vec{y}_t^p(\vec{y}_t))), \vec{y}_{t+1}) \phi(\vec{y}_{t+1} | \vec{y}_t) d\vec{y}_{t+1}. \quad (10)$$

Finally, if consumption rules and value functions $\{\mathbf{c}_t(a_t^p, \vec{y}_t^p), W_t(a_t^p, \vec{y}_t^p)\}_{t=0}^{T-1}$ and $\{V_t(a_t, \vec{y}_t)\}_{t=0}^T$ satisfy (9)–(10) and the boundary $V_T(a_T, \vec{y}_T) = v(a_T + y_T)$, they then coincide with the corresponding objects in the unique intrapersonal equilibrium.

(9) is a recursive version of each self t 's optimality condition in (4): she chooses her optimal consumption to maximize the sum of her current consumption utility and her subjective expected future value. Note that the objective value function at $t+1$, instead of the corresponding subjective value function, appears in (9). This is because, as mentioned above, each self t can correctly predict her future selves' consumption rules and misperception of wealth. She then correctly understands the mapping from the future states to the objective value function. The sole friction is her misperception of the current state. Finally, (10) expresses the objective value at period t recursively, given each self's consumption rule and perception function and the budget constraints. To the best of my knowledge, Proposition 1 is the first result to establish a form of principle of

optimality that allows for misperception of both the endogenous and exogenous states. The result can be applied to other Markov decision problems.¹³

Evidence on imperfect perception of wealth. The consumer’s imperfect perception of her current wealth can come from bounded recall (Kahana, 2012). That is, the consumer may not perfectly recall her past consumption and saving decisions, and thus not perfectly know her current wealth. The imperfect perception of wealth can also come from selective retrieval from memory (Anderson, 2009; Gennaioli and Shleifer, 2010; Bordalo, Gennaioli and Shleifer, 2017). That is, even if the information to calculate the consumer’s total wealth is stored somewhere in her memory, when she decides on her consumption and savings she may not retrieve this information. As a result, she may not have her total wealth level at the front of her mind when making her consumption decisions. In sum, it is hard for a typical consumer to track her total wealth. It is even harder for her to have the exact total wealth at the front of her mind when making each consumption and saving decisions, as the standard model suggests.

There is ample empirical support for imperfect perception of wealth and its influence on economic decisions. The credit card literature, e.g. Agarwal et al. (2008) and Stango and Zinman (2014), finds that consumers often neglect their credit card balances, and this neglect often leads to suboptimal credit card usage. Brunnermeier and Nagel (2008) and Alvarez, Guiso and Lippi (2012) find that consumers often have imperfect knowledge of their financial wealth changes and fail to adjust accordingly.

Moreover, the recent literature on Fintech shows that providing information about a consumer’s total wealth by aggregating her financial account will change her consumption behavior. Levi (2015) conducts an experiment in which he provides the participants with account aggregation tools that display their current total wealth. Participants significantly change their consumption and saving after seeing their wealth, implying that they have imperfect perception of wealth without the tool. Likewise, Carlin, Olafsson and Pagel (2017) study the introduction of a financial app that consolidates all of its users’ bank account information and transaction histories. They show that the app significantly reduces its users’ interest expenses on consumer debt as well as other bank fees.

Another strand of empirical literature documents that the MPC out of current income is significantly larger than the MPC out of financial wealth (Paiella and Pistaferri, 2017; Di Maggio,

¹³Compared to Gabaix (2016a), there are two key differences: first, Proposition 1 here allows imperfect perception of both the exogenous and endogenous states, while Gabaix (2016a) focuses on the case of imperfect perception of the exogenous state; second, I start from the sequential forms in (7) and (8) and establish a form of principle of optimality; for the behavioral agent, Gabaix (2016a) directly works with a recursive form.

Kermani and Majlesi, 2018; Fagereng et al., 2019; Chodorow-Reich, Nenov and Simsek, 2019). This pattern is a direct consequence of imperfect perception of wealth (e.g. part 2 of Theorem 1). On the other hand, as discussed in detail in the next section, it is not easy for canonical models, including two-asset models, to generate such an empirical pattern, especially for high-liquidity consumers. Together, the evidence further points to the existence of imperfect perception of wealth.

3 Main Results

The main specification. In this section, I lay out main analytical results about consumption with imperfect perception of wealth. For the problem from the previous section to be analytically solvable, I consider the following main specification throughout the section. First, I follow Hall (1978): I let the utility functions $u(\cdot)$ and $v(\cdot)$ be quadratic and concave functions. With quadratic utility, each self's optimal consumption rule is linear in her perceived state. This guarantees that the ensuing excess sensitivity result has nothing to do with liquidity considerations. I can also establish parallel results under more general utilities by working with linearized consumption rules. In the calibration exercise in Section 5, I also show that key analytical results hold with more general utilities, without any linear approximation.

Second, I let each self t 's perceived wealth be a weighted average of her actual wealth and a default. To isolate the friction of interest, I also let each self t perfectly perceive her current income state \vec{y}_t .¹⁴ That is, for $t \in \{0, \dots, T-1\}$,

$$\mathbf{a}_t^p(a_t) = (1 - \lambda)a_t + \lambda a_t^D \quad \text{and} \quad \vec{\mathbf{y}}_t^p(\vec{y}_t) = \vec{y}_t, \quad (11)$$

where a_t^D is the default wealth for self t . A higher $\lambda \in [0, 1]$ means a higher weight on the default and a higher degree of imperfect perception of wealth. The standard model without friction is nested by $\lambda = 0$. In the main analysis, for simplicity, I let the default wealth a_t^D be the value of a_t in the frictionless case when the stochastic income in each period is fixed at their respective averages. With quadratic utility and linear consumption rules here, this value of the default wealth a_t^D is equal to the average wealth level for self t in the intrapersonal equilibrium (over realizations of the stochastic incomes).¹⁵ In other words, the default wealth here is effectively the prior about

¹⁴Because each self t understands the the law of motion of the income process, she also correctly forms expectations about her future income given the current \vec{y}_t .

¹⁵It is worth noting that, with the quadratic utility and linear consumption rules here, the average value of each

wealth in period t in the noisy-signal approach studied in Corollary 3 below. Moreover, the main Theorem 1 below holds more generally: the default wealth a_t^D can be an arbitrary exogenous function of the income process (see Appendix B).

Finally, for notation simplicity, I impose $\beta R = 1$ in the main text. The main theorem below can be easily extended to the case with $\beta R \neq 1$ (see Appendix B).

The main theorem. The main theorem summarizes the key analytical results about consumption with imperfect perception of wealth. To facilitate comparisons with empirical results, I express consumption as a function of current income, expected future income, and current (liquid) wealth.¹⁶ In the theorem, a hat over a variable denotes its deviation from its value in the frictionless case when the stochastic incomes are fixed at their respective averages, which is also equal to its average value in the intrapersonal equilibrium (over realizations of the stochastic incomes).

Theorem 1. *There exists deterministic scalars $\{\psi_t^y, \psi_t^a, \delta_{t,l}\}$ such that, for each $t \in \{0, \dots, T-1\}$, self t 's consumption can be characterized by*

$$\hat{c}_t = \underbrace{\psi_t^y}_{MPC_t^y} \hat{x}_t + \underbrace{\psi_t^a}_{MPC_t^a} \hat{a}_t, \quad (12)$$

where \hat{x}_t is the **modified permanent income** defined as

$$\hat{x}_t \equiv \hat{y}_t + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} E_t [\hat{y}_{t+l}], \quad (13)$$

where $E_t [\hat{y}_{t+l}]$ captures the (rationally) expected future income given the current income state \vec{y}_t . Furthermore, for each $t \in \{0, \dots, T-1\}$, self t 's consumption displays:

- (i) **Excess sensitivity to current income:** ψ_t^y increases with the friction λ .
- (ii) **A smaller MPC out of liquid wealth than out of current income:** ψ_t^a satisfies

$$\psi_t^a = (1 - \lambda) \psi_t^y. \quad (14)$$

(iii) **Excess discounting of future income:** Given t , for $l \in \{1, \dots, T-t\}$, $\delta_{t,l} \leq 1$ decreases with the friction λ and decreases with the distance from receiving the income, l .

For each $t \in \{0, \dots, T-1\}$, ψ_t^y in (12) captures both the MPC out of current income (holding

income, consumption, and wealth is independent of the degree of frictions λ . The degree of frictions (λ) instead governs the consumption response to current and future income *shocks*.

¹⁶That is, Theorem 1 studies the object $\mathbf{c}_t(\mathbf{a}_t^P(a_t), \vec{\mathbf{y}}_t^P(\vec{y}_t))$.

future income fixed) and the MPC out of the modified permanent income \hat{x}_t . I will call it the MPC out of current income throughout. Moreover, ψ_t^a in (12) captures the MPC out of (liquid) wealth. Furthermore, given t , for $l \in \{1, \dots, T-t\}$, $\delta_{t,l} \leq 1$ captures the extra discounting of income that arrives l periods later when calculating the modified permanent income in (13). Finally, it is worth noting that the formulas for ψ_t^y , ψ_t^a and $\delta_{t,l}$ in Theorem 1 do not depend on the stochastic properties of the income process. They are only functions of the interest rate R (hence the discount factor β), the degree of friction λ , and the relevant time horizons (t, l, T) .¹⁷

Theorem 1 shows how imperfect perception of wealth can jointly explain the deviations of consumption behavior from the predictions of the permanent income hypothesis. The first part of Theorem 1 shows that the consumer with imperfect perception exhibits excess sensitivity to current income, without relying on liquidity constraints and precautionary saving motives. A larger friction λ leads to a larger MPC out of current income. The second part of Theorem 1 shows that the consumer exhibits a smaller MPC out of liquid wealth than out of current income: her MPC out of liquid wealth is only $1 - \lambda$ times of her MPC out of current income. The third part of Theorem 1 shows that the consumer exhibits excess discounting of future income, and a larger friction λ leads to more discounting. Moreover, the consumer exhibits more discounting when responding to income shocks further in the future.

Together, the theorem shows how the consumer with imperfect perception of wealth violates the fungibility principle, i.e. different components of income and wealth cannot be collapsed into a single number representing their combined present value. The model then answers Thaler (1990)'s call for a consumption model that treats the MPCs out of current income, expected future income, and wealth differently. By contrast, with perfect perception of wealth ($\lambda = 0$), $\psi_t^y = \psi_t^a$ and $\delta_{t,l} = 1$. The consumption in (12) and (13) becomes $\hat{c}_t = \psi_t^y \left(\hat{y}_t + \sum_{l=1}^{T-t} R^{-l} E_t [\hat{y}_{t+l}] + \hat{a}_t \right)$. That is, only the total present value of income and wealth matters.

A smaller MPC out of liquid wealth than out of current income. Let me first explain the straightforward part of the result, part (ii) of Theorem 1. It shows that, with imperfect perception of wealth, the consumer exhibits a smaller MPC out of liquid wealth than out of current income. This result comes directly from the current self's imperfect perception of wealth. As an extra dollar in self t 's wealth only increases her perceived wealth by $1 - \lambda$, her MPC out of wealth is only $1 - \lambda$ times of her MPC out of the perfectly perceived current income. This is the equation in (14). In the rest of the section, I will explain the economic mechanisms behind the

¹⁷To calculate the MPC out of a persistent income shock, note that such a shock changes the modified permanent income in (13) through changes in both \hat{y}_t and future $E_t [\hat{y}_{t+l}]$. This will be illustrated in the AR (1) case in Corollary 5.

other two, more “interesting,” parts of Theorem 1.

3.1 Excess Sensitivity to Current Income

Let me now explain the most important part of the theorem, excess sensitivity to current income. For concreteness, one can think about the case where income is drawn independently across periods. In this case, the modified permanent income \hat{x}_t in (13) is equal to current income \hat{y}_t and Theorem 1 can be written as

$$\hat{c}_t = \underbrace{\psi_t^y}_{\text{MPC}_t^y} \hat{y}_t + \underbrace{\psi_t^a}_{\text{MPC}_t^a} \hat{a}_t. \quad (15)$$

But all the intuition and results below hold exactly the same for the general case.

Excess concavity of the value function. To explain why imperfect perception of wealth leads to excess sensitivity to current income, let me first introduce a lemma.

Lemma 1. *For each $t \in \{0, \dots, T\}$, there exists a scalar $\Gamma_t > 0$ that captures the “concavity” of the consumer’s objective value function in (7) with respect to wealth. That is,*

$$\frac{\partial^2 V_t(a_t, \vec{y}_t)}{\partial a_t^2} = u'' \cdot \Gamma_t \quad \forall a_t, \vec{y}_t,$$

where $u'' < 0$ is the second derivative of the utility function¹⁸ and a larger Γ_t means a more concave objective value function V_t .

For $t \in \{0, \dots, T-1\}$, the objective value function with imperfect perception of wealth exhibits **excess concavity**: Γ_t strictly increases with the friction λ .¹⁹

The intuition behind the excess concavity is as follows: since a larger friction λ leads to a more “mistaken” perception of wealth, consumption will respond more inefficiently to changes in wealth a_t . As a result, the marginal value of wealth $\frac{\partial V_t}{\partial a_t}$ decreases faster with the wealth a_t and the value function V_t becomes more concave.

Excess sensitivity to current income. I now explain the excess sensitivity to current income. First note that the FOC of the optimal consumption for each self $t \in \{0, \dots, T-1\}$ in (9) is given by

$$u'(c_t) = \tilde{E}_t \left[\frac{\partial V_{t+1}}{\partial a_{t+1}}(a_{t+1}, \vec{y}_{t+1}) \right]. \quad (16)$$

¹⁸Note that since u is quadratic, u'' is a constant.

¹⁹In the last period, $\Gamma_T = \frac{v''}{u''}$.

That is, each self t equates the marginal utility of consuming now with her expected marginal value of saving for the next period.

Consider a temporary positive shock to current income y_t . As self t understands that her future selves have misperception of wealth, from Lemma 1, she knows that the marginal value of saving $\frac{\partial V_{t+1}}{\partial a_{t+1}}$ decreases faster with a_{t+1} . As a result, she is less willing to increase her savings after the positive income shock, and instead increases her current consumption more. Intuitively, if the current self saves the additional income, her future selves will not fully perceive the increase in their wealth and will not increase their consumption enough. The current self then increase current consumption more. Similarly, consider a negative shock to y_t . Because of the excess concavity in Lemma 1, self t is less willing to decrease her savings after the negative income shock. She instead decreases her current consumption more. In sum, the consumer's current consumption exhibits excess sensitivity to current income.

Importantly, such excess sensitivity to current income is driven by the current self's concern about her future selves' misperception of wealth, instead of the current self's misperception of current wealth per se. This can be seen from the FOC in (16), where the current self's MPC is determined by the marginal value of wealth in the future. To illustrate this point more clearly, let me temporarily introduce *time-varying* misperception of wealth. That is, for $t \in \{0, \dots, T-1\}$, the perception function of wealth in (11) becomes

$$\mathbf{a}_t^p(a_t) = (1 - \lambda_t) a_t + \lambda_t a_t^D, \quad (17)$$

where $\lambda_t \in [0, 1]$ captures self t 's misperception of wealth, and a higher λ_t means a larger friction. I can then re-state the excess sensitivity result in Theorem 1 as:

Corollary 1. *With time-varying misperception of wealth in (17), for each $t \in \{0, \dots, T-2\}$, self t 's MPC ψ_t^y strictly increases with her future selves' frictions, that is, strictly increases with each λ_{t+l} for $l \in \{1, \dots, T-1-t\}$. On the other hand, ψ_t^y is independent from current λ_t .*

Even if the current self can perfectly perceive her wealth ($\lambda_t = 0$), as long as she is worried about her future selves' misperception, her consumption will exhibit excess sensitivity to current income. In this sense, the consumer's deviation from the frictionless benchmark is self-fulfilling: if the current self is worried that her future selves will deviate from the frictionless benchmark, she will deviate too.

Despite the clarification offered in Corollary 1 with time-varying misperception of wealth, I maintain the time-invariant friction λ in (11) throughout the main analysis. The main rationale

is twofold. First, by introducing a *single* psychology-based parameter λ as championed by Rabin (2013), I minimize the additional degrees of freedom. The existing model can be nested by the case $\lambda = 0$. Second, the single friction λ can lead to predictions along multiple dimensions, such as all three parts of Theorem 1. This also helps me calibrate the degree of friction in the following calibration exercise: with a single psychology-based parameter λ , one can find a sufficient statistic to calibrate the size of the friction λ based on the model’s predictions in other dimensions; one can then gauge how imperfect perception of wealth influences the MPC out of current income.

The $T \rightarrow \infty$ limit. The MPCs out of current income and wealth, ψ_t^y and ψ_t^a in (15), converge to simple limits when the consumer’s horizon T goes to infinity.

Corollary 2. *As long as $\lambda < R^{-1/2}$, as $T \rightarrow +\infty$,*

$$\psi_t^y \rightarrow \psi^y \equiv \frac{1 - R^{-1}}{1 - \lambda^2} \quad \text{and} \quad \psi_t^a \rightarrow \psi^a \equiv \frac{1 - R^{-1}}{1 + \lambda}. \quad (18)$$

With perfect perception of wealth ($\lambda = 0$), the MPC out of current income converges to $1 - R^{-1}$, its familiar counterpart in the infinite horizon permanent income hypothesis benchmark. It is well known that, with the discount factor β (and interest rate R) close to one, the MPC is close to zero and too small compared to its empirical counterpart. With imperfect perception of wealth ($\lambda > 0$), the MPC out of current income ψ^y increases. When $\lambda \rightarrow (R^{-1/2})^-$, the MPC out of current income achieves its upper bound, $\lim_{\lambda \rightarrow (R^{-1/2})^-} \psi^y \equiv 1$. That is, when the misperception of wealth is severe enough, the current self is so worried about her future selves’ misperception that her consumption follows a simple rule of thumb: she consumes all changes in her current income. In other words, she is effectively “hand-to-mouth” with respect to *changes* in current income, even though her total consumption does not need to track her total current income ($c_t \neq y_t$).

Connections to the empirical evidence. The most important economic lesson behind part (i) of Theorem 1 is that consumption can exhibit excess sensitivity to current income without liquidity constraints or precautionary saving motives. The result then fills a gap in the literature, providing an explanation for the emerging empirical evidence on excess sensitivity for consumers *with high income and high liquidity*. For example, Fagereng, Holm and Natvik (2019) study consumption responses to unexpected Norwegian lottery prizes, and find the MPC out of current income remains high among liquid winners: their estimates of the MPC out of current income for the group with the highest liquid asset balance is 0.459 dollars per year. They argue that conventional models based on liquidity constraints do not imply such large magnitudes for high-liquidity winners. Kueng (2018) documents excess sensitivity of the consumption response to the

Alaska Permanent Fund payments, and finds the excess sensitivity is largely driven by high-income households with substantial liquid assets. Relatedly, Stephens and Unayama (2011), Parker (2017), Olafsson and Pagel (2018), and Ganong and Noel (2019) also question whether liquidity motives can explain their findings on the excess sensitivity of consumption to income. Parker (2017) further finds that lack of consumption smoothing is associated with a measure of lack of financial planning, consistent with a behavioral mechanism.²⁰

Let me now turn to the prediction of part (ii) of Theorem 1: with imperfect perception of wealth, the consumer's MPC out of liquid wealth is only a portion of her MPC out of current income. This is consistent with Thaler (1990)'s observation in a classical JEP paper. In the recent empirical literature, estimates for the MPC out of financial wealth are also typically much smaller than those for the MPC out of current income (Baker, Nagel and Wurgler, 2007; Paiella and Pistaferri, 2017; Di Maggio, Kermani and Majlesi, 2018; Fagereng et al., 2019). For example, in a study that I will explain in detail in Section 5, Di Maggio, Kermani and Majlesi (2018) find that, for the top half of wealth distribution, the MPC out of final wealth is \$0.05 per year, while the MPC out of current income is \$0.35 per year.

It is not easy for canonical models based on liquidity constraints to explain the small MPC out of liquid wealth compared to the MPC out of current income. Even for two-asset models with illiquid assets (Kaplan and Violante, 2014), the MPCs out of current income and liquid wealth are the same. One remaining concern is that the empirical estimates of the MPC out of financial wealth may not exactly capture the MPC out of liquid wealth. There are two responses to this concern. First, the empirical estimates are mostly based on the MPC out of capital gains on direct holdings of stocks. They are quite liquid and are typically treated as liquid assets in the literature, such as in Kaplan and Violante (2014) and Kaplan, Violante and Weidner (2014). Second, in standard two-asset models for consumers with positive liquidity, if anything, the MPC out of illiquid wealth is larger than, instead of smaller than, the MPC out of liquid wealth and current income. This is because illiquid wealth has a higher return, and consumers with positive liquidity can freely adjust their liquid balances to consume out of changes in illiquid wealth. Together, even if financial wealth is not fully liquid, it is still hard for standard models to explain the empirical evidence on the small MPC out of financial wealth for rich households (e.g. Di Maggio, Kermani and Majlesi, 2018 and Fagereng et al., 2019). This empirical evidence then further supports my friction of interest, i.e. imperfect perception of wealth.

²⁰Misra and Surico (2014) also find that high-income consumers exhibit excess sensitivity to US Federal economic stimulus payments.

3.2 Excess Discounting of Future Income

I now turn to explain part (iii) of Theorem 1, excess discounting of future income. From (12) and (13) in Theorem 1, for each $t \in \{0, \dots, T-1\}$, self t 's consumption can be written by

$$\hat{c}_t = \psi_t^y \left(\hat{y}_t + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} E_t [\hat{y}_{t+l}] \right) + \psi_t^a \hat{a}_t, \quad (19)$$

where $\delta_{t,l} \leq 1$ in (19) captures the extra discounting of income that arrives l periods later. Part (iii) of Theorem 1 states that $\delta_{t,l}$ decreases with the friction λ and decreases with the distance from receiving the income, l .

Excess discounting of future income. To understand the intuition behind excess discounting, note that the current self's response to expected future income shocks requires changes in wealth. In the frictionless case, if the current self t receives positive (negative) news about future income, she increases (decreases) her consumption now, which means a decrease (increase) of a_{t+1} . With imperfect perception of wealth, however, future selves will not perfectly perceive or respond to the changes in wealth. Worried about this friction, the current self is less willing to change her consumption (and thus wealth) in response to expected future income shocks. Similar to the excess sensitivity to current income in part (i) of Theorem 1, the excess discounting of future income here is driven by the current self's concern about future selves' misperception of wealth, but not the current self's own misperception.

Part (iii) of Theorem 1 further establishes a "distance effect": there is more discounting when the consumer responds to an income shock further in the future. This distance effect arises because the frictionless consumption response to such a shock requires more coordination across different selves. If the current self t receives news about income y_{t+l} and wants to respond to it, selves $t+1, \dots, t+l$ all need to perceive the ensuing wealth changes and respond accordingly. With imperfect perception of wealth, such coordination across different selves is harder. As a result, the current self is less willing to change her consumption in response to income shocks further in the future and exhibits higher extra discounting. To see this "distance effect" clearly, let us consider the $T \rightarrow +\infty$ limit. In this case, the following corollary establishes that the extra discounting takes a geometric discounting form with respect to the distance l .

Corollary 3. *As long as $\lambda < R^{-1/2}$, with $T \rightarrow +\infty$, for all t and l ,*

$$\delta_{t,l} \rightarrow \delta^l \quad \text{and} \quad \delta \equiv 1 - \frac{(R-1)\lambda^2}{1-\lambda^2} \leq 1. \quad (20)$$

When $\lambda \rightarrow (R^{-1/2})^-$, we have $\delta = 1 - \frac{(R-1)\lambda^2}{1-\lambda^2} \rightarrow 0$. That is, when the friction is severe enough, the current self is so worried about her future selves' misperception that she does not respond to changes in expected future income. Instead, her consumption just tracks changes in her current income.²¹

Empirical support. Consistent with excess discounting of future income, empirical studies typically find very limited consumption response to news about future income, i.e. a very limited “announcement effect.” Papers that find this pattern cannot be fully explained by liquidity constraints include Stephens and Unayama (2011), Parker (2017), Olafsson and Pagel (2018), and Kueng (2018).

