Barcelona GSE Summer Forum
Balmes Building (UPF) – Balmes 132, Barcelona

STATISTICS, JUMP PROCESSES AND MALLIAVIN CALCULUS:
RECENT APPLICATIONS
June 25-27, 2014

JUNE 25

Session 1
10:00-11:00  PETER TANKOV (Université Paris 7)
“Asymptotic methods for portfolio risk management”
11:00-11:30  Coffee-break*

Session 2
11:30-13:30  ARTURO KOHATSU-HIGA (Ritsumeikan University)
“Stochastic differential equations with irregular coefficients”
MARTA SANZ-SOLÉ (Universitat de Barcelona)
“Support theorem for a stochastic wave equation in dimension three”
13:30-14:30  Lunch*

Session 3
14:30-16:00  DAVID NUALART (University of Kansas)
“Convergence of densities for random variables on a finite Wiener chaos”
ROLANDO D. NAVARRO, JR. (Purdue University)
“Mean-variance hedging with partial information using the Clark-Ocone representation with the change of measure for Lévy process”
16:00-16:30  Coffee-break*
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“Optimal portfolio problems with price dynamics driven by time-changed Lévy noises”  
ELISA ALÒS (Universitat Pompeu Fabra and Barcelona GSE)  
“On the closed-form approximation of short-time random strike options” |
| 11:00-11:30 | Coffee-break*                                                                                                                                   |
| 11:30-13:30 | Session 2                                                                                                                                   |
| 11:30-13:30 | ANDRÉ SUESS (Universitat de Barcelona)  
“Integration theory for infinite dimensional volatility modulated Volterra processes”  
JOSEP VIVES (Universitat de Barcelona)  
“A Hull and White formula for a stochastic volatility Lévy model with infinite activity” |
| 13:30-14:30 | Lunch*                                                                                                                                       |

**Organizers:** Eulàlia Nualart (Universitat Pompeu Fabra and Barcelona GSE)

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* Meals are provided by the organization
Abstracts:

PETER TANKOV (Université Paris 7)
‘Asymptotic methods for portfolio risk management’

We shall present new findings regarding the tail behavior of the sum of dependent positive random variables. This problem has received considerable attention in the literature, but mainly in the insurance context, where the random variables represent losses from individual claims, and one is interested in the right tail asymptotics of their sum. By contrast, we focus on the finance context, where the random variables represent the prices of individual assets, and to estimate the probability of a very large loss, one needs to focus on the left tail asymptotics of their sum. Owing to the positivity of the variables, these asymptotics turn out to be very different from those of the right tail. In particular, the dependence and diversification effects play a major role.

In the talk, we shall present two kinds of results:
- logarithmic ‘large deviations’ asymptotics for the distribution function of the sum of positive random variables under quite general assumptions;
- sharp asymptotics for the distribution function of the sum of exponentials of components of a multidimensional Brownian motion time-changed with an independent increasing stochastic process. This setting covers the multidimensional versions of the commonly used exponential Lévy models, as well as stochastic volatility models with no correlation between the volatility and the stock.

These results have a wide range of applications in risk analysis of long only portfolios. Among other issues, we shall consider
- Variance reduction methods for precise estimation of tail event probabilities by Monte Carlo.
- Asymptotic formulas for implied volatility of basket options.
- Behavior of long only portfolios under market downturns and systematic design of stress tests for such portfolios.

ARTURO KOHATSU-HIGA (Ritsumeikan University)
‘Stochastic differential equations with irregular coefficients’

In this talk we will give some results regarding the existence and regularity of densities for stochastic differential equations (sde’s) with irregular coefficients starting with Holder coefficients and then bounded and measurable. If time allows we will also discuss issues related to the simulation schemes for such sde’s.

MARTA SANZ-SOLÉ (Universitat de Barcelona)
‘Support theorem for a stochastic wave equation in dimension three’

The connection between the characterization of the topological support of the law of a random vector and approximation schemes is already visible in the classical result for diffusion processes by Stroock and Varadhan (1972). In the framework of an abstract Wiener space, this is set up more explicitly by Aida, Kusuoka and Stroock (1993). In this talk, we will consider a class of stochastic wave equations driven by a Gaussian spatially stationary noise. We will present an approximation result by a sequence of stochastic partial differential equations obtained by smoothing the noise. As a consequence, a characterization of the
support of the law of the solution in Hölder norm will be derived. We shall also briefly report on other applications, like the asymptotics of the density of small perturbations of the initial equation. This is joint work with F. Delgado-Vences.

DAVID NUALART (University of Kansas)
‘Convergence of densities for random variables on a finite Wiener chaos’

The aim of this talk is to present some recent results on the convergence of densities for a sequence of d-dimensional random vectors whose components belong to a finite sum of Wiener chaos. First, using techniques of Malliavin calculus one can show that the convergence in law implies the convergence in total variation, assuming that the determinant of the Malliavin matrix is bounded away from zero. On the other hand, for one-dimensional random variables on a fixed chaos, the densities converge uniformly, assuming a uniform lower bound on the expectation of negative powers of norm of the Malliavin derivative. We will discuss some applications of this result in the framework of the Breuer-Major theorem.

ANDRÉ SUESS (Universitat de Barcelona)
‘Integration theory for infinite dimensional volatility modulated Volterra processes’

A pricing measure to explain the risk premium in power markets

ROLANDO D. NAVARRO, JR. (Purdue University)
‘Mean-variance hedging with partial information using the Clark-Ocone representation with the change of measure for Lévy process’

YACINE AIT-SAHALIA (Princeton University)
‘High Frequency Traders: Taking Advantage of Speed’

We propose a model of dynamic trading where a strategic high frequency trader receives an imperfect signal about the future order flow, and exploits his speed advantage to act as a market maker. We determine the provision of liquidity, order cancellations, and impact on low frequency traders. The model predicts that volatility leads high frequency traders to reduce their provision of liquidity. Next, we analyze the problem when the high frequency trader competes with another market maker. Finally, we provide the first formal, model-based analysis of the impact of various policies designed to regulate high frequency trading. Joint paper with Meghmet Saglam (University of Cincinnati).

MARK PODOLSKIJ (University of Heidelberg)
‘Limit theorems for Lévy moving average processes’

In this talk we present some new results on infill asymptotics for power variation Lévy moving average processes. Depending on the power, behavior of the kernel function at 0 and the Blumenthal-Getoor index of the driving Lévy process, there are five different limits in the first
order asymptotic. In particular, one of them is rather non-standard and unexpected. We will also present a discussion of a central limit theorem in certain cases.

MATHIEU ROSENBAUM (Université Pierre et Marie Curie)
‘Limit theorems for nearly unstable Hawkes processes’

Because of their tractability and their natural interpretations in term of market quantities, Hawkes processes are nowadays widely used in high frequency finance. However, in practice, the statistical estimation results seem to show that very often, only nearly unstable Hawkes processes are able to fit the data properly. By nearly unstable, we mean that the L1 norm of their kernel is close to unity. We study in this work such processes for which the stability condition is almost violated. Our main result states that after suitable rescaling, they asymptotically behave like integrated Cox Ingersoll Ross models. Thus, modeling financial order flows as nearly unstable Hawkes processes may be a good way to reproduce both their high and low frequency stylized facts. We then extend this result to the Hawkes based price model introduced by Bacry et al. We show that under a similar criticality condition, this process converges to a Heston model. Again, we recover well known stylized acts of prices, both at the microstructure level and at the macroscopic scale. Joint work with Thibault Jaisson (Ecole Polytechnique Paris).

VLAD BALLY (Université Paris Est Marne la Vallée)
‘Integration by parts, convergence in total variation and Central Limit Theorem’

We consider random variables with laws, which are not singular, more precisely, they have an absolute continuous component with a density which is lower semi-continuous. Then one may use the Nummelin splitting in order to represent such a random variable as the sum of two independent random variables, one of which has a smooth density. Based on this smooth density one may settle a Malliavin type calculus, which permits to build integration by parts formulas. We use these integration by parts formulas in order to prove that (under some appropriate hypotheses) convergence in law implies convergence in total variation. Based on this strategy we obtain estimates of the error in the CLT in the total variation distance. Recently, Bobkov, Chistyakov and Götze obtained similar results in relative entropy distance; but the class of random variables for which the two types of results work is different. Moreover, Nourdin and Poly used similar methods in order to prove the convergence in total variation for the non-linear CLT; but they do not obtain estimates of the error.