Moreover, those empirical studies also find that the consumption response when the consumer actually *receives* the income is much larger than when the news is announced, i.e. excess sensitivity to *anticipated* income shocks. This is also consistent with the prediction of imperfect perception of wealth, jointly explained by excess sensitivity to current income and excess discounting of future income. In fact, one can apply Theorem 1 to calculate the impulse responses of consumption to news about future income (after taking into consideration of the endogenous response of wealth in each period). One can then see that imperfect perception of wealth can generate the empirical pattern of a larger consumption response when the consumer receives the income than when the news arrives.

Corollary 4. *Let $T \rightarrow +\infty$. Consider news about future income y_{t+k} ($k \geq 1$) that arrives at period t .²² Let \mathcal{C}_l^k denote the impulse response of c_{t+l} to the news about y_{t+k} . With imperfect perception of wealth ($\lambda > 0$), the consumption response when the consumer actually receives the income is the largest*

$$\mathcal{C}_k^k > \max \{ \mathcal{C}_0^k, \dots, \mathcal{C}_{k-1}^k \},$$

while without imperfect perception of wealth ($\lambda = 0$),

$$\mathcal{C}_k^k = \mathcal{C}_0^k = \dots = \mathcal{C}_{k-1}^k.$$

When the friction is severe enough, i.e. $\lambda \rightarrow (R^{-1/2})^-$, as discussed above, current consumption does not respond to news about future income and just tracks changes in her current income. In

²¹The formulas for ψ^y and ψ^a in Corollary 1 still hold. So we know that when $\lambda \rightarrow (R^{-1/2})^-$, $\psi^y \rightarrow 1$. That is, $\hat{c}_t = \hat{y}_t$.

²²Specifically, consider the case that $y_{t+k} = y_{t+k}^P + y_{t+k}^U$, where y_{t+k}^P is realized at t and y_{t+k}^U is realized at $t+k$ and is unpredictable beforehand. The first component, y_{t+k}^P , can be interpreted as the news about future income that arrives at period t . \mathcal{C}_l^k here then denotes the impulse response of c_{t+l} with respect to y_{t+k}^P .

this case, consumption responds one-to-one to changes in *anticipated* current income: $C_k^k = 1$. On the other hand, there is no announcement effect: $C_0^k = \dots = C_{k-1}^k = 0$.

3.3 Additional Results

The AR (1) case. Now, I turn to an application of the main Theorem 1: the case where income is distributed as an AR(1) process: $y_t = \rho y_{t-1} + \epsilon_t$ with $\rho \in (0, 1]$ and ϵ_t independent from y_{t-1} .²³ I study the MPC out of the innovation in current income. As mentioned above, the formulas for ψ_t^y , ψ_t^a and $\delta_{t,l}$ in Theorem 1 do not depend on the stochastic properties on the income process. To calculate the MPC out of the persistent income innovation, one should first calculate how the shock changes the modified permanent income in (13) through changes in both \hat{y}_t and future $E_t[\hat{y}_{t+l}]$. One can then find the MPC out of this persistent shock by multiplying ψ_t^y by the change in the modified permanent income. I can then establish:

Corollary 5. *Fix $\rho \in (0, 1]$, with $T \rightarrow +\infty$, there exists a scalar ψ_ρ^y such that*

$$\hat{c}_t = \psi_\rho^y \hat{y}_t + \psi^a \hat{a}_t \quad \text{and} \quad \psi_\rho^y = \frac{\psi^y}{1 - \rho + \rho \psi^y}, \quad (21)$$

where ψ^y and ψ^a are the $T \rightarrow +\infty$ limits of the MPCs in Theorem 1, given by (18). From (21), ψ_ρ^y still increases with the friction λ .

With a persistent income process, from (19), we know two countervailing forces govern the frictional MPC out of the income innovation. First, the excess sensitivity of ψ^y pushes ψ_ρ^y upwards. Second, since the income innovation is persistent, the excess discounting of future income pushes ψ_ρ^y downwards. Together, Corollary 5 shows that the first channel dominates, and the friction still leads to excess sensitivity.

Now consider the corner case $\rho = 1$. That is, the income process is a random walk. The MPC in (21) becomes $\psi_1^y = 1$. It coincides with its frictionless benchmark and is independent of the degree of friction λ . That is, for a perfectly persistent income innovation, the excess sensitivity to current income and the excess discounting of future income cancel out. In fact, in this case, each self consumes all the innovation in y_t , which leaves wealth a_{t+1} unchanged. As a result, the future self's imperfect perception of wealth will not influence the current self's MPC. This result is also consistent with the empirical evidence in Blundell, Pistaferri and Preston (2008): consumption does *not* exhibit excess sensitivity to shocks to the permanent component of the income process.

²³In this case, we still have $\vec{y}_t = (y_t)$, as the current income y_t serves as a summary statistics of the knowledge about future income available at period t .

A modified Euler equation. Finally, one can use a modified Euler equation to summarize the excess sensitivity to current income in part (i) of Theorem 1 and the excess discounting of future income in part (iii) of Theorem 1.

Proposition 2. *For each $t \in \{0, \dots, T - 2\}$, one can find a scalar $\omega_t \in (0, 1]$ such that*

$$\hat{c}_t = \omega_t \tilde{E}_t [\hat{c}_{t+1}] + (1 - \omega_t) \tilde{E}_t [\hat{a}_t + \hat{y}_t]. \quad (22)$$

Moreover, ω_t decreases with the friction λ .

In the frictionless benchmark, ω_t in (22) equals 1, and (22) becomes the familiar Euler equation, $\hat{c}_t = \tilde{E}_t [\hat{c}_{t+1}]$. With frictions, $\omega_t \in (0, 1)$ captures the excess discounting of the impact of future related to part (iii) of Theorem 1, while $1 - \omega_t \in (0, 1)$ captures the excess impact from perceived current income and wealth related to part (i) of Theorem 1. Compared to the discounted Euler equation in Angeletos and Lian (2018) and Gabaix (2016b), the modified Euler equation in (22) has two differences. First, (22) is a discounted Euler equation at the *individual* level, while Angeletos and Lian (2018) and Gabaix (2016b) focus on discounted Euler equation at the *aggregate* level. Second, the last term in (22) about the excess current impact does not appear in Angeletos and Lian (2018) and Gabaix (2016b).

4 Discussion

A mental account interpretation. One way to interpret my model is that the consumer has two separate mental accounts: one current income account that she perfectly knows, and one wealth account of which she has imperfect perception. The current self's separate mental accounts can explain the consumer's smaller MPC out of liquid wealth than out of current income. The current self's concern about her future selves' separate mental accounts then leads to excess sensitivity to current income and excess discounting of future income. In Appendix B, I further explore and formalize this mental account interpretation.

The existence of such separate mental accounts then contributes to the similarity between the consumer behavior here in an one-asset context and that in two-asset models (Kaplan and Violante, 2014), such as excess sensitivity to current income. However, there is a crucial difference from the two-asset model. There, as discussed above, the consumer only exhibits such frictional behavior when she is close to the liquidity constraint. High-liquidity consumer's behavior instead proxies that of the permanent income hypothesis. Here, however, the consumer's frictional behavior

does not depend on liquidity constraints. Even high-income, high-liquidity consumers can behave as “hand-to-mouth” consumers. My approach also avoids the need for high adjustment costs embedded in some two-asset models.

Comparison with other approaches. Hyperbolic discounting (Laibson, 1997) also leads to excess sensitivity to current income (Barro, 1999; Ganong and Noel, 2019). However, there are a few key differences between the hyperbolic discounting and my approach within the context of one-asset models without borrowing constraints studied here. First, the fungibility principle is maintained under hyperbolic discounting. That is, all components of income and wealth can still be collapsed into a single number by combining their present values. As a result, hyperbolic discounting alone cannot generate different MPCs out of current income versus liquid wealth or excess discounting of future income. Second, with hyperbolic discounting, the consumer always over-consumes compared to her frictionless benchmark. On the other hand, based on my approach, the consumer may only exhibit frictional responses to shocks. On average, the consumer’s behavior can coincide with the frictionless benchmark.

In another strand of literature on hyperbolic discounting, Laibson (1997) and Angeletos et al. (2001) study the implications of hyperbolic discounting for a two-asset model with an illiquid asset and liquidity constraints. There, the consumer may use the illiquid asset as the commitment device, and may endogenously choose to be at the liquidity constraint to avoid over-spending. In this case, the liquidity-constrained consumer displays excess sensitivity to current income. However, this mechanism cannot explain the evidence on high-liquidity consumers’ excess sensitivity to current income. Moreover, the consumer in these hyperbolic discounting models still displays the same MPC out of current income and liquid wealth.

Several papers incorporate inattention to exogenous income into intertemporal consumption and saving problems (e.g. Sims, 2003; Reis, 2006; Luo, 2008; Luo and Young, 2010; Maćkowiak and Wiederholt, 2015; Gabaix, 2016a). Those papers can explain the evidence on the sluggish response of aggregate consumption to aggregate shocks. At the individual level, however, it cannot generate the excess sensitivity to current income as in part (i) of Theorem 1.

Another strand of literature focuses on inattention to the exogenous return on wealth (Gabaix and Laibson, 2002; Alvarez, Guiso and Lippi, 2012; Abel, Eberly and Panageas, 2007, 2013). Consider a variant of Alvarez, Guiso and Lippi (2012) in my environment with time-varying interest rates: the law of motion of wealth can be written as $a_{t+1} = R'_t(a_t + y_t - c_t)$, but the consumer cannot perfectly perceive the movement of the return R'_t . This consumer, however, has perfect recall: she remembers her past consumption and saving decisions. In this case, the law of iterated

expectation still holds, and the canonical Euler equation, $\hat{c}_t = \tilde{E}_t[-\sigma^{-1}R_t + \hat{c}_{t+1}]$,²⁴ holds (see Appendix B for details). This consumer’s MPC out of current income is then the same as its frictionless counterpart. This comparison also emphasizes what drives the excess sensitivity result in my approach: the current self is worried that future selves cannot perfectly perceive and respond to the current self’s consumption and saving decisions.

The role of sophistication. As discussed before, the results about excess sensitivity to current income and excess discounting of future income are driven by the current self’s concern about her future selves’ misperception of wealth, instead of the current self’s misperception of current wealth. Using the language of O’Donoghue and Rabin (1999, 2001), “sophistication,” i.e. the current self’s understanding about her future selves’ mistakes, plays an important role. There are several related points worth mentioning. First, even though Theorem 1 is established with full sophistication, the main results are still true with partial sophistication. In Appendix B, I follow O’Donoghue and Rabin (2001) and introduce partial sophistication: each self t ’s misperception of her current wealth is still captured by (11), with a degree of friction λ ; however she now thinks that her future selves have a degree of friction $\lambda' \in [\lambda, 1]$. As long as the consumer is not completely naive about her future misperception ($\lambda' < 1$), the results about excess sensitivity to current income and excess discounting of future income are still true.

Second, the consumer’s knowledge about her other selves’ imperfect perception of wealth can come naturally from her own experiences in the past. Based on similar situations before, the consumer knows that her consumption may not be very sensitive to changes in wealth. The excess sensitivity and excess discounting in Theorem 1 can then be viewed as a second-best response to the known failure of consumption smoothing.

Third, each self’s “sophistication” about her future selves’ misperception of wealth does not necessarily lead to “complicated” behavior. In fact, as the discussion after Corollaries 2 and 3 shows, when the degree of imperfect perception of wealth is large enough, the consumption behavior can become very simple: the consumer just consumes all changes in her current income.

Fourth, there is also empirical evidence that consumers are indeed concerned about their future selves’ frictions. For example, in the credit card literature, Jiang et al. (2018) find that many consumers choose to pay their credit card balance in advance. They provide further evidence that these early payments are driven by consumers’ concern that their future selves may be inattentive to their credit card balances: consistent with the prediction of sophistication, they find that there are more early payments when the revolving interest rate is higher.

²⁴ $\sigma \equiv -\frac{u''(c^*)}{c^*u'''(c^*)}$ is the inverse of elasticity of intertemporal substitution.

Noisy signals about wealth. In the main analysis, each self mistakenly thinks her wealth is her perceived wealth a_t^p with certainty. An alternative way to model misperception of wealth is through noisy signals (Sims, 2003). Specifically, consider the environment in Section 3 with quadratic utility and perfect perception of the income state. Unlike in the main analysis, each self t 's knowledge of the current wealth is now summarized by a noisy signal $s_t = a_t + \epsilon_t$. In this case, each self understands that her signal is noisy and tries to infer her actual wealth from the signal. With linear consumption rules (true under quadratic utilities), as long as noises and incomes are Normally distributed, Theorem 1 is still true:

Proposition 3. *Let each self t 's knowledge of her current wealth be summarized by a noisy signal $s_t = a_t + \epsilon_t$, where ϵ_t and a_t are independent. As long as the noise ϵ_t and the current and expected future income y_t and $E[y_{t+l} | \vec{y}_t]$ are Normally distributed, each self t 's consumption can be written as*

$$\hat{c}_t = \psi_t^y \hat{x}_t + \psi_t^a (\hat{a}_t + \epsilon_t),$$

where \hat{x}_t is still defined as in (13) and ψ_t^y , ψ_t^a and $\delta_{t,l}$ share the formula from the time-varying λ_t version of Theorem 1 in the proof of Corollary 1. Each self's $\lambda_t = \frac{\text{Var}(\epsilon_t)}{\text{Var}(a_t) + \text{Var}(\epsilon_t)} \in [0, 1]$ depends negatively on the signal-to-noise ratio of her signal about a_t .

To see this intuition behind the equivalence, note that, with Normally distributed fundamentals and noises, self t 's belief about her wealth is given by:

$$E[\hat{a}_t | \hat{s}_t] = (1 - \lambda_t) (a_t + \epsilon_t),$$

which takes a similar form as the perception function in (11).

Robustness. Main results in Theorem 1 can be easily extended to four more general cases, which I explore in Appendix B. First, Theorem 1 does not depend on the simplifying assumption $\beta R = 1$, and can be easily extended to the case where $\beta R \neq 1$. Second, Theorem 1 still holds when the default wealth levels $\{a_t^D\}_{t=0}^{T-1}$ are arbitrary exogenous functions of the income process. Third, I study the case in which imperfect perception of income is also allowed ($\vec{y}_t^p(\vec{y}_t) \neq \vec{y}_t$). The main results in Theorem 1 can then be recast as excess sensitivity to perceived current income and excess discounting of perceived future income. Fourth, Theorem 1 can also be extended to the case that $\lambda < 0$, that is, when each self t 's perceived wealth moves more than one to one with her actual wealth. This is true because the excess sensitivity to current income and the excess

discounting of future income come from the current self's concern that her future selves will not respond efficiently to changes in wealth. These inefficient responses happen regardless of whether $\lambda < 0$ or $\lambda > 0$.

The length of a period as the length of perfect recall. Mapping my model to the data faces a natural challenge: how long is of a period in the model? To answer this question, I work out a continuous-time version of the model. In this model, the consumer can perfectly coordinate her consumption and saving decisions within each time interval of length T_p , but has bounded recall about what happened before. In other words, I can divide the timeline into intervals $[0, T_p]$, $[T_p, 2T_p]$, \dots . The consumer can perfectly recall her consumption and saving decisions within each interval, but has difficulty recalling her past consumption and saving decisions in previous time intervals. Similar to the discrete-time model, her memory about the past is summarized by her imperfect perception of wealth at the start of her current interval. I show this continuous-time version of the model is equivalent to the discrete-time model studied above, and one can think of each time-interval as a period in the discrete-time version. One can then interpret the length of a period in the discrete-time model as the length of her perfect recall about her past consumption and saving decisions.

Endogenize the degree of frictions λ . In the main analysis, I treat the perception function as given. That is, I let $\mathbf{a}_t^p(a_t) = (1-\lambda)a_t + \lambda a_t^D$, where the degree of frictions $\lambda \in [0, 1]$ is exogenous. Here, I endogenize the degree of frictions λ . Specifically, I add a stage 0, before all periods. In that stage, the consumer endogenously chooses her degree of friction λ to maximize her expected utility function, subject to a perception cost:

$$\max_{\lambda} E \left[\sum_{t=0}^{T-1} \beta^t u(c_t) + \beta^T v(a_T + y_T) \right] - C(1 - \lambda),$$

where the perception cost $C(\cdot)$ is increasing and convex, and $E[\cdot]$ averages over realizations of the stochastic incomes.²⁵ Note that a lower friction λ leads to a higher perception cost $C(1 - \lambda)$. There are two main results. First, the consumer with a less concave utility function (lower $|u''|$) endogenously chooses a higher friction λ , and then exhibits a higher MPC out of current income. Intuitively, this is because, with a less concave utility, the utility loss driven by any degree of imperfect perception of wealth is smaller. This result seems to be consistent with the evidence in Di Maggio, Kermani and Majlesi (2018), who find that the ratio between the MPC out of wealth

²⁵As in Section 3, I let $u(\cdot)$ be a quadratic concave function, assume there are no borrowing constraints, and let the consumer have perfect perception about the current income state $\bar{\mathbf{y}}_t^p(\bar{\mathbf{y}}_t) = \bar{\mathbf{y}}_t$.

and the MPC out of current income (which equals $1 - \lambda$ in my model) decreases with wealth.²⁶ This result can then potentially explain the stunning finding in Kueng (2018) that the MPC out of current income can increase with wealth.

Second, the consumer who faces income shocks with a higher variance chooses a smaller friction λ , and then exhibits a lower MPC out of current income. This result might be able to speak to the evidence that the MPC out of current income decreases with the size of the income shock (Fagereng, Holm and Natvik, 2019). See Appendix B for details.

Durable consumption, illiquid assets, and learning. In Appendix B, I study three additional extensions. First, I show that, with imperfect perception of wealth, durable consumption exhibits less excess sensitivity to current income. This is because the current self's purchase of durable goods automatically raises her future selves' durable consumption, even if they do not have perfect perception of wealth. Second, I study the case with an additional illiquid asset. The main results in Theorem 1 remains and a new result emerges: as long as the consumer has more imperfect perception of the illiquid asset balance than of the liquid asset balance, she is less willing to adjust her illiquid wealth balance in response to income shocks compared to the frictionless benchmark. Finally, I study how the main results change when the default wealth also depends on the previous selves' perceived wealth, effectively a form of learning.

5 A Calibration Exercise

In this section, to illustrate the quantitative potential of imperfect perception of wealth, I conduct a calibration exercise with a more general utility function (CRRA) and a borrowing constraint.

5.1 The Environment

Utility. I introduce a more general utility function (CRRA):

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \quad \forall t \in \{0, \dots, T-1\},$$

where $\sigma > 0$ is the coefficient of relative risk aversion. To present the results most cleanly, I take the infinite horizon limit $T \rightarrow \infty$ throughout this section and do not introduce additional degrees of freedom related to life-cycle considerations and retirement.

²⁶For commonly used utilities, $|u''(c^*)|$ decreases with the consumption level.

Perception functions. For consistency, I maintain the same perception functions (11) as the analytical Section 3. That is, I let each self's perceived wealth be a weighted average of the actual state and a default wealth: $\mathbf{a}_t^p(a_t) = (1 - \lambda)a_t + \lambda a_t^D$, where a higher $\lambda \in [0, 1]$ means a larger friction. On the other hand, to isolate the friction of interest, I let each self perfectly perceive her income state $\vec{\mathbf{y}}_t^p(\vec{y}_t) = \vec{y}_t$ and form rational expectations about her exogenous income process. About the default wealth a_t^D , same as Section 3, I let the default a_t^D be the average value of a_t in the frictionless case.²⁷

Borrowing constraints. I introduce the following borrowing constraint:

$$a_{t+1} = R(a_t + y_t - c_t) \geq -\underline{b}_{t+1} \equiv 0 \quad \forall t \in \{0, \dots, T-1\}. \quad (23)$$

Here, I require the consumer's wealth to be non-negative. I also consider robustness checks with positive borrowing.

A natural challenge with bounded rationality is guaranteeing that the borrowing constraint (23) always holds despite imperfect perception of wealth. If the actual wealth is low but the consumer's perceived wealth is not as low, she may over-consume so that her borrowing constraint is violated. To avoid such a violation, I impose the following rule about the consumer's actual consumption:

$$c_t = \min \left\{ c_t(a_t^p, \vec{y}_t^p), a_t + y_t + \frac{b_{t+1}}{R} \right\}. \quad (24)$$

To interpret (24), note that if the consumer's consumption decision $c_t(a_t^p, \vec{y}_t^p)$ given her perceived state (a_t^p, \vec{y}_t^p) does not violate the borrowing constraint, $c_t(a_t^p, \vec{y}_t^p)$ is her actual consumption. If the borrowing constraint is violated, instead, the consumer consumes to the extent that the borrowing constraint binds $\left(c_t = a_t + y_t + \frac{b_{t+1}}{R}\right)$. One can interpret (24) as a credit card spending limit: the bank only allows the consumer to consume up to her limit.^{28'29}

²⁷Note that, without quadratic utilities, the property that the average wealth level is independent of the degree of frictions (λ) no longer holds. My choice then maintains the consistency across different calibrations about the degree of frictions λ . I also consider robustness checks in which the default a_t^D is the average wealth level given each calibration about the degree of frictions λ .

²⁸An alternative way to deal with the borrowing constraint is to let the consumer re-decide on her consumption after she learns that she hits the borrowing constraint. However, in this case, the consumption rule will have the unappealing property of being discontinuous in actual wealth.

²⁹(24) does lead to a kink in the consumption rule. In the hyperbolic discounting literature (e.g. Harris and Laibson, 2001), the kink in the future self's consumption rule may lead to a discontinuous current consumption rule. However, at least given my calibration of the income process below, I do not find that the consumption rule here is discontinuous. There are two reasons. First, similar to the hyperbolic discounting literature, these discontinuities are alleviated when future incomes are stochastic. Second, here, at a low wealth level, the consumer's perceived wealth is larger than her actual wealth. She then expects her future self not to be stuck in the region in which the borrowing constraint binds and her consumption rule has a kink.

Standard parameters. As discussed in Section 4, one should interpret the length of a period in the discrete-time model here as the length of the consumer’s perfect recall about her past consumption and saving decisions. Because of the prevalence of annual financial budgets, in my main analysis I treat each period as a year and compare the MPCs in the model to the annualized MPCs in empirical studies. I also explore quarterly calibration as a robustness check.

I henceforth use standard annual parameters to calibrate my model. I let the real interest rate be $R = 1.03$ (Kaplan and Violante, 2010). I no longer impose $\beta R = 1$. Instead, I follow the standard practice in the literature on one-asset models (e.g. Fuster, Kaplan and Zafar, 2018) and calibrate β to match the average wealth to average income ratio, 3.5. In the main analysis, $\beta = 0.967$.³⁰ Finally, I let the degree of relative risk aversion be $\sigma = 1$ (Krueger, Mitman and Perri, 2016). I also explore alternative calibrations in robustness checks.

Income process. I use the canonical income process with both transitory and persistent shocks:

$$\log y_t = \underbrace{p_t}_{\text{permanent}} + \underbrace{\epsilon_t}_{\text{temporary}} \quad (25)$$

$$p_{t+1} = \phi p_t + \eta_t,$$

where ϕ is the persistence, ϵ_t is the (normal) transitory income shock and η_t is the (normal) persistent income shock. I follow Krueger, Mitman and Perri (2016) and let

$$(\phi, \sigma_\eta^2, \sigma_\epsilon^2) = (0.9695, 0.0384, 0.0522).$$

As in Section 3, to isolate the friction of interest, I let each self t perfectly perceive her income state $\vec{y}_t = (y_t, p_t)$.

Given the income process, I can study the MPC out of the temporary current income shocks ϵ_t (as in part (i) of Theorem 1) and the MPC out of the persistent income shocks η_t (as in Corollary 5). Later, in an extension, I also introduce news about future temporary current income shocks and study the MPC out of future income shock (as in part (iii) of Theorem 1).

Calibrating the degree of frictions λ . This is a challenging problem. The cleanest method I have found is a sufficient statistic approach. As mentioned in Section 3, I can use part (ii) of Theorem 1 to develop a sufficient statistic for $\lambda = 1 - \psi_t^a / \psi_t^y$, based on the ratio between the

³⁰To maintain consistency of β across the different calibrations of λ , I choose β to match the average wealth to average income ratio in the frictionless case.

MPC out of wealth and the MPC out of current income. I then study how imperfect perception of wealth changes consumers' MPCs. Note that the relationship $\lambda = 1 - \psi_t^a / \psi_t^y$ still holds in the more general set up here as long as the borrowing constraint does not bind exactly. This relationship holds because the consumer's perception function here is still given by (11).

This sufficient statistic approach has two main advantages. First, estimates for those MPCs are directly available from empirical studies. Second, as discussed in detail above, it is not easy for canonical rational models, including two-asset models, to generate a smaller MPC out of wealth than the MPC out of current income, especially for consumers with positive liquidity. Note that this method does leverage the fact that there is a single psychology-based parameter λ , and assumes that each consumer knows her future selves' misperception as in Theorem 1. If the consumer is partially sophisticated, the results about excess sensitivity to current income for high-income, high-liquidity consumer below can be viewed as an upper bound on how much the friction can drive up ψ_t^y .

Di Maggio, Kermani and Majlesi (2018) estimate the MPC out of wealth and the MPC out of current income for rich households from the same dataset. Specifically, they estimate ψ_t^a based on the consumption response to the capital gain on direct stock holdings.³¹ They estimate ψ_t^y based on the consumption response to dividend income. There are two points worth mentioning about their estimates.

First, one may be worried that direct stock holdings are not fully liquid, even though they are indeed treated as liquid assets in the literature (Kaplan and Violante, 2014; Kaplan, Violante and Weidner, 2014). As discussed in Section 3, with perfect perception of wealth, for consumers with positive liquidity, if anything, the MPC out of illiquid wealth is larger than, instead of smaller than, the MPC out of liquid wealth and current income. This is because illiquid wealth has a higher return, and consumers with positive liquidity can freely adjust their liquid balances to consume out of changes in illiquid wealth. The formula $\lambda = 1 - \psi_t^a / \psi_t^y$ then reflects a lower bound on the degree of imperfect perception of wealth if one is concerned that the direct stock holdings are slightly illiquid.

Second, although dividend incomes are typically persistent, their estimates are designed to measure the MPC out of current income only. This is because Di Maggio, Kermani and Majlesi (2018) study the consumption response to both the current dividend income and the capital gain on stock holdings. Since changes in future dividends will be reflected in stock prices and thus in

³¹As stock prices approximate random walk, the capital gains are nearly independent over time. As a result, their empirical estimate of ψ_t^a approximates the term in Theorem 1.

the capital gain of stock holdings, the coefficient on current dividend income will reflect the MPC out of current income.

In their estimates, for consumers in the top half of wealth distribution, the MPC out of wealth is \$0.05 per year, and the MPC out of current income for rich households is \$0.35 per year. Together, they imply a sizable friction: $\lambda = 1 - \psi_t^a / \psi_t^y = 1 - 0.05 / 0.35 \approx 6/7$.³²

Di Maggio, Kermani and Majlesi (2018)'s estimates reflect a general theme in the recent empirical literature: the estimates of MPC out of wealth are typically much smaller than the estimates of the MPC out of current income. Using other estimates of the MPC out of wealth and the MPC out of current income, I can get similar, if not larger, estimates of the degree of frictions λ . For example, Chodorow-Reich, Nenov and Simsek (2019)'s estimate of the MPC out of financial wealth ψ_t^a is only \$0.028 per year, smaller than Di Maggio, Kermani and Majlesi (2018)'s. Fagereng et al. (2019) also find that rich households consume very little out of capital gains and have a savings rate out of capital gains close to one hundred percent.

One may wonder whether it is possible to calibrate the degree of imperfect perception of wealth λ by directly asking about participants' knowledge of their own wealth in surveys. There are several challenges to this approach. First, in my model, what drives the frictional MPCs is imperfect perception of wealth *changes*. The friction still matters even if consumers have good knowledge about their average wealth. Through surveys, it is not easy to disentangle imperfect perception about wealth changes from imperfect perception about average wealth. Second, what drives the frictional MPCs is the imperfect perception of wealth *when consumers make consumption decisions*. It is possible that a consumer can perfectly recall her total wealth when forced to (e.g. through a specific survey question), but she may not have wealth at the front of her mind when making consumption decisions. This distinction highlights the difference between bounded recall and selective retrieval from memory in cognitive psychology (Anderson, 2009, Kahneman, 2011). One may also wonder whether I can use the evidence in the Fintech literature to calibrate the degree of imperfect perception of wealth λ . As the existing research (e.g. Levi, 2015) focuses on the cross-sectional comparison between the treatment group (with access to the account aggregation tool) and the control group (without access), I face the same difficulty in disentangling imperfect perception about wealth changes from imperfect perception about average wealth. Given these limitations, I use the above sufficient statistic approach to calibrate the degree of frictions λ in this paper. In future work, I hope to develop a satisfactory way to calibrate λ through surveys

³²In their published version (Maggio, Kermani and Majlesi, 2019), their estimates of MPC out of current income are slightly larger than their NBER version above, which implies an even higher degree of frictions λ . To be conservative, I use their estimates in the NBER working paper version above.

and Fintech experiments.

The solution method. Note that borrowing constraints combined with imperfect perception of wealth lead to kinks in consumption rules (24). These kinks leave open the possibility of non-concave value functions. Extra-care then needs to be taken to numerically solve the model. Specifically, Euler-equation-based methods may not be valid with non-concave value functions (e.g. policy function iteration and endogenous grid methods). I instead use value function iteration to solve the model in this section, which is robust to non-concave value functions despite its relatively slow speed. Practically, it seems that the non-concavity of the value functions is alleviated when the variances of income shocks are large enough.

5.2 Results

Excess sensitivity of a high-income, high-liquidity consumer. Motivated by the empirical evidence on excess sensitivity of high-income, high-liquidity consumers, I first study the behavior of a consumer with high income and high liquidity. Specifically, I consider a consumer whose wealth is at the 87.5 percentile³³ and who has the highest income realization (I discretize the income process based on the Rouwenhorst method). Since this consumer is far from the borrowing constraint, liquidity concerns and precautionary behavior should not be of significant interest for her and I can isolate the impact from imperfect perception of wealth. Her behavior should then mimic the consumer studied in the analytical section who does not face borrowing constraints.

I first study the consumer's MPC out of temporary current income shocks in Table 1. Note that since I take the infinite horizon limit $T \rightarrow \infty$ throughout this section, the MPC will not depend on the exact period t . With perfect perception of wealth ($\lambda = 0$), the high income, high liquidity consumer's MPC out of a dollar of a temporary current income shock is only 0.037 dollars per year. This low MPC is consistent with the permanent income hypothesis with a patient agent ($\beta = 0.967$). With imperfect perception of wealth ($\lambda = 6/7$), the MPC out of a dollar of a temporary current income shock becomes 0.146 dollars per year, around 3.89 times the frictionless benchmark. This is similar to the analytical case studied in Corollary 2 in Section 3: based on (18) and given the calibration of λ here, the MPC with imperfect perception of wealth is 3.77 times the frictionless counterpart. In other columns of panel (a) of Table 1, I also explore alternative calibrations about the degree of frictions λ . Together, the result corroborates that

³³To maintain consistency across different calibrations about the degree of frictions λ , the considered wealth level is the 87.5% percentile of wealth without frictions ($\lambda = 0$). This means a wealth level that is 8.32 times the average income.

Table 1: Excess Sensitivity of a High-income, High-liquidity Consumer³⁵

	Frictionless ($\lambda = 1$)	Benchmark ($\lambda = 6/7$)	Mild frictions ($\lambda = 1/2$)	Large frictions ($\lambda = 19/20$)	Data (Fagereng, Holm and Natvik, 2019)
MPC (\$ per year)	0.037	0.146	0.052	0.348	0.459

(a) The Main Specification

	$\beta = 0.92$	$\sigma = 2$	Alternative \underline{b}_t	Alternative a_t^D	Quarter
MPC (\$ per year)	0.147	0.132	0.141	0.140	0.153

(b) Robustness Checks

imperfect perception of wealth can generate a quantitatively important increase in the MPC of a high-income, high-liquidity consumer.

The MPC out of temporary current income shocks studied here should be compared to the empirical estimates of the MPC out of unexpected temporary income shocks. Fagereng, Holm and Natvik (2019) study consumption responses to unexpected Norwegian lottery prizes, and find the MPC remains high among *liquid* winners (0.459 dollars per year for the group with the highest liquidity). It is true that the MPC with imperfect perception of wealth in my model (0.146 dollars per year) is not as high as this empirical counterpart. However, their estimate is probably considered at the high end of the MPC estimates³⁴. Moreover, I can easily increase the MPC in my model by shortening the horizon. In the end, my main goal is to illustrate that the imperfect perception of wealth can generate a sizable increase in the MPC of a high-income, high-liquidity consumer.

The result that imperfect perception of wealth can generate a quantitatively important increase in the MPC of a high-income, high-liquidity consumer is robust to alternative specifications. Specially, in panel (b) of Table 1, I consider five robustness checks. First, I consider a higher discount rate $\beta = 0.92$. Second, I study a different degree of relative risk aversion, $\sigma = 2$. Third, I look at the case with positive borrowing, $\underline{b}_t = \underline{b} = 0.5E[y_s]$ for all t , where $E[y_s]$ is the average income. Fourth, I let the default a_t^D be the average wealth level given the calibration $\lambda = 6/7$.³⁶ Fifth, I work with a quarterly calibration (the length of a period, i.e. the length of memory, is a quar-

³⁴One reason is that it includes durable consumption.

³⁵The table displays the (annualized) MPC out of temporary current income shocks for a consumer whose wealth is at the 87.5 percentile and who has the highest income realization.

³⁶As discussed above, in the main numerical results, I let the default a_t^D be the average wealth level without frictions ($\lambda = 0$) to maintain consistency across different calibrations about the degree of frictions λ .

ter.)³⁷ I find the (annualized) MPC of a high-income, high-liquidity consumer is not very sensitive to these alternative specifications.

I then study the MPC out of persistent income shocks. To facilitate the comparison with the MPC out of temporary current income shocks, I study the MPC out of a persistent income shock that increases the consumer's total present value of income by 1 dollar. With perfect perception of wealth ($\lambda = 0$), the MPC out of the persistent income shock is 0.0356 dollars per year. With imperfect perception of wealth ($\lambda = 6/7$), the MPC goes up to 0.0680 dollars per year. This result is consistent with Corollary 5 above, which studies the response to persistent income shocks in the analytical case. The consumer with imperfect perception of wealth still exhibits excess sensitivity to persistent income shocks, but less so than to transitory income shocks. This is because the excess discounting of future income partially cancels out the excess sensitivity to current income.

Imperfect perception of wealth interacts with the borrowing constraint. I now study how imperfect perception of wealth interacts with the borrowing constraint. Due to the space constraint, let me briefly summarize two main results here and refer the reader to Appendix B for details.

First, the consumer with imperfect perception of wealth has a flattened MPC-wealth curve. This is different from the case with perfect perception of wealth, in which the consumer's MPC out of temporary current income shocks decreases with wealth (Carroll and Kimball, 1996). With imperfect perception of wealth, we already know that, at a high wealth level the consumer exhibits a higher MPC at a high wealth level than her counterpart without imperfect perception of wealth. At a low wealth level, a new channel emerges: as long as the consumer's borrowing constraint does not bind exactly (and her wealth level is below the default wealth), her perceived wealth will be larger than her actual wealth. As a result, the consumer thinks she is further away from the borrowing constraint than she actually is. She then displays a lower MPC at a low wealth level than she does with perfect perception of wealth. Together, the consumer with imperfect perception of wealth has a flattened MPC-wealth curve. There is also empirical support for such a flattened MPC-wealth curve (Kueng, 2018; Fagereng, Holm and Natvik, 2019).

Second, imperfect perception of wealth increases the probability that a consumer hits her borrowing constraint. The intuition is similar: at a low wealth level, since the consumer's perceived wealth is larger than her actual wealth, she over-consumes. As a result, she is more likely to hit the borrowing constraint. In other words, the consumer with imperfect perception of wealth may fall into a "poverty trap." In fact, it is well known that one shortcoming of the traditional one-asset

³⁷I adjust the discount rate, interest rate and income process accordingly.

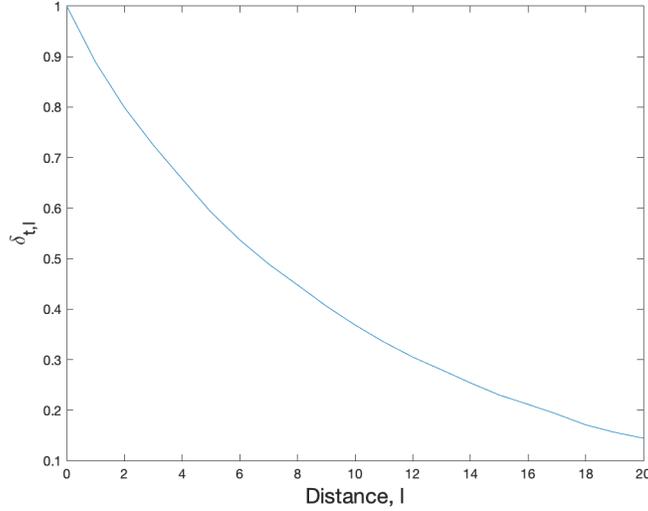


Figure 1: Excess Discounting of Future Income

model of income fluctuations is that it does not generate enough liquidity-constrained households (Kaplan and Violante, 2014). Imperfect perception of wealth then provides a way to boost the number of liquidity-constrained households without the complications of adding an illiquid asset.

A potential follow-up project is to study the quantitative implication of imperfect perception of wealth on wealth distribution. For such a purpose, note that the above analysis about how imperfect perception of wealth interacts with the borrowing constraint treats the degree of friction λ as a constant, independent from the actual wealth level. Such an approach is consistent with the rest of the paper to minimize the degrees of freedom. On the other hand, the consumer may want to pay more attention to her wealth level if she is closer to the borrowing constraint. This feature might be important in matching the empirical wealth distribution.

Excess discounting of future income. Now I consider an extension and introduce news about temporary future income shocks. As in Part (ii) of Theorem 1, the consumer with imperfect perception of wealth here still exhibits excess discounting of future income. For a high-income, high-liquidity consumer,³⁸ Figure 1 plots the extra discounting of income that will be received l periods later (the counterpart of $\lim_{T \rightarrow +\infty} \delta_{t,l}$ in Corollary 3). We can see that the extra discounting decreases with the distance l .

One can further try to match the empirical evidence on the impulse responses of consumption to news about future income: consumption does not respond much to news about future income; instead, the consumption response when the consumer actually receives the income is much larger

³⁸Similar to the study of the MPC out of current income at the start of the section, Figure 1 studies a high-income, high-liquidity consumer whose wealth is at the 87.5 percentile and who has the highest income realization.

(excess sensitivity to anticipated income shocks). For high-income, high-liquidity households, see Stephens and Unayama (2011) and Kueng (2018). As Corollary 4 shows, imperfect perception of wealth can indeed qualitatively generate this empirical pattern. There are, however, several challenges to quantitatively match the model predictions with the empirical evidence. First, it is often unclear when the news about future income arrives exactly. Second, it is unclear how many consumers actually pay attention to the news (note that in the model, to isolate the friction of interest, I eliminate inattention to the income process). With these caveats in mind, in Appendix B, I study how my model can match the estimates in Kueng (2018). It seems that imperfect perception of wealth alone can indeed generate the empirical pattern of excess sensitivity to anticipated income shocks when the income arrives. Nevertheless, to fully match the near-zero consumption responses when the news about future income arrives, one also needs some inattention to the income news.

6 Macro Implications

As studied above, imperfect perception of wealth has a significant impact on the consumption and saving behavior of a consumer. The friction can also have important macro implications. First, as studied above, with imperfect perception of wealth, even consumers with high liquidity can deviate from the permanent income hypotheses and exhibit excess sensitivity to current income. Imperfect perception of wealth then decouples such behavior from the existence of liquidity constraints: rich consumers can also behave as “hand-to-mouth” consumers. Such frictional behavior of the rich can be important in determining aggregate demand. For example, the redistributive policy from the rich towards the poor to stimulate aggregate demand may not be as powerful.

Second, imperfect perception of wealth more generally increases the importance of GE income multiplier because of the excess sensitivity to income. However, as Angeletos and Lian (2018), Farhi and Werning (2017) and Gabaix (2016*b*) show, the strength of the GE effects requires unrealistic coordination among consumers. A larger weight on the GE effects with imperfect perception of wealth then strengthens any GE dampening mechanisms studied in Angeletos and Lian (2018), Farhi and Werning (2017) and Gabaix (2016*b*). The rest of this section illustrates this point by studying the classical GE question about how aggregate demand responds to monetary policy.

Environment. I am interested in how imperfect perception of wealth influences the response of aggregate demand to monetary policy. For this purpose, I follow the seminal approach of Werning (2015): I focus on the demand block and study how aggregate demand y_t responds to exogenous movement in the real interest rate path $\{R_t\}$. The demand block can be combined with

a new-Keynesian “supply block.” However, the details about the supply block are irrelevant to the question of interest.

Specifically, consider a general equilibrium economy with a measure one of consumers. As in Section 2, the consumer has concave utilities but does not face a borrowing constraint. For a typical consumer i , her utility and budget constraints are given by

$$\begin{aligned}
U_{i,0} &\equiv \sum_{t=0}^{T-1} \beta^t u(c_{i,t}) + \beta^T v(a_{i,T} + y_{i,T}) \\
\text{s.t. } a_{i,t+1} &= R_t (a_{i,t} + y_{i,t} - c_t) \quad \forall t \in \{0, \dots, T-1\} \\
y_{i,t} &= \delta_i y_t \quad \forall t \in \{0, \dots, T\},
\end{aligned}$$

where δ_i captures the share of aggregate income that household i receives, with $\int \delta_i di = 1$. There is no capital so aggregate saving $\int a_{i,t} di = 0$ for all t . Similar to the analysis above, each consumer’s subjective belief $\tilde{E}_{i,t}[\cdot]$ is based on imperfect perception of wealth, i.e. $\mathbf{a}_{i,t}^P(a_{i,t}) = (1-\lambda)a_{i,t} + \lambda a_t^D$, but she has perfect perception of income and interest rates (which are exogenous to her).³⁹ In other words, the consumer solves the same problem as in the main analysis, with time-varying interest rates. Following Werning (2015), to simplify notation, I let the aggregate uncertainty about the entire interest rate path $\{R_t\}$ be resolved at period 0. Since each consumer has perfect perception of income and interest rates, for all t and l , $\tilde{E}_{i,t}[R_{t+l}] = R_{t+l}$ and $\tilde{E}_{i,t}[y_{t+l}] = y_{t+l}$.

Aggregate demand. Here, I work with a linearization for analytical results. Similar to above, I use a hat over a variable to denote its deviation from the deterministic steady state in which all interest rates are fixed at $R_t = R \equiv \beta^{-1}$.

Proposition 4. *Consider the $T \rightarrow +\infty$ limit. Aggregate consumption $\hat{c}_t = \int \hat{c}_{i,t} di$ in each period t satisfies*

$$\hat{c}_t = -\psi^R \left(\hat{R}_t + \sum_{l=1}^{+\infty} \delta^l R^{-l} \hat{R}_{t+l} \right) + \psi^y \left(\hat{y}_t + \sum_{l=1}^{+\infty} \delta^l R^{-l} \hat{y}_{t+l} \right), \quad (26)$$

where $\psi^R > 0$ parametrizes the PE effect of interest rates on aggregate consumption, $\psi^y > 0$ parametrizes the GE effect through the income multiplier and $\delta \in [0, 1]$ captures the excess aggregate discounting.

(i) **Smaller PE effect:** ψ^R decreases with the friction λ ;

(ii) **Larger GE effect:** ψ^y increases with the friction λ ;

³⁹Similar to above, I let the default wealth a_t^D be the value of a_t in the deterministic steady state in which all interest rates are fixed at $R_t = R \equiv \beta^{-1}$.

(iii) **Excess discounting:** $\delta < 1$ decreases with friction λ .

The first part of Proposition 4 shows that imperfect perception of wealth dampens the PE effect of interest rates on consumption. By PE effect I mean the impact of interest rates on the average consumer's consumption as if in a single-agent problem, holding other consumers' decisions fixed. The intuition is similar to Theorem 1: the response of current consumption to interest rate changes leads to changes in savings; with imperfect perception of wealth, worrying that her future selves cannot perfectly perceive and respond to such wealth changes, the current self is less willing to respond to interest rate changes.⁴⁰ A smaller ψ^R also seems to be consistent with empirical evidence on the small response of consumption to interest rate changes (Hall, 1988; Campbell and Mankiw, 1989; Havránek, 2015).

The second and the third parts of Proposition 4 are essentially the GE version of the results about excess sensitivity to current income and excess discounting of future income in Theorem 1. Here, the excess sensitivity to current income in Theorem 1 manifests as a larger GE effect through the income multiplier.

The impact of interest rates on aggregate demand. I now turn to how aggregate demand responds to changes in interest rates. Imposing market clearing $\hat{c}_t = \hat{y}_t$ in (26), the elasticity of aggregate demand to interest rates is given by

$$\epsilon_l \equiv \frac{d(\hat{y}_t/y^*)}{d(\hat{R}_{t+l}/R)} = -\sigma \quad \forall l \geq 0, \quad (27)$$

where $\sigma = -\frac{u'(c^*)}{c^*u''(c^*)}$ is the elasticity of intertemporal substitution and $c^* = y^*$ is the steady state aggregate spending when all interest rates are fixed at $R_t = R \equiv \beta^{-1}$. As in Werning (2015), the smaller PE effect of interest rates in (26) is offset by the larger GE effect of interest rates in (26), and the total impact of interest rates on aggregate demand remains is the same. In Werning (2015), the smaller PE effect and the larger GE effect of interest rates are driven by liquidity constraints, while here the smaller PE effect and the larger GE effect of interest rates are driven by imperfect perception of wealth.

Nevertheless, as the follow up literature (Angeletos and Lian, 2018; Farhi and Werning, 2017; Gabaix, 2016b) to Werning (2015) shows, the strong GE effect imbedded in (27) requires unrealistic coordination among consumers: each consumer not only needs to perfectly perceive the interest

⁴⁰Note that here aggregate saving is zero. The impact of interest rates on aggregate consumption comes solely from the intertemporal substitution motive, and Proposition 4 shows that the intertemporal substitution motive is dampened by imperfect perception of wealth. On the other hand, imperfect perception of wealth might amplify the income effect of interest rates on consumption, similar to the excess sensitivity to income result in Theorem 1.

rate changes, she also needs to believe that all other consumers perfectly perceive the interest rate changes and respond frictionlessly. Without such perfect coordination, the GE effects will be attenuated, and the aggregate response will depend predominantly on the PE effects. Here, as imperfect perception of wealth leads to a smaller PE effect, with any degree of GE imperfection, the considered friction will attenuate the total impact of interest rates on aggregate demand.

To illustrate, I consider a simple twist to the problem above to capture such GE imperfection. As discussed in Angeletos and Sastry (2018), this approach introduces GE dampening by effectively capturing a form of anchored higher-order beliefs or limited higher-level reasoning. Specifically, I let each consumer perfectly perceive the exogenous interest rate path $\{R_t\}$. However, when she calculates her current and future incomes which depend on the general equilibrium aggregate spending, she believes that a portion $\mu \in [0, 1]$ of other consumers do not perceive the changes and do not change their consumption, and only the rest $1 - \mu$ of consumers perfectly perceive and respond to the interest rate changes.⁴¹ A larger $\mu \in [0, 1]$ means a larger coordination friction and a larger GE imperfection. With such GE imperfection, I can establish:

Proposition 5. *Using $\epsilon_l \equiv \frac{d(\hat{y}_t/y^*)}{d(\hat{R}_{t+l}/R)}$ to denote the elasticity of aggregate demand with respect to interest rate changes l periods later. As long as there is any GE imperfection ($\mu > 0$),*

- 1) *The response $|\epsilon_l| \leq \sigma$ decreases with the degree of imperfect perception of wealth λ .*
- 2) *The response $|\epsilon_l| \leq \sigma$ decreases with the distance l from the interest rate changes.*

In other words, as long as the GE coordination is not perfect, imperfect perception of wealth attenuates the response of aggregate demand to interest rates. A larger degree of imperfect perception of wealth λ leads to a larger attenuation. Moreover, similar to the “distance effect” in part (iii) of Theorem 1, for a given λ , the attenuation increases with the distance from the interest rate changes. In general equilibrium, this “distance effect” helps explain the forward guidance puzzle.