FREDERI VIENS (Purdue University)
‘Third moment theorem for functionals of stationary Gaussian sequences’

Bierme, Bonami, Nourdin, and Peccati recently gave sharp general quantitative bounds to complement the well-known fourth moment theorem of Nualart and Peccati, by which a sequence in a fixed Wiener chaos converges to a normal law if and only if its fourth cumulant converges to 0. The bounds show that the speed of convergence is precisely of order the maximum of the fourth cumulant and the absolutely value of the third cumulant. Specializing to the case of normalized centered quadratic variations for stationary Gaussian sequences, we show that a third moment theorem holds: convergence occurs if and only if the sequence’s third moments tend to 0. This is proved for sequences with general decreasing covariance. We finding exact speeds of convergence as intrinsic functions of the covariance
itself, which helps puts in perspective the notion of critical Hurst parameters when studying the convergence of fractional Brownian motion’s quadratic variation. We also study the speed of convergence when the limit is not Gaussian but rather a second-Wiener-chaos law, recovering a classical result of Dobrushin-Major/Taqqu whereby the limit is a Rosenblatt law, and proving that the price to pay to obtain a Rosenblatt limit despite a slowly varying modulation is a very slow convergence speed, roughly of the same order as the modulation. This is joint work with Leo Neufcourt.

NGOC KHUE TRAN (Université Paris 13)

‘LAN property for some jump diffusion processes with discrete observations’

In this talk, we will study the local asymptotic normality property for a class of ergodic diffusion process with jumps when the process is observed discretely at high frequency. To obtain this result, Malliavin calculus and Girsanov’s theorem are applied in order to write the log-likelihood ratio in terms of sums of conditional expectations, for which a central limit theorem for triangular arrays can be applied. Based on joint work with A. Kohatsu-Higa and E. Nualart.

JAMES-MICHAEL LEAHY (University of Edinburgh)

‘Finite Difference Schemes For Linear Stochastic Integro-Differential Equations’

We study the rate of convergence of an explicit and an implicit-explicit finite difference scheme for linear stochastic integro-differential equations of parabolic type arising in non-linear filtering of jump-diffusion processes. We show that the rate is of order one in space and order one-half in time.

GIULIA DI NUNNO (University of Oslo)

‘Optimal portfolio problems with price dynamics driven by time-changed Levy noises’

Time changed Lévy noises and doubly stochastic Poisson random measure appear in risk theory, in the study of ruin probabilities in insurance, and in insurance-linked security pricing. Also they are suggested in stochastic volatility models and option pricing. Here we consider dynamics driven by these noises from a stochastic calculus perspective, aiming at stochastic control. In particular, we study the chaos generated by these noises and the stochastic anticipating and non-anticipating derivatives keeping stochastic integral representations in view. This will then be applied to study portfolio optimisation problems either in the form of expected utility of the final wealth or in the form of minimal variance hedging. This is based on joint works with Steffen Sjursen.

ELISA ALÒS (Universitat Pompeu Fabra and Barcelona GSE)

‘On the closed-form approximation of short-time random strike options’

In this paper we propose a general technique to develop first and second order closed-form approximation formulas for short-time options with random strikes. Our method is based on Malliavin calculus techniques, which allow us to obtain simple closed-form approximation formulas depending on the derivative operator. The numerical analysis shows that these
Formulas are extremely accurate and improve some previous approaches on two-assets and three-assets spread options as Kirk’s formula or the decomposition method presented in Alòs, Eydeland and Laurence (2011) (joint work with J. A. León).

JOSÉ ENRIQUE FIGUEROA-LOPEZ (Purdue University)

‘Short-time expansions for close-to-the-money options under a Lévy jump model with stochastic volatility’

Due to their importance for model calibration and testing, small-time asymptotics of option prices have received considerable attention in recent years. In recent work, a second order expansion for the ATM option prices of a large class of exponential Lévy models, with or without a Brownian component, was developed. In this talk, we relax the regularity conditions imposed on the Lévy density to the minimal possible conditions for such an expansion to make sense. Our approach is based on approximating the option price by the option price of a process satisfying the more stringent regularity conditions. We also show that the formulas extend both to the case of “close-to-the-money” strikes and to the case where the continuous Brownian component is replaced by an independent stochastic volatility process with leverage effect. This is joint work with Sveinn Ólafsson from Purdue University.

JOSEP VIVES (Universitat de Barcelona)

‘A Hull and White formula for a stochastic volatility Lévy model with infinite activity’

By using techniques of Malliavin calculus for Lévy processes, we obtain an anticipating Itô formula for an infinite activity Lévy process. As an application we derive a Hull and White formula for an infinite activity stochastic volatility Lévy model. There are no assumptions on the Lévy measure and only basic Malliavin calculus assumptions are considered on the stochastic volatility process.