In Figure 2, I plot how imperfect perception of wealth changes the elasticity of aggregate demand with respect to interest rates, i.e. each ϵ_l .⁴² Without imperfect perception of wealth ($\lambda = 0$), the GE dampening ($\mu > 0$) per se does not attenuate the elasticity ϵ_l that much. However, with imperfect perception of wealth ($\lambda > 0$), GE dampening is significantly strengthened and ϵ_l is attenuated more. Moreover, with imperfect perception of wealth, the elasticity ϵ_l decreases quickly with respect to the distance l .

⁴¹The fact that each consumer knows the interest rate path but believes only $1 - \mu$ of consumers perfectly perceive the interest rate path is common knowledge.

⁴²In the figure, $\sigma = 1$, $\mu = 0.2$ and $\lambda = 6/7$ (from the sufficient statistic method developed in Section 5).

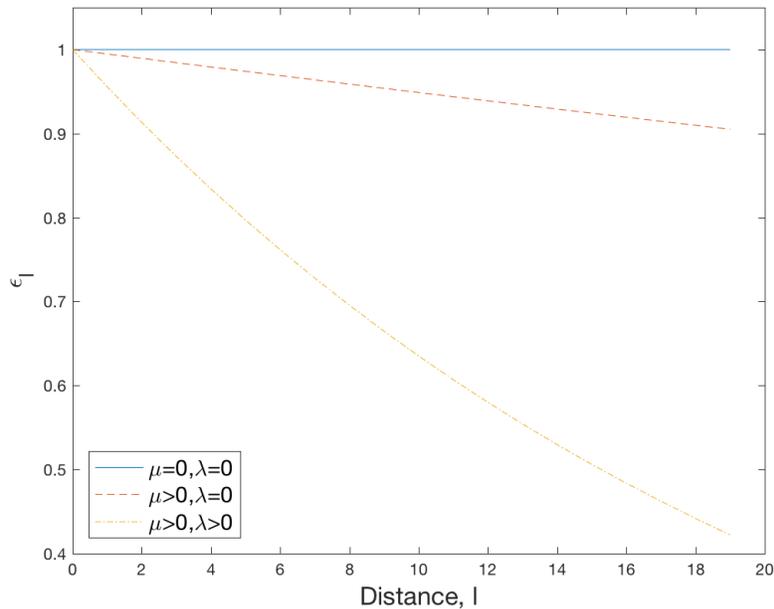


Figure 2: The Impact of Interest Rates on Aggregate Demand

7 Conclusion

In this paper, I develop a new theory motivated by the observation that consumers often make their consumption and saving decisions with imperfect perception of wealth. I show how this friction can jointly explain several empirical puzzles about consumption: excess sensitivity to current income, smaller MPCs out of liquid wealth than out of current income, and excess discounting of future income. I also illustrate the quantitative potential of the friction and discuss the potential macro implications.

There are multiple follow-up research questions related to imperfect perception of wealth, some of which I am pursuing in ongoing work. First, as discussed in Section 4, interesting new economic mechanisms can arise in the case with multiple assets. Imperfect perception of the balance of a particular asset can also lead to the illiquidity of that asset itself. Second, to avoid additional degrees of freedom, I do not allow the degree of imperfect perception of wealth (λ) to move with the wealth level. Nevertheless, it seems natural to study a variant in which the consumer has better knowledge about her wealth when she is close to the borrowing constraint. This variant is also useful for a full quantitative analysis of general equilibrium implications, such as for wealth inequality and redistributive policy. Third, it is useful to further explore the psychological foundations of imperfect perception of wealth: why does the consumer want to know more about her income than her wealth? Fourth, I hope to develop a satisfactory way to calibrate the degree of

imperfect perception of wealth λ through surveys and Fintech experiments. Fifth, the method developed in the paper to incorporate imperfect perception of the endogenous state in Markov decision problems can be applied to other settings, such as investment and price setting.

Appendix A: Proofs

Proof of Proposition 1. First, let $\{\mathbf{c}_t(a_t^p, \vec{y}_t^p), W_t(a_t^p, \vec{y}_t^p)\}_{t=0}^{T-1}$ and $\{V_t(a_t, \vec{y}_t)\}_{t=0}^T$ be the consumption rules and value functions in the intrapersonal equilibrium. For $t \in \{0, \dots, T-1\}$, we first notice that, as each self t understands the transitional density function for the exogenous income process, we have $\tilde{E}_t[E_{t+k}[\cdot]] = \tilde{E}_t[\cdot]$ for all $k \in \{0, \dots, T-t-1\}$, where, as in (7), $E_{t+k}[\cdot]$ averages over all potential realizations of the exogenous income process (after $t+k$) given \vec{y}_{t+k} . As a result, from (4), we know

$$\begin{aligned} c_t(a_t^p, \vec{y}_t^p) &= \arg \max_{c_t} u(c_t) + \beta \tilde{E}_t \left[\sum_{k=1}^{T-t-1} \beta^k u(\mathbf{c}_{t+k}(\mathbf{a}_{t+k}^p(a_{t+k}), \vec{\mathbf{y}}_{t+k}^p(\vec{y}_{t+k}))) + \beta^{T-t} v(a_T + y_T) \right] \\ &= \arg \max_{c_t} u(c_t) + \beta \tilde{E}_t [V_{t+1}(a_{t+1}, \vec{y}_{t+1})] \\ &= \arg \max_{c_t} u(c_t) + \beta \int V_{t+1}(R(a_t^p + y_t^p - c_t), \vec{y}_{t+1}) \phi(\vec{y}_{t+1} | \vec{y}_t^p) d\vec{y}_{t+1}, \end{aligned}$$

where I use (5) and (7). By the same token, we have

$$W_t(a_t^p, \vec{y}_t^p) = \max_{c_t} u(c_t) + \beta \int V_{t+1}(R(a_t^p + y_t^p - c_t), \vec{y}_{t+1}) \phi(\vec{y}_{t+1} | \vec{y}_t^p) d\vec{y}_{t+1}.$$

This proves (9). Similarly, for $t \in \{0, \dots, T-1\}$, from (7), we have

$$\begin{aligned} V_t(a_t, \vec{y}_t) &= u(\mathbf{c}_t(\mathbf{a}_t^p(a_t), \vec{\mathbf{y}}_t^p(\vec{y}_t))) + \beta E_t \left[\sum_{k=1}^{T-t-1} \beta^k u(\mathbf{c}_{t+k}(\mathbf{a}_{t+k}^p(a_{t+k}), \vec{\mathbf{y}}_{t+k}^p(\vec{y}_{t+k}))) + \beta^{T-t} v(a_T + y_T) \right] \\ &= u(c_t) + \beta \int V_{t+1}(R(a_t + y_t - \mathbf{c}_t(\mathbf{a}_t^p(a_t), \vec{\mathbf{y}}_t^p(\vec{y}_t))), \vec{y}_{t+1}) \phi(\vec{y}_{t+1} | \vec{y}_t) d\vec{y}_{t+1}, \end{aligned}$$

where I use the law of iteration for $E_t[\cdot]$. This proves (10).

Now, consider consumption rules and value functions $\{\mathbf{c}_t(a_t^p, \vec{y}_t^p), W_t(a_t^p, \vec{y}_t^p)\}_{t=0}^{T-1}$ and $\{V_t(a_t, \vec{y}_t)\}_{t=0}^T$ that satisfy (9) and (10) and the boundary condition $V_T(a_T, \vec{y}_T) = v(a_T + y_T)$. First note that given the consumption rules $\{\mathbf{c}_t(a_t^p, \vec{y}_t^p)\}_{t=0}^{T-1}$ and the exogenous perception functions $\{\mathbf{a}_t^p(a_t), \vec{\mathbf{y}}_t^p(\vec{y}_t)\}_{t=0}^{T-1}$, one can back out the law of motion of the exogenous and endogenous states. One can then back out each self t 's subjective expectation $\tilde{E}_t[\cdot]$ based on her perceived state (a_t^p, \vec{y}_t^p) . Then, from (9)

and (10), we have, for $t \in \{0, \dots, T-1\}$,

$$\begin{aligned} c_t(a_t^p, \vec{y}_t^p) &= \arg \max_{c_t} u(c_t) + \beta \int V_{t+1}(R(a_t^p + y_t^p - c_t), \vec{y}_{t+1}) \phi(\vec{y}_{t+1} | \vec{y}_t^p) d\vec{y}_{t+1}, \\ &= \arg \max_{c_t} u(c_t) + \beta \tilde{E}_t[V_{t+1}(a_{t+1}, \vec{y}_{t+1})] \\ &= \arg \max_{c_t} u(c_t) + \beta \tilde{E}_t \left[\sum_{k=1}^{T-t-1} \beta^k u(\mathbf{c}_{t+k}(\mathbf{a}_{t+k}^p(a_{t+k}), \vec{\mathbf{y}}_{t+k}^p(\vec{y}_{t+k}))) + \beta^{T-t} v(a_T + y_T) \right], \end{aligned}$$

where I again use $\tilde{E}_t[E_{t+k}[\cdot]] = \tilde{E}_t[\cdot]$ for all $k \in \{0, \dots, T-t-1\}$.

As a result, each self t 's consumption rule solves (4). The consumption rules and value functions $\{\mathbf{c}_t(a_t^p, \vec{y}_t^p), W_t(a_t^p, \vec{y}_t^p)\}_{t=0}^{T-1}$ and $\{V_t(a_t, \vec{y}_t)\}_{t=0}^T$ then coincide with the corresponding objects in the unique intrapersonal equilibrium. Note that the intrapersonal equilibrium is always unique: as utilities are concave, one can always solve (4) backward directly, without going through the recursive forms studied here.

Proof of Theorem 1. We solve each self's consumption uniquely through backward induction, based on Proposition 1 and the fact that $V_T(a_T, \vec{y}_T) = v(a_T + y_T)$. Specifically, I take a guess and verify approach. For $t \in \{0, \dots, T-1\}$, I conjecture that the consumption in each period takes the form of (12). That is,

$$\hat{c}_t = \psi_t^a \hat{a}_t + \psi_t^y \left(\hat{y}_t + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} E_t[\hat{y}_{t+l}] \right), \quad (28)$$

where a hat over a variable denotes its value in the frictionless case when the stochastic incomes are fixed at their respective averages. Note that, with the quadratic utility and linear consumption rules here, this value is equal to the average value of the variable intrapersonal equilibrium (over realizations of the stochastic incomes).⁴³

Moreover, for $t \in \{0, \dots, T-1\}$, I conjecture that

$$\left(\frac{\partial \hat{V}_t}{\partial a_t} \right) = u'' \cdot \left(\Gamma_t \hat{a}_t + \Gamma_t^y \left(\hat{y}_t + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} E_t[\hat{y}_{t+l}] \right) \right), \quad (29)$$

where $\Gamma_t > 0$ captures the concavity of the value function V_t over a_t . We can also extend the expression of (29) to the last period T , where $V_T(a_T, \vec{y}_T) = v(a_T + y_T)$:

⁴³With the quadratic utility and linear consumption rules here, the average value of each income, consumption, and wealth is independent of the degree of frictions λ .

$$\left(\frac{\partial \hat{V}_T}{\partial a_T}\right) = u'' \cdot (\Gamma_T \hat{a}_T + \Gamma_T^y \hat{y}_T),$$

where

$$\Gamma_T = \Gamma_T^y = \frac{v''}{u''} > 0. \quad (30)$$

For $t \in \{0, \dots, T-1\}$, we use backward induction to prove the conjectures are true. Suppose (28) and (29) are true from $t+1$ onward. For t , based on the optimality condition in (9), we have

$$u'(c_t) = \tilde{E}_t \left[\frac{\partial V_{t+1}(a_{t+1}, \vec{y}_{t+1})}{\partial a_{t+1}} \right].$$

Note that u is quadratic and based on (29), we have

$$\hat{c}_t = \Gamma_{t+1} R (\hat{a}_t^p + \hat{y}_t - \hat{c}_t) + E_t \left[\Gamma_{t+1}^y \left(\hat{y}_{t+1} + \sum_{l=1}^{T-t-1} \delta_{t+1,l} R^{-l} \hat{y}_{t+1+l} \right) \right], \quad (31)$$

where I use (5) and the fact that $\tilde{E}_t [y_{t+l}] = E [y_{t+l}]$ for all $l \in \{0, \dots, T-t\}$ because $\vec{y}_t^p(\vec{y}_t) = \vec{y}_t$. Then,

$$\hat{c}_t = \Gamma_{t+1} R ((1-\lambda) \hat{a}_t + \hat{y}_t - \hat{c}_t) + E_t \left[\Gamma_{t+1}^y \left(\hat{y}_{t+1} + \sum_{l=1}^{T-t-1} \delta_{t+1,l} R^{-l} \hat{y}_{t+1+l} \right) \right],$$

where I use the perception function in (11). Collecting terms,

$$\hat{c}_t = \frac{\Gamma_{t+1} R}{1 + \Gamma_{t+1} R} ((1-\lambda) \hat{a}_t + \hat{y}_t) + \frac{1}{1 + \Gamma_{t+1} R} E_t \left[\Gamma_{t+1}^y \left(\hat{y}_{t+1} + \sum_{l=1}^{T-t-1} \delta_{t+1,l} R^{-l} \hat{y}_{t+1+l} \right) \right]. \quad (32)$$

We can then prove the conjecture (28) is true at t , with

$$\psi_t^y = \frac{\Gamma_{t+1} R}{1 + \Gamma_{t+1} R} \quad \text{and} \quad \psi_t^a = (1-\lambda) \psi_t^y, \quad (33)$$

and

$$\delta_{t,1} = \frac{\Gamma_{t+1}^y}{1 + \Gamma_{t+1} R} / (\psi_t^y R^{-1}) = \frac{\Gamma_{t+1}^y}{\Gamma_{t+1}}, \quad (34)$$

and

$$\delta_{t,l} = \frac{\Gamma_{t+1}^y R^{-(l-1)} \delta_{t+1,l-1}}{1 + \Gamma_{t+1} R} / (\psi_t^y R^{-l}) = \frac{\Gamma_{t+1}^y \delta_{t+1,l-1}}{\Gamma_{t+1}} \quad \forall l \in \{2, \dots, T-t\}. \quad (35)$$

Now, from (10), we know that, for $t \in \{0, \dots, T-1\}$,

$$\frac{\partial V_t(a_t, \vec{y}_t)}{\partial a_t} = \psi_t^a u'(c_t) + (1 - \psi_t^a) E_t \left[\frac{\partial V_{t+1}(a_{t+1}, \vec{y}_{t+1})}{\partial a_{t+1}} \right], \quad (36)$$

where I use the fact that $\frac{\partial(c_t(\mathbf{a}_t^p(a_t), \vec{y}_t^p(\vec{y}_t)))}{\partial a_t} = \psi_t^a$ based on the conjecture (28). As a result,

$$\left(\frac{\partial \hat{V}_t}{\partial a_t} \right) = u'' \cdot \psi_t^a \hat{c}_t + (1 - \psi_t^a) E_t \left[\left(\frac{\partial \hat{V}_{t+1}}{\partial a_{t+1}} \right) \right]. \quad (37)$$

From (37) and the conjecture (29) for $t+1$, we have

$$\begin{aligned} \left(\frac{\partial \hat{V}_t}{\partial a_t} \right) &= u'' \cdot \left(\psi_t^a \hat{c}_t + (1 - \psi_t^a) E_t \left[\Gamma_{t+1} R (\hat{a}_t + \hat{y}_t - \hat{c}_t) + \Gamma_{t+1}^y \left(\hat{y}_{t+1} + \sum_{l=1}^{T-t-1} \delta_{t+1,l} R^{-l} \hat{y}_{t+1+l} \right) \right] \right), \\ &= u'' \cdot (\hat{c}_t + (1 - \psi_t^a) \lambda \Gamma_{t+1} R \hat{a}_t), \end{aligned} \quad (38)$$

where I use (31) for the last equation. The conjecture (29) is then true at t , with

$$\Gamma_t = \psi_t^a + (1 - \psi_t^a) \lambda \Gamma_{t+1} R = (\psi_t^a)^2 + (1 - \psi_t^a)^2 \Gamma_{t+1} R = \frac{(R \Gamma_{t+1})^2}{1 + R \Gamma_{t+1}} \lambda^2 + \frac{R \Gamma_{t+1}}{1 + R \Gamma_{t+1}} \quad \text{and} \quad \Gamma_t^y = \psi_t^y, \quad (39)$$

where I use (33).

We now turn to prove part (i) - (iii) of Theorem 1. Part (ii) follows directly from (33). From (39), we know that Γ_t increases in λ for all $t \in \{0, \dots, T-1\}$ by induction. Part (i) then also follows from (33). To prove part (iii), first note that from (33) and (39), we have, for all $t \in \{0, \dots, T-1\}$,

$$\Gamma_t = \lambda^2 \frac{(\psi_t^y)^2}{1 - \psi_t^y} + \psi_t^y = \Gamma_t^y \left(\lambda^2 \frac{\psi_t^y}{1 - \psi_t^y} + 1 \right).$$

From (30), (34) and (35), we have

$$\delta_{t,1} = \frac{1}{1 + \lambda^2 \frac{\psi_{t+1}^y}{1 - \psi_{t+1}^y}} \quad \text{and} \quad \delta_{t,l} = \frac{1}{1 + \lambda^2 \frac{\psi_{t+1}^y}{1 - \psi_{t+1}^y}} \cdots \frac{1}{1 + \lambda^2 \frac{\psi_{t+l}^y}{1 - \psi_{t+l}^y}} \quad \forall l \in \{2, \dots, T-t-1\}. \quad (40)$$

Moreover, we have $\delta_{T-1,1} = 1$ and $\delta_{t,T-t} = \delta_{t,T-t-1}$ for $t \in \{2, \dots, T\}$, as $\Gamma_T^y = \Gamma_T$. Part (iii) then follows from part (i).

Proof of Lemma 1. See the proof of Theorem 1.

Proof of Corollary 1. We can modify the proof of Theorem 1 as follows. For $t \in \{0, \dots, T-1\}$,

the optimal consumption in (32) and the expression for $\frac{\partial V_t}{\partial a_t}$ in (38) become

$$\hat{c}_t = \frac{\Gamma_{t+1}R}{1 + \Gamma_{t+1}R} ((1 - \lambda_t) \hat{a}_t + \hat{y}_t) + \frac{1}{1 + \Gamma_{t+1}R} E_t \left[\Gamma_{t+1}^y \left(\hat{y}_{t+1} + \sum_{l=1}^{T-t-1} \delta_{t+1,l} R^{-l} \hat{y}_{t+1+l} \right) \right]; \quad (41)$$

$$\left(\frac{\partial \hat{V}_t}{\partial a_t} \right) = u''(c^*) (\hat{c}_t + (1 - \psi_t^a) \lambda_t \Gamma_{t+1} R \hat{a}_t). \quad (42)$$

We then have

$$\psi_t^y = \frac{\Gamma_{t+1}R}{1 + \Gamma_{t+1}R} \quad \text{and} \quad \psi_t^a = (1 - \lambda_t) \psi_t^y, \quad (43)$$

and

$$\Gamma_t = \psi_t^a + (1 - \psi_t^a) \lambda_t \Gamma_{t+1} R = \frac{(R\Gamma_{t+1})^2}{1 + R\Gamma_{t+1}} \lambda_t^2 + \frac{R\Gamma_{t+1}}{1 + R\Gamma_{t+1}} \quad \text{and} \quad \Gamma_t^y = \psi_t^y. \quad (44)$$

As a result, Γ_{t+1} strictly increases in λ_{t+l} for $l \in \{1, \dots, T-1-t\}$, but independent from λ_t . From $\psi_t^y = \frac{\Gamma_{t+1}R}{1 + \Gamma_{t+1}R}$, we then know ψ_t^y strictly increases with with each λ_{t+l} for $l \in \{1, \dots, T-1-t\}$. On the other hand, ψ_t^y is independent from current λ_t .

Also notice that, we still have

$$\delta_{t,1} = \frac{\Gamma_{t+1}^y}{1 + \Gamma_{t+1}R} / (\psi_t^y R^{-1}) = \frac{\Gamma_{t+1}^y}{\Gamma_{t+1}},$$

and

$$\delta_{t,l} = \frac{\Gamma_{t+1}^y R^{-(l-1)} \delta_{t+1,l-1}}{1 + \Gamma_{t+1}R} / (\psi_t^y R^{-l}) = \frac{\Gamma_{t+1}^y \delta_{t+1,l-1}}{\Gamma_{t+1}} \quad \forall l \in \{2, \dots, T-t\}.$$

As a result, similar to (40), we have

$$\delta_{t,1} = \frac{1}{1 + \lambda_{t+1}^2 \frac{\psi_{t+1}^y}{1 - \psi_{t+1}^y}} \quad \text{and} \quad \delta_{t,l} = \frac{1}{1 + \lambda_{t+1}^2 \frac{\psi_{t+1}^y}{1 - \psi_{t+1}^y}} \dots \frac{1}{1 + \lambda_{t+l}^2 \frac{\psi_{t+l}^y}{1 - \psi_{t+l}^y}} \quad \forall l \in \{2, \dots, T-t-1\},$$

Moreover, we have $\delta_{T-1,1} = 1$ and $\delta_{t,T-t} = \delta_{t,T-t-1}$ for $t \in \{2, \dots, T\}$, as $\Gamma_T^y = \Gamma_T$. As a result, $\delta_{t,l}$ is decreasing in $\{\lambda_{t+1}, \dots, \lambda_{T-1}\}$ but independent from λ_t .

Proof of Corollaries 2 and 3. From (39), we know that $\Gamma_t = \frac{(R\Gamma_{t+1})^2}{1 + R\Gamma_{t+1}} \lambda_t^2 + \frac{R\Gamma_{t+1}}{1 + R\Gamma_{t+1}} \equiv f(\Gamma_{t+1})$, with $f(x) \equiv \frac{Rx}{1+Rx} + \frac{(Rx)^2}{1+Rx} \lambda^2 = \frac{Rx}{1+Rx} (1 + \lambda^2 Rx)$. We also know that $\Gamma_T = \frac{v''}{u''} > 0$.

Let $\Gamma = \frac{R-1}{R(1-R\lambda^2)}$ denote the fix point of f . That is $f(\Gamma) = \Gamma$. Moreover, as long as $0 \leq \lambda < R^{-1/2}$, we have $\Gamma > f(x) > x$ if $0 < x < \Gamma$; and $\Gamma < f(x) < x$ if $x > \Gamma$. We then have two cases:

1) If $\Gamma > \frac{v''}{u''} = \Gamma_T$. We have $\Gamma > \Gamma_t = f^{(T-t)}(\Gamma_T) > f^{(T-t-1)}(\Gamma_T) > \dots > \frac{v''}{u''} = \Gamma_T$. As a result, $\Gamma_t = f^{(T-t)}(\Gamma_T)$ converges to the fix point Γ with $T \rightarrow +\infty$.

2) If $\Gamma < \frac{v''}{u''} = \Gamma_T$. We have $\Gamma < \Gamma_t = f^{(T-t)}(\Gamma_T) < f^{(T-t-1)}(\Gamma_T) < \dots < \frac{v''}{u''} = \Gamma_T$. As a result, $\Gamma_t = f^{(T-t)}(\Gamma_T)$ converges to the fix point Γ with $T \rightarrow +\infty$.

Together, one way or another, as long as $0 \leq \lambda < R^{-1/2}$, $\Gamma_t \rightarrow \Gamma$ with $T \rightarrow +\infty$. From (33), we then have, with $T \rightarrow +\infty$.

$$\psi_t^y \rightarrow \psi^y \equiv \frac{\Gamma R}{1 + \Gamma R} = \frac{1 - R^{-1}}{1 - \lambda^2} \quad \text{and} \quad \psi_t^a \rightarrow \psi^a \equiv (1 - \lambda) \psi^y = \frac{1 - R^{-1}}{1 + \lambda}.$$

By the same token, from (34) and (35), we know, for all $l \geq 1$, with $T \rightarrow +\infty$.

$$\delta_{t,l} \rightarrow \delta^l \equiv \left(\frac{\Gamma^y}{\Gamma} \right)^l = \left(\frac{\psi^y}{\Gamma} \right)^l = \left(1 - \frac{(R-1)\lambda^2}{1-\lambda^2} \right)^l. \quad (45)$$

Proof of Proposition 2. Combining (16) and (38), we have, for each $t \in \{0, \dots, T-2\}$,

$$\hat{c}_t = \tilde{E}_t [\hat{c}_{t+1} + (1 - \psi_{t+1}^a) \lambda \Gamma_{t+2} R \hat{a}_{t+1}]. \quad (46)$$

Using (5), we have

$$\hat{c}_t = \tilde{E}_t [\hat{c}_{t+1}] + (1 - \psi_{t+1}^a) \lambda \Gamma_{t+2} R^2 \left(\tilde{E}_t [\hat{a}_t + \hat{y}_t] - \hat{c}_t \right).$$

Collecting terms, we have

$$\hat{c}_t = \frac{1}{1 + (1 - \psi_{t+1}^a) \lambda \Gamma_{t+2} R^2} \tilde{E}_t [\hat{c}_{t+1}] + \frac{(1 - \psi_{t+1}^a) \lambda \Gamma_{t+2} R^2}{1 + (1 - \psi_{t+1}^a) \lambda \Gamma_{t+2} R^2} \tilde{E}_t [\hat{a}_t + \hat{y}_t].$$

This proves (22) with

$$\begin{aligned} \omega_t &= \frac{1}{1 + (1 - \psi_{t+1}^a) \lambda \Gamma_{t+2} R^2} = \frac{1}{1 + \left(1 - (1 - \lambda) \frac{\Gamma_{t+2} R}{1 + \Gamma_{t+2} R} \right) \lambda \Gamma_{t+2} R^2} \\ &= \frac{1}{1 + \frac{\lambda \Gamma_{t+2} R^2}{1 + \Gamma_{t+2} R} + \frac{\lambda^2 \Gamma_{t+2}^2 R^3}{1 + \Gamma_{t+2} R}}. \end{aligned}$$

We then know $\omega_t \in (0, 1]$ and decreases in λ . This proves Proposition 2.

Proof of Corollary 4. From the proof of Corollaries 2 and 3, we know that, with $T \rightarrow +\infty$,

$$\hat{c}_t = \psi^y \left(\hat{y}_t + \sum_{l=1}^T (\delta R^{-1})^l E_t [\hat{y}_{t+l}] \right) + \psi^a \hat{a}_t, \quad (47)$$

where ψ^y and ψ^a are given by (18) and δ is given by (45).

Consider news about future income y_{t+k} ($k \geq 1$) that arrives at period t . Let me use \mathcal{C}_l^k and \mathcal{A}_l^k to denote the impose response of c_{t+l} and a_{t+l} with respect to the news about y_{t+k} .⁴⁴ From (2) and (47), we know

$$\mathcal{C}_0^k = (\delta R^{-1})^k \psi^y \quad \text{and} \quad \mathcal{A}_0^k = 0,$$

and, for $l \in \{1, \dots, T\}$,

$$\mathcal{A}_l^k = R(\mathcal{A}_{l-1}^k - \mathcal{C}_{l-1}^k) \quad \text{and} \quad \mathcal{C}_l^k = (\delta R^{-1})^{k-l} \psi^y + \psi^a \mathcal{A}_l^k.$$

Together and note the fact that, from (33) and (34), $\delta R^{-1} = \frac{\psi^y}{\Gamma} R^{-1} = \frac{1}{1+\Gamma R} = 1 - \psi^y$, we have, for $l \in \{0, \dots, T\}$,

$$\mathcal{C}_l^k = (1 - \psi^y)^{k-l} \psi^y \left(\frac{1 - R(1 - \psi^y) + \psi^a R^{l+1} (1 - \psi^a)^l (1 - \psi^y)^{l+1}}{1 - R(1 - \psi^a)(1 - \psi^y)} \right).$$

From this formula, one can prove that

$$\mathcal{C}_k^k > \mathcal{C}_{k-1}^k > \dots > \mathcal{C}_0^k,$$

as long as $R(1 - \psi^y) < 1$. The latter inequality is true as long as $\lambda > 0$ based on Corollary 2. This proves Corollary 4.

Proof of Corollary 5. From the proof of Corollaries 2 and 3, we know that, with $T \rightarrow +\infty$,

$$\hat{c}_t = \psi^y \left(\hat{y}_t + \sum_{l=1}^{+\infty} (\delta R^{-1})^l E_t [\hat{y}_{t+l}] \right) + \psi^a \hat{a}_t,$$

where ψ^y and ψ^a are given by (18) and δ is given by (45). With an AR(1) process: $y_{t+1} = \rho y_t + \epsilon_{t+1}$,

$$\hat{c}_t = \frac{\psi^y}{1 - \rho \delta R^{-1}} \hat{y}_t + \psi^a \hat{a}_t = \psi^y_\rho \hat{y}_t + \psi^a \hat{a}_t,$$

⁴⁴Specifically, consider the case that $y_{t+k} = y_{t+k}^P + y_{t+k}^U$, where y_{t+k}^P is realized at t and y_{t+k}^U is realized at $t+k$ and is unpredictable prior to that point. The first component, y_{t+k}^P , can then be interpreted as the news about future income that arrives at period t . \mathcal{C}_l^k and \mathcal{A}_l^k here then denote the impose response of c_{t+l} and a_{t+l} with respect to y_{t+k}^P .

with $\psi_\rho^y = \frac{\psi^y}{1 - \rho\delta R^{-1}}$. Further note that, from (33), (34), and (39)

$$\delta = \frac{\Gamma^y}{\Gamma} = \frac{\psi^y}{\Gamma} = \frac{R}{1 + \Gamma R} = R(1 - \psi^y).$$

As a result,

$$\psi_\rho^y = \frac{\psi^y}{1 - \rho(1 - \psi^y)}.$$

This proves the Corollary.

Proof of Proposition 3. Here, I explore an alternative way to model misperception of the current state, through noisy signals (Sims, 2003). In each period $t \in \{0, \dots, T - 1\}$, I use a noisy signal about wealth, $s_t = a_t + \epsilon_t$, to summarize each self t 's knowledge of the current wealth, where the noise ϵ_t has p.d.f. $\phi_t^\epsilon(\epsilon_t)$ and is independent from a_t and all incomes and other noises in the economy.

Three things are worth noting. First, each self $t \in \{0, \dots, T - 1\}$ here understands that her signal is noisy and tries to infer her actual state based on the signal. Second, I assume that s_t summarizes self t 's knowledge about her wealth, so self t will not try to recall past states and decisions to infer a_t . The Markov property of the problem is maintained. Third, each self understands that her future selves' knowledge about their wealth is also summarized by noisy signals.

Similar to the main analysis, I let each self have perfect knowledge about her income state \vec{y}_t . The principle of optimality in Proposition 1 can be written as: for $t \in \{0, \dots, T - 1\}$, each self t 's consumption $\mathbf{c}_t(s_t, \vec{y}_t)$ and perceived value $W_t(s_t, \vec{y}_t)$ are, respectively, the optimal policy and the maximized value of the following problem:

$$\max_{c_t} u(c_t) + \beta E_t \left[\int V_{t+1}(R(a_t + y_t - c_t), \vec{y}_{t+1}) \phi(\vec{y}_{t+1} | \vec{y}_t) d\vec{y}_{t+1} | (s_t, \vec{y}_t) \right].$$

The objective value $V_t(a_t, \vec{y}_t)$ satisfies

$$V_t(a_t, \vec{y}_t) = \int \left\{ \mathbf{c}_t(a_t + \epsilon_t, \vec{y}_t) + \beta \int V_{t+1}(R(a_t + y_t - \mathbf{c}_t(a_t + \epsilon_t, \vec{y}_t)), \vec{y}_{t+1}) \phi(\vec{y}_{t+1} | \vec{y}_t) d\vec{y}_{t+1} \right\} \phi_t^\epsilon(\epsilon_t) dt.$$

Now, I will prove the main result Theorem 1 remains to be true, with Normally distributed fundamentals and noises. Specially, I assume ϵ_t , y_t and $E[y_{t+1} | \vec{y}_t]$ are all normally distributed. As consumption rules are linear (as below), each endogenous a_t for $t \in \{1, \dots, T\}$ is also normally distributed. As a result, we have $\tilde{E}_t[a_t] = E_t[a_t | s_t] = (1 - \lambda_t)s_t + \lambda_t a_t^*$, where $\lambda_t = \frac{\text{Var}(\epsilon_t)}{\text{Var}(a_t) + \text{Var}(\epsilon_t)} \in$

$(0, 1)$ and a_t^* denotes self t 's prior about a_t , i.e. the average wealth at the period t (over realizations of the stochastic incomes). Similar to the main text, using a hat over a variable still denotes its deviation from its average value (over realizations of the stochastic incomes),⁴⁵ we have

$$\tilde{E}_t [\hat{a}_t] = E_t [\hat{a}_t \mid \hat{s}_t] = (1 - \lambda_t) \hat{s}_t = (1 - \lambda_t) (\hat{a}_t + \epsilon_t).$$

I now modify the proof of Theorem 1 as follows. For $t \in \{0, \dots, T-1\}$, I conjecture each self t 's consumption can be characterized by

$$\hat{c}_t = \psi_t^y \hat{x}_t + \psi_t^a (\hat{a}_t + \epsilon_t),$$

where \hat{x}_t is still defined as in (13) and ψ_t^y , ψ_t^a and $\delta_{t,l}$ share the same value as those in the time-varying λ_t version of Theorem 1 in Corollary 1. Similarly, the conjectured $\left(\frac{\partial \hat{V}_t}{\partial a_t}\right)$ is the same as (29), with the same Γ_t in the time-varying λ_t version of Theorem 1 in Corollary 1.

We still use backward induction to prove the conjectures are true. Specifically, the optimal consumption rule in (41) remains to be true, except \hat{a}_t is replaced with $\hat{a}_t + \epsilon_t$. The expression for $\frac{\partial \hat{V}_t}{\partial a_t}$ in (42) remains to be true. The formula for ψ_t^y , ψ_t^a , $\delta_{t,l}$ and Γ_t are then exactly the same as those in Corollary 1. the time-varying λ_t version of Theorem 1 then follows.

Proof of Proposition 4. We first solve the individual problem (for a finite T). The final period utility is still given by $v(a_T + y_T)$. As a result,

$$\frac{\partial \hat{V}_T}{\partial a_T} = u'' (\Gamma_T \hat{a}_T + \Gamma_T^y \hat{y}_T),$$

where $\Gamma_T = \Gamma_T^y = \frac{v''}{u''} > 0$. For $t \in \{0, \dots, T-1\}$, we conjecture:

$$\hat{c}_{i,t} = \psi_t^a \hat{a}_{i,t} + \psi_t^y \left(\hat{y}_{i,t} + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} \hat{y}_{i,t+l} \right) - \psi_t^R \left(\hat{R}_t + \sum_{l=1}^{T-t-1} \delta_{t,l}^R R^{-l} \hat{R}_{t+l} \right) \quad (48)$$

$$\left(\frac{\partial \hat{V}_t}{\partial a_{i,t}} \right) = u'' (c^*) \left(\Gamma_t \hat{a}_t + \Gamma_t^y \left(\hat{y}_{i,t} + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} \hat{y}_{i,t+l} \right) - \Gamma_t^R \left(\hat{R}_t + \sum_{l=1}^{T-t-1} \delta_{t,l}^R R^{-l} \hat{R}_{t+l} \right) \right), \quad (49)$$

where a hat over a variable denotes its deviation from the deterministic steady state in which all interest rates are fixed at $R_t = R \equiv \beta^{-1}$. Note that as all aggregate uncertainty is resolved at

⁴⁵Similar to the main text, with the quadratic utility and linear consumption rules here, the average value of each variable here coincides with the value of the variable in the frictionless case when the stochastic incomes are fixed at their respective averages.

period 0 here, we have $\tilde{E}_{i,t}[R_{t+l}] = R_{t+l}$ and $\tilde{E}_{i,t}[y_{i,t+l}] = y_{i,t+l}$.

Similar to the proof of Theorem 1, we use backward induction to prove the conjectures are true. Suppose (48) and (49) are true from $t + 1$ onward. For $t \in \{0, \dots, T - 1\}$, note that the optimality condition in (16) becomes

$$u'(c_{i,t}) = \tilde{E}_{i,t} \left[\frac{R_t \frac{\partial V_{t+1}}{R} (a_{i,t+1}, \bar{y}_{i,t+1}, \vec{R}_{t+1})}{\partial a_{i,t+1}} \right].$$

Taking a linear approximation, we have

$$u''(c^*) \hat{c}_{i,t} = -\sigma \frac{c^*}{R} \hat{R}_t + \tilde{E}_{i,t} \left[\frac{\partial \hat{V}_{t+1}}{\partial a_{i,t+1}} \right], \quad (50)$$

where $\sigma = -\frac{u'(c^*)}{c^* u''(c^*)}$ is the elasticity of intertemporal substitution and $c^* = y^*$ is the steady state aggregate spending when all interest rates are fixed at $R_t = R \equiv \beta^{-1}$. Together with (49), one can prove the conjecture in (48) is true, with (33), (34), (35),

$$\psi_t^R = \sigma \frac{c^*}{R} (1 - \psi_t^y) \quad \forall t \in \{0, \dots, T - 1\}, \quad (51)$$

$$\delta_{t,1}^R = \frac{\Gamma_{t+1}^R}{1 + \Gamma_{t+1}^R R} / (\psi_t^R R^{-1}) = \frac{R \Gamma_{t+1}^R}{\sigma \frac{c^*}{R}} \quad \forall t \in \{0, \dots, T - 2\}, \quad (52)$$

$$\delta_{t,l}^R = \frac{\psi_{t+1}^R R^{-(l-1)} \delta_{t+1,l-1}^R}{1 + \Gamma_{t+1}^R R} / (\psi_t^R R^{-l}) = \frac{R \Gamma_{t+1}^R}{\sigma \frac{c^*}{R}} \delta_{t+1,l-1}^R \quad \forall l \in \{2, \dots, T - t - 1\}. \quad (53)$$

Now, $t \in \{0, \dots, T - 1\}$, the equation about $\frac{\partial V_t}{\partial a_t}$ in (37) become (after a linear approximation),

$$\left(\frac{\partial \hat{V}_t}{\partial a_{i,t}} \right) = \psi_t^a u''(c^*) \hat{c}_{i,t} + (1 - \psi_t^a) \left(-u''(c^*) \sigma \frac{c^*}{R} \hat{R}_t + E_t \left[\left(\frac{\partial \hat{V}_{t+1}}{\partial a_{i,t+1}} \right) \right] \right).$$

Together with the conjecture (49) for $t + 1$ and (50), we still have

$$\left(\frac{\partial \hat{V}_t}{\partial a_{i,t}} \right) = u''(c^*) (\hat{c}_{i,t} + (1 - \psi_t^a) \lambda \Gamma_{t+1}^R \hat{a}_{i,t}).$$

Therefore, the conjecture (49) is true at t , with (39) and

$$\Gamma_t^R = \psi_t^R \quad \forall t \in \{0, \dots, T - 1\}. \quad (54)$$

In sum, ψ_t^a , ψ_t^y , Γ_t and $\delta_{t,l}$ in (48) and (49) are the same as those in Theorem 1. ψ_t^R , $\delta_{t,l}^R$, and Γ_t^R are then determined by (51), (52), (53) and (54).

We now take $T \rightarrow +\infty$ limit. From the proof of Corollaries 2 and 3, (51), (52), and (53), we have

$$\psi_t^y = \Gamma_t^y \rightarrow \psi^y, \quad \psi_t^a \rightarrow \psi^a, \quad \psi_t^R = \Gamma_t^R \rightarrow \psi^R, \quad \delta_{t,l} \rightarrow \delta^l, \quad \delta_{t,l}^R \rightarrow \delta^l,$$

where $\psi^R = \sigma \frac{c^*}{R} (1 - \psi^y)$ and I use the fact $\delta = \frac{\Gamma^y}{\Gamma} = \frac{\psi^y}{\Gamma} = \frac{R}{1+\Gamma R} = R(1 - \psi^y)$. Together, we have

$$\hat{c}_{i,t} = \psi^a \hat{a}_{i,t} - \psi^R \left(\hat{R}_t + \sum_{l=1}^{+\infty} \delta^l R^{-l} \hat{R}_{t+l} \right) + \psi^y \left(\hat{y}_{i,t} + \sum_{l=1}^{+\infty} \delta^l R^{-l} \hat{y}_{i,t+l} \right).$$

Aggregating over i , we have (26). The three parts of Proposition 4 then follow directly from the formula of ψ^y , ψ^R and δ in the proof of Corollaries 2 and 3.

Proof of Proposition 5. In this variant, for all t and l , $\tilde{E}_{i,t} [\hat{R}_{t+l}] = \hat{R}_{t+l}$ and $\tilde{E}_{i,t} [\hat{y}_{t+l}] = (1 - \mu) \hat{y}_{t+l}$. The later equation is true because when each consumer i forms expectation about other consumers' spending $\hat{c}_{j,t+l}$, she thinks that only a $1 - \mu$ of consumers perceive the interest rate changes and adjusts their spending. Moreover, consumer i thinks those who do change their spending change their spending $\hat{c}_{j,t+l}$ to the aggregate level $\hat{c}_{t+l} = \hat{y}_{t+l}$. This is true because of the fact that each consumer knows the interest rate path but believes only $1 - \mu$ of consumers perfectly perceive the interest rate path is common knowledge (footnote 41): consumer i knows she will change her spending to the level \hat{c}_{t+l} , and she expects those who do change spending form the same expectation as she does and adjust their spending also to \hat{c}_{t+l} .⁴⁶

(26) becomes

$$\hat{c}_t = -\psi^R \left(\hat{R}_t + \sum_{l=1}^{+\infty} \delta^l R^{-l} \hat{R}_{t+l} \right) + (1 - \mu) \psi^y \left(\hat{y}_t + \sum_{l=1}^{+\infty} \delta^l R^{-l} \hat{y}_{t+l} \right).$$

Imposing $\hat{c}_t = \hat{y}_t$ for all t , we can write the above equation recursively

$$\hat{y}_t = -\frac{\psi^R}{1 - (1 - \mu) \psi^y} \hat{R}_t + \frac{1 - \psi^y}{1 - (1 - \mu) \psi^y} \hat{y}_{t+1},$$

⁴⁶See Angeletos and Sastry (2018) for a similar approach.

where I use the fact that $\delta R^{-1} = \frac{\psi^y R^{-1}}{\Gamma} = \frac{1}{1+\Gamma R} = 1 - \psi^y$. Using $\psi^R = \sigma \frac{y^*}{R} (1 - \psi^y)$, we then have

$$\epsilon_l \equiv \frac{d(\hat{y}_t/y^*)}{d(\hat{R}_{t+l}/R)} = -\sigma \left(\frac{1 - \psi^y}{1 - (1 - \mu)\psi^y} \right)^l.$$

When $\mu > 0$, we have $\frac{1 - \psi^y}{1 - (1 - \mu)\psi^y} < 1$ and decrease in ψ^y . The two parts of the Proposition 5 then follows.

Appendix B: Additional Results

A mental account interpretation.

One way to interpret the main analysis in Section 3 is that the consumer has two separate mental accounts: one current income account which she perfectly knows, and one wealth account of which she has imperfect perception. The current self's separate mental accounts can then explain the smaller MPC out of liquid wealth than out of current income. Her knowledge about her future selves' separate mental accounts then leads to excess sensitivity to current income and excess discounting of future income. Note that in the main analysis the income account only includes the current-period income. As the current-period income is incorporated into the next-period wealth, it goes to the wealth account in which the consumer has imperfect perception of. Here, I consider an extension in which some of the current-period income can stay in the income account of the next period in which the next self perfectly knows. The main results in Theorem 1 remain to be true.

Specifically, consider the environment in Section 3 and allows the news about future income. The consumer has imperfect perception of the “wealth account,” which has the law of motion

$$a_{t+1} = R(a_t - c_t) + R(1 - \mu^b) b_t.$$

On the other hand, she perfectly perceives the “income account,” which has the law of motion

$$b_{t+1} = y_{t+1} + R\mu^b b_t.$$

In other words, the income account here not only includes the current income, but also partially includes the balance in the income account in the previous period, and $\mu^b \in [0, 1)$ captures such

dependence. The main analysis can be summarized as $\mu^b = 0$. Here, I want to prove that the main theorem in Theorem 1 remains to be true as long as $\mu^b < R^{-1}$.⁴⁷

Specifically, I let each self at period t has perfect knowledge about the income account balance b_t and the income state \vec{y}_t . For the wealth account, for $t \in \{0, \dots, T-1\}$, similar to the main analysis, the perception function takes the form of $\mathbf{a}_t^P(a_t) = (1 - \lambda)a_t + \lambda a_t^D$, where $\lambda \in [0, 1)$ and a_t^D is average wealth account balance for self t in the intrapersonal equilibrium. The last period utility is given by $v(a_T + b_T)$.

I will now show that, for $t \in \{0, \dots, T-1\}$, the consumption function in this case still takes the form of

$$\hat{c}_t = \phi_t^y \left(\hat{b}_t + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} E_t [\hat{y}_{t+l}] \right) + \phi_t^a \hat{a}_t, \quad (55)$$

where a hat over a variable still denotes its value in the frictionless case when the stochastic incomes are fixed at their respective averages. Same as the main analysis, with the quadratic utility and linear consumption rules here, this value is equal to the average value of the variable intrapersonal equilibrium (over realizations of the stochastic incomes).

To prove it, similar to (7), I define $V_t(a_t, b_t, \vec{y}_t)$ as the objective value at each period t . I then complement (55) with conjectures of $\left(\frac{\partial \hat{V}_t}{\partial a_t}\right)$ and $\left(\frac{\partial \hat{V}_t}{\partial b_t}\right)$. For $t \in \{0, \dots, T-1\}$,

$$\left(\frac{\partial \hat{V}_t}{\partial a_t}\right) = u'' \cdot \left(\Gamma_t \hat{a}_t + \Gamma_t^y \left(\hat{b}_t + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} E_t [\hat{y}_{t+l}] \right) \right), \quad (56)$$

$$\left(\frac{\partial \hat{V}_t}{\partial b_t}\right) = u'' \cdot \left(\Gamma_t^y \hat{a}_t + \Gamma_t^y \left(\hat{b}_t + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} E_t [\hat{y}_{t+l}] \right) \right). \quad (57)$$

The above expressions can also be extended to the last period T with utility $v(a_T + b_T)$:

$$\frac{\partial \hat{V}_T}{\partial a_T} = u'' \left(\Gamma_T \hat{a}_T + \Gamma_T^y \hat{b}_T \right) \quad \text{and} \quad \frac{\partial \hat{V}_T}{\partial b_T} = u'' \left(\Gamma_T^y \hat{a}_T + \Gamma_T^y \hat{b}_T \right),$$

where $\Gamma_T = \Gamma_T^y = 1$.

Similar to the proof of Theorem 1, I still prove that conjectures are right by backward induction. where $\Gamma_T = \Gamma_T^y = \frac{v''}{u''} > 0$. Assume the above conjectures are true from $t+1$ onward. For

⁴⁷Note that when $\mu^b > R^{-1}$, the balance of the income account becomes explosive.

$t \in \{0, \dots, T-1\}$, the optimal consumption in (31) becomes

$$\hat{c}_t = \Gamma_{t+1} R \left((1-\lambda) \hat{a}_t + R(1-\mu^b) \hat{b}_t - \hat{c}_t \right) + \Gamma_{t+1}^y R \mu^b \hat{b}_t + E_t \left[\Gamma_{t+1}^y \left(\hat{y}_{t+1} + \sum_{l=1}^{T-t-1} \delta_{t+1,l} R^{-l} \hat{y}_{t+1+l} \right) \right].$$

We can then prove the conjecture (55) is true at t , with

$$\psi_t^y = \frac{[(1-\mu^b) \Gamma_{t+1} + \mu^b \Gamma_{t+1}^y] R}{1 + \Gamma_{t+1} R} \quad \text{and} \quad \psi_t^a = \frac{(1-\lambda) \Gamma_{t+1} R}{1 + \Gamma_{t+1} R}, \quad (58)$$

and

$$\delta_{t,1} = \frac{\Gamma_{t+1}^y}{1 + \Gamma_{t+1} R} / (\psi_t^y R^{-1}) = \frac{\Gamma_{t+1}^y}{(1-\mu^b) \Gamma_{t+1} + \mu^b \psi_{t+1}^y}, \quad (59)$$

and

$$\delta_{t,l} = \frac{\Gamma_{t+1}^y R^{-(l-1)} \delta_{t+1,l-1}}{1 + \Gamma_{t+1} R} / (\psi_t^y R^{-l}) = \frac{\Gamma_{t+1}^y \delta_{t+1,l-1}}{(1-\mu^b) \Gamma_{t+1} + \mu^b \psi_{t+1}^y} \quad \forall l \in \{2, \dots, T-t\}. \quad (60)$$

Now, similar to (37) and (38), we have, $t \in \{0, \dots, T-1\}$,

$$\begin{aligned} \frac{\partial V_t(a_t, b_t, \vec{y}_t)}{\partial a_t} &= \psi_t^a u'(c_t) + (1-\psi_t^a) E_t \left[\frac{\partial V_{t+1}(a_{t+1}, b_{t+1}, \vec{y}_{t+1})}{\partial a_{t+1}} \right] \\ \left(\frac{\partial \hat{V}_t}{\partial a_t} \right) &= u'' \cdot (\hat{c}_t + (1-\psi_t^a) \lambda \Gamma_{t+1} R \hat{a}_t), \end{aligned}$$

and

$$\begin{aligned} \frac{\partial V_t(a_t, b_t, \vec{y}_t)}{\partial b_t} &= \psi_t^y u'(c_t) + (1-\mu^b - \psi_t^y) E_t \left[\frac{\partial V_{t+1}(a_{t+1}, b_{t+1}, \vec{y}_{t+1})}{\partial a_{t+1}} \right] + \mu^b E_t \left[\frac{\partial V_{t+1}(a_{t+1}, b_{t+1}, \vec{y}_{t+1})}{\partial b_{t+1}} \right] \\ \left(\frac{\partial \hat{V}_t}{\partial b_t} \right) &= \hat{c}_t + (1-\psi_t^a) \lambda \Gamma_{t+1} R \hat{a}_t - \mu^b (\Gamma_{t+1} - \Gamma_{t+1}^y) R (1-\psi_t^a) \hat{a}_t. \end{aligned}$$

Together, it means the conjectures in (56) and (57) are true with

$$\Gamma_t = \psi_t^a + (1-\psi_t^a) \lambda \Gamma_{t+1} R = \frac{(R\Gamma_{t+1})^2}{1 + R\Gamma_{t+1}} \lambda^2 + \frac{R\Gamma_{t+1}}{1 + R\Gamma_{t+1}} \quad \text{and} \quad \Gamma_t^y = \psi_t^y,$$

where I use the fact that

$$\psi_t^a + (1-\psi_t^a) \lambda \Gamma_{t+1} R - \mu^b (\Gamma_{t+1} - \Gamma_{t+1}^y) R (1-\psi_t^a) = \phi_t^y = \Gamma_t^y$$

based on (58). We can see that Γ_t here is the same as that in Theorem 1.

For simplicity, consider the $T \rightarrow \infty$ limit (for $\lambda < R^{-1/2}$), we have

$$\begin{aligned}\Gamma_t &\rightarrow \Gamma = \frac{R-1}{R(1-R\lambda^2)} \\ \psi_t^y &\rightarrow \psi^y = \frac{(1-\mu^b)\Gamma R}{(1-\mu^b R + \Gamma R)} \\ \delta_{t,l} &\rightarrow \delta^l = \left(\frac{\psi^y}{(1-\mu^b)\Gamma + \mu^b \psi^y} \right)^l.\end{aligned}$$

One can then see easily, similar to Theorem 1, as long as $\mu^b \in [0, R^{-1})$, Γ and ψ^y increase with λ and δ decreases with λ .

The role of sophistication.

Here, I follow O’Donoghue and Rabin (2001) and introduce “partial sophistication.” I will show that, as long as the consumer is not completely naive about her future misperception ($\lambda' < 1$), the results about excess sensitivity to current income and excess discounting of future income remain to be true.

Specifically, each self t ’s misperception of her current wealth is still captured by (11), with a degree of friction λ ; however, she now thinks that each of her future self $t+l$, for $l \in \{1, \dots, T-1-t\}$, has a degree of friction $\lambda' \in [0, 1]$, which may differ from λ . Moreover, self t thinks that each of her future self $t+l$ shares with her belief about the behavior of other selves further in the future. This is the “dynamically consistent” belief requirement in O’Donoghue and Rabin (2001). Together, it means that, from self t ’s perspective, she thinks the value of her saving in the next period is given by the objective value function in (7), $V_{t+1}^{\lambda'}(a_{t+1}, \vec{y}_{t+1})$, with a degree of friction λ' . Self t ’s optimal consumption is then given by

$$u'(c_t) = \tilde{E}_t \left[\frac{\partial V_{t+1}^{\lambda'}}{\partial a_{t+1}}(a_{t+1}, \vec{y}_{t+1}) \right].$$

As a result, self t ’s consumption function is the same as the one in the time-varying λ s case (with full sophistication) studied in Corollary 1, with $\lambda_t = \lambda$ and $\lambda_{t+l} = \lambda'$ for $l \in \{1, \dots, T-t-1\}$. We then have, for each $t \in \{0, \dots, T-1\}$, self t ’s consumption in the partial sophistication case

is characterized by

$$\hat{c}_t = \psi_t^y \left(\hat{y}_t + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} E_t [\hat{y}_{t+l}] \right) + \psi_t^a \hat{a}_t,$$

where ψ_t^y , ψ_t^a and $\delta_{t,l}$ are given in the proof of Corollary 1. We then have:

Corollary 6. *Consider a variant in which I decouple each self's misperception of wealth $\lambda \in [0, 1]$ from her concern about her future selves' misperception of wealth $\lambda' \in [0, 1]$. For each $t \in \{0, \dots, T-2\}$, self t 's MPC ψ_t^y strictly increases with her concern about her future self's misperception of wealth λ' . On the other hand, ψ_t^y is independent of her own misperception of wealth λ . Moreover, for $l \in \{1, \dots, T-t\}$, the extra $\delta_{t,l} \leq 1$ is independent of λ but decreases with λ' . Finally, ψ_t^y and $\delta_{t,l}$ approach their frictionless counterparts when $\lambda' = 1$.*

In other words, as long as the consumer is not completely naive about her future misperception ($\lambda' < 1$), the results about excess sensitivity to current income and excess discounting of future income remain to be true.

The case with $\beta R \neq 1$.

With $\beta R \neq 1$, Theorem 1 remains to be true, with slightly different formula for ψ_t^y , ψ_t^a and $\delta_{t,l}$.

Specifically, we change the proof of Theorem 1 as follows. We still use a hat over a variable denotes its value in the frictionless case when the stochastic incomes are fixed at their respective averages. Note that, with the quadratic utility and linear consumption rules here, this value is equal to the average value of the variable intrapersonal equilibrium (over realizations of the stochastic incomes).

For $t \in \{0, \dots, T-1\}$, I still conjecture the consumption takes the form of (28):

$$\hat{c}_t = \psi_t^a \hat{a}_t + \psi_t^y \left(\hat{y}_t + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} E_t [\hat{y}_{t+l}] \right).$$

And $\left(\frac{\partial \hat{V}_t}{\partial a_t} \right)$ takes the form of (29):

$$\left(\frac{\partial \hat{V}_t}{\partial a_t} \right) = u'' \cdot \left(\Gamma_t \hat{a}_t + \Gamma_t^y \left(\hat{y}_t + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} E_t [\hat{y}_{t+l}] \right) \right).$$

We can still prove the conjectures are true by backward induction from the last period utility

$V(a_T, \vec{y}_T) = v(a_T + y_T)$. Specifically, suppose the above conjectures are true from $t + 1$ onwards. For $t \in \{0, \dots, T - 1\}$, the first order condition in (31) becomes

$$\hat{c}_t = \beta R \left\{ \Gamma_{t+1} R (\hat{a}_t^p + \hat{y}_t - \hat{c}_t) + E_t \left[\Gamma_{t+1}^y \left(\hat{y}_{t+1} + \sum_{l=1}^{T-t-1} \delta_{t+1,l} R^{-l} \hat{y}_{t+1+l} \right) \right] \right\}.$$

Collecting terms, we know the conjecture about \hat{c}_t is true at t , with

$$\psi_t^y = \frac{\beta R^2 \Gamma_{t+1}}{1 + \beta R^2 \Gamma_{t+1}} \quad \text{and} \quad \psi_t^a = (1 - \lambda) \psi_t^y, \quad (61)$$

and

$$\delta_{t,1} = \frac{\beta R \Gamma_{t+1}^y}{1 + \beta R^2 \Gamma_{t+1}} / (\psi_t^y R^{-1}) = \frac{\Gamma_{t+1}^y}{\Gamma_{t+1}}, \quad (62)$$

and

$$\delta_{t,l} = \frac{\beta R \Gamma_{t+1}^y R^{-(l-1)} \delta_{t+1,l-1}}{1 + \beta R^2 \Gamma_{t+1}} / (\psi_t^y R^{-l}) = \frac{\Gamma_{t+1}^y \delta_{t+1,l-1}}{\Gamma_{t+1}} \quad \forall l \in \{2, \dots, T - t\}. \quad (63)$$

Condition (37) becomes

$$\left(\frac{\partial \hat{V}_t}{\partial a_t} \right) = u'' \cdot \psi_t^a \hat{c}_t + \beta R (1 - \psi_t^a) E_t \left[\left(\frac{\partial \hat{V}_{t+1}}{\partial a_{t+1}} \right) \right],$$

and condition (38) becomes

$$\left(\frac{\partial \hat{V}_t}{\partial a_t} \right) = u'' \cdot (\hat{c}_t + (1 - \psi_t^a) \lambda \Gamma_{t+1} \beta R^2 \hat{a}_t).$$

As a result, the conjecture about $\left(\frac{\partial \hat{V}_t}{\partial a_t} \right)$ is true, with

$$\Gamma_t = \psi_t^a + (1 - \psi_t^a) \lambda \Gamma_{t+1} \beta R^2 = \beta R \frac{(R \Gamma_{t+1})^2}{1 + R \Gamma_{t+1}} \lambda^2 + \frac{R \Gamma_{t+1}}{1 + R \Gamma_{t+1}} \quad \text{and} \quad \Gamma_t^y = \psi_t^y. \quad (64)$$

From (61) - (64), we can easily see that the three parts of Theorem 1 remain to be true with $\beta R \neq 1$.

Alternative default wealth level.

In the main text, I let the default wealth a_t^D be the value of a_t in the frictionless case when the stochastic income at each period is fixed at their respective averages. With the quadratic utility

and linear consumption rules here, such a value of the default wealth a_t^D is equal to average wealth level for self t in the intrapersonal equilibrium (over realizations of the stochastic incomes). Here, I allow the default wealth level a_t^D for each self $t \in \{0, \dots, T-1\}$ be any exogenous functions of the income process, and show that the three main results in Theorem 1 remain to be true.

I still use a hat over a variable denotes its deviation from the value of the variable in the frictionless case when the stochastic incomes are fixed at their respective averages.⁴⁸ I also define $\hat{a}_t^D \equiv a_t^D - a_t^*$, that is, the deviation of the default wealth level from the wealth at that period in the frictionless case when the stochastic incomes are fixed at their respective averages.

I change the proof of Theorem 1 as follows. For $t \in \{0, \dots, T-1\}$, I conjecture the consumption takes the form:

$$\hat{c}_t = \psi_t^a \hat{a}_t + \psi_t^y \left(\hat{y}_t + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} E_t [\hat{y}_{t+l}] \right) + \sum_{l=0}^{T-t-1} \psi_{t,l}^{\bar{a}} \hat{a}_{t+l}^d.$$

And $\left(\frac{\partial \hat{V}_t}{\partial a_t} \right)$ takes the form:

$$\left(\frac{\partial \hat{V}_t}{\partial a_t} \right) = u'' \cdot \left(\Gamma_t \hat{a}_t + \Gamma_t^y \left(\hat{y}_t + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} E_t [\hat{y}_{t+l}] \right) + \sum_{l=0}^{T-t-1} \Gamma_{t,l}^{\bar{a}} \hat{a}_{t+l}^d \right).$$

We can still prove the above conjectures are true by backward induction from the last period utility $V(a_T, \vec{y}_T) = v(a_T + y_T)$. Specifically, suppose the above conjectures are true from $t+1$ onwards. For t , the first order condition in (31) becomes

$$\begin{aligned} \hat{c}_t &= \Gamma_{t+1} R \left((1-\lambda) \hat{a}_t + \lambda \hat{a}_t^D + \hat{y}_t - \hat{c}_t \right) + E_t \left[\Gamma_{t+1}^y \left(\hat{y}_{t+1} + \sum_{l=1}^{T-t-1} \delta_{t+1,l} R^{-l} \hat{y}_{t+1+l} \right) \right] \\ &+ \sum_{l=0}^{T-t-2} \Gamma_{t+1,l}^{\bar{a}} \hat{a}_{t+1+l}^d. \end{aligned}$$

As a result, the formula for ψ_t^y , ψ_t^a and $\delta_{t,l}$ in (33) - (35) remain to be true. Moreover, we have

$$\psi_{t,0}^{\bar{a}} = (1-\lambda) \psi_t^y \quad \text{and} \quad \psi_{t,l}^{\bar{a}} = \frac{\Gamma_{t+1,l-1}^{\bar{a}}}{1 + \Gamma_{t+1} R} \quad \forall l \in \{1, \dots, T-t-1\}.$$

⁴⁸Note that here with arbitrary defaults, the average consumption and wealth for each self may not coincide with this value.

About $\left(\frac{\hat{\partial V}_t}{\partial a_t}\right)$, condition (37) remains to be true. (38) becomes

$$\left(\frac{\hat{\partial V}_t}{\partial a_t}\right) = u'' \cdot (\hat{c}_t + (1 - \psi_t^a) \lambda \Gamma_{t+1} R (\hat{a}_t - \bar{a}_t)).$$

As a result, the formula for Γ_t and Γ_t^y in (39) remains to be true. Moreover, we have

$$\Gamma_{t,0}^{\bar{a}} = \psi_{t,0}^{\bar{a}} - (1 - \psi_t^a) \lambda \Gamma_{t+1} R \quad \text{and} \quad \Gamma_{t,l}^{\bar{a}} = \psi_{t,l}^{\bar{a}} \quad \forall l \in \{1, \dots, T - t - 1\}.$$

In sum, the formula for ψ_t^y , ψ_t^a , Γ_t and $\delta_{t,l}$ are the same as those in Theorem 1. As a result, the three main results in Theorem 1 remain to be true.

Allowing imperfect perception of income.

Now I allow the possibility of imperfect perception of income: $\vec{y}_t^p(\vec{y}_t) \neq \vec{y}_t$. Note that the modified principle of optimality in Proposition 1 still covers this case.

Here, I try to extend Theorem 1 to this case. Specifically, similar to Section 3, I let $u(\cdot)$ be a quadratic and concave function. For the perception functions, for $t \in \{0, \dots, T - 1\}$, I assume

$$\mathbf{a}_t^p(a_t) = (1 - \lambda) a_t + \lambda a_t^D \quad \text{and} \quad \vec{y}_t^p(\vec{y}_t) = (1 - \lambda^y) \vec{y}_t + \lambda^y \vec{y}_t^D,$$

where $\lambda, \lambda^y \in [0, 1]$ capture the imperfect perception of wealth and income, respectively. Also similar to Section 3, I let the default a_t^D and \vec{y}_t^D be the corresponding unconditional averages (over realizations of the stochastic incomes). The last self T can perfectly perceive her wealth and income and $c_T = a_T + y_T$. Finally, I also assume expected future income is a linear function of the current income state. That is, $E_t[y_{t+l}|\vec{y}_t]$ is linear in \vec{y}_t .

Let me still use a hat over a variable denotes its deviation from its unconditional average, which coincides with the value of the variable in the frictionless case when the stochastic incomes are fixed at their respective averages. Note that perceived current income is $\tilde{E}_t[\hat{y}_t] = \hat{y}_t^p = (1 - \lambda^y) \hat{y}_t$, and the perceived future income $\tilde{E}_t[\hat{y}_{t+l}] = E_t[\hat{y}_{t+l}|\vec{y}_t^p] = (1 - \lambda^y) E_t[\hat{y}_{t+l}|\vec{y}_t]$.

I will now show that, for $t \in \{0, \dots, T - 1\}$, the consumption function in this case takes the form of

$$\hat{c}_t = \psi_t^a \hat{a}_t + \psi_t^y (1 - \lambda^y) \left(\hat{y}_t + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} E_t[\hat{y}_{t+l}] \right), \quad (65)$$

where ψ_t^a still captures MPC out of liquid wealth and ψ_t^y now captures the MPC out of *perceived*

current income. Moreover ψ_t^a and ψ_t^y have the exact same expression as in Theorem 1, independent from λ^y . As a result, the excess sensitivity to current income in Theorem 1 becomes excess sensitivity to perceived current income. Moreover, even though the formula for $\delta_{t,l}$ will be a bit different from that in Theorem 1, $\delta_{t,l}$ still decreases with λ and we still have $\delta_{t,l} < 1$ if $\lambda^y < \lambda$. That is, imperfect perception of wealth still leads to excess discounting of perceived future income. However, $\delta_{t,l}$ increases with λ^y . That is, concern about future selves' imperfect perception of *income* may lead to less excess discounting: if future selves will not perceptively perceive future incomes, the current self may want to consume more out of news about future income.

To prove the conjecture in (65), let me complement it with the conjecture of $\left(\frac{\partial \hat{V}_t}{\partial a_t}\right)$:

$$\left(\frac{\partial \hat{V}_t}{\partial a_t}\right) = u'' \cdot \left(\Gamma_t \hat{a}_t + \Gamma_t^y \left(\hat{y}_t + \sum_{l=1}^{T-t} \delta_{t,l} R^{-l} E_t [\hat{y}_{t+l}] \right) \right). \quad (66)$$

I still prove that conjectures are right by backward induction. At T , we have $V(a_T, \vec{y}_T) = v(a_T + y_T)$. Assume the above conjectures are true for $t+1, \dots, T$. For $t \in \{0, \dots, T-1\}$, the optimal consumption in (31) becomes

$$\hat{c}_t = \Gamma_{t+1} R ((1-\lambda) \hat{a}_t + (1-\lambda^y) \hat{y}_t - \hat{c}_t) + (1-\lambda^y) E_t \left[\Gamma_{t+1}^y \left(\hat{y}_{t+1} + \sum_{l=1}^{T-t-1} \delta_{t+1,l} R^{-l} \hat{y}_{t+l+1} \right) \right]. \quad (67)$$

We can then prove the conjecture (65) is true at t , with

$$\psi_t^y = \frac{\Gamma_{t+1} R}{1 + \Gamma_{t+1} R} \quad \text{and} \quad \psi_t^a = \frac{(1-\lambda) \Gamma_{t+1} R}{1 + \Gamma_{t+1} R}, \quad (68)$$

and

$$\delta_{t,1} = \frac{\Gamma_{t+1}^y}{1 + \Gamma_{t+1} R} / (\psi_t^y R^{-1}) = \frac{\Gamma_{t+1}^y}{\Gamma_{t+1}}, \quad (69)$$

and

$$\delta_{t,l} = \frac{\Gamma_{t+1}^y R^{-(l-1)} \delta_{t+1,l-1}}{1 + \Gamma_{t+1} R} / (\psi_t^y R^{-l}) = \frac{\Gamma_{t+1}^y \delta_{t+1,l-1}}{\Gamma_{t+1}} \quad \forall l \in \{2, \dots, T-t\}. \quad (70)$$

Now, from (10) and (67), we have

$$\begin{aligned}
\left(\frac{\partial \hat{V}_t}{\partial a_t}\right) &= u'' \cdot \psi_t^a \hat{c}_t + (1 - \psi_t^a) E_t \left[\left(\frac{\partial \hat{V}_{t+1}}{\partial a_{t+1}} \right) \right] \\
&= u'' \cdot \left(\psi_t^a \hat{c}_t + (1 - \psi_t^a) \left[\Gamma_{t+1} R (\hat{a}_t + \hat{y}_t - \hat{c}_t) + \Gamma_{t+1}^y \left(\hat{y}_{t+1} + \sum_{l=1}^{T-t-1} \delta_{t+1,l} R^{-l} E_{t+1} [\hat{y}_{t+1+l}] \right) \right] \right) \\
&= u'' \cdot \left(\hat{c}_t + (1 - \psi_t^a) \left(\lambda \Gamma_{t+1} R \hat{a}_t + \lambda^y \left(\Gamma_{t+1} R \hat{y}_t + \Gamma_{t+1}^y \left(\hat{y}_{t+1} + \sum_{l=1}^{T-t-1} \delta_{t+1,l} R^{-l} E_{t+1} [\hat{y}_{t+1+l}] \right) \right) \right) \right).
\end{aligned}$$

The conjecture (66) is then true with

$$\Gamma_t = \psi_t^a + (1 - \psi_t^a) \lambda \Gamma_{t+1} R = \frac{(R \Gamma_{t+1})^2}{1 + R \Gamma_{t+1}} \lambda^2 + \frac{R \Gamma_{t+1}}{1 + R \Gamma_{t+1}}. \quad (71)$$

and

$$\Gamma_t^y = (1 - \lambda^y) \psi_t^y + (1 - \psi_t^a) \lambda^y \Gamma_{t+1} R = \frac{(R \Gamma_{t+1})^2}{1 + R \Gamma_{t+1}} \lambda \lambda^y + \frac{R \Gamma_{t+1}}{1 + R \Gamma_{t+1}}. \quad (72)$$

From (68) to (72), we first notice that the formula for ψ_t^y, ψ_t^a and Γ_t remain the same as those in Theorem 1. We then notice that, for all $t \in \{0, \dots, T-1\}$,

$$\frac{\Gamma_t^y}{\Gamma_t} = \frac{1 + \lambda^y \lambda \Gamma_{t+1} R}{1 + \lambda^2 \Gamma_{t+1} R} = 1 - \frac{(\lambda - \lambda^y)}{\lambda} \frac{\lambda^2 \Gamma_{t+1} R}{1 + \lambda^2 \Gamma_{t+1} R}.$$

As a result, $\frac{\Gamma_t^y}{\Gamma_t} < 1$ when $\lambda > \lambda^y$. Moreover, $\frac{\Gamma_t^y}{\Gamma_t}$ decreases in λ and increases in λ^y . We then have $\delta_{t,l} < 1$ when $\lambda > \lambda^y$, decreases in λ and increases in λ^y , for all $t \in \{0, \dots, T-1\}$ and $l \in \{0, \dots, T-t\}$.

The case with $\lambda < 0$.

Recall that $\mathbf{a}_t^p(a_t) = (1 - \lambda) a_t + \lambda a_t^D$. The case with $\lambda < 0$ means each self t 's perceived wealth moves more than one to one with respect to actual wealth. In this case, the main consumption formula (12) and (13) in Theorem 1 and the formula for $\psi_t^y, \delta_{t,l}$ and Γ_t in (33), (34), (35) and (39) remain to be the same. From these formulas, we know that ψ_t^y increases with λ^2 and $\delta_{t,l}$ decreases with λ^2 . As a result, a larger friction (which leads to a larger λ^2) still leads to excess sensitivity to current income and excess discounting of future income.

Comparison with inattention about the return on wealth.

Here, I study a variant of Alvarez, Guiso and Lippi (2012) in which the consumer cannot perfectly perceive the the movement of the return R_t . Such a consumer, however, has perfect recall: she remembers her past consumption (and income). I will show that such a consumer's MPC out of current income is the same as its frictionless counterpart.

Specifically, consider the environment in Section 2, but with time-varying interest rate. That is, the budget constraint in (2) becomes $a_{t+1} = R_t (a_t + y_t - c_t)$, where each R_t is a random variable with mean $R = 1/\beta$. The consumer has perfect knowledge about her current income state \vec{y}_t , and has perfect recall: she remembers her past consumption (and income). However, she has imperfect perception of the movement in R_t . In fact, for simplicity, I assume the consumer has no knowledge about shocks in R_t beyond the prior R . That is, she understands that there is uncertainty about R_t , but is inattentive to it. Because such a consumer has perfect recall, the law of iterated expectations holds for her subjective expectation $\tilde{E}_t[\cdot]$.

In this case, the consumer's perceived wealth $a_t^p \equiv \tilde{E}_t[a_t^p]$ evolves according to

$$a_{t+1}^p = R (a_t^p + y_t - c_t) \quad \forall t \in \{0, \dots, T-2\}. \quad (73)$$

As the main analysis, I still let $a_T^p = a_T$ and for simplicity I let $a_0^p = a_0$.

Now let me define each self t 's subjective value in the current environment. As a_t^p is no longer a function of a_t , I add a_t^p as a state variable. Specifically, I use a_t^p , $g_t = a_t - a_t^p$ and \vec{y}_t as states (note that $\vec{y}_t^p = \vec{y}_t$), and define the objective value at each period t as

$$V_t(a_t^p, \vec{y}_t, g_t) \equiv u(\mathbf{c}_t(a_t^p, \vec{y}_t)) + E_t \left[\sum_{k=1}^{T-t-1} \beta^k u(\mathbf{c}_{t+k}(a_{t+k}^p, \vec{y}_{t+k})) + \beta^{T-t} v(a_T, \vec{y}_T) \right],$$

subject to the budget constraint

$$a_{t+1} = a_{t+1}^p + g_{t+1} = R_t (a_t + y_t - c_t) = R_t (a_t^p + g_t + y_t - c_t) \quad (74)$$

and (73).

The principle of optimality in Proposition 1 becomes: $\mathbf{c}_t(a_t^p, \vec{y}_t)$ and perceived value $W_t(a_t^p, \vec{y}_t)$ are the optimal policy and the maximized value of the following problem

$$\max_{c_t} u(c_t) + \beta \tilde{E}_t [V_{t+1}(a_{t+1}^p, \vec{y}_{t+1}, g_{t+1})], \quad (75)$$

subject to (3) and (74). The objective value $V_t(a_t^p, \vec{y}_t, g_t)$ satisfied

$$V_t(a_t^p, \vec{y}_t, g_t) \equiv u(\mathbf{c}_t(a_t^p, \vec{y}_t)) + \beta \int V_{t+1}(R(a_t^p + y_t - \mathbf{c}_t(a_t^p, \vec{y}_t)), \vec{y}_{t+1}, g_{t+1}) \phi(\vec{y}_{t+1}|\vec{y}_t) d\vec{y}_{t+1}, \quad (76)$$

with $g_{t+1} = R_t(a_t^p + g_t + y_t - c_t) - R(a_t^p + y_t - c_t)$.

The consumer's first order condition of (75) is:

$$u'(\mathbf{c}_t(a_t^p, \vec{y}_t)) = \tilde{E}_t \left[\frac{\partial V_{t+1}(a_{t+1}^p, \vec{y}_{t+1}, g_{t+1})}{a_{t+1}^p} + \beta \frac{\partial V_{t+1}(a_{t+1}^p, \vec{y}_{t+1}, g_{t+1})}{g_{t+1}} (R_t - R) \right].$$

Taking a first order approximation and use the fact that $\tilde{E}_t[R_t - R] = 0$, we have

$$u'(\mathbf{c}_t(a_t^p, \vec{y}_t)) = \tilde{E}_t \left[\frac{\partial V_{t+1}(a_{t+1}^p, \vec{y}_{t+1}, g_{t+1})}{a_{t+1}^p} \right]. \quad (77)$$

Take of a partial derivative of V_t with respect to a_t^p in (76)

$$\begin{aligned} \frac{\partial V_t(a_t^p, \vec{y}_t, g_t)}{\partial a_t^p} &\equiv u'(\mathbf{c}_t(a_t^p, \vec{y}_t)) \frac{\partial \mathbf{c}_t(a_t^p, \vec{y}_t)}{\partial a_t^p} \\ &+ \left(1 - \frac{\partial \mathbf{c}_t(a_t^p, \vec{y}_t)}{\partial a_t^p} \right) \int \frac{\partial V_{t+1}(R(a_t^p + y_t - \mathbf{c}_t(a_t^p, \vec{y}_t)), \vec{y}_{t+1}, g_{t+1})}{\partial a_{t+1}^p} \phi(\vec{y}_{t+1}|\vec{y}_t) d\vec{y}_{t+1}. \end{aligned}$$

Applying $\tilde{E}_t[\cdot]$ for the above expression at $t + 1$, using (77) at $t + 1$ and using the law of iterated expectation for $\tilde{E}_t[\cdot]$, we have

$$u'(\mathbf{c}_t(a_t^p, \vec{y}_t)) = \tilde{E}_{t+1} [u'(\mathbf{c}_{t+1}(a_{t+1}^p, \vec{y}_{t+1}))]$$

and

$$\hat{c}_t = \tilde{E}_{t+1} [\hat{c}_{t+1}].$$

As a result, the standard Euler equation holds, and the MPC out of current income is the same as the frictionless benchmark.

The length of a period as the length of perfect recall.

Here, I work out a continuous-time version of the model. The consumer's utility is

$$U_0 = \int_{t=0}^{T \cdot T_p} e^{-\rho t} u(c_t) dt + v(a_{T \cdot T_p}),$$

where ρ is the discount rate and the utility function $u(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$ is strictly increasing and concave. Her budget constraint is given by $\dot{a}_t = ra_t + y_t - c_t$, with $r = \rho$ is the (net) interest rate and a_0 is given.

In this model, T_p captures the length of perfect recall. Specifically, I can divide the timeline into intervals such as $[0, T_p]$, $[T_p, 2 \cdot T_p]$, \dots , $[(T-1) \cdot T_p, T \cdot T_p]$. For simplicity, I let the income uncertainty in each interval be resolved at the start of that interval, which the consumer in that interval perfect knowledge about.

The consumer can perfectly recall her past income and consumption within each time interval, but have bounded recall about what happened before. Similar to the discrete-time model, her memory about previous intervals is summarized by her imperfect perception of the wealth at the start of her current interval. Specifically, during the interval $[(k-1) \cdot T_p, k \cdot T_p]$, the consumer's knowledge about what happened before is summarized by her imperfect knowledge about the wealth at the start of her current interval, $A_{k-1}^p = \mathbf{a}_{k-1}^p(a_{(k-1) \cdot T_p})$.

I now show this continuous-time version of the model is equivalent to the discrete-time model studied in the main analysis. One can then think each time-interval as a period in the discrete-time version and interpret the length of a period in the discrete-time model as the length of perfect recall.

Specifically, define

$$U(C) \equiv \max \int_{s=0}^{T_p} e^{-\rho s} u(c_s) ds \quad s.t. \quad C = \int_{s=0}^{s=T_p} e^{-rs} c_s ds,$$

$C_k \equiv \int_{s=0}^{s=T_p} e^{-rs} c_{k+s} ds$, $Y_k \equiv \int_{s=0}^{s=T_p} e^{-rs} y_{k+s} ds$ and $A_k \equiv a_{k \cdot T_p}$ for $k \in \{0, \dots, T\}$. The above continuous-time version of problem is then equivalent to the following discrete-time version with utility

$$U_0 = \sum_{k=0}^{T-1} e^{-\rho k} U(C_k) + v(A_T),$$

and budget $A_{k+1} = R(A_k + Y_k - C_k)$. In this discrete-time version, each self $k \in \{0, \dots, T-1\}$ decides on consumption C_k . She has perfect knowledge about income Y_k . Her perceived wealth is $A_k^p = \mathbf{a}_k^p(A_k)$. This problem is then exactly the same as the one set up in Section 2.

Endogeneize the degree of frictions λ .

Consider the environment in Section 2. In the main analysis, I treat the perception function as given. That is, I let $\mathbf{a}_t^p(a_t) = (1 - \lambda)a_t + \lambda a_t^D$, where the degree of imperfect perception of wealth

$\lambda \in [0, 1]$ is exogenous. Here, I try to endogenize such λ . Specifically, I add a stage 0, before all periods $t = \{0, \dots, T\}$. In that stage, the consumer endogenously chooses her degree of friction λ to maximize her expected utility function, subject to a perception cost:

$$\lambda^{\text{Endo}} = \arg \max_{\lambda} E[U_0] - C(1 - \lambda), \quad (78)$$

where the perception cost $C(\cdot)$ is increasing and convex, and $E[\cdot]$ averages over realizations of stochastic income. Note that a lower friction λ leads to a higher perception cost $C(1 - \lambda)$. I can establish the following results about the endogenously chosen λ^{Endo} .

Proposition 6. *Consider a stationary environment: $T \rightarrow +\infty$ and y_t is i.i.d with variance $\text{Var}(y)$. We have:*

1). *The degree of friction λ^{Endo} , and the MPC out of current income ψ^y , decreases with the concavity of utility function $|u''|$.*

2). *The degree of friction λ^{Endo} , and the MPC out of current income ψ^y , decreases with the variance of the income shock $\text{Var}(y)$.*

As in the main text, let me use superscript $*$ to denote the unconditional average of a variable (over realizations of stochastic incomes). To prove Proposition 1, note that from the proof of Theorem 1 and Corollaries 2 and 3:

$$E[U_0] = E[V(a_0, y)] = V(a_0^*, y_0^*) + \frac{1}{2}u'' \cdot \Gamma^y \text{Var}(y), \quad (79)$$

where $\Gamma = \frac{1}{u''} \frac{\partial^2 V}{\partial a^2} = \frac{1-R^{-1}}{1-R\lambda^2}$, $\Gamma^y \equiv \frac{1}{u''} \frac{\partial^2 V}{\partial y^2} = \psi^y = \frac{1-R^{-1}}{1-\lambda^2}$ and $a_0^* = a_0$ (the exogenous initial wealth).

Similarly, for all t ,

$$V(a_t^*, y_t^*) = u(c^*) + \beta E[V(a_{t+1}^*, y_{t+1}^*)] = u(c^*) + \beta V(a_{t+1}^*, y_{t+1}^*) + \frac{1}{2}\beta u'' \cdot \Gamma^y \text{Var}(y).$$

Using it recursively and together with (79), we have

$$E[U_0] = \frac{u(c^*)}{1-R^{-1}} + \frac{1}{2} \frac{u''}{1-R^{-1}} \cdot \Gamma^y \text{Var}(y).$$

We now turn to the problem in (78). Using the formula for Γ and Γ^y , we know the FOC of

(78) is

$$\frac{\lambda^{\text{Endo}}}{\left(1 - (\lambda^{\text{Endo}})^2\right)^2} |u''| \text{Var}(y) + \frac{\lambda^{\text{Endo}} (R - 1)}{\left(1 - R(\lambda^{\text{Endo}})^2\right)^2} |u''| \text{Var}(y) = C' (1 - \lambda^{\text{Endo}}).$$

As C is convex, Proposition 6 follows directly.

Durable consumption.

The main analysis focuses on non-durable consumption. Here, I introduce durable consumption following the classical model in Mankiw (1982). Specifically, consider the same one-asset environment in Section 2 with and $T \rightarrow \infty$. The only difference is that the consumer's utility depends on the consumption of durables

$$\sum_{t=0}^{+\infty} \beta^t U(k_t),$$

where k_t captures her durable consumption at period t . Her budget constraint is given by

$$a_{t+1} = (1 + r)(a_t + y_t - (k_t - (1 - \delta)k_{t-1})),$$

where δ is the depreciation rate of the durables. The case with non-durable consumption studied in the main text is nested by $\delta = 1$. When $\delta < 1$, the durable consumption stock becomes an additional state variable and the analysis becomes much more involved. Here, for a clean contrast with the non-durable consumption, I consider the case with no depreciation ($\delta = 0$) and independently distributed income.

First, consider the frictionless case without imperfect perception of wealth. Similar to the main text, let me use a hat over a variable to denote its deviation from its value in the frictionless case when the stochastic incomes are fixed at their respective averages. In this case, the optimal durable consumption in each period is given by:

$$\hat{K}_t = \hat{a}_t + \hat{y}_t + \hat{K}_{t-1}.$$

That is, the consumer's durable MPC out of current income is one. Importantly, this means that changes in current income will not result in changes in the next period wealth. As a result, in the frictional case with imperfect perception of wealth, concerns about future selves' imperfect perception of wealth will not change the consumer's durable MPC out of current income. It is still

one.

In sum, the current self's purchase of durable goods automatically raises her future selves' durable consumption, even if they do have perfect perception of wealth. The durable consumption effectively serves a commitment device for the consumer to smooth her consumption increases across periods.

Illiquid Asset.

Here, I study a two-asset variant with a liquid and an illiquid asset, and the consumer faces an adjustment cost in adjusting the illiquid asset balance. Because of the additional complexity driven by the new state variable, I consider a three-period version, with $t = 0, 1, 2$. Her utility is given by

$$U_0 = u(c_0) + \beta u(c_1) + \beta^2 u(c_2),$$

where the utility function $u(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$ is strictly increasing, concave and quadratic. At $t = 0$, the consumer has initial wealth a_0 and receives a stochastic income y_0 . She decides on her consumption c_0 and investment in the illiquid asset i_0 , and the rest of her money goes to the liquid account. When she invests in the illiquid asset, she is subject to an adjustment cost $\frac{i_0^2}{2(a_0 + y_0)}$. That is, if she wants to put too much portion of her cash on hand to the illiquid asset, she needs to pay a higher adjustment cost. The gross return on the illiquid asset is R^b per period, and the gross return on the liquid asset is $R = \frac{1}{\beta} < R^b$. For simplicity, the consumer will not receive new income in period 1 and 2. In period $t = 1$, the consumer can adjust her liquid asset balance to consume, but cannot get access to her illiquid asset balance. In the last period $t = 2$, the consumer consumes all her receipts from the illiquid and liquid assets. Let me use a_t and b_t to denote the balance of her liquid asset and illiquid asset at the beginning of each period. Let $a_0 = a_0^*$ be the exogenous initial asset, and $b_0 = 0$. The budget constraint can be written as

$$a_1 = R \left(a_0 + y_0 - c_0 - i_0 - \frac{i_0^2}{2(a_0 + y_0)} \right) \quad \text{and} \quad b_1 = R^b i_0$$

$$a_2 = R(a_1 - c_1) \quad \text{and} \quad b_2 = R^b b_1.$$

At the last period 2, we have

$$c_2 = a_2 + b_2.$$

At period 1, the consumer has imperfect perception of illiquid and liquid asset balance. Similar

to (11), we have

$$a_1^p = \mathbf{a}_1^p(a_1) = (1 - \lambda^a) a_1 + \lambda^a a_1^D \quad \text{and} \quad b_1^p = \mathbf{b}_1^p(b_1) = (1 - \lambda^b) b_1 + \lambda^b b_1^D,$$

where a_1^D and b_1^D are the default levels and a higher $\lambda^a \in [0, 1]$ and $\lambda^b \in [0, 1]$ means a higher degree of imperfect perception of the liquid and illiquid asset balances. Similar to the main analysis, I let the defaults a_1^D and b_1^D be the respective balance of a_1 and b_1 when the stochastic income y_0 is at its average y_0^* . Finally, similar to the main analysis, I let the consumer have perfect knowledge about the stochastic income y_0 at period 0 (and the exogenous and deterministic $a_0 = a_0^*$).

At period 1, optimal consumption decision leads to

$$c_1 = \frac{R}{R+1} \left(a_1^p + y_1 + \frac{R^b b_1^p}{R} \right).$$

Based on the budget constraints, we then have

$$c_2 = \frac{R}{R+1} y_1 + R \left(a_1 - \frac{R}{R+1} a_1^p \right) + R^b b_1 - \frac{R}{R+1} R^b b_1^p.$$

Similar to (7), let me use $V_1(a_1, b_1)$ to denote the *objective* value at period 1, based on the consumer's consumption rule and perception function above, we have

$$\begin{aligned} V_1(a_1, b_1) = & u \left(\frac{R}{R+1} \left(\mathbf{a}_1^p(a_1) + y_1 + \frac{R^b \mathbf{b}_1^p(b_1)}{R} \right) \right) \\ & + \beta u \left(\frac{R}{R+1} y_1 + R \left(a_1 - \frac{R}{R+1} \mathbf{a}_1^p(a_1) \right) + R^b b_1 - \frac{R}{R+1} R^b \mathbf{b}_1^p(b_1) \right). \end{aligned} \quad (80)$$

Let me now turn to period 0, the optimal consumption and investment lead to

$$\begin{aligned} u'(c_0) &= \beta R \frac{\partial V_1}{\partial a_1}(a_1, b_1) \\ u'(c_0) \left(1 + \frac{i_0}{a_0 + y_0} \right) &= \beta R^b \frac{\partial V_1}{\partial b_1}(a_1, b_1), \end{aligned} \quad (81)$$

and thus

$$\frac{i_0}{a_0 + y_0} = \frac{R^b}{R} \left(\frac{\partial V_1}{\partial b_1}(a_1, b_1) - \frac{R^b}{R} \right) + \left(\frac{R^b}{R} \right)^2 - 1. \quad (82)$$

Now we work with a linearization, and similar to Section 3, let me use a hat over a variable denotes its deviation from its value in the frictionless case when the stochastic income y_0 is at its average

level y_0^* .⁴⁹ From the value function in (80), we have

$$\begin{aligned}\frac{\partial \hat{V}_1}{\partial a_1} / u'' &= \left[\left(\frac{R(1-\lambda^a)}{R+1} \right)^2 + R \left(\frac{1+\lambda^a R}{R+1} \right)^2 \right] \hat{a}_1 \\ &+ \frac{R^b}{R} \left(\frac{R(1-\lambda^a)}{R+1} \frac{R(1-\lambda^b)}{R+1} + R \left(\frac{1+\lambda^a R}{R+1} \right) \left(\frac{1+\lambda^b R}{R+1} \right) \right) \hat{b}_1 \\ \frac{R}{R^b} \frac{\partial \hat{V}_1}{\partial b_1} / u'' &= \left[\frac{R(1-\lambda^a)}{R+1} \frac{R(1-\lambda^b)}{R+1} + R \left(\frac{1+\lambda^a R}{R+1} \right) \left(\frac{1+\lambda^b R}{R+1} \right) \right] \hat{a}_1 \\ &+ \frac{R^b}{R} \left(\left(\frac{R(1-\lambda^b)}{R+1} \right)^2 + R \left(\frac{1+\lambda^b R}{R+1} \right)^2 \right) \hat{b}_1.\end{aligned}$$

From the budget constraints, we have

$$\hat{a}_1 = R \left(\hat{a}_0 + \hat{y}_0 - \hat{c}_0 - \hat{i}_0 - \frac{i_0^*}{a_0^* + y_0^*} \hat{i}_0 + \frac{1}{2} \left(\frac{i_0^*}{a_0^* + y_0^*} \right)^2 (\hat{a}_0 + \hat{y}_0) \right) \quad \text{and} \quad \hat{b}_1 = R^b \hat{i}_0,$$

where a superscript * denotes the value of a variable when the stochastic income is at its average y_0^* . Optimal consumption (81) and optimal investment (82) lead to

$$\hat{c}_0 = \left[\left(\frac{R(1-\lambda^a)}{R+1} \right)^2 + R \left(\frac{1+\lambda^a R}{R+1} \right)^2 \right] \hat{a}_1 + \frac{R^b}{R} \left(\frac{R(1-\lambda^a)}{R+1} \frac{R(1-\lambda^b)}{R+1} + R \left(\frac{1+\lambda^a R}{R+1} \right) \left(\frac{1+\lambda^b R}{R+1} \right) \right) \hat{b}_1$$

and

$$\frac{\hat{i}_0}{a^* + y^*} - \frac{i^*}{(a^* + y^*)^2} (\hat{a}_0 + \hat{y}_0) = - \left(\frac{R^b}{R} \right)^2 \frac{|u''|}{u'(c^*)} \frac{R(\lambda^b - \lambda^a)}{R+1} (R\lambda_a \hat{a}_1 + R^b \lambda_b \hat{b}_1).$$

To solve the first period consumption and investment, we take a guess and verify approach. I guess that

$$\hat{c}_0 = \psi^y (\hat{a}_0 + \hat{y}_0) \quad \text{and} \quad \hat{i}_0 = \iota (\hat{a}_0 + \hat{y}_0).$$

From the above linearized consumption and investment decisions and the budget constraints, we

⁴⁹This value may deviate from the unconditional average of the variable here, because of the non-linearity of the adjustment cost.

have

$$\psi^y = \frac{\left(\frac{R(1-\lambda^a)}{R+1}\right)^2 + R\left(\frac{1+\lambda^a R}{R+1}\right)^2}{1 + R\left[\left(\frac{R(1-\lambda^a)}{R+1}\right)^2 + R\left(\frac{1+\lambda^a R}{R+1}\right)^2\right]} R \left(1 - \iota - \frac{i_0^*}{a_0^* + y_0^*} \iota + \frac{1}{2} \left(\frac{i_0^*}{a_0^* + y_0^*}\right)^2\right)$$

$$+ \frac{\left(\frac{R(1-\lambda^a)}{R+1} \frac{R(1-\lambda^b)}{R+1} + R\left(\frac{1+\lambda^a R}{R+1}\right) \left(\frac{1+\lambda^b R}{R+1}\right)\right) (R^b)^2}{1 + R\left[\left(\frac{R(1-\lambda^a)}{R+1}\right)^2 + R\left(\frac{1+\lambda^a R}{R+1}\right)^2\right]} \frac{(R^b)^2}{R} \iota$$

$$\frac{\iota - \frac{i_0^*}{a_0^* + y_0^*}}{a_0^* + y_0^*} = - \left(\frac{R^b}{R}\right)^2 \frac{|u''|}{u'(c^*)} \frac{R(\lambda^b - \lambda^a)}{R+1} \left\{ R\lambda_a R \left(1 - \psi^y - \iota - \frac{i_0^*}{a_0^* + y_0^*} \iota + \frac{1}{2} \left(\frac{i_0^*}{a_0^* + y_0^*}\right)^2\right) + (R^b)^2 \lambda_b \iota \right\}.$$

The above two equations then pin down the equilibrium ψ^y and ι . As the system is highly nonlinear, I seek help from Matlab. For whatever parameter value I try for R^b , R , $\frac{|u''|}{u'(c^*)}$, i_0^* , a_0^* , y_0^* , $\lambda^a \in [0, 1]$ and $\lambda^b \in [0, 1]$, I have the following two observations. First, the main result about excess sensitivity to current income remains to hold: when λ^a , $\lambda^b > 0$, ψ^y is larger than its frictionless counterpart. Second, as long as the consumer has more imperfect perception of the illiquid asset balance than of the liquid asset balance ($\lambda^b > \lambda^a$), she is less willing to adjust her illiquid wealth balance in response to incomes shocks compared to the frictionless counterpart. That is, ι is smaller than its frictionless counterpart.

Learning.

Here, I study how the main results change when the default wealth also depends on the previous selves' perceived wealth, effectively a form of learning. Consider the environment in Section 3, in which the utility is quadratic and each self t has perfect knowledge about the income state \vec{y}_t . In the perception function (11) used in the paper, each self's perceived wealth depends on her actual wealth and the default wealth a_t^d . The default wealth, $a_t = a_t^*$, is assumed to be the unconditional average wealth level at period t (over realizations of income), an exogenous function of the stochastic income process. For this approach, the default wealth at each t does not depend on previous selves' knowledge. One advantage of such an approach is that the perception function can capture the core friction (imperfect perception of wealth) without introducing any new state variable.

Here, I let the default wealth also depend on the previous selves' perceived wealth. For $t \in$

$\{0, \dots, T-1\}$,

$$a_t^d = \mu' R (a_{t-1}^p + y_{t-1} - c_{t-1}) + (1 - \mu') a_t^*, \quad (83)$$

where $R (a_{t-1}^p + y_{t-1} - c_{t-1})$ captures the previous self $t-1$'s perceived wealth a_t , a_t^* is still the unconditional average wealth level at period t (over realizations of income) and $\mu' \in [0, 1]$ captures the weight on the former. Now, $t \in \{0, \dots, T-1\}$, each self t 's perceived wealth is given by:

$$\begin{aligned} a_t^p &= (1 - \lambda) a_t + \lambda a_t^d, \\ &= (1 - \lambda) a_t + \mu R (a_{t-1}^p + y_{t-1} - c_{t-1}) + (\lambda - \mu) a_t^*, \end{aligned} \quad (84)$$

where $\lambda \in [0, 1]$ still captures the weight on the default wealth and $\mu = \lambda \mu' \in [0, \lambda]$ captures how much the current self's perceived wealth depends on the previous self's perception through (83).

Effectively, one can interpret (83) and (84) together capture a form of learning with bounded recall. a_t^d in (83) effectively captures the prior of self t when she tries to learn the actual a_t : μ' captures how much the prior depends on the the previous self $t-1$'s perceived wealth a_t , and $1 - \mu'$ captures how much the prior depends on the the unconditional average wealth level a_t^* . When $\mu' < 1$, it means self t partially forgets her previous self's knowledge and retraces back to the unconditional average wealth level a_t^* at period t to form the prior. The case studied in main text is nested by $\mu' = 0$.

Even for the exogenous learning rule in (84), the introduction of the perceived wealth as an additional state variable makes the analysis much more involved. One could alternatively study an “optimal” learning problem with filtering. However, the analysis of that case will depend on the stochastic process of income and becomes completely intractable. To make progress, let us focus on (84) and the case with independent income, with p.d.f. $\phi_t(y_t)$.

Specifically, let me define $g_t = a_t^p - a_t$. For $t \in \{0, \dots, T-2\}$, we have

$$\begin{aligned} g_{t+1} &= -\lambda R (a_t + y_t - c_t) + \mu R (a_t^p + y_t - c_t) + (\lambda - \mu) a_{t+1}^d \\ &= -(\lambda - \mu) (R (a_t + y_t - c_t) - a_{t+1}^d) + \mu R g_t. \end{aligned} \quad (85)$$

and note that $\tilde{E}_t [a_t] = a_t^p$ and $\tilde{E}_t [g_t] = 0$, we have each self t believe with certainty that

$$\tilde{E}_t [g_{t+1}] = -(\lambda - \mu) (R (a_t^p + y_t - c_t) - a_{t+1}^d).$$

Now, use $c_t(a_t^p, y_t)$ to capture self t 's optimal consumption rule. Define the objective value

$V_t(a_t, g_t, y_t)$ at each period t as in (7), given each self's misperception and consumption rules. The optimal consumption rule is then determined by

$$c_t(a_t^p, y_t) = \arg \max_{c_t} u(c_t) + \beta \tilde{E}_t \left[\int V_{t+1}(a_{t+1}, g_{t+1}, y_{t+1}) \right],$$

where the last term is equal to

$$\int V_{t+1}(R(a_t^p + y_t - c_t), -(\lambda - \mu)(R(a_t^p + y_t - c_t) - a_{t+1}^d), y_{t+1}) \phi(y_{t+1}) dy_{t+1}.$$

The FOC for the optimal consumption is given by

$$u'(c_t(a_t^p, z_t)) = \tilde{E}_t \left[\frac{\partial V_{t+1}(a_{t+1}, g_{t+1}, y_{t+1})}{\partial a_{t+1}} - (\lambda - \mu) \frac{\partial V_{t+1}(a_{t+1}, g_{t+1}, z_{t+1})}{\partial g_{t+1}} \right]. \quad (86)$$

Similar to Proposition 1, one can write the objective value recursively

$$V_t(a_t, g_t, y_t) = u(c_t(a_t^p, y_t)) + \beta E_t \left[\int V_{t+1}(a_{t+1}, g_{t+1}, y_{t+1}) \right],$$

subject to the budget constraint and (85). Taking partial derivative with respect to a_t , y_t and g_t , we have

$$\frac{\partial V_t(a_t, g_t, y_t)}{\partial y_t} = \frac{\partial V_t(a_t, g_t, y_t)}{\partial a_t} = u'(c_t(a_t + g_t, y_t)) \psi_t^y \quad (87)$$

$$+ (1 - \psi_t^y) E_t \left[\frac{\partial V_{t+1}(a_{t+1}, g_{t+1}, y_{t+1})}{\partial a_{t+1}} - (\lambda - \mu) \frac{\partial V_{t+1}(a_{t+1}, g_{t+1}, y_{t+1})}{\partial g_{t+1}} \right]$$

$$\frac{\partial V_t(a_t, g_t, y_t)}{\partial g_t} = u'(c_t(a_t + g_t, y_t)) \psi_t^y \quad (88)$$

$$+ E_t \left[(-\psi_t^y) \frac{\partial V_{t+1}(a_{t+1}, g_{t+1}, y_{t+1})}{\partial a_{t+1}} + [(\lambda - \mu) \psi_t^y + \mu] \frac{\partial V_{t+1}(a_{t+1}, g_{t+1}, y_{t+1})}{\partial g_{t+1}} \right],$$

where $\psi_t^y = \frac{\partial c_t}{\partial a_t^p} = \frac{\partial c_t}{\partial y_t}$. Note that, different from that the main analysis, $\frac{\partial V_t(a_t, g_t, y_t)}{\partial a_t}$ here is equal to $\frac{\partial V_t(a_t, g_t, y_t)}{\partial y_t}$. This is because the partial derivatives here hold the difference between a_t^p and a_t , g_t , fixed.

As u is quadratic, $V_t(a_t, g_t, y_t)$ is quadratic and $c_t(a_t^p, y_t)$, $\frac{\partial V_t(a_t, g_t, y_t)}{\partial a_t}$ and $\frac{\partial V_t(a_t, g_t, y_t)}{\partial g_t}$ are linear. Using a hat over a variable denotes its deviation from its average value (over realizations of stochastic incomes), we conjecture

$$\hat{c}_t = \psi_t^y (\hat{a}_t + \hat{g}_t + \hat{y}_t)$$

and

$$\begin{aligned}\left(\frac{\partial \hat{V}_t}{\partial a_t}\right) &= \left(\frac{\partial \hat{V}_t}{\partial y_t}\right) = u'' \cdot \{\Gamma_t (\hat{a}_t + \hat{y}_t) + \Gamma_t^{ag} \hat{g}_t\} \\ \left(\frac{\partial \hat{V}_t}{\partial g_t}\right) &= u'' \cdot \{\Gamma_t^{ag} (\hat{a}_t + \hat{y}_t) + \Gamma_t^g \hat{g}_t\},\end{aligned}$$

where $\Gamma_t \equiv \frac{\partial^2 V_t}{\partial a_t^2}$, $\Gamma_t^{ag} \equiv \frac{\partial^2 V_t}{\partial a_t \partial g_t}$ and $\Gamma_t^g = \frac{\partial^2 V_t}{\partial g_t^2}$.

From the optimal consumption in (86), we have

$$\begin{aligned}\hat{c}_t &= \Gamma_{t+1} R (\hat{a}_t^p + \hat{y}_t - \hat{c}_t) - \Gamma_{t+1}^{ag} (\lambda - \mu) R (\hat{a}_t^p + \hat{y}_t - \hat{c}_t) \\ &\quad - (\lambda - \mu) (\Gamma_{t+1}^{ag} R (\hat{a}_t^p + \hat{y}_t - \hat{c}_t) - \Gamma_{t+1}^g (\lambda - \mu) R (\hat{a}_t^p + \hat{y}_t - \hat{c}_t)),\end{aligned}\tag{89}$$

and thus

$$\psi_t^y = \frac{R [\Gamma_{t+1} - 2\Gamma_{t+1}^{ag} (\lambda - \mu) + \Gamma_{t+1}^g (\lambda - \mu)^2]}{1 + R [\Gamma_{t+1} - 2\Gamma_{t+1}^{ag} (\lambda - \mu) + \Gamma_{t+1}^g (\lambda - \mu)^2]}.$$

Together with (87), we have

$$\left(\frac{\partial \hat{V}_t}{\partial a_t}\right) = \hat{c}_t + (1 - \psi_t^y) (-\Gamma_{t+1} + 2\Gamma_{t+1}^{ag} (\lambda - \mu) - \Gamma_{t+1}^g (\lambda - \mu)^2 + (\Gamma_{t+1}^{ag} - (\lambda - \mu) \Gamma_{t+1}^g) \mu) R \hat{g}_t.$$

As a result, we have

$$\Gamma_t = \psi_t^y$$

and

$$\begin{aligned}\Gamma_t^{ag} &= \psi_t^y + (1 - \psi_t^y) (-\Gamma_{t+1} + 2\Gamma_{t+1}^{ag} (\lambda - \mu) - \Gamma_{t+1}^g (\lambda - \mu)^2 + (\Gamma_{t+1}^{ag} - (\lambda - \mu) \Gamma_{t+1}^g) \mu) R \\ &= (1 - \psi_t^y) (\Gamma_{ag,t+1} - (\lambda - \mu) \Gamma_{g,t+1}) \mu R.\end{aligned}$$

(89) together with (88), we have

$$\begin{aligned}\left(\frac{\partial \hat{V}_t}{\partial g_t}\right) &= -\psi_t^y (-\Gamma_{t+1} + 2\Gamma_{t+1}^{ag} (\lambda - \mu) - \Gamma_{t+1}^g (\lambda - \mu)^2 + (\Gamma_{t+1}^{ag} - (\lambda - \mu) \Gamma_{t+1}^g) \mu) R \hat{g}_t \\ &\quad + \mu (\Gamma_{t+1}^{ag} R (\hat{a}_t + \hat{y}_t - \hat{c}_t) + \Gamma_{t+1}^g (-(\lambda - \mu) R (\hat{a}_t + \hat{y}_t - \hat{c}_t) + \mu R \hat{g}_t)),\end{aligned}$$

and thus

$$\begin{aligned}
\Gamma_t^g &= -\psi_t^y \left(-\Gamma_{t+1} + 2\Gamma_{t+1}^{ag} (\lambda - \mu) - \Gamma_{t+1}^g (\lambda - \mu)^2 + (\Gamma_{t+1}^{ag} - (\lambda - \mu) \Gamma_{t+1}^g) \mu \right) R \\
&\quad + \mu \left(\Gamma_{t+1}^{ag} R (-\psi_t^y) + \Gamma_{t+1}^g (- (\lambda - \mu) R (-\psi_t^y) + \mu R) \right) \\
&= \psi_t^y \frac{\psi_t^y}{1 - \psi_t^y} - 2 \frac{\psi_t^y}{1 - \psi_t^y} \Gamma_{t+1}^{ag} + \Gamma_{t+1}^g \mu^2 R.
\end{aligned}$$

Consider a $T \rightarrow +\infty$ stationary limit. We have

$$\begin{aligned}
\psi^y = \Gamma &= \frac{R [\Gamma - 2\Gamma^{ag} (\lambda - \mu) + \Gamma^g (\lambda - \mu)^2]}{1 + R [\Gamma - 2\Gamma^{ag} (\lambda - \mu) + \Gamma^g (\lambda - \mu)^2]} \\
\Gamma^{ag} &= (1 - \psi^y) (\Gamma_{ag} - (\lambda - \mu) \Gamma_g) \mu R \\
\Gamma^g &= \psi^y \frac{\psi^y}{1 - \psi^y} - 2\psi^y \Gamma^{ag} + \Gamma^g \mu^2 R.
\end{aligned}$$

One can solve $\psi^y, \Gamma, \Gamma^{ag}$ and Γ^g as functions of parameters λ, μ and R . As the system is highly nonlinear, I seek help from Matlab. For whatever parameter value I try, I have the following observations. First, as long as $\mu' < 1$ (so $\lambda > \mu$), that is, the weight of the default wealth on the unconditional average wealth is non-zero, the main result on how the friction leads to excess sensitivity to current income remains to be true. Second, holding λ constant, the dependence on the previous self's perceived wealth μ (or equivalently μ') has two countervailing impacts on the MPC out of current income. First, the dependence on past perception (a larger μ) provides each self with additional knowledge about her current wealth. As a result, each self perceives and responds to wealth changes better, and the MPC out of current income is lower. On the other hand, the dependence on past perception (a larger μ) also makes any perception error persistent, which leads to persistent mistakes in response to wealth changes. This channel pushes the MPC out of current income higher.

Holding constant λ and increasing μ , it seems that the second channel dominates initially (MPC increases initially), but the first channel dominates eventually (MPC decreases eventually). The following figure plots the MPC out of current income ψ^y against μ when $\lambda = 0.7$ and $R = 1/0.98$ and illustrates this observation.

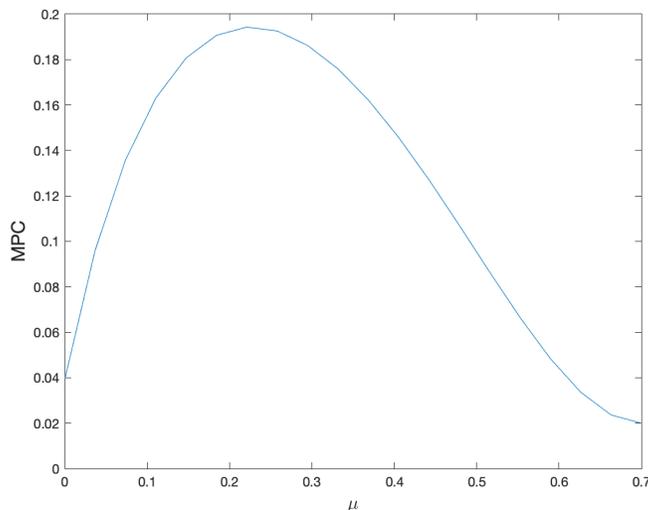


Figure 3: Learning

Imperfect Perception of Wealth Interacts with the Borrowing Constraint.

New Result I: a flattened MPC-wealth curve. In the environment in Section 5, I study how imperfect perception of wealth interacts with the borrowing constraint. Without imperfect perception of wealth ($\lambda = 0$), the consumer's MPC out of temporary current income shocks decreases with wealth (Carroll and Kimball, 1996). When the consumer's wealth is low, her consumption is mostly driven by liquidity concerns and precautionary motives. As a result, her consumption is quite sensitive to liquidity and she exhibits high MPC (out of the temporary current income shocks). When the consumer's wealth is high, her consumption instead proxies the prediction of the permanent income hypothesis, and she exhibits a low MPC. Such a downward sloping MPC-wealth curve (when $\lambda = 0$) is corroborated in Figure 4, which plots how the MPC out of temporary current income shocks of a consumer with average income moves with respect to her wealth.

With imperfect perception of wealth, however, the consumer has a flattened MPC-wealth curve (Figure 4 shows both the main calibration $\lambda = 6/7$, and the case with a milder friction $\lambda = 1/2$).⁵⁰ When the consumer's wealth is low, her MPC is still mostly driven by liquidity concerns and precautionary motives. However, as long as the consumer's borrowing constraint does not bind exactly (and her wealth level is below the default wealth), her perceived wealth will be larger than her actual wealth. As a result, the consumer thinks she is further away from the borrowing

⁵⁰Note that Figure 4 excludes the case that the consumer is exactly at her borrowing constraint $a_t = 0$. In that case, the MPC out of temporary current income is 1 for all λ s.

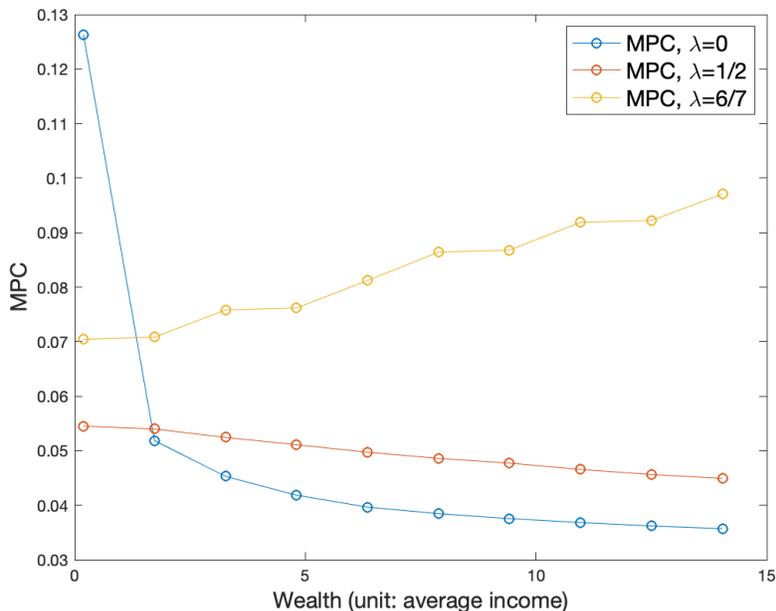


Figure 4: Excess sensitivity and Excess Discounting

constraint than she actually is. She then displays a lower MPC at a low wealth level than that without imperfect perception of wealth. On the other hand, at a high wealth level, her MPC is mostly driven by imperfect perception of wealth studied in this paper. She then exhibits a higher MPC at a high wealth level than her counterpart without imperfect perception of wealth. Together, the consumer with imperfect perception of wealth has a flattened MPC-wealth curve. In fact, from Figure 4, we know that the MPC-wealth curve may become upward sloping when the cognitive friction is large enough.⁵¹

There is also empirical support for such a flattened MPC-wealth curve. Fagereng, Holm and Natvik (2019) find that the differences between the MPC (out of the temporary income shocks) of the rich consumer and that of the poor consumer is much closer than its theoretical counterpart in benchmark models. In their estimates, the MPC of the group with lowest liquidity is only 34% larger than the MPC of group with highest liquidity, far smaller than the frictionless counterpart. Kueng (2018) even finds some evidence that the MPC increases with liquid assets (and also income). Furthermore, as mentioned at Section 4, if I let consumers endogenously choose the degree of frictions λ , a richer household may endogenously choose a higher friction λ , and exhibits a higher

⁵¹I suspect such an upward sloping MPC-wealth curve can be explained as follows. Note that the excess sensitivity to current income comes from the fact that the marginal value of wealth decreases faster with wealth (Lemma 1). Here, with CRRA utility, at a very high wealth level, the marginal utility of wealth may not have much room to decrease further. As a result, the impact of the friction on the MPC may not be as large at a very high wealth level.

Table 2: Proportion of Constrained Agents

Frictionless ($\lambda = 1$)	Benchmark ($\lambda = 6/7$)	Mild frictions ($\lambda = 1/2$)	Alternative \underline{b}_t
0	39%	26%	38%

MPC out of current income. This mechanism contributes to another reason for a flattened, or even increasing, MPC-wealth curve.

Finally, it is worth noting that the literature (e.g. Carroll, 2001) often studies the MPC as a function of the one-dimensional state variable, the ratio between liquid wealth and the permanent income. However, the model here is non-homothetic, and such a ratio is no longer a one-dimensional state variable. This is because the default wealth does not move one by one with the realization of the permanent income.

New Result II: a higher probability to hit the borrowing constraints. Imperfect perception of wealth can increase the probability that a consumer hits her borrowing constraint. The intuition is similar to the one above: at a low wealth level, as the consumer’s perceived wealth is larger than her actual wealth, she over-consumes. As a result, she is more likely to hit the borrowing constraint. In other words, the consumer with imperfect perception of wealth may fall into a “poverty trap.” Table 2 studies a consumer whose initial wealth is at the default level and whose income is subject to the risks in (25). Table 2 shows the probability of such a consumer hitting her borrowing constraint after 30 periods, In my calibration, without imperfect perception of wealth ($\lambda = 0$), the consumer’s borrowing constraint never binds because of the relatively high $\beta = 0.967$. However, with imperfect perception of wealth ($\lambda = 6/7$), she hits her borrowing constraint with a 39% probability. With a milder friction ($\lambda = 1/2$), she still hits her borrowing constraint with a 26% probability. The result is also not sensitive to the borrowing constraint $\underline{b}_{t+1} = 0$. In fact, I also consider the case with positive borrowing: $\underline{b}_t = \underline{b} = 0.5E[y_s]$ for all t , where $E[y_s]$ is the average income. In this case, with imperfect perception of wealth ($\lambda = 6/7$), she hits her borrowing constraint with a 38% probability.

In fact, it is well known that one shortcoming of the traditional one-asset model of income fluctuations is that it does not generate enough liquidity-constrained households (Kaplan and Violante, 2014). Imperfect perception of wealth then provides a way to boost the amount of liquidity-constrained households without the complications of adding an illiquid asset.

A potential follow-up project is to study the implication of imperfect perception of wealth on wealth distribution. For such a purpose, note that the above analysis about how imperfect perception of wealth interacts with the borrowing constraint treats the degree of friction λ as a

constant, independent from the actual wealth level. Such an approach is consistent with the rest of the paper to minimize the degree of freedom. On the other hand, the consumer may want to pay more attention to her wealth level if she is closer to the borrowing constraint. This feature might be important in matching the empirical wealth distribution.

The Impulse Responses of Consumption to News about Future Income in the Calibration Exercise.

Consider the environment in Section 5 and introduce news about temporary future income shocks. Here I try to match the empirical evidence that the consumption response when the consumer actually *receives* the income is larger than the consumption response when the news about future income arrives (for high-income, high-liquidity households, see Stephens and Unayama, 2011, Kueng, 2018). As mentioned in main text, there are several conceptual challenges to quantitatively match the model predictions with the empirical evidence. First, it is often unclear when the news about future income arrives exactly. Second, it is unclear how many consumers actually pay attention to the news (note that in the model, to isolate the friction of interest, I eliminate imperfect perception about the income process.) Using Kueng (2018) as an example, who studies consumption responses to the Alaska Permanent Fund payment. The news about the payment size is announced one month before the payment, but there are ample media predictions about the payment size throughout the entire year before the payment. Moreover, even though there is evidence that some households are paying attention to the news based on Google search results, it is unclear the portion of households choose to do so.

With these caveats in mind, one can consider the following exercise in the model to proxy the setting in Kueng (2018). Consider a piece of news about future temporary income shocks that arrives one period beforehand. Without imperfect perception of wealth ($\lambda = 0$), the canonical model fails to generate the excess sensitivity to anticipated income shocks when the income arrives. For high-liquidity households, the consumption response to a one-dollar payment is 0.0366 dollars per year in the model versus 0.341 dollars per year in Kueng (2018). Now, with imperfect perception of wealth ($\lambda = 6/7$) but perfect perception about the income news (as all analysis above), the model can generate the excess sensitivity to anticipated income shocks when the income arrives (for high-liquidity households, 0.1429 dollars per year in the model). However, with perfect perception about the income news, the model also generates counterfactual strong consumption responses when the *news* arrives (for high-liquidity households, 0.1343 dollars per year). This is because the

excess discounting of future income in Part (ii) of Theorem 1 and Figure 1 here is not particularly strong when there is only one period between the announcement and the arrival of the income. If I further introduce imperfect perception about the income news (e.g. $\lambda^y = 3/4$), the consumption response when the news arrives drops significantly to 0.0336 dollars per year. On the other hand, the consumption response when the income arrives remains to be high (0.1451 dollars per year).

